Computation of positively graded filiform Lie algebras over C

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Summary table (starting on the next page)

Explanation of table

- Column 1 (search) - A character string for text searching purposes
- Column 2 (algebra) - The subclass of positively graded filiform Lie algebra(s)
- Column 3 (Jac) - A check indicates that the class is nonempty
- Column 4 (sol) - Number of Lie algebras in the class

search	algebra	Jac	sol
m1A25	$\mathfrak{m}_{1A}(2,5)$	$\surd$	1
m2A26	$\mathfrak{m}_{2A}(2,6)$	$\surd$	1
m1A36	$\mathfrak{m}_{1A}(3,6)$	$\surd$	1
m1A27	$\mathfrak{m}_{1A}(2,7)$	$\surd$	1
m3A27	$\mathfrak{m}_{3A}(2,7)$	$\surd$	∞
m2A37	$\mathfrak{m}_{2A}(3,7)$	$\surd$	1
m1A47	$\mathfrak{m}_{1A}(4,7)$	$\surd$	1
m2A28	$\mathfrak{m}_{2A}(2,8)$	$\surd$	1
m4A28	$\mathfrak{m}_{4A}(2,8)$	$\surd$	∞
m1A38	$\mathfrak{m}_{1A}(3,8)$	$\surd$	1
m3A38	$\mathfrak{m}_{3A}(3,8)$	$\surd$	∞
m2A48	$\mathfrak{m}_{2A}(4,8)$	$\surd$	1
m1A58	$\mathfrak{m}_{1A}(5,8)$	$\surd$	1
m1A29	$\mathfrak{m}_{1A}(2,9)$	$\surd$	1
m3A29	$\mathfrak{m}_{3A}(2,9)$	$\surd$	1
m5A29	$\mathfrak{m}_{5A}(2,9)$	$\surd$	∞
m2A39	$\mathfrak{m}_{2A}(3,9)$	$\surd$	1
m4A39	$\mathfrak{m}_{4A}(3,9)$	$\surd$	∞
m1A49	$\mathfrak{m}_{1A}(4,9)$	$\surd$	1
m3A49	$\mathfrak{m}_{3A}(4,9)$	$\surd$	∞
m2A59	$\mathfrak{m}_{2A}(5,9)$	$\surd$	1
m1A69	$\mathfrak{m}_{1A}(6,9)$	$\surd$	1
m2A210	$\mathfrak{m}_{2A}(2,10)$	$\surd$	1
m4A210	$\mathfrak{m}_{4A}(2,10)$	$\surd$	1
m6A210	$\mathfrak{m}_{6A}(2,10)$	$\surd$	∞
m1A310	$\mathfrak{m}_{1A}(3,10)$	$\surd$	1
m3A310	$\mathfrak{m}_{3A}(3,10)$	$\surd$	∞
m5A310	$\mathfrak{m}_{5A}(3,10)$	$\surd$	∞
m2A410	$\mathfrak{m}_{2A}(4,10)$	$\surd$	1
m4A410	$\mathfrak{m}_{4A}(4,10)$	$\surd$	∞
m1A510	$\mathfrak{m}_{1A}(5,10)$	$\surd$	1
m3A510	$\mathfrak{m}_{3A}(5,10)$	$\surd$	∞
m2A610	$\mathfrak{m}_{2A}(6,10)$	$\surd$	1
m1A710	$\mathfrak{m}_{1A}(7,10)$	$\surd$	1
m1A211	$\mathfrak{m}_{1A}(2,11)$	$\surd$	1
m3A211	$\mathfrak{m}_{3A}(2,11)$	$\surd$	1
m5A211	$\mathfrak{m}_{5A}(2,11)$	$\surd$	1
m7A211	$\mathfrak{m}_{7A}(2,11)$	$\surd$	∞
m2A311	$\mathfrak{m}_{2A}(3,11)$	$\surd$	1
m4A311	$\mathfrak{m}_{4A}(3,11)$	$\surd$	1
m6A311	$\mathfrak{m}_{6A}(3,11)$	$\surd$	∞
m1A411	$\mathfrak{m}_{1A}(4,11)$	$\surd$	1

search	algebra	Jac	sol
m3A411	$\mathfrak{m}_{3A}(4,11)$	$\surd$	∞
m5A411	$\mathfrak{m}_{5A}(4,11)$	$\surd$	∞
m2A511	$\mathfrak{m}_{2A}(5,11)$	$\surd$	1
m4A511	$\mathfrak{m}_{4A}(5,11)$	$\surd$	∞
m1A611	$\mathfrak{m}_{1A}(6,11)$	$\surd$	1
m3A611	$\mathfrak{m}_{3A}(6,11)$	$\surd$	∞
m2A711	$\mathfrak{m}_{2A}(7,11)$	$\surd$	1
m1A811	$\mathfrak{m}_{1A}(8,11)$	$\surd$	1
m2A212	$\mathfrak{m}_{2A}(2,12)$	$\surd$	1
m4A212	$\mathfrak{m}_{4A}(2,12)$		0
m6A212	$\mathfrak{m}_{6A}(2,12)$		0
m8A212	$\mathfrak{m}_{8A}(2,12)$	$\surd$	2
m1A312	$\mathfrak{m}_{1A}(3,12)$	$\surd$	1
m3A312	$\mathfrak{m}_{3A}(3,12)$	$\surd$	∞
m5A312	$\mathfrak{m}_{5A}(3,12)$	$\surd$	∞
m7A312	$\mathfrak{m}_{7A}(3,12)$	$\surd$	∞
m2A412	$\mathfrak{m}_{2A}(4,12)$	$\surd$	1
m4A412	$\mathfrak{m}_{4A}(4,12)$	$\surd$	∞
m6A412	$\mathfrak{m}_{6A}(4,12)$	$\surd$	∞
m1A512	$\mathfrak{m}_{1A}(5,12)$	$\surd$	1
m3A512	$\mathfrak{m}_{3A}(5,12)$	$\surd$	∞
m5A512	$\mathfrak{m}_{5A}(5,12)$	$\surd$	∞
m2A612	$\mathfrak{m}_{2A}(6,12)$	$\surd$	1
m4A612	$\mathfrak{m}_{4A}(6,12)$	$\surd$	∞
m1A712	$\mathfrak{m}_{1A}(7,12)$	$\surd$	1
m3A712	$\mathfrak{m}_{3A}(7,12)$	$\surd$	∞
m2A812	$\mathfrak{m}_{2A}(8,12)$	$\surd$	1
m1A912	$\mathfrak{m}_{1A}(9,12)$	$\surd$	1
m1A213	$\mathfrak{m}_{1A}(2,13)$	$\surd$	1
m3A213	$\mathfrak{m}_{3A}(2,13)$	$\surd$	1
m9A213	$\mathfrak{m}_{9A}(2,13)$	$\surd$	2
m2A313	$\mathfrak{m}_{2A}(3,13)$	$\surd$	1
m4A313	$\mathfrak{m}_{4A}(3,13)$	$\surd$	1
m6A313	$\mathfrak{m}_{6A}(3,13)$	$\surd$	1
m8A313	$\mathfrak{m}_{8A}(3,13)$	$\surd$	∞
m1A413	$\mathfrak{m}_{1A}(4,13)$	$\surd$	1
m3A413	$\mathfrak{m}_{3A}(4,13)$	$\surd$	∞
m5A413	$\mathfrak{m}_{5A}(4,13)$	$\surd$	∞
m7A413	$\mathfrak{m}_{7A}(4,13)$	$\surd$	∞
m2A513	$\mathfrak{m}_{2A}(5,13)$	$\surd$	1
m4A513	$\mathfrak{m}_{4A}(5,13)$	$\surd$	∞
m6A513	$\mathfrak{m}_{6A}(5,13)$	$\surd$	∞

search	algebra	Jac	sol
m1A613	$\mathfrak{m}_{1A}(6,13)$	$\surd$	1
m3A613	$\mathfrak{m}_{3A}(6,13)$	$\surd$	∞
m5A613	$\mathfrak{m}_{5A}(6,13)$	$\surd$	∞
m2A713	$\mathfrak{m}_{2A}(7,13)$	$\surd$	1
m4A713	$\mathfrak{m}_{4A}(7,13)$	$\surd$	∞
m1A813	$\mathfrak{m}_{1A}(8,13)$	$\surd$	1
m3A813	$\mathfrak{m}_{3A}(8,13)$	$\surd$	∞
m2A913	$\mathfrak{m}_{2A}(9,13)$	$\surd$	1
m1A1013	$\mathfrak{m}_{1A}(10,13)$	$\surd$	1
m2A214	$\mathfrak{m}_{2A}(2,14)$	$\surd$	1
m4A214	$\mathfrak{m}_{4A}(2,14)$		0
m10A214	$\mathfrak{m}_{10A}(2,14)$	$\surd$	2
m1A314	$\mathfrak{m}_{1A}(3,14)$	$\surd$	1
m3A314	$\mathfrak{m}_{3A}(3,14)$	$\surd$	∞
m5A314	$\mathfrak{m}_{5A}(3,14)$	$\surd$	1
m7A314	$\mathfrak{m}_{7A}(3,14)$	$\surd$	1
m9A314	$\mathfrak{m}_{9A}(3,14)$	$\surd$	∞
m2A414	$\mathfrak{m}_{2A}(4,14)$	$\surd$	1
m4A414	$\mathfrak{m}_{4A}(4,14)$	$\surd$	∞
m6A414	$\mathfrak{m}_{6A}(4,14)$	$\surd$	∞
m8A414	$\mathfrak{m}_{8A}(4,14)$	$\surd$	∞
m1A514	$\mathfrak{m}_{1A}(5,14)$	$\surd$	1
m3A514	$\mathfrak{m}_{3A}(5,14)$	$\surd$	∞
m5A514	$\mathfrak{m}_{5A}(5,14)$	$\surd$	∞
m7A514	$\mathfrak{m}_{7A}(5,14)$	$\surd$	∞
m2A614	$\mathfrak{m}_{2A}(6,14)$	$\surd$	1
m4A614	$\mathfrak{m}_{4A}(6,14)$	$\surd$	∞
m6A614	$\mathfrak{m}_{6A}(6,14)$	$\surd$	∞
m1A714	$\mathfrak{m}_{1A}(7,14)$	$\surd$	1
m3A714	$\mathfrak{m}_{3A}(7,14)$	$\surd$	∞
m5A714	$\mathfrak{m}_{5A}(7,14)$	$\surd$	∞
m2A814	$\mathfrak{m}_{2A}(8,14)$	$\surd$	1
m4A814	$\mathfrak{m}_{4A}(8,14)$	$\surd$	∞
m1A914	$\mathfrak{m}_{1A}(9,14)$	$\surd$	1
m3A914	$\mathfrak{m}_{3A}(9,14)$	$\surd$	∞
m2A1014	$\mathfrak{m}_{2A}(10,14)$	$\surd$	1
m1A1114	$\mathfrak{m}_{1A}(11,14)$	$\surd$	1
m2B26	$\mathfrak{m}_{2B}(2,6)$	$\surd$	1
m2B28	$\mathfrak{m}_{2B}(2,8)$		0
m4B28	$\mathfrak{m}_{4B}(2,8)$	$\surd$	1
m3B38	$\mathfrak{m}_{3B}(3,8)$	$\surd$	1
m2B48	$\mathfrak{m}_{2B}(4,8)$	$\surd$	1

search	algebra	Jac	sol
m2B210	$\mathfrak{m}_{2B}(2,10)$		0
m4B210	$\mathfrak{m}_{4B}(2,10)$		0
m6B210	$\mathfrak{m}_{6B}(2,10)$	$\surd$	2
m3B310	$\mathfrak{m}_{3B}(3,10)$	$\surd$	1
m5B310	$\mathfrak{m}_{5B}(3,10)$	$\surd$	∞
m2B410	$\mathfrak{m}_{2B}(4,10)$		0
m4B410	$\mathfrak{m}_{4B}(4,10)$	$\surd$	1
m3B510	$\mathfrak{m}_{3B}(5,10)$	$\surd$	1
m2B610	$\mathfrak{m}_{2B}(6,10)$	$\surd$	1
m2B212	$\mathfrak{m}_{2B}(2,12)$		0
m4B212	$\mathfrak{m}_{4B}(2,12)$		0
m6B212	$\mathfrak{m}_{6B}(2,12)$		0
m8B212	$\mathfrak{m}_{8B}(2,12)$	$\surd$	4
m3B312	$\mathfrak{m}_{3B}(3,12)$		0
m5B312	$\mathfrak{m}_{5B}(3,12)$		0
m7B312	$\mathfrak{m}_{7B}(3,12)$	$\surd$	2
m2B412	$\mathfrak{m}_{2B}(4,12)$		0
m4B412	$\mathfrak{m}_{4B}(4,12)$	$\surd$	1
m6B412	$\mathfrak{m}_{6B}(4,12)$	$\surd$	∞
m3B512	$\mathfrak{m}_{3B}(5,12)$	$\surd$	1
m5B512	$\mathfrak{m}_{5B}(5,12)$	$\surd$	∞
m2B612	$\mathfrak{m}_{2B}(6,12)$		0
m4B612	$\mathfrak{m}_{4B}(6,12)$	$\surd$	1
m3B712	$\mathfrak{m}_{3B}(7,12)$	$\surd$	1
m2B812	$\mathfrak{m}_{2B}(8,12)$	$\surd$	1
m2B214	$\mathfrak{m}_{2B}(2,14)$		0
m4B214	$\mathfrak{m}_{4B}(2,14)$		0
m10B214	$\mathfrak{m}_{10B}(2,14)$		0
m3B314	$\mathfrak{m}_{3B}(3,14)$		0
m5B314	$\mathfrak{m}_{5B}(3,14)$		0
m7B314	$\mathfrak{m}_{7B}(3,14)$		0
m9B314	$\mathfrak{m}_{9B}(3,14)$	$\surd$	4
m2B414	$\mathfrak{m}_{2B}(4,14)$		0
m4B414	$\mathfrak{m}_{4B}(4,14)$	$\surd$	1
m6B414	$\mathfrak{m}_{6B}(4,14)$	$\surd$	1
m8B414	$\mathfrak{m}_{8B}(4,14)$	$\surd$	∞
m3B514	$\mathfrak{m}_{3B}(5,14)$		0
m5B514	$\mathfrak{m}_{5B}(5,14)$	$\surd$	1
m7B514	$\mathfrak{m}_{7B}(5,14)$	$\surd$	∞
m2B614	$\mathfrak{m}_{2B}(6,14)$		0
m4B614	$\mathfrak{m}_{4B}(6,14)$	$\surd$	1
m6B614	$\mathfrak{m}_{6B}(6,14)$	$\surd$	∞

search	algebra	Jac	sol
m3B714	$\mathfrak{m}_{3B}(7,14)$	$\surd$	1
m5B714	$\mathfrak{m}_{5B}(7,14)$	$\surd$	∞
m2B814	$\mathfrak{m}_{2B}(8,14)$		0
m4B814	$\mathfrak{m}_{4B}(8,14)$	$\surd$	1
m3B914	$\mathfrak{m}_{3B}(9,14)$	$\surd$	1
m2B1014	$\mathfrak{m}_{2B}(10,14)$	$\surd$	1

Algebra details

$\mathfrak{m}_{1A}(2,5)$

m1A25 (this line included for string searching purposes)
Original brackets:

$$[e_1,e_2]=e_3 \qquad [e_1,e_3]=e_4$$
$$[e_1,e_4]=e_5 \qquad [e_2,e_3]=e_5$$

No non-trivial Jacobi tests

$\mathfrak{m}_{2A}(2,6)$

m2A26 (this line included for string searching purposes)
Original brackets:

$$[e_1,e_2]=e_3 \qquad [e_1,e_3]=e_4$$
$$[e_1,e_4]=e_5 \qquad [e_1,e_5]=e_6$$
$$[e_2,e_3]=e_5 \qquad [e_2,e_4]=e_6$$

No non-trivial Jacobi tests

$\mathfrak{m}_{1A}(3,6)$

m1A36 (this line included for string searching purposes)
Original brackets:

$$[e_1,e_2]=e_3 \qquad [e_1,e_3]=e_4$$
$$[e_1,e_4]=e_5 \qquad [e_1,e_5]=e_6$$
$$[e_2,e_3]=e_6$$

No non-trivial Jacobi tests

$\mathfrak{m}_{1A}(2,7)$

m1A27 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
&[e_1,e_2]=e_3 && [e_1,e_3]=e_4\\
&[e_1,e_4]=e_5 && [e_1,e_5]=e_6\\
&[e_1,e_6]=e_7 && [e_2,e_5]=e_7\\
&[e_3,e_4]=-e_7
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{3A}(2,7)$

m3A27 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
&[e_1,e_2]=e_3 && [e_1,e_3]=e_4\\
&[e_1,e_4]=e_5 && [e_1,e_5]=e_6\\
&[e_1,e_6]=e_7 && [e_2,e_3]=e_5\\
&[e_2,e_4]=e_6 && [e_2,e_5]=\alpha_{2,5}^{7}e_7\\
&[e_3,e_4]=\alpha_{3,4}^{7}e_7
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$(e_1,e_2,e_4):\quad -\alpha_{2,5}^{7}-\alpha_{3,4}^{7}+1 = 0$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{3,4}^{7}\to x_1$$
$$\alpha_{2,5}^{7}\to x_2$$

Jacobi Tests

$$(e_1,e_2,e_4):\quad -x_1-x_2+1 = 0$$

Groebner basis (2 variables, 1 linear, 0 nonlinear)

$$x_1+x_2-1=0$$

$\mathfrak{m}_{2A}(3,7)$

m2A37 (this line included for string searching purposes)
Original brackets:

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_2, e_3] = e_6$$
$$[e_2, e_4] = e_7$$

No non-trivial Jacobi tests

$\mathfrak{m}_{1A}(4,7)$

m1A47 (this line included for string searching purposes)
Original brackets:

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_2, e_3] = e_7$$

No non-trivial Jacobi tests

$\mathfrak{m}_{2A}(2,8)$

m2A28 (this line included for string searching purposes)
Original brackets:

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_2, e_5] = e_7 \qquad [e_2, e_6] = 2e_8$$
$$[e_3, e_4] = -e_7 \qquad [e_3, e_5] = -e_8$$

No non-trivial Jacobi tests

$\mathfrak{m}_{4A}(2,8)$

m4A28 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_2, e_3] &= e_5 & [e_2, e_4] &= e_6 \\
[e_2, e_5] &= \alpha_{2,5}^{7} e_7 & [e_2, e_6] &= \alpha_{2,6}^{8} e_8 \\
[e_3, e_4] &= \alpha_{3,4}^{7} e_7 & [e_3, e_5] &= \alpha_{3,5}^{8} e_8
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_4): &\quad -\alpha_{2,5}^{7} - \alpha_{3,4}^{7} + 1 &= 0 \\
(e_1, e_2, e_5): &\quad \alpha_{2,5}^{7} - \alpha_{2,6}^{8} - \alpha_{3,5}^{8} &= 0 \\
(e_1, e_3, e_4): &\quad \alpha_{3,4}^{7} - \alpha_{3,5}^{8} &= 0
\end{aligned}
$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{3,4}^{7} \to x_1$$

$$\alpha_{3,5}^{8} \to x_2$$

$$\alpha_{2,6}^{8} \to x_3$$

$$\alpha_{2,5}^{7} \to x_4$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_4): &\quad -x_1 - x_4 + 1 &= 0 \\
(e_1, e_2, e_5): &\quad -x_2 - x_3 + x_4 &= 0 \\
(e_1, e_3, e_4): &\quad x_1 - x_2 &= 0
\end{aligned}
$$

Groebner basis (4 variables, 3 linear, 0 nonlinear)

$$x_1 + x_4 - 1 = 0$$

$$x_2 + x_4 - 1 = 0$$

$$x_3 - 2x_4 + 1 = 0$$

$\mathfrak{m}_{1A}(3,8)$

m1A38 (this line included for string searching purposes)
Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4 \\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6 \\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8 \\
[e_2,e_5] &= e_8 & [e_3,e_4] &= -e_8
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{3A}(3,8)$

m3A38 (this line included for string searching purposes)
Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4 \\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6 \\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8 \\
[e_2,e_3] &= e_6 & [e_2,e_4] &= e_7 \\
[e_2,e_5] &= \alpha_{2,5}^8 e_8 & [e_3,e_4] &= \alpha_{3,4}^8 e_8
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$(e_1,e_2,e_4): \quad -\alpha_{2,5}^8 - \alpha_{3,4}^8 + 1 \qquad = 0$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$
\begin{aligned}
\alpha_{3,4}^8 &\to x_1 \\
\alpha_{2,5}^8 &\to x_2
\end{aligned}
$$

Jacobi Tests

$$(e_1,e_2,e_4): \quad -x_1 - x_2 + 1 \qquad = 0$$

Groebner basis (2 variables, 1 linear, 0 nonlinear)

$$x_1 + x_2 - 1 = 0$$

$\mathfrak{m}_{2A}(4,8)$

m2A48 (this line included for string searching purposes)
Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4 \\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6 \\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8 \\
[e_2,e_3] &= e_7 & [e_2,e_4] &= e_8
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{1A}(5,8)$

m1A58 (this line included for string searching purposes)
Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4 \\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6 \\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8 \\
[e_2,e_3] &= e_8
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{1A}(2,9)$

m1A29 (this line included for string searching purposes)
Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4 \\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6 \\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8 \\
[e_1,e_8] &= e_9 & [e_2,e_7] &= e_9 \\
[e_3,e_6] &= -e_9 & [e_4,e_5] &= e_9
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{3A}(2,9)$

m3A29 (this line included for string searching purposes)

Solution 1

$$
\begin{aligned}
&[e_1, e_2] = e_3 && [e_1, e_3] = e_4 \\
&[e_1, e_4] = e_5 && [e_1, e_5] = e_6 \\
&[e_1, e_6] = e_7 && [e_1, e_7] = e_8 \\
&[e_1, e_8] = e_9 && [e_2, e_5] = e_7 \\
&[e_2, e_6] = 2e_8 && [e_2, e_7] = 0 \\
&[e_3, e_4] = -e_7 && [e_3, e_5] = -e_8 \\
&[e_3, e_6] = 2e_9 && [e_4, e_5] = -3e_9
\end{aligned}
$$

Original brackets:

$$
\begin{aligned}
&[e_1, e_2] = e_3 && [e_1, e_3] = e_4 \\
&[e_1, e_4] = e_5 && [e_1, e_5] = e_6 \\
&[e_1, e_6] = e_7 && [e_1, e_7] = e_8 \\
&[e_1, e_8] = e_9 && [e_2, e_5] = e_7 \\
&[e_2, e_6] = 2e_8 && [e_2, e_7] = \alpha_{2,7}^9 e_9 \\
&[e_3, e_4] = -e_7 && [e_3, e_5] = -e_8 \\
&[e_3, e_6] = \alpha_{3,6}^9 e_9 && [e_4, e_5] = \alpha_{4,5}^9 e_9
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_6): &\quad -\alpha_{2,7}^9 - \alpha_{3,6}^9 + 2 && = 0 \\
(e_1, e_3, e_5): &\quad -\alpha_{3,6}^9 - \alpha_{4,5}^9 - 1 && = 0 \\
(e_2, e_3, e_4): &\quad -\alpha_{2,7}^9 && = 0
\end{aligned}
$$

Solution 1:

$$
\begin{aligned}
\alpha_{4,5}^9 &= -3 \\
\alpha_{2,7}^9 &= 0 \\
\alpha_{3,6}^9 &= 2
\end{aligned}
$$

How the solution(s) were or were not found:
Change variables

$$\alpha_{4,5}^9 \to x_1$$

$$\alpha^9_{2,7} \to x_2$$

$$\alpha^9_{3,6} \to x_3$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_6): \quad & -x_2 - x_3 + 2 & = 0 \\
(e_1, e_3, e_5): \quad & -x_1 - x_3 - 1 & = 0 \\
(e_2, e_3, e_4): \quad & -x_2 & = 0
\end{aligned}$$

Groebner basis (3 variables, 3 linear, 0 nonlinear)

$$x_1 + 3 = 0$$

$$x_2 = 0$$

$$x_3 - 2 = 0$$

Solution 1:

$$x_1 = -3$$

$$x_2 = 0$$

$$x_3 = 2$$

$\mathfrak{m}_{5A}(2, 9)$

m5A29 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_2, e_3] &= e_5 \\
[e_2, e_4] &= e_6 & [e_2, e_5] &= \alpha^7_{2,5} e_7 \\
[e_2, e_6] &= \alpha^8_{2,6} e_8 & [e_2, e_7] &= \alpha^9_{2,7} e_9 \\
[e_3, e_4] &= \alpha^7_{3,4} e_7 & [e_3, e_5] &= \alpha^8_{3,5} e_8 \\
[e_3, e_6] &= \alpha^9_{3,6} e_9 & [e_4, e_5] &= \alpha^9_{4,5} e_9
\end{aligned}$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1, e_2, e_4): &\quad -\alpha_{2,5}^7 - \alpha_{3,4}^7 + 1 &&= 0\\
(e_1, e_2, e_5): &\quad \alpha_{2,5}^7 - \alpha_{2,6}^8 - \alpha_{3,5}^8 &&= 0\\
(e_1, e_3, e_4): &\quad \alpha_{3,4}^7 - \alpha_{3,5}^8 &&= 0\\
(e_1, e_2, e_6): &\quad \alpha_{2,6}^8 - \alpha_{2,7}^9 - \alpha_{3,6}^9 &&= 0\\
(e_1, e_3, e_5): &\quad \alpha_{3,5}^8 - \alpha_{3,6}^9 - \alpha_{4,5}^9 &&= 0\\
(e_2, e_3, e_4): &\quad \alpha_{2,7}^9 \alpha_{3,4}^7 - \alpha_{3,6}^9 + \alpha_{4,5}^9 &&= 0
\end{aligned}$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\begin{gathered}
\alpha_{3,4}^7 \to x_1\\
\alpha_{2,7}^9 \to x_2\\
\alpha_{3,6}^9 \to x_3\\
\alpha_{2,5}^7 \to x_4\\
\alpha_{2,6}^8 \to x_5\\
\alpha_{4,5}^9 \to x_6\\
\alpha_{3,5}^8 \to x_7
\end{gathered}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4): &\quad -x_1 - x_4 + 1 &&= 0\\
(e_1, e_2, e_5): &\quad x_4 - x_5 - x_7 &&= 0\\
(e_1, e_3, e_4): &\quad x_1 - x_7 &&= 0\\
(e_1, e_2, e_6): &\quad -x_2 - x_3 + x_5 &&= 0\\
(e_1, e_3, e_5): &\quad -x_3 - x_6 + x_7 &&= 0\\
(e_2, e_3, e_4): &\quad x_1 x_2 - x_3 + x_6 &&= 0
\end{aligned}$$

Groebner basis (7 variables, 5 linear, 1 nonlinear)

$$\begin{gathered}
x_1 - x_7 = 0\\
x_2 - x_6 + 3x_7 - 1 = 0\\
x_3 + x_6 - x_7 = 0\\
x_4 + x_7 - 1 = 0\\
x_5 + 2x_7 - 1 = 0\\
x_6 x_7 + 2x_6 - 3x_7^2 = 0
\end{gathered}$$

$\mathfrak{m}_{2A}(3,9)$

m2A39 (this line included for string searching purposes)
Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4 \\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6 \\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8 \\
[e_1,e_8] &= e_9 & [e_2,e_5] &= e_8 \\
[e_2,e_6] &= 2e_9 & [e_3,e_4] &= -e_8 \\
[e_3,e_5] &= -e_9
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{4A}(3,9)$

m4A39 (this line included for string searching purposes)
Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4 \\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6 \\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8 \\
[e_1,e_8] &= e_9 & [e_2,e_3] &= e_6 \\
[e_2,e_4] &= e_7 & [e_2,e_5] &= \alpha_{2,5}^{8} e_8 \\
[e_2,e_6] &= \alpha_{2,6}^{9} e_9 & [e_3,e_4] &= \alpha_{3,4}^{8} e_8 \\
[e_3,e_5] &= \alpha_{3,5}^{9} e_9
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1,e_2,e_4): &\quad -\alpha_{2,5}^{8} - \alpha_{3,4}^{8} + 1 &= 0 \\
(e_1,e_2,e_5): &\quad \alpha_{2,5}^{8} - \alpha_{2,6}^{9} - \alpha_{3,5}^{9} &= 0 \\
(e_1,e_3,e_4): &\quad \alpha_{3,4}^{8} - \alpha_{3,5}^{9} &= 0
\end{aligned}
$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{3,4}^{8} \to x_1$$

$$\alpha_{3,5}^{9} \to x_2$$

$$\alpha_{2,5}^{8} \to x_3$$

$$\alpha^9_{2,6} \to x_4$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4): &\quad -x_1 - x_3 + 1 &= 0\\
(e_1, e_2, e_5): &\quad -x_2 + x_3 - x_4 &= 0\\
(e_1, e_3, e_4): &\quad x_1 - x_2 &= 0
\end{aligned}$$

Groebner basis (4 variables, 3 linear, 0 nonlinear)

$$2x_1 + x_4 - 1 = 0$$

$$2x_2 + x_4 - 1 = 0$$

$$2x_3 - x_4 - 1 = 0$$

$\mathfrak{m}_{1A}(4, 9)$

m1A49 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4\\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6\\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8\\
[e_1, e_8] &= e_9 & [e_2, e_5] &= e_9\\
[e_3, e_4] &= -e_9
\end{aligned}$$

No non-trivial Jacobi tests

$\mathfrak{m}_{3A}(4, 9)$

m3A49 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4\\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6\\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8\\
[e_1, e_8] &= e_9 & [e_2, e_3] &= e_7\\
[e_2, e_4] &= e_8 & [e_2, e_5] &= \alpha^9_{2,5} e_9\\
[e_3, e_4] &= \alpha^9_{3,4} e_9
\end{aligned}$$

Non-trivial Jacobi Tests:

$$(e_1, e_2, e_4): \quad -\alpha_{2,5}^9 - \alpha_{3,4}^9 + 1 \qquad = 0$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{2,5}^9 \to x_1$$
$$\alpha_{3,4}^9 \to x_2$$

Jacobi Tests

$$(e_1, e_2, e_4): \quad -x_1 - x_2 + 1 \qquad = 0$$

Groebner basis (2 variables, 1 linear, 0 nonlinear)

$$x_1 + x_2 - 1 = 0$$

$\mathfrak{m}_{2A}(5, 9)$

m2A59 (this line included for string searching purposes)
Original brackets:

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_2, e_3] = e_8$$
$$[e_2, e_4] = e_9$$

No non-trivial Jacobi tests

$\mathfrak{m}_{1A}(6, 9)$

m1A69 (this line included for string searching purposes)
Original brackets:

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_2, e_3] = e_9$$

No non-trivial Jacobi tests

$\mathfrak{m}_{2A}(2,10)$

m2A210 (this line included for string searching purposes)
Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4\\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6\\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8\\
[e_1,e_8] &= e_9 & [e_1,e_9] &= e_{10}\\
[e_2,e_7] &= e_9 & [e_2,e_8] &= 3e_{10}\\
[e_3,e_6] &= -e_9 & [e_3,e_7] &= -2e_{10}\\
[e_4,e_5] &= e_9 & [e_4,e_6] &= e_{10}
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{4A}(2,10)$

m4A210 (this line included for string searching purposes)
Solution 1

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4\\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6\\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8\\
[e_1,e_8] &= e_9 & [e_1,e_9] &= e_{10}\\
[e_2,e_5] &= e_7 & [e_2,e_6] &= 2e_8\\
[e_2,e_7] &= 0 & [e_2,e_8] &= -5e_{10}\\
[e_3,e_4] &= -e_7 & [e_3,e_5] &= -e_8\\
[e_3,e_6] &= 2e_9 & [e_3,e_7] &= 5e_{10}\\
[e_4,e_5] &= -3e_9 & [e_4,e_6] &= -3e_{10}
\end{aligned}
$$

Original brackets:

$$
\begin{aligned}
&[e_1, e_2] = e_3 && [e_1, e_3] = e_4 \\
&[e_1, e_4] = e_5 && [e_1, e_5] = e_6 \\
&[e_1, e_6] = e_7 && [e_1, e_7] = e_8 \\
&[e_1, e_8] = e_9 && [e_1, e_9] = e_{10} \\
&[e_2, e_5] = e_7 && [e_2, e_6] = 2e_8 \\
&[e_2, e_7] = \alpha_{2,7}^{9} e_9 && [e_2, e_8] = \alpha_{2,8}^{10} e_{10} \\
&[e_3, e_4] = -e_7 && [e_3, e_5] = -e_8 \\
&[e_3, e_6] = \alpha_{3,6}^{9} e_9 && [e_3, e_7] = \alpha_{3,7}^{10} e_{10} \\
&[e_4, e_5] = \alpha_{4,5}^{9} e_9 && [e_4, e_6] = \alpha_{4,6}^{10} e_{10}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
&(e_1, e_2, e_6): && -\alpha_{2,7}^{9} - \alpha_{3,6}^{9} + 2 && = 0 \\
&(e_1, e_3, e_5): && -\alpha_{3,6}^{9} - \alpha_{4,5}^{9} - 1 && = 0 \\
&(e_2, e_3, e_4): && -\alpha_{2,7}^{9} && = 0 \\
&(e_1, e_2, e_7): && \alpha_{2,7}^{9} - \alpha_{2,8}^{10} - \alpha_{3,7}^{10} && = 0 \\
&(e_1, e_3, e_6): && \alpha_{3,6}^{9} - \alpha_{3,7}^{10} - \alpha_{4,6}^{10} && = 0 \\
&(e_1, e_4, e_5): && \alpha_{4,5}^{9} - \alpha_{4,6}^{10} && = 0 \\
&(e_2, e_3, e_5): && -\alpha_{2,8}^{10} - \alpha_{3,7}^{10} && = 0
\end{aligned}
$$

Solution 1:

$$
\begin{aligned}
\alpha_{2,7}^{9} &= 0 \\
\alpha_{2,8}^{10} &= -5 \\
\alpha_{3,7}^{10} &= 5 \\
\alpha_{3,6}^{9} &= 2 \\
\alpha_{4,5}^{9} &= -3 \\
\alpha_{4,6}^{10} &= -3
\end{aligned}
$$

How the solution(s) were or were not found:
Change variables

$$\alpha_{2,7}^{9} \to x_1$$

$$\alpha^{10}_{2,8} \to x_2$$
$$\alpha^{10}_{3,7} \to x_3$$
$$\alpha^{9}_{3,6} \to x_4$$
$$\alpha^{9}_{4,5} \to x_5$$
$$\alpha^{10}_{4,6} \to x_6$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_6): &\quad -x_1 - x_4 + 2 &= 0\\
(e_1, e_3, e_5): &\quad -x_4 - x_5 - 1 &= 0\\
(e_2, e_3, e_4): &\quad -x_1 &= 0\\
(e_1, e_2, e_7): &\quad x_1 - x_2 - x_3 &= 0\\
(e_1, e_3, e_6): &\quad -x_3 + x_4 - x_6 &= 0\\
(e_1, e_4, e_5): &\quad x_5 - x_6 &= 0\\
(e_2, e_3, e_5): &\quad -x_2 - x_3 &= 0
\end{aligned}$$

Groebner basis (6 variables, 6 linear, 0 nonlinear)

$$x_1 = 0$$
$$x_2 + 5 = 0$$
$$x_3 - 5 = 0$$
$$x_4 - 2 = 0$$
$$x_5 + 3 = 0$$
$$x_6 + 3 = 0$$

Solution 1:

$$x_1 = 0$$
$$x_2 = -5$$
$$x_3 = 5$$
$$x_4 = 2$$
$$x_5 = -3$$
$$x_6 = -3$$

$\mathfrak{m}_{6A}(2,10)$

m6A210 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4\\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6\\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8\\
[e_1,e_8] &= e_9 & [e_1,e_9] &= e_{10}\\
[e_2,e_3] &= e_5 & [e_2,e_4] &= e_6\\
[e_2,e_5] &= \alpha_{2,5}^{7}e_7 & [e_2,e_6] &= \alpha_{2,6}^{8}e_8\\
[e_2,e_7] &= \alpha_{2,7}^{9}e_9 & [e_2,e_8] &= \alpha_{2,8}^{10}e_{10}\\
[e_3,e_4] &= \alpha_{3,4}^{7}e_7 & [e_3,e_5] &= \alpha_{3,5}^{8}e_8\\
[e_3,e_6] &= \alpha_{3,6}^{9}e_9 & [e_3,e_7] &= \alpha_{3,7}^{10}e_{10}\\
[e_4,e_5] &= \alpha_{4,5}^{9}e_9 & [e_4,e_6] &= \alpha_{4,6}^{10}e_{10}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1,e_2,e_4):&\quad -\alpha_{2,5}^{7}-\alpha_{3,4}^{7}+1 &&=0\\
(e_1,e_2,e_5):&\quad \alpha_{2,5}^{7}-\alpha_{2,6}^{8}-\alpha_{3,5}^{8} &&=0\\
(e_1,e_3,e_4):&\quad \alpha_{3,4}^{7}-\alpha_{3,5}^{8} &&=0\\
(e_1,e_2,e_6):&\quad \alpha_{2,6}^{8}-\alpha_{2,7}^{9}-\alpha_{3,6}^{9} &&=0\\
(e_1,e_3,e_5):&\quad \alpha_{3,5}^{8}-\alpha_{3,6}^{9}-\alpha_{4,5}^{9} &&=0\\
(e_2,e_3,e_4):&\quad \alpha_{2,7}^{9}\alpha_{3,4}^{7}-\alpha_{3,6}^{9}+\alpha_{4,5}^{9} &&=0\\
(e_1,e_2,e_7):&\quad \alpha_{2,7}^{9}-\alpha_{2,8}^{10}-\alpha_{3,7}^{10} &&=0\\
(e_1,e_3,e_6):&\quad \alpha_{3,6}^{9}-\alpha_{3,7}^{10}-\alpha_{4,6}^{10} &&=0\\
(e_1,e_4,e_5):&\quad \alpha_{4,5}^{9}-\alpha_{4,6}^{10} &&=0\\
(e_2,e_3,e_5):&\quad -\alpha_{2,5}^{7}\alpha_{3,7}^{10}+\alpha_{2,8}^{10}\alpha_{3,5}^{8} &&=0
\end{aligned}
$$

Infinite number of solutions.

How the solution(s) were or were not found:

Change variables

$$\alpha_{3,4}^{7}\to x_1$$

$$\alpha_{2,7}^{9}\to x_2$$

$$\alpha_{2,8}^{10}\to x_3$$

$$\alpha^9_{3,6} \to x_4$$
$$\alpha^7_{2,5} \to x_5$$
$$\alpha^{10}_{3,7} \to x_6$$
$$\alpha^8_{2,6} \to x_7$$
$$\alpha^9_{4,5} \to x_8$$
$$\alpha^{10}_{4,6} \to x_9$$
$$\alpha^8_{3,5} \to x_{10}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4): &\quad -x_1 - x_5 + 1 &&= 0\\
(e_1, e_2, e_5): &\quad -x_{10} + x_5 - x_7 &&= 0\\
(e_1, e_3, e_4): &\quad x_1 - x_{10} &&= 0\\
(e_1, e_2, e_6): &\quad -x_2 - x_4 + x_7 &&= 0\\
(e_1, e_3, e_5): &\quad x_{10} - x_4 - x_8 &&= 0\\
(e_2, e_3, e_4): &\quad x_1 x_2 - x_4 + x_8 &&= 0\\
(e_1, e_2, e_7): &\quad x_2 - x_3 - x_6 &&= 0\\
(e_1, e_3, e_6): &\quad x_4 - x_6 - x_9 &&= 0\\
(e_1, e_4, e_5): &\quad x_8 - x_9 &&= 0\\
(e_2, e_3, e_5): &\quad x_{10} x_3 - x_5 x_6 &&= 0
\end{aligned}$$

Groebner basis (10 variables, 8 linear, 1 nonlinear)

$$x_1 - x_{10} = 0$$
$$3x_{10} + x_2 - x_9 - 1 = 0$$
$$4x_{10} + x_3 - 3x_9 - 1 = 0$$
$$-x_{10} + x_4 + x_9 = 0$$
$$x_{10} + x_5 - 1 = 0$$
$$-x_{10} + x_6 + 2x_9 = 0$$
$$2x_{10} + x_7 - 1 = 0$$
$$x_8 - x_9 = 0$$
$$-3x_{10}^2 + x_{10} x_9 + 2x_9 = 0$$

$\mathfrak{m}_{1A}(3, 10)$

m1A310 (this line included for string searching purposes)
Original brackets:

$$
\begin{aligned}
&[e_1, e_2] = e_3 && [e_1, e_3] = e_4 \\
&[e_1, e_4] = e_5 && [e_1, e_5] = e_6 \\
&[e_1, e_6] = e_7 && [e_1, e_7] = e_8 \\
&[e_1, e_8] = e_9 && [e_1, e_9] = e_{10} \\
&[e_2, e_7] = e_{10} && [e_3, e_6] = -e_{10} \\
&[e_4, e_5] = e_{10}
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{3A}(3, 10)$

m3A310 (this line included for string searching purposes)
Original brackets:

$$
\begin{aligned}
&[e_1, e_2] = e_3 && [e_1, e_3] = e_4 \\
&[e_1, e_4] = e_5 && [e_1, e_5] = e_6 \\
&[e_1, e_6] = e_7 && [e_1, e_7] = e_8 \\
&[e_1, e_8] = e_9 && [e_1, e_9] = e_{10} \\
&[e_2, e_5] = e_8 && [e_2, e_6] = 2e_9 \\
&[e_2, e_7] = \alpha_{2,7}^{10} e_{10} && [e_3, e_4] = -e_8 \\
&[e_3, e_5] = -e_9 && [e_3, e_6] = \alpha_{3,6}^{10} e_{10} \\
&[e_4, e_5] = \alpha_{4,5}^{10} e_{10}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_6): \quad & -\alpha_{2,7}^{10} - \alpha_{3,6}^{10} + 2 && = 0 \\
(e_1, e_3, e_5): \quad & -\alpha_{3,6}^{10} - \alpha_{4,5}^{10} - 1 && = 0
\end{aligned}
$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$
\begin{aligned}
\alpha_{2,7}^{10} &\to x_1 \\
\alpha_{3,6}^{10} &\to x_2 \\
\alpha_{4,5}^{10} &\to x_3
\end{aligned}
$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_6): &\quad -x_1 - x_2 + 2 &= 0\\
(e_1, e_3, e_5): &\quad -x_2 - x_3 - 1 &= 0
\end{aligned}$$

Groebner basis (3 variables, 2 linear, 0 nonlinear)

$$x_1 - x_3 - 3 = 0$$
$$x_2 + x_3 + 1 = 0$$

$\mathfrak{m}_{5A}(3, 10)$

m5A310 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4\\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6\\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8\\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10}\\
[e_2, e_3] &= e_6 & [e_2, e_4] &= e_7\\
[e_2, e_5] &= \alpha_{2,5}^{8} e_8 & [e_2, e_6] &= \alpha_{2,6}^{9} e_9\\
[e_2, e_7] &= \alpha_{2,7}^{10} e_{10} & [e_3, e_4] &= \alpha_{3,4}^{8} e_8\\
[e_3, e_5] &= \alpha_{3,5}^{9} e_9 & [e_3, e_6] &= \alpha_{3,6}^{10} e_{10}\\
[e_4, e_5] &= \alpha_{4,5}^{10} e_{10}
\end{aligned}$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1, e_2, e_4): &\quad -\alpha_{2,5}^{8} - \alpha_{3,4}^{8} + 1 &= 0\\
(e_1, e_2, e_5): &\quad \alpha_{2,5}^{8} - \alpha_{2,6}^{9} - \alpha_{3,5}^{9} &= 0\\
(e_1, e_3, e_4): &\quad \alpha_{3,4}^{8} - \alpha_{3,5}^{9} &= 0\\
(e_1, e_2, e_6): &\quad \alpha_{2,6}^{9} - \alpha_{2,7}^{10} - \alpha_{3,6}^{10} &= 0\\
(e_1, e_3, e_5): &\quad \alpha_{3,5}^{9} - \alpha_{3,6}^{10} - \alpha_{4,5}^{10} &= 0
\end{aligned}$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{3,5}^{9} \to x_1$$

$$\alpha_{2,5}^{8} \to x_2$$
$$\alpha_{2,6}^{9} \to x_3$$
$$\alpha_{2,7}^{10} \to x_4$$
$$\alpha_{4,5}^{10} \to x_5$$
$$\alpha_{3,4}^{8} \to x_6$$
$$\alpha_{3,6}^{10} \to x_7$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4): &\quad -x_2 - x_6 + 1 &&= 0\\
(e_1, e_2, e_5): &\quad -x_1 + x_2 - x_3 &&= 0\\
(e_1, e_3, e_4): &\quad -x_1 + x_6 &&= 0\\
(e_1, e_2, e_6): &\quad x_3 - x_4 - x_7 &&= 0\\
(e_1, e_3, e_5): &\quad x_1 - x_5 - x_7 &&= 0
\end{aligned}$$

Groebner basis (7 variables, 5 linear, 0 nonlinear)

$$x_1 - x_6 = 0$$
$$x_2 + x_6 - 1 = 0$$
$$x_3 + 2x_6 - 1 = 0$$
$$x_4 + 2x_6 + x_7 - 1 = 0$$
$$x_5 - x_6 + x_7 = 0$$

$\mathfrak{m}_{2A}(4, 10)$

m2A410 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4\\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6\\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8\\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10}\\
[e_2, e_5] &= e_9 & [e_2, e_6] &= 2e_{10}\\
[e_3, e_4] &= -e_9 & [e_3, e_5] &= -e_{10}
\end{aligned}$$

No non-trivial Jacobi tests

$\mathfrak{m}_{4A}(4,10)$

m4A410 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4\\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6\\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8\\
[e_1,e_8] &= e_9 & [e_1,e_9] &= e_{10}\\
[e_2,e_3] &= e_7 & [e_2,e_4] &= e_8\\
[e_2,e_5] &= \alpha_{2,5}^{9}e_9 & [e_2,e_6] &= \alpha_{2,6}^{10}e_{10}\\
[e_3,e_4] &= \alpha_{3,4}^{9}e_9 & [e_3,e_5] &= \alpha_{3,5}^{10}e_{10}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1,e_2,e_4):&\quad -\alpha_{2,5}^{9}-\alpha_{3,4}^{9}+1 &= 0\\
(e_1,e_2,e_5):&\quad \alpha_{2,5}^{9}-\alpha_{2,6}^{10}-\alpha_{3,5}^{10} &= 0\\
(e_1,e_3,e_4):&\quad \alpha_{3,4}^{9}-\alpha_{3,5}^{10} &= 0
\end{aligned}
$$

Infinite number of solutions.

How the solution(s) were or were not found:

Change variables

$$
\begin{aligned}
\alpha_{2,6}^{10} &\to x_1\\
\alpha_{2,5}^{9} &\to x_2\\
\alpha_{3,4}^{9} &\to x_3\\
\alpha_{3,5}^{10} &\to x_4
\end{aligned}
$$

Jacobi Tests

$$
\begin{aligned}
(e_1,e_2,e_4):&\quad -x_2-x_3+1 &= 0\\
(e_1,e_2,e_5):&\quad -x_1+x_2-x_4 &= 0\\
(e_1,e_3,e_4):&\quad x_3-x_4 &= 0
\end{aligned}
$$

Groebner basis (4 variables, 3 linear, 0 nonlinear)

$$
\begin{aligned}
x_1+2x_4-1&=0\\
x_2+x_4-1&=0\\
x_3-x_4&=0
\end{aligned}
$$

$\mathfrak{m}_{1A}(5, 10)$

m1A510 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
&[e_1, e_2] = e_3 && [e_1, e_3] = e_4 \\
&[e_1, e_4] = e_5 && [e_1, e_5] = e_6 \\
&[e_1, e_6] = e_7 && [e_1, e_7] = e_8 \\
&[e_1, e_8] = e_9 && [e_1, e_9] = e_{10} \\
&[e_2, e_5] = e_{10} && [e_3, e_4] = -e_{10}
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{3A}(5, 10)$

m3A510 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
&[e_1, e_2] = e_3 && [e_1, e_3] = e_4 \\
&[e_1, e_4] = e_5 && [e_1, e_5] = e_6 \\
&[e_1, e_6] = e_7 && [e_1, e_7] = e_8 \\
&[e_1, e_8] = e_9 && [e_1, e_9] = e_{10} \\
&[e_2, e_3] = e_8 && [e_2, e_4] = e_9 \\
&[e_2, e_5] = \alpha_{2,5}^{10} e_{10} && [e_3, e_4] = \alpha_{3,4}^{10} e_{10}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$(e_1, e_2, e_4): \quad -\alpha_{2,5}^{10} - \alpha_{3,4}^{10} + 1 \qquad = 0$$

Infinite number of solutions.

How the solution(s) were or were not found:

Change variables

$$
\begin{aligned}
\alpha_{3,4}^{10} &\to x_1 \\
\alpha_{2,5}^{10} &\to x_2
\end{aligned}
$$

Jacobi Tests

$$(e_1, e_2, e_4): \quad -x_1 - x_2 + 1 \qquad = 0$$

Groebner basis (2 variables, 1 linear, 0 nonlinear)

$$x_1 + x_2 - 1 = 0$$

$\mathfrak{m}_{2A}(6, 10)$

m2A610 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned} [e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\ [e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\ [e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\ [e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\ [e_2, e_3] &= e_9 & [e_2, e_4] &= e_{10} \end{aligned}$$

No non-trivial Jacobi tests

$\mathfrak{m}_{1A}(7, 10)$

m1A710 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned} [e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\ [e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\ [e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\ [e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\ [e_2, e_3] &= e_{10} \end{aligned}$$

No non-trivial Jacobi tests

$\mathfrak{m}_{1A}(2, 11)$

m1A211 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned} [e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\ [e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\ [e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\ [e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\ [e_1, e_{10}] &= e_{11} & [e_2, e_9] &= e_{11} \\ [e_3, e_8] &= -e_{11} & [e_4, e_7] &= e_{11} \\ [e_5, e_6] &= -e_{11} \end{aligned}$$

No non-trivial Jacobi tests

$\mathfrak{m}_{3A}(2,11)$

m3A211 (this line included for string searching purposes)

Solution 1

$$
\begin{aligned}
&[e_1,e_2]=e_3 && [e_1,e_3]=e_4\\
&[e_1,e_4]=e_5 && [e_1,e_5]=e_6\\
&[e_1,e_6]=e_7 && [e_1,e_7]=e_8\\
&[e_1,e_8]=e_9 && [e_1,e_9]=e_{10}\\
&[e_1,e_{10}]=e_{11} && [e_2,e_7]=e_9\\
&[e_2,e_8]=3e_{10} && [e_2,e_9]=0\\
&[e_3,e_6]=-e_9 && [e_3,e_7]=-2e_{10}\\
&[e_3,e_8]=3e_{11} && [e_4,e_5]=e_9\\
&[e_4,e_6]=e_{10} && [e_4,e_7]=-5e_{11}\\
&[e_5,e_6]=6e_{11}
\end{aligned}
$$

Original brackets:

$$
\begin{aligned}
&[e_1,e_2]=e_3 && [e_1,e_3]=e_4\\
&[e_1,e_4]=e_5 && [e_1,e_5]=e_6\\
&[e_1,e_6]=e_7 && [e_1,e_7]=e_8\\
&[e_1,e_8]=e_9 && [e_1,e_9]=e_{10}\\
&[e_1,e_{10}]=e_{11} && [e_2,e_7]=e_9\\
&[e_2,e_8]=3e_{10} && [e_2,e_9]=\alpha_{2,9}^{11}e_{11}\\
&[e_3,e_6]=-e_9 && [e_3,e_7]=-2e_{10}\\
&[e_3,e_8]=\alpha_{3,8}^{11}e_{11} && [e_4,e_5]=e_9\\
&[e_4,e_6]=e_{10} && [e_4,e_7]=\alpha_{4,7}^{11}e_{11}\\
&[e_5,e_6]=\alpha_{5,6}^{11}e_{11}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1,e_2,e_8):&\quad -\alpha_{2,9}^{11}-\alpha_{3,8}^{11}+3 &&=0\\
(e_1,e_3,e_7):&\quad -\alpha_{3,8}^{11}-\alpha_{4,7}^{11}-2 &&=0\\
(e_1,e_4,e_6):&\quad -\alpha_{4,7}^{11}-\alpha_{5,6}^{11}+1 &&=0\\
(e_2,e_3,e_6):&\quad -\alpha_{2,9}^{11} &&=0\\
(e_2,e_4,e_5):&\quad \alpha_{2,9}^{11} &&=0
\end{aligned}
$$

Solution 1:

$$\alpha^{11}_{4,7} = -5$$
$$\alpha^{11}_{5,6} = 6$$
$$\alpha^{11}_{3,8} = 3$$
$$\alpha^{11}_{2,9} = 0$$

How the solution(s) were or were not found:
Change variables

$$\alpha^{11}_{4,7} \to x_1$$
$$\alpha^{11}_{5,6} \to x_2$$
$$\alpha^{11}_{3,8} \to x_3$$
$$\alpha^{11}_{2,9} \to x_4$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_8): &\quad -x_3 - x_4 + 3 &&= 0\\
(e_1, e_3, e_7): &\quad -x_1 - x_3 - 2 &&= 0\\
(e_1, e_4, e_6): &\quad -x_1 - x_2 + 1 &&= 0\\
(e_2, e_3, e_6): &\quad -x_4 &&= 0\\
(e_2, e_4, e_5): &\quad x_4 &&= 0
\end{aligned}$$

Groebner basis (4 variables, 4 linear, 0 nonlinear)

$$x_1 + 5 = 0$$
$$x_2 - 6 = 0$$
$$x_3 - 3 = 0$$
$$x_4 = 0$$

Solution 1:

$$x_1 = -5$$
$$x_2 = 6$$
$$x_3 = 3$$
$$x_4 = 0$$

$\mathfrak{m}_{5A}(2,11)$

m5A211 (this line included for string searching purposes)

Solution 1

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4 \\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6 \\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8 \\
[e_1,e_8] &= e_9 & [e_1,e_9] &= e_{10} \\
[e_1,e_{10}] &= e_{11} & [e_2,e_5] &= e_7 \\
[e_2,e_6] &= 2e_8 & [e_2,e_7] &= 0 \\
[e_2,e_8] &= -5e_{10} & [e_2,e_9] &= -\frac{5e_{11}}{2} \\
[e_3,e_4] &= -e_7 & [e_3,e_5] &= -e_8 \\
[e_3,e_6] &= 2e_9 & [e_3,e_7] &= 5e_{10} \\
[e_3,e_8] &= -\frac{5e_{11}}{2} & [e_4,e_5] &= -3e_9 \\
[e_4,e_6] &= -3e_{10} & [e_4,e_7] &= \frac{15e_{11}}{2} \\
[e_5,e_6] &= -\frac{21e_{11}}{2}
\end{aligned}
$$

Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4 \\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6 \\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8 \\
[e_1,e_8] &= e_9 & [e_1,e_9] &= e_{10} \\
[e_1,e_{10}] &= e_{11} & [e_2,e_5] &= e_7 \\
[e_2,e_6] &= 2e_8 & [e_2,e_7] &= \alpha_{2,7}^{9}e_9 \\
[e_2,e_8] &= \alpha_{2,8}^{10}e_{10} & [e_2,e_9] &= \alpha_{2,9}^{11}e_{11} \\
[e_3,e_4] &= -e_7 & [e_3,e_5] &= -e_8 \\
[e_3,e_6] &= \alpha_{3,6}^{9}e_9 & [e_3,e_7] &= \alpha_{3,7}^{10}e_{10} \\
[e_3,e_8] &= \alpha_{3,8}^{11}e_{11} & [e_4,e_5] &= \alpha_{4,5}^{9}e_9 \\
[e_4,e_6] &= \alpha_{4,6}^{10}e_{10} & [e_4,e_7] &= \alpha_{4,7}^{11}e_{11} \\
[e_5,e_6] &= \alpha_{5,6}^{11}e_{11}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_6): &\quad -\alpha_{2,7}^{9} - \alpha_{3,6}^{9} + 2 &= 0\\
(e_1, e_3, e_5): &\quad -\alpha_{3,6}^{9} - \alpha_{4,5}^{9} - 1 &= 0\\
(e_2, e_3, e_4): &\quad -\alpha_{2,7}^{9} &= 0\\
(e_1, e_2, e_7): &\quad \alpha_{2,7}^{9} - \alpha_{2,8}^{10} - \alpha_{3,7}^{10} &= 0\\
(e_1, e_3, e_6): &\quad \alpha_{3,6}^{9} - \alpha_{3,7}^{10} - \alpha_{4,6}^{10} &= 0\\
(e_1, e_4, e_5): &\quad \alpha_{4,5}^{9} - \alpha_{4,6}^{10} &= 0\\
(e_2, e_3, e_5): &\quad -\alpha_{2,8}^{10} - \alpha_{3,7}^{10} &= 0\\
(e_1, e_2, e_8): &\quad \alpha_{2,8}^{10} - \alpha_{2,9}^{11} - \alpha_{3,8}^{11} &= 0\\
(e_1, e_3, e_7): &\quad \alpha_{3,7}^{10} - \alpha_{3,8}^{11} - \alpha_{4,7}^{11} &= 0\\
(e_1, e_4, e_6): &\quad \alpha_{4,6}^{10} - \alpha_{4,7}^{11} - \alpha_{5,6}^{11} &= 0\\
(e_2, e_3, e_6): &\quad \alpha_{2,9}^{11}\alpha_{3,6}^{9} - 2\alpha_{3,8}^{11} &= 0\\
(e_2, e_4, e_5): &\quad \alpha_{2,9}^{11}\alpha_{4,5}^{9} - \alpha_{4,7}^{11} &= 0
\end{aligned}
$$

Solution 1:

$$
\begin{aligned}
\alpha_{2,7}^{9} &= 0\\
\alpha_{2,8}^{10} &= -5\\
\alpha_{2,9}^{11} &= -5/2\\
\alpha_{3,7}^{10} &= 5\\
\alpha_{3,6}^{9} &= 2\\
\alpha_{5,6}^{11} &= -21/2\\
\alpha_{4,7}^{11} &= 15/2\\
\alpha_{4,5}^{9} &= -3\\
\alpha_{4,6}^{10} &= -3\\
\alpha_{3,8}^{11} &= -5/2
\end{aligned}
$$

How the solution(s) were or were not found:
Change variables

$$
\begin{aligned}
\alpha_{2,7}^{9} &\to x_1\\
\alpha_{2,8}^{10} &\to x_2\\
\alpha_{2,9}^{11} &\to x_3
\end{aligned}
$$

$$\alpha_{3,7}^{10} \to x_4$$

$$\alpha_{3,6}^{9} \to x_5$$

$$\alpha_{5,6}^{11} \to x_6$$

$$\alpha_{4,7}^{11} \to x_7$$

$$\alpha_{4,5}^{9} \to x_8$$

$$\alpha_{4,6}^{10} \to x_9$$

$$\alpha_{3,8}^{11} \to x_{10}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_6): &\quad -x_1 - x_5 + 2 && = 0 \\
(e_1, e_3, e_5): &\quad -x_5 - x_8 - 1 && = 0 \\
(e_2, e_3, e_4): &\quad -x_1 && = 0 \\
(e_1, e_2, e_7): &\quad x_1 - x_2 - x_4 && = 0 \\
(e_1, e_3, e_6): &\quad -x_4 + x_5 - x_9 && = 0 \\
(e_1, e_4, e_5): &\quad x_8 - x_9 && = 0 \\
(e_2, e_3, e_5): &\quad -x_2 - x_4 && = 0 \\
(e_1, e_2, e_8): &\quad -x_{10} + x_2 - x_3 && = 0 \\
(e_1, e_3, e_7): &\quad -x_{10} + x_4 - x_7 && = 0 \\
(e_1, e_4, e_6): &\quad -x_6 - x_7 + x_9 && = 0 \\
(e_2, e_3, e_6): &\quad -2x_{10} + x_3 x_5 && = 0 \\
(e_2, e_4, e_5): &\quad x_3 x_8 - x_7 && = 0
\end{aligned}$$

Groebner basis (10 variables, 10 linear, 0 nonlinear)

$$x_1 = 0$$

$$x_2 + 5 = 0$$

$$2x_3 + 5 = 0$$

$$x_4 - 5 = 0$$

$$x_5 - 2 = 0$$

$$2x_6 + 21 = 0$$

$$2x_7 - 15 = 0$$

$$x_8 + 3 = 0$$

$$x_9 + 3 = 0$$

$$2x_{10} + 5 = 0$$

Solution 1:

$$x_1 = 0$$

$$x_2 = -5$$

$$x_3 = -5/2$$

$$x_4 = 5$$

$$x_5 = 2$$

$$x_6 = -21/2$$

$$x_7 = 15/2$$

$$x_8 = -3$$

$$x_9 = -3$$

$$x_10 = -5/2$$

$\mathfrak{m}_{7A}(2, 11)$

m7A211 (this line included for string searching purposes)

Original brackets:

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_2, e_3] = e_5$$
$$[e_2, e_4] = e_6 \qquad [e_2, e_5] = \alpha_{2,5}^{7} e_7$$
$$[e_2, e_6] = \alpha_{2,6}^{8} e_8 \qquad [e_2, e_7] = \alpha_{2,7}^{9} e_9$$
$$[e_2, e_8] = \alpha_{2,8}^{10} e_{10} \qquad [e_2, e_9] = \alpha_{2,9}^{11} e_{11}$$
$$[e_3, e_4] = \alpha_{3,4}^{7} e_7 \qquad [e_3, e_5] = \alpha_{3,5}^{8} e_8$$
$$[e_3, e_6] = \alpha_{3,6}^{9} e_9 \qquad [e_3, e_7] = \alpha_{3,7}^{10} e_{10}$$
$$[e_3, e_8] = \alpha_{3,8}^{11} e_{11} \qquad [e_4, e_5] = \alpha_{4,5}^{9} e_9$$
$$[e_4, e_6] = \alpha_{4,6}^{10} e_{10} \qquad [e_4, e_7] = \alpha_{4,7}^{11} e_{11}$$
$$[e_5, e_6] = \alpha_{5,6}^{11} e_{11}$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_4): &\quad -\alpha^7_{2,5} - \alpha^7_{3,4} + 1 &&= 0\\
(e_1, e_2, e_5): &\quad \alpha^7_{2,5} - \alpha^8_{2,6} - \alpha^8_{3,5} &&= 0\\
(e_1, e_3, e_4): &\quad \alpha^7_{3,4} - \alpha^8_{3,5} &&= 0\\
(e_1, e_2, e_6): &\quad \alpha^8_{2,6} - \alpha^9_{2,7} - \alpha^9_{3,6} &&= 0\\
(e_1, e_3, e_5): &\quad \alpha^8_{3,5} - \alpha^9_{3,6} - \alpha^9_{4,5} &&= 0\\
(e_2, e_3, e_4): &\quad \alpha^9_{2,7}\alpha^7_{3,4} - \alpha^9_{3,6} + \alpha^9_{4,5} &&= 0\\
(e_1, e_2, e_7): &\quad \alpha^9_{2,7} - \alpha^{10}_{2,8} - \alpha^{10}_{3,7} &&= 0\\
(e_1, e_3, e_6): &\quad \alpha^9_{3,6} - \alpha^{10}_{3,7} - \alpha^{10}_{4,6} &&= 0\\
(e_1, e_4, e_5): &\quad \alpha^9_{4,5} - \alpha^{10}_{4,6} &&= 0\\
(e_2, e_3, e_5): &\quad -\alpha^7_{2,5}\alpha^{10}_{3,7} + \alpha^{10}_{2,8}\alpha^8_{3,5} &&= 0\\
(e_1, e_2, e_8): &\quad \alpha^{10}_{2,8} - \alpha^{11}_{2,9} - \alpha^{11}_{3,8} &&= 0\\
(e_1, e_3, e_7): &\quad \alpha^{10}_{3,7} - \alpha^{11}_{3,8} - \alpha^{11}_{4,7} &&= 0\\
(e_1, e_4, e_6): &\quad \alpha^{10}_{4,6} - \alpha^{11}_{4,7} - \alpha^{11}_{5,6} &&= 0\\
(e_2, e_3, e_6): &\quad -\alpha^8_{2,6}\alpha^{11}_{3,8} + \alpha^{11}_{2,9}\alpha^9_{3,6} - \alpha^{11}_{5,6} &&= 0\\
(e_2, e_4, e_5): &\quad -\alpha^7_{2,5}\alpha^{11}_{4,7} + \alpha^{11}_{2,9}\alpha^9_{4,5} + \alpha^{11}_{5,6} &&= 0
\end{aligned}
$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha^7_{3,4} \to x_1$$

$$\alpha^9_{2,7} \to x_2$$

$$\alpha^{10}_{2,8} \to x_3$$

$$\alpha^{11}_{2,9} \to x_4$$

$$\alpha^9_{3,6} \to x_5$$

$$\alpha^7_{2,5} \to x_6$$

$$\alpha^{10}_{3,7} \to x_7$$

$$\alpha^{11}_{5,6} \to x_8$$

$$\alpha^{11}_{3,8} \to x_9$$

$$\alpha^8_{2,6} \to x_{10}$$

$$\alpha^{11}_{4,7} \to x_{11}$$

$$\alpha_{4,5}^{9} \to x_{12}$$
$$\alpha_{4,6}^{10} \to x_{13}$$
$$\alpha_{3,5}^{8} \to x_{14}$$

Jacobi Tests

$$\begin{aligned}
(e_1,e_2,e_4):&\quad -x_1 - x_6 + 1 &&= 0\\
(e_1,e_2,e_5):&\quad -x_{10} - x_{14} + x_6 &&= 0\\
(e_1,e_3,e_4):&\quad x_1 - x_{14} &&= 0\\
(e_1,e_2,e_6):&\quad x_{10} - x_2 - x_5 &&= 0\\
(e_1,e_3,e_5):&\quad -x_{12} + x_{14} - x_5 &&= 0\\
(e_2,e_3,e_4):&\quad x_1x_2 + x_{12} - x_5 &&= 0\\
(e_1,e_2,e_7):&\quad x_2 - x_3 - x_7 &&= 0\\
(e_1,e_3,e_6):&\quad -x_{13} + x_5 - x_7 &&= 0\\
(e_1,e_4,e_5):&\quad x_{12} - x_{13} &&= 0\\
(e_2,e_3,e_5):&\quad x_{14}x_3 - x_6x_7 &&= 0\\
(e_1,e_2,e_8):&\quad x_3 - x_4 - x_9 &&= 0\\
(e_1,e_3,e_7):&\quad -x_{11} + x_7 - x_9 &&= 0\\
(e_1,e_4,e_6):&\quad -x_{11} + x_{13} - x_8 &&= 0\\
(e_2,e_3,e_6):&\quad -x_{10}x_9 + x_4x_5 - x_8 &&= 0\\
(e_2,e_4,e_5):&\quad -x_{11}x_6 + x_{12}x_4 + x_8 &&= 0
\end{aligned}$$

Groebner basis (14 variables, 11 linear, 3 nonlinear)

$$x_1 - x_{14} = 0$$
$$-x_{13} + 3x_{14} + x_2 - 1 = 0$$
$$-3x_{13} + 4x_{14} + x_3 - 1 = 0$$
$$-x_{11} - 5x_{13} + 5x_{14} + x_4 - 1 = 0$$
$$x_{13} - x_{14} + x_5 = 0$$
$$x_{14} + x_6 - 1 = 0$$
$$2x_{13} - x_{14} + x_7 = 0$$
$$x_{11} - x_{13} + x_8 = 0$$
$$x_{11} + 2x_{13} - x_{14} + x_9 = 0$$
$$x_{10} + 2x_{14} - 1 = 0$$
$$x_{11}x_{13} + x_{11}x_{14} - 2x_{11} + 5x_{13}^2 + 12x_{13} - 15x_{14}^2 = 0$$
$$2x_{11}x_{14}^2 - 2x_{11} + 30x_{13} + 15x_{14}^3 - 42x_{14}^2 = 0$$
$$x_{12} - x_{13} = 0$$
$$x_{13}x_{14} + 2x_{13} - 3x_{14}^2 = 0$$

$\mathfrak{m}_{2A}(3, 11)$

m2A311 (this line included for string searching purposes)

Original brackets:

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_2, e_7] = e_{10}$$
$$[e_2, e_8] = 3e_{11} \qquad [e_3, e_6] = -e_{10}$$
$$[e_3, e_7] = -2e_{11} \qquad [e_4, e_5] = e_{10}$$
$$[e_4, e_6] = e_{11}$$

No non-trivial Jacobi tests

$\mathfrak{m}_{4A}(3, 11)$

m4A311 (this line included for string searching purposes)

Solution 1

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_2, e_5] = e_8$$
$$[e_2, e_6] = 2e_9 \qquad [e_2, e_7] = \frac{5e_{10}}{3}$$
$$[e_2, e_8] = 0 \qquad [e_3, e_4] = -e_8$$
$$[e_3, e_5] = -e_9 \qquad [e_3, e_6] = \frac{e_{10}}{3}$$
$$[e_3, e_7] = \frac{5e_{11}}{3} \qquad [e_4, e_5] = -\frac{4e_{10}}{3}$$
$$[e_4, e_6] = -\frac{4e_{11}}{3}$$

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_2, e_5] &= e_8 \\
[e_2, e_6] &= 2e_9 & [e_2, e_7] &= \alpha_{2,7}^{10} e_{10} \\
[e_2, e_8] &= \alpha_{2,8}^{11} e_{11} & [e_3, e_4] &= -e_8 \\
[e_3, e_5] &= -e_9 & [e_3, e_6] &= \alpha_{3,6}^{10} e_{10} \\
[e_3, e_7] &= \alpha_{3,7}^{11} e_{11} & [e_4, e_5] &= \alpha_{4,5}^{10} e_{10} \\
[e_4, e_6] &= \alpha_{4,6}^{11} e_{11}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_6) :& \quad -\alpha_{2,7}^{10} - \alpha_{3,6}^{10} + 2 &= 0 \\
(e_1, e_3, e_5) :& \quad -\alpha_{3,6}^{10} - \alpha_{4,5}^{10} - 1 &= 0 \\
(e_1, e_2, e_7) :& \quad \alpha_{2,7}^{10} - \alpha_{2,8}^{11} - \alpha_{3,7}^{11} &= 0 \\
(e_1, e_3, e_6) :& \quad \alpha_{3,6}^{10} - \alpha_{3,7}^{11} - \alpha_{4,6}^{11} &= 0 \\
(e_1, e_4, e_5) :& \quad \alpha_{4,5}^{10} - \alpha_{4,6}^{11} &= 0 \\
(e_2, e_3, e_4) :& \quad -\alpha_{2,8}^{11} &= 0
\end{aligned}
$$

Solution 1:

$$
\begin{aligned}
\alpha_{3,7}^{11} &= 5/3 \\
\alpha_{2,8}^{11} &= 0 \\
\alpha_{2,7}^{10} &= 5/3 \\
\alpha_{4,6}^{11} &= -4/3 \\
\alpha_{4,5}^{10} &= -4/3 \\
\alpha_{3,6}^{10} &= 1/3
\end{aligned}
$$

How the solution(s) were or were not found:
Change variables

$$\alpha_{3,7}^{11} \to x_1$$

$$\alpha_{2,8}^{11} \to x_2$$

$$\alpha_{2,7}^{10} \to x_3$$

$$\alpha_{4,6}^{11} \to x_4$$

$$\alpha_{4,5}^{10} \to x_5$$

$$\alpha_{3,6}^{10} \to x_6$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_6) :& \quad -x_3 - x_6 + 2 &&= 0\\
(e_1, e_3, e_5) :& \quad -x_5 - x_6 - 1 &&= 0\\
(e_1, e_2, e_7) :& \quad -x_1 - x_2 + x_3 &&= 0\\
(e_1, e_3, e_6) :& \quad -x_1 - x_4 + x_6 &&= 0\\
(e_1, e_4, e_5) :& \quad -x_4 + x_5 &&= 0\\
(e_2, e_3, e_4) :& \quad -x_2 &&= 0
\end{aligned}$$

Groebner basis (6 variables, 6 linear, 0 nonlinear)

$$3x_1 - 5 = 0$$

$$x_2 = 0$$

$$3x_3 - 5 = 0$$

$$3x_4 + 4 = 0$$

$$3x_5 + 4 = 0$$

$$3x_6 - 1 = 0$$

Solution 1:

$$x_1 = 5/3$$

$$x_2 = 0$$

$$x_3 = 5/3$$

$$x_4 = -4/3$$

$$x_5 = -4/3$$

$$x_6 = 1/3$$

$\mathfrak{m}_{6A}(3,11)$

m6A311 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4 \\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6 \\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8 \\
[e_1,e_8] &= e_9 & [e_1,e_9] &= e_{10} \\
[e_1,e_{10}] &= e_{11} & [e_2,e_3] &= e_6 \\
[e_2,e_4] &= e_7 & [e_2,e_5] &= \alpha_{2,5}^{8} e_8 \\
[e_2,e_6] &= \alpha_{2,6}^{9} e_9 & [e_2,e_7] &= \alpha_{2,7}^{10} e_{10} \\
[e_2,e_8] &= \alpha_{2,8}^{11} e_{11} & [e_3,e_4] &= \alpha_{3,4}^{8} e_8 \\
[e_3,e_5] &= \alpha_{3,5}^{9} e_9 & [e_3,e_6] &= \alpha_{3,6}^{10} e_{10} \\
[e_3,e_7] &= \alpha_{3,7}^{11} e_{11} & [e_4,e_5] &= \alpha_{4,5}^{10} e_{10} \\
[e_4,e_6] &= \alpha_{4,6}^{11} e_{11}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1,e_2,e_4):&\quad -\alpha_{2,5}^{8}-\alpha_{3,4}^{8}+1 &= 0 \\
(e_1,e_2,e_5):&\quad \alpha_{2,5}^{8}-\alpha_{2,6}^{9}-\alpha_{3,5}^{9} &= 0 \\
(e_1,e_3,e_4):&\quad \alpha_{3,4}^{8}-\alpha_{3,5}^{9} &= 0 \\
(e_1,e_2,e_6):&\quad \alpha_{2,6}^{9}-\alpha_{2,7}^{10}-\alpha_{3,6}^{10} &= 0 \\
(e_1,e_3,e_5):&\quad \alpha_{3,5}^{9}-\alpha_{3,6}^{10}-\alpha_{4,5}^{10} &= 0 \\
(e_1,e_2,e_7):&\quad \alpha_{2,7}^{10}-\alpha_{2,8}^{11}-\alpha_{3,7}^{11} &= 0 \\
(e_1,e_3,e_6):&\quad \alpha_{3,6}^{10}-\alpha_{3,7}^{11}-\alpha_{4,6}^{11} &= 0 \\
(e_1,e_4,e_5):&\quad \alpha_{4,5}^{10}-\alpha_{4,6}^{11} &= 0 \\
(e_2,e_3,e_4):&\quad \alpha_{2,8}^{11}\alpha_{3,4}^{8}-\alpha_{3,7}^{11}+\alpha_{4,6}^{11} &= 0
\end{aligned}
$$

Infinite number of solutions.

How the solution(s) were or were not found:

Change variables

$$
\begin{aligned}
\alpha_{3,5}^{9} &\to x_1 \\
\alpha_{2,5}^{8} &\to x_2 \\
\alpha_{3,7}^{11} &\to x_3 \\
\alpha_{2,8}^{11} &\to x_4
\end{aligned}
$$

$$\alpha^{9}_{2,6} \to x_5$$

$$\alpha^{10}_{2,7} \to x_6$$

$$\alpha^{11}_{4,6} \to x_7$$

$$\alpha^{10}_{4,5} \to x_8$$

$$\alpha^{8}_{3,4} \to x_9$$

$$\alpha^{10}_{3,6} \to x_{10}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4):\quad & -x_2 - x_9 + 1 && = 0\\
(e_1, e_2, e_5):\quad & -x_1 + x_2 - x_5 && = 0\\
(e_1, e_3, e_4):\quad & -x_1 + x_9 && = 0\\
(e_1, e_2, e_6):\quad & -x_{10} + x_5 - x_6 && = 0\\
(e_1, e_3, e_5):\quad & x_1 - x_{10} - x_8 && = 0\\
(e_1, e_2, e_7):\quad & -x_3 - x_4 + x_6 && = 0\\
(e_1, e_3, e_6):\quad & x_{10} - x_3 - x_7 && = 0\\
(e_1, e_4, e_5):\quad & -x_7 + x_8 && = 0\\
(e_2, e_3, e_4):\quad & -x_3 + x_4x_9 + x_7 && = 0
\end{aligned}$$

Groebner basis (10 variables, 8 linear, 1 nonlinear)

$$x_1 - x_9 = 0$$

$$x_2 + x_9 - 1 = 0$$

$$-2x_{10} + x_3 + x_9 = 0$$

$$3x_{10} + x_4 + x_9 - 1 = 0$$

$$x_5 + 2x_9 - 1 = 0$$

$$x_{10} + x_6 + 2x_9 - 1 = 0$$

$$x_{10} + x_7 - x_9 = 0$$

$$x_{10} + x_8 - x_9 = 0$$

$$3x_{10}x_9 + 3x_{10} + x_9^2 - 3x_9 = 0$$

$\mathfrak{m}_{1A}(4, 11)$

m1A411 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_2, e_7] &= e_{11} \\
[e_3, e_6] &= -e_{11} & [e_4, e_5] &= e_{11}
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{3A}(4, 11)$

m3A411 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_2, e_5] &= e_9 \\
[e_2, e_6] &= 2e_{10} & [e_2, e_7] &= \alpha_{2,7}^{11} e_{11} \\
[e_3, e_4] &= -e_9 & [e_3, e_5] &= -e_{10} \\
[e_3, e_6] &= \alpha_{3,6}^{11} e_{11} & [e_4, e_5] &= \alpha_{4,5}^{11} e_{11}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_6): &\quad -\alpha_{2,7}^{11} - \alpha_{3,6}^{11} + 2 &&= 0 \\
(e_1, e_3, e_5): &\quad -\alpha_{3,6}^{11} - \alpha_{4,5}^{11} - 1 &&= 0
\end{aligned}
$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$
\begin{aligned}
\alpha_{2,7}^{11} &\to x_1 \\
\alpha_{4,5}^{11} &\to x_2
\end{aligned}
$$

$$\alpha_{3,6}^{11} \to x_3$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_6): &\quad -x_1 - x_3 + 2 &&= 0\\
(e_1, e_3, e_5): &\quad -x_2 - x_3 - 1 &&= 0
\end{aligned}$$

Groebner basis (3 variables, 2 linear, 0 nonlinear)

$$x_1 + x_3 - 2 = 0$$

$$x_2 + x_3 + 1 = 0$$

$\mathfrak{m}_{5A}(4, 11)$

m5A411 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4\\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6\\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8\\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10}\\
[e_1, e_{10}] &= e_{11} & [e_2, e_3] &= e_7\\
[e_2, e_4] &= e_8 & [e_2, e_5] &= \alpha_{2,5}^{9} e_9\\
[e_2, e_6] &= \alpha_{2,6}^{10} e_{10} & [e_2, e_7] &= \alpha_{2,7}^{11} e_{11}\\
[e_3, e_4] &= \alpha_{3,4}^{9} e_9 & [e_3, e_5] &= \alpha_{3,5}^{10} e_{10}\\
[e_3, e_6] &= \alpha_{3,6}^{11} e_{11} & [e_4, e_5] &= \alpha_{4,5}^{11} e_{11}
\end{aligned}$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1, e_2, e_4): &\quad -\alpha_{2,5}^{9} - \alpha_{3,4}^{9} + 1 &&= 0\\
(e_1, e_2, e_5): &\quad \alpha_{2,5}^{9} - \alpha_{2,6}^{10} - \alpha_{3,5}^{10} &&= 0\\
(e_1, e_3, e_4): &\quad \alpha_{3,4}^{9} - \alpha_{3,5}^{10} &&= 0\\
(e_1, e_2, e_6): &\quad \alpha_{2,6}^{10} - \alpha_{2,7}^{11} - \alpha_{3,6}^{11} &&= 0\\
(e_1, e_3, e_5): &\quad \alpha_{3,5}^{10} - \alpha_{3,6}^{11} - \alpha_{4,5}^{11} &&= 0
\end{aligned}$$

Infinite number of solutions.

How the solution(s) were or were not found:

Change variables

$$\alpha_{2,7}^{11} \to x_1$$
$$\alpha_{2,5}^{9} \to x_2$$
$$\alpha_{3,5}^{10} \to x_3$$
$$\alpha_{4,5}^{11} \to x_4$$
$$\alpha_{3,4}^{9} \to x_5$$
$$\alpha_{2,6}^{10} \to x_6$$
$$\alpha_{3,6}^{11} \to x_7$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4): &\quad -x_2 - x_5 + 1 &= 0\\
(e_1, e_2, e_5): &\quad x_2 - x_3 - x_6 &= 0\\
(e_1, e_3, e_4): &\quad -x_3 + x_5 &= 0\\
(e_1, e_2, e_6): &\quad -x_1 + x_6 - x_7 &= 0\\
(e_1, e_3, e_5): &\quad x_3 - x_4 - x_7 &= 0
\end{aligned}$$

Groebner basis (7 variables, 5 linear, 0 nonlinear)

$$x_1 - x_6 + x_7 = 0$$
$$2x_2 - x_6 - 1 = 0$$
$$2x_3 + x_6 - 1 = 0$$
$$2x_4 + x_6 + 2x_7 - 1 = 0$$
$$2x_5 + x_6 - 1 = 0$$

$\mathfrak{m}_{2A}(5, 11)$

m2A511 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4\\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6\\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8\\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10}\\
[e_1, e_{10}] &= e_{11} & [e_2, e_5] &= e_{10}\\
[e_2, e_6] &= 2e_{11} & [e_3, e_4] &= -e_{10}\\
[e_3, e_5] &= -e_{11}
\end{aligned}$$

No non-trivial Jacobi tests

$\mathfrak{m}_{4A}(5,11)$

m4A511 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4\\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6\\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8\\
[e_1,e_8] &= e_9 & [e_1,e_9] &= e_{10}\\
[e_1,e_{10}] &= e_{11} & [e_2,e_3] &= e_8\\
[e_2,e_4] &= e_9 & [e_2,e_5] &= \alpha_{2,5}^{10}e_{10}\\
[e_2,e_6] &= \alpha_{2,6}^{11}e_{11} & [e_3,e_4] &= \alpha_{3,4}^{10}e_{10}\\
[e_3,e_5] &= \alpha_{3,5}^{11}e_{11}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1,e_2,e_4):&\quad -\alpha_{2,5}^{10}-\alpha_{3,4}^{10}+1 &&=0\\
(e_1,e_2,e_5):&\quad \alpha_{2,5}^{10}-\alpha_{2,6}^{11}-\alpha_{3,5}^{11} &&=0\\
(e_1,e_3,e_4):&\quad \alpha_{3,4}^{10}-\alpha_{3,5}^{11} &&=0
\end{aligned}
$$

Infinite number of solutions.

How the solution(s) were or were not found:

Change variables

$$
\begin{aligned}
\alpha_{3,4}^{10} &\to x_1\\
\alpha_{2,6}^{11} &\to x_2\\
\alpha_{3,5}^{11} &\to x_3\\
\alpha_{2,5}^{10} &\to x_4
\end{aligned}
$$

Jacobi Tests

$$
\begin{aligned}
(e_1,e_2,e_4):&\quad -x_1-x_4+1 &&=0\\
(e_1,e_2,e_5):&\quad -x_2-x_3+x_4 &&=0\\
(e_1,e_3,e_4):&\quad x_1-x_3 &&=0
\end{aligned}
$$

Groebner basis (4 variables, 3 linear, 0 nonlinear)

$$
\begin{aligned}
x_1+x_4-1&=0\\
x_2-2x_4+1&=0\\
x_3+x_4-1&=0
\end{aligned}
$$

$\mathfrak{m}_{1A}(6,11)$

m1A611 (this line included for string searching purposes)
Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4 \\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6 \\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8 \\
[e_1,e_8] &= e_9 & [e_1,e_9] &= e_{10} \\
[e_1,e_{10}] &= e_{11} & [e_2,e_5] &= e_{11} \\
[e_3,e_4] &= -e_{11}
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{3A}(6,11)$

m3A611 (this line included for string searching purposes)
Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4 \\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6 \\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8 \\
[e_1,e_8] &= e_9 & [e_1,e_9] &= e_{10} \\
[e_1,e_{10}] &= e_{11} & [e_2,e_3] &= e_9 \\
[e_2,e_4] &= e_{10} & [e_2,e_5] &= \alpha_{2,5}^{11} e_{11} \\
[e_3,e_4] &= \alpha_{3,4}^{11} e_{11}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$(e_1,e_2,e_4): \quad -\alpha_{2,5}^{11} - \alpha_{3,4}^{11} + 1 \qquad = 0$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$
\begin{aligned}
\alpha_{2,5}^{11} &\to x_1 \\
\alpha_{3,4}^{11} &\to x_2
\end{aligned}
$$

Jacobi Tests

$$(e_1,e_2,e_4): \quad -x_1 - x_2 + 1 \qquad = 0$$

Groebner basis (2 variables, 1 linear, 0 nonlinear)

$$x_1 + x_2 - 1 = 0$$

$\mathfrak{m}_{2A}(7, 11)$

m2A711 (this line included for string searching purposes)
Original brackets:

$$\begin{aligned} [e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\ [e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\ [e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\ [e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\ [e_1, e_{10}] &= e_{11} & [e_2, e_3] &= e_{10} \\ [e_2, e_4] &= e_{11} \end{aligned}$$

No non-trivial Jacobi tests

$\mathfrak{m}_{1A}(8, 11)$

m1A811 (this line included for string searching purposes)
Original brackets:

$$\begin{aligned} [e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\ [e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\ [e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\ [e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\ [e_1, e_{10}] &= e_{11} & [e_2, e_3] &= e_{11} \end{aligned}$$

No non-trivial Jacobi tests

$\mathfrak{m}_{2A}(2, 12)$

m2A212 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_2, e_9] &= e_{11} & [e_2, e_{10}] &= 4e_{12} \\
[e_3, e_8] &= -e_{11} & [e_3, e_9] &= -3e_{12} \\
[e_4, e_7] &= e_{11} & [e_4, e_8] &= 2e_{12} \\
[e_5, e_6] &= -e_{11} & [e_5, e_7] &= -e_{12}
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{4A}(2, 12)$

m4A212 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_2, e_7] &= e_9 & [e_2, e_8] &= 3e_{10} \\
[e_2, e_9] &= \alpha_{2,9}^{11} e_{11} & [e_2, e_{10}] &= \alpha_{2,10}^{12} e_{12} \\
[e_3, e_6] &= -e_9 & [e_3, e_7] &= -2e_{10} \\
[e_3, e_8] &= \alpha_{3,8}^{11} e_{11} & [e_3, e_9] &= \alpha_{3,9}^{12} e_{12} \\
[e_4, e_5] &= e_9 & [e_4, e_6] &= e_{10} \\
[e_4, e_7] &= \alpha_{4,7}^{11} e_{11} & [e_4, e_8] &= \alpha_{4,8}^{12} e_{12} \\
[e_5, e_6] &= \alpha_{5,6}^{11} e_{11} & [e_5, e_7] &= \alpha_{5,7}^{12} e_{12}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_8): &\quad -\alpha_{2,9}^{11} - \alpha_{3,8}^{11} + 3 &&= 0\\
(e_1, e_3, e_7): &\quad -\alpha_{3,8}^{11} - \alpha_{4,7}^{11} - 2 &&= 0\\
(e_1, e_4, e_6): &\quad -\alpha_{4,7}^{11} - \alpha_{5,6}^{11} + 1 &&= 0\\
(e_2, e_3, e_6): &\quad -\alpha_{2,9}^{11} &&= 0\\
(e_2, e_4, e_5): &\quad \alpha_{2,9}^{11} &&= 0\\
(e_1, e_2, e_9): &\quad -\alpha_{2,10}^{12} + \alpha_{2,9}^{11} - \alpha_{3,9}^{12} &&= 0\\
(e_1, e_3, e_8): &\quad \alpha_{3,8}^{11} - \alpha_{3,9}^{12} - \alpha_{4,8}^{12} &&= 0\\
(e_1, e_4, e_7): &\quad \alpha_{4,7}^{11} - \alpha_{4,8}^{12} - \alpha_{5,7}^{12} &&= 0\\
(e_1, e_5, e_6): &\quad \alpha_{5,6}^{11} - \alpha_{5,7}^{12} &&= 0\\
(e_2, e_3, e_7): &\quad -2\alpha_{2,10}^{12} - \alpha_{3,9}^{12} &&= 0\\
(e_2, e_4, e_6): &\quad \alpha_{2,10}^{12} &&= 0\\
(e_3, e_4, e_5): &\quad \alpha_{3,9}^{12} &&= 0
\end{aligned}
$$

No solutions.
How the solution(s) were or were not found:
Change variables

$$
\begin{aligned}
\alpha_{2,10}^{12} &\to x_1\\
\alpha_{5,7}^{12} &\to x_2\\
\alpha_{4,8}^{12} &\to x_3\\
\alpha_{2,9}^{11} &\to x_4\\
\alpha_{5,6}^{11} &\to x_5\\
\alpha_{4,7}^{11} &\to x_6\\
\alpha_{3,9}^{12} &\to x_7\\
\alpha_{3,8}^{11} &\to x_8
\end{aligned}
$$

Jacobi Tests

$$
\begin{aligned}
(e_1,e_2,e_8): &\quad -x_4-x_8+3 &&=0\\
(e_1,e_3,e_7): &\quad -x_6-x_8-2 &&=0\\
(e_1,e_4,e_6): &\quad -x_5-x_6+1 &&=0\\
(e_2,e_3,e_6): &\quad -x_4 &&=0\\
(e_2,e_4,e_5): &\quad x_4 &&=0\\
(e_1,e_2,e_9): &\quad -x_1+x_4-x_7 &&=0\\
(e_1,e_3,e_8): &\quad -x_3-x_7+x_8 &&=0\\
(e_1,e_4,e_7): &\quad -x_2-x_3+x_6 &&=0\\
(e_1,e_5,e_6): &\quad -x_2+x_5 &&=0\\
(e_2,e_3,e_7): &\quad -2x_1-x_7 &&=0\\
(e_2,e_4,e_6): &\quad x_1 &&=0\\
(e_3,e_4,e_5): &\quad x_7 &&=0
\end{aligned}
$$

Groebner basis (8 variables, 1 linear, 0 nonlinear)

$$1=0$$

$\mathfrak{m}_{6A}(2,12)$

m6A212 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
&[e_1, e_2] = e_3 && [e_1, e_3] = e_4 \\
&[e_1, e_4] = e_5 && [e_1, e_5] = e_6 \\
&[e_1, e_6] = e_7 && [e_1, e_7] = e_8 \\
&[e_1, e_8] = e_9 && [e_1, e_9] = e_{10} \\
&[e_1, e_{10}] = e_{11} && [e_1, e_{11}] = e_{12} \\
&[e_2, e_5] = e_7 && [e_2, e_6] = 2e_8 \\
&[e_2, e_7] = \alpha_{2,7}^{9} e_9 && [e_2, e_8] = \alpha_{2,8}^{10} e_{10} \\
&[e_2, e_9] = \alpha_{2,9}^{11} e_{11} && [e_2, e_{10}] = \alpha_{2,10}^{12} e_{12} \\
&[e_3, e_4] = -e_7 && [e_3, e_5] = -e_8 \\
&[e_3, e_6] = \alpha_{3,6}^{9} e_9 && [e_3, e_7] = \alpha_{3,7}^{10} e_{10} \\
&[e_3, e_8] = \alpha_{3,8}^{11} e_{11} && [e_3, e_9] = \alpha_{3,9}^{12} e_{12} \\
&[e_4, e_5] = \alpha_{4,5}^{9} e_9 && [e_4, e_6] = \alpha_{4,6}^{10} e_{10} \\
&[e_4, e_7] = \alpha_{4,7}^{11} e_{11} && [e_4, e_8] = \alpha_{4,8}^{12} e_{12} \\
&[e_5, e_6] = \alpha_{5,6}^{11} e_{11} && [e_5, e_7] = \alpha_{5,7}^{12} e_{12}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_6): &\quad -\alpha_{2,7}^{9} - \alpha_{3,6}^{9} + 2 &&= 0\\
(e_1, e_3, e_5): &\quad -\alpha_{3,6}^{9} - \alpha_{4,5}^{9} - 1 &&= 0\\
(e_2, e_3, e_4): &\quad -\alpha_{2,7}^{9} &&= 0\\
(e_1, e_2, e_7): &\quad \alpha_{2,7}^{9} - \alpha_{2,8}^{10} - \alpha_{3,7}^{10} &&= 0\\
(e_1, e_3, e_6): &\quad \alpha_{3,6}^{9} - \alpha_{3,7}^{10} - \alpha_{4,6}^{10} &&= 0\\
(e_1, e_4, e_5): &\quad \alpha_{4,5}^{9} - \alpha_{4,6}^{10} &&= 0\\
(e_2, e_3, e_5): &\quad -\alpha_{2,8}^{10} - \alpha_{3,7}^{10} &&= 0\\
(e_1, e_2, e_8): &\quad \alpha_{2,8}^{10} - \alpha_{2,9}^{11} - \alpha_{3,8}^{11} &&= 0\\
(e_1, e_3, e_7): &\quad \alpha_{3,7}^{10} - \alpha_{3,8}^{11} - \alpha_{4,7}^{11} &&= 0\\
(e_1, e_4, e_6): &\quad \alpha_{4,6}^{10} - \alpha_{4,7}^{11} - \alpha_{5,6}^{11} &&= 0\\
(e_2, e_3, e_6): &\quad \alpha_{2,9}^{11}\alpha_{3,6}^{9} - 2\alpha_{3,8}^{11} &&= 0\\
(e_2, e_4, e_5): &\quad \alpha_{2,9}^{11}\alpha_{4,5}^{9} - \alpha_{4,7}^{11} &&= 0\\
(e_1, e_2, e_9): &\quad -\alpha_{2,10}^{12} + \alpha_{2,9}^{11} - \alpha_{3,9}^{12} &&= 0\\
(e_1, e_3, e_8): &\quad \alpha_{3,8}^{11} - \alpha_{3,9}^{12} - \alpha_{4,8}^{12} &&= 0\\
(e_1, e_4, e_7): &\quad \alpha_{4,7}^{11} - \alpha_{4,8}^{12} - \alpha_{5,7}^{12} &&= 0\\
(e_1, e_5, e_6): &\quad \alpha_{5,6}^{11} - \alpha_{5,7}^{12} &&= 0\\
(e_2, e_3, e_7): &\quad \alpha_{2,10}^{12}\alpha_{3,7}^{10} - \alpha_{2,7}^{9}\alpha_{3,9}^{12} &&= 0\\
(e_2, e_4, e_6): &\quad \alpha_{2,10}^{12}\alpha_{4,6}^{10} - 2\alpha_{4,8}^{12} &&= 0\\
(e_3, e_4, e_5): &\quad \alpha_{3,9}^{12}\alpha_{4,5}^{9} + \alpha_{4,8}^{12} - \alpha_{5,7}^{12} &&= 0
\end{aligned}
$$

No solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{2,7}^{9} \to x_1$$

$$\alpha_{2,10}^{12} \to x_2$$

$$\alpha_{2,8}^{10} \to x_3$$

$$\alpha_{5,7}^{12} \to x_4$$

$$\alpha_{4,8}^{12} \to x_5$$

$$\alpha_{2,9}^{11} \to x_6$$

$$\alpha_{3,7}^{10} \to x_7$$

$$\alpha_{3,6}^{9} \to x_8$$

$$\alpha_{5,6}^{11} \to x_9$$
$$\alpha_{4,7}^{11} \to x_{10}$$
$$\alpha_{4,5}^{9} \to x_{11}$$
$$\alpha_{4,6}^{10} \to x_{12}$$
$$\alpha_{3,9}^{12} \to x_{13}$$
$$\alpha_{3,8}^{11} \to x_{14}$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_6): &\quad -x_1 - x_8 + 2 &&= 0\\
(e_1, e_3, e_5): &\quad -x_{11} - x_8 - 1 &&= 0\\
(e_2, e_3, e_4): &\quad -x_1 &&= 0\\
(e_1, e_2, e_7): &\quad x_1 - x_3 - x_7 &&= 0\\
(e_1, e_3, e_6): &\quad -x_{12} - x_7 + x_8 &&= 0\\
(e_1, e_4, e_5): &\quad x_{11} - x_{12} &&= 0\\
(e_2, e_3, e_5): &\quad -x_3 - x_7 &&= 0\\
(e_1, e_2, e_8): &\quad -x_{14} + x_3 - x_6 &&= 0\\
(e_1, e_3, e_7): &\quad -x_{10} - x_{14} + x_7 &&= 0\\
(e_1, e_4, e_6): &\quad -x_{10} + x_{12} - x_9 &&= 0\\
(e_2, e_3, e_6): &\quad -2x_{14} + x_6 x_8 &&= 0\\
(e_2, e_4, e_5): &\quad -x_{10} + x_{11} x_6 &&= 0\\
(e_1, e_2, e_9): &\quad -x_{13} - x_2 + x_6 &&= 0\\
(e_1, e_3, e_8): &\quad -x_{13} + x_{14} - x_5 &&= 0\\
(e_1, e_4, e_7): &\quad x_{10} - x_4 - x_5 &&= 0\\
(e_1, e_5, e_6): &\quad -x_4 + x_9 &&= 0\\
(e_2, e_3, e_7): &\quad -x_1 x_{13} + x_2 x_7 &&= 0\\
(e_2, e_4, e_6): &\quad x_{12} x_2 - 2x_5 &&= 0\\
(e_3, e_4, e_5): &\quad x_{11} x_{13} - x_4 + x_5 &&= 0
\end{aligned}
$$

Groebner basis (14 variables, 1 linear, 0 nonlinear)

$$1 = 0$$

$\mathfrak{m}_{8A}(2, 12)$

m8A212 (this line included for string searching purposes)

Solution 1

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_1, e_{11}] = e_{12}$$
$$[e_2, e_3] = e_5 \qquad [e_2, e_4] = e_6$$
$$[e_2, e_5] = e_7 \qquad [e_2, e_6] = e_8$$
$$[e_2, e_7] = e_9 \qquad [e_2, e_8] = e_{10}$$
$$[e_2, e_9] = e_{11} \qquad [e_2, e_{10}] = e_{12}$$
$$[e_3, e_4] = 0 \qquad [e_3, e_5] = 0$$
$$[e_3, e_6] = 0 \qquad [e_3, e_7] = 0$$
$$[e_3, e_8] = 0 \qquad [e_3, e_9] = 0$$
$$[e_4, e_5] = 0 \qquad [e_4, e_6] = 0$$
$$[e_4, e_7] = 0 \qquad [e_4, e_8] = 0$$
$$[e_5, e_6] = 0 \qquad [e_5, e_7] = 0$$

Solution 2

$$
\begin{aligned}
&[e_1, e_2] = e_3 & &[e_1, e_3] = e_4 \\
&[e_1, e_4] = e_5 & &[e_1, e_5] = e_6 \\
&[e_1, e_6] = e_7 & &[e_1, e_7] = e_8 \\
&[e_1, e_8] = e_9 & &[e_1, e_9] = e_{10} \\
&[e_1, e_{10}] = e_{11} & &[e_1, e_{11}] = e_{12} \\
&[e_2, e_3] = e_5 & &[e_2, e_4] = e_6 \\
&[e_2, e_5] = \frac{9e_7}{10} & &[e_2, e_6] = \frac{4e_8}{5} \\
&[e_2, e_7] = \frac{5e_9}{7} & &[e_2, e_8] = \frac{9e_{10}}{14} \\
&[e_2, e_9] = \frac{7e_{11}}{12} & &[e_2, e_{10}] = \frac{8e_{12}}{15} \\
&[e_3, e_4] = \frac{e_7}{10} & &[e_3, e_5] = \frac{e_8}{10} \\
&[e_3, e_6] = \frac{3e_9}{35} & &[e_3, e_7] = \frac{e_{10}}{14} \\
&[e_3, e_8] = \frac{5e_{11}}{84} & &[e_3, e_9] = \frac{e_{12}}{20} \\
&[e_4, e_5] = \frac{e_9}{70} & &[e_4, e_6] = \frac{e_{10}}{70} \\
&[e_4, e_7] = \frac{e_{11}}{84} & &[e_4, e_8] = \frac{e_{12}}{105} \\
&[e_5, e_6] = \frac{e_{11}}{420} & &[e_5, e_7] = \frac{e_{12}}{420}
\end{aligned}
$$

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_2, e_3] &= e_5 & [e_2, e_4] &= e_6 \\
[e_2, e_5] &= \alpha_{2,5}^{7} e_7 & [e_2, e_6] &= \alpha_{2,6}^{8} e_8 \\
[e_2, e_7] &= \alpha_{2,7}^{9} e_9 & [e_2, e_8] &= \alpha_{2,8}^{10} e_{10} \\
[e_2, e_9] &= \alpha_{2,9}^{11} e_{11} & [e_2, e_{10}] &= \alpha_{2,10}^{12} e_{12} \\
[e_3, e_4] &= \alpha_{3,4}^{7} e_7 & [e_3, e_5] &= \alpha_{3,5}^{8} e_8 \\
[e_3, e_6] &= \alpha_{3,6}^{9} e_9 & [e_3, e_7] &= \alpha_{3,7}^{10} e_{10} \\
[e_3, e_8] &= \alpha_{3,8}^{11} e_{11} & [e_3, e_9] &= \alpha_{3,9}^{12} e_{12} \\
[e_4, e_5] &= \alpha_{4,5}^{9} e_9 & [e_4, e_6] &= \alpha_{4,6}^{10} e_{10} \\
[e_4, e_7] &= \alpha_{4,7}^{11} e_{11} & [e_4, e_8] &= \alpha_{4,8}^{12} e_{12} \\
[e_5, e_6] &= \alpha_{5,6}^{11} e_{11} & [e_5, e_7] &= \alpha_{5,7}^{12} e_{12}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_4): &\quad -\alpha_{2,5}^{7} - \alpha_{3,4}^{7} + 1 &&= 0\\
(e_1, e_2, e_5): &\quad \alpha_{2,5}^{7} - \alpha_{2,6}^{8} - \alpha_{3,5}^{8} &&= 0\\
(e_1, e_3, e_4): &\quad \alpha_{3,4}^{7} - \alpha_{3,5}^{8} &&= 0\\
(e_1, e_2, e_6): &\quad \alpha_{2,6}^{8} - \alpha_{2,7}^{9} - \alpha_{3,6}^{9} &&= 0\\
(e_1, e_3, e_5): &\quad \alpha_{3,5}^{8} - \alpha_{3,6}^{9} - \alpha_{4,5}^{9} &&= 0\\
(e_2, e_3, e_4): &\quad \alpha_{2,7}^{9}\alpha_{3,4}^{7} - \alpha_{3,6}^{9} + \alpha_{4,5}^{9} &&= 0\\
(e_1, e_2, e_7): &\quad \alpha_{2,7}^{9} - \alpha_{2,8}^{10} - \alpha_{3,7}^{10} &&= 0\\
(e_1, e_3, e_6): &\quad \alpha_{3,6}^{9} - \alpha_{3,7}^{10} - \alpha_{4,6}^{10} &&= 0\\
(e_1, e_4, e_5): &\quad \alpha_{4,5}^{9} - \alpha_{4,6}^{10} &&= 0\\
(e_2, e_3, e_5): &\quad -\alpha_{2,5}^{7}\alpha_{3,7}^{10} + \alpha_{2,8}^{10}\alpha_{3,5}^{8} &&= 0\\
(e_1, e_2, e_8): &\quad \alpha_{2,8}^{10} - \alpha_{2,9}^{11} - \alpha_{3,8}^{11} &&= 0\\
(e_1, e_3, e_7): &\quad \alpha_{3,7}^{10} - \alpha_{3,8}^{11} - \alpha_{4,7}^{11} &&= 0\\
(e_1, e_4, e_6): &\quad \alpha_{4,6}^{10} - \alpha_{4,7}^{11} - \alpha_{5,6}^{11} &&= 0\\
(e_2, e_3, e_6): &\quad -\alpha_{2,6}^{8}\alpha_{3,8}^{11} + \alpha_{2,9}^{11}\alpha_{3,6}^{9} - \alpha_{5,6}^{11} &&= 0\\
(e_2, e_4, e_5): &\quad -\alpha_{2,5}^{7}\alpha_{4,7}^{11} + \alpha_{2,9}^{11}\alpha_{4,5}^{9} + \alpha_{5,6}^{11} &&= 0\\
(e_1, e_2, e_9): &\quad -\alpha_{2,10}^{12} + \alpha_{2,9}^{11} - \alpha_{3,9}^{12} &&= 0\\
(e_1, e_3, e_8): &\quad \alpha_{3,8}^{11} - \alpha_{3,9}^{12} - \alpha_{4,8}^{12} &&= 0\\
(e_1, e_4, e_7): &\quad \alpha_{4,7}^{11} - \alpha_{4,8}^{12} - \alpha_{5,7}^{12} &&= 0\\
(e_1, e_5, e_6): &\quad \alpha_{5,6}^{11} - \alpha_{5,7}^{12} &&= 0\\
(e_2, e_3, e_7): &\quad \alpha_{2,10}^{12}\alpha_{3,7}^{10} - \alpha_{2,7}^{9}\alpha_{3,9}^{12} - \alpha_{5,7}^{12} &&= 0\\
(e_2, e_4, e_6): &\quad \alpha_{2,10}^{12}\alpha_{4,6}^{10} - \alpha_{2,6}^{8}\alpha_{4,8}^{12} &&= 0\\
(e_3, e_4, e_5): &\quad \alpha_{3,4}^{7}\alpha_{5,7}^{12} - \alpha_{3,5}^{8}\alpha_{4,8}^{12} + \alpha_{3,9}^{12}\alpha_{4,5}^{9} &&= 0
\end{aligned}
$$

Solution 1:

$$\alpha_{3,4}^{7} = 0$$
$$\alpha_{2,7}^{9} = 1$$
$$\alpha_{2,10}^{12} = 1$$
$$\alpha_{2,8}^{10} = 1$$
$$\alpha_{5,7}^{12} = 0$$
$$\alpha_{4,8}^{12} = 0$$
$$\alpha_{2,9}^{11} = 1$$
$$\alpha_{3,6}^{9} = 0$$
$$\alpha_{2,5}^{7} = 1$$
$$\alpha_{3,7}^{10} = 0$$
$$\alpha_{5,6}^{11} = 0$$
$$\alpha_{3,8}^{11} = 0$$
$$\alpha_{2,6}^{8} = 1$$
$$\alpha_{4,7}^{11} = 0$$
$$\alpha_{4,5}^{9} = 0$$
$$\alpha_{4,6}^{10} = 0$$
$$\alpha_{3,9}^{12} = 0$$
$$\alpha_{3,5}^{8} = 0$$

Solution 2:

$$\alpha^{7}_{3,4} = 1/10$$
$$\alpha^{9}_{2,7} = 5/7$$
$$\alpha^{12}_{2,10} = 8/15$$
$$\alpha^{10}_{2,8} = 9/14$$
$$\alpha^{12}_{5,7} = 1/420$$
$$\alpha^{12}_{4,8} = 1/105$$
$$\alpha^{11}_{2,9} = 7/12$$
$$\alpha^{9}_{3,6} = 3/35$$
$$\alpha^{7}_{2,5} = 9/10$$
$$\alpha^{10}_{3,7} = 1/14$$
$$\alpha^{11}_{5,6} = 1/420$$
$$\alpha^{11}_{3,8} = 5/84$$
$$\alpha^{8}_{2,6} = 4/5$$
$$\alpha^{11}_{4,7} = 1/84$$
$$\alpha^{9}_{4,5} = 1/70$$
$$\alpha^{10}_{4,6} = 1/70$$
$$\alpha^{12}_{3,9} = 1/20$$
$$\alpha^{8}_{3,5} = 1/10$$

How the solution(s) were or were not found:
Change variables

$$\alpha^{7}_{3,4} \to x_1$$
$$\alpha^{9}_{2,7} \to x_2$$
$$\alpha^{12}_{2,10} \to x_3$$
$$\alpha^{10}_{2,8} \to x_4$$
$$\alpha^{12}_{5,7} \to x_5$$
$$\alpha^{12}_{4,8} \to x_6$$
$$\alpha^{11}_{2,9} \to x_7$$
$$\alpha^{9}_{3,6} \to x_8$$
$$\alpha^{7}_{2,5} \to x_9$$

$$\alpha_{3,7}^{10} \to x_{10}$$
$$\alpha_{5,6}^{11} \to x_{11}$$
$$\alpha_{3,8}^{11} \to x_{12}$$
$$\alpha_{2,6}^{8} \to x_{13}$$
$$\alpha_{4,7}^{11} \to x_{14}$$
$$\alpha_{4,5}^{9} \to x_{15}$$
$$\alpha_{4,6}^{10} \to x_{16}$$
$$\alpha_{3,9}^{12} \to x_{17}$$
$$\alpha_{3,5}^{8} \to x_{18}$$

Jacobi Tests

$$\begin{aligned}
(e_1,e_2,e_4):\quad & -x_1 - x_9 + 1 && = 0\\
(e_1,e_2,e_5):\quad & -x_{13} - x_{18} + x_9 && = 0\\
(e_1,e_3,e_4):\quad & x_1 - x_{18} && = 0\\
(e_1,e_2,e_6):\quad & x_{13} - x_2 - x_8 && = 0\\
(e_1,e_3,e_5):\quad & -x_{15} + x_{18} - x_8 && = 0\\
(e_2,e_3,e_4):\quad & x_1x_2 + x_{15} - x_8 && = 0\\
(e_1,e_2,e_7):\quad & -x_{10} + x_2 - x_4 && = 0\\
(e_1,e_3,e_6):\quad & -x_{10} - x_{16} + x_8 && = 0\\
(e_1,e_4,e_5):\quad & x_{15} - x_{16} && = 0\\
(e_2,e_3,e_5):\quad & -x_{10}x_9 + x_{18}x_4 && = 0\\
(e_1,e_2,e_8):\quad & -x_{12} + x_4 - x_7 && = 0\\
(e_1,e_3,e_7):\quad & x_{10} - x_{12} - x_{14} && = 0\\
(e_1,e_4,e_6):\quad & -x_{11} - x_{14} + x_{16} && = 0\\
(e_2,e_3,e_6):\quad & -x_{11} - x_{12}x_{13} + x_7x_8 && = 0\\
(e_2,e_4,e_5):\quad & x_{11} - x_{14}x_9 + x_{15}x_7 && = 0\\
(e_1,e_2,e_9):\quad & -x_{17} - x_3 + x_7 && = 0\\
(e_1,e_3,e_8):\quad & x_{12} - x_{17} - x_6 && = 0\\
(e_1,e_4,e_7):\quad & x_{14} - x_5 - x_6 && = 0\\
(e_1,e_5,e_6):\quad & x_{11} - x_5 && = 0\\
(e_2,e_3,e_7):\quad & x_{10}x_3 - x_{17}x_2 - x_5 && = 0\\
(e_2,e_4,e_6):\quad & -x_{13}x_6 + x_{16}x_3 && = 0\\
(e_3,e_4,e_5):\quad & x_1x_5 + x_{15}x_{17} - x_{18}x_6 && = 0
\end{aligned}$$

Groebner basis (18 variables, 3 linear, 15 nonlinear)

$$x_1 - x_{18} = 0$$
$$10x_{18}^5 - 21x_{18}^4 + 42x_{18}^3 - 84x_{18}^2 + 168x_{18} + 56x_2 - 56 = 0$$
$$290x_{18}^5 - 369x_{18}^4 + 234x_{18}^3 - 180x_{18}^2 + 72x_{18} + 12x_3 - 12 = 0$$
$$30x_{18}^5 - 63x_{18}^4 + 126x_{18}^3 - 252x_{18}^2 + 224x_{18} + 56x_4 - 56 = 0$$
$$-470x_{18}^5 + 567x_{18}^4 - 252x_{18}^3 + 84x_5 = 0$$
$$1910x_{18}^5 - 2331x_{18}^4 + 1134x_{18}^3 - 252x_{18}^2 + 168x_6 = 0$$
$$40x_{18}^5 - 54x_{18}^4 + 45x_{18}^3 - 54x_{18}^2 + 30x_{18} + 6x_7 - 6 = 0$$
$$-10x_{18}^5 + 21x_{18}^4 - 42x_{18}^3 + 84x_{18}^2 - 56x_{18} + 56x_8 = 0$$
$$x_{18} + x_9 - 1 = 0$$
$$28x_{10} - 10x_{18}^5 + 21x_{18}^4 - 42x_{18}^3 + 84x_{18}^2 - 28x_{18} = 0$$
$$84x_{11} - 470x_{18}^5 + 567x_{18}^4 - 252x_{18}^3 = 0$$
$$168x_{12} - 1030x_{18}^5 + 1323x_{18}^4 - 882x_{18}^3 + 756x_{18}^2 - 168x_{18} = 0$$
$$x_{13} + 2x_{18} - 1 = 0$$
$$168x_{14} + 970x_{18}^5 - 1197x_{18}^4 + 630x_{18}^3 - 252x_{18}^2 = 0$$
$$56x_{15} + 10x_{18}^5 - 21x_{18}^4 + 42x_{18}^3 - 84x_{18}^2 = 0$$
$$56x_{16} + 10x_{18}^5 - 21x_{18}^4 + 42x_{18}^3 - 84x_{18}^2 = 0$$
$$4x_{17} - 70x_{18}^5 + 87x_{18}^4 - 48x_{18}^3 + 24x_{18}^2 - 4x_{18} = 0$$
$$10x_{18}^6 - x_{18}^5 = 0$$

Solution 1:

$$x_1 = 0$$
$$x_2 = 1$$
$$x_3 = 1$$
$$x_4 = 1$$
$$x_5 = 0$$
$$x_6 = 0$$
$$x_7 = 1$$
$$x_8 = 0$$
$$x_9 = 1$$
$$x_10 = 0$$
$$x_11 = 0$$

$$x_12 = 0$$
$$x_13 = 1$$
$$x_14 = 0$$
$$x_15 = 0$$
$$x_16 = 0$$
$$x_17 = 0$$
$$x_18 = 0$$

Solution 2:

$$x_1 = 1/10$$
$$x_2 = 5/7$$
$$x_3 = 8/15$$
$$x_4 = 9/14$$
$$x_5 = 1/420$$
$$x_6 = 1/105$$
$$x_7 = 7/12$$
$$x_8 = 3/35$$
$$x_9 = 9/10$$
$$x_10 = 1/14$$
$$x_11 = 1/420$$
$$x_12 = 5/84$$
$$x_13 = 4/5$$
$$x_14 = 1/84$$
$$x_15 = 1/70$$
$$x_16 = 1/70$$
$$x_17 = 1/20$$
$$x_18 = 1/10$$

$\mathfrak{m}_{1A}(3,12)$

m1A312 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4 \\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6 \\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8 \\
[e_1,e_8] &= e_9 & [e_1,e_9] &= e_{10} \\
[e_1,e_{10}] &= e_{11} & [e_1,e_{11}] &= e_{12} \\
[e_2,e_9] &= e_{12} & [e_3,e_8] &= -e_{12} \\
[e_4,e_7] &= e_{12} & [e_5,e_6] &= -e_{12}
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{3A}(3,12)$

m3A312 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4 \\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6 \\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8 \\
[e_1,e_8] &= e_9 & [e_1,e_9] &= e_{10} \\
[e_1,e_{10}] &= e_{11} & [e_1,e_{11}] &= e_{12} \\
[e_2,e_7] &= e_{10} & [e_2,e_8] &= 3e_{11} \\
[e_2,e_9] &= \alpha_{2,9}^{12}e_{12} & [e_3,e_6] &= -e_{10} \\
[e_3,e_7] &= -2e_{11} & [e_3,e_8] &= \alpha_{3,8}^{12}e_{12} \\
[e_4,e_5] &= e_{10} & [e_4,e_6] &= e_{11} \\
[e_4,e_7] &= \alpha_{4,7}^{12}e_{12} & [e_5,e_6] &= \alpha_{5,6}^{12}e_{12}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1,e_2,e_8): &\quad -\alpha_{2,9}^{12} - \alpha_{3,8}^{12} + 3 &&= 0 \\
(e_1,e_3,e_7): &\quad -\alpha_{3,8}^{12} - \alpha_{4,7}^{12} - 2 &&= 0 \\
(e_1,e_4,e_6): &\quad -\alpha_{4,7}^{12} - \alpha_{5,6}^{12} + 1 &&= 0
\end{aligned}
$$

Infinite number of solutions.

How the solution(s) were or were not found:

Change variables

$$\alpha_{2,9}^{12} \to x_1$$
$$\alpha_{4,7}^{12} \to x_2$$
$$\alpha_{3,8}^{12} \to x_3$$
$$\alpha_{5,6}^{12} \to x_4$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_8): &\quad -x_1 - x_3 + 3 &= 0\\
(e_1, e_3, e_7): &\quad -x_2 - x_3 - 2 &= 0\\
(e_1, e_4, e_6): &\quad -x_2 - x_4 + 1 &= 0
\end{aligned}$$

Groebner basis (4 variables, 3 linear, 0 nonlinear)

$$x_1 + x_4 - 6 = 0$$
$$x_2 + x_4 - 1 = 0$$
$$x_3 - x_4 + 3 = 0$$

$\mathfrak{m}_{5A}(3, 12)$

m5A312 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4\\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6\\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8\\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10}\\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12}\\
[e_2, e_5] &= e_8 & [e_2, e_6] &= 2e_9\\
[e_2, e_7] &= \alpha_{2,7}^{10} e_{10} & [e_2, e_8] &= \alpha_{2,8}^{11} e_{11}\\
[e_2, e_9] &= \alpha_{2,9}^{12} e_{12} & [e_3, e_4] &= -e_8\\
[e_3, e_5] &= -e_9 & [e_3, e_6] &= \alpha_{3,6}^{10} e_{10}\\
[e_3, e_7] &= \alpha_{3,7}^{11} e_{11} & [e_3, e_8] &= \alpha_{3,8}^{12} e_{12}\\
[e_4, e_5] &= \alpha_{4,5}^{10} e_{10} & [e_4, e_6] &= \alpha_{4,6}^{11} e_{11}\\
[e_4, e_7] &= \alpha_{4,7}^{12} e_{12} & [e_5, e_6] &= \alpha_{5,6}^{12} e_{12}
\end{aligned}$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_6): &\quad -\alpha_{2,7}^{10} - \alpha_{3,6}^{10} + 2 &&= 0 \\
(e_1, e_3, e_5): &\quad -\alpha_{3,6}^{10} - \alpha_{4,5}^{10} - 1 &&= 0 \\
(e_1, e_2, e_7): &\quad \alpha_{2,7}^{10} - \alpha_{2,8}^{11} - \alpha_{3,7}^{11} &&= 0 \\
(e_1, e_3, e_6): &\quad \alpha_{3,6}^{10} - \alpha_{3,7}^{11} - \alpha_{4,6}^{11} &&= 0 \\
(e_1, e_4, e_5): &\quad \alpha_{4,5}^{10} - \alpha_{4,6}^{11} &&= 0 \\
(e_2, e_3, e_4): &\quad -\alpha_{2,8}^{11} &&= 0 \\
(e_1, e_2, e_8): &\quad \alpha_{2,8}^{11} - \alpha_{2,9}^{12} - \alpha_{3,8}^{12} &&= 0 \\
(e_1, e_3, e_7): &\quad \alpha_{3,7}^{11} - \alpha_{3,8}^{12} - \alpha_{4,7}^{12} &&= 0 \\
(e_1, e_4, e_6): &\quad \alpha_{4,6}^{11} - \alpha_{4,7}^{12} - \alpha_{5,6}^{12} &&= 0 \\
(e_2, e_3, e_5): &\quad -\alpha_{2,9}^{12} - \alpha_{3,8}^{12} &&= 0
\end{aligned}
$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{3,7}^{11} \to x_1$$

$$\alpha_{2,8}^{11} \to x_2$$

$$\alpha_{4,7}^{12} \to x_3$$

$$\alpha_{3,8}^{12} \to x_4$$

$$\alpha_{2,7}^{10} \to x_5$$

$$\alpha_{4,6}^{11} \to x_6$$

$$\alpha_{4,5}^{10} \to x_7$$

$$\alpha_{5,6}^{12} \to x_8$$

$$\alpha_{2,9}^{12} \to x_9$$

$$\alpha_{3,6}^{10} \to x_{10}$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_6): \quad & -x_{10} - x_5 + 2 && = 0 \\
(e_1, e_3, e_5): \quad & -x_{10} - x_7 - 1 && = 0 \\
(e_1, e_2, e_7): \quad & -x_1 - x_2 + x_5 && = 0 \\
(e_1, e_3, e_6): \quad & -x_1 + x_{10} - x_6 && = 0 \\
(e_1, e_4, e_5): \quad & -x_6 + x_7 && = 0 \\
(e_2, e_3, e_4): \quad & -x_2 && = 0 \\
(e_1, e_2, e_8): \quad & x_2 - x_4 - x_9 && = 0 \\
(e_1, e_3, e_7): \quad & x_1 - x_3 - x_4 && = 0 \\
(e_1, e_4, e_6): \quad & -x_3 + x_6 - x_8 && = 0 \\
(e_2, e_3, e_5): \quad & -x_4 - x_9 && = 0
\end{aligned}
$$

Groebner basis (10 variables, 9 linear, 0 nonlinear)

$$3x_1 - 5 = 0$$

$$x_2 = 0$$

$$3x_3 - 3x_9 - 5 = 0$$

$$x_4 + x_9 = 0$$

$$3x_5 - 5 = 0$$

$$3x_6 + 4 = 0$$

$$3x_7 + 4 = 0$$

$$x_8 + x_9 + 3 = 0$$

$$3x_{10} - 1 = 0$$

$\mathfrak{m}_{7A}(3, 12)$

m7A312 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_2, e_3] &= e_6 & [e_2, e_4] &= e_7 \\
[e_2, e_5] &= \alpha_{2,5}^{8} e_8 & [e_2, e_6] &= \alpha_{2,6}^{9} e_9 \\
[e_2, e_7] &= \alpha_{2,7}^{10} e_{10} & [e_2, e_8] &= \alpha_{2,8}^{11} e_{11} \\
[e_2, e_9] &= \alpha_{2,9}^{12} e_{12} & [e_3, e_4] &= \alpha_{3,4}^{8} e_8 \\
[e_3, e_5] &= \alpha_{3,5}^{9} e_9 & [e_3, e_6] &= \alpha_{3,6}^{10} e_{10} \\
[e_3, e_7] &= \alpha_{3,7}^{11} e_{11} & [e_3, e_8] &= \alpha_{3,8}^{12} e_{12} \\
[e_4, e_5] &= \alpha_{4,5}^{10} e_{10} & [e_4, e_6] &= \alpha_{4,6}^{11} e_{11} \\
[e_4, e_7] &= \alpha_{4,7}^{12} e_{12} & [e_5, e_6] &= \alpha_{5,6}^{12} e_{12}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_4): &\quad -\alpha_{2,5}^{8} - \alpha_{3,4}^{8} + 1 &= 0 \\
(e_1, e_2, e_5): &\quad \alpha_{2,5}^{8} - \alpha_{2,6}^{9} - \alpha_{3,5}^{9} &= 0 \\
(e_1, e_3, e_4): &\quad \alpha_{3,4}^{8} - \alpha_{3,5}^{9} &= 0 \\
(e_1, e_2, e_6): &\quad \alpha_{2,6}^{9} - \alpha_{2,7}^{10} - \alpha_{3,6}^{10} &= 0 \\
(e_1, e_3, e_5): &\quad \alpha_{3,5}^{9} - \alpha_{3,6}^{10} - \alpha_{4,5}^{10} &= 0 \\
(e_1, e_2, e_7): &\quad \alpha_{2,7}^{10} - \alpha_{2,8}^{11} - \alpha_{3,7}^{11} &= 0 \\
(e_1, e_3, e_6): &\quad \alpha_{3,6}^{10} - \alpha_{3,7}^{11} - \alpha_{4,6}^{11} &= 0 \\
(e_1, e_4, e_5): &\quad \alpha_{4,5}^{10} - \alpha_{4,6}^{11} &= 0 \\
(e_2, e_3, e_4): &\quad \alpha_{2,8}^{11}\alpha_{3,4}^{8} - \alpha_{3,7}^{11} + \alpha_{4,6}^{11} &= 0 \\
(e_1, e_2, e_8): &\quad \alpha_{2,8}^{11} - \alpha_{2,9}^{12} - \alpha_{3,8}^{12} &= 0 \\
(e_1, e_3, e_7): &\quad \alpha_{3,7}^{11} - \alpha_{3,8}^{12} - \alpha_{4,7}^{12} &= 0 \\
(e_1, e_4, e_6): &\quad \alpha_{4,6}^{11} - \alpha_{4,7}^{12} - \alpha_{5,6}^{12} &= 0 \\
(e_2, e_3, e_5): &\quad -\alpha_{2,5}^{8}\alpha_{3,8}^{12} + \alpha_{2,9}^{12}\alpha_{3,5}^{9} + \alpha_{5,6}^{12} &= 0
\end{aligned}
$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{3,5}^{9} \to x_1$$

$$\alpha^{8}_{2,5} \to x_2$$

$$\alpha^{11}_{3,7} \to x_3$$

$$\alpha^{11}_{2,8} \to x_4$$

$$\alpha^{12}_{3,8} \to x_5$$

$$\alpha^{9}_{2,6} \to x_6$$

$$\alpha^{12}_{4,7} \to x_7$$

$$\alpha^{10}_{2,7} \to x_8$$

$$\alpha^{11}_{4,6} \to x_9$$

$$\alpha^{10}_{4,5} \to x_{10}$$

$$\alpha^{12}_{5,6} \to x_{11}$$

$$\alpha^{8}_{3,4} \to x_{12}$$

$$\alpha^{12}_{2,9} \to x_{13}$$

$$\alpha^{10}_{3,6} \to x_{14}$$

Jacobi Tests

$$\begin{aligned}
(e_1,e_2,e_4):&\quad -x_{12}-x_2+1 &&=0\\
(e_1,e_2,e_5):&\quad -x_1+x_2-x_6 &&=0\\
(e_1,e_3,e_4):&\quad -x_1+x_{12} &&=0\\
(e_1,e_2,e_6):&\quad -x_{14}+x_6-x_8 &&=0\\
(e_1,e_3,e_5):&\quad x_1-x_{10}-x_{14} &&=0\\
(e_1,e_2,e_7):&\quad -x_3-x_4+x_8 &&=0\\
(e_1,e_3,e_6):&\quad x_{14}-x_3-x_9 &&=0\\
(e_1,e_4,e_5):&\quad x_{10}-x_9 &&=0\\
(e_2,e_3,e_4):&\quad x_{12}x_4-x_3+x_9 &&=0\\
(e_1,e_2,e_8):&\quad -x_{13}+x_4-x_5 &&=0\\
(e_1,e_3,e_7):&\quad x_3-x_5-x_7 &&=0\\
(e_1,e_4,e_6):&\quad -x_{11}-x_7+x_9 &&=0\\
(e_2,e_3,e_5):&\quad x_1x_{13}+x_{11}-x_2x_5 &&=0
\end{aligned}$$

Groebner basis (14 variables, 11 linear, 1 nonlinear)

$$x_1 - x_{12} = 0$$

$$x_{12} + x_2 - 1 = 0$$

$$x_{12} - 2x_{14} + x_3 = 0$$
$$x_{12} + 3x_{14} + x_4 - 1 = 0$$
$$x_{12} + x_{13} + 3x_{14} + x_5 - 1 = 0$$
$$2x_{12} + x_6 - 1 = 0$$
$$-x_{13} - 5x_{14} + x_7 + 1 = 0$$
$$2x_{12} + x_{14} + x_8 - 1 = 0$$
$$-x_{12} + x_{14} + x_9 = 0$$
$$x_{10} - x_{12} + x_{14} = 0$$
$$x_{11} - x_{12} + x_{13} + 6x_{14} - 1 = 0$$
$$x_{12}^2 + 3x_{12}x_{14} - 3x_{12} + 3x_{14} = 0$$

$\mathfrak{m}_{2A}(4, 12)$

m2A412 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_2, e_7] &= e_{11} & [e_2, e_8] &= 3e_{12} \\
[e_3, e_6] &= -e_{11} & [e_3, e_7] &= -2e_{12} \\
[e_4, e_5] &= e_{11} & [e_4, e_6] &= e_{12}
\end{aligned}$$

No non-trivial Jacobi tests

$\mathfrak{m}_{4A}(4, 12)$

m4A412 (this line included for string searching purposes)

Original brackets:

$$
\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_1, e_9] = e_{10} \\
[e_1, e_{10}] = e_{11} & [e_1, e_{11}] = e_{12} \\
[e_2, e_5] = e_9 & [e_2, e_6] = 2e_{10} \\
[e_2, e_7] = \alpha_{2,7}^{11} e_{11} & [e_2, e_8] = \alpha_{2,8}^{12} e_{12} \\
[e_3, e_4] = -e_9 & [e_3, e_5] = -e_{10} \\
[e_3, e_6] = \alpha_{3,6}^{11} e_{11} & [e_3, e_7] = \alpha_{3,7}^{12} e_{12} \\
[e_4, e_5] = \alpha_{4,5}^{11} e_{11} & [e_4, e_6] = \alpha_{4,6}^{12} e_{12}
\end{array}
$$

Non-trivial Jacobi Tests:

$$
\begin{array}{lll}
(e_1, e_2, e_6): & -\alpha_{2,7}^{11} - \alpha_{3,6}^{11} + 2 & = 0 \\
(e_1, e_3, e_5): & -\alpha_{3,6}^{11} - \alpha_{4,5}^{11} - 1 & = 0 \\
(e_1, e_2, e_7): & \alpha_{2,7}^{11} - \alpha_{2,8}^{12} - \alpha_{3,7}^{12} & = 0 \\
(e_1, e_3, e_6): & \alpha_{3,6}^{11} - \alpha_{3,7}^{12} - \alpha_{4,6}^{12} & = 0 \\
(e_1, e_4, e_5): & \alpha_{4,5}^{11} - \alpha_{4,6}^{12} & = 0
\end{array}
$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$
\begin{array}{c}
\alpha_{2,7}^{11} \to x_1 \\
\alpha_{2,8}^{12} \to x_2 \\
\alpha_{4,6}^{12} \to x_3 \\
\alpha_{4,5}^{11} \to x_4 \\
\alpha_{3,7}^{12} \to x_5 \\
\alpha_{3,6}^{11} \to x_6
\end{array}
$$

Jacobi Tests

$$
\begin{aligned}
(e_1,e_2,e_6):&\quad -x_1-x_6+2 &&=0\\
(e_1,e_3,e_5):&\quad -x_4-x_6-1 &&=0\\
(e_1,e_2,e_7):&\quad x_1-x_2-x_5 &&=0\\
(e_1,e_3,e_6):&\quad -x_3-x_5+x_6 &&=0\\
(e_1,e_4,e_5):&\quad -x_3+x_4 &&=0
\end{aligned}
$$

Groebner basis (6 variables, 5 linear, 0 nonlinear)

$$x_1+x_6-2=0$$
$$x_2+3x_6-1=0$$
$$x_3+x_6+1=0$$
$$x_4+x_6+1=0$$
$$x_5-2x_6-1=0$$

$\mathfrak{m}_{6A}(4,12)$

m6A412 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1,e_2]&=e_3 & [e_1,e_3]&=e_4\\
[e_1,e_4]&=e_5 & [e_1,e_5]&=e_6\\
[e_1,e_6]&=e_7 & [e_1,e_7]&=e_8\\
[e_1,e_8]&=e_9 & [e_1,e_9]&=e_{10}\\
[e_1,e_{10}]&=e_{11} & [e_1,e_{11}]&=e_{12}\\
[e_2,e_3]&=e_7 & [e_2,e_4]&=e_8\\
[e_2,e_5]&=\alpha_{2,5}^{9}e_9 & [e_2,e_6]&=\alpha_{2,6}^{10}e_{10}\\
[e_2,e_7]&=\alpha_{2,7}^{11}e_{11} & [e_2,e_8]&=\alpha_{2,8}^{12}e_{12}\\
[e_3,e_4]&=\alpha_{3,4}^{9}e_9 & [e_3,e_5]&=\alpha_{3,5}^{10}e_{10}\\
[e_3,e_6]&=\alpha_{3,6}^{11}e_{11} & [e_3,e_7]&=\alpha_{3,7}^{12}e_{12}\\
[e_4,e_5]&=\alpha_{4,5}^{11}e_{11} & [e_4,e_6]&=\alpha_{4,6}^{12}e_{12}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1, e_2, e_4):& \quad -\alpha_{2,5}^{9} - \alpha_{3,4}^{9} + 1 &= 0\\
(e_1, e_2, e_5):& \quad \alpha_{2,5}^{9} - \alpha_{2,6}^{10} - \alpha_{3,5}^{10} &= 0\\
(e_1, e_3, e_4):& \quad \alpha_{3,4}^{9} - \alpha_{3,5}^{10} &= 0\\
(e_1, e_2, e_6):& \quad \alpha_{2,6}^{10} - \alpha_{2,7}^{11} - \alpha_{3,6}^{11} &= 0\\
(e_1, e_3, e_5):& \quad \alpha_{3,5}^{10} - \alpha_{3,6}^{11} - \alpha_{4,5}^{11} &= 0\\
(e_1, e_2, e_7):& \quad \alpha_{2,7}^{11} - \alpha_{2,8}^{12} - \alpha_{3,7}^{12} &= 0\\
(e_1, e_3, e_6):& \quad \alpha_{3,6}^{11} - \alpha_{3,7}^{12} - \alpha_{4,6}^{12} &= 0\\
(e_1, e_4, e_5):& \quad \alpha_{4,5}^{11} - \alpha_{4,6}^{12} &= 0
\end{aligned}$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\begin{gathered}
\alpha_{2,7}^{11} \to x_1\\
\alpha_{2,5}^{9} \to x_2\\
\alpha_{3,5}^{10} \to x_3\\
\alpha_{2,8}^{12} \to x_4\\
\alpha_{4,6}^{12} \to x_5\\
\alpha_{4,5}^{11} \to x_6\\
\alpha_{3,7}^{12} \to x_7\\
\alpha_{3,4}^{9} \to x_8\\
\alpha_{2,6}^{10} \to x_9\\
\alpha_{3,6}^{11} \to x_{10}
\end{gathered}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4):& \quad -x_2 - x_8 + 1 &= 0\\
(e_1, e_2, e_5):& \quad x_2 - x_3 - x_9 &= 0\\
(e_1, e_3, e_4):& \quad -x_3 + x_8 &= 0\\
(e_1, e_2, e_6):& \quad -x_1 - x_{10} + x_9 &= 0\\
(e_1, e_3, e_5):& \quad -x_{10} + x_3 - x_6 &= 0\\
(e_1, e_2, e_7):& \quad x_1 - x_4 - x_7 &= 0\\
(e_1, e_3, e_6):& \quad x_{10} - x_5 - x_7 &= 0\\
(e_1, e_4, e_5):& \quad -x_5 + x_6 &= 0
\end{aligned}$$

Groebner basis (10 variables, 8 linear, 0 nonlinear)

$$x_1 + x_{10} - x_9 = 0$$
$$2x_2 - x_9 - 1 = 0$$
$$2x_3 + x_9 - 1 = 0$$
$$6x_{10} + 2x_4 - x_9 - 1 = 0$$
$$2x_{10} + 2x_5 + x_9 - 1 = 0$$
$$2x_{10} + 2x_6 + x_9 - 1 = 0$$
$$-4x_{10} + 2x_7 - x_9 + 1 = 0$$
$$2x_8 + x_9 - 1 = 0$$

$\mathfrak{m}_{1A}(5, 12)$

m1A512 (this line included for string searching purposes)

Original brackets:

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_1, e_{11}] = e_{12}$$
$$[e_2, e_7] = e_{12} \qquad [e_3, e_6] = -e_{12}$$
$$[e_4, e_5] = e_{12}$$

No non-trivial Jacobi tests

$\mathfrak{m}_{3A}(5, 12)$

m3A512 (this line included for string searching purposes)

Original brackets:

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_1, e_{11}] = e_{12}$$
$$[e_2, e_5] = e_{10} \qquad [e_2, e_6] = 2e_{11}$$
$$[e_2, e_7] = \alpha_{2,7}^{12} e_{12} \qquad [e_3, e_4] = -e_{10}$$
$$[e_3, e_5] = -e_{11} \qquad [e_3, e_6] = \alpha_{3,6}^{12} e_{12}$$
$$[e_4, e_5] = \alpha_{4,5}^{12} e_{12}$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_6):& \quad -\alpha_{2,7}^{12} - \alpha_{3,6}^{12} + 2 &= 0 \\
(e_1, e_3, e_5):& \quad -\alpha_{3,6}^{12} - \alpha_{4,5}^{12} - 1 &= 0
\end{aligned}
$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{3,6}^{12} \to x_1$$
$$\alpha_{2,7}^{12} \to x_2$$
$$\alpha_{4,5}^{12} \to x_3$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_6):& \quad -x_1 - x_2 + 2 &= 0 \\
(e_1, e_3, e_5):& \quad -x_1 - x_3 - 1 &= 0
\end{aligned}
$$

Groebner basis (3 variables, 2 linear, 0 nonlinear)

$$x_1 + x_3 + 1 = 0$$
$$x_2 - x_3 - 3 = 0$$

$\mathfrak{m}_{5A}(5, 12)$

m5A512 (this line included for string searching purposes)
Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_2, e_3] &= e_8 & [e_2, e_4] &= e_9 \\
[e_2, e_5] &= \alpha_{2,5}^{10} e_{10} & [e_2, e_6] &= \alpha_{2,6}^{11} e_{11} \\
[e_2, e_7] &= \alpha_{2,7}^{12} e_{12} & [e_3, e_4] &= \alpha_{3,4}^{10} e_{10} \\
[e_3, e_5] &= \alpha_{3,5}^{11} e_{11} & [e_3, e_6] &= \alpha_{3,6}^{12} e_{12} \\
[e_4, e_5] &= \alpha_{4,5}^{12} e_{12}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_4): &\quad -\alpha_{2,5}^{10} - \alpha_{3,4}^{10} + 1 &&= 0\\
(e_1, e_2, e_5): &\quad \alpha_{2,5}^{10} - \alpha_{2,6}^{11} - \alpha_{3,5}^{11} &&= 0\\
(e_1, e_3, e_4): &\quad \alpha_{3,4}^{10} - \alpha_{3,5}^{11} &&= 0\\
(e_1, e_2, e_6): &\quad \alpha_{2,6}^{11} - \alpha_{2,7}^{12} - \alpha_{3,6}^{12} &&= 0\\
(e_1, e_3, e_5): &\quad \alpha_{3,5}^{11} - \alpha_{3,6}^{12} - \alpha_{4,5}^{12} &&= 0
\end{aligned}
$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{2,6}^{11} \to x_1$$

$$\alpha_{2,7}^{12} \to x_2$$

$$\alpha_{3,6}^{12} \to x_3$$

$$\alpha_{3,5}^{11} \to x_4$$

$$\alpha_{4,5}^{12} \to x_5$$

$$\alpha_{3,4}^{10} \to x_6$$

$$\alpha_{2,5}^{10} \to x_7$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_4): &\quad -x_6 - x_7 + 1 &&= 0\\
(e_1, e_2, e_5): &\quad -x_1 - x_4 + x_7 &&= 0\\
(e_1, e_3, e_4): &\quad -x_4 + x_6 &&= 0\\
(e_1, e_2, e_6): &\quad x_1 - x_2 - x_3 &&= 0\\
(e_1, e_3, e_5): &\quad -x_3 + x_4 - x_5 &&= 0
\end{aligned}
$$

Groebner basis (7 variables, 5 linear, 0 nonlinear)

$$x_1 - 2x_7 + 1 = 0$$

$$x_2 - x_5 - 3x_7 + 2 = 0$$

$$x_3 + x_5 + x_7 - 1 = 0$$

$$x_4 + x_7 - 1 = 0$$

$$x_6 + x_7 - 1 = 0$$

$\mathfrak{m}_{2A}(6, 12)$

m2A612 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_2, e_5] &= e_{11} & [e_2, e_6] &= 2e_{12} \\
[e_3, e_4] &= -e_{11} & [e_3, e_5] &= -e_{12}
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{4A}(6, 12)$

m4A612 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_2, e_3] &= e_9 & [e_2, e_4] &= e_{10} \\
[e_2, e_5] &= \alpha_{2,5}^{11} e_{11} & [e_2, e_6] &= \alpha_{2,6}^{12} e_{12} \\
[e_3, e_4] &= \alpha_{3,4}^{11} e_{11} & [e_3, e_5] &= \alpha_{3,5}^{12} e_{12}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_4) :& \quad -\alpha_{2,5}^{11} - \alpha_{3,4}^{11} + 1 &= 0 \\
(e_1, e_2, e_5) :& \quad \alpha_{2,5}^{11} - \alpha_{2,6}^{12} - \alpha_{3,5}^{12} &= 0 \\
(e_1, e_3, e_4) :& \quad \alpha_{3,4}^{11} - \alpha_{3,5}^{12} &= 0
\end{aligned}
$$

Infinite number of solutions.

How the solution(s) were or were not found:

Change variables

$$\alpha_{2,6}^{12} \to x_1$$

$$\alpha^{12}_{3,5} \to x_2$$

$$\alpha^{11}_{2,5} \to x_3$$

$$\alpha^{11}_{3,4} \to x_4$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4): &\quad -x_3 - x_4 + 1 &= 0\\
(e_1, e_2, e_5): &\quad -x_1 - x_2 + x_3 &= 0\\
(e_1, e_3, e_4): &\quad -x_2 + x_4 &= 0
\end{aligned}$$

Groebner basis (4 variables, 3 linear, 0 nonlinear)

$$x_1 + 2x_4 - 1 = 0$$

$$x_2 - x_4 = 0$$

$$x_3 + x_4 - 1 = 0$$

$\mathfrak{m}_{1A}(7, 12)$

m1A712 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4\\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6\\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8\\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10}\\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12}\\
[e_2, e_5] &= e_{12} & [e_3, e_4] &= -e_{12}
\end{aligned}$$

No non-trivial Jacobi tests

$\mathfrak{m}_{3A}(7, 12)$

m3A712 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_2, e_3] &= e_{10} & [e_2, e_4] &= e_{11} \\
[e_2, e_5] &= \alpha_{2,5}^{12} e_{12} & [e_3, e_4] &= \alpha_{3,4}^{12} e_{12}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$(e_1, e_2, e_4): \quad -\alpha_{2,5}^{12} - \alpha_{3,4}^{12} + 1 = 0$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$
\begin{aligned}
\alpha_{3,4}^{12} &\to x_1 \\
\alpha_{2,5}^{12} &\to x_2
\end{aligned}
$$

Jacobi Tests

$$(e_1, e_2, e_4): \quad -x_1 - x_2 + 1 = 0$$

Groebner basis (2 variables, 1 linear, 0 nonlinear)

$$x_1 + x_2 - 1 = 0$$

$\mathfrak{m}_{2A}(8, 12)$

m2A812 (this line included for string searching purposes)
Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_2, e_3] &= e_{11} & [e_2, e_4] &= e_{12}
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{1A}(9, 12)$

m1A912 (this line included for string searching purposes)
Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_2, e_3] &= e_{12}
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{1A}(2, 13)$

m1A213 (this line included for string searching purposes)
Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_2, e_{11}] &= e_{13} \\
[e_3, e_{10}] &= -e_{13} & [e_4, e_9] &= e_{13} \\
[e_5, e_8] &= -e_{13} & [e_6, e_7] &= e_{13}
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{3A}(2, 13)$

m3A213 (this line included for string searching purposes)

Solution 1

$$
\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_1, e_9] = e_{10} \\
[e_1, e_{10}] = e_{11} & [e_1, e_{11}] = e_{12} \\
[e_1, e_{12}] = e_{13} & [e_2, e_9] = e_{11} \\
[e_2, e_{10}] = 4e_{12} & [e_2, e_{11}] = 0 \\
[e_3, e_8] = -e_{11} & [e_3, e_9] = -3e_{12} \\
[e_3, e_{10}] = 4e_{13} & [e_4, e_7] = e_{11} \\
[e_4, e_8] = 2e_{12} & [e_4, e_9] = -7e_{13} \\
[e_5, e_6] = -e_{11} & [e_5, e_7] = -e_{12} \\
[e_5, e_8] = 9e_{13} & [e_6, e_7] = -10e_{13}
\end{array}
$$

Original brackets:

$$
\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_1, e_9] = e_{10} \\
[e_1, e_{10}] = e_{11} & [e_1, e_{11}] = e_{12} \\
[e_1, e_{12}] = e_{13} & [e_2, e_9] = e_{11} \\
[e_2, e_{10}] = 4e_{12} & [e_2, e_{11}] = \alpha_{2,11}^{13} e_{13} \\
[e_3, e_8] = -e_{11} & [e_3, e_9] = -3e_{12} \\
[e_3, e_{10}] = \alpha_{3,10}^{13} e_{13} & [e_4, e_7] = e_{11} \\
[e_4, e_8] = 2e_{12} & [e_4, e_9] = \alpha_{4,9}^{13} e_{13} \\
[e_5, e_6] = -e_{11} & [e_5, e_7] = -e_{12} \\
[e_5, e_8] = \alpha_{5,8}^{13} e_{13} & [e_6, e_7] = \alpha_{6,7}^{13} e_{13}
\end{array}
$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1, e_2, e_{10}):& \quad -\alpha^{13}_{2,11} - \alpha^{13}_{3,10} + 4 &&= 0\\
(e_1, e_3, e_9):& \quad -\alpha^{13}_{3,10} - \alpha^{13}_{4,9} - 3 &&= 0\\
(e_1, e_4, e_8):& \quad -\alpha^{13}_{4,9} - \alpha^{13}_{5,8} + 2 &&= 0\\
(e_1, e_5, e_7):& \quad -\alpha^{13}_{5,8} - \alpha^{13}_{6,7} - 1 &&= 0\\
(e_2, e_3, e_8):& \quad -\alpha^{13}_{2,11} &&= 0\\
(e_2, e_4, e_7):& \quad \alpha^{13}_{2,11} &&= 0\\
(e_2, e_5, e_6):& \quad -\alpha^{13}_{2,11} &&= 0
\end{aligned}$$

Solution 1:

$$\begin{aligned}
\alpha^{13}_{6,7} &= -10\\
\alpha^{13}_{4,9} &= -7\\
\alpha^{13}_{3,10} &= 4\\
\alpha^{13}_{5,8} &= 9\\
\alpha^{13}_{2,11} &= 0
\end{aligned}$$

How the solution(s) were or were not found:
Change variables

$$\begin{aligned}
\alpha^{13}_{6,7} &\to x_1\\
\alpha^{13}_{4,9} &\to x_2\\
\alpha^{13}_{3,10} &\to x_3\\
\alpha^{13}_{5,8} &\to x_4\\
\alpha^{13}_{2,11} &\to x_5
\end{aligned}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_{10}):& \quad -x_3 - x_5 + 4 &&= 0\\
(e_1, e_3, e_9):& \quad -x_2 - x_3 - 3 &&= 0\\
(e_1, e_4, e_8):& \quad -x_2 - x_4 + 2 &&= 0\\
(e_1, e_5, e_7):& \quad -x_1 - x_4 - 1 &&= 0\\
(e_2, e_3, e_8):& \quad -x_5 &&= 0\\
(e_2, e_4, e_7):& \quad x_5 &&= 0\\
(e_2, e_5, e_6):& \quad -x_5 &&= 0
\end{aligned}$$

Groebner basis (5 variables, 5 linear, 0 nonlinear)

$$x_1 + 10 = 0$$
$$x_2 + 7 = 0$$
$$x_3 - 4 = 0$$
$$x_4 - 9 = 0$$
$$x_5 = 0$$

Solution 1:

$$x_1 = -10$$
$$x_2 = -7$$
$$x_3 = 4$$
$$x_4 = 9$$
$$x_5 = 0$$

$\mathfrak{m}_{9A}(2, 13)$

m9A213 (this line included for string searching purposes)

Solution 1

$[e_1, e_2] = e_3$	$[e_1, e_3] = e_4$
$[e_1, e_4] = e_5$	$[e_1, e_5] = e_6$
$[e_1, e_6] = e_7$	$[e_1, e_7] = e_8$
$[e_1, e_8] = e_9$	$[e_1, e_9] = e_{10}$
$[e_1, e_{10}] = e_{11}$	$[e_1, e_{11}] = e_{12}$
$[e_1, e_{12}] = e_{13}$	$[e_2, e_3] = e_5$
$[e_2, e_4] = e_6$	$[e_2, e_5] = e_7$
$[e_2, e_6] = e_8$	$[e_2, e_7] = e_9$
$[e_2, e_8] = e_{10}$	$[e_2, e_9] = e_{11}$
$[e_2, e_{10}] = e_{12}$	$[e_2, e_{11}] = e_{13}$
$[e_3, e_4] = 0$	$[e_3, e_5] = 0$
$[e_3, e_6] = 0$	$[e_3, e_7] = 0$
$[e_3, e_8] = 0$	$[e_3, e_9] = 0$
$[e_3, e_{10}] = 0$	$[e_4, e_5] = 0$
$[e_4, e_6] = 0$	$[e_4, e_7] = 0$
$[e_4, e_8] = 0$	$[e_4, e_9] = 0$
$[e_5, e_6] = 0$	$[e_5, e_7] = 0$
$[e_5, e_8] = 0$	$[e_6, e_7] = 0$

Solution 2

$$[e_1,e_2]=e_3 \qquad [e_1,e_3]=e_4$$
$$[e_1,e_4]=e_5 \qquad [e_1,e_5]=e_6$$
$$[e_1,e_6]=e_7 \qquad [e_1,e_7]=e_8$$
$$[e_1,e_8]=e_9 \qquad [e_1,e_9]=e_{10}$$
$$[e_1,e_{10}]=e_{11} \qquad [e_1,e_{11}]=e_{12}$$
$$[e_1,e_{12}]=e_{13} \qquad [e_2,e_3]=e_5$$
$$[e_2,e_4]=e_6 \qquad [e_2,e_5]=\frac{9e_7}{10}$$
$$[e_2,e_6]=\frac{4e_8}{5} \qquad [e_2,e_7]=\frac{5e_9}{7}$$
$$[e_2,e_8]=\frac{9e_{10}}{14} \qquad [e_2,e_9]=\frac{7e_{11}}{12}$$
$$[e_2,e_{10}]=\frac{8e_{12}}{15} \qquad [e_2,e_{11}]=\frac{27e_{13}}{55}$$
$$[e_3,e_4]=\frac{e_7}{10} \qquad [e_3,e_5]=\frac{e_8}{10}$$
$$[e_3,e_6]=\frac{3e_9}{35} \qquad [e_3,e_7]=\frac{e_{10}}{14}$$
$$[e_3,e_8]=\frac{5e_{11}}{84} \qquad [e_3,e_9]=\frac{e_{12}}{20}$$
$$[e_3,e_{10}]=\frac{7e_{13}}{165} \qquad [e_4,e_5]=\frac{e_9}{70}$$
$$[e_4,e_6]=\frac{e_{10}}{70} \qquad [e_4,e_7]=\frac{e_{11}}{84}$$
$$[e_4,e_8]=\frac{e_{12}}{105} \qquad [e_4,e_9]=\frac{e_{13}}{132}$$
$$[e_5,e_6]=\frac{e_{11}}{420} \qquad [e_5,e_7]=\frac{e_{12}}{420}$$
$$[e_5,e_8]=\frac{3e_{13}}{1540} \qquad [e_6,e_7]=\frac{e_{13}}{2310}$$

Original brackets:

$$
\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_1, e_9] = e_{10} \\
[e_1, e_{10}] = e_{11} & [e_1, e_{11}] = e_{12} \\
[e_1, e_{12}] = e_{13} & [e_2, e_3] = e_5 \\
[e_2, e_4] = e_6 & [e_2, e_5] = \alpha_{2,5}^{7} e_7 \\
[e_2, e_6] = \alpha_{2,6}^{8} e_8 & [e_2, e_7] = \alpha_{2,7}^{9} e_9 \\
[e_2, e_8] = \alpha_{2,8}^{10} e_{10} & [e_2, e_9] = \alpha_{2,9}^{11} e_{11} \\
[e_2, e_{10}] = \alpha_{2,10}^{12} e_{12} & [e_2, e_{11}] = \alpha_{2,11}^{13} e_{13} \\
[e_3, e_4] = \alpha_{3,4}^{7} e_7 & [e_3, e_5] = \alpha_{3,5}^{8} e_8 \\
[e_3, e_6] = \alpha_{3,6}^{9} e_9 & [e_3, e_7] = \alpha_{3,7}^{10} e_{10} \\
[e_3, e_8] = \alpha_{3,8}^{11} e_{11} & [e_3, e_9] = \alpha_{3,9}^{12} e_{12} \\
[e_3, e_{10}] = \alpha_{3,10}^{13} e_{13} & [e_4, e_5] = \alpha_{4,5}^{9} e_9 \\
[e_4, e_6] = \alpha_{4,6}^{10} e_{10} & [e_4, e_7] = \alpha_{4,7}^{11} e_{11} \\
[e_4, e_8] = \alpha_{4,8}^{12} e_{12} & [e_4, e_9] = \alpha_{4,9}^{13} e_{13} \\
[e_5, e_6] = \alpha_{5,6}^{11} e_{11} & [e_5, e_7] = \alpha_{5,7}^{12} e_{12} \\
[e_5, e_8] = \alpha_{5,8}^{13} e_{13} & [e_6, e_7] = \alpha_{6,7}^{13} e_{13}
\end{array}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1,e_2,e_4):&\quad -\alpha^7_{2,5}-\alpha^7_{3,4}+1 &&=0\\
(e_1,e_2,e_5):&\quad \alpha^7_{2,5}-\alpha^8_{2,6}-\alpha^8_{3,5} &&=0\\
(e_1,e_3,e_4):&\quad \alpha^7_{3,4}-\alpha^8_{3,5} &&=0\\
(e_1,e_2,e_6):&\quad \alpha^8_{2,6}-\alpha^9_{2,7}-\alpha^9_{3,6} &&=0\\
(e_1,e_3,e_5):&\quad \alpha^8_{3,5}-\alpha^9_{3,6}-\alpha^9_{4,5} &&=0\\
(e_2,e_3,e_4):&\quad \alpha^9_{2,7}\alpha^7_{3,4}-\alpha^9_{3,6}+\alpha^9_{4,5} &&=0\\
(e_1,e_2,e_7):&\quad \alpha^9_{2,7}-\alpha^{10}_{2,8}-\alpha^{10}_{3,7} &&=0\\
(e_1,e_3,e_6):&\quad \alpha^9_{3,6}-\alpha^{10}_{3,7}-\alpha^{10}_{4,6} &&=0\\
(e_1,e_4,e_5):&\quad \alpha^9_{4,5}-\alpha^{10}_{4,6} &&=0\\
(e_2,e_3,e_5):&\quad -\alpha^7_{2,5}\alpha^{10}_{3,7}+\alpha^{10}_{2,8}\alpha^8_{3,5} &&=0\\
(e_1,e_2,e_8):&\quad \alpha^{10}_{2,8}-\alpha^{11}_{2,9}-\alpha^{11}_{3,8} &&=0\\
(e_1,e_3,e_7):&\quad \alpha^{10}_{3,7}-\alpha^{11}_{3,8}-\alpha^{11}_{4,7} &&=0\\
(e_1,e_4,e_6):&\quad \alpha^{10}_{4,6}-\alpha^{11}_{4,7}-\alpha^{11}_{5,6} &&=0\\
(e_2,e_3,e_6):&\quad -\alpha^8_{2,6}\alpha^{11}_{3,8}+\alpha^{11}_{2,9}\alpha^9_{3,6}-\alpha^{11}_{5,6} &&=0\\
(e_2,e_4,e_5):&\quad -\alpha^7_{2,5}\alpha^{11}_{4,7}+\alpha^{11}_{2,9}\alpha^9_{4,5}+\alpha^{11}_{5,6} &&=0\\
(e_1,e_2,e_9):&\quad -\alpha^{12}_{2,10}+\alpha^{11}_{2,9}-\alpha^{12}_{3,9} &&=0\\
(e_1,e_3,e_8):&\quad \alpha^{11}_{3,8}-\alpha^{12}_{3,9}-\alpha^{12}_{4,8} &&=0\\
(e_1,e_4,e_7):&\quad \alpha^{11}_{4,7}-\alpha^{12}_{4,8}-\alpha^{12}_{5,7} &&=0\\
(e_1,e_5,e_6):&\quad \alpha^{11}_{5,6}-\alpha^{12}_{5,7} &&=0\\
(e_2,e_3,e_7):&\quad \alpha^{12}_{2,10}\alpha^{10}_{3,7}-\alpha^9_{2,7}\alpha^{12}_{3,9}-\alpha^{12}_{5,7} &&=0\\
(e_2,e_4,e_6):&\quad \alpha^{12}_{2,10}\alpha^{10}_{4,6}-\alpha^8_{2,6}\alpha^{12}_{4,8} &&=0\\
(e_3,e_4,e_5):&\quad \alpha^7_{3,4}\alpha^{12}_{5,7}-\alpha^8_{3,5}\alpha^{12}_{4,8}+\alpha^{12}_{3,9}\alpha^9_{4,5} &&=0\\
(e_1,e_2,e_{10}):&\quad \alpha^{12}_{2,10}-\alpha^{13}_{2,11}-\alpha^{13}_{3,10} &&=0\\
(e_1,e_3,e_9):&\quad -\alpha^{13}_{3,10}+\alpha^{12}_{3,9}-\alpha^{13}_{4,9} &&=0\\
(e_1,e_4,e_8):&\quad \alpha^{12}_{4,8}-\alpha^{13}_{4,9}-\alpha^{13}_{5,8} &&=0\\
(e_1,e_5,e_7):&\quad \alpha^{12}_{5,7}-\alpha^{13}_{5,8}-\alpha^{13}_{6,7} &&=0\\
(e_2,e_3,e_8):&\quad \alpha^{13}_{2,11}\alpha^{11}_{3,8}-\alpha^{10}_{2,8}\alpha^{13}_{3,10}-\alpha^{13}_{5,8} &&=0\\
(e_2,e_4,e_7):&\quad \alpha^{13}_{2,11}\alpha^{11}_{4,7}-\alpha^9_{2,7}\alpha^{13}_{4,9}-\alpha^{13}_{6,7} &&=0\\
(e_2,e_5,e_6):&\quad \alpha^{13}_{2,11}\alpha^{11}_{5,6}+\alpha^7_{2,5}\alpha^{13}_{6,7}-\alpha^8_{2,6}\alpha^{13}_{5,8} &&=0\\
(e_3,e_4,e_6):&\quad \alpha^{13}_{3,10}\alpha^{10}_{4,6}+\alpha^7_{3,4}\alpha^{13}_{6,7}-\alpha^9_{3,6}\alpha^{13}_{4,9} &&=0
\end{aligned}
$$

Solution 1:

$$\alpha^{13}_{6,7} = 0$$
$$\alpha^{9}_{3,6} = 0$$
$$\alpha^{7}_{2,5} = 1$$
$$\alpha^{13}_{5,8} = 0$$
$$\alpha^{11}_{4,7} = 0$$
$$\alpha^{10}_{4,6} = 0$$
$$\alpha^{12}_{3,9} = 0$$
$$\alpha^{11}_{3,8} = 0$$
$$\alpha^{9}_{2,7} = 1$$
$$\alpha^{12}_{2,10} = 1$$
$$\alpha^{12}_{5,7} = 0$$
$$\alpha^{11}_{2,9} = 1$$
$$\alpha^{10}_{2,8} = 1$$
$$\alpha^{10}_{3,7} = 0$$
$$\alpha^{13}_{3,10} = 0$$
$$\alpha^{13}_{4,9} = 0$$
$$\alpha^{11}_{5,6} = 0$$
$$\alpha^{8}_{2,6} = 1$$
$$\alpha^{9}_{4,5} = 0$$
$$\alpha^{7}_{3,4} = 0$$
$$\alpha^{12}_{4,8} = 0$$
$$\alpha^{13}_{2,11} = 1$$
$$\alpha^{8}_{3,5} = 0$$

Solution 2:

$$\alpha^{13}_{6,7} = 1/2310$$
$$\alpha^{9}_{3,6} = 3/35$$
$$\alpha^{7}_{2,5} = 9/10$$
$$\alpha^{13}_{5,8} = 3/1540$$
$$\alpha^{11}_{4,7} = 1/84$$
$$\alpha^{10}_{4,6} = 1/70$$
$$\alpha^{12}_{3,9} = 1/20$$
$$\alpha^{11}_{3,8} = 5/84$$
$$\alpha^{9}_{2,7} = 5/7$$
$$\alpha^{12}_{2,10} = 8/15$$
$$\alpha^{12}_{5,7} = 1/420$$
$$\alpha^{11}_{2,9} = 7/12$$
$$\alpha^{10}_{2,8} = 9/14$$
$$\alpha^{10}_{3,7} = 1/14$$
$$\alpha^{13}_{3,10} = 7/165$$
$$\alpha^{13}_{4,9} = 1/132$$
$$\alpha^{11}_{5,6} = 1/420$$
$$\alpha^{8}_{2,6} = 4/5$$
$$\alpha^{9}_{4,5} = 1/70$$
$$\alpha^{7}_{3,4} = 1/10$$
$$\alpha^{12}_{4,8} = 1/105$$
$$\alpha^{13}_{2,11} = 27/55$$
$$\alpha^{8}_{3,5} = 1/10$$

How the solution(s) were or were not found:
Change variables

$$\alpha^{13}_{6,7} \to x_1$$
$$\alpha^{9}_{3,6} \to x_2$$
$$\alpha^{7}_{2,5} \to x_3$$
$$\alpha^{13}_{5,8} \to x_4$$
$$\alpha^{11}_{4,7} \to x_5$$

$$\alpha_{4,6}^{10} \to x_6$$

$$\alpha_{3,9}^{12} \to x_7$$

$$\alpha_{3,8}^{11} \to x_8$$

$$\alpha_{2,7}^{9} \to x_9$$

$$\alpha_{2,10}^{12} \to x_{10}$$

$$\alpha_{5,7}^{12} \to x_{11}$$

$$\alpha_{2,9}^{11} \to x_{12}$$

$$\alpha_{2,8}^{10} \to x_{13}$$

$$\alpha_{3,7}^{10} \to x_{14}$$

$$\alpha_{3,10}^{13} \to x_{15}$$

$$\alpha_{4,9}^{13} \to x_{16}$$

$$\alpha_{5,6}^{11} \to x_{17}$$

$$\alpha_{2,6}^{8} \to x_{18}$$

$$\alpha_{4,5}^{9} \to x_{19}$$

$$\alpha_{3,4}^{7} \to x_{20}$$

$$\alpha_{4,8}^{12} \to x_{21}$$

$$\alpha_{2,11}^{13} \to x_{22}$$

$$\alpha_{3,5}^{8} \to x_{23}$$

Jacobi Tests

$$
\begin{aligned}
(e_1,e_2,e_4):&\quad -x_{20}-x_3+1 &&=0\\
(e_1,e_2,e_5):&\quad -x_{18}-x_{23}+x_3 &&=0\\
(e_1,e_3,e_4):&\quad x_{20}-x_{23} &&=0\\
(e_1,e_2,e_6):&\quad x_{18}-x_2-x_9 &&=0\\
(e_1,e_3,e_5):&\quad -x_{19}-x_2+x_{23} &&=0\\
(e_2,e_3,e_4):&\quad x_{19}-x_2+x_{20}x_9 &&=0\\
(e_1,e_2,e_7):&\quad -x_{13}-x_{14}+x_9 &&=0\\
(e_1,e_3,e_6):&\quad -x_{14}+x_2-x_6 &&=0\\
(e_1,e_4,e_5):&\quad x_{19}-x_6 &&=0\\
(e_2,e_3,e_5):&\quad x_{13}x_{23}-x_{14}x_3 &&=0\\
(e_1,e_2,e_8):&\quad -x_{12}+x_{13}-x_8 &&=0\\
(e_1,e_3,e_7):&\quad x_{14}-x_5-x_8 &&=0\\
(e_1,e_4,e_6):&\quad -x_{17}-x_5+x_6 &&=0\\
(e_2,e_3,e_6):&\quad x_{12}x_2-x_{17}-x_{18}x_8 &&=0\\
(e_2,e_4,e_5):&\quad x_{12}x_{19}+x_{17}-x_3x_5 &&=0\\
(e_1,e_2,e_9):&\quad -x_{10}+x_{12}-x_7 &&=0\\
(e_1,e_3,e_8):&\quad -x_{21}-x_7+x_8 &&=0\\
(e_1,e_4,e_7):&\quad -x_{11}-x_{21}+x_5 &&=0\\
(e_1,e_5,e_6):&\quad -x_{11}+x_{17} &&=0\\
(e_2,e_3,e_7):&\quad x_{10}x_{14}-x_{11}-x_7x_9 &&=0\\
(e_2,e_4,e_6):&\quad x_{10}x_6-x_{18}x_{21} &&=0\\
(e_3,e_4,e_5):&\quad x_{11}x_{20}+x_{19}x_7-x_{21}x_{23} &&=0\\
(e_1,e_2,e_{10}):&\quad x_{10}-x_{15}-x_{22} &&=0\\
(e_1,e_3,e_9):&\quad -x_{15}-x_{16}+x_7 &&=0\\
(e_1,e_4,e_8):&\quad -x_{16}+x_{21}-x_4 &&=0\\
(e_1,e_5,e_7):&\quad -x_1+x_{11}-x_4 &&=0\\
(e_2,e_3,e_8):&\quad -x_{13}x_{15}+x_{22}x_8-x_4 &&=0\\
(e_2,e_4,e_7):&\quad -x_1-x_{16}x_9+x_{22}x_5 &&=0\\
(e_2,e_5,e_6):&\quad x_1x_3+x_{17}x_{22}-x_{18}x_4 &&=0\\
(e_3,e_4,e_6):&\quad x_1x_{20}+x_{15}x_6-x_{16}x_2 &&=0
\end{aligned}
$$

Groebner basis (23 variables, 3 linear, 20 nonlinear)

$$462x_1+14650x_{23}^5-3465x_{23}^4=0$$
$$56x_2-10x_{23}^5+21x_{23}^4-42x_{23}^3+84x_{23}^2-56x_{23}=0$$

$$x_{23} + x_3 - 1 = 0$$
$$-11490x_{23}^5 + 4389x_{23}^4 - 924x_{23}^3 + 308x_4 = 0$$
$$970x_{23}^5 - 1197x_{23}^4 + 630x_{23}^3 - 252x_{23}^2 + 168x_5 = 0$$
$$10x_{23}^5 - 21x_{23}^4 + 42x_{23}^3 - 84x_{23}^2 + 56x_6 = 0$$
$$-70x_{23}^5 + 87x_{23}^4 - 48x_{23}^3 + 24x_{23}^2 - 4x_{23} + 4x_7 = 0$$
$$-1030x_{23}^5 + 1323x_{23}^4 - 882x_{23}^3 + 756x_{23}^2 - 168x_{23} + 168x_8 = 0$$
$$10x_{23}^5 - 21x_{23}^4 + 42x_{23}^3 - 84x_{23}^2 + 168x_{23} + 56x_9 - 56 = 0$$
$$12x_{10} + 290x_{23}^5 - 369x_{23}^4 + 234x_{23}^3 - 180x_{23}^2 + 72x_{23} - 12 = 0$$
$$84x_{11} - 470x_{23}^5 + 567x_{23}^4 - 252x_{23}^3 = 0$$
$$6x_{12} + 40x_{23}^5 - 54x_{23}^4 + 45x_{23}^3 - 54x_{23}^2 + 30x_{23} - 6 = 0$$
$$56x_{13} + 30x_{23}^5 - 63x_{23}^4 + 126x_{23}^3 - 252x_{23}^2 + 224x_{23} - 56 = 0$$
$$28x_{14} - 10x_{23}^5 + 21x_{23}^4 - 42x_{23}^3 + 84x_{23}^2 - 28x_{23} = 0$$
$$264x_{15} - 17470x_{23}^5 + 13167x_{23}^4 - 5742x_{23}^3 + 1980x_{23}^2 - 264x_{23} = 0$$
$$264x_{16} + 12850x_{23}^5 - 7425x_{23}^4 + 2574x_{23}^3 - 396x_{23}^2 = 0$$
$$84x_{17} - 470x_{23}^5 + 567x_{23}^4 - 252x_{23}^3 = 0$$
$$x_{18} + 2x_{23} - 1 = 0$$
$$56x_{19} + 10x_{23}^5 - 21x_{23}^4 + 42x_{23}^3 - 84x_{23}^2 = 0$$
$$x_{20} - x_{23} = 0$$
$$168x_{21} + 1910x_{23}^5 - 2331x_{23}^4 + 1134x_{23}^3 - 252x_{23}^2 = 0$$
$$88x_{22} + 7950x_{23}^5 - 7095x_{23}^4 + 3630x_{23}^3 - 1980x_{23}^2 + 616x_{23} - 88 = 0$$
$$10x_{23}^6 - x_{23}^5 = 0$$

Solution 1:

$$x_1 = 0$$
$$x_2 = 0$$
$$x_3 = 1$$
$$x_4 = 0$$
$$x_5 = 0$$
$$x_6 = 0$$
$$x_7 = 0$$
$$x_8 = 0$$
$$x_9 = 1$$
$$x_10 = 1$$

$$x_11 = 0$$
$$x_12 = 1$$
$$x_13 = 1$$
$$x_14 = 0$$
$$x_15 = 0$$
$$x_16 = 0$$
$$x_17 = 0$$
$$x_18 = 1$$
$$x_19 = 0$$
$$x_20 = 0$$
$$x_21 = 0$$
$$x_22 = 1$$
$$x_23 = 0$$

Solution 2:

$$x_1 = 1/2310$$
$$x_2 = 3/35$$
$$x_3 = 9/10$$
$$x_4 = 3/1540$$
$$x_5 = 1/84$$
$$x_6 = 1/70$$
$$x_7 = 1/20$$
$$x_8 = 5/84$$
$$x_9 = 5/7$$
$$x_10 = 8/15$$
$$x_11 = 1/420$$
$$x_12 = 7/12$$
$$x_13 = 9/14$$
$$x_14 = 1/14$$
$$x_15 = 7/165$$
$$x_16 = 1/132$$
$$x_17 = 1/420$$
$$x_18 = 4/5$$
$$x_19 = 1/70$$
$$x_20 = 1/10$$
$$x_21 = 1/105$$
$$x_22 = 27/55$$
$$x_23 = 1/10$$

$\mathfrak{m}_{2A}(3,13)$

m2A313 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4\\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6\\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8\\
[e_1,e_8] &= e_9 & [e_1,e_9] &= e_{10}\\
[e_1,e_{10}] &= e_{11} & [e_1,e_{11}] &= e_{12}\\
[e_1,e_{12}] &= e_{13} & [e_2,e_9] &= e_{12}\\
[e_2,e_{10}] &= 4e_{13} & [e_3,e_8] &= -e_{12}\\
[e_3,e_9] &= -3e_{13} & [e_4,e_7] &= e_{12}\\
[e_4,e_8] &= 2e_{13} & [e_5,e_6] &= -e_{12}\\
[e_5,e_7] &= -e_{13}
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{4A}(3,13)$

m4A313 (this line included for string searching purposes)

Solution 1

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4\\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6\\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8\\
[e_1,e_8] &= e_9 & [e_1,e_9] &= e_{10}\\
[e_1,e_{10}] &= e_{11} & [e_1,e_{11}] &= e_{12}\\
[e_1,e_{12}] &= e_{13} & [e_2,e_7] &= e_{10}\\
[e_2,e_8] &= 3e_{11} & [e_2,e_9] &= \frac{7e_{12}}{2}\\
[e_2,e_{10}] &= 0 & [e_3,e_6] &= -e_{10}\\
[e_3,e_7] &= -2e_{11} & [e_3,e_8] &= -\frac{e_{12}}{2}\\
[e_3,e_9] &= \frac{7e_{13}}{2} & [e_4,e_5] &= e_{10}\\
[e_4,e_6] &= e_{11} & [e_4,e_7] &= -\frac{3e_{12}}{2}\\
[e_4,e_8] &= -4e_{13} & [e_5,e_6] &= \frac{5e_{12}}{2}\\
[e_5,e_7] &= \frac{5e_{13}}{2}
\end{aligned}
$$

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_2, e_7] &= e_{10} \\
[e_2, e_8] &= 3e_{11} & [e_2, e_9] &= \alpha_{2,9}^{12} e_{12} \\
[e_2, e_{10}] &= \alpha_{2,10}^{13} e_{13} & [e_3, e_6] &= -e_{10} \\
[e_3, e_7] &= -2e_{11} & [e_3, e_8] &= \alpha_{3,8}^{12} e_{12} \\
[e_3, e_9] &= \alpha_{3,9}^{13} e_{13} & [e_4, e_5] &= e_{10} \\
[e_4, e_6] &= e_{11} & [e_4, e_7] &= \alpha_{4,7}^{12} e_{12} \\
[e_4, e_8] &= \alpha_{4,8}^{13} e_{13} & [e_5, e_6] &= \alpha_{5,6}^{12} e_{12} \\
[e_5, e_7] &= \alpha_{5,7}^{13} e_{13}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_8) &: \quad -\alpha_{2,9}^{12} - \alpha_{3,8}^{12} + 3 &&= 0 \\
(e_1, e_3, e_7) &: \quad -\alpha_{3,8}^{12} - \alpha_{4,7}^{12} - 2 &&= 0 \\
(e_1, e_4, e_6) &: \quad -\alpha_{4,7}^{12} - \alpha_{5,6}^{12} + 1 &&= 0 \\
(e_1, e_2, e_9) &: \quad -\alpha_{2,10}^{13} + \alpha_{2,9}^{12} - \alpha_{3,9}^{13} &&= 0 \\
(e_1, e_3, e_8) &: \quad \alpha_{3,8}^{12} - \alpha_{3,9}^{13} - \alpha_{4,8}^{13} &&= 0 \\
(e_1, e_4, e_7) &: \quad \alpha_{4,7}^{12} - \alpha_{4,8}^{13} - \alpha_{5,7}^{13} &&= 0 \\
(e_1, e_5, e_6) &: \quad \alpha_{5,6}^{12} - \alpha_{5,7}^{13} &&= 0 \\
(e_2, e_3, e_6) &: \quad -\alpha_{2,10}^{13} &&= 0 \\
(e_2, e_4, e_5) &: \quad \alpha_{2,10}^{13} &&= 0
\end{aligned}
$$

Solution 1:

$$\alpha^{13}_{3,9} = 7/2$$
$$\alpha^{13}_{2,10} = 0$$
$$\alpha^{13}_{4,8} = -4$$
$$\alpha^{12}_{4,7} = -3/2$$
$$\alpha^{12}_{3,8} = -1/2$$
$$\alpha^{12}_{5,6} = 5/2$$
$$\alpha^{12}_{2,9} = 7/2$$
$$\alpha^{13}_{5,7} = 5/2$$

How the solution(s) were or were not found:
Change variables

$$\alpha^{13}_{3,9} \to x_1$$
$$\alpha^{13}_{2,10} \to x_2$$
$$\alpha^{13}_{4,8} \to x_3$$
$$\alpha^{12}_{4,7} \to x_4$$
$$\alpha^{12}_{3,8} \to x_5$$
$$\alpha^{12}_{5,6} \to x_6$$
$$\alpha^{12}_{2,9} \to x_7$$
$$\alpha^{13}_{5,7} \to x_8$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_8): &\quad -x_5 - x_7 + 3 &&= 0\\
(e_1, e_3, e_7): &\quad -x_4 - x_5 - 2 &&= 0\\
(e_1, e_4, e_6): &\quad -x_4 - x_6 + 1 &&= 0\\
(e_1, e_2, e_9): &\quad -x_1 - x_2 + x_7 &&= 0\\
(e_1, e_3, e_8): &\quad -x_1 - x_3 + x_5 &&= 0\\
(e_1, e_4, e_7): &\quad -x_3 + x_4 - x_8 &&= 0\\
(e_1, e_5, e_6): &\quad x_6 - x_8 &&= 0\\
(e_2, e_3, e_6): &\quad -x_2 &&= 0\\
(e_2, e_4, e_5): &\quad x_2 &&= 0
\end{aligned}$$

Groebner basis (8 variables, 8 linear, 0 nonlinear)

$$2x_1 - 7 = 0$$
$$x_2 = 0$$
$$x_3 + 4 = 0$$
$$2x_4 + 3 = 0$$
$$2x_5 + 1 = 0$$
$$2x_6 - 5 = 0$$
$$2x_7 - 7 = 0$$
$$2x_8 - 5 = 0$$

Solution 1:

$$x_1 = 7/2$$
$$x_2 = 0$$
$$x_3 = -4$$
$$x_4 = -3/2$$
$$x_5 = -1/2$$
$$x_6 = 5/2$$
$$x_7 = 7/2$$
$$x_8 = 5/2$$

$\mathfrak{m}_{6A}(3, 13)$

m6A313 (this line included for string searching purposes)

Solution 1

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_1, e_{11}] = e_{12}$$
$$[e_1, e_{12}] = e_{13} \qquad [e_2, e_5] = e_8$$
$$[e_2, e_6] = 2e_9 \qquad [e_2, e_7] = \frac{5e_{10}}{3}$$
$$[e_2, e_8] = 0 \qquad [e_2, e_9] = -\frac{49e_{12}}{33}$$
$$[e_2, e_{10}] = -\frac{14e_{13}}{11} \qquad [e_3, e_4] = -e_8$$
$$[e_3, e_5] = -e_9 \qquad [e_3, e_6] = \frac{e_{10}}{3}$$
$$[e_3, e_7] = \frac{5e_{11}}{3} \qquad [e_3, e_8] = \frac{49e_{12}}{33}$$
$$[e_3, e_9] = -\frac{7e_{13}}{33} \qquad [e_4, e_5] = -\frac{4e_{10}}{3}$$
$$[e_4, e_6] = -\frac{4e_{11}}{3} \qquad [e_4, e_7] = \frac{2e_{12}}{11}$$
$$[e_4, e_8] = \frac{56e_{13}}{33} \qquad [e_5, e_6] = -\frac{50e_{12}}{33}$$
$$[e_5, e_7] = -\frac{50e_{13}}{33}$$

Original brackets:

$$
\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_1, e_9] = e_{10} \\
[e_1, e_{10}] = e_{11} & [e_1, e_{11}] = e_{12} \\
[e_1, e_{12}] = e_{13} & [e_2, e_5] = e_8 \\
[e_2, e_6] = 2e_9 & [e_2, e_7] = \alpha_{2,7}^{10} e_{10} \\
[e_2, e_8] = \alpha_{2,8}^{11} e_{11} & [e_2, e_9] = \alpha_{2,9}^{12} e_{12} \\
[e_2, e_{10}] = \alpha_{2,10}^{13} e_{13} & [e_3, e_4] = -e_8 \\
[e_3, e_5] = -e_9 & [e_3, e_6] = \alpha_{3,6}^{10} e_{10} \\
[e_3, e_7] = \alpha_{3,7}^{11} e_{11} & [e_3, e_8] = \alpha_{3,8}^{12} e_{12} \\
[e_3, e_9] = \alpha_{3,9}^{13} e_{13} & [e_4, e_5] = \alpha_{4,5}^{10} e_{10} \\
[e_4, e_6] = \alpha_{4,6}^{11} e_{11} & [e_4, e_7] = \alpha_{4,7}^{12} e_{12} \\
[e_4, e_8] = \alpha_{4,8}^{13} e_{13} & [e_5, e_6] = \alpha_{5,6}^{12} e_{12} \\
[e_5, e_7] = \alpha_{5,7}^{13} e_{13} &
\end{array}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1,e_2,e_6): &\quad -\alpha_{2,7}^{10}-\alpha_{3,6}^{10}+2 &&=0\\
(e_1,e_3,e_5): &\quad -\alpha_{3,6}^{10}-\alpha_{4,5}^{10}-1 &&=0\\
(e_1,e_2,e_7): &\quad \alpha_{2,7}^{10}-\alpha_{2,8}^{11}-\alpha_{3,7}^{11} &&=0\\
(e_1,e_3,e_6): &\quad \alpha_{3,6}^{10}-\alpha_{3,7}^{11}-\alpha_{4,6}^{11} &&=0\\
(e_1,e_4,e_5): &\quad \alpha_{4,5}^{10}-\alpha_{4,6}^{11} &&=0\\
(e_2,e_3,e_4): &\quad -\alpha_{2,8}^{11} &&=0\\
(e_1,e_2,e_8): &\quad \alpha_{2,8}^{11}-\alpha_{2,9}^{12}-\alpha_{3,8}^{12} &&=0\\
(e_1,e_3,e_7): &\quad \alpha_{3,7}^{11}-\alpha_{3,8}^{12}-\alpha_{4,7}^{12} &&=0\\
(e_1,e_4,e_6): &\quad \alpha_{4,6}^{11}-\alpha_{4,7}^{12}-\alpha_{5,6}^{12} &&=0\\
(e_2,e_3,e_5): &\quad -\alpha_{2,9}^{12}-\alpha_{3,8}^{12} &&=0\\
(e_1,e_2,e_9): &\quad -\alpha_{2,10}^{13}+\alpha_{2,9}^{12}-\alpha_{3,9}^{13} &&=0\\
(e_1,e_3,e_8): &\quad \alpha_{3,8}^{12}-\alpha_{3,9}^{13}-\alpha_{4,8}^{13} &&=0\\
(e_1,e_4,e_7): &\quad \alpha_{4,7}^{12}-\alpha_{4,8}^{13}-\alpha_{5,7}^{13} &&=0\\
(e_1,e_5,e_6): &\quad \alpha_{5,6}^{12}-\alpha_{5,7}^{13} &&=0\\
(e_2,e_3,e_6): &\quad \alpha_{2,10}^{13}\alpha_{3,6}^{10}-2\alpha_{3,9}^{13} &&=0\\
(e_2,e_4,e_5): &\quad \alpha_{2,10}^{13}\alpha_{4,5}^{10}-\alpha_{4,8}^{13} &&=0
\end{aligned}
$$

Solution 1:

$$\alpha^{13}_{3,9} = -7/33$$
$$\alpha^{13}_{5,7} = -50/33$$
$$\alpha^{13}_{2,10} = -14/11$$
$$\alpha^{13}_{4,8} = 56/33$$
$$\alpha^{11}_{3,7} = 5/3$$
$$\alpha^{11}_{2,8} = 0$$
$$\alpha^{12}_{4,7} = 2/11$$
$$\alpha^{12}_{3,8} = 49/33$$
$$\alpha^{10}_{2,7} = 5/3$$
$$\alpha^{11}_{4,6} = -4/3$$
$$\alpha^{10}_{4,5} = -4/3$$
$$\alpha^{12}_{5,6} = -50/33$$
$$\alpha^{12}_{2,9} = -49/33$$
$$\alpha^{10}_{3,6} = 1/3$$

How the solution(s) were or were not found:
Change variables

$$\alpha^{13}_{3,9} \to x_1$$
$$\alpha^{13}_{5,7} \to x_2$$
$$\alpha^{13}_{2,10} \to x_3$$
$$\alpha^{13}_{4,8} \to x_4$$
$$\alpha^{11}_{3,7} \to x_5$$
$$\alpha^{11}_{2,8} \to x_6$$
$$\alpha^{12}_{4,7} \to x_7$$
$$\alpha^{12}_{3,8} \to x_8$$
$$\alpha^{10}_{2,7} \to x_9$$
$$\alpha^{11}_{4,6} \to x_{10}$$
$$\alpha^{10}_{4,5} \to x_{11}$$
$$\alpha^{12}_{5,6} \to x_{12}$$
$$\alpha^{12}_{2,9} \to x_{13}$$

$$\alpha_{3,6}^{10} \to x_{14}$$

Jacobi Tests

$$\begin{array}{lll}
(e_1, e_2, e_6): & -x_{14} - x_9 + 2 & = 0 \\
(e_1, e_3, e_5): & -x_{11} - x_{14} - 1 & = 0 \\
(e_1, e_2, e_7): & -x_5 - x_6 + x_9 & = 0 \\
(e_1, e_3, e_6): & -x_{10} + x_{14} - x_5 & = 0 \\
(e_1, e_4, e_5): & -x_{10} + x_{11} & = 0 \\
(e_2, e_3, e_4): & -x_6 & = 0 \\
(e_1, e_2, e_8): & -x_{13} + x_6 - x_8 & = 0 \\
(e_1, e_3, e_7): & x_5 - x_7 - x_8 & = 0 \\
(e_1, e_4, e_6): & x_{10} - x_{12} - x_7 & = 0 \\
(e_2, e_3, e_5): & -x_{13} - x_8 & = 0 \\
(e_1, e_2, e_9): & -x_1 + x_{13} - x_3 & = 0 \\
(e_1, e_3, e_8): & -x_1 - x_4 + x_8 & = 0 \\
(e_1, e_4, e_7): & -x_2 - x_4 + x_7 & = 0 \\
(e_1, e_5, e_6): & x_{12} - x_2 & = 0 \\
(e_2, e_3, e_6): & -2x_1 + x_{14}x_3 & = 0 \\
(e_2, e_4, e_5): & x_{11}x_3 - x_4 & = 0
\end{array}$$

Groebner basis (14 variables, 14 linear, 0 nonlinear)

$$33x_1 + 7 = 0$$

$$33x_2 + 50 = 0$$

$$11x_3 + 14 = 0$$

$$33x_4 - 56 = 0$$

$$3x_5 - 5 = 0$$

$$x_6 = 0$$

$$11x_7 - 2 = 0$$

$$33x_8 - 49 = 0$$

$$3x_9 - 5 = 0$$

$$3x_{10} + 4 = 0$$

$$3x_{11} + 4 = 0$$

$$33x_{12} + 50 = 0$$

$$33x_{13} + 49 = 0$$
$$3x_{14} - 1 = 0$$

Solution 1:

$$x_1 = -7/33$$
$$x_2 = -50/33$$
$$x_3 = -14/11$$
$$x_4 = 56/33$$
$$x_5 = 5/3$$
$$x_6 = 0$$
$$x_7 = 2/11$$
$$x_8 = 49/33$$
$$x_9 = 5/3$$
$$x_10 = -4/3$$
$$x_11 = -4/3$$
$$x_12 = -50/33$$
$$x_13 = -49/33$$
$$x_14 = 1/3$$

$\mathfrak{m}_{8A}(3, 13)$

m8A313 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
&[e_1, e_2] = e_3 && [e_1, e_3] = e_4 \\
&[e_1, e_4] = e_5 && [e_1, e_5] = e_6 \\
&[e_1, e_6] = e_7 && [e_1, e_7] = e_8 \\
&[e_1, e_8] = e_9 && [e_1, e_9] = e_{10} \\
&[e_1, e_{10}] = e_{11} && [e_1, e_{11}] = e_{12} \\
&[e_1, e_{12}] = e_{13} && [e_2, e_3] = e_6 \\
&[e_2, e_4] = e_7 && [e_2, e_5] = \alpha_{2,5}^{8} e_8 \\
&[e_2, e_6] = \alpha_{2,6}^{9} e_9 && [e_2, e_7] = \alpha_{2,7}^{10} e_{10} \\
&[e_2, e_8] = \alpha_{2,8}^{11} e_{11} && [e_2, e_9] = \alpha_{2,9}^{12} e_{12} \\
&[e_2, e_{10}] = \alpha_{2,10}^{13} e_{13} && [e_3, e_4] = \alpha_{3,4}^{8} e_8 \\
&[e_3, e_5] = \alpha_{3,5}^{9} e_9 && [e_3, e_6] = \alpha_{3,6}^{10} e_{10} \\
&[e_3, e_7] = \alpha_{3,7}^{11} e_{11} && [e_3, e_8] = \alpha_{3,8}^{12} e_{12} \\
&[e_3, e_9] = \alpha_{3,9}^{13} e_{13} && [e_4, e_5] = \alpha_{4,5}^{10} e_{10} \\
&[e_4, e_6] = \alpha_{4,6}^{11} e_{11} && [e_4, e_7] = \alpha_{4,7}^{12} e_{12} \\
&[e_4, e_8] = \alpha_{4,8}^{13} e_{13} && [e_5, e_6] = \alpha_{5,6}^{12} e_{12} \\
&[e_5, e_7] = \alpha_{5,7}^{13} e_{13} &&
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_4): &\quad -\alpha_{2,5}^{8} - \alpha_{3,4}^{8} + 1 &&= 0\\
(e_1, e_2, e_5): &\quad \alpha_{2,5}^{8} - \alpha_{2,6}^{9} - \alpha_{3,5}^{9} &&= 0\\
(e_1, e_3, e_4): &\quad \alpha_{3,4}^{8} - \alpha_{3,5}^{9} &&= 0\\
(e_1, e_2, e_6): &\quad \alpha_{2,6}^{9} - \alpha_{2,7}^{10} - \alpha_{3,6}^{10} &&= 0\\
(e_1, e_3, e_5): &\quad \alpha_{3,5}^{9} - \alpha_{3,6}^{10} - \alpha_{4,5}^{10} &&= 0\\
(e_1, e_2, e_7): &\quad \alpha_{2,7}^{10} - \alpha_{2,8}^{11} - \alpha_{3,7}^{11} &&= 0\\
(e_1, e_3, e_6): &\quad \alpha_{3,6}^{10} - \alpha_{3,7}^{11} - \alpha_{4,6}^{11} &&= 0\\
(e_1, e_4, e_5): &\quad \alpha_{4,5}^{10} - \alpha_{4,6}^{11} &&= 0\\
(e_2, e_3, e_4): &\quad \alpha_{2,8}^{11}\alpha_{3,4}^{8} - \alpha_{3,7}^{11} + \alpha_{4,6}^{11} &&= 0\\
(e_1, e_2, e_8): &\quad \alpha_{2,8}^{11} - \alpha_{2,9}^{12} - \alpha_{3,8}^{12} &&= 0\\
(e_1, e_3, e_7): &\quad \alpha_{3,7}^{11} - \alpha_{3,8}^{12} - \alpha_{4,7}^{12} &&= 0\\
(e_1, e_4, e_6): &\quad \alpha_{4,6}^{11} - \alpha_{4,7}^{12} - \alpha_{5,6}^{12} &&= 0\\
(e_2, e_3, e_5): &\quad -\alpha_{2,5}^{8}\alpha_{3,8}^{12} + \alpha_{2,9}^{12}\alpha_{3,5}^{9} + \alpha_{5,6}^{12} &&= 0\\
(e_1, e_2, e_9): &\quad -\alpha_{2,10}^{13} + \alpha_{2,9}^{12} - \alpha_{3,9}^{13} &&= 0\\
(e_1, e_3, e_8): &\quad \alpha_{3,8}^{12} - \alpha_{3,9}^{13} - \alpha_{4,8}^{13} &&= 0\\
(e_1, e_4, e_7): &\quad \alpha_{4,7}^{12} - \alpha_{4,8}^{13} - \alpha_{5,7}^{13} &&= 0\\
(e_1, e_5, e_6): &\quad \alpha_{5,6}^{12} - \alpha_{5,7}^{13} &&= 0\\
(e_2, e_3, e_6): &\quad \alpha_{2,10}^{13}\alpha_{3,6}^{10} - \alpha_{2,6}^{9}\alpha_{3,9}^{13} &&= 0\\
(e_2, e_4, e_5): &\quad \alpha_{2,10}^{13}\alpha_{4,5}^{10} - \alpha_{2,5}^{8}\alpha_{4,8}^{13} + \alpha_{5,7}^{13} &&= 0
\end{aligned}
$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{3,5}^{9} \to x_1$$

$$\alpha_{3,9}^{13} \to x_2$$

$$\alpha_{5,7}^{13} \to x_3$$

$$\alpha_{2,10}^{13} \to x_4$$

$$\alpha_{4,8}^{13} \to x_5$$

$$\alpha_{2,5}^{8} \to x_6$$

$$\alpha_{3,7}^{11} \to x_7$$

$$\alpha_{2,8}^{11} \to x_8$$

$$\alpha^{12}_{3,8} \to x_9$$

$$\alpha^{9}_{2,6} \to x_{10}$$

$$\alpha^{12}_{4,7} \to x_{11}$$

$$\alpha^{10}_{2,7} \to x_{12}$$

$$\alpha^{11}_{4,6} \to x_{13}$$

$$\alpha^{10}_{4,5} \to x_{14}$$

$$\alpha^{12}_{5,6} \to x_{15}$$

$$\alpha^{8}_{3,4} \to x_{16}$$

$$\alpha^{12}_{2,9} \to x_{17}$$

$$\alpha^{10}_{3,6} \to x_{18}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4):& \quad -x_{16} - x_6 + 1 && = 0 \\
(e_1, e_2, e_5):& \quad -x_1 - x_{10} + x_6 && = 0 \\
(e_1, e_3, e_4):& \quad -x_1 + x_{16} && = 0 \\
(e_1, e_2, e_6):& \quad x_{10} - x_{12} - x_{18} && = 0 \\
(e_1, e_3, e_5):& \quad x_1 - x_{14} - x_{18} && = 0 \\
(e_1, e_2, e_7):& \quad x_{12} - x_7 - x_8 && = 0 \\
(e_1, e_3, e_6):& \quad -x_{13} + x_{18} - x_7 && = 0 \\
(e_1, e_4, e_5):& \quad -x_{13} + x_{14} && = 0 \\
(e_2, e_3, e_4):& \quad x_{13} + x_{16}x_8 - x_7 && = 0 \\
(e_1, e_2, e_8):& \quad -x_{17} + x_8 - x_9 && = 0 \\
(e_1, e_3, e_7):& \quad -x_{11} + x_7 - x_9 && = 0 \\
(e_1, e_4, e_6):& \quad -x_{11} + x_{13} - x_{15} && = 0 \\
(e_2, e_3, e_5):& \quad x_1x_{17} + x_{15} - x_6x_9 && = 0 \\
(e_1, e_2, e_9):& \quad x_{17} - x_2 - x_4 && = 0 \\
(e_1, e_3, e_8):& \quad -x_2 - x_5 + x_9 && = 0 \\
(e_1, e_4, e_7):& \quad x_{11} - x_3 - x_5 && = 0 \\
(e_1, e_5, e_6):& \quad x_{15} - x_3 && = 0 \\
(e_2, e_3, e_6):& \quad -x_{10}x_2 + x_{18}x_4 && = 0 \\
(e_2, e_4, e_5):& \quad x_{14}x_4 + x_3 - x_5x_6 && = 0
\end{aligned}$$

Groebner basis (18 variables, 15 linear, 4 nonlinear)

$$x_1 - x_{16} = 0$$
$$3x_{17} + 14x_{18} + x_2 - 3 = 0$$
$$-x_{16} + x_{17} + 6x_{18} + x_3 - 1 = 0$$
$$-4x_{17} - 14x_{18} + x_4 + 3 = 0$$
$$x_{16} - 2x_{17} - 11x_{18} + x_5 + 2 = 0$$
$$x_{16} + x_6 - 1 = 0$$
$$x_{16} - 2x_{18} + x_7 = 0$$
$$x_{16} + 3x_{18} + x_8 - 1 = 0$$
$$x_{16} + x_{17} + 3x_{18} + x_9 - 1 = 0$$
$$x_{10} + 2x_{16} - 1 = 0$$
$$x_{11} - x_{17} - 5x_{18} + 1 = 0$$
$$x_{12} + 2x_{16} + x_{18} - 1 = 0$$
$$x_{13} - x_{16} + x_{18} = 0$$
$$x_{14} - x_{16} + x_{18} = 0$$
$$x_{15} - x_{16} + x_{17} + 6x_{18} - 1 = 0$$
$$x_{16}^2 + 3x_{16}x_{18} - 3x_{16} + 3x_{18} = 0$$
$$6x_{16}x_{17} + 28x_{16}x_{18} - 6x_{16} - 4x_{17}x_{18} - 3x_{17} - 14x_{18}^2 - 11x_{18} + 3 = 0$$
$$28x_{16}x_{18}^2 - 6x_{16}x_{18} - 88x_{17}x_{18}^2 - 114x_{17}x_{18} + 45x_{17} - 308x_{18}^3 - 536x_{18}^2 + 339x_{18} - 45 = 0$$
$$88x_{17}^2x_{18}^2 + 114x_{17}^2x_{18} - 45x_{17}^2 + 700x_{17}x_{18}^3 + 970x_{17}x_{18}^2 - 660x_{17}x_{18} + 90x_{17} + 1372x_{18}^4 + 2156x_{18}^3 - 2093x_{18}^2 + 546x_{18} - 4$$

$\mathfrak{m}_{1A}(4, 13)$

m1A413 (this line included for string searching purposes)

Original brackets:

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_1, e_{11}] = e_{12}$$
$$[e_1, e_{12}] = e_{13} \qquad [e_2, e_9] = e_{13}$$
$$[e_3, e_8] = -e_{13} \qquad [e_4, e_7] = e_{13}$$
$$[e_5, e_6] = -e_{13}$$

No non-trivial Jacobi tests

$\mathfrak{m}_{3A}(4,13)$

m3A413 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
&[e_1,e_2]=e_3 && [e_1,e_3]=e_4\\
&[e_1,e_4]=e_5 && [e_1,e_5]=e_6\\
&[e_1,e_6]=e_7 && [e_1,e_7]=e_8\\
&[e_1,e_8]=e_9 && [e_1,e_9]=e_{10}\\
&[e_1,e_{10}]=e_{11} && [e_1,e_{11}]=e_{12}\\
&[e_1,e_{12}]=e_{13} && [e_2,e_7]=e_{11}\\
&[e_2,e_8]=3e_{12} && [e_2,e_9]=\alpha_{2,9}^{13}e_{13}\\
&[e_3,e_6]=-e_{11} && [e_3,e_7]=-2e_{12}\\
&[e_3,e_8]=\alpha_{3,8}^{13}e_{13} && [e_4,e_5]=e_{11}\\
&[e_4,e_6]=e_{12} && [e_4,e_7]=\alpha_{4,7}^{13}e_{13}\\
&[e_5,e_6]=\alpha_{5,6}^{13}e_{13}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1,e_2,e_8):&\quad -\alpha_{2,9}^{13}-\alpha_{3,8}^{13}+3 &&=0\\
(e_1,e_3,e_7):&\quad -\alpha_{3,8}^{13}-\alpha_{4,7}^{13}-2 &&=0\\
(e_1,e_4,e_6):&\quad -\alpha_{4,7}^{13}-\alpha_{5,6}^{13}+1 &&=0
\end{aligned}
$$

Infinite number of solutions.

How the solution(s) were or were not found:

Change variables

$$
\begin{aligned}
\alpha_{2,9}^{13}&\to x_1\\
\alpha_{4,7}^{13}&\to x_2\\
\alpha_{3,8}^{13}&\to x_3\\
\alpha_{5,6}^{13}&\to x_4
\end{aligned}
$$

Jacobi Tests

$$
\begin{aligned}
(e_1,e_2,e_8):&\quad -x_1-x_3+3 &&=0\\
(e_1,e_3,e_7):&\quad -x_2-x_3-2 &&=0\\
(e_1,e_4,e_6):&\quad -x_2-x_4+1 &&=0
\end{aligned}
$$

Groebner basis (4 variables, 3 linear, 0 nonlinear)

$$x_1 + x_4 - 6 = 0$$
$$x_2 + x_4 - 1 = 0$$
$$x_3 - x_4 + 3 = 0$$

$\mathfrak{m}_{5A}(4, 13)$

m5A413 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_2, e_5] &= e_9 \\
[e_2, e_6] &= 2e_{10} & [e_2, e_7] &= \alpha_{2,7}^{11} e_{11} \\
[e_2, e_8] &= \alpha_{2,8}^{12} e_{12} & [e_2, e_9] &= \alpha_{2,9}^{13} e_{13} \\
[e_3, e_4] &= -e_9 & [e_3, e_5] &= -e_{10} \\
[e_3, e_6] &= \alpha_{3,6}^{11} e_{11} & [e_3, e_7] &= \alpha_{3,7}^{12} e_{12} \\
[e_3, e_8] &= \alpha_{3,8}^{13} e_{13} & [e_4, e_5] &= \alpha_{4,5}^{11} e_{11} \\
[e_4, e_6] &= \alpha_{4,6}^{12} e_{12} & [e_4, e_7] &= \alpha_{4,7}^{13} e_{13} \\
[e_5, e_6] &= \alpha_{5,6}^{13} e_{13}
\end{aligned}$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1, e_2, e_6) &: \quad -\alpha_{2,7}^{11} - \alpha_{3,6}^{11} + 2 &&= 0 \\
(e_1, e_3, e_5) &: \quad -\alpha_{3,6}^{11} - \alpha_{4,5}^{11} - 1 &&= 0 \\
(e_1, e_2, e_7) &: \quad \alpha_{2,7}^{11} - \alpha_{2,8}^{12} - \alpha_{3,7}^{12} &&= 0 \\
(e_1, e_3, e_6) &: \quad \alpha_{3,6}^{11} - \alpha_{3,7}^{12} - \alpha_{4,6}^{12} &&= 0 \\
(e_1, e_4, e_5) &: \quad \alpha_{4,5}^{11} - \alpha_{4,6}^{12} &&= 0 \\
(e_1, e_2, e_8) &: \quad \alpha_{2,8}^{12} - \alpha_{2,9}^{13} - \alpha_{3,8}^{13} &&= 0 \\
(e_1, e_3, e_7) &: \quad \alpha_{3,7}^{12} - \alpha_{3,8}^{13} - \alpha_{4,7}^{13} &&= 0 \\
(e_1, e_4, e_6) &: \quad \alpha_{4,6}^{12} - \alpha_{4,7}^{13} - \alpha_{5,6}^{13} &&= 0 \\
(e_2, e_3, e_4) &: \quad -\alpha_{2,9}^{13} &&= 0
\end{aligned}$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha^{11}_{2,7} \to x_1$$
$$\alpha^{12}_{2,8} \to x_2$$
$$\alpha^{12}_{4,6} \to x_3$$
$$\alpha^{13}_{5,6} \to x_4$$
$$\alpha^{11}_{4,5} \to x_5$$
$$\alpha^{13}_{3,8} \to x_6$$
$$\alpha^{12}_{3,7} \to x_7$$
$$\alpha^{13}_{2,9} \to x_8$$
$$\alpha^{13}_{4,7} \to x_9$$
$$\alpha^{11}_{3,6} \to x_{10}$$

Jacobi Tests

$$\begin{aligned}
(e_1,e_2,e_6):&\quad -x_1-x_{10}+2 &&=0\\
(e_1,e_3,e_5):&\quad -x_{10}-x_5-1 &&=0\\
(e_1,e_2,e_7):&\quad x_1-x_2-x_7 &&=0\\
(e_1,e_3,e_6):&\quad x_{10}-x_3-x_7 &&=0\\
(e_1,e_4,e_5):&\quad -x_3+x_5 &&=0\\
(e_1,e_2,e_8):&\quad x_2-x_6-x_8 &&=0\\
(e_1,e_3,e_7):&\quad -x_6+x_7-x_9 &&=0\\
(e_1,e_4,e_6):&\quad x_3-x_4-x_9 &&=0\\
(e_2,e_3,e_4):&\quad -x_8 &&=0
\end{aligned}$$

Groebner basis (10 variables, 9 linear, 0 nonlinear)

$$x_1+x_{10}-2=0$$
$$3x_{10}+x_2-1=0$$
$$x_{10}+x_3+1=0$$
$$6x_{10}+x_4+1=0$$
$$x_{10}+x_5+1=0$$
$$3x_{10}+x_6-1=0$$
$$-2x_{10}+x_7-1=0$$
$$x_8=0$$
$$-5x_{10}+x_9=0$$

$\mathfrak{m}_{7A}(4, 13)$

m7A413 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_2, e_3] &= e_7 \\
[e_2, e_4] &= e_8 & [e_2, e_5] &= \alpha_{2,5}^{9} e_9 \\
[e_2, e_6] &= \alpha_{2,6}^{10} e_{10} & [e_2, e_7] &= \alpha_{2,7}^{11} e_{11} \\
[e_2, e_8] &= \alpha_{2,8}^{12} e_{12} & [e_2, e_9] &= \alpha_{2,9}^{13} e_{13} \\
[e_3, e_4] &= \alpha_{3,4}^{9} e_9 & [e_3, e_5] &= \alpha_{3,5}^{10} e_{10} \\
[e_3, e_6] &= \alpha_{3,6}^{11} e_{11} & [e_3, e_7] &= \alpha_{3,7}^{12} e_{12} \\
[e_3, e_8] &= \alpha_{3,8}^{13} e_{13} & [e_4, e_5] &= \alpha_{4,5}^{11} e_{11} \\
[e_4, e_6] &= \alpha_{4,6}^{12} e_{12} & [e_4, e_7] &= \alpha_{4,7}^{13} e_{13} \\
[e_5, e_6] &= \alpha_{5,6}^{13} e_{13}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_4) &: \quad -\alpha_{2,5}^{9} - \alpha_{3,4}^{9} + 1 && = 0 \\
(e_1, e_2, e_5) &: \quad \alpha_{2,5}^{9} - \alpha_{2,6}^{10} - \alpha_{3,5}^{10} && = 0 \\
(e_1, e_3, e_4) &: \quad \alpha_{3,4}^{9} - \alpha_{3,5}^{10} && = 0 \\
(e_1, e_2, e_6) &: \quad \alpha_{2,6}^{10} - \alpha_{2,7}^{11} - \alpha_{3,6}^{11} && = 0 \\
(e_1, e_3, e_5) &: \quad \alpha_{3,5}^{10} - \alpha_{3,6}^{11} - \alpha_{4,5}^{11} && = 0 \\
(e_1, e_2, e_7) &: \quad \alpha_{2,7}^{11} - \alpha_{2,8}^{12} - \alpha_{3,7}^{12} && = 0 \\
(e_1, e_3, e_6) &: \quad \alpha_{3,6}^{11} - \alpha_{3,7}^{12} - \alpha_{4,6}^{12} && = 0 \\
(e_1, e_4, e_5) &: \quad \alpha_{4,5}^{11} - \alpha_{4,6}^{12} && = 0 \\
(e_1, e_2, e_8) &: \quad \alpha_{2,8}^{12} - \alpha_{2,9}^{13} - \alpha_{3,8}^{13} && = 0 \\
(e_1, e_3, e_7) &: \quad \alpha_{3,7}^{12} - \alpha_{3,8}^{13} - \alpha_{4,7}^{13} && = 0 \\
(e_1, e_4, e_6) &: \quad \alpha_{4,6}^{12} - \alpha_{4,7}^{13} - \alpha_{5,6}^{13} && = 0 \\
(e_2, e_3, e_4) &: \quad \alpha_{2,9}^{13}\alpha_{3,4}^{9} - \alpha_{3,8}^{13} + \alpha_{4,7}^{13} && = 0
\end{aligned}
$$

Infinite number of solutions.

How the solution(s) were or were not found:

Change variables

$$\alpha_{2,7}^{11} \to x_1$$
$$\alpha_{2,5}^{9} \to x_2$$
$$\alpha_{3,5}^{10} \to x_3$$
$$\alpha_{2,8}^{12} \to x_4$$
$$\alpha_{4,6}^{12} \to x_5$$
$$\alpha_{5,6}^{13} \to x_6$$
$$\alpha_{4,5}^{11} \to x_7$$
$$\alpha_{3,8}^{13} \to x_8$$
$$\alpha_{3,7}^{12} \to x_9$$
$$\alpha_{3,4}^{9} \to x_{10}$$
$$\alpha_{2,9}^{13} \to x_{11}$$
$$\alpha_{2,6}^{10} \to x_{12}$$
$$\alpha_{4,7}^{13} \to x_{13}$$
$$\alpha_{3,6}^{11} \to x_{14}$$

Jacobi Tests

$$\begin{aligned}
(e_1,e_2,e_4):&\quad -x_{10}-x_2+1 &&=0\\
(e_1,e_2,e_5):&\quad -x_{12}+x_2-x_3 &&=0\\
(e_1,e_3,e_4):&\quad x_{10}-x_3 &&=0\\
(e_1,e_2,e_6):&\quad -x_1+x_{12}-x_{14} &&=0\\
(e_1,e_3,e_5):&\quad -x_{14}+x_3-x_7 &&=0\\
(e_1,e_2,e_7):&\quad x_1-x_4-x_9 &&=0\\
(e_1,e_3,e_6):&\quad x_{14}-x_5-x_9 &&=0\\
(e_1,e_4,e_5):&\quad -x_5+x_7 &&=0\\
(e_1,e_2,e_8):&\quad -x_{11}+x_4-x_8 &&=0\\
(e_1,e_3,e_7):&\quad -x_{13}-x_8+x_9 &&=0\\
(e_1,e_4,e_6):&\quad -x_{13}+x_5-x_6 &&=0\\
(e_2,e_3,e_4):&\quad x_{10}x_{11}+x_{13}-x_8 &&=0
\end{aligned}$$

Groebner basis (14 variables, 11 linear, 1 nonlinear)

$$x_1-x_{12}+x_{14}=0$$

$$-x_{12} + 2x_2 - 1 = 0$$
$$x_{12} + 2x_3 - 1 = 0$$
$$-x_{12} + 6x_{14} + 2x_4 - 1 = 0$$
$$x_{12} + 2x_{14} + 2x_5 - 1 = 0$$
$$x_{12} + 2x_{13} + 2x_{14} + 2x_6 - 1 = 0$$
$$x_{12} + 2x_{14} + 2x_7 - 1 = 0$$
$$-x_{12} + 2x_{13} - 4x_{14} + 2x_8 + 1 = 0$$
$$-x_{12} - 4x_{14} + 2x_9 + 1 = 0$$
$$2x_{10} + x_{12} - 1 = 0$$
$$x_{11} - x_{13} + 5x_{14} - 1 = 0$$
$$x_{12}x_{13} - 5x_{12}x_{14} + 2x_{12} - 5x_{13} + 9x_{14} - 2 = 0$$

$\mathfrak{m}_{2A}(5, 13)$

m2A513 (this line included for string searching purposes)

Original brackets:

$$
\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_1, e_9] = e_{10} \\
[e_1, e_{10}] = e_{11} & [e_1, e_{11}] = e_{12} \\
[e_1, e_{12}] = e_{13} & [e_2, e_7] = e_{12} \\
[e_2, e_8] = 3e_{13} & [e_3, e_6] = -e_{12} \\
[e_3, e_7] = -2e_{13} & [e_4, e_5] = e_{12} \\
[e_4, e_6] = e_{13} &
\end{array}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{4A}(5, 13)$

m4A513 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1,e_2]&=e_3 & [e_1,e_3]&=e_4\\
[e_1,e_4]&=e_5 & [e_1,e_5]&=e_6\\
[e_1,e_6]&=e_7 & [e_1,e_7]&=e_8\\
[e_1,e_8]&=e_9 & [e_1,e_9]&=e_{10}\\
[e_1,e_{10}]&=e_{11} & [e_1,e_{11}]&=e_{12}\\
[e_1,e_{12}]&=e_{13} & [e_2,e_5]&=e_{10}\\
[e_2,e_6]&=2e_{11} & [e_2,e_7]&=\alpha_{2,7}^{12}e_{12}\\
[e_2,e_8]&=\alpha_{2,8}^{13}e_{13} & [e_3,e_4]&=-e_{10}\\
[e_3,e_5]&=-e_{11} & [e_3,e_6]&=\alpha_{3,6}^{12}e_{12}\\
[e_3,e_7]&=\alpha_{3,7}^{13}e_{13} & [e_4,e_5]&=\alpha_{4,5}^{12}e_{12}\\
[e_4,e_6]&=\alpha_{4,6}^{13}e_{13}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1,e_2,e_6):&\quad -\alpha_{2,7}^{12}-\alpha_{3,6}^{12}+2 &&=0\\
(e_1,e_3,e_5):&\quad -\alpha_{3,6}^{12}-\alpha_{4,5}^{12}-1 &&=0\\
(e_1,e_2,e_7):&\quad \alpha_{2,7}^{12}-\alpha_{2,8}^{13}-\alpha_{3,7}^{13} &&=0\\
(e_1,e_3,e_6):&\quad \alpha_{3,6}^{12}-\alpha_{3,7}^{13}-\alpha_{4,6}^{13} &&=0\\
(e_1,e_4,e_5):&\quad \alpha_{4,5}^{12}-\alpha_{4,6}^{13} &&=0
\end{aligned}
$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$
\begin{aligned}
\alpha_{3,7}^{13}&\to x_1\\
\alpha_{2,7}^{12}&\to x_2\\
\alpha_{4,6}^{13}&\to x_3\\
\alpha_{3,6}^{12}&\to x_4\\
\alpha_{4,5}^{12}&\to x_5\\
\alpha_{2,8}^{13}&\to x_6
\end{aligned}
$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_6):&\quad -x_2 - x_4 + 2 &= 0\\
(e_1, e_3, e_5):&\quad -x_4 - x_5 - 1 &= 0\\
(e_1, e_2, e_7):&\quad -x_1 + x_2 - x_6 &= 0\\
(e_1, e_3, e_6):&\quad -x_1 - x_3 + x_4 &= 0\\
(e_1, e_4, e_5):&\quad -x_3 + x_5 &= 0
\end{aligned}
$$

Groebner basis (6 variables, 5 linear, 0 nonlinear)

$$3x_1 + 2x_6 - 5 = 0$$

$$3x_2 - x_6 - 5 = 0$$

$$3x_3 - x_6 + 4 = 0$$

$$3x_4 + x_6 - 1 = 0$$

$$3x_5 - x_6 + 4 = 0$$

$\mathfrak{m}_{6A}(5, 13)$

m6A513 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4\\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6\\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8\\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10}\\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12}\\
[e_1, e_{12}] &= e_{13} & [e_2, e_3] &= e_8\\
[e_2, e_4] &= e_9 & [e_2, e_5] &= \alpha_{2,5}^{10} e_{10}\\
[e_2, e_6] &= \alpha_{2,6}^{11} e_{11} & [e_2, e_7] &= \alpha_{2,7}^{12} e_{12}\\
[e_2, e_8] &= \alpha_{2,8}^{13} e_{13} & [e_3, e_4] &= \alpha_{3,4}^{10} e_{10}\\
[e_3, e_5] &= \alpha_{3,5}^{11} e_{11} & [e_3, e_6] &= \alpha_{3,6}^{12} e_{12}\\
[e_3, e_7] &= \alpha_{3,7}^{13} e_{13} & [e_4, e_5] &= \alpha_{4,5}^{12} e_{12}\\
[e_4, e_6] &= \alpha_{4,6}^{13} e_{13}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_4): &\quad -\alpha_{2,5}^{10} - \alpha_{3,4}^{10} + 1 &&= 0\\
(e_1, e_2, e_5): &\quad \alpha_{2,5}^{10} - \alpha_{2,6}^{11} - \alpha_{3,5}^{11} &&= 0\\
(e_1, e_3, e_4): &\quad \alpha_{3,4}^{10} - \alpha_{3,5}^{11} &&= 0\\
(e_1, e_2, e_6): &\quad \alpha_{2,6}^{11} - \alpha_{2,7}^{12} - \alpha_{3,6}^{12} &&= 0\\
(e_1, e_3, e_5): &\quad \alpha_{3,5}^{11} - \alpha_{3,6}^{12} - \alpha_{4,5}^{12} &&= 0\\
(e_1, e_2, e_7): &\quad \alpha_{2,7}^{12} - \alpha_{2,8}^{13} - \alpha_{3,7}^{13} &&= 0\\
(e_1, e_3, e_6): &\quad \alpha_{3,6}^{12} - \alpha_{3,7}^{13} - \alpha_{4,6}^{13} &&= 0\\
(e_1, e_4, e_5): &\quad \alpha_{4,5}^{12} - \alpha_{4,6}^{13} &&= 0
\end{aligned}
$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$
\begin{aligned}
\alpha_{2,6}^{11} &\to x_1\\
\alpha_{3,7}^{13} &\to x_2\\
\alpha_{2,7}^{12} &\to x_3\\
\alpha_{4,6}^{13} &\to x_4\\
\alpha_{3,6}^{12} &\to x_5\\
\alpha_{3,5}^{11} &\to x_6\\
\alpha_{4,5}^{12} &\to x_7\\
\alpha_{3,4}^{10} &\to x_8\\
\alpha_{2,8}^{13} &\to x_9\\
\alpha_{2,5}^{10} &\to x_{10}
\end{aligned}
$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_4): &\quad -x_{10} - x_8 + 1 &&= 0\\
(e_1, e_2, e_5): &\quad -x_1 + x_{10} - x_6 &&= 0\\
(e_1, e_3, e_4): &\quad -x_6 + x_8 &&= 0\\
(e_1, e_2, e_6): &\quad x_1 - x_3 - x_5 &&= 0\\
(e_1, e_3, e_5): &\quad -x_5 + x_6 - x_7 &&= 0\\
(e_1, e_2, e_7): &\quad -x_2 + x_3 - x_9 &&= 0\\
(e_1, e_3, e_6): &\quad -x_2 - x_4 + x_5 &&= 0\\
(e_1, e_4, e_5): &\quad -x_4 + x_7 &&= 0
\end{aligned}
$$

Groebner basis (10 variables, 8 linear, 0 nonlinear)

$$x_1 - 2x_{10} + 1 = 0$$
$$-5x_{10} + 3x_2 + 2x_9 + 3 = 0$$
$$-5x_{10} + 3x_3 - x_9 + 3 = 0$$
$$4x_{10} + 3x_4 - x_9 - 3 = 0$$
$$-x_{10} + 3x_5 + x_9 = 0$$
$$x_{10} + x_6 - 1 = 0$$
$$4x_{10} + 3x_7 - x_9 - 3 = 0$$
$$x_{10} + x_8 - 1 = 0$$

$\mathfrak{m}_{1A}(6, 13)$

m1A613 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_2, e_7] &= e_{13} \\
[e_3, e_6] &= -e_{13} & [e_4, e_5] &= e_{13}
\end{aligned}$$

No non-trivial Jacobi tests

$\mathfrak{m}_{3A}(6, 13)$

m3A613 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_2, e_5] &= e_{11} \\
[e_2, e_6] &= 2e_{12} & [e_2, e_7] &= \alpha_{2,7}^{13} e_{13} \\
[e_3, e_4] &= -e_{11} & [e_3, e_5] &= -e_{12} \\
[e_3, e_6] &= \alpha_{3,6}^{13} e_{13} & [e_4, e_5] &= \alpha_{4,5}^{13} e_{13}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_6): &\quad -\alpha_{2,7}^{13} - \alpha_{3,6}^{13} + 2 &= 0 \\
(e_1, e_3, e_5): &\quad -\alpha_{3,6}^{13} - \alpha_{4,5}^{13} - 1 &= 0
\end{aligned}
$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{2,7}^{13} \to x_1$$

$$\alpha_{3,6}^{13} \to x_2$$

$$\alpha_{4,5}^{13} \to x_3$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_6): &\quad -x_1 - x_2 + 2 &= 0 \\
(e_1, e_3, e_5): &\quad -x_2 - x_3 - 1 &= 0
\end{aligned}
$$

Groebner basis (3 variables, 2 linear, 0 nonlinear)

$$x_1 - x_3 - 3 = 0$$

$$x_2 + x_3 + 1 = 0$$

$\mathfrak{m}_{5A}(6,13)$

m5A613 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4\\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6\\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8\\
[e_1,e_8] &= e_9 & [e_1,e_9] &= e_{10}\\
[e_1,e_{10}] &= e_{11} & [e_1,e_{11}] &= e_{12}\\
[e_1,e_{12}] &= e_{13} & [e_2,e_3] &= e_9\\
[e_2,e_4] &= e_{10} & [e_2,e_5] &= \alpha_{2,5}^{11}e_{11}\\
[e_2,e_6] &= \alpha_{2,6}^{12}e_{12} & [e_2,e_7] &= \alpha_{2,7}^{13}e_{13}\\
[e_3,e_4] &= \alpha_{3,4}^{11}e_{11} & [e_3,e_5] &= \alpha_{3,5}^{12}e_{12}\\
[e_3,e_6] &= \alpha_{3,6}^{13}e_{13} & [e_4,e_5] &= \alpha_{4,5}^{13}e_{13}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1,e_2,e_4):&\quad -\alpha_{2,5}^{11}-\alpha_{3,4}^{11}+1 &&= 0\\
(e_1,e_2,e_5):&\quad \alpha_{2,5}^{11}-\alpha_{2,6}^{12}-\alpha_{3,5}^{12} &&= 0\\
(e_1,e_3,e_4):&\quad \alpha_{3,4}^{11}-\alpha_{3,5}^{12} &&= 0\\
(e_1,e_2,e_6):&\quad \alpha_{2,6}^{12}-\alpha_{2,7}^{13}-\alpha_{3,6}^{13} &&= 0\\
(e_1,e_3,e_5):&\quad \alpha_{3,5}^{12}-\alpha_{3,6}^{13}-\alpha_{4,5}^{13} &&= 0
\end{aligned}
$$

Infinite number of solutions.

How the solution(s) were or were not found:

Change variables

$$\alpha_{3,6}^{13} \to x_1$$

$$\alpha_{4,5}^{13} \to x_2$$

$$\alpha_{2,7}^{13} \to x_3$$

$$\alpha_{3,5}^{12} \to x_4$$

$$\alpha_{2,5}^{11} \to x_5$$

$$\alpha_{2,6}^{12} \to x_6$$

$$\alpha_{3,4}^{11} \to x_7$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_4): &\quad -x_5 - x_7 + 1 &&= 0\\
(e_1, e_2, e_5): &\quad -x_4 + x_5 - x_6 &&= 0\\
(e_1, e_3, e_4): &\quad -x_4 + x_7 &&= 0\\
(e_1, e_2, e_6): &\quad -x_1 - x_3 + x_6 &&= 0\\
(e_1, e_3, e_5): &\quad -x_1 - x_2 + x_4 &&= 0
\end{aligned}
$$

Groebner basis (7 variables, 5 linear, 0 nonlinear)

$$x_1 + x_3 + 2x_7 - 1 = 0$$
$$x_2 - x_3 - 3x_7 + 1 = 0$$
$$x_4 - x_7 = 0$$
$$x_5 + x_7 - 1 = 0$$
$$x_6 + 2x_7 - 1 = 0$$

$\mathfrak{m}_{2A}(7, 13)$

m2A713 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4\\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6\\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8\\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10}\\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12}\\
[e_1, e_{12}] &= e_{13} & [e_2, e_5] &= e_{12}\\
[e_2, e_6] &= 2e_{13} & [e_3, e_4] &= -e_{12}\\
[e_3, e_5] &= -e_{13}
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{4A}(7, 13)$

m4A713 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_2, e_3] &= e_{10} \\
[e_2, e_4] &= e_{11} & [e_2, e_5] &= \alpha_{2,5}^{12} e_{12} \\
[e_2, e_6] &= \alpha_{2,6}^{13} e_{13} & [e_3, e_4] &= \alpha_{3,4}^{12} e_{12} \\
[e_3, e_5] &= \alpha_{3,5}^{13} e_{13}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_4): &\quad -\alpha_{2,5}^{12} - \alpha_{3,4}^{12} + 1 &= 0 \\
(e_1, e_2, e_5): &\quad \alpha_{2,5}^{12} - \alpha_{2,6}^{13} - \alpha_{3,5}^{13} &= 0 \\
(e_1, e_3, e_4): &\quad \alpha_{3,4}^{12} - \alpha_{3,5}^{13} &= 0
\end{aligned}
$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$
\begin{aligned}
\alpha_{3,4}^{12} &\to x_1 \\
\alpha_{2,6}^{13} &\to x_2 \\
\alpha_{2,5}^{12} &\to x_3 \\
\alpha_{3,5}^{13} &\to x_4
\end{aligned}
$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_4): &\quad -x_1 - x_3 + 1 &= 0 \\
(e_1, e_2, e_5): &\quad -x_2 + x_3 - x_4 &= 0 \\
(e_1, e_3, e_4): &\quad x_1 - x_4 &= 0
\end{aligned}
$$

Groebner basis (4 variables, 3 linear, 0 nonlinear)

$$
\begin{aligned}
x_1 - x_4 &= 0 \\
x_2 + 2x_4 - 1 &= 0 \\
x_3 + x_4 - 1 &= 0
\end{aligned}
$$

$\mathfrak{m}_{1A}(8,13)$

m1A813 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4 \\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6 \\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8 \\
[e_1,e_8] &= e_9 & [e_1,e_9] &= e_{10} \\
[e_1,e_{10}] &= e_{11} & [e_1,e_{11}] &= e_{12} \\
[e_1,e_{12}] &= e_{13} & [e_2,e_5] &= e_{13} \\
[e_3,e_4] &= -e_{13}
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{3A}(8,13)$

m3A813 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4 \\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6 \\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8 \\
[e_1,e_8] &= e_9 & [e_1,e_9] &= e_{10} \\
[e_1,e_{10}] &= e_{11} & [e_1,e_{11}] &= e_{12} \\
[e_1,e_{12}] &= e_{13} & [e_2,e_3] &= e_{11} \\
[e_2,e_4] &= e_{12} & [e_2,e_5] &= \alpha_{2,5}^{13} e_{13} \\
[e_3,e_4] &= \alpha_{3,4}^{13} e_{13}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$(e_1,e_2,e_4): \quad -\alpha_{2,5}^{13} - \alpha_{3,4}^{13} + 1 = 0$$

Infinite number of solutions.

How the solution(s) were or were not found:

Change variables

$$
\begin{aligned}
\alpha_{3,4}^{13} &\to x_1 \\
\alpha_{2,5}^{13} &\to x_2
\end{aligned}
$$

Jacobi Tests

$$(e_1, e_2, e_4): \quad -x_1 - x_2 + 1 \qquad = 0$$

Groebner basis (2 variables, 1 linear, 0 nonlinear)

$$x_1 + x_2 - 1 = 0$$

$\mathfrak{m}_{2A}(9, 13)$

m2A913 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_2, e_3] &= e_{12} \\
[e_2, e_4] &= e_{13}
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{1A}(10, 13)$

m1A1013 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_2, e_3] &= e_{13}
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{2A}(2,14)$

m2A214 (this line included for string searching purposes)
Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4\\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6\\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8\\
[e_1,e_8] &= e_9 & [e_1,e_9] &= e_{10}\\
[e_1,e_{10}] &= e_{11} & [e_1,e_{11}] &= e_{12}\\
[e_1,e_{12}] &= e_{13} & [e_1,e_{13}] &= e_{14}\\
[e_2,e_{11}] &= e_{13} & [e_2,e_{12}] &= 5e_{14}\\
[e_3,e_{10}] &= -e_{13} & [e_3,e_{11}] &= -4e_{14}\\
[e_4,e_9] &= e_{13} & [e_4,e_{10}] &= 3e_{14}\\
[e_5,e_8] &= -e_{13} & [e_5,e_9] &= -2e_{14}\\
[e_6,e_7] &= e_{13} & [e_6,e_8] &= e_{14}
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{4A}(2,14)$

m4A214 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
&[e_1, e_2] = e_3 && [e_1, e_3] = e_4 \\
&[e_1, e_4] = e_5 && [e_1, e_5] = e_6 \\
&[e_1, e_6] = e_7 && [e_1, e_7] = e_8 \\
&[e_1, e_8] = e_9 && [e_1, e_9] = e_{10} \\
&[e_1, e_{10}] = e_{11} && [e_1, e_{11}] = e_{12} \\
&[e_1, e_{12}] = e_{13} && [e_1, e_{13}] = e_{14} \\
&[e_2, e_9] = e_{11} && [e_2, e_{10}] = 4e_{12} \\
&[e_2, e_{11}] = \alpha_{2,11}^{13} e_{13} && [e_2, e_{12}] = \alpha_{2,12}^{14} e_{14} \\
&[e_3, e_8] = -e_{11} && [e_3, e_9] = -3e_{12} \\
&[e_3, e_{10}] = \alpha_{3,10}^{13} e_{13} && [e_3, e_{11}] = \alpha_{3,11}^{14} e_{14} \\
&[e_4, e_7] = e_{11} && [e_4, e_8] = 2e_{12} \\
&[e_4, e_9] = \alpha_{4,9}^{13} e_{13} && [e_4, e_{10}] = \alpha_{4,10}^{14} e_{14} \\
&[e_5, e_6] = -e_{11} && [e_5, e_7] = -e_{12} \\
&[e_5, e_8] = \alpha_{5,8}^{13} e_{13} && [e_5, e_9] = \alpha_{5,9}^{14} e_{14} \\
&[e_6, e_7] = \alpha_{6,7}^{13} e_{13} && [e_6, e_8] = \alpha_{6,8}^{14} e_{14}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_{10}): \quad & -\alpha_{2,11}^{13} - \alpha_{3,10}^{13} + 4 && = 0 \\
(e_1, e_3, e_9): \quad & -\alpha_{3,10}^{13} - \alpha_{4,9}^{13} - 3 && = 0 \\
(e_1, e_4, e_8): \quad & -\alpha_{4,9}^{13} - \alpha_{5,8}^{13} + 2 && = 0 \\
(e_1, e_5, e_7): \quad & -\alpha_{5,8}^{13} - \alpha_{6,7}^{13} - 1 && = 0 \\
(e_2, e_3, e_8): \quad & -\alpha_{2,11}^{13} && = 0 \\
(e_2, e_4, e_7): \quad & \alpha_{2,11}^{13} && = 0 \\
(e_2, e_5, e_6): \quad & -\alpha_{2,11}^{13} && = 0 \\
(e_1, e_2, e_{11}): \quad & \alpha_{2,11}^{13} - \alpha_{2,12}^{14} - \alpha_{3,11}^{14} && = 0 \\
(e_1, e_3, e_{10}): \quad & \alpha_{3,10}^{13} - \alpha_{3,11}^{14} - \alpha_{4,10}^{14} && = 0 \\
(e_1, e_4, e_9): \quad & -\alpha_{4,10}^{14} + \alpha_{4,9}^{13} - \alpha_{5,9}^{14} && = 0 \\
(e_1, e_5, e_8): \quad & \alpha_{5,8}^{13} - \alpha_{5,9}^{14} - \alpha_{6,8}^{14} && = 0 \\
(e_1, e_6, e_7): \quad & \alpha_{6,7}^{13} - \alpha_{6,8}^{14} && = 0 \\
(e_2, e_3, e_9): \quad & -3\alpha_{2,12}^{14} - \alpha_{3,11}^{14} && = 0 \\
(e_2, e_4, e_8): \quad & 2\alpha_{2,12}^{14} && = 0 \\
(e_2, e_5, e_7): \quad & -\alpha_{2,12}^{14} && = 0 \\
(e_3, e_4, e_7): \quad & \alpha_{3,11}^{14} && = 0 \\
(e_3, e_5, e_6): \quad & -\alpha_{3,11}^{14} && = 0
\end{aligned}
$$

No solutions.
How the solution(s) were or were not found:
Change variables

$$
\begin{gathered}
\alpha_{5,9}^{14} \to x_1 \\
\alpha_{6,7}^{13} \to x_2 \\
\alpha_{4,9}^{13} \to x_3 \\
\alpha_{3,10}^{13} \to x_4 \\
\alpha_{6,8}^{14} \to x_5 \\
\alpha_{5,8}^{13} \to x_6 \\
\alpha_{4,10}^{14} \to x_7 \\
\alpha_{3,11}^{14} \to x_8 \\
\alpha_{2,11}^{13} \to x_9 \\
\alpha_{2,12}^{14} \to x_{10}
\end{gathered}
$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_{10}): &\quad -x_4 - x_9 + 4 &&= 0\\
(e_1, e_3, e_9): &\quad -x_3 - x_4 - 3 &&= 0\\
(e_1, e_4, e_8): &\quad -x_3 - x_6 + 2 &&= 0\\
(e_1, e_5, e_7): &\quad -x_2 - x_6 - 1 &&= 0\\
(e_2, e_3, e_8): &\quad -x_9 &&= 0\\
(e_2, e_4, e_7): &\quad x_9 &&= 0\\
(e_2, e_5, e_6): &\quad -x_9 &&= 0\\
(e_1, e_2, e_{11}): &\quad -x_{10} - x_8 + x_9 &&= 0\\
(e_1, e_3, e_{10}): &\quad x_4 - x_7 - x_8 &&= 0\\
(e_1, e_4, e_9): &\quad -x_1 + x_3 - x_7 &&= 0\\
(e_1, e_5, e_8): &\quad -x_1 - x_5 + x_6 &&= 0\\
(e_1, e_6, e_7): &\quad x_2 - x_5 &&= 0\\
(e_2, e_3, e_9): &\quad -3x_{10} - x_8 &&= 0\\
(e_2, e_4, e_8): &\quad 2x_{10} &&= 0\\
(e_2, e_5, e_7): &\quad -x_{10} &&= 0\\
(e_3, e_4, e_7): &\quad x_8 &&= 0\\
(e_3, e_5, e_6): &\quad -x_8 &&= 0
\end{aligned}
$$

Groebner basis (10 variables, 1 linear, 0 nonlinear)

$$1 = 0$$

$\mathfrak{m}_{10A}(2, 14)$

m10A214 (this line included for string searching purposes)

Solution 1

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_1, e_{13}] &= e_{14} \\
[e_2, e_3] &= e_5 & [e_2, e_4] &= e_6 \\
[e_2, e_5] &= e_7 & [e_2, e_6] &= e_8 \\
[e_2, e_7] &= e_9 & [e_2, e_8] &= e_{10} \\
[e_2, e_9] &= e_{11} & [e_2, e_{10}] &= e_{12} \\
[e_2, e_{11}] &= e_{13} & [e_2, e_{12}] &= e_{14} \\
[e_3, e_4] &= 0 & [e_3, e_5] &= 0 \\
[e_3, e_6] &= 0 & [e_3, e_7] &= 0 \\
[e_3, e_8] &= 0 & [e_3, e_9] &= 0 \\
[e_3, e_{10}] &= 0 & [e_3, e_{11}] &= 0 \\
[e_4, e_5] &= 0 & [e_4, e_6] &= 0 \\
[e_4, e_7] &= 0 & [e_4, e_8] &= 0 \\
[e_4, e_9] &= 0 & [e_4, e_{10}] &= 0 \\
[e_5, e_6] &= 0 & [e_5, e_7] &= 0 \\
[e_5, e_8] &= 0 & [e_5, e_9] &= 0 \\
[e_6, e_7] &= 0 & [e_6, e_8] &= 0
\end{aligned}
$$

Solution 2

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_1, e_{11}] = e_{12}$$
$$[e_1, e_{12}] = e_{13} \qquad [e_1, e_{13}] = e_{14}$$
$$[e_2, e_3] = e_5 \qquad [e_2, e_4] = e_6$$
$$[e_2, e_5] = \frac{9e_7}{10} \qquad [e_2, e_6] = \frac{4e_8}{5}$$
$$[e_2, e_7] = \frac{5e_9}{7} \qquad [e_2, e_8] = \frac{9e_{10}}{14}$$
$$[e_2, e_9] = \frac{7e_{11}}{12} \qquad [e_2, e_{10}] = \frac{8e_{12}}{15}$$
$$[e_2, e_{11}] = \frac{27e_{13}}{55} \qquad [e_2, e_{12}] = \frac{5e_{14}}{11}$$
$$[e_3, e_4] = \frac{e_7}{10} \qquad [e_3, e_5] = \frac{e_8}{10}$$
$$[e_3, e_6] = \frac{3e_9}{35} \qquad [e_3, e_7] = \frac{e_{10}}{14}$$
$$[e_3, e_8] = \frac{5e_{11}}{84} \qquad [e_3, e_9] = \frac{e_{12}}{20}$$
$$[e_3, e_{10}] = \frac{7e_{13}}{165} \qquad [e_3, e_{11}] = \frac{2e_{14}}{55}$$
$$[e_4, e_5] = \frac{e_9}{70} \qquad [e_4, e_6] = \frac{e_{10}}{70}$$
$$[e_4, e_7] = \frac{e_{11}}{84} \qquad [e_4, e_8] = \frac{e_{12}}{105}$$
$$[e_4, e_9] = \frac{e_{13}}{132} \qquad [e_4, e_{10}] = \frac{e_{14}}{165}$$
$$[e_5, e_6] = \frac{e_{11}}{420} \qquad [e_5, e_7] = \frac{e_{12}}{420}$$
$$[e_5, e_8] = \frac{3e_{13}}{1540} \qquad [e_5, e_9] = \frac{e_{14}}{660}$$
$$[e_6, e_7] = \frac{e_{13}}{2310} \qquad [e_6, e_8] = \frac{e_{14}}{2310}$$

Original brackets:

$$
\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_1, e_9] = e_{10} \\
[e_1, e_{10}] = e_{11} & [e_1, e_{11}] = e_{12} \\
[e_1, e_{12}] = e_{13} & [e_1, e_{13}] = e_{14} \\
[e_2, e_3] = e_5 & [e_2, e_4] = e_6 \\
[e_2, e_5] = \alpha_{2,5}^{7} e_7 & [e_2, e_6] = \alpha_{2,6}^{8} e_8 \\
[e_2, e_7] = \alpha_{2,7}^{9} e_9 & [e_2, e_8] = \alpha_{2,8}^{10} e_{10} \\
[e_2, e_9] = \alpha_{2,9}^{11} e_{11} & [e_2, e_{10}] = \alpha_{2,10}^{12} e_{12} \\
[e_2, e_{11}] = \alpha_{2,11}^{13} e_{13} & [e_2, e_{12}] = \alpha_{2,12}^{14} e_{14} \\
[e_3, e_4] = \alpha_{3,4}^{7} e_7 & [e_3, e_5] = \alpha_{3,5}^{8} e_8 \\
[e_3, e_6] = \alpha_{3,6}^{9} e_9 & [e_3, e_7] = \alpha_{3,7}^{10} e_{10} \\
[e_3, e_8] = \alpha_{3,8}^{11} e_{11} & [e_3, e_9] = \alpha_{3,9}^{12} e_{12} \\
[e_3, e_{10}] = \alpha_{3,10}^{13} e_{13} & [e_3, e_{11}] = \alpha_{3,11}^{14} e_{14} \\
[e_4, e_5] = \alpha_{4,5}^{9} e_9 & [e_4, e_6] = \alpha_{4,6}^{10} e_{10} \\
[e_4, e_7] = \alpha_{4,7}^{11} e_{11} & [e_4, e_8] = \alpha_{4,8}^{12} e_{12} \\
[e_4, e_9] = \alpha_{4,9}^{13} e_{13} & [e_4, e_{10}] = \alpha_{4,10}^{14} e_{14} \\
[e_5, e_6] = \alpha_{5,6}^{11} e_{11} & [e_5, e_7] = \alpha_{5,7}^{12} e_{12} \\
[e_5, e_8] = \alpha_{5,8}^{13} e_{13} & [e_5, e_9] = \alpha_{5,9}^{14} e_{14} \\
[e_6, e_7] = \alpha_{6,7}^{13} e_{13} & [e_6, e_8] = \alpha_{6,8}^{14} e_{14}
\end{array}
$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1,e_2,e_4):&\quad -\alpha^{7}_{2,5}-\alpha^{7}_{3,4}+1 &&=0\\
(e_1,e_2,e_5):&\quad \alpha^{7}_{2,5}-\alpha^{8}_{2,6}-\alpha^{8}_{3,5} &&=0\\
(e_1,e_3,e_4):&\quad \alpha^{7}_{3,4}-\alpha^{8}_{3,5} &&=0\\
(e_1,e_2,e_6):&\quad \alpha^{8}_{2,6}-\alpha^{9}_{2,7}-\alpha^{9}_{3,6} &&=0\\
(e_1,e_3,e_5):&\quad \alpha^{8}_{3,5}-\alpha^{9}_{3,6}-\alpha^{9}_{4,5} &&=0\\
(e_2,e_3,e_4):&\quad \alpha^{9}_{2,7}\alpha^{7}_{3,4}-\alpha^{9}_{3,6}+\alpha^{9}_{4,5} &&=0\\
(e_1,e_2,e_7):&\quad \alpha^{9}_{2,7}-\alpha^{10}_{2,8}-\alpha^{10}_{3,7} &&=0\\
(e_1,e_3,e_6):&\quad \alpha^{9}_{3,6}-\alpha^{10}_{3,7}-\alpha^{10}_{4,6} &&=0\\
(e_1,e_4,e_5):&\quad \alpha^{9}_{4,5}-\alpha^{10}_{4,6} &&=0\\
(e_2,e_3,e_5):&\quad -\alpha^{7}_{2,5}\alpha^{10}_{3,7}+\alpha^{10}_{2,8}\alpha^{8}_{3,5} &&=0\\
(e_1,e_2,e_8):&\quad \alpha^{10}_{2,8}-\alpha^{11}_{2,9}-\alpha^{11}_{3,8} &&=0\\
(e_1,e_3,e_7):&\quad \alpha^{10}_{3,7}-\alpha^{11}_{3,8}-\alpha^{11}_{4,7} &&=0\\
(e_1,e_4,e_6):&\quad \alpha^{10}_{4,6}-\alpha^{11}_{4,7}-\alpha^{11}_{5,6} &&=0\\
(e_2,e_3,e_6):&\quad -\alpha^{8}_{2,6}\alpha^{11}_{3,8}+\alpha^{11}_{2,9}\alpha^{9}_{3,6}-\alpha^{11}_{5,6} &&=0\\
(e_2,e_4,e_5):&\quad -\alpha^{7}_{2,5}\alpha^{11}_{4,7}+\alpha^{11}_{2,9}\alpha^{9}_{4,5}+\alpha^{11}_{5,6} &&=0\\
(e_1,e_2,e_9):&\quad -\alpha^{12}_{2,10}+\alpha^{11}_{2,9}-\alpha^{12}_{3,9} &&=0\\
(e_1,e_3,e_8):&\quad \alpha^{11}_{3,8}-\alpha^{12}_{3,9}-\alpha^{12}_{4,8} &&=0\\
(e_1,e_4,e_7):&\quad \alpha^{11}_{4,7}-\alpha^{12}_{4,8}-\alpha^{12}_{5,7} &&=0\\
(e_1,e_5,e_6):&\quad \alpha^{11}_{5,6}-\alpha^{12}_{5,7} &&=0\\
(e_2,e_3,e_7):&\quad \alpha^{12}_{2,10}\alpha^{10}_{3,7}-\alpha^{9}_{2,7}\alpha^{12}_{3,9}-\alpha^{12}_{5,7} &&=0\\
(e_2,e_4,e_6):&\quad \alpha^{12}_{2,10}\alpha^{10}_{4,6}-\alpha^{8}_{2,6}\alpha^{12}_{4,8} &&=0\\
(e_3,e_4,e_5):&\quad \alpha^{7}_{3,4}\alpha^{12}_{5,7}-\alpha^{8}_{3,5}\alpha^{12}_{4,8}+\alpha^{12}_{3,9}\alpha^{9}_{4,5} &&=0\\
(e_1,e_2,e_{10}):&\quad \alpha^{12}_{2,10}-\alpha^{13}_{2,11}-\alpha^{13}_{3,10} &&=0\\
(e_1,e_3,e_9):&\quad -\alpha^{13}_{3,10}+\alpha^{12}_{3,9}-\alpha^{13}_{4,9} &&=0\\
(e_1,e_4,e_8):&\quad \alpha^{12}_{4,8}-\alpha^{13}_{4,9}-\alpha^{13}_{5,8} &&=0\\
(e_1,e_5,e_7):&\quad \alpha^{12}_{5,7}-\alpha^{13}_{5,8}-\alpha^{13}_{6,7} &&=0\\
(e_2,e_3,e_8):&\quad \alpha^{13}_{2,11}\alpha^{11}_{3,8}-\alpha^{10}_{2,8}\alpha^{13}_{3,10}-\alpha^{13}_{5,8} &&=0\\
(e_2,e_4,e_7):&\quad \alpha^{13}_{2,11}\alpha^{11}_{4,7}-\alpha^{9}_{2,7}\alpha^{13}_{4,9}-\alpha^{13}_{6,7} &&=0\\
(e_2,e_5,e_6):&\quad \alpha^{13}_{2,11}\alpha^{11}_{5,6}+\alpha^{7}_{2,5}\alpha^{13}_{6,7}-\alpha^{8}_{2,6}\alpha^{13}_{5,8} &&=0\\
(e_3,e_4,e_6):&\quad \alpha^{13}_{3,10}\alpha^{10}_{4,6}+\alpha^{7}_{3,4}\alpha^{13}_{6,7}-\alpha^{9}_{3,6}\alpha^{13}_{4,9} &&=0\\
(e_1,e_2,e_{11}):&\quad \alpha^{13}_{2,11}-\alpha^{14}_{2,12}-\alpha^{14}_{3,11} &&=0\\
(e_1,e_3,e_{10}):&\quad \alpha^{13}_{3,10}-\alpha^{14}_{3,11}-\alpha^{14}_{4,10} &&=0\\
(e_1,e_4,e_9):&\quad -\alpha^{14}_{4,10}+\alpha^{13}_{4,9}-\alpha^{14}_{5,9} &&=0\\
(e_1,e_5,e_8):&\quad \alpha^{13}_{5,8}-\alpha^{14}_{5,9}-\alpha^{14}_{6,8} &&=0\\
(e_1,e_6,e_7):&\quad \alpha^{13}_{6,7}-\alpha^{14}_{6,8} &&=0\\
(e_2,e_3,e_9):&\quad \alpha^{14}_{2,12}\alpha^{12}_{3,9}-\alpha^{11}_{2,9}\alpha^{14}_{3,11}-\alpha^{14}_{5,9} &&=0\\
(e_2,e_4,e_8):&\quad \alpha^{14}_{2,12}\alpha^{12}_{4,8}-\alpha^{10}_{2,8}\alpha^{14}_{4,10}-\alpha^{14}_{6,8} &&=0\\
(e_2,e_5,e_7):&\quad \alpha^{14}_{2,12}\alpha^{12}_{5,7}-\alpha^{9}_{2,7}\alpha^{14}_{5,9} &&=0\\
(e_3,e_4,e_7):&\quad \alpha^{14}_{3,11}\alpha^{11}_{4,7}-\alpha^{10}_{3,7}\alpha^{14}_{4,10} &&=0\\
(e_3,e_5,e_6):&\quad \alpha^{14}_{3,11}\alpha^{11}_{5,6}+\alpha^{8}_{3,5}\alpha^{14}_{6,8}-\alpha^{9}_{3,6}\alpha^{14}_{5,9} &&=0
\end{aligned}$$

Solution 1:

$$\alpha^{14}_{5,9} = 0$$
$$\alpha^{13}_{6,7} = 0$$
$$\alpha^{9}_{3,6} = 0$$
$$\alpha^{7}_{2,5} = 1$$
$$\alpha^{13}_{5,8} = 0$$
$$\alpha^{11}_{4,7} = 0$$
$$\alpha^{14}_{3,11} = 0$$
$$\alpha^{10}_{4,6} = 0$$
$$\alpha^{12}_{3,9} = 0$$
$$\alpha^{11}_{3,8} = 0$$
$$\alpha^{9}_{2,7} = 1$$
$$\alpha^{12}_{2,10} = 1$$
$$\alpha^{12}_{5,7} = 0$$
$$\alpha^{11}_{2,9} = 1$$
$$\alpha^{14}_{6,8} = 0$$
$$\alpha^{10}_{2,8} = 1$$
$$\alpha^{10}_{3,7} = 0$$
$$\alpha^{13}_{3,10} = 0$$
$$\alpha^{13}_{4,9} = 0$$
$$\alpha^{11}_{5,6} = 0$$
$$\alpha^{8}_{2,6} = 1$$
$$\alpha^{14}_{4,10} = 0$$
$$\alpha^{9}_{4,5} = 0$$
$$\alpha^{14}_{2,12} = 1$$
$$\alpha^{7}_{3,4} = 0$$
$$\alpha^{12}_{4,8} = 0$$
$$\alpha^{13}_{2,11} = 1$$
$$\alpha^{8}_{3,5} = 0$$

Solution 2:

$$\alpha^{14}_{5,9} = 1/660$$
$$\alpha^{13}_{6,7} = 1/2310$$
$$\alpha^{9}_{3,6} = 3/35$$
$$\alpha^{7}_{2,5} = 9/10$$
$$\alpha^{13}_{5,8} = 3/1540$$
$$\alpha^{11}_{4,7} = 1/84$$
$$\alpha^{14}_{3,11} = 2/55$$
$$\alpha^{10}_{4,6} = 1/70$$
$$\alpha^{12}_{3,9} = 1/20$$
$$\alpha^{11}_{3,8} = 5/84$$
$$\alpha^{9}_{2,7} = 5/7$$
$$\alpha^{12}_{2,10} = 8/15$$
$$\alpha^{12}_{5,7} = 1/420$$
$$\alpha^{11}_{2,9} = 7/12$$
$$\alpha^{14}_{6,8} = 1/2310$$
$$\alpha^{10}_{2,8} = 9/14$$
$$\alpha^{10}_{3,7} = 1/14$$
$$\alpha^{13}_{3,10} = 7/165$$
$$\alpha^{13}_{4,9} = 1/132$$
$$\alpha^{11}_{5,6} = 1/420$$
$$\alpha^{8}_{2,6} = 4/5$$
$$\alpha^{14}_{4,10} = 1/165$$
$$\alpha^{9}_{4,5} = 1/70$$
$$\alpha^{14}_{2,12} = 5/11$$
$$\alpha^{7}_{3,4} = 1/10$$
$$\alpha^{12}_{4,8} = 1/105$$
$$\alpha^{13}_{2,11} = 27/55$$
$$\alpha^{8}_{3,5} = 1/10$$

How the solution(s) were or were not found:
Change variables

$$\alpha^{14}_{5,9} \to x_1$$

$$\alpha^{13}_{6,7} \to x_2$$

$$\alpha^{9}_{3,6} \to x_3$$

$$\alpha^{7}_{2,5} \to x_4$$

$$\alpha^{13}_{5,8} \to x_5$$

$$\alpha^{11}_{4,7} \to x_6$$

$$\alpha^{14}_{3,11} \to x_7$$

$$\alpha^{10}_{4,6} \to x_8$$

$$\alpha^{12}_{3,9} \to x_9$$

$$\alpha^{11}_{3,8} \to x_{10}$$

$$\alpha^{9}_{2,7} \to x_{11}$$

$$\alpha^{12}_{2,10} \to x_{12}$$

$$\alpha^{12}_{5,7} \to x_{13}$$

$$\alpha^{11}_{2,9} \to x_{14}$$

$$\alpha^{14}_{6,8} \to x_{15}$$

$$\alpha^{10}_{2,8} \to x_{16}$$

$$\alpha^{10}_{3,7} \to x_{17}$$

$$\alpha^{13}_{3,10} \to x_{18}$$

$$\alpha^{13}_{4,9} \to x_{19}$$

$$\alpha^{11}_{5,6} \to x_{20}$$

$$\alpha^{8}_{2,6} \to x_{21}$$

$$\alpha^{14}_{4,10} \to x_{22}$$

$$\alpha^{9}_{4,5} \to x_{23}$$

$$\alpha^{14}_{2,12} \to x_{24}$$

$$\alpha^{7}_{3,4} \to x_{25}$$

$$\alpha^{12}_{4,8} \to x_{26}$$

$$\alpha^{13}_{2,11} \to x_{27}$$

$$\alpha^{8}_{3,5} \to x_{28}$$

Jacobi Tests

$$
\begin{aligned}
(e_1,e_2,e_4):&\quad -x_{25}-x_4+1 &&=0\\
(e_1,e_2,e_5):&\quad -x_{21}-x_{28}+x_4 &&=0\\
(e_1,e_3,e_4):&\quad x_{25}-x_{28} &&=0\\
(e_1,e_2,e_6):&\quad -x_{11}+x_{21}-x_3 &&=0\\
(e_1,e_3,e_5):&\quad -x_{23}+x_{28}-x_3 &&=0\\
(e_2,e_3,e_4):&\quad x_{11}x_{25}+x_{23}-x_3 &&=0\\
(e_1,e_2,e_7):&\quad x_{11}-x_{16}-x_{17} &&=0\\
(e_1,e_3,e_6):&\quad -x_{17}+x_3-x_8 &&=0\\
(e_1,e_4,e_5):&\quad x_{23}-x_8 &&=0\\
(e_2,e_3,e_5):&\quad x_{16}x_{28}-x_{17}x_4 &&=0\\
(e_1,e_2,e_8):&\quad -x_{10}-x_{14}+x_{16} &&=0\\
(e_1,e_3,e_7):&\quad -x_{10}+x_{17}-x_6 &&=0\\
(e_1,e_4,e_6):&\quad -x_{20}-x_6+x_8 &&=0\\
(e_2,e_3,e_6):&\quad -x_{10}x_{21}+x_{14}x_3-x_{20} &&=0\\
(e_2,e_4,e_5):&\quad x_{14}x_{23}+x_{20}-x_4x_6 &&=0\\
(e_1,e_2,e_9):&\quad -x_{12}+x_{14}-x_9 &&=0\\
(e_1,e_3,e_8):&\quad x_{10}-x_{26}-x_9 &&=0\\
(e_1,e_4,e_7):&\quad -x_{13}-x_{26}+x_6 &&=0\\
(e_1,e_5,e_6):&\quad -x_{13}+x_{20} &&=0\\
(e_2,e_3,e_7):&\quad -x_{11}x_9+x_{12}x_{17}-x_{13} &&=0\\
(e_2,e_4,e_6):&\quad x_{12}x_8-x_{21}x_{26} &&=0\\
(e_3,e_4,e_5):&\quad x_{13}x_{25}+x_{23}x_9-x_{26}x_{28} &&=0\\
(e_1,e_2,e_{10}):&\quad x_{12}-x_{18}-x_{27} &&=0\\
(e_1,e_3,e_9):&\quad -x_{18}-x_{19}+x_9 &&=0\\
(e_1,e_4,e_8):&\quad -x_{19}+x_{26}-x_5 &&=0\\
(e_1,e_5,e_7):&\quad x_{13}-x_2-x_5 &&=0\\
(e_2,e_3,e_8):&\quad x_{10}x_{27}-x_{16}x_{18}-x_5 &&=0\\
(e_2,e_4,e_7):&\quad -x_{11}x_{19}-x_2+x_{27}x_6 &&=0\\
(e_2,e_5,e_6):&\quad x_2x_4+x_{20}x_{27}-x_{21}x_5 &&=0\\
(e_3,e_4,e_6):&\quad x_{18}x_8-x_{19}x_3+x_2x_{25} &&=0\\
(e_1,e_2,e_{11}):&\quad -x_{24}+x_{27}-x_7 &&=0\\
(e_1,e_3,e_{10}):&\quad x_{18}-x_{22}-x_7 &&=0\\
(e_1,e_4,e_9):&\quad -x_1+x_{19}-x_{22} &&=0\\
(e_1,e_5,e_8):&\quad -x_1-x_{15}+x_5 &&=0\\
(e_1,e_6,e_7):&\quad -x_{15}+x_2 &&=0\\
(e_2,e_3,e_9):&\quad -x_1-x_{14}x_7+x_{24}x_9 &&=0\\
(e_2,e_4,e_8):&\quad -x_{15}-x_{16}x_{22}+x_{24}x_{26} &&=0\\
(e_2,e_5,e_7):&\quad -x_1x_{11}+x_{13}x_{24} &&=0\\
(e_3,e_4,e_7):&\quad -x_{17}x_{22}+x_6x_7 &&=0\\
(e_3,e_5,e_6):&\quad -x_1x_3+x_{15}x_{28}+x_{20}x_7 &&=0
\end{aligned}
$$

Groebner basis (28 variables, 3 linear, 25 nonlinear)

$$132x_1 - 9110x_{28}^5 + 2871x_{28}^4 - 396x_{28}^3 = 0$$
$$462x_2 + 14650x_{28}^5 - 3465x_{28}^4 = 0$$
$$-10x_{28}^5 + 21x_{28}^4 - 42x_{28}^3 + 84x_{28}^2 - 56x_{28} + 56x_3 = 0$$
$$x_{28} + x_4 - 1 = 0$$
$$-11490x_{28}^5 + 4389x_{28}^4 - 924x_{28}^3 + 308x_5 = 0$$
$$970x_{28}^5 - 1197x_{28}^4 + 630x_{28}^3 - 252x_{28}^2 + 168x_6 = 0$$
$$-8090x_{28}^5 + 4389x_{28}^4 - 1518x_{28}^3 + 396x_{28}^2 - 44x_{28} + 44x_7 = 0$$
$$10x_{28}^5 - 21x_{28}^4 + 42x_{28}^3 - 84x_{28}^2 + 56x_8 = 0$$
$$-70x_{28}^5 + 87x_{28}^4 - 48x_{28}^3 + 24x_{28}^2 - 4x_{28} + 4x_9 = 0$$
$$168x_{10} - 1030x_{28}^5 + 1323x_{28}^4 - 882x_{28}^3 + 756x_{28}^2 - 168x_{28} = 0$$
$$56x_{11} + 10x_{28}^5 - 21x_{28}^4 + 42x_{28}^3 - 84x_{28}^2 + 168x_{28} - 56 = 0$$
$$12x_{12} + 290x_{28}^5 - 369x_{28}^4 + 234x_{28}^3 - 180x_{28}^2 + 72x_{28} - 12 = 0$$
$$84x_{13} - 470x_{28}^5 + 567x_{28}^4 - 252x_{28}^3 = 0$$
$$6x_{14} + 40x_{28}^5 - 54x_{28}^4 + 45x_{28}^3 - 54x_{28}^2 + 30x_{28} - 6 = 0$$
$$462x_{15} + 14650x_{28}^5 - 3465x_{28}^4 = 0$$
$$56x_{16} + 30x_{28}^5 - 63x_{28}^4 + 126x_{28}^3 - 252x_{28}^2 + 224x_{28} - 56 = 0$$
$$28x_{17} - 10x_{28}^5 + 21x_{28}^4 - 42x_{28}^3 + 84x_{28}^2 - 28x_{28} = 0$$
$$264x_{18} - 17470x_{28}^5 + 13167x_{28}^4 - 5742x_{28}^3 + 1980x_{28}^2 - 264x_{28} = 0$$
$$264x_{19} + 12850x_{28}^5 - 7425x_{28}^4 + 2574x_{28}^3 - 396x_{28}^2 = 0$$
$$84x_{20} - 470x_{28}^5 + 567x_{28}^4 - 252x_{28}^3 = 0$$
$$x_{21} + 2x_{28} - 1 = 0$$
$$264x_{22} + 31070x_{28}^5 - 13167x_{28}^4 + 3366x_{28}^3 - 396x_{28}^2 = 0$$
$$56x_{23} + 10x_{28}^5 - 21x_{28}^4 + 42x_{28}^3 - 84x_{28}^2 = 0$$
$$88x_{24} + 24130x_{28}^5 - 15873x_{28}^4 + 6666x_{28}^3 - 2772x_{28}^2 + 704x_{28} - 88 = 0$$
$$x_{25} - x_{28} = 0$$
$$168x_{26} + 1910x_{28}^5 - 2331x_{28}^4 + 1134x_{28}^3 - 252x_{28}^2 = 0$$
$$88x_{27} + 7950x_{28}^5 - 7095x_{28}^4 + 3630x_{28}^3 - 1980x_{28}^2 + 616x_{28} - 88 = 0$$
$$10x_{28}^6 - x_{28}^5 = 0$$

Solution 1:

$$x_1 = 0$$

$$x_2 = 0$$
$$x_3 = 0$$
$$x_4 = 1$$
$$x_5 = 0$$
$$x_6 = 0$$
$$x_7 = 0$$
$$x_8 = 0$$
$$x_9 = 0$$
$$x_10 = 0$$
$$x_11 = 1$$
$$x_12 = 1$$
$$x_13 = 0$$
$$x_14 = 1$$
$$x_15 = 0$$
$$x_16 = 1$$
$$x_17 = 0$$
$$x_18 = 0$$
$$x_19 = 0$$
$$x_20 = 0$$
$$x_21 = 1$$
$$x_22 = 0$$
$$x_23 = 0$$
$$x_24 = 1$$
$$x_25 = 0$$
$$x_26 = 0$$
$$x_27 = 1$$
$$x_28 = 0$$

Solution 2:

$$x_1 = 1/660$$
$$x_2 = 1/2310$$
$$x_3 = 3/35$$
$$x_4 = 9/10$$

$$x_5 = 3/1540$$
$$x_6 = 1/84$$
$$x_7 = 2/55$$
$$x_8 = 1/70$$
$$x_9 = 1/20$$
$$x_10 = 5/84$$
$$x_11 = 5/7$$
$$x_12 = 8/15$$
$$x_13 = 1/420$$
$$x_14 = 7/12$$
$$x_15 = 1/2310$$
$$x_16 = 9/14$$
$$x_17 = 1/14$$
$$x_18 = 7/165$$
$$x_19 = 1/132$$
$$x_20 = 1/420$$
$$x_21 = 4/5$$
$$x_22 = 1/165$$
$$x_23 = 1/70$$
$$x_24 = 5/11$$
$$x_25 = 1/10$$
$$x_26 = 1/105$$
$$x_27 = 27/55$$
$$x_28 = 1/10$$

$\mathfrak{m}_{1A}(3,14)$

m1A314 (this line included for string searching purposes)
Original brackets:

$$[e_1,e_2]=e_3 \qquad [e_1,e_3]=e_4$$
$$[e_1,e_4]=e_5 \qquad [e_1,e_5]=e_6$$
$$[e_1,e_6]=e_7 \qquad [e_1,e_7]=e_8$$
$$[e_1,e_8]=e_9 \qquad [e_1,e_9]=e_{10}$$
$$[e_1,e_{10}]=e_{11} \qquad [e_1,e_{11}]=e_{12}$$
$$[e_1,e_{12}]=e_{13} \qquad [e_1,e_{13}]=e_{14}$$
$$[e_2,e_{11}]=e_{14} \qquad [e_3,e_{10}]=-e_{14}$$
$$[e_4,e_9]=e_{14} \qquad [e_5,e_8]=-e_{14}$$
$$[e_6,e_7]=e_{14}$$

No non-trivial Jacobi tests

$\mathfrak{m}_{3A}(3,14)$

m3A314 (this line included for string searching purposes)
Original brackets:

$$[e_1,e_2]=e_3 \qquad [e_1,e_3]=e_4$$
$$[e_1,e_4]=e_5 \qquad [e_1,e_5]=e_6$$
$$[e_1,e_6]=e_7 \qquad [e_1,e_7]=e_8$$
$$[e_1,e_8]=e_9 \qquad [e_1,e_9]=e_{10}$$
$$[e_1,e_{10}]=e_{11} \qquad [e_1,e_{11}]=e_{12}$$
$$[e_1,e_{12}]=e_{13} \qquad [e_1,e_{13}]=e_{14}$$
$$[e_2,e_9]=e_{12} \qquad [e_2,e_{10}]=4e_{13}$$
$$[e_2,e_{11}]=\alpha_{2,11}^{14}e_{14} \qquad [e_3,e_8]=-e_{12}$$
$$[e_3,e_9]=-3e_{13} \qquad [e_3,e_{10}]=\alpha_{3,10}^{14}e_{14}$$
$$[e_4,e_7]=e_{12} \qquad [e_4,e_8]=2e_{13}$$
$$[e_4,e_9]=\alpha_{4,9}^{14}e_{14} \qquad [e_5,e_6]=-e_{12}$$
$$[e_5,e_7]=-e_{13} \qquad [e_5,e_8]=\alpha_{5,8}^{14}e_{14}$$
$$[e_6,e_7]=\alpha_{6,7}^{14}e_{14}$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1, e_2, e_{10}):& \quad -\alpha^{14}_{2,11} - \alpha^{14}_{3,10} + 4 &&= 0\\
(e_1, e_3, e_9):& \quad -\alpha^{14}_{3,10} - \alpha^{14}_{4,9} - 3 &&= 0\\
(e_1, e_4, e_8):& \quad -\alpha^{14}_{4,9} - \alpha^{14}_{5,8} + 2 &&= 0\\
(e_1, e_5, e_7):& \quad -\alpha^{14}_{5,8} - \alpha^{14}_{6,7} - 1 &&= 0
\end{aligned}$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha^{14}_{4,9} \to x_1$$

$$\alpha^{14}_{2,11} \to x_2$$

$$\alpha^{14}_{3,10} \to x_3$$

$$\alpha^{14}_{6,7} \to x_4$$

$$\alpha^{14}_{5,8} \to x_5$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_{10}):& \quad -x_2 - x_3 + 4 &&= 0\\
(e_1, e_3, e_9):& \quad -x_1 - x_3 - 3 &&= 0\\
(e_1, e_4, e_8):& \quad -x_1 - x_5 + 2 &&= 0\\
(e_1, e_5, e_7):& \quad -x_4 - x_5 - 1 &&= 0
\end{aligned}$$

Groebner basis (5 variables, 4 linear, 0 nonlinear)

$$x_1 + x_5 - 2 = 0$$

$$x_2 + x_5 - 9 = 0$$

$$x_3 - x_5 + 5 = 0$$

$$x_4 + x_5 + 1 = 0$$

$\mathfrak{m}_{5A}(3,14)$

m5A314 (this line included for string searching purposes)

Solution 1

$$[e_1,e_2]=e_3 \qquad [e_1,e_3]=e_4$$
$$[e_1,e_4]=e_5 \qquad [e_1,e_5]=e_6$$
$$[e_1,e_6]=e_7 \qquad [e_1,e_7]=e_8$$
$$[e_1,e_8]=e_9 \qquad [e_1,e_9]=e_{10}$$
$$[e_1,e_{10}]=e_{11} \qquad [e_1,e_{11}]=e_{12}$$
$$[e_1,e_{12}]=e_{13} \qquad [e_1,e_{13}]=e_{14}$$
$$[e_2,e_7]=e_{10} \qquad [e_2,e_8]=3e_{11}$$
$$[e_2,e_9]=\frac{7e_{12}}{2} \qquad [e_2,e_{10}]=0$$
$$[e_2,e_{11}]=0 \qquad [e_3,e_6]=-e_{10}$$
$$[e_3,e_7]=-2e_{11} \qquad [e_3,e_8]=-\frac{e_{12}}{2}$$
$$[e_3,e_9]=\frac{7e_{13}}{2} \qquad [e_3,e_{10}]=0$$
$$[e_4,e_5]=e_{10} \qquad [e_4,e_6]=e_{11}$$
$$[e_4,e_7]=-\frac{3e_{12}}{2} \qquad [e_4,e_8]=-4e_{13}$$
$$[e_4,e_9]=\frac{7e_{14}}{2} \qquad [e_5,e_6]=\frac{5e_{12}}{2}$$
$$[e_5,e_7]=\frac{5e_{13}}{2} \qquad [e_5,e_8]=-\frac{15e_{14}}{2}$$
$$[e_6,e_7]=10e_{14}$$

Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4\\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6\\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8\\
[e_1,e_8] &= e_9 & [e_1,e_9] &= e_{10}\\
[e_1,e_{10}] &= e_{11} & [e_1,e_{11}] &= e_{12}\\
[e_1,e_{12}] &= e_{13} & [e_1,e_{13}] &= e_{14}\\
[e_2,e_7] &= e_{10} & [e_2,e_8] &= 3e_{11}\\
[e_2,e_9] &= \alpha_{2,9}^{12}e_{12} & [e_2,e_{10}] &= \alpha_{2,10}^{13}e_{13}\\
[e_2,e_{11}] &= \alpha_{2,11}^{14}e_{14} & [e_3,e_6] &= -e_{10}\\
[e_3,e_7] &= -2e_{11} & [e_3,e_8] &= \alpha_{3,8}^{12}e_{12}\\
[e_3,e_9] &= \alpha_{3,9}^{13}e_{13} & [e_3,e_{10}] &= \alpha_{3,10}^{14}e_{14}\\
[e_4,e_5] &= e_{10} & [e_4,e_6] &= e_{11}\\
[e_4,e_7] &= \alpha_{4,7}^{12}e_{12} & [e_4,e_8] &= \alpha_{4,8}^{13}e_{13}\\
[e_4,e_9] &= \alpha_{4,9}^{14}e_{14} & [e_5,e_6] &= \alpha_{5,6}^{12}e_{12}\\
[e_5,e_7] &= \alpha_{5,7}^{13}e_{13} & [e_5,e_8] &= \alpha_{5,8}^{14}e_{14}\\
[e_6,e_7] &= \alpha_{6,7}^{14}e_{14}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_8): &\quad -\alpha_{2,9}^{12} - \alpha_{3,8}^{12} + 3 && = 0 \\
(e_1, e_3, e_7): &\quad -\alpha_{3,8}^{12} - \alpha_{4,7}^{12} - 2 && = 0 \\
(e_1, e_4, e_6): &\quad -\alpha_{4,7}^{12} - \alpha_{5,6}^{12} + 1 && = 0 \\
(e_1, e_2, e_9): &\quad -\alpha_{2,10}^{13} + \alpha_{2,9}^{12} - \alpha_{3,9}^{13} && = 0 \\
(e_1, e_3, e_8): &\quad \alpha_{3,8}^{12} - \alpha_{3,9}^{13} - \alpha_{4,8}^{13} && = 0 \\
(e_1, e_4, e_7): &\quad \alpha_{4,7}^{12} - \alpha_{4,8}^{13} - \alpha_{5,7}^{13} && = 0 \\
(e_1, e_5, e_6): &\quad \alpha_{5,6}^{12} - \alpha_{5,7}^{13} && = 0 \\
(e_2, e_3, e_6): &\quad -\alpha_{2,10}^{13} && = 0 \\
(e_2, e_4, e_5): &\quad \alpha_{2,10}^{13} && = 0 \\
(e_1, e_2, e_{10}): &\quad \alpha_{2,10}^{13} - \alpha_{2,11}^{14} - \alpha_{3,10}^{14} && = 0 \\
(e_1, e_3, e_9): &\quad -\alpha_{3,10}^{14} + \alpha_{3,9}^{13} - \alpha_{4,9}^{14} && = 0 \\
(e_1, e_4, e_8): &\quad \alpha_{4,8}^{13} - \alpha_{4,9}^{14} - \alpha_{5,8}^{14} && = 0 \\
(e_1, e_5, e_7): &\quad \alpha_{5,7}^{13} - \alpha_{5,8}^{14} - \alpha_{6,7}^{14} && = 0 \\
(e_2, e_3, e_7): &\quad -2\alpha_{2,11}^{14} - \alpha_{3,10}^{14} && = 0 \\
(e_2, e_4, e_6): &\quad \alpha_{2,11}^{14} && = 0 \\
(e_3, e_4, e_5): &\quad \alpha_{3,10}^{14} && = 0
\end{aligned}
$$

Solution 1:

$$\alpha^{13}_{3,9} = 7/2$$
$$\alpha^{13}_{2,10} = 0$$
$$\alpha^{13}_{4,8} = -4$$
$$\alpha^{14}_{4,9} = 7/2$$
$$\alpha^{12}_{4,7} = -3/2$$
$$\alpha^{12}_{3,8} = -1/2$$
$$\alpha^{14}_{6,7} = 10$$
$$\alpha^{14}_{2,11} = 0$$
$$\alpha^{14}_{3,10} = 0$$
$$\alpha^{12}_{5,6} = 5/2$$
$$\alpha^{12}_{2,9} = 7/2$$
$$\alpha^{14}_{5,8} = -15/2$$
$$\alpha^{13}_{5,7} = 5/2$$

How the solution(s) were or were not found:
Change variables

$$\alpha^{13}_{3,9} \to x_1$$
$$\alpha^{13}_{2,10} \to x_2$$
$$\alpha^{13}_{4,8} \to x_3$$
$$\alpha^{14}_{4,9} \to x_4$$
$$\alpha^{12}_{4,7} \to x_5$$
$$\alpha^{12}_{3,8} \to x_6$$
$$\alpha^{14}_{6,7} \to x_7$$
$$\alpha^{14}_{2,11} \to x_8$$
$$\alpha^{14}_{3,10} \to x_9$$
$$\alpha^{12}_{5,6} \to x_{10}$$
$$\alpha^{12}_{2,9} \to x_{11}$$
$$\alpha^{14}_{5,8} \to x_{12}$$
$$\alpha^{13}_{5,7} \to x_{13}$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_8): &\quad -x_{11} - x_6 + 3 &&= 0\\
(e_1, e_3, e_7): &\quad -x_5 - x_6 - 2 &&= 0\\
(e_1, e_4, e_6): &\quad -x_{10} - x_5 + 1 &&= 0\\
(e_1, e_2, e_9): &\quad -x_1 + x_{11} - x_2 &&= 0\\
(e_1, e_3, e_8): &\quad -x_1 - x_3 + x_6 &&= 0\\
(e_1, e_4, e_7): &\quad -x_{13} - x_3 + x_5 &&= 0\\
(e_1, e_5, e_6): &\quad x_{10} - x_{13} &&= 0\\
(e_2, e_3, e_6): &\quad -x_2 &&= 0\\
(e_2, e_4, e_5): &\quad x_2 &&= 0\\
(e_1, e_2, e_{10}): &\quad x_2 - x_8 - x_9 &&= 0\\
(e_1, e_3, e_9): &\quad x_1 - x_4 - x_9 &&= 0\\
(e_1, e_4, e_8): &\quad -x_{12} + x_3 - x_4 &&= 0\\
(e_1, e_5, e_7): &\quad -x_{12} + x_{13} - x_7 &&= 0\\
(e_2, e_3, e_7): &\quad -2x_8 - x_9 &&= 0\\
(e_2, e_4, e_6): &\quad x_8 &&= 0\\
(e_3, e_4, e_5): &\quad x_9 &&= 0
\end{aligned}
$$

Groebner basis (13 variables, 13 linear, 0 nonlinear)

$$2x_1 - 7 = 0$$

$$x_2 = 0$$

$$x_3 + 4 = 0$$

$$2x_4 - 7 = 0$$

$$2x_5 + 3 = 0$$

$$2x_6 + 1 = 0$$

$$x_7 - 10 = 0$$

$$x_8 = 0$$

$$x_9 = 0$$

$$2x_{10} - 5 = 0$$

$$2x_{11} - 7 = 0$$

$$2x_{12} + 15 = 0$$

$$2x_{13} - 5 = 0$$

Solution 1:

$$x_1 = 7/2$$
$$x_2 = 0$$
$$x_3 = -4$$
$$x_4 = 7/2$$
$$x_5 = -3/2$$
$$x_6 = -1/2$$
$$x_7 = 10$$
$$x_8 = 0$$
$$x_9 = 0$$
$$x_10 = 5/2$$
$$x_11 = 7/2$$
$$x_12 = -15/2$$
$$x_13 = 5/2$$

$\mathfrak{m}_{7A}(3, 14)$

m7A314 (this line included for string searching purposes)

Solution 1

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_1, e_{11}] = e_{12}$$
$$[e_1, e_{12}] = e_{13} \qquad [e_1, e_{13}] = e_{14}$$
$$[e_2, e_5] = e_8 \qquad [e_2, e_6] = 2e_9$$
$$[e_2, e_7] = \frac{5e_{10}}{3} \qquad [e_2, e_8] = 0$$
$$[e_2, e_9] = -\frac{49e_{12}}{33} \qquad [e_2, e_{10}] = -\frac{14e_{13}}{11}$$
$$[e_2, e_{11}] = -\frac{7e_{14}}{11} \qquad [e_3, e_4] = -e_8$$
$$[e_3, e_5] = -e_9 \qquad [e_3, e_6] = \frac{e_{10}}{3}$$
$$[e_3, e_7] = \frac{5e_{11}}{3} \qquad [e_3, e_8] = \frac{49e_{12}}{33}$$
$$[e_3, e_9] = -\frac{7e_{13}}{33} \qquad [e_3, e_{10}] = -\frac{7e_{14}}{11}$$
$$[e_4, e_5] = -\frac{4e_{10}}{3} \qquad [e_4, e_6] = -\frac{4e_{11}}{3}$$
$$[e_4, e_7] = \frac{2e_{12}}{11} \qquad [e_4, e_8] = \frac{56e_{13}}{33}$$
$$[e_4, e_9] = \frac{14e_{14}}{33} \qquad [e_5, e_6] = -\frac{50e_{12}}{33}$$
$$[e_5, e_7] = -\frac{50e_{13}}{33} \qquad [e_5, e_8] = \frac{14e_{14}}{11}$$
$$[e_6, e_7] = -\frac{92e_{14}}{33}$$

Original brackets:

$$
\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_1, e_9] = e_{10} \\
[e_1, e_{10}] = e_{11} & [e_1, e_{11}] = e_{12} \\
[e_1, e_{12}] = e_{13} & [e_1, e_{13}] = e_{14} \\
[e_2, e_5] = e_8 & [e_2, e_6] = 2e_9 \\
[e_2, e_7] = \alpha_{2,7}^{10} e_{10} & [e_2, e_8] = \alpha_{2,8}^{11} e_{11} \\
[e_2, e_9] = \alpha_{2,9}^{12} e_{12} & [e_2, e_{10}] = \alpha_{2,10}^{13} e_{13} \\
[e_2, e_{11}] = \alpha_{2,11}^{14} e_{14} & [e_3, e_4] = -e_8 \\
[e_3, e_5] = -e_9 & [e_3, e_6] = \alpha_{3,6}^{10} e_{10} \\
[e_3, e_7] = \alpha_{3,7}^{11} e_{11} & [e_3, e_8] = \alpha_{3,8}^{12} e_{12} \\
[e_3, e_9] = \alpha_{3,9}^{13} e_{13} & [e_3, e_{10}] = \alpha_{3,10}^{14} e_{14} \\
[e_4, e_5] = \alpha_{4,5}^{10} e_{10} & [e_4, e_6] = \alpha_{4,6}^{11} e_{11} \\
[e_4, e_7] = \alpha_{4,7}^{12} e_{12} & [e_4, e_8] = \alpha_{4,8}^{13} e_{13} \\
[e_4, e_9] = \alpha_{4,9}^{14} e_{14} & [e_5, e_6] = \alpha_{5,6}^{12} e_{12} \\
[e_5, e_7] = \alpha_{5,7}^{13} e_{13} & [e_5, e_8] = \alpha_{5,8}^{14} e_{14} \\
[e_6, e_7] = \alpha_{6,7}^{14} e_{14} &
\end{array}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_6):&\quad -\alpha_{2,7}^{10} - \alpha_{3,6}^{10} + 2 &&= 0\\
(e_1, e_3, e_5):&\quad -\alpha_{3,6}^{10} - \alpha_{4,5}^{10} - 1 &&= 0\\
(e_1, e_2, e_7):&\quad \alpha_{2,7}^{10} - \alpha_{2,8}^{11} - \alpha_{3,7}^{11} &&= 0\\
(e_1, e_3, e_6):&\quad \alpha_{3,6}^{10} - \alpha_{3,7}^{11} - \alpha_{4,6}^{11} &&= 0\\
(e_1, e_4, e_5):&\quad \alpha_{4,5}^{10} - \alpha_{4,6}^{11} &&= 0\\
(e_2, e_3, e_4):&\quad -\alpha_{2,8}^{11} &&= 0\\
(e_1, e_2, e_8):&\quad \alpha_{2,8}^{11} - \alpha_{2,9}^{12} - \alpha_{3,8}^{12} &&= 0\\
(e_1, e_3, e_7):&\quad \alpha_{3,7}^{11} - \alpha_{3,8}^{12} - \alpha_{4,7}^{12} &&= 0\\
(e_1, e_4, e_6):&\quad \alpha_{4,6}^{11} - \alpha_{4,7}^{12} - \alpha_{5,6}^{12} &&= 0\\
(e_2, e_3, e_5):&\quad -\alpha_{2,9}^{12} - \alpha_{3,8}^{12} &&= 0\\
(e_1, e_2, e_9):&\quad -\alpha_{2,10}^{13} + \alpha_{2,9}^{12} - \alpha_{3,9}^{13} &&= 0\\
(e_1, e_3, e_8):&\quad \alpha_{3,8}^{12} - \alpha_{3,9}^{13} - \alpha_{4,8}^{13} &&= 0\\
(e_1, e_4, e_7):&\quad \alpha_{4,7}^{12} - \alpha_{4,8}^{13} - \alpha_{5,7}^{13} &&= 0\\
(e_1, e_5, e_6):&\quad \alpha_{5,6}^{12} - \alpha_{5,7}^{13} &&= 0\\
(e_2, e_3, e_6):&\quad \alpha_{2,10}^{13}\alpha_{3,6}^{10} - 2\alpha_{3,9}^{13} &&= 0\\
(e_2, e_4, e_5):&\quad \alpha_{2,10}^{13}\alpha_{4,5}^{10} - \alpha_{4,8}^{13} &&= 0\\
(e_1, e_2, e_{10}):&\quad \alpha_{2,10}^{13} - \alpha_{2,11}^{14} - \alpha_{3,10}^{14} &&= 0\\
(e_1, e_3, e_9):&\quad -\alpha_{3,10}^{14} + \alpha_{3,9}^{13} - \alpha_{4,9}^{14} &&= 0\\
(e_1, e_4, e_8):&\quad \alpha_{4,8}^{13} - \alpha_{4,9}^{14} - \alpha_{5,8}^{14} &&= 0\\
(e_1, e_5, e_7):&\quad \alpha_{5,7}^{13} - \alpha_{5,8}^{14} - \alpha_{6,7}^{14} &&= 0\\
(e_2, e_3, e_7):&\quad \alpha_{2,11}^{14}\alpha_{3,7}^{11} - \alpha_{2,7}^{10}\alpha_{3,10}^{14} &&= 0\\
(e_2, e_4, e_6):&\quad \alpha_{2,11}^{14}\alpha_{4,6}^{11} - 2\alpha_{4,9}^{14} &&= 0\\
(e_3, e_4, e_5):&\quad \alpha_{3,10}^{14}\alpha_{4,5}^{10} + \alpha_{4,9}^{14} - \alpha_{5,8}^{14} &&= 0
\end{aligned}
$$

Solution 1:

$$\alpha^{13}_{3,9} = -7/33$$
$$\alpha^{13}_{5,7} = -50/33$$
$$\alpha^{13}_{2,10} = -14/11$$
$$\alpha^{13}_{4,8} = 56/33$$
$$\alpha^{14}_{4,9} = 14/33$$
$$\alpha^{11}_{3,7} = 5/3$$
$$\alpha^{11}_{2,8} = 0$$
$$\alpha^{12}_{4,7} = 2/11$$
$$\alpha^{12}_{3,8} = 49/33$$
$$\alpha^{14}_{6,7} = -92/33$$
$$\alpha^{10}_{2,7} = 5/3$$
$$\alpha^{14}_{2,11} = -7/11$$
$$\alpha^{14}_{3,10} = -7/11$$
$$\alpha^{11}_{4,6} = -4/3$$
$$\alpha^{10}_{4,5} = -4/3$$
$$\alpha^{12}_{5,6} = -50/33$$
$$\alpha^{12}_{2,9} = -49/33$$
$$\alpha^{14}_{5,8} = 14/11$$
$$\alpha^{10}_{3,6} = 1/3$$

How the solution(s) were or were not found:
Change variables

$$\alpha^{13}_{3,9} \to x_1$$
$$\alpha^{13}_{5,7} \to x_2$$
$$\alpha^{13}_{2,10} \to x_3$$
$$\alpha^{13}_{4,8} \to x_4$$
$$\alpha^{14}_{4,9} \to x_5$$
$$\alpha^{11}_{3,7} \to x_6$$
$$\alpha^{11}_{2,8} \to x_7$$
$$\alpha^{12}_{4,7} \to x_8$$

$$\alpha_{3,8}^{12} \to x_9$$

$$\alpha_{6,7}^{14} \to x_{10}$$

$$\alpha_{2,7}^{10} \to x_{11}$$

$$\alpha_{2,11}^{14} \to x_{12}$$

$$\alpha_{3,10}^{14} \to x_{13}$$

$$\alpha_{4,6}^{11} \to x_{14}$$

$$\alpha_{4,5}^{10} \to x_{15}$$

$$\alpha_{5,6}^{12} \to x_{16}$$

$$\alpha_{2,9}^{12} \to x_{17}$$

$$\alpha_{5,8}^{14} \to x_{18}$$

$$\alpha_{3,6}^{10} \to x_{19}$$

Jacobi Tests

$$
\begin{aligned}
(e_1,e_2,e_6):&\quad -x_{11}-x_{19}+2 &&=0\\
(e_1,e_3,e_5):&\quad -x_{15}-x_{19}-1 &&=0\\
(e_1,e_2,e_7):&\quad x_{11}-x_6-x_7 &&=0\\
(e_1,e_3,e_6):&\quad -x_{14}+x_{19}-x_6 &&=0\\
(e_1,e_4,e_5):&\quad -x_{14}+x_{15} &&=0\\
(e_2,e_3,e_4):&\quad -x_7 &&=0\\
(e_1,e_2,e_8):&\quad -x_{17}+x_7-x_9 &&=0\\
(e_1,e_3,e_7):&\quad x_6-x_8-x_9 &&=0\\
(e_1,e_4,e_6):&\quad x_{14}-x_{16}-x_8 &&=0\\
(e_2,e_3,e_5):&\quad -x_{17}-x_9 &&=0\\
(e_1,e_2,e_9):&\quad -x_1+x_{17}-x_3 &&=0\\
(e_1,e_3,e_8):&\quad -x_1-x_4+x_9 &&=0\\
(e_1,e_4,e_7):&\quad -x_2-x_4+x_8 &&=0\\
(e_1,e_5,e_6):&\quad x_{16}-x_2 &&=0\\
(e_2,e_3,e_6):&\quad -2x_1+x_{19}x_3 &&=0\\
(e_2,e_4,e_5):&\quad x_{15}x_3-x_4 &&=0\\
(e_1,e_2,e_{10}):&\quad -x_{12}-x_{13}+x_3 &&=0\\
(e_1,e_3,e_9):&\quad x_1-x_{13}-x_5 &&=0\\
(e_1,e_4,e_8):&\quad -x_{18}+x_4-x_5 &&=0\\
(e_1,e_5,e_7):&\quad -x_{10}-x_{18}+x_2 &&=0\\
(e_2,e_3,e_7):&\quad -x_{11}x_{13}+x_{12}x_6 &&=0\\
(e_2,e_4,e_6):&\quad x_{12}x_{14}-2x_5 &&=0\\
(e_3,e_4,e_5):&\quad x_{13}x_{15}-x_{18}+x_5 &&=0
\end{aligned}
$$

Groebner basis (19 variables, 19 linear, 0 nonlinear)

$$33x_1+7=0$$

$$33x_2+50=0$$

$$11x_3+14=0$$

$$33x_4-56=0$$

$$33x_5-14=0$$

$$3x_6-5=0$$

$$x_7=0$$

$$11x_8-2=0$$

$$33x_9 - 49 = 0$$
$$33x_{10} + 92 = 0$$
$$3x_{11} - 5 = 0$$
$$11x_{12} + 7 = 0$$
$$11x_{13} + 7 = 0$$
$$3x_{14} + 4 = 0$$
$$3x_{15} + 4 = 0$$
$$33x_{16} + 50 = 0$$
$$33x_{17} + 49 = 0$$
$$11x_{18} - 14 = 0$$
$$3x_{19} - 1 = 0$$

Solution 1:

$$x_1 = -7/33$$
$$x_2 = -50/33$$
$$x_3 = -14/11$$
$$x_4 = 56/33$$
$$x_5 = 14/33$$
$$x_6 = 5/3$$
$$x_7 = 0$$
$$x_8 = 2/11$$
$$x_9 = 49/33$$
$$x_10 = -92/33$$
$$x_11 = 5/3$$
$$x_12 = -7/11$$
$$x_13 = -7/11$$
$$x_14 = -4/3$$
$$x_15 = -4/3$$
$$x_16 = -50/33$$
$$x_17 = -49/33$$
$$x_18 = 14/11$$
$$x_19 = 1/3$$

$\mathfrak{m}_{9A}(3, 14)$

m9A314 (this line included for string searching purposes)

Original brackets:

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_1, e_{11}] = e_{12}$$
$$[e_1, e_{12}] = e_{13} \qquad [e_1, e_{13}] = e_{14}$$
$$[e_2, e_3] = e_6 \qquad [e_2, e_4] = e_7$$
$$[e_2, e_5] = \alpha_{2,5}^{8} e_8 \qquad [e_2, e_6] = \alpha_{2,6}^{9} e_9$$
$$[e_2, e_7] = \alpha_{2,7}^{10} e_{10} \qquad [e_2, e_8] = \alpha_{2,8}^{11} e_{11}$$
$$[e_2, e_9] = \alpha_{2,9}^{12} e_{12} \qquad [e_2, e_{10}] = \alpha_{2,10}^{13} e_{13}$$
$$[e_2, e_{11}] = \alpha_{2,11}^{14} e_{14} \qquad [e_3, e_4] = \alpha_{3,4}^{8} e_8$$
$$[e_3, e_5] = \alpha_{3,5}^{9} e_9 \qquad [e_3, e_6] = \alpha_{3,6}^{10} e_{10}$$
$$[e_3, e_7] = \alpha_{3,7}^{11} e_{11} \qquad [e_3, e_8] = \alpha_{3,8}^{12} e_{12}$$
$$[e_3, e_9] = \alpha_{3,9}^{13} e_{13} \qquad [e_3, e_{10}] = \alpha_{3,10}^{14} e_{14}$$
$$[e_4, e_5] = \alpha_{4,5}^{10} e_{10} \qquad [e_4, e_6] = \alpha_{4,6}^{11} e_{11}$$
$$[e_4, e_7] = \alpha_{4,7}^{12} e_{12} \qquad [e_4, e_8] = \alpha_{4,8}^{13} e_{13}$$
$$[e_4, e_9] = \alpha_{4,9}^{14} e_{14} \qquad [e_5, e_6] = \alpha_{5,6}^{12} e_{12}$$
$$[e_5, e_7] = \alpha_{5,7}^{13} e_{13} \qquad [e_5, e_8] = \alpha_{5,8}^{14} e_{14}$$
$$[e_6, e_7] = \alpha_{6,7}^{14} e_{14}$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1,e_2,e_4):&\quad -\alpha_{2,5}^{8}-\alpha_{3,4}^{8}+1 &&=0\\
(e_1,e_2,e_5):&\quad \alpha_{2,5}^{8}-\alpha_{2,6}^{9}-\alpha_{3,5}^{9} &&=0\\
(e_1,e_3,e_4):&\quad \alpha_{3,4}^{8}-\alpha_{3,5}^{9} &&=0\\
(e_1,e_2,e_6):&\quad \alpha_{2,6}^{9}-\alpha_{2,7}^{10}-\alpha_{3,6}^{10} &&=0\\
(e_1,e_3,e_5):&\quad \alpha_{3,5}^{9}-\alpha_{3,6}^{10}-\alpha_{4,5}^{10} &&=0\\
(e_1,e_2,e_7):&\quad \alpha_{2,7}^{10}-\alpha_{2,8}^{11}-\alpha_{3,7}^{11} &&=0\\
(e_1,e_3,e_6):&\quad \alpha_{3,6}^{10}-\alpha_{3,7}^{11}-\alpha_{4,6}^{11} &&=0\\
(e_1,e_4,e_5):&\quad \alpha_{4,5}^{10}-\alpha_{4,6}^{11} &&=0\\
(e_2,e_3,e_4):&\quad \alpha_{2,8}^{11}\alpha_{3,4}^{8}-\alpha_{3,7}^{11}+\alpha_{4,6}^{11} &&=0\\
(e_1,e_2,e_8):&\quad \alpha_{2,8}^{11}-\alpha_{2,9}^{12}-\alpha_{3,8}^{12} &&=0\\
(e_1,e_3,e_7):&\quad \alpha_{3,7}^{11}-\alpha_{3,8}^{12}-\alpha_{4,7}^{12} &&=0\\
(e_1,e_4,e_6):&\quad \alpha_{4,6}^{11}-\alpha_{4,7}^{12}-\alpha_{5,6}^{12} &&=0\\
(e_2,e_3,e_5):&\quad -\alpha_{2,5}^{8}\alpha_{3,8}^{12}+\alpha_{2,9}^{12}\alpha_{3,5}^{9}+\alpha_{5,6}^{12} &&=0\\
(e_1,e_2,e_9):&\quad -\alpha_{2,10}^{13}+\alpha_{2,9}^{12}-\alpha_{3,9}^{13} &&=0\\
(e_1,e_3,e_8):&\quad \alpha_{3,8}^{12}-\alpha_{3,9}^{13}-\alpha_{4,8}^{13} &&=0\\
(e_1,e_4,e_7):&\quad \alpha_{4,7}^{12}-\alpha_{4,8}^{13}-\alpha_{5,7}^{13} &&=0\\
(e_1,e_5,e_6):&\quad \alpha_{5,6}^{12}-\alpha_{5,7}^{13} &&=0\\
(e_2,e_3,e_6):&\quad \alpha_{2,10}^{13}\alpha_{3,6}^{10}-\alpha_{2,6}^{9}\alpha_{3,9}^{13} &&=0\\
(e_2,e_4,e_5):&\quad \alpha_{2,10}^{13}\alpha_{4,5}^{10}-\alpha_{2,5}^{8}\alpha_{4,8}^{13}+\alpha_{5,7}^{13} &&=0\\
(e_1,e_2,e_{10}):&\quad \alpha_{2,10}^{13}-\alpha_{2,11}^{14}-\alpha_{3,10}^{14} &&=0\\
(e_1,e_3,e_9):&\quad -\alpha_{3,10}^{14}+\alpha_{3,9}^{13}-\alpha_{4,9}^{14} &&=0\\
(e_1,e_4,e_8):&\quad \alpha_{4,8}^{13}-\alpha_{4,9}^{14}-\alpha_{5,8}^{14} &&=0\\
(e_1,e_5,e_7):&\quad \alpha_{5,7}^{13}-\alpha_{5,8}^{14}-\alpha_{6,7}^{14} &&=0\\
(e_2,e_3,e_7):&\quad \alpha_{2,11}^{14}\alpha_{3,7}^{11}-\alpha_{2,7}^{10}\alpha_{3,10}^{14}-\alpha_{6,7}^{14} &&=0\\
(e_2,e_4,e_6):&\quad \alpha_{2,11}^{14}\alpha_{4,6}^{11}-\alpha_{2,6}^{9}\alpha_{4,9}^{14}+\alpha_{6,7}^{14} &&=0\\
(e_3,e_4,e_5):&\quad \alpha_{3,10}^{14}\alpha_{4,5}^{10}+\alpha_{3,4}^{8}\alpha_{5,8}^{14}-\alpha_{3,5}^{9}\alpha_{4,9}^{14} &&=0
\end{aligned}$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{3,5}^{9}\to x_1$$

$$\alpha_{3,9}^{13}\to x_2$$

$$\alpha^{13}_{2,10} \to x_3$$
$$\alpha^{11}_{3,7} \to x_4$$
$$\alpha^{9}_{2,6} \to x_5$$
$$\alpha^{14}_{3,10} \to x_6$$
$$\alpha^{8}_{3,4} \to x_7$$
$$\alpha^{13}_{4,8} \to x_8$$
$$\alpha^{14}_{4,9} \to x_9$$
$$\alpha^{14}_{2,11} \to x_{10}$$
$$\alpha^{11}_{4,6} \to x_{11}$$
$$\alpha^{12}_{5,6} \to x_{12}$$
$$\alpha^{13}_{5,7} \to x_{13}$$
$$\alpha^{14}_{6,7} \to x_{14}$$
$$\alpha^{10}_{4,5} \to x_{15}$$
$$\alpha^{8}_{2,5} \to x_{16}$$
$$\alpha^{12}_{3,8} \to x_{17}$$
$$\alpha^{12}_{2,9} \to x_{18}$$
$$\alpha^{14}_{5,8} \to x_{19}$$
$$\alpha^{11}_{2,8} \to x_{20}$$
$$\alpha^{10}_{2,7} \to x_{21}$$
$$\alpha^{12}_{4,7} \to x_{22}$$
$$\alpha^{10}_{3,6} \to x_{23}$$

Jacobi Tests

$$
\begin{aligned}
(e_1,e_2,e_4):&\quad -x_{16}-x_7+1 &&=0\\
(e_1,e_2,e_5):&\quad -x_1+x_{16}-x_5 &&=0\\
(e_1,e_3,e_4):&\quad -x_1+x_7 &&=0\\
(e_1,e_2,e_6):&\quad -x_{21}-x_{23}+x_5 &&=0\\
(e_1,e_3,e_5):&\quad x_1-x_{15}-x_{23} &&=0\\
(e_1,e_2,e_7):&\quad -x_{20}+x_{21}-x_4 &&=0\\
(e_1,e_3,e_6):&\quad -x_{11}+x_{23}-x_4 &&=0\\
(e_1,e_4,e_5):&\quad -x_{11}+x_{15} &&=0\\
(e_2,e_3,e_4):&\quad x_{11}+x_{20}x_7-x_4 &&=0\\
(e_1,e_2,e_8):&\quad -x_{17}-x_{18}+x_{20} &&=0\\
(e_1,e_3,e_7):&\quad -x_{17}-x_{22}+x_4 &&=0\\
(e_1,e_4,e_6):&\quad x_{11}-x_{12}-x_{22} &&=0\\
(e_2,e_3,e_5):&\quad x_1x_{18}+x_{12}-x_{16}x_{17} &&=0\\
(e_1,e_2,e_9):&\quad x_{18}-x_2-x_3 &&=0\\
(e_1,e_3,e_8):&\quad x_{17}-x_2-x_8 &&=0\\
(e_1,e_4,e_7):&\quad -x_{13}+x_{22}-x_8 &&=0\\
(e_1,e_5,e_6):&\quad x_{12}-x_{13} &&=0\\
(e_2,e_3,e_6):&\quad -x_2x_5+x_{23}x_3 &&=0\\
(e_2,e_4,e_5):&\quad x_{13}+x_{15}x_3-x_{16}x_8 &&=0\\
(e_1,e_2,e_{10}):&\quad -x_{10}+x_3-x_6 &&=0\\
(e_1,e_3,e_9):&\quad x_2-x_6-x_9 &&=0\\
(e_1,e_4,e_8):&\quad -x_{19}+x_8-x_9 &&=0\\
(e_1,e_5,e_7):&\quad x_{13}-x_{14}-x_{19} &&=0\\
(e_2,e_3,e_7):&\quad x_{10}x_4-x_{14}-x_{21}x_6 &&=0\\
(e_2,e_4,e_6):&\quad x_{10}x_{11}+x_{14}-x_5x_9 &&=0\\
(e_3,e_4,e_5):&\quad -x_1x_9+x_{15}x_6+x_{19}x_7 &&=0
\end{aligned}
$$

Groebner basis (23 variables, 19 linear, 7 nonlinear)

$$2x_1+x_{21}+x_{23}-1=0$$

$$x_2+3x_{22}-x_{23}=0$$

$$-4x_{22}+6x_{23}+x_3-1=0$$

$$-x_{21}-5x_{23}+2x_4+1=0$$

$$-x_{21}-x_{23}+x_5=0$$

$$-2x_{19}+x_{21}+10x_{22}+x_{23}+2x_6-1=0$$
$$x_{21}+x_{23}+2x_7-1=0$$
$$-x_{21}-4x_{22}-3x_{23}+2x_8+1=0$$
$$2x_{19}-x_{21}-4x_{22}-3x_{23}+2x_9+1=0$$
$$2x_{10}+2x_{19}-x_{21}-18x_{22}+11x_{23}-1=0$$
$$2x_{11}+x_{21}+3x_{23}-1=0$$
$$2x_{12}+x_{21}+2x_{22}+3x_{23}-1=0$$
$$2x_{13}+x_{21}+2x_{22}+3x_{23}-1=0$$
$$2x_{14}+2x_{19}+x_{21}+2x_{22}+3x_{23}-1=0$$
$$2x_{15}+x_{21}+3x_{23}-1=0$$
$$2x_{16}-x_{21}-x_{23}-1=0$$
$$2x_{17}-x_{21}+2x_{22}-5x_{23}+1=0$$
$$x_{18}-x_{22}+5x_{23}-1=0$$
$$9x_{19}x_{21}+15x_{19}x_{23}-9x_{19}-31x_{21}x_{23}+18x_{21}-2x_{22}x_{23}+21x_{22}-73x_{23}^2+85x_{23}-18=0$$
$$9x_{19}x_{22}-144x_{19}x_{23}+46x_{21}x_{23}-176x_{22}^2x_{23}+39x_{22}^2+360x_{22}x_{23}^2+200x_{22}x_{23}-12x_{22}-144x_{23}^3+154x_{23}^2-46x_{23}=0$$
$$35x_{19}x_{23}^2+63x_{19}x_{23}-23x_{21}x_{23}-135x_{22}x_{23}^2-x_{22}x_{23}-30x_{22}-60x_{23}^3-29x_{23}^2+23x_{23}=0$$
$$2x_{20}-x_{21}+5x_{23}-1=0$$
$$x_{21}^2-4x_{21}x_{23}+4x_{21}-5x_{23}^2+22x_{23}-5=0$$
$$3x_{21}x_{22}-x_{21}x_{23}+7x_{22}x_{23}-7x_{23}^2+x_{23}=0$$
$$14x_{21}x_{23}^2-3x_{21}x_{23}+88x_{22}x_{23}^2+114x_{22}x_{23}-45x_{22}-118x_{23}^3+37x_{23}^2+3x_{23}=0$$
$$88x_{22}^2x_{23}^2+114x_{22}^2x_{23}-45x_{22}^2-180x_{22}x_{23}^3+6x_{22}x_{23}^2+18x_{22}x_{23}+72x_{23}^4-24x_{23}^3=0$$

$\mathfrak{m}_{2A}(4,14)$

m2A414 (this line included for string searching purposes)

Original brackets:

$$
\begin{array}{ll}
[e_1,e_2]=e_3 & [e_1,e_3]=e_4 \\
[e_1,e_4]=e_5 & [e_1,e_5]=e_6 \\
[e_1,e_6]=e_7 & [e_1,e_7]=e_8 \\
[e_1,e_8]=e_9 & [e_1,e_9]=e_{10} \\
[e_1,e_{10}]=e_{11} & [e_1,e_{11}]=e_{12} \\
[e_1,e_{12}]=e_{13} & [e_1,e_{13}]=e_{14} \\
[e_2,e_9]=e_{13} & [e_2,e_{10}]=4e_{14} \\
[e_3,e_8]=-e_{13} & [e_3,e_9]=-3e_{14} \\
[e_4,e_7]=e_{13} & [e_4,e_8]=2e_{14} \\
[e_5,e_6]=-e_{13} & [e_5,e_7]=-e_{14}
\end{array}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{4A}(4,14)$

m4A414 (this line included for string searching purposes)

Original brackets:

$$
\begin{array}{ll}
[e_1,e_2]=e_3 & [e_1,e_3]=e_4 \\
[e_1,e_4]=e_5 & [e_1,e_5]=e_6 \\
[e_1,e_6]=e_7 & [e_1,e_7]=e_8 \\
[e_1,e_8]=e_9 & [e_1,e_9]=e_{10} \\
[e_1,e_{10}]=e_{11} & [e_1,e_{11}]=e_{12} \\
[e_1,e_{12}]=e_{13} & [e_1,e_{13}]=e_{14} \\
[e_2,e_7]=e_{11} & [e_2,e_8]=3e_{12} \\
[e_2,e_9]=\alpha_{2,9}^{13}e_{13} & [e_2,e_{10}]=\alpha_{2,10}^{14}e_{14} \\
[e_3,e_6]=-e_{11} & [e_3,e_7]=-2e_{12} \\
[e_3,e_8]=\alpha_{3,8}^{13}e_{13} & [e_3,e_9]=\alpha_{3,9}^{14}e_{14} \\
[e_4,e_5]=e_{11} & [e_4,e_6]=e_{12} \\
[e_4,e_7]=\alpha_{4,7}^{13}e_{13} & [e_4,e_8]=\alpha_{4,8}^{14}e_{14} \\
[e_5,e_6]=\alpha_{5,6}^{13}e_{13} & [e_5,e_7]=\alpha_{5,7}^{14}e_{14}
\end{array}
$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1, e_2, e_8): &\quad -\alpha_{2,9}^{13} - \alpha_{3,8}^{13} + 3 &= 0\\
(e_1, e_3, e_7): &\quad -\alpha_{3,8}^{13} - \alpha_{4,7}^{13} - 2 &= 0\\
(e_1, e_4, e_6): &\quad -\alpha_{4,7}^{13} - \alpha_{5,6}^{13} + 1 &= 0\\
(e_1, e_2, e_9): &\quad -\alpha_{2,10}^{14} + \alpha_{2,9}^{13} - \alpha_{3,9}^{14} &= 0\\
(e_1, e_3, e_8): &\quad \alpha_{3,8}^{13} - \alpha_{3,9}^{14} - \alpha_{4,8}^{14} &= 0\\
(e_1, e_4, e_7): &\quad \alpha_{4,7}^{13} - \alpha_{4,8}^{14} - \alpha_{5,7}^{14} &= 0\\
(e_1, e_5, e_6): &\quad \alpha_{5,6}^{13} - \alpha_{5,7}^{14} &= 0
\end{aligned}$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{4,8}^{14} \to x_1$$

$$\alpha_{5,6}^{13} \to x_2$$

$$\alpha_{3,8}^{13} \to x_3$$

$$\alpha_{5,7}^{14} \to x_4$$

$$\alpha_{2,10}^{14} \to x_5$$

$$\alpha_{2,9}^{13} \to x_6$$

$$\alpha_{3,9}^{14} \to x_7$$

$$\alpha_{4,7}^{13} \to x_8$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_8): &\quad -x_3 - x_6 + 3 &= 0\\
(e_1, e_3, e_7): &\quad -x_3 - x_8 - 2 &= 0\\
(e_1, e_4, e_6): &\quad -x_2 - x_8 + 1 &= 0\\
(e_1, e_2, e_9): &\quad -x_5 + x_6 - x_7 &= 0\\
(e_1, e_3, e_8): &\quad -x_1 + x_3 - x_7 &= 0\\
(e_1, e_4, e_7): &\quad -x_1 - x_4 + x_8 &= 0\\
(e_1, e_5, e_6): &\quad x_2 - x_4 &= 0
\end{aligned}$$

Groebner basis (8 variables, 7 linear, 0 nonlinear)

$$x_1 - 2x_8 + 1 = 0$$

$$x_2 + x_8 - 1 = 0$$
$$x_3 + x_8 + 2 = 0$$
$$x_4 + x_8 - 1 = 0$$
$$x_5 - 4x_8 - 6 = 0$$
$$x_6 - x_8 - 5 = 0$$
$$x_7 + 3x_8 + 1 = 0$$

$\mathfrak{m}_{6A}(4, 14)$

m6A414 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
&[e_1, e_2] = e_3 && [e_1, e_3] = e_4 \\
&[e_1, e_4] = e_5 && [e_1, e_5] = e_6 \\
&[e_1, e_6] = e_7 && [e_1, e_7] = e_8 \\
&[e_1, e_8] = e_9 && [e_1, e_9] = e_{10} \\
&[e_1, e_{10}] = e_{11} && [e_1, e_{11}] = e_{12} \\
&[e_1, e_{12}] = e_{13} && [e_1, e_{13}] = e_{14} \\
&[e_2, e_5] = e_9 && [e_2, e_6] = 2e_{10} \\
&[e_2, e_7] = \alpha_{2,7}^{11} e_{11} && [e_2, e_8] = \alpha_{2,8}^{12} e_{12} \\
&[e_2, e_9] = \alpha_{2,9}^{13} e_{13} && [e_2, e_{10}] = \alpha_{2,10}^{14} e_{14} \\
&[e_3, e_4] = -e_9 && [e_3, e_5] = -e_{10} \\
&[e_3, e_6] = \alpha_{3,6}^{11} e_{11} && [e_3, e_7] = \alpha_{3,7}^{12} e_{12} \\
&[e_3, e_8] = \alpha_{3,8}^{13} e_{13} && [e_3, e_9] = \alpha_{3,9}^{14} e_{14} \\
&[e_4, e_5] = \alpha_{4,5}^{11} e_{11} && [e_4, e_6] = \alpha_{4,6}^{12} e_{12} \\
&[e_4, e_7] = \alpha_{4,7}^{13} e_{13} && [e_4, e_8] = \alpha_{4,8}^{14} e_{14} \\
&[e_5, e_6] = \alpha_{5,6}^{13} e_{13} && [e_5, e_7] = \alpha_{5,7}^{14} e_{14}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_6): &\quad -\alpha_{2,7}^{11} - \alpha_{3,6}^{11} + 2 && = 0 \\
(e_1, e_3, e_5): &\quad -\alpha_{3,6}^{11} - \alpha_{4,5}^{11} - 1 && = 0 \\
(e_1, e_2, e_7): &\quad \alpha_{2,7}^{11} - \alpha_{2,8}^{12} - \alpha_{3,7}^{12} && = 0 \\
(e_1, e_3, e_6): &\quad \alpha_{3,6}^{11} - \alpha_{3,7}^{12} - \alpha_{4,6}^{12} && = 0 \\
(e_1, e_4, e_5): &\quad \alpha_{4,5}^{11} - \alpha_{4,6}^{12} && = 0 \\
(e_1, e_2, e_8): &\quad \alpha_{2,8}^{12} - \alpha_{2,9}^{13} - \alpha_{3,8}^{13} && = 0 \\
(e_1, e_3, e_7): &\quad \alpha_{3,7}^{12} - \alpha_{3,8}^{13} - \alpha_{4,7}^{13} && = 0 \\
(e_1, e_4, e_6): &\quad \alpha_{4,6}^{12} - \alpha_{4,7}^{13} - \alpha_{5,6}^{13} && = 0 \\
(e_2, e_3, e_4): &\quad -\alpha_{2,9}^{13} && = 0 \\
(e_1, e_2, e_9): &\quad -\alpha_{2,10}^{14} + \alpha_{2,9}^{13} - \alpha_{3,9}^{14} && = 0 \\
(e_1, e_3, e_8): &\quad \alpha_{3,8}^{13} - \alpha_{3,9}^{14} - \alpha_{4,8}^{14} && = 0 \\
(e_1, e_4, e_7): &\quad \alpha_{4,7}^{13} - \alpha_{4,8}^{14} - \alpha_{5,7}^{14} && = 0 \\
(e_1, e_5, e_6): &\quad \alpha_{5,6}^{13} - \alpha_{5,7}^{14} && = 0 \\
(e_2, e_3, e_5): &\quad -\alpha_{2,10}^{14} - \alpha_{3,9}^{14} && = 0
\end{aligned}
$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{4,8}^{14} \to x_1$$

$$\alpha_{2,7}^{11} \to x_2$$

$$\alpha_{2,8}^{12} \to x_3$$

$$\alpha_{4,6}^{12} \to x_4$$

$$\alpha_{5,6}^{13} \to x_5$$

$$\alpha_{4,5}^{11} \to x_6$$

$$\alpha_{3,8}^{13} \to x_7$$

$$\alpha_{3,7}^{12} \to x_8$$

$$\alpha_{5,7}^{14} \to x_9$$

$$\alpha_{2,10}^{14} \to x_{10}$$

$$\alpha_{2,9}^{13} \to x_{11}$$

$$\alpha_{3,9}^{14} \to x_{12}$$

$$\alpha^{13}_{4,7} \to x_{13}$$

$$\alpha^{11}_{3,6} \to x_{14}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_6):&\quad -x_{14} - x_2 + 2 &= 0\\
(e_1, e_3, e_5):&\quad -x_{14} - x_6 - 1 &= 0\\
(e_1, e_2, e_7):&\quad x_2 - x_3 - x_8 &= 0\\
(e_1, e_3, e_6):&\quad x_{14} - x_4 - x_8 &= 0\\
(e_1, e_4, e_5):&\quad -x_4 + x_6 &= 0\\
(e_1, e_2, e_8):&\quad -x_{11} + x_3 - x_7 &= 0\\
(e_1, e_3, e_7):&\quad -x_{13} - x_7 + x_8 &= 0\\
(e_1, e_4, e_6):&\quad -x_{13} + x_4 - x_5 &= 0\\
(e_2, e_3, e_4):&\quad -x_{11} &= 0\\
(e_1, e_2, e_9):&\quad -x_{10} + x_{11} - x_{12} &= 0\\
(e_1, e_3, e_8):&\quad -x_1 - x_{12} + x_7 &= 0\\
(e_1, e_4, e_7):&\quad -x_1 + x_{13} - x_9 &= 0\\
(e_1, e_5, e_6):&\quad x_5 - x_9 &= 0\\
(e_2, e_3, e_5):&\quad -x_{10} - x_{12} &= 0
\end{aligned}$$

Groebner basis (14 variables, 13 linear, 0 nonlinear)

$$x_1 - 11x_{14} - 1 = 0$$

$$x_{14} + x_2 - 2 = 0$$

$$3x_{14} + x_3 - 1 = 0$$

$$x_{14} + x_4 + 1 = 0$$

$$6x_{14} + x_5 + 1 = 0$$

$$x_{14} + x_6 + 1 = 0$$

$$3x_{14} + x_7 - 1 = 0$$

$$-2x_{14} + x_8 - 1 = 0$$

$$6x_{14} + x_9 + 1 = 0$$

$$x_{10} - 14x_{14} = 0$$

$$x_{11} = 0$$

$$x_{12} + 14x_{14} = 0$$

$$x_{13} - 5x_{14} = 0$$

$\mathfrak{m}_{8A}(4, 14)$

m8A414 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
&[e_1, e_2] = e_3 && [e_1, e_3] = e_4 \\
&[e_1, e_4] = e_5 && [e_1, e_5] = e_6 \\
&[e_1, e_6] = e_7 && [e_1, e_7] = e_8 \\
&[e_1, e_8] = e_9 && [e_1, e_9] = e_{10} \\
&[e_1, e_{10}] = e_{11} && [e_1, e_{11}] = e_{12} \\
&[e_1, e_{12}] = e_{13} && [e_1, e_{13}] = e_{14} \\
&[e_2, e_3] = e_7 && [e_2, e_4] = e_8 \\
&[e_2, e_5] = \alpha_{2,5}^{9} e_9 && [e_2, e_6] = \alpha_{2,6}^{10} e_{10} \\
&[e_2, e_7] = \alpha_{2,7}^{11} e_{11} && [e_2, e_8] = \alpha_{2,8}^{12} e_{12} \\
&[e_2, e_9] = \alpha_{2,9}^{13} e_{13} && [e_2, e_{10}] = \alpha_{2,10}^{14} e_{14} \\
&[e_3, e_4] = \alpha_{3,4}^{9} e_9 && [e_3, e_5] = \alpha_{3,5}^{10} e_{10} \\
&[e_3, e_6] = \alpha_{3,6}^{11} e_{11} && [e_3, e_7] = \alpha_{3,7}^{12} e_{12} \\
&[e_3, e_8] = \alpha_{3,8}^{13} e_{13} && [e_3, e_9] = \alpha_{3,9}^{14} e_{14} \\
&[e_4, e_5] = \alpha_{4,5}^{11} e_{11} && [e_4, e_6] = \alpha_{4,6}^{12} e_{12} \\
&[e_4, e_7] = \alpha_{4,7}^{13} e_{13} && [e_4, e_8] = \alpha_{4,8}^{14} e_{14} \\
&[e_5, e_6] = \alpha_{5,6}^{13} e_{13} && [e_5, e_7] = \alpha_{5,7}^{14} e_{14}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_4):\quad & -\alpha_{2,5}^{9} - \alpha_{3,4}^{9} + 1 &&= 0\\
(e_1, e_2, e_5):\quad & \alpha_{2,5}^{9} - \alpha_{2,6}^{10} - \alpha_{3,5}^{10} &&= 0\\
(e_1, e_3, e_4):\quad & \alpha_{3,4}^{9} - \alpha_{3,5}^{10} &&= 0\\
(e_1, e_2, e_6):\quad & \alpha_{2,6}^{10} - \alpha_{2,7}^{11} - \alpha_{3,6}^{11} &&= 0\\
(e_1, e_3, e_5):\quad & \alpha_{3,5}^{10} - \alpha_{3,6}^{11} - \alpha_{4,5}^{11} &&= 0\\
(e_1, e_2, e_7):\quad & \alpha_{2,7}^{11} - \alpha_{2,8}^{12} - \alpha_{3,7}^{12} &&= 0\\
(e_1, e_3, e_6):\quad & \alpha_{3,6}^{11} - \alpha_{3,7}^{12} - \alpha_{4,6}^{12} &&= 0\\
(e_1, e_4, e_5):\quad & \alpha_{4,5}^{11} - \alpha_{4,6}^{12} &&= 0\\
(e_1, e_2, e_8):\quad & \alpha_{2,8}^{12} - \alpha_{2,9}^{13} - \alpha_{3,8}^{13} &&= 0\\
(e_1, e_3, e_7):\quad & \alpha_{3,7}^{12} - \alpha_{3,8}^{13} - \alpha_{4,7}^{13} &&= 0\\
(e_1, e_4, e_6):\quad & \alpha_{4,6}^{12} - \alpha_{4,7}^{13} - \alpha_{5,6}^{13} &&= 0\\
(e_2, e_3, e_4):\quad & \alpha_{2,9}^{13}\alpha_{3,4}^{9} - \alpha_{3,8}^{13} + \alpha_{4,7}^{13} &&= 0\\
(e_1, e_2, e_9):\quad & -\alpha_{2,10}^{14} + \alpha_{2,9}^{13} - \alpha_{3,9}^{14} &&= 0\\
(e_1, e_3, e_8):\quad & \alpha_{3,8}^{13} - \alpha_{3,9}^{14} - \alpha_{4,8}^{14} &&= 0\\
(e_1, e_4, e_7):\quad & \alpha_{4,7}^{13} - \alpha_{4,8}^{14} - \alpha_{5,7}^{14} &&= 0\\
(e_1, e_5, e_6):\quad & \alpha_{5,6}^{13} - \alpha_{5,7}^{14} &&= 0\\
(e_2, e_3, e_5):\quad & \alpha_{2,10}^{14}\alpha_{3,5}^{10} - \alpha_{2,5}^{9}\alpha_{3,9}^{14} + \alpha_{5,7}^{14} &&= 0
\end{aligned}
$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{4,8}^{14} \to x_1$$

$$\alpha_{2,7}^{11} \to x_2$$

$$\alpha_{2,5}^{9} \to x_3$$

$$\alpha_{3,5}^{10} \to x_4$$

$$\alpha_{2,8}^{12} \to x_5$$

$$\alpha_{4,6}^{12} \to x_6$$

$$\alpha_{5,6}^{13} \to x_7$$

$$\alpha_{4,5}^{11} \to x_8$$

$$\alpha_{3,8}^{13} \to x_9$$

$$\alpha_{3,7}^{12} \to x_{10}$$

$$\alpha^{14}_{5,7} \to x_{11}$$

$$\alpha^{14}_{2,10} \to x_{12}$$

$$\alpha^{9}_{3,4} \to x_{13}$$

$$\alpha^{13}_{2,9} \to x_{14}$$

$$\alpha^{14}_{3,9} \to x_{15}$$

$$\alpha^{10}_{2,6} \to x_{16}$$

$$\alpha^{13}_{4,7} \to x_{17}$$

$$\alpha^{11}_{3,6} \to x_{18}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4): &\quad -x_{13} - x_3 + 1 &&= 0\\
(e_1, e_2, e_5): &\quad -x_{16} + x_3 - x_4 &&= 0\\
(e_1, e_3, e_4): &\quad x_{13} - x_4 &&= 0\\
(e_1, e_2, e_6): &\quad x_{16} - x_{18} - x_2 &&= 0\\
(e_1, e_3, e_5): &\quad -x_{18} + x_4 - x_8 &&= 0\\
(e_1, e_2, e_7): &\quad -x_{10} + x_2 - x_5 &&= 0\\
(e_1, e_3, e_6): &\quad -x_{10} + x_{18} - x_6 &&= 0\\
(e_1, e_4, e_5): &\quad -x_6 + x_8 &&= 0\\
(e_1, e_2, e_8): &\quad -x_{14} + x_5 - x_9 &&= 0\\
(e_1, e_3, e_7): &\quad x_{10} - x_{17} - x_9 &&= 0\\
(e_1, e_4, e_6): &\quad -x_{17} + x_6 - x_7 &&= 0\\
(e_2, e_3, e_4): &\quad x_{13}x_{14} + x_{17} - x_9 &&= 0\\
(e_1, e_2, e_9): &\quad -x_{12} + x_{14} - x_{15} &&= 0\\
(e_1, e_3, e_8): &\quad -x_1 - x_{15} + x_9 &&= 0\\
(e_1, e_4, e_7): &\quad -x_1 - x_{11} + x_{17} &&= 0\\
(e_1, e_5, e_6): &\quad -x_{11} + x_7 &&= 0\\
(e_2, e_3, e_5): &\quad x_{11} + x_{12}x_4 - x_{15}x_3 &&= 0
\end{aligned}$$

Groebner basis (18 variables, 15 linear, 1 nonlinear)

$$2x_1 - x_{16} - 4x_{17} - 2x_{18} + 1 = 0$$

$$-x_{16} + x_{18} + x_2 = 0$$

$$-x_{16} + 2x_3 - 1 = 0$$

$$x_{16} + 2x_4 - 1 = 0$$

$$-x_{16} + 6x_{18} + 2x_5 - 1 = 0$$
$$x_{16} + 2x_{18} + 2x_6 - 1 = 0$$
$$x_{16} + 2x_{17} + 2x_{18} + 2x_7 - 1 = 0$$
$$x_{16} + 2x_{18} + 2x_8 - 1 = 0$$
$$-x_{16} + 2x_{17} - 4x_{18} + 2x_9 + 1 = 0$$
$$2x_{10} - x_{16} - 4x_{18} + 1 = 0$$
$$2x_{11} + x_{16} + 2x_{17} + 2x_{18} - 1 = 0$$
$$x_{12} - 4x_{17} + 6x_{18} - 1 = 0$$
$$2x_{13} + x_{16} - 1 = 0$$
$$x_{14} - x_{17} + 5x_{18} - 1 = 0$$
$$x_{15} + 3x_{17} - x_{18} = 0$$
$$x_{16}x_{17} - 5x_{16}x_{18} + 2x_{16} - 5x_{17} + 9x_{18} - 2 = 0$$

$\mathfrak{m}_{1A}(5, 14)$

m1A514 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_1, e_{13}] &= e_{14} \\
[e_2, e_9] &= e_{14} & [e_3, e_8] &= -e_{14} \\
[e_4, e_7] &= e_{14} & [e_5, e_6] &= -e_{14}
\end{aligned}$$

No non-trivial Jacobi tests

$\mathfrak{m}_{3A}(5,14)$

m3A514 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4\\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6\\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8\\
[e_1,e_8] &= e_9 & [e_1,e_9] &= e_{10}\\
[e_1,e_{10}] &= e_{11} & [e_1,e_{11}] &= e_{12}\\
[e_1,e_{12}] &= e_{13} & [e_1,e_{13}] &= e_{14}\\
[e_2,e_7] &= e_{12} & [e_2,e_8] &= 3e_{13}\\
[e_2,e_9] &= \alpha_{2,9}^{14}e_{14} & [e_3,e_6] &= -e_{12}\\
[e_3,e_7] &= -2e_{13} & [e_3,e_8] &= \alpha_{3,8}^{14}e_{14}\\
[e_4,e_5] &= e_{12} & [e_4,e_6] &= e_{13}\\
[e_4,e_7] &= \alpha_{4,7}^{14}e_{14} & [e_5,e_6] &= \alpha_{5,6}^{14}e_{14}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1,e_2,e_8):&\quad -\alpha_{2,9}^{14}-\alpha_{3,8}^{14}+3 &&= 0\\
(e_1,e_3,e_7):&\quad -\alpha_{3,8}^{14}-\alpha_{4,7}^{14}-2 &&= 0\\
(e_1,e_4,e_6):&\quad -\alpha_{4,7}^{14}-\alpha_{5,6}^{14}+1 &&= 0
\end{aligned}
$$

Infinite number of solutions.

How the solution(s) were or were not found:

Change variables

$$
\begin{aligned}
\alpha_{4,7}^{14} &\to x_1\\
\alpha_{2,9}^{14} &\to x_2\\
\alpha_{5,6}^{14} &\to x_3\\
\alpha_{3,8}^{14} &\to x_4
\end{aligned}
$$

Jacobi Tests

$$
\begin{aligned}
(e_1,e_2,e_8):&\quad -x_2-x_4+3 &&= 0\\
(e_1,e_3,e_7):&\quad -x_1-x_4-2 &&= 0\\
(e_1,e_4,e_6):&\quad -x_1-x_3+1 &&= 0
\end{aligned}
$$

Groebner basis (4 variables, 3 linear, 0 nonlinear)

$$x_1 + x_4 + 2 = 0$$
$$x_2 + x_4 - 3 = 0$$
$$x_3 - x_4 - 3 = 0$$

$\mathfrak{m}_{5A}(5, 14)$

m5A514 (this line included for string searching purposes)

Original brackets:

$$\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_1, e_9] = e_{10} \\
[e_1, e_{10}] = e_{11} & [e_1, e_{11}] = e_{12} \\
[e_1, e_{12}] = e_{13} & [e_1, e_{13}] = e_{14} \\
[e_2, e_5] = e_{10} & [e_2, e_6] = 2e_{11} \\
[e_2, e_7] = \alpha_{2,7}^{12} e_{12} & [e_2, e_8] = \alpha_{2,8}^{13} e_{13} \\
[e_2, e_9] = \alpha_{2,9}^{14} e_{14} & [e_3, e_4] = -e_{10} \\
[e_3, e_5] = -e_{11} & [e_3, e_6] = \alpha_{3,6}^{12} e_{12} \\
[e_3, e_7] = \alpha_{3,7}^{13} e_{13} & [e_3, e_8] = \alpha_{3,8}^{14} e_{14} \\
[e_4, e_5] = \alpha_{4,5}^{12} e_{12} & [e_4, e_6] = \alpha_{4,6}^{13} e_{13} \\
[e_4, e_7] = \alpha_{4,7}^{14} e_{14} & [e_5, e_6] = \alpha_{5,6}^{14} e_{14}
\end{array}$$

Non-trivial Jacobi Tests:

$$\begin{array}{lll}
(e_1, e_2, e_6): & -\alpha_{2,7}^{12} - \alpha_{3,6}^{12} + 2 & = 0 \\
(e_1, e_3, e_5): & -\alpha_{3,6}^{12} - \alpha_{4,5}^{12} - 1 & = 0 \\
(e_1, e_2, e_7): & \alpha_{2,7}^{12} - \alpha_{2,8}^{13} - \alpha_{3,7}^{13} & = 0 \\
(e_1, e_3, e_6): & \alpha_{3,6}^{12} - \alpha_{3,7}^{13} - \alpha_{4,6}^{13} & = 0 \\
(e_1, e_4, e_5): & \alpha_{4,5}^{12} - \alpha_{4,6}^{13} & = 0 \\
(e_1, e_2, e_8): & \alpha_{2,8}^{13} - \alpha_{2,9}^{14} - \alpha_{3,8}^{14} & = 0 \\
(e_1, e_3, e_7): & \alpha_{3,7}^{13} - \alpha_{3,8}^{14} - \alpha_{4,7}^{14} & = 0 \\
(e_1, e_4, e_6): & \alpha_{4,6}^{13} - \alpha_{4,7}^{14} - \alpha_{5,6}^{14} & = 0
\end{array}$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha^{13}_{3,7} \to x_1$$
$$\alpha^{12}_{2,7} \to x_2$$
$$\alpha^{13}_{4,6} \to x_3$$
$$\alpha^{14}_{5,6} \to x_4$$
$$\alpha^{14}_{4,7} \to x_5$$
$$\alpha^{12}_{3,6} \to x_6$$
$$\alpha^{12}_{4,5} \to x_7$$
$$\alpha^{14}_{2,9} \to x_8$$
$$\alpha^{14}_{3,8} \to x_9$$
$$\alpha^{13}_{2,8} \to x_{10}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_6):& \quad -x_2 - x_6 + 2 &&= 0\\
(e_1, e_3, e_5):& \quad -x_6 - x_7 - 1 &&= 0\\
(e_1, e_2, e_7):& \quad -x_1 - x_{10} + x_2 &&= 0\\
(e_1, e_3, e_6):& \quad -x_1 - x_3 + x_6 &&= 0\\
(e_1, e_4, e_5):& \quad -x_3 + x_7 &&= 0\\
(e_1, e_2, e_8):& \quad x_{10} - x_8 - x_9 &&= 0\\
(e_1, e_3, e_7):& \quad x_1 - x_5 - x_9 &&= 0\\
(e_1, e_4, e_6):& \quad x_3 - x_4 - x_5 &&= 0
\end{aligned}$$

Groebner basis (10 variables, 8 linear, 0 nonlinear)

$$3x_1 + 2x_{10} - 5 = 0$$
$$-x_{10} + 3x_2 - 5 = 0$$
$$-x_{10} + 3x_3 + 4 = 0$$
$$-x_{10} + x_4 - x_9 + 3 = 0$$
$$2x_{10} + 3x_5 + 3x_9 - 5 = 0$$
$$x_{10} + 3x_6 - 1 = 0$$
$$-x_{10} + 3x_7 + 4 = 0$$
$$-x_{10} + x_8 + x_9 = 0$$

$\mathfrak{m}_{7A}(5, 14)$

m7A514 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
&[e_1, e_2] = e_3 && [e_1, e_3] = e_4 \\
&[e_1, e_4] = e_5 && [e_1, e_5] = e_6 \\
&[e_1, e_6] = e_7 && [e_1, e_7] = e_8 \\
&[e_1, e_8] = e_9 && [e_1, e_9] = e_{10} \\
&[e_1, e_{10}] = e_{11} && [e_1, e_{11}] = e_{12} \\
&[e_1, e_{12}] = e_{13} && [e_1, e_{13}] = e_{14} \\
&[e_2, e_3] = e_8 && [e_2, e_4] = e_9 \\
&[e_2, e_5] = \alpha_{2,5}^{10} e_{10} && [e_2, e_6] = \alpha_{2,6}^{11} e_{11} \\
&[e_2, e_7] = \alpha_{2,7}^{12} e_{12} && [e_2, e_8] = \alpha_{2,8}^{13} e_{13} \\
&[e_2, e_9] = \alpha_{2,9}^{14} e_{14} && [e_3, e_4] = \alpha_{3,4}^{10} e_{10} \\
&[e_3, e_5] = \alpha_{3,5}^{11} e_{11} && [e_3, e_6] = \alpha_{3,6}^{12} e_{12} \\
&[e_3, e_7] = \alpha_{3,7}^{13} e_{13} && [e_3, e_8] = \alpha_{3,8}^{14} e_{14} \\
&[e_4, e_5] = \alpha_{4,5}^{12} e_{12} && [e_4, e_6] = \alpha_{4,6}^{13} e_{13} \\
&[e_4, e_7] = \alpha_{4,7}^{14} e_{14} && [e_5, e_6] = \alpha_{5,6}^{14} e_{14}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
&(e_1, e_2, e_4): && -\alpha_{2,5}^{10} - \alpha_{3,4}^{10} + 1 && = 0 \\
&(e_1, e_2, e_5): && \alpha_{2,5}^{10} - \alpha_{2,6}^{11} - \alpha_{3,5}^{11} && = 0 \\
&(e_1, e_3, e_4): && \alpha_{3,4}^{10} - \alpha_{3,5}^{11} && = 0 \\
&(e_1, e_2, e_6): && \alpha_{2,6}^{11} - \alpha_{2,7}^{12} - \alpha_{3,6}^{12} && = 0 \\
&(e_1, e_3, e_5): && \alpha_{3,5}^{11} - \alpha_{3,6}^{12} - \alpha_{4,5}^{12} && = 0 \\
&(e_1, e_2, e_7): && \alpha_{2,7}^{12} - \alpha_{2,8}^{13} - \alpha_{3,7}^{13} && = 0 \\
&(e_1, e_3, e_6): && \alpha_{3,6}^{12} - \alpha_{3,7}^{13} - \alpha_{4,6}^{13} && = 0 \\
&(e_1, e_4, e_5): && \alpha_{4,5}^{12} - \alpha_{4,6}^{13} && = 0 \\
&(e_1, e_2, e_8): && \alpha_{2,8}^{13} - \alpha_{2,9}^{14} - \alpha_{3,8}^{14} && = 0 \\
&(e_1, e_3, e_7): && \alpha_{3,7}^{13} - \alpha_{3,8}^{14} - \alpha_{4,7}^{14} && = 0 \\
&(e_1, e_4, e_6): && \alpha_{4,6}^{13} - \alpha_{4,7}^{14} - \alpha_{5,6}^{14} && = 0
\end{aligned}
$$

Infinite number of solutions.

How the solution(s) were or were not found:

Change variables

$$\alpha_{2,6}^{11} \to x_1$$
$$\alpha_{3,7}^{13} \to x_2$$
$$\alpha_{2,7}^{12} \to x_3$$
$$\alpha_{4,6}^{13} \to x_4$$
$$\alpha_{5,6}^{14} \to x_5$$
$$\alpha_{4,7}^{14} \to x_6$$
$$\alpha_{3,6}^{12} \to x_7$$
$$\alpha_{3,5}^{11} \to x_8$$
$$\alpha_{4,5}^{12} \to x_9$$
$$\alpha_{2,9}^{14} \to x_{10}$$
$$\alpha_{3,8}^{14} \to x_{11}$$
$$\alpha_{3,4}^{10} \to x_{12}$$
$$\alpha_{2,8}^{13} \to x_{13}$$
$$\alpha_{2,5}^{10} \to x_{14}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4): &\quad -x_{12} - x_{14} + 1 && = 0 \\
(e_1, e_2, e_5): &\quad -x_1 + x_{14} - x_8 && = 0 \\
(e_1, e_3, e_4): &\quad x_{12} - x_8 && = 0 \\
(e_1, e_2, e_6): &\quad x_1 - x_3 - x_7 && = 0 \\
(e_1, e_3, e_5): &\quad -x_7 + x_8 - x_9 && = 0 \\
(e_1, e_2, e_7): &\quad -x_{13} - x_2 + x_3 && = 0 \\
(e_1, e_3, e_6): &\quad -x_2 - x_4 + x_7 && = 0 \\
(e_1, e_4, e_5): &\quad -x_4 + x_9 && = 0 \\
(e_1, e_2, e_8): &\quad -x_{10} - x_{11} + x_{13} && = 0 \\
(e_1, e_3, e_7): &\quad -x_{11} + x_2 - x_6 && = 0 \\
(e_1, e_4, e_6): &\quad x_4 - x_5 - x_6 && = 0
\end{aligned}$$

Groebner basis (14 variables, 11 linear, 0 nonlinear)

$$x_1 - 2x_{14} + 1 = 0$$

$$2x_{13} - 5x_{14} + 3x_2 + 3 = 0$$
$$-x_{13} - 5x_{14} + 3x_3 + 3 = 0$$
$$-x_{13} + 4x_{14} + 3x_4 - 3 = 0$$
$$-x_{11} - x_{13} + 3x_{14} + x_5 - 2 = 0$$
$$3x_{11} + 2x_{13} - 5x_{14} + 3x_6 + 3 = 0$$
$$x_{13} - x_{14} + 3x_7 = 0$$
$$x_{14} + x_8 - 1 = 0$$
$$-x_{13} + 4x_{14} + 3x_9 - 3 = 0$$
$$x_{10} + x_{11} - x_{13} = 0$$
$$x_{12} + x_{14} - 1 = 0$$

$\mathfrak{m}_{2A}(6, 14)$

m2A614 (this line included for string searching purposes)
Original brackets:

$$\begin{aligned} [e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\ [e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\ [e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\ [e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\ [e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\ [e_1, e_{12}] &= e_{13} & [e_1, e_{13}] &= e_{14} \\ [e_2, e_7] &= e_{13} & [e_2, e_8] &= 3e_{14} \\ [e_3, e_6] &= -e_{13} & [e_3, e_7] &= -2e_{14} \\ [e_4, e_5] &= e_{13} & [e_4, e_6] &= e_{14} \end{aligned}$$

No non-trivial Jacobi tests

$\mathfrak{m}_{4A}(6, 14)$

m4A614 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4 \\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6 \\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8 \\
[e_1,e_8] &= e_9 & [e_1,e_9] &= e_{10} \\
[e_1,e_{10}] &= e_{11} & [e_1,e_{11}] &= e_{12} \\
[e_1,e_{12}] &= e_{13} & [e_1,e_{13}] &= e_{14} \\
[e_2,e_5] &= e_{11} & [e_2,e_6] &= 2e_{12} \\
[e_2,e_7] &= \alpha_{2,7}^{13} e_{13} & [e_2,e_8] &= \alpha_{2,8}^{14} e_{14} \\
[e_3,e_4] &= -e_{11} & [e_3,e_5] &= -e_{12} \\
[e_3,e_6] &= \alpha_{3,6}^{13} e_{13} & [e_3,e_7] &= \alpha_{3,7}^{14} e_{14} \\
[e_4,e_5] &= \alpha_{4,5}^{13} e_{13} & [e_4,e_6] &= \alpha_{4,6}^{14} e_{14}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1,e_2,e_6): &\quad -\alpha_{2,7}^{13} - \alpha_{3,6}^{13} + 2 &&= 0 \\
(e_1,e_3,e_5): &\quad -\alpha_{3,6}^{13} - \alpha_{4,5}^{13} - 1 &&= 0 \\
(e_1,e_2,e_7): &\quad \alpha_{2,7}^{13} - \alpha_{2,8}^{14} - \alpha_{3,7}^{14} &&= 0 \\
(e_1,e_3,e_6): &\quad \alpha_{3,6}^{13} - \alpha_{3,7}^{14} - \alpha_{4,6}^{14} &&= 0 \\
(e_1,e_4,e_5): &\quad \alpha_{4,5}^{13} - \alpha_{4,6}^{14} &&= 0
\end{aligned}
$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{2,8}^{14} \to x_1$$

$$\alpha_{3,6}^{13} \to x_2$$

$$\alpha_{4,5}^{13} \to x_3$$

$$\alpha_{2,7}^{13} \to x_4$$

$$\alpha_{4,6}^{14} \to x_5$$

$$\alpha_{3,7}^{14} \to x_6$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_6): &\quad -x_2 - x_4 + 2 &= 0\\
(e_1, e_3, e_5): &\quad -x_2 - x_3 - 1 &= 0\\
(e_1, e_2, e_7): &\quad -x_1 + x_4 - x_6 &= 0\\
(e_1, e_3, e_6): &\quad x_2 - x_5 - x_6 &= 0\\
(e_1, e_4, e_5): &\quad x_3 - x_5 &= 0
\end{aligned}
$$

Groebner basis (6 variables, 5 linear, 0 nonlinear)

$$2x_1 + 3x_6 - 5 = 0$$

$$2x_2 - x_6 + 1 = 0$$

$$2x_3 + x_6 + 1 = 0$$

$$2x_4 + x_6 - 5 = 0$$

$$2x_5 + x_6 + 1 = 0$$

$\mathfrak{m}_{6A}(6, 14)$

m6A614 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4\\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6\\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8\\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10}\\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12}\\
[e_1, e_{12}] &= e_{13} & [e_1, e_{13}] &= e_{14}\\
[e_2, e_3] &= e_9 & [e_2, e_4] &= e_{10}\\
[e_2, e_5] &= \alpha_{2,5}^{11} e_{11} & [e_2, e_6] &= \alpha_{2,6}^{12} e_{12}\\
[e_2, e_7] &= \alpha_{2,7}^{13} e_{13} & [e_2, e_8] &= \alpha_{2,8}^{14} e_{14}\\
[e_3, e_4] &= \alpha_{3,4}^{11} e_{11} & [e_3, e_5] &= \alpha_{3,5}^{12} e_{12}\\
[e_3, e_6] &= \alpha_{3,6}^{13} e_{13} & [e_3, e_7] &= \alpha_{3,7}^{14} e_{14}\\
[e_4, e_5] &= \alpha_{4,5}^{13} e_{13} & [e_4, e_6] &= \alpha_{4,6}^{14} e_{14}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_4): &\quad -\alpha_{2,5}^{11} - \alpha_{3,4}^{11} + 1 &&= 0\\
(e_1, e_2, e_5): &\quad \alpha_{2,5}^{11} - \alpha_{2,6}^{12} - \alpha_{3,5}^{12} &&= 0\\
(e_1, e_3, e_4): &\quad \alpha_{3,4}^{11} - \alpha_{3,5}^{12} &&= 0\\
(e_1, e_2, e_6): &\quad \alpha_{2,6}^{12} - \alpha_{2,7}^{13} - \alpha_{3,6}^{13} &&= 0\\
(e_1, e_3, e_5): &\quad \alpha_{3,5}^{12} - \alpha_{3,6}^{13} - \alpha_{4,5}^{13} &&= 0\\
(e_1, e_2, e_7): &\quad \alpha_{2,7}^{13} - \alpha_{2,8}^{14} - \alpha_{3,7}^{14} &&= 0\\
(e_1, e_3, e_6): &\quad \alpha_{3,6}^{13} - \alpha_{3,7}^{14} - \alpha_{4,6}^{14} &&= 0\\
(e_1, e_4, e_5): &\quad \alpha_{4,5}^{13} - \alpha_{4,6}^{14} &&= 0
\end{aligned}
$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$
\begin{aligned}
\alpha_{2,8}^{14} &\to x_1\\
\alpha_{3,6}^{13} &\to x_2\\
\alpha_{4,5}^{13} &\to x_3\\
\alpha_{2,7}^{13} &\to x_4\\
\alpha_{4,6}^{14} &\to x_5\\
\alpha_{3,5}^{12} &\to x_6\\
\alpha_{2,5}^{11} &\to x_7\\
\alpha_{2,6}^{12} &\to x_8\\
\alpha_{3,7}^{14} &\to x_9\\
\alpha_{3,4}^{11} &\to x_{10}
\end{aligned}
$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_4): &\quad -x_{10} - x_7 + 1 &&= 0\\
(e_1, e_2, e_5): &\quad -x_6 + x_7 - x_8 &&= 0\\
(e_1, e_3, e_4): &\quad x_{10} - x_6 &&= 0\\
(e_1, e_2, e_6): &\quad -x_2 - x_4 + x_8 &&= 0\\
(e_1, e_3, e_5): &\quad -x_2 - x_3 + x_6 &&= 0\\
(e_1, e_2, e_7): &\quad -x_1 + x_4 - x_9 &&= 0\\
(e_1, e_3, e_6): &\quad x_2 - x_5 - x_9 &&= 0\\
(e_1, e_4, e_5): &\quad x_3 - x_5 &&= 0
\end{aligned}
$$

Groebner basis (10 variables, 8 linear, 0 nonlinear)

$$2x_1 + 5x_{10} + 3x_9 - 2 = 0$$
$$-x_{10} + 2x_2 - x_9 = 0$$
$$-x_{10} + 2x_3 + x_9 = 0$$
$$5x_{10} + 2x_4 + x_9 - 2 = 0$$
$$-x_{10} + 2x_5 + x_9 = 0$$
$$-x_{10} + x_6 = 0$$
$$x_{10} + x_7 - 1 = 0$$
$$2x_{10} + x_8 - 1 = 0$$

$\mathfrak{m}_{1A}(7, 14)$

m1A714 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_1, e_{13}] &= e_{14} \\
[e_2, e_7] &= e_{14} & [e_3, e_6] &= -e_{14} \\
[e_4, e_5] &= e_{14}
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{3A}(7, 14)$

m3A714 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4\\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6\\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8\\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10}\\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12}\\
[e_1, e_{12}] &= e_{13} & [e_1, e_{13}] &= e_{14}\\
[e_2, e_5] &= e_{12} & [e_2, e_6] &= 2e_{13}\\
[e_2, e_7] &= \alpha_{2,7}^{14} e_{14} & [e_3, e_4] &= -e_{12}\\
[e_3, e_5] &= -e_{13} & [e_3, e_6] &= \alpha_{3,6}^{14} e_{14}\\
[e_4, e_5] &= \alpha_{4,5}^{14} e_{14}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_6): &\quad -\alpha_{2,7}^{14} - \alpha_{3,6}^{14} + 2 &&= 0\\
(e_1, e_3, e_5): &\quad -\alpha_{3,6}^{14} - \alpha_{4,5}^{14} - 1 &&= 0
\end{aligned}
$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$
\alpha_{2,7}^{14} \to x_1
$$

$$
\alpha_{4,5}^{14} \to x_2
$$

$$
\alpha_{3,6}^{14} \to x_3
$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_6): &\quad -x_1 - x_3 + 2 &&= 0\\
(e_1, e_3, e_5): &\quad -x_2 - x_3 - 1 &&= 0
\end{aligned}
$$

Groebner basis (3 variables, 2 linear, 0 nonlinear)

$$
x_1 + x_3 - 2 = 0
$$

$$
x_2 + x_3 + 1 = 0
$$

$\mathfrak{m}_{5A}(7,14)$

m5A714 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4 \\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6 \\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8 \\
[e_1,e_8] &= e_9 & [e_1,e_9] &= e_{10} \\
[e_1,e_{10}] &= e_{11} & [e_1,e_{11}] &= e_{12} \\
[e_1,e_{12}] &= e_{13} & [e_1,e_{13}] &= e_{14} \\
[e_2,e_3] &= e_{10} & [e_2,e_4] &= e_{11} \\
[e_2,e_5] &= \alpha_{2,5}^{12} e_{12} & [e_2,e_6] &= \alpha_{2,6}^{13} e_{13} \\
[e_2,e_7] &= \alpha_{2,7}^{14} e_{14} & [e_3,e_4] &= \alpha_{3,4}^{12} e_{12} \\
[e_3,e_5] &= \alpha_{3,5}^{13} e_{13} & [e_3,e_6] &= \alpha_{3,6}^{14} e_{14} \\
[e_4,e_5] &= \alpha_{4,5}^{14} e_{14}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1,e_2,e_4):&\quad -\alpha_{2,5}^{12} - \alpha_{3,4}^{12} + 1 &= 0 \\
(e_1,e_2,e_5):&\quad \alpha_{2,5}^{12} - \alpha_{2,6}^{13} - \alpha_{3,5}^{13} &= 0 \\
(e_1,e_3,e_4):&\quad \alpha_{3,4}^{12} - \alpha_{3,5}^{13} &= 0 \\
(e_1,e_2,e_6):&\quad \alpha_{2,6}^{13} - \alpha_{2,7}^{14} - \alpha_{3,6}^{14} &= 0 \\
(e_1,e_3,e_5):&\quad \alpha_{3,5}^{13} - \alpha_{3,6}^{14} - \alpha_{4,5}^{14} &= 0
\end{aligned}
$$

Infinite number of solutions.

How the solution(s) were or were not found:

Change variables

$$\alpha_{2,6}^{13} \to x_1$$

$$\alpha_{3,5}^{13} \to x_2$$

$$\alpha_{3,4}^{12} \to x_3$$

$$\alpha_{2,5}^{12} \to x_4$$

$$\alpha_{4,5}^{14} \to x_5$$

$$\alpha_{2,7}^{14} \to x_6$$

$$\alpha_{3,6}^{14} \to x_7$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_4): & \quad -x_3 - x_4 + 1 && = 0 \\
(e_1, e_2, e_5): & \quad -x_1 - x_2 + x_4 && = 0 \\
(e_1, e_3, e_4): & \quad -x_2 + x_3 && = 0 \\
(e_1, e_2, e_6): & \quad x_1 - x_6 - x_7 && = 0 \\
(e_1, e_3, e_5): & \quad x_2 - x_5 - x_7 && = 0
\end{aligned}
$$

Groebner basis (7 variables, 5 linear, 0 nonlinear)

$$x_1 - x_6 - x_7 = 0$$
$$2x_2 + x_6 + x_7 - 1 = 0$$
$$2x_3 + x_6 + x_7 - 1 = 0$$
$$2x_4 - x_6 - x_7 - 1 = 0$$
$$2x_5 + x_6 + 3x_7 - 1 = 0$$

$\mathfrak{m}_{2A}(8, 14)$

m2A814 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_1, e_{13}] &= e_{14} \\
[e_2, e_5] &= e_{13} & [e_2, e_6] &= 2e_{14} \\
[e_3, e_4] &= -e_{13} & [e_3, e_5] &= -e_{14}
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{4A}(8, 14)$

m4A814 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_1, e_{13}] &= e_{14} \\
[e_2, e_3] &= e_{11} & [e_2, e_4] &= e_{12} \\
[e_2, e_5] &= \alpha_{2,5}^{13} e_{13} & [e_2, e_6] &= \alpha_{2,6}^{14} e_{14} \\
[e_3, e_4] &= \alpha_{3,4}^{13} e_{13} & [e_3, e_5] &= \alpha_{3,5}^{14} e_{14}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_4): &\quad -\alpha_{2,5}^{13} - \alpha_{3,4}^{13} + 1 &&= 0 \\
(e_1, e_2, e_5): &\quad \alpha_{2,5}^{13} - \alpha_{2,6}^{14} - \alpha_{3,5}^{14} &&= 0 \\
(e_1, e_3, e_4): &\quad \alpha_{3,4}^{13} - \alpha_{3,5}^{14} &&= 0
\end{aligned}
$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$
\begin{aligned}
\alpha_{3,4}^{13} &\to x_1 \\
\alpha_{2,5}^{13} &\to x_2 \\
\alpha_{2,6}^{14} &\to x_3 \\
\alpha_{3,5}^{14} &\to x_4
\end{aligned}
$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_4): &\quad -x_1 - x_2 + 1 &&= 0 \\
(e_1, e_2, e_5): &\quad x_2 - x_3 - x_4 &&= 0 \\
(e_1, e_3, e_4): &\quad x_1 - x_4 &&= 0
\end{aligned}
$$

Groebner basis (4 variables, 3 linear, 0 nonlinear)

$$
\begin{aligned}
x_1 - x_4 &= 0 \\
x_2 + x_4 - 1 &= 0 \\
x_3 + 2x_4 - 1 &= 0
\end{aligned}
$$

$\mathfrak{m}_{1A}(9,14)$

m1A914 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4\\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6\\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8\\
[e_1,e_8] &= e_9 & [e_1,e_9] &= e_{10}\\
[e_1,e_{10}] &= e_{11} & [e_1,e_{11}] &= e_{12}\\
[e_1,e_{12}] &= e_{13} & [e_1,e_{13}] &= e_{14}\\
[e_2,e_5] &= e_{14} & [e_3,e_4] &= -e_{14}
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{3A}(9,14)$

m3A914 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4\\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6\\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8\\
[e_1,e_8] &= e_9 & [e_1,e_9] &= e_{10}\\
[e_1,e_{10}] &= e_{11} & [e_1,e_{11}] &= e_{12}\\
[e_1,e_{12}] &= e_{13} & [e_1,e_{13}] &= e_{14}\\
[e_2,e_3] &= e_{12} & [e_2,e_4] &= e_{13}\\
[e_2,e_5] &= \alpha_{2,5}^{14} e_{14} & [e_3,e_4] &= \alpha_{3,4}^{14} e_{14}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$(e_1,e_2,e_4): \quad -\alpha_{2,5}^{14} - \alpha_{3,4}^{14} + 1 \qquad = 0$$

Infinite number of solutions.

How the solution(s) were or were not found:

Change variables

$$\alpha_{3,4}^{14} \to x_1$$

$$\alpha_{2,5}^{14} \to x_2$$

Jacobi Tests

$$(e_1, e_2, e_4): \quad -x_1 - x_2 + 1 \qquad = 0$$

Groebner basis (2 variables, 1 linear, 0 nonlinear)

$$x_1 + x_2 - 1 = 0$$

$\mathfrak{m}_{2A}(10, 14)$

m2A1014 (this line included for string searching purposes)
Original brackets:

$$\begin{aligned} [e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\ [e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\ [e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\ [e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\ [e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\ [e_1, e_{12}] &= e_{13} & [e_1, e_{13}] &= e_{14} \\ [e_2, e_3] &= e_{13} & [e_2, e_4] &= e_{14} \end{aligned}$$

No non-trivial Jacobi tests

$\mathfrak{m}_{1A}(11, 14)$

m1A1114 (this line included for string searching purposes)
Original brackets:

$$\begin{aligned} [e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\ [e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\ [e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\ [e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\ [e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\ [e_1, e_{12}] &= e_{13} & [e_1, e_{13}] &= e_{14} \\ [e_2, e_3] &= e_{14} \end{aligned}$$

No non-trivial Jacobi tests

$\mathfrak{m}_{2B}(2,6)$

m2B26 (this line included for string searching purposes)
Solution 1

$$
\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_2, e_3] = e_5 \\
[e_2, e_5] = e_6 & [e_3, e_4] = -e_6
\end{array}
$$

Original brackets:

$$
\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_2, e_3] = e_5 \\
[e_2, e_5] = e_6 & [e_3, e_4] = \alpha_{3,4}^6 e_6
\end{array}
$$

Non-trivial Jacobi Tests:

$$(e_1, e_2, e_4): \quad -\alpha_{3,4}^6 - 1 = 0$$

Solution 1:

$$\alpha_{3,4}^6 = -1$$

How the solution(s) were or were not found:
Change variables

$$\alpha_{3,4}^6 \to x_1$$

Jacobi Tests

$$(e_1, e_2, e_4): \quad -x_1 - 1 = 0$$

Groebner basis (1 variables, 1 linear, 0 nonlinear)

$$x_1 + 1 = 0$$

Solution 1:

$$x_1 = -1$$

$\mathfrak{m}_{2B}(2,8)$

m2B28 (this line included for string searching purposes)
Original brackets:

$$
\begin{aligned}
&[e_1,e_2]=e_3 && [e_1,e_3]=e_4\\
&[e_1,e_4]=e_5 && [e_1,e_5]=e_6\\
&[e_1,e_6]=e_7 && [e_2,e_5]=e_7\\
&[e_2,e_7]=e_8 && [e_3,e_4]=-e_7\\
&[e_3,e_6]=\alpha_{3,6}^{8}e_8 && [e_4,e_5]=\alpha_{4,5}^{8}e_8
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
&(e_1,e_2,e_6): && -\alpha_{3,6}^{8}-1 &&=0\\
&(e_1,e_3,e_5): && -\alpha_{3,6}^{8}-\alpha_{4,5}^{8} &&=0\\
&(e_2,e_3,e_4): && \text{no solutions}
\end{aligned}
$$

There are no solutions.

$\mathfrak{m}_{4B}(2,8)$

m4B28 (this line included for string searching purposes)
Solution 1

$$
\begin{aligned}
&[e_1,e_2]=e_3 && [e_1,e_3]=e_4\\
&[e_1,e_4]=e_5 && [e_1,e_5]=e_6\\
&[e_1,e_6]=e_7 && [e_2,e_3]=e_5\\
&[e_2,e_4]=e_6 && [e_2,e_5]=3e_7\\
&[e_2,e_7]=e_8 && [e_3,e_4]=-2e_7\\
&[e_3,e_6]=-e_8 && [e_4,e_5]=e_8
\end{aligned}
$$

Original brackets:

$$
\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_2, e_3] = e_5 \\
[e_2, e_4] = e_6 & [e_2, e_5] = \alpha_{2,5}^{7} e_7 \\
[e_2, e_7] = e_8 & [e_3, e_4] = \alpha_{3,4}^{7} e_7 \\
[e_3, e_6] = \alpha_{3,6}^{8} e_8 & [e_4, e_5] = \alpha_{4,5}^{8} e_8
\end{array}
$$

Non-trivial Jacobi Tests:

$$
\begin{array}{lll}
(e_1, e_2, e_4): & -\alpha_{2,5}^{7} - \alpha_{3,4}^{7} + 1 & = 0 \\
(e_1, e_2, e_6): & -\alpha_{3,6}^{8} - 1 & = 0 \\
(e_1, e_3, e_5): & -\alpha_{3,6}^{8} - \alpha_{4,5}^{8} & = 0 \\
(e_2, e_3, e_4): & \alpha_{3,4}^{7} - \alpha_{3,6}^{8} + \alpha_{4,5}^{8} & = 0
\end{array}
$$

Solution 1:

$$
\begin{aligned}
\alpha_{4,5}^{8} &= 1 \\
\alpha_{3,4}^{7} &= -2 \\
\alpha_{3,6}^{8} &= -1 \\
\alpha_{2,5}^{7} &= 3
\end{aligned}
$$

How the solution(s) were or were not found:
Change variables

$$
\begin{aligned}
\alpha_{4,5}^{8} &\to x_1 \\
\alpha_{3,4}^{7} &\to x_2 \\
\alpha_{3,6}^{8} &\to x_3 \\
\alpha_{2,5}^{7} &\to x_4
\end{aligned}
$$

Jacobi Tests

$$
\begin{array}{lll}
(e_1, e_2, e_4): & -x_2 - x_4 + 1 & = 0 \\
(e_1, e_2, e_6): & -x_3 - 1 & = 0 \\
(e_1, e_3, e_5): & -x_1 - x_3 & = 0 \\
(e_2, e_3, e_4): & x_1 + x_2 - x_3 & = 0
\end{array}
$$

Groebner basis (4 variables, 4 linear, 0 nonlinear)

$$x_1 - 1 = 0$$
$$x_2 + 2 = 0$$
$$x_3 + 1 = 0$$
$$x_4 - 3 = 0$$

Solution 1:

$$x_1 = 1$$
$$x_2 = -2$$
$$x_3 = -1$$
$$x_4 = 3$$

$\mathfrak{m}_{3B}(3, 8)$

m3B38 (this line included for string searching purposes)

Solution 1

$$\begin{aligned} &[e_1, e_2] = e_3 && [e_1, e_3] = e_4 \\ &[e_1, e_4] = e_5 && [e_1, e_5] = e_6 \\ &[e_1, e_6] = e_7 && [e_2, e_3] = e_6 \\ &[e_2, e_4] = e_7 && [e_2, e_7] = e_8 \\ &[e_3, e_6] = -e_8 && [e_4, e_5] = e_8 \end{aligned}$$

Original brackets:

$$\begin{aligned} &[e_1, e_2] = e_3 && [e_1, e_3] = e_4 \\ &[e_1, e_4] = e_5 && [e_1, e_5] = e_6 \\ &[e_1, e_6] = e_7 && [e_2, e_3] = e_6 \\ &[e_2, e_4] = e_7 && [e_2, e_7] = e_8 \\ &[e_3, e_6] = \alpha_{3,6}^{8} e_8 && [e_4, e_5] = \alpha_{4,5}^{8} e_8 \end{aligned}$$

Non-trivial Jacobi Tests:

$$\begin{aligned} &(e_1, e_2, e_6): && -\alpha_{3,6}^{8} - 1 && = 0 \\ &(e_1, e_3, e_5): && -\alpha_{3,6}^{8} - \alpha_{4,5}^{8} && = 0 \end{aligned}$$

Solution 1:

$$\alpha_{4,5}^{8} = 1$$
$$\alpha_{3,6}^{8} = -1$$

How the solution(s) were or were not found:
Change variables

$$\alpha_{4,5}^{8} \to x_1$$
$$\alpha_{3,6}^{8} \to x_2$$

Jacobi Tests

$$\begin{aligned} (e_1, e_2, e_6): & \quad -x_2 - 1 & = 0 \\ (e_1, e_3, e_5): & \quad -x_1 - x_2 & = 0 \end{aligned}$$

Groebner basis (2 variables, 2 linear, 0 nonlinear)

$$x_1 - 1 = 0$$
$$x_2 + 1 = 0$$

Solution 1:

$$x_1 = 1$$
$$x_2 = -1$$

$\mathfrak{m}_{2B}(4,8)$

m2B48 (this line included for string searching purposes)

Solution 1

$$\begin{aligned} [e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\ [e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\ [e_1, e_6] &= e_7 & [e_2, e_3] &= e_7 \\ [e_2, e_7] &= e_8 & [e_3, e_6] &= -e_8 \\ [e_4, e_5] &= e_8 \end{aligned}$$

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_2, e_3] &= e_7 \\
[e_2, e_7] &= e_8 & [e_3, e_6] &= \alpha_{3,6}^{8} e_8 \\
[e_4, e_5] &= \alpha_{4,5}^{8} e_8
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_6): &\quad -\alpha_{3,6}^{8} - 1 &= 0 \\
(e_1, e_3, e_5): &\quad -\alpha_{3,6}^{8} - \alpha_{4,5}^{8} &= 0
\end{aligned}
$$

Solution 1:

$$
\begin{aligned}
\alpha_{4,5}^{8} &= 1 \\
\alpha_{3,6}^{8} &= -1
\end{aligned}
$$

How the solution(s) were or were not found:
Change variables

$$
\alpha_{4,5}^{8} \to x_1
$$

$$
\alpha_{3,6}^{8} \to x_2
$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_6): &\quad -x_2 - 1 &= 0 \\
(e_1, e_3, e_5): &\quad -x_1 - x_2 &= 0
\end{aligned}
$$

Groebner basis (2 variables, 2 linear, 0 nonlinear)

$$
x_1 - 1 = 0
$$

$$
x_2 + 1 = 0
$$

Solution 1:

$$
x_1 = 1
$$

$$
x_2 = -1
$$

$\mathfrak{m}_{2B}(2,10)$

m2B210 (this line included for string searching purposes)

Original brackets:

$$
\begin{array}{ll}
[e_1,e_2]=e_3 & [e_1,e_3]=e_4 \\
[e_1,e_4]=e_5 & [e_1,e_5]=e_6 \\
[e_1,e_6]=e_7 & [e_1,e_7]=e_8 \\
[e_1,e_8]=e_9 & [e_2,e_7]=e_9 \\
[e_2,e_9]=e_{10} & [e_3,e_6]=-e_9 \\
[e_3,e_8]=\alpha_{3,8}^{10}e_{10} & [e_4,e_5]=e_9 \\
[e_4,e_7]=\alpha_{4,7}^{10}e_{10} & [e_5,e_6]=\alpha_{5,6}^{10}e_{10}
\end{array}
$$

Non-trivial Jacobi Tests:

$$
\begin{array}{lll}
(e_1,e_2,e_8): & -\alpha_{3,8}^{10}-1 & =0 \\
(e_1,e_3,e_7): & -\alpha_{3,8}^{10}-\alpha_{4,7}^{10} & =0 \\
(e_1,e_4,e_6): & -\alpha_{4,7}^{10}-\alpha_{5,6}^{10} & =0 \\
(e_2,e_3,e_6): & \text{no solutions} & \\
(e_2,e_4,e_5): & \text{no solutions} &
\end{array}
$$

There are no solutions.

$\mathfrak{m}_{4B}(2,10)$

m4B210 (this line included for string searching purposes)

Original brackets:

$$
\begin{array}{ll}
[e_1,e_2]=e_3 & [e_1,e_3]=e_4 \\
[e_1,e_4]=e_5 & [e_1,e_5]=e_6 \\
[e_1,e_6]=e_7 & [e_1,e_7]=e_8 \\
[e_1,e_8]=e_9 & [e_2,e_5]=e_7 \\
[e_2,e_6]=2e_8 & [e_2,e_7]=\alpha_{2,7}^{9}e_9 \\
[e_2,e_9]=e_{10} & [e_3,e_4]=-e_7 \\
[e_3,e_5]=-e_8 & [e_3,e_6]=\alpha_{3,6}^{9}e_9 \\
[e_3,e_8]=\alpha_{3,8}^{10}e_{10} & [e_4,e_5]=\alpha_{4,5}^{9}e_9 \\
[e_4,e_7]=\alpha_{4,7}^{10}e_{10} & [e_5,e_6]=\alpha_{5,6}^{10}e_{10}
\end{array}
$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1, e_2, e_6): &\quad -\alpha_{2,7}^{9} - \alpha_{3,6}^{9} + 2 &= 0\\
(e_1, e_3, e_5): &\quad -\alpha_{3,6}^{9} - \alpha_{4,5}^{9} - 1 &= 0\\
(e_2, e_3, e_4): &\quad -\alpha_{2,7}^{9} &= 0\\
(e_1, e_2, e_8): &\quad -\alpha_{3,8}^{10} - 1 &= 0\\
(e_1, e_3, e_7): &\quad -\alpha_{3,8}^{10} - \alpha_{4,7}^{10} &= 0\\
(e_1, e_4, e_6): &\quad -\alpha_{4,7}^{10} - \alpha_{5,6}^{10} &= 0\\
(e_2, e_3, e_6): &\quad \alpha_{3,6}^{9} - 2\alpha_{3,8}^{10} &= 0\\
(e_2, e_4, e_5): &\quad \alpha_{4,5}^{9} - \alpha_{4,7}^{10} &= 0
\end{aligned}$$

No solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{4,7}^{10} \to x_1$$

$$\alpha_{2,7}^{9} \to x_2$$

$$\alpha_{3,6}^{9} \to x_3$$

$$\alpha_{5,6}^{10} \to x_4$$

$$\alpha_{3,8}^{10} \to x_5$$

$$\alpha_{4,5}^{9} \to x_6$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_6): &\quad -x_2 - x_3 + 2 &= 0\\
(e_1, e_3, e_5): &\quad -x_3 - x_6 - 1 &= 0\\
(e_2, e_3, e_4): &\quad -x_2 &= 0\\
(e_1, e_2, e_8): &\quad -x_5 - 1 &= 0\\
(e_1, e_3, e_7): &\quad -x_1 - x_5 &= 0\\
(e_1, e_4, e_6): &\quad -x_1 - x_4 &= 0\\
(e_2, e_3, e_6): &\quad x_3 - 2x_5 &= 0\\
(e_2, e_4, e_5): &\quad -x_1 + x_6 &= 0
\end{aligned}$$

Groebner basis (6 variables, 1 linear, 0 nonlinear)

$$1 = 0$$

$\mathfrak{m}_{6B}(2, 10)$

m6B210 (this line included for string searching purposes)

Solution 1

$[e_1, e_2] = e_3$	$[e_1, e_3] = e_4$
$[e_1, e_4] = e_5$	$[e_1, e_5] = e_6$
$[e_1, e_6] = e_7$	$[e_1, e_7] = e_8$
$[e_1, e_8] = e_9$	$[e_2, e_3] = e_5$
$[e_2, e_4] = e_6$	$[e_2, e_5] = 2e_7$
$[e_2, e_6] = 3e_8$	$[e_2, e_7] = 7e_9$
$[e_2, e_9] = e_{10}$	$[e_3, e_4] = -e_7$
$[e_3, e_5] = -e_8$	$[e_3, e_6] = -4e_9$
$[e_3, e_8] = -e_{10}$	$[e_4, e_5] = 3e_9$
$[e_4, e_7] = e_{10}$	$[e_5, e_6] = -e_{10}$

Solution 2

$[e_1, e_2] = e_3$	$[e_1, e_3] = e_4$
$[e_1, e_4] = e_5$	$[e_1, e_5] = e_6$
$[e_1, e_6] = e_7$	$[e_1, e_7] = e_8$
$[e_1, e_8] = e_9$	$[e_2, e_3] = e_5$
$[e_2, e_4] = e_6$	$[e_2, e_5] = 0$
$[e_2, e_6] = -e_8$	$[e_2, e_7] = -e_9$
$[e_2, e_9] = e_{10}$	$[e_3, e_4] = e_7$
$[e_3, e_5] = e_8$	$[e_3, e_6] = 0$
$[e_3, e_8] = -e_{10}$	$[e_4, e_5] = e_9$
$[e_4, e_7] = e_{10}$	$[e_5, e_6] = -e_{10}$

Original brackets:

$$
\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_2, e_3] = e_5 \\
[e_2, e_4] = e_6 & [e_2, e_5] = \alpha_{2,5}^{7} e_7 \\
[e_2, e_6] = \alpha_{2,6}^{8} e_8 & [e_2, e_7] = \alpha_{2,7}^{9} e_9 \\
[e_2, e_9] = e_{10} & [e_3, e_4] = \alpha_{3,4}^{7} e_7 \\
[e_3, e_5] = \alpha_{3,5}^{8} e_8 & [e_3, e_6] = \alpha_{3,6}^{9} e_9 \\
[e_3, e_8] = \alpha_{3,8}^{10} e_{10} & [e_4, e_5] = \alpha_{4,5}^{9} e_9 \\
[e_4, e_7] = \alpha_{4,7}^{10} e_{10} & [e_5, e_6] = \alpha_{5,6}^{10} e_{10}
\end{array}
$$

Non-trivial Jacobi Tests:

$$
\begin{array}{lll}
(e_1, e_2, e_4): & -\alpha_{2,5}^{7} - \alpha_{3,4}^{7} + 1 & = 0 \\
(e_1, e_2, e_5): & \alpha_{2,5}^{7} - \alpha_{2,6}^{8} - \alpha_{3,5}^{8} & = 0 \\
(e_1, e_3, e_4): & \alpha_{3,4}^{7} - \alpha_{3,5}^{8} & = 0 \\
(e_1, e_2, e_6): & \alpha_{2,6}^{8} - \alpha_{2,7}^{9} - \alpha_{3,6}^{9} & = 0 \\
(e_1, e_3, e_5): & \alpha_{3,5}^{8} - \alpha_{3,6}^{9} - \alpha_{4,5}^{9} & = 0 \\
(e_2, e_3, e_4): & \alpha_{2,7}^{9}\alpha_{3,4}^{7} - \alpha_{3,6}^{9} + \alpha_{4,5}^{9} & = 0 \\
(e_1, e_2, e_8): & -\alpha_{3,8}^{10} - 1 & = 0 \\
(e_1, e_3, e_7): & -\alpha_{3,8}^{10} - \alpha_{4,7}^{10} & = 0 \\
(e_1, e_4, e_6): & -\alpha_{4,7}^{10} - \alpha_{5,6}^{10} & = 0 \\
(e_2, e_3, e_6): & -\alpha_{2,6}^{8}\alpha_{3,8}^{10} + \alpha_{3,6}^{9} - \alpha_{5,6}^{10} & = 0 \\
(e_2, e_4, e_5): & -\alpha_{2,5}^{7}\alpha_{4,7}^{10} + \alpha_{4,5}^{9} + \alpha_{5,6}^{10} & = 0
\end{array}
$$

Solution 1:

$$\alpha^{10}_{4,7} = 1$$
$$\alpha^{7}_{3,4} = -1$$
$$\alpha^{9}_{2,7} = 7$$
$$\alpha^{9}_{3,6} = -4$$
$$\alpha^{7}_{2,5} = 2$$
$$\alpha^{8}_{2,6} = 3$$
$$\alpha^{10}_{5,6} = -1$$
$$\alpha^{10}_{3,8} = -1$$
$$\alpha^{9}_{4,5} = 3$$
$$\alpha^{8}_{3,5} = -1$$

Solution 2:

$$\alpha^{10}_{4,7} = 1$$
$$\alpha^{7}_{3,4} = 1$$
$$\alpha^{9}_{2,7} = -1$$
$$\alpha^{9}_{3,6} = 0$$
$$\alpha^{7}_{2,5} = 0$$
$$\alpha^{8}_{2,6} = -1$$
$$\alpha^{10}_{5,6} = -1$$
$$\alpha^{10}_{3,8} = -1$$
$$\alpha^{9}_{4,5} = 1$$
$$\alpha^{8}_{3,5} = 1$$

How the solution(s) were or were not found:
Change variables

$$\alpha^{10}_{4,7} \to x_1$$
$$\alpha^{7}_{3,4} \to x_2$$
$$\alpha^{9}_{2,7} \to x_3$$
$$\alpha^{9}_{3,6} \to x_4$$
$$\alpha^{7}_{2,5} \to x_5$$

$$\alpha_{2,6}^{8} \to x_6$$

$$\alpha_{5,6}^{10} \to x_7$$

$$\alpha_{3,8}^{10} \to x_8$$

$$\alpha_{4,5}^{9} \to x_9$$

$$\alpha_{3,5}^{8} \to x_{10}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4):&\quad -x_2 - x_5 + 1 &&= 0\\
(e_1, e_2, e_5):&\quad -x_{10} + x_5 - x_6 &&= 0\\
(e_1, e_3, e_4):&\quad -x_{10} + x_2 &&= 0\\
(e_1, e_2, e_6):&\quad -x_3 - x_4 + x_6 &&= 0\\
(e_1, e_3, e_5):&\quad x_{10} - x_4 - x_9 &&= 0\\
(e_2, e_3, e_4):&\quad x_2x_3 - x_4 + x_9 &&= 0\\
(e_1, e_2, e_8):&\quad -x_8 - 1 &&= 0\\
(e_1, e_3, e_7):&\quad -x_1 - x_8 &&= 0\\
(e_1, e_4, e_6):&\quad -x_1 - x_7 &&= 0\\
(e_2, e_3, e_6):&\quad x_4 - x_6x_8 - x_7 &&= 0\\
(e_2, e_4, e_5):&\quad -x_1x_5 + x_7 + x_9 &&= 0
\end{aligned}$$

Groebner basis (10 variables, 9 linear, 1 nonlinear)

$$x_1 - 1 = 0$$

$$-x_{10} + x_2 = 0$$

$$4x_{10} + x_3 - 3 = 0$$

$$-2x_{10} + x_4 + 2 = 0$$

$$x_{10} + x_5 - 1 = 0$$

$$2x_{10} + x_6 - 1 = 0$$

$$x_7 + 1 = 0$$

$$x_8 + 1 = 0$$

$$x_{10} + x_9 - 2 = 0$$

$$x_{10}^2 - 1 = 0$$

Solution 1:

$$x_1 = 1$$

$$x_2 = -1$$
$$x_3 = 7$$
$$x_4 = -4$$
$$x_5 = 2$$
$$x_6 = 3$$
$$x_7 = -1$$
$$x_8 = -1$$
$$x_9 = 3$$
$$x_10 = -1$$

Solution 2:

$$x_1 = 1$$
$$x_2 = 1$$
$$x_3 = -1$$
$$x_4 = 0$$
$$x_5 = 0$$
$$x_6 = -1$$
$$x_7 = -1$$
$$x_8 = -1$$
$$x_9 = 1$$
$$x_10 = 1$$

$\mathfrak{m}_{3B}(3, 10)$

m3B310 (this line included for string searching purposes)

Solution 1

$$
\begin{aligned}
&[e_1, e_2] = e_3 && [e_1, e_3] = e_4 \\
&[e_1, e_4] = e_5 && [e_1, e_5] = e_6 \\
&[e_1, e_6] = e_7 && [e_1, e_7] = e_8 \\
&[e_1, e_8] = e_9 && [e_2, e_5] = e_8 \\
&[e_2, e_6] = 2e_9 && [e_2, e_9] = e_{10} \\
&[e_3, e_4] = -e_8 && [e_3, e_5] = -e_9 \\
&[e_3, e_8] = -e_{10} && [e_4, e_7] = e_{10} \\
&[e_5, e_6] = -e_{10} &&
\end{aligned}
$$

Original brackets:

$$
\begin{aligned}
&[e_1, e_2] = e_3 && [e_1, e_3] = e_4 \\
&[e_1, e_4] = e_5 && [e_1, e_5] = e_6 \\
&[e_1, e_6] = e_7 && [e_1, e_7] = e_8 \\
&[e_1, e_8] = e_9 && [e_2, e_5] = e_8 \\
&[e_2, e_6] = 2e_9 && [e_2, e_9] = e_{10} \\
&[e_3, e_4] = -e_8 && [e_3, e_5] = -e_9 \\
&[e_3, e_8] = \alpha_{3,8}^{10} e_{10} && [e_4, e_7] = \alpha_{4,7}^{10} e_{10} \\
&[e_5, e_6] = \alpha_{5,6}^{10} e_{10}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_8): &\quad -\alpha_{3,8}^{10} - 1 &&= 0 \\
(e_1, e_3, e_7): &\quad -\alpha_{3,8}^{10} - \alpha_{4,7}^{10} &&= 0 \\
(e_1, e_4, e_6): &\quad -\alpha_{4,7}^{10} - \alpha_{5,6}^{10} &&= 0 \\
(e_2, e_3, e_5): &\quad -\alpha_{3,8}^{10} - 1 &&= 0
\end{aligned}
$$

Solution 1:

$$
\begin{aligned}
\alpha_{4,7}^{10} &= 1 \\
\alpha_{3,8}^{10} &= -1 \\
\alpha_{5,6}^{10} &= -1
\end{aligned}
$$

How the solution(s) were or were not found:
Change variables

$$
\begin{aligned}
\alpha_{4,7}^{10} &\to x_1 \\
\alpha_{3,8}^{10} &\to x_2 \\
\alpha_{5,6}^{10} &\to x_3
\end{aligned}
$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_8): &\quad -x_2 - 1 &&= 0 \\
(e_1, e_3, e_7): &\quad -x_1 - x_2 &&= 0 \\
(e_1, e_4, e_6): &\quad -x_1 - x_3 &&= 0 \\
(e_2, e_3, e_5): &\quad -x_2 - 1 &&= 0
\end{aligned}
$$

Groebner basis (3 variables, 3 linear, 0 nonlinear)

$$x_1 - 1 = 0$$
$$x_2 + 1 = 0$$
$$x_3 + 1 = 0$$

Solution 1:

$$x_1 = 1$$
$$x_2 = -1$$
$$x_3 = -1$$

$\mathfrak{m}_{5B}(3, 10)$

m5B310 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_2, e_3] &= e_6 \\
[e_2, e_4] &= e_7 & [e_2, e_5] &= \alpha_{2,5}^{8} e_8 \\
[e_2, e_6] &= \alpha_{2,6}^{9} e_9 & [e_2, e_9] &= e_{10} \\
[e_3, e_4] &= \alpha_{3,4}^{8} e_8 & [e_3, e_5] &= \alpha_{3,5}^{9} e_9 \\
[e_3, e_8] &= \alpha_{3,8}^{10} e_{10} & [e_4, e_7] &= \alpha_{4,7}^{10} e_{10} \\
[e_5, e_6] &= \alpha_{5,6}^{10} e_{10}
\end{aligned}$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1, e_2, e_4): &\quad -\alpha_{2,5}^{8} - \alpha_{3,4}^{8} + 1 &= 0 \\
(e_1, e_2, e_5): &\quad \alpha_{2,5}^{8} - \alpha_{2,6}^{9} - \alpha_{3,5}^{9} &= 0 \\
(e_1, e_3, e_4): &\quad \alpha_{3,4}^{8} - \alpha_{3,5}^{9} &= 0 \\
(e_1, e_2, e_8): &\quad -\alpha_{3,8}^{10} - 1 &= 0 \\
(e_1, e_3, e_7): &\quad -\alpha_{3,8}^{10} - \alpha_{4,7}^{10} &= 0 \\
(e_1, e_4, e_6): &\quad -\alpha_{4,7}^{10} - \alpha_{5,6}^{10} &= 0 \\
(e_2, e_3, e_5): &\quad -\alpha_{2,5}^{8}\alpha_{3,8}^{10} + \alpha_{3,5}^{9} + \alpha_{5,6}^{10} &= 0
\end{aligned}$$

Infinite number of solutions.

How the solution(s) were or were not found:

Change variables

$$\alpha_{3,5}^{9} \to x_1$$
$$\alpha_{4,7}^{10} \to x_2$$
$$\alpha_{2,5}^{8} \to x_3$$
$$\alpha_{2,6}^{9} \to x_4$$
$$\alpha_{5,6}^{10} \to x_5$$
$$\alpha_{3,4}^{8} \to x_6$$
$$\alpha_{3,8}^{10} \to x_7$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4): &\quad -x_3 - x_6 + 1 && = 0 \\
(e_1, e_2, e_5): &\quad -x_1 + x_3 - x_4 && = 0 \\
(e_1, e_3, e_4): &\quad -x_1 + x_6 && = 0 \\
(e_1, e_2, e_8): &\quad -x_7 - 1 && = 0 \\
(e_1, e_3, e_7): &\quad -x_2 - x_7 && = 0 \\
(e_1, e_4, e_6): &\quad -x_2 - x_5 && = 0 \\
(e_2, e_3, e_5): &\quad x_1 - x_3 x_7 + x_5 && = 0
\end{aligned}$$

Groebner basis (7 variables, 6 linear, 0 nonlinear)

$$x_1 - x_6 = 0$$
$$x_2 - 1 = 0$$
$$x_3 + x_6 - 1 = 0$$
$$x_4 + 2x_6 - 1 = 0$$
$$x_5 + 1 = 0$$
$$x_7 + 1 = 0$$

$\mathfrak{m}_{2B}(4,10)$

m2B410 (this line included for string searching purposes)

Original brackets:

$$
\begin{array}{ll}
[e_1,e_2]=e_3 & [e_1,e_3]=e_4 \\
[e_1,e_4]=e_5 & [e_1,e_5]=e_6 \\
[e_1,e_6]=e_7 & [e_1,e_7]=e_8 \\
[e_1,e_8]=e_9 & [e_2,e_5]=e_9 \\
[e_2,e_9]=e_{10} & [e_3,e_4]=-e_9 \\
[e_3,e_8]=\alpha_{3,8}^{10}e_{10} & [e_4,e_7]=\alpha_{4,7}^{10}e_{10} \\
[e_5,e_6]=\alpha_{5,6}^{10}e_{10} &
\end{array}
$$

Non-trivial Jacobi Tests:

$$
\begin{array}{lll}
(e_1,e_2,e_8): & -\alpha_{3,8}^{10}-1 & =0 \\
(e_1,e_3,e_7): & -\alpha_{3,8}^{10}-\alpha_{4,7}^{10} & =0 \\
(e_1,e_4,e_6): & -\alpha_{4,7}^{10}-\alpha_{5,6}^{10} & =0 \\
(e_2,e_3,e_4): & \text{no solutions} &
\end{array}
$$

There are no solutions.

$\mathfrak{m}_{4B}(4,10)$

m4B410 (this line included for string searching purposes)

Solution 1

$$
\begin{array}{ll}
[e_1,e_2]=e_3 & [e_1,e_3]=e_4 \\
[e_1,e_4]=e_5 & [e_1,e_5]=e_6 \\
[e_1,e_6]=e_7 & [e_1,e_7]=e_8 \\
[e_1,e_8]=e_9 & [e_2,e_3]=e_7 \\
[e_2,e_4]=e_8 & [e_2,e_5]=3e_9 \\
[e_2,e_9]=e_{10} & [e_3,e_4]=-2e_9 \\
[e_3,e_8]=-e_{10} & [e_4,e_7]=e_{10} \\
[e_5,e_6]=-e_{10} &
\end{array}
$$

Original brackets:

$$
\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_2, e_3] = e_7 \\
[e_2, e_4] = e_8 & [e_2, e_5] = \alpha_{2,5}^{9} e_9 \\
[e_2, e_9] = e_{10} & [e_3, e_4] = \alpha_{3,4}^{9} e_9 \\
[e_3, e_8] = \alpha_{3,8}^{10} e_{10} & [e_4, e_7] = \alpha_{4,7}^{10} e_{10} \\
[e_5, e_6] = \alpha_{5,6}^{10} e_{10} &
\end{array}
$$

Non-trivial Jacobi Tests:

$$
\begin{array}{lll}
(e_1, e_2, e_4): & -\alpha_{2,5}^{9} - \alpha_{3,4}^{9} + 1 & = 0 \\
(e_1, e_2, e_8): & -\alpha_{3,8}^{10} - 1 & = 0 \\
(e_1, e_3, e_7): & -\alpha_{3,8}^{10} - \alpha_{4,7}^{10} & = 0 \\
(e_1, e_4, e_6): & -\alpha_{4,7}^{10} - \alpha_{5,6}^{10} & = 0 \\
(e_2, e_3, e_4): & \alpha_{3,4}^{9} - \alpha_{3,8}^{10} + \alpha_{4,7}^{10} & = 0
\end{array}
$$

Solution 1:

$$
\begin{aligned}
\alpha_{4,7}^{10} &= 1 \\
\alpha_{2,5}^{9} &= 3 \\
\alpha_{3,4}^{9} &= -2 \\
\alpha_{5,6}^{10} &= -1 \\
\alpha_{3,8}^{10} &= -1
\end{aligned}
$$

How the solution(s) were or were not found:
Change variables

$$
\begin{aligned}
\alpha_{4,7}^{10} &\to x_1 \\
\alpha_{2,5}^{9} &\to x_2 \\
\alpha_{3,4}^{9} &\to x_3 \\
\alpha_{5,6}^{10} &\to x_4 \\
\alpha_{3,8}^{10} &\to x_5
\end{aligned}
$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4): &\quad -x_2 - x_3 + 1 &&= 0\\
(e_1, e_2, e_8): &\quad -x_5 - 1 &&= 0\\
(e_1, e_3, e_7): &\quad -x_1 - x_5 &&= 0\\
(e_1, e_4, e_6): &\quad -x_1 - x_4 &&= 0\\
(e_2, e_3, e_4): &\quad x_1 + x_3 - x_5 &&= 0
\end{aligned}$$

Groebner basis (5 variables, 5 linear, 0 nonlinear)

$$x_1 - 1 = 0$$
$$x_2 - 3 = 0$$
$$x_3 + 2 = 0$$
$$x_4 + 1 = 0$$
$$x_5 + 1 = 0$$

Solution 1:

$$x_1 = 1$$
$$x_2 = 3$$
$$x_3 = -2$$
$$x_4 = -1$$
$$x_5 = -1$$

$\mathfrak{m}_{3B}(5, 10)$

m3B510 (this line included for string searching purposes)

Solution 1

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4\\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6\\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8\\
[e_1, e_8] &= e_9 & [e_2, e_3] &= e_8\\
[e_2, e_4] &= e_9 & [e_2, e_9] &= e_{10}\\
[e_3, e_8] &= -e_{10} & [e_4, e_7] &= e_{10}\\
[e_5, e_6] &= -e_{10}
\end{aligned}$$

Original brackets:

$$\begin{aligned} [e_1,e_2] &= e_3 & [e_1,e_3] &= e_4 \\ [e_1,e_4] &= e_5 & [e_1,e_5] &= e_6 \\ [e_1,e_6] &= e_7 & [e_1,e_7] &= e_8 \\ [e_1,e_8] &= e_9 & [e_2,e_3] &= e_8 \\ [e_2,e_4] &= e_9 & [e_2,e_9] &= e_{10} \\ [e_3,e_8] &= \alpha_{3,8}^{10} e_{10} & [e_4,e_7] &= \alpha_{4,7}^{10} e_{10} \\ [e_5,e_6] &= \alpha_{5,6}^{10} e_{10} \end{aligned}$$

Non-trivial Jacobi Tests:

$$\begin{aligned} (e_1,e_2,e_8): &\quad -\alpha_{3,8}^{10} - 1 &= 0 \\ (e_1,e_3,e_7): &\quad -\alpha_{3,8}^{10} - \alpha_{4,7}^{10} &= 0 \\ (e_1,e_4,e_6): &\quad -\alpha_{4,7}^{10} - \alpha_{5,6}^{10} &= 0 \end{aligned}$$

Solution 1:

$$\begin{aligned} \alpha_{4,7}^{10} &= 1 \\ \alpha_{3,8}^{10} &= -1 \\ \alpha_{5,6}^{10} &= -1 \end{aligned}$$

How the solution(s) were or were not found:
Change variables

$$\begin{aligned} \alpha_{4,7}^{10} &\to x_1 \\ \alpha_{3,8}^{10} &\to x_2 \\ \alpha_{5,6}^{10} &\to x_3 \end{aligned}$$

Jacobi Tests

$$\begin{aligned} (e_1,e_2,e_8): &\quad -x_2 - 1 &= 0 \\ (e_1,e_3,e_7): &\quad -x_1 - x_2 &= 0 \\ (e_1,e_4,e_6): &\quad -x_1 - x_3 &= 0 \end{aligned}$$

Groebner basis (3 variables, 3 linear, 0 nonlinear)

$$x_1 - 1 = 0$$

$$x_2 + 1 = 0$$

$$x_3 + 1 = 0$$

Solution 1:

$$x_1 = 1$$

$$x_2 = -1$$

$$x_3 = -1$$

$\mathfrak{m}_{2B}(6, 10)$

m2B610 (this line included for string searching purposes)

Solution 1

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_2, e_3] &= e_9 \\
[e_2, e_9] &= e_{10} & [e_3, e_8] &= -e_{10} \\
[e_4, e_7] &= e_{10} & [e_5, e_6] &= -e_{10}
\end{aligned}$$

Original brackets:

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_2, e_3] &= e_9 \\
[e_2, e_9] &= e_{10} & [e_3, e_8] &= \alpha_{3,8}^{10} e_{10} \\
[e_4, e_7] &= \alpha_{4,7}^{10} e_{10} & [e_5, e_6] &= \alpha_{5,6}^{10} e_{10}
\end{aligned}$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1, e_2, e_8): &\quad -\alpha_{3,8}^{10} - 1 &= 0 \\
(e_1, e_3, e_7): &\quad -\alpha_{3,8}^{10} - \alpha_{4,7}^{10} &= 0 \\
(e_1, e_4, e_6): &\quad -\alpha_{4,7}^{10} - \alpha_{5,6}^{10} &= 0
\end{aligned}$$

Solution 1:

$$\alpha_{4,7}^{10} = 1$$
$$\alpha_{3,8}^{10} = -1$$
$$\alpha_{5,6}^{10} = -1$$

How the solution(s) were or were not found:
Change variables

$$\alpha_{4,7}^{10} \to x_1$$
$$\alpha_{3,8}^{10} \to x_2$$
$$\alpha_{5,6}^{10} \to x_3$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_8) : \quad & -x_2 - 1 && = 0 \\
(e_1, e_3, e_7) : \quad & -x_1 - x_2 && = 0 \\
(e_1, e_4, e_6) : \quad & -x_1 - x_3 && = 0
\end{aligned}$$

Groebner basis (3 variables, 3 linear, 0 nonlinear)

$$x_1 - 1 = 0$$
$$x_2 + 1 = 0$$
$$x_3 + 1 = 0$$

Solution 1:

$$x_1 = 1$$
$$x_2 = -1$$
$$x_3 = -1$$

$\mathfrak{m}_{2B}(2, 12)$

m2B212 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_2, e_9] &= e_{11} \\
[e_2, e_{11}] &= e_{12} & [e_3, e_8] &= -e_{11} \\
[e_3, e_{10}] &= \alpha_{3,10}^{12} e_{12} & [e_4, e_7] &= e_{11} \\
[e_4, e_9] &= \alpha_{4,9}^{12} e_{12} & [e_5, e_6] &= -e_{11} \\
[e_5, e_8] &= \alpha_{5,8}^{12} e_{12} & [e_6, e_7] &= \alpha_{6,7}^{12} e_{12}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_{10}) &: \quad -\alpha_{3,10}^{12} - 1 &= 0 \\
(e_1, e_3, e_9) &: \quad -\alpha_{3,10}^{12} - \alpha_{4,9}^{12} &= 0 \\
(e_1, e_4, e_8) &: \quad -\alpha_{4,9}^{12} - \alpha_{5,8}^{12} &= 0 \\
(e_1, e_5, e_7) &: \quad -\alpha_{5,8}^{12} - \alpha_{6,7}^{12} &= 0 \\
(e_2, e_3, e_8) &: \quad \text{no solutions} \\
(e_2, e_4, e_7) &: \quad \text{no solutions} \\
(e_2, e_5, e_6) &: \quad \text{no solutions}
\end{aligned}
$$

There are no solutions.

$\mathfrak{m}_{4B}(2, 12)$

m4B212 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_2, e_7] &= e_9 \\
[e_2, e_8] &= 3e_{10} & [e_2, e_9] &= \alpha_{2,9}^{11} e_{11} \\
[e_2, e_{11}] &= e_{12} & [e_3, e_6] &= -e_9 \\
[e_3, e_7] &= -2e_{10} & [e_3, e_8] &= \alpha_{3,8}^{11} e_{11} \\
[e_3, e_{10}] &= \alpha_{3,10}^{12} e_{12} & [e_4, e_5] &= e_9 \\
[e_4, e_6] &= e_{10} & [e_4, e_7] &= \alpha_{4,7}^{11} e_{11} \\
[e_4, e_9] &= \alpha_{4,9}^{12} e_{12} & [e_5, e_6] &= \alpha_{5,6}^{11} e_{11} \\
[e_5, e_8] &= \alpha_{5,8}^{12} e_{12} & [e_6, e_7] &= \alpha_{6,7}^{12} e_{12}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_8): &\quad -\alpha_{2,9}^{11} - \alpha_{3,8}^{11} + 3 &&= 0 \\
(e_1, e_3, e_7): &\quad -\alpha_{3,8}^{11} - \alpha_{4,7}^{11} - 2 &&= 0 \\
(e_1, e_4, e_6): &\quad -\alpha_{4,7}^{11} - \alpha_{5,6}^{11} + 1 &&= 0 \\
(e_2, e_3, e_6): &\quad -\alpha_{2,9}^{11} &&= 0 \\
(e_2, e_4, e_5): &\quad \alpha_{2,9}^{11} &&= 0 \\
(e_1, e_2, e_{10}): &\quad -\alpha_{3,10}^{12} - 1 &&= 0 \\
(e_1, e_3, e_9): &\quad -\alpha_{3,10}^{12} - \alpha_{4,9}^{12} &&= 0 \\
(e_1, e_4, e_8): &\quad -\alpha_{4,9}^{12} - \alpha_{5,8}^{12} &&= 0 \\
(e_1, e_5, e_7): &\quad -\alpha_{5,8}^{12} - \alpha_{6,7}^{12} &&= 0 \\
(e_2, e_3, e_8): &\quad -3\alpha_{3,10}^{12} + \alpha_{3,8}^{11} &&= 0 \\
(e_2, e_4, e_7): &\quad \alpha_{4,7}^{11} - \alpha_{4,9}^{12} &&= 0 \\
(e_2, e_5, e_6): &\quad \alpha_{5,6}^{11} &&= 0 \\
(e_3, e_4, e_6): &\quad \alpha_{3,10}^{12} + \alpha_{4,9}^{12} &&= 0
\end{aligned}
$$

No solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{4,9}^{12} \to x_1$$

$$\alpha^{12}_{6,7} \to x_2$$

$$\alpha^{11}_{2,9} \to x_3$$

$$\alpha^{12}_{5,8} \to x_4$$

$$\alpha^{11}_{5,6} \to x_5$$

$$\alpha^{12}_{3,10} \to x_6$$

$$\alpha^{11}_{4,7} \to x_7$$

$$\alpha^{11}_{3,8} \to x_8$$

Jacobi Tests

$$\begin{array}{rll}
(e_1, e_2, e_8): & -x_3 - x_8 + 3 & = 0 \\
(e_1, e_3, e_7): & -x_7 - x_8 - 2 & = 0 \\
(e_1, e_4, e_6): & -x_5 - x_7 + 1 & = 0 \\
(e_2, e_3, e_6): & -x_3 & = 0 \\
(e_2, e_4, e_5): & x_3 & = 0 \\
(e_1, e_2, e_{10}): & -x_6 - 1 & = 0 \\
(e_1, e_3, e_9): & -x_1 - x_6 & = 0 \\
(e_1, e_4, e_8): & -x_1 - x_4 & = 0 \\
(e_1, e_5, e_7): & -x_2 - x_4 & = 0 \\
(e_2, e_3, e_8): & -3x_6 + x_8 & = 0 \\
(e_2, e_4, e_7): & -x_1 + x_7 & = 0 \\
(e_2, e_5, e_6): & x_5 & = 0 \\
(e_3, e_4, e_6): & x_1 + x_6 & = 0
\end{array}$$

Groebner basis (8 variables, 1 linear, 0 nonlinear)

$$1 = 0$$

$\mathfrak{m}_{6B}(2, 12)$

m6B212 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_2, e_5] &= e_7 \\
[e_2, e_6] &= 2e_8 & [e_2, e_7] &= \alpha_{2,7}^{9} e_9 \\
[e_2, e_8] &= \alpha_{2,8}^{10} e_{10} & [e_2, e_9] &= \alpha_{2,9}^{11} e_{11} \\
[e_2, e_{11}] &= e_{12} & [e_3, e_4] &= -e_7 \\
[e_3, e_5] &= -e_8 & [e_3, e_6] &= \alpha_{3,6}^{9} e_9 \\
[e_3, e_7] &= \alpha_{3,7}^{10} e_{10} & [e_3, e_8] &= \alpha_{3,8}^{11} e_{11} \\
[e_3, e_{10}] &= \alpha_{3,10}^{12} e_{12} & [e_4, e_5] &= \alpha_{4,5}^{9} e_9 \\
[e_4, e_6] &= \alpha_{4,6}^{10} e_{10} & [e_4, e_7] &= \alpha_{4,7}^{11} e_{11} \\
[e_4, e_9] &= \alpha_{4,9}^{12} e_{12} & [e_5, e_6] &= \alpha_{5,6}^{11} e_{11} \\
[e_5, e_8] &= \alpha_{5,8}^{12} e_{12} & [e_6, e_7] &= \alpha_{6,7}^{12} e_{12}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_6): &\quad -\alpha_{2,7}^{9} - \alpha_{3,6}^{9} + 2 && = 0\\
(e_1, e_3, e_5): &\quad -\alpha_{3,6}^{9} - \alpha_{4,5}^{9} - 1 && = 0\\
(e_2, e_3, e_4): &\quad -\alpha_{2,7}^{9} && = 0\\
(e_1, e_2, e_7): &\quad \alpha_{2,7}^{9} - \alpha_{2,8}^{10} - \alpha_{3,7}^{10} && = 0\\
(e_1, e_3, e_6): &\quad \alpha_{3,6}^{9} - \alpha_{3,7}^{10} - \alpha_{4,6}^{10} && = 0\\
(e_1, e_4, e_5): &\quad \alpha_{4,5}^{9} - \alpha_{4,6}^{10} && = 0\\
(e_2, e_3, e_5): &\quad -\alpha_{2,8}^{10} - \alpha_{3,7}^{10} && = 0\\
(e_1, e_2, e_8): &\quad \alpha_{2,8}^{10} - \alpha_{2,9}^{11} - \alpha_{3,8}^{11} && = 0\\
(e_1, e_3, e_7): &\quad \alpha_{3,7}^{10} - \alpha_{3,8}^{11} - \alpha_{4,7}^{11} && = 0\\
(e_1, e_4, e_6): &\quad \alpha_{4,6}^{10} - \alpha_{4,7}^{11} - \alpha_{5,6}^{11} && = 0\\
(e_2, e_3, e_6): &\quad \alpha_{2,9}^{11}\alpha_{3,6}^{9} - 2\alpha_{3,8}^{11} && = 0\\
(e_2, e_4, e_5): &\quad \alpha_{2,9}^{11}\alpha_{4,5}^{9} - \alpha_{4,7}^{11} && = 0\\
(e_1, e_2, e_{10}): &\quad -\alpha_{3,10}^{12} - 1 && = 0\\
(e_1, e_3, e_9): &\quad -\alpha_{3,10}^{12} - \alpha_{4,9}^{12} && = 0\\
(e_1, e_4, e_8): &\quad -\alpha_{4,9}^{12} - \alpha_{5,8}^{12} && = 0\\
(e_1, e_5, e_7): &\quad -\alpha_{5,8}^{12} - \alpha_{6,7}^{12} && = 0\\
(e_2, e_3, e_8): &\quad -\alpha_{2,8}^{10}\alpha_{3,10}^{12} + \alpha_{3,8}^{11} && = 0\\
(e_2, e_4, e_7): &\quad -\alpha_{2,7}^{9}\alpha_{4,9}^{12} + \alpha_{4,7}^{11} && = 0\\
(e_2, e_5, e_6): &\quad \alpha_{5,6}^{11} - 2\alpha_{5,8}^{12} + \alpha_{6,7}^{12} && = 0\\
(e_3, e_4, e_6): &\quad \alpha_{3,10}^{12}\alpha_{4,6}^{10} - \alpha_{3,6}^{9}\alpha_{4,9}^{12} - \alpha_{6,7}^{12} && = 0
\end{aligned}
$$

No solutions.
How the solution(s) were or were not found:
Change variables

$$
\begin{gathered}
\alpha_{2,7}^{9} \to x_1\\
\alpha_{4,9}^{12} \to x_2\\
\alpha_{6,7}^{12} \to x_3\\
\alpha_{2,8}^{10} \to x_4\\
\alpha_{2,9}^{11} \to x_5\\
\alpha_{3,7}^{10} \to x_6\\
\alpha_{3,6}^{9} \to x_7
\end{gathered}
$$

$$\alpha_{5,8}^{12} \to x_8$$

$$\alpha_{5,6}^{11} \to x_9$$

$$\alpha_{3,10}^{12} \to x_{10}$$

$$\alpha_{4,7}^{11} \to x_{11}$$

$$\alpha_{4,5}^{9} \to x_{12}$$

$$\alpha_{4,6}^{10} \to x_{13}$$

$$\alpha_{3,8}^{11} \to x_{14}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_6): &\quad -x_1 - x_7 + 2 &&= 0\\
(e_1, e_3, e_5): &\quad -x_{12} - x_7 - 1 &&= 0\\
(e_2, e_3, e_4): &\quad -x_1 &&= 0\\
(e_1, e_2, e_7): &\quad x_1 - x_4 - x_6 &&= 0\\
(e_1, e_3, e_6): &\quad -x_{13} - x_6 + x_7 &&= 0\\
(e_1, e_4, e_5): &\quad x_{12} - x_{13} &&= 0\\
(e_2, e_3, e_5): &\quad -x_4 - x_6 &&= 0\\
(e_1, e_2, e_8): &\quad -x_{14} + x_4 - x_5 &&= 0\\
(e_1, e_3, e_7): &\quad -x_{11} - x_{14} + x_6 &&= 0\\
(e_1, e_4, e_6): &\quad -x_{11} + x_{13} - x_9 &&= 0\\
(e_2, e_3, e_6): &\quad -2x_{14} + x_5 x_7 &&= 0\\
(e_2, e_4, e_5): &\quad -x_{11} + x_{12} x_5 &&= 0\\
(e_1, e_2, e_{10}): &\quad -x_{10} - 1 &&= 0\\
(e_1, e_3, e_9): &\quad -x_{10} - x_2 &&= 0\\
(e_1, e_4, e_8): &\quad -x_2 - x_8 &&= 0\\
(e_1, e_5, e_7): &\quad -x_3 - x_8 &&= 0\\
(e_2, e_3, e_8): &\quad -x_{10} x_4 + x_{14} &&= 0\\
(e_2, e_4, e_7): &\quad -x_1 x_2 + x_{11} &&= 0\\
(e_2, e_5, e_6): &\quad x_3 - 2x_8 + x_9 &&= 0\\
(e_3, e_4, e_6): &\quad x_{10} x_{13} - x_2 x_7 - x_3 &&= 0
\end{aligned}$$

Groebner basis (14 variables, 1 linear, 0 nonlinear)

$$1 = 0$$

$\mathfrak{m}_{8B}(2, 12)$

m8B212 (this line included for string searching purposes)

Solution 1

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_2, e_3] = e_5$$
$$[e_2, e_4] = e_6 \qquad [e_2, e_5] = e_7 \left(\frac{\sqrt{10}}{5} + 1\right)$$
$$[e_2, e_6] = e_8 \left(1 + \frac{2\sqrt{10}}{5}\right) \qquad [e_2, e_7] = e_9 \left(\frac{5}{3} + \frac{2\sqrt{10}}{3}\right)$$
$$[e_2, e_8] = e_{10} \left(3 + \sqrt{10}\right) \qquad [e_2, e_9] = e_{11} \left(2\sqrt{10} + 7\right)$$
$$[e_2, e_{11}] = e_{12} \qquad [e_3, e_4] = -\frac{\sqrt{10}e_7}{5}$$
$$[e_3, e_5] = -\frac{\sqrt{10}e_8}{5} \qquad [e_3, e_6] = e_9 \left(-\frac{4\sqrt{10}}{15} - \frac{2}{3}\right)$$
$$[e_3, e_7] = e_{10} \left(-\frac{4}{3} - \frac{\sqrt{10}}{3}\right) \qquad [e_3, e_8] = e_{11} \left(-4 - \sqrt{10}\right)$$
$$[e_3, e_{10}] = -e_{12} \qquad [e_4, e_5] = e_9 \left(\frac{\sqrt{10}}{15} + \frac{2}{3}\right)$$
$$[e_4, e_6] = e_{10} \left(\frac{\sqrt{10}}{15} + \frac{2}{3}\right) \qquad [e_4, e_7] = e_{11} \left(\frac{2\sqrt{10}}{3} + \frac{8}{3}\right)$$
$$[e_4, e_9] = e_{12} \qquad [e_5, e_6] = e_{11} \left(-2 - \frac{3\sqrt{10}}{5}\right)$$
$$[e_5, e_8] = -e_{12} \qquad [e_6, e_7] = e_{12}$$

Solution 2

$$
\begin{aligned}
&[e_1, e_2] = e_3 && [e_1, e_3] = e_4 \\
&[e_1, e_4] = e_5 && [e_1, e_5] = e_6 \\
&[e_1, e_6] = e_7 && [e_1, e_7] = e_8 \\
&[e_1, e_8] = e_9 && [e_1, e_9] = e_{10} \\
&[e_1, e_{10}] = e_{11} && [e_2, e_3] = e_5 \\
&[e_2, e_4] = e_6 && [e_2, e_5] = e_7 \left(1 - \frac{\sqrt{10}}{5}\right) \\
&[e_2, e_6] = e_8 \left(1 - \frac{2\sqrt{10}}{5}\right) && [e_2, e_7] = e_9 \left(\frac{5}{3} - \frac{2\sqrt{10}}{3}\right) \\
&[e_2, e_8] = e_{10} \left(3 - \sqrt{10}\right) && [e_2, e_9] = e_{11} \left(7 - 2\sqrt{10}\right) \\
&[e_2, e_{11}] = e_{12} && [e_3, e_4] = \frac{\sqrt{10}e_7}{5} \\
&[e_3, e_5] = \frac{\sqrt{10}e_8}{5} && [e_3, e_6] = e_9 \left(-\frac{2}{3} + \frac{4\sqrt{10}}{15}\right) \\
&[e_3, e_7] = e_{10} \left(-\frac{4}{3} + \frac{\sqrt{10}}{3}\right) && [e_3, e_8] = e_{11} \left(-4 + \sqrt{10}\right) \\
&[e_3, e_{10}] = -e_{12} && [e_4, e_5] = e_9 \left(\frac{2}{3} - \frac{\sqrt{10}}{15}\right) \\
&[e_4, e_6] = e_{10} \left(\frac{2}{3} - \frac{\sqrt{10}}{15}\right) && [e_4, e_7] = e_{11} \left(\frac{8}{3} - \frac{2\sqrt{10}}{3}\right) \\
&[e_4, e_9] = e_{12} && [e_5, e_6] = e_{11} \left(-2 + \frac{3\sqrt{10}}{5}\right) \\
&[e_5, e_8] = -e_{12} && [e_6, e_7] = e_{12}
\end{aligned}
$$

Solution 3

$$
\begin{aligned}
&[e_1,e_2]=e_3 && [e_1,e_3]=e_4\\
&[e_1,e_4]=e_5 && [e_1,e_5]=e_6\\
&[e_1,e_6]=e_7 && [e_1,e_7]=e_8\\
&[e_1,e_8]=e_9 && [e_1,e_9]=e_{10}\\
&[e_1,e_{10}]=e_{11} && [e_2,e_3]=e_5\\
&[e_2,e_4]=e_6 && [e_2,e_5]=e_7\left(\frac{1}{3}+\frac{2\sqrt{2}i}{3}\right)\\
&[e_2,e_6]=e_8\left(-\frac{1}{3}+\frac{4\sqrt{2}i}{3}\right) && [e_2,e_7]=e_9\left(-1+\sqrt{2}i\right)\\
&[e_2,e_8]=e_{10}\left(-\frac{5}{3}-\frac{\sqrt{2}i}{3}\right) && [e_2,e_9]=e_{11}\left(-\frac{7}{3}-\frac{2\sqrt{2}i}{3}\right)\\
&[e_2,e_{11}]=e_{12} && [e_3,e_4]=e_7\left(\frac{2}{3}-\frac{2\sqrt{2}i}{3}\right)\\
&[e_3,e_5]=e_8\left(\frac{2}{3}-\frac{2\sqrt{2}i}{3}\right) && [e_3,e_6]=e_9\left(\frac{2}{3}+\frac{\sqrt{2}i}{3}\right)\\
&[e_3,e_7]=e_{10}\left(\frac{2}{3}+\frac{4\sqrt{2}i}{3}\right) && [e_3,e_8]=e_{11}\left(\frac{2}{3}+\frac{\sqrt{2}i}{3}\right)\\
&[e_3,e_{10}]=-e_{12} && [e_4,e_5]=-\sqrt{2}ie_9\\
&[e_4,e_6]=-\sqrt{2}ie_{10} && [e_4,e_7]=\sqrt{2}ie_{11}\\
&[e_4,e_9]=e_{12} && [e_5,e_6]=-2\sqrt{2}ie_{11}\\
&[e_5,e_8]=-e_{12} && [e_6,e_7]=e_{12}
\end{aligned}
$$

Solution 4

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_2, e_3] = e_5$$
$$[e_2, e_4] = e_6 \qquad [e_2, e_5] = e_7 \left(\frac{1}{3} - \frac{2\sqrt{2}i}{3}\right)$$
$$[e_2, e_6] = e_8 \left(-\frac{1}{3} - \frac{4\sqrt{2}i}{3}\right) \qquad [e_2, e_7] = e_9 \left(-1 - \sqrt{2}i\right)$$
$$[e_2, e_8] = e_{10} \left(-\frac{5}{3} + \frac{\sqrt{2}i}{3}\right) \qquad [e_2, e_9] = e_{11} \left(-\frac{7}{3} + \frac{2\sqrt{2}i}{3}\right)$$
$$[e_2, e_{11}] = e_{12} \qquad [e_3, e_4] = e_7 \left(\frac{2}{3} + \frac{2\sqrt{2}i}{3}\right)$$
$$[e_3, e_5] = e_8 \left(\frac{2}{3} + \frac{2\sqrt{2}i}{3}\right) \qquad [e_3, e_6] = e_9 \left(\frac{2}{3} - \frac{\sqrt{2}i}{3}\right)$$
$$[e_3, e_7] = e_{10} \left(\frac{2}{3} - \frac{4\sqrt{2}i}{3}\right) \qquad [e_3, e_8] = e_{11} \left(\frac{2}{3} - \frac{\sqrt{2}i}{3}\right)$$
$$[e_3, e_{10}] = -e_{12} \qquad [e_4, e_5] = \sqrt{2}ie_9$$
$$[e_4, e_6] = \sqrt{2}ie_{10} \qquad [e_4, e_7] = -\sqrt{2}ie_{11}$$
$$[e_4, e_9] = e_{12} \qquad [e_5, e_6] = 2\sqrt{2}ie_{11}$$
$$[e_5, e_8] = -e_{12} \qquad [e_6, e_7] = e_{12}$$

Original brackets:

$$
\begin{aligned}
&[e_1, e_2] = e_3 && [e_1, e_3] = e_4 \\
&[e_1, e_4] = e_5 && [e_1, e_5] = e_6 \\
&[e_1, e_6] = e_7 && [e_1, e_7] = e_8 \\
&[e_1, e_8] = e_9 && [e_1, e_9] = e_{10} \\
&[e_1, e_{10}] = e_{11} && [e_2, e_3] = e_5 \\
&[e_2, e_4] = e_6 && [e_2, e_5] = \alpha_{2,5}^{7} e_7 \\
&[e_2, e_6] = \alpha_{2,6}^{8} e_8 && [e_2, e_7] = \alpha_{2,7}^{9} e_9 \\
&[e_2, e_8] = \alpha_{2,8}^{10} e_{10} && [e_2, e_9] = \alpha_{2,9}^{11} e_{11} \\
&[e_2, e_{11}] = e_{12} && [e_3, e_4] = \alpha_{3,4}^{7} e_7 \\
&[e_3, e_5] = \alpha_{3,5}^{8} e_8 && [e_3, e_6] = \alpha_{3,6}^{9} e_9 \\
&[e_3, e_7] = \alpha_{3,7}^{10} e_{10} && [e_3, e_8] = \alpha_{3,8}^{11} e_{11} \\
&[e_3, e_{10}] = \alpha_{3,10}^{12} e_{12} && [e_4, e_5] = \alpha_{4,5}^{9} e_9 \\
&[e_4, e_6] = \alpha_{4,6}^{10} e_{10} && [e_4, e_7] = \alpha_{4,7}^{11} e_{11} \\
&[e_4, e_9] = \alpha_{4,9}^{12} e_{12} && [e_5, e_6] = \alpha_{5,6}^{11} e_{11} \\
&[e_5, e_8] = \alpha_{5,8}^{12} e_{12} && [e_6, e_7] = \alpha_{6,7}^{12} e_{12}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1, e_2, e_4):&\quad -\alpha_{2,5}^{7} - \alpha_{3,4}^{7} + 1 &= 0\\
(e_1, e_2, e_5):&\quad \alpha_{2,5}^{7} - \alpha_{2,6}^{8} - \alpha_{3,5}^{8} &= 0\\
(e_1, e_3, e_4):&\quad \alpha_{3,4}^{7} - \alpha_{3,5}^{8} &= 0\\
(e_1, e_2, e_6):&\quad \alpha_{2,6}^{8} - \alpha_{2,7}^{9} - \alpha_{3,6}^{9} &= 0\\
(e_1, e_3, e_5):&\quad \alpha_{3,5}^{8} - \alpha_{3,6}^{9} - \alpha_{4,5}^{9} &= 0\\
(e_2, e_3, e_4):&\quad \alpha_{2,7}^{9}\alpha_{3,4}^{7} - \alpha_{3,6}^{9} + \alpha_{4,5}^{9} &= 0\\
(e_1, e_2, e_7):&\quad \alpha_{2,7}^{9} - \alpha_{2,8}^{10} - \alpha_{3,7}^{10} &= 0\\
(e_1, e_3, e_6):&\quad \alpha_{3,6}^{9} - \alpha_{3,7}^{10} - \alpha_{4,6}^{10} &= 0\\
(e_1, e_4, e_5):&\quad \alpha_{4,5}^{9} - \alpha_{4,6}^{10} &= 0\\
(e_2, e_3, e_5):&\quad -\alpha_{2,5}^{7}\alpha_{3,7}^{10} + \alpha_{2,8}^{10}\alpha_{3,5}^{8} &= 0\\
(e_1, e_2, e_8):&\quad \alpha_{2,8}^{10} - \alpha_{2,9}^{11} - \alpha_{3,8}^{11} &= 0\\
(e_1, e_3, e_7):&\quad \alpha_{3,7}^{10} - \alpha_{3,8}^{11} - \alpha_{4,7}^{11} &= 0\\
(e_1, e_4, e_6):&\quad \alpha_{4,6}^{10} - \alpha_{4,7}^{11} - \alpha_{5,6}^{11} &= 0\\
(e_2, e_3, e_6):&\quad -\alpha_{2,6}^{8}\alpha_{3,8}^{11} + \alpha_{2,9}^{11}\alpha_{3,6}^{9} - \alpha_{5,6}^{11} &= 0\\
(e_2, e_4, e_5):&\quad -\alpha_{2,5}^{7}\alpha_{4,7}^{11} + \alpha_{2,9}^{11}\alpha_{4,5}^{9} + \alpha_{5,6}^{11} &= 0\\
(e_1, e_2, e_{10}):&\quad -\alpha_{3,10}^{12} - 1 &= 0\\
(e_1, e_3, e_9):&\quad -\alpha_{3,10}^{12} - \alpha_{4,9}^{12} &= 0\\
(e_1, e_4, e_8):&\quad -\alpha_{4,9}^{12} - \alpha_{5,8}^{12} &= 0\\
(e_1, e_5, e_7):&\quad -\alpha_{5,8}^{12} - \alpha_{6,7}^{12} &= 0\\
(e_2, e_3, e_8):&\quad -\alpha_{2,8}^{10}\alpha_{3,10}^{12} + \alpha_{3,8}^{11} - \alpha_{5,8}^{12} &= 0\\
(e_2, e_4, e_7):&\quad -\alpha_{2,7}^{9}\alpha_{4,9}^{12} + \alpha_{4,7}^{11} - \alpha_{6,7}^{12} &= 0\\
(e_2, e_5, e_6):&\quad \alpha_{2,5}^{7}\alpha_{6,7}^{12} - \alpha_{2,6}^{8}\alpha_{5,8}^{12} + \alpha_{5,6}^{11} &= 0\\
(e_3, e_4, e_6):&\quad \alpha_{3,10}^{12}\alpha_{4,6}^{10} + \alpha_{3,4}^{7}\alpha_{6,7}^{12} - \alpha_{3,6}^{9}\alpha_{4,9}^{12} &= 0
\end{aligned}$$

Solution 1:

$$\alpha_{3,4}^{7} = -sqrt(10)/5$$
$$\alpha_{2,7}^{9} = 5/3 + 2 * sqrt(10)/3$$
$$\alpha_{4,9}^{12} = 1$$
$$\alpha_{6,7}^{12} = 1$$
$$\alpha_{2,8}^{10} = 3 + sqrt(10)$$
$$\alpha_{2,9}^{11} = 2 * sqrt(10) + 7$$
$$\alpha_{3,6}^{9} = -4 * sqrt(10)/15 - 2/3$$
$$\alpha_{2,5}^{7} = sqrt(10)/5 + 1$$
$$\alpha_{3,7}^{10} = -4/3 - sqrt(10)/3$$
$$\alpha_{5,6}^{11} = -2 - 3 * sqrt(10)/5$$
$$\alpha_{3,8}^{11} = -4 - sqrt(10)$$
$$\alpha_{3,10}^{12} = -1$$
$$\alpha_{5,8}^{12} = -1$$
$$\alpha_{2,6}^{8} = 1 + 2 * sqrt(10)/5$$
$$\alpha_{4,7}^{11} = 2 * sqrt(10)/3 + 8/3$$
$$\alpha_{4,5}^{9} = sqrt(10)/15 + 2/3$$
$$\alpha_{4,6}^{10} = sqrt(10)/15 + 2/3$$
$$\alpha_{3,5}^{8} = -sqrt(10)/5$$

Solution 2:

$$\alpha_{3,4}^{7} = sqrt(10)/5$$
$$\alpha_{2,7}^{9} = 5/3 - 2 * sqrt(10)/3$$
$$\alpha_{4,9}^{12} = 1$$
$$\alpha_{6,7}^{12} = 1$$
$$\alpha_{2,8}^{10} = 3 - sqrt(10)$$
$$\alpha_{2,9}^{11} = 7 - 2 * sqrt(10)$$
$$\alpha_{3,6}^{9} = -2/3 + 4 * sqrt(10)/15$$
$$\alpha_{2,5}^{7} = 1 - sqrt(10)/5$$
$$\alpha_{3,7}^{10} = -4/3 + sqrt(10)/3$$
$$\alpha_{5,6}^{11} = -2 + 3 * sqrt(10)/5$$
$$\alpha_{3,8}^{11} = -4 + sqrt(10)$$
$$\alpha_{3,10}^{12} = -1$$
$$\alpha_{5,8}^{12} = -1$$
$$\alpha_{2,6}^{8} = 1 - 2 * sqrt(10)/5$$
$$\alpha_{4,7}^{11} = 8/3 - 2 * sqrt(10)/3$$
$$\alpha_{4,5}^{9} = 2/3 - sqrt(10)/15$$
$$\alpha_{4,6}^{10} = 2/3 - sqrt(10)/15$$
$$\alpha_{3,5}^{8} = sqrt(10)/5$$

Solution 3:

$$\alpha^{7}_{3,4} = 2/3 - 2 * sqrt(2) * I/3$$
$$\alpha^{9}_{2,7} = -1 + sqrt(2) * I$$
$$\alpha^{12}_{4,9} = 1$$
$$\alpha^{12}_{6,7} = 1$$
$$\alpha^{10}_{2,8} = -5/3 - sqrt(2) * I/3$$
$$\alpha^{11}_{2,9} = -7/3 - 2 * sqrt(2) * I/3$$
$$\alpha^{9}_{3,6} = 2/3 + sqrt(2) * I/3$$
$$\alpha^{7}_{2,5} = 1/3 + 2 * sqrt(2) * I/3$$
$$\alpha^{10}_{3,7} = 2/3 + 4 * sqrt(2) * I/3$$
$$\alpha^{11}_{5,6} = -2 * sqrt(2) * I$$
$$\alpha^{11}_{3,8} = 2/3 + sqrt(2) * I/3$$
$$\alpha^{12}_{3,10} = -1$$
$$\alpha^{12}_{5,8} = -1$$
$$\alpha^{8}_{2,6} = -1/3 + 4 * sqrt(2) * I/3$$
$$\alpha^{11}_{4,7} = sqrt(2) * I$$
$$\alpha^{9}_{4,5} = -sqrt(2) * I$$
$$\alpha^{10}_{4,6} = -sqrt(2) * I$$
$$\alpha^{8}_{3,5} = 2/3 - 2 * sqrt(2) * I/3$$

Solution 4:

$$\alpha^{7}_{3,4} = 2/3 + 2 * sqrt(2) * I/3$$
$$\alpha^{9}_{2,7} = -1 - sqrt(2) * I$$
$$\alpha^{12}_{4,9} = 1$$
$$\alpha^{12}_{6,7} = 1$$
$$\alpha^{10}_{2,8} = -5/3 + sqrt(2) * I/3$$
$$\alpha^{11}_{2,9} = -7/3 + 2 * sqrt(2) * I/3$$
$$\alpha^{9}_{3,6} = 2/3 - sqrt(2) * I/3$$
$$\alpha^{7}_{2,5} = 1/3 - 2 * sqrt(2) * I/3$$
$$\alpha^{10}_{3,7} = 2/3 - 4 * sqrt(2) * I/3$$
$$\alpha^{11}_{5,6} = 2 * sqrt(2) * I$$
$$\alpha^{11}_{3,8} = 2/3 - sqrt(2) * I/3$$
$$\alpha^{12}_{3,10} = -1$$
$$\alpha^{12}_{5,8} = -1$$
$$\alpha^{8}_{2,6} = -1/3 - 4 * sqrt(2) * I/3$$
$$\alpha^{11}_{4,7} = -sqrt(2) * I$$
$$\alpha^{9}_{4,5} = sqrt(2) * I$$
$$\alpha^{10}_{4,6} = sqrt(2) * I$$
$$\alpha^{8}_{3,5} = 2/3 + 2 * sqrt(2) * I/3$$

How the solution(s) were or were not found:
Change variables

$$\alpha^{7}_{3,4} \to x_1$$
$$\alpha^{9}_{2,7} \to x_2$$
$$\alpha^{12}_{4,9} \to x_3$$
$$\alpha^{12}_{6,7} \to x_4$$
$$\alpha^{10}_{2,8} \to x_5$$
$$\alpha^{11}_{2,9} \to x_6$$
$$\alpha^{9}_{3,6} \to x_7$$
$$\alpha^{7}_{2,5} \to x_8$$
$$\alpha^{10}_{3,7} \to x_9$$

$$\alpha_{5,6}^{11} \to x_{10}$$
$$\alpha_{3,8}^{11} \to x_{11}$$
$$\alpha_{3,10}^{12} \to x_{12}$$
$$\alpha_{5,8}^{12} \to x_{13}$$
$$\alpha_{2,6}^{8} \to x_{14}$$
$$\alpha_{4,7}^{11} \to x_{15}$$
$$\alpha_{4,5}^{9} \to x_{16}$$
$$\alpha_{4,6}^{10} \to x_{17}$$
$$\alpha_{3,5}^{8} \to x_{18}$$

Jacobi Tests

$$\begin{aligned}
(e_1,e_2,e_4):&\quad -x_1-x_8+1 &&=0\\
(e_1,e_2,e_5):&\quad -x_{14}-x_{18}+x_8 &&=0\\
(e_1,e_3,e_4):&\quad x_1-x_{18} &&=0\\
(e_1,e_2,e_6):&\quad x_{14}-x_2-x_7 &&=0\\
(e_1,e_3,e_5):&\quad -x_{16}+x_{18}-x_7 &&=0\\
(e_2,e_3,e_4):&\quad x_1x_2+x_{16}-x_7 &&=0\\
(e_1,e_2,e_7):&\quad x_2-x_5-x_9 &&=0\\
(e_1,e_3,e_6):&\quad -x_{17}+x_7-x_9 &&=0\\
(e_1,e_4,e_5):&\quad x_{16}-x_{17} &&=0\\
(e_2,e_3,e_5):&\quad x_{18}x_5-x_8x_9 &&=0\\
(e_1,e_2,e_8):&\quad -x_{11}+x_5-x_6 &&=0\\
(e_1,e_3,e_7):&\quad -x_{11}-x_{15}+x_9 &&=0\\
(e_1,e_4,e_6):&\quad -x_{10}-x_{15}+x_{17} &&=0\\
(e_2,e_3,e_6):&\quad -x_{10}-x_{11}x_{14}+x_6x_7 &&=0\\
(e_2,e_4,e_5):&\quad x_{10}-x_{15}x_8+x_{16}x_6 &&=0\\
(e_1,e_2,e_{10}):&\quad -x_{12}-1 &&=0\\
(e_1,e_3,e_9):&\quad -x_{12}-x_3 &&=0\\
(e_1,e_4,e_8):&\quad -x_{13}-x_3 &&=0\\
(e_1,e_5,e_7):&\quad -x_{13}-x_4 &&=0\\
(e_2,e_3,e_8):&\quad x_{11}-x_{12}x_5-x_{13} &&=0\\
(e_2,e_4,e_7):&\quad x_{15}-x_2x_3-x_4 &&=0\\
(e_2,e_5,e_6):&\quad x_{10}-x_{13}x_{14}+x_4x_8 &&=0\\
(e_3,e_4,e_6):&\quad x_1x_4+x_{12}x_{17}-x_3x_7 &&=0
\end{aligned}$$

Groebner basis (18 variables, 8 linear, 10 nonlinear)

$$x_1 - x_{18} = 0$$
$$15x_{18}^3 - 50x_{18}^2 + 114x_{18} + 36x_2 - 40 = 0$$
$$x_3 - 1 = 0$$
$$x_4 - 1 = 0$$
$$15x_{18}^3 - 50x_{18}^2 + 54x_{18} + 12x_5 - 16 = 0$$
$$15x_{18}^3 - 50x_{18}^2 + 54x_{18} + 6x_6 - 22 = 0$$
$$-15x_{18}^3 + 50x_{18}^2 - 42x_{18} + 36x_7 + 4 = 0$$
$$x_{18} + x_8 - 1 = 0$$
$$-15x_{18}^3 + 50x_{18}^2 - 24x_{18} + 18x_9 + 4 = 0$$
$$x_{10} - 3x_{18} + 2 = 0$$
$$12x_{11} - 15x_{18}^3 + 50x_{18}^2 - 54x_{18} + 28 = 0$$
$$x_{12} + 1 = 0$$
$$x_{13} + 1 = 0$$
$$x_{14} + 2x_{18} - 1 = 0$$
$$36x_{15} + 15x_{18}^3 - 50x_{18}^2 + 114x_{18} - 76 = 0$$
$$36x_{16} + 15x_{18}^3 - 50x_{18}^2 + 6x_{18} - 4 = 0$$
$$36x_{17} + 15x_{18}^3 - 50x_{18}^2 + 6x_{18} - 4 = 0$$
$$15x_{18}^4 - 20x_{18}^3 + 14x_{18}^2 + 8x_{18} - 8 = 0$$

Solution 1:

$$x_1 = -sqrt(10)/5$$
$$x_2 = 5/3 + 2 * sqrt(10)/3$$
$$x_3 = 1$$
$$x_4 = 1$$
$$x_5 = 3 + sqrt(10)$$
$$x_6 = 2 * sqrt(10) + 7$$
$$x_7 = -4 * sqrt(10)/15 - 2/3$$
$$x_8 = sqrt(10)/5 + 1$$
$$x_9 = -4/3 - sqrt(10)/3$$
$$x_10 = -2 - 3 * sqrt(10)/5$$
$$x_11 = -4 - sqrt(10)$$

$$x_12 = -1$$
$$x_13 = -1$$
$$x_14 = 1 + 2 * sqrt(10)/5$$
$$x_15 = 2 * sqrt(10)/3 + 8/3$$
$$x_16 = sqrt(10)/15 + 2/3$$
$$x_17 = sqrt(10)/15 + 2/3$$
$$x_18 = -sqrt(10)/5$$

Solution 2:

$$x_1 = sqrt(10)/5$$
$$x_2 = 5/3 - 2 * sqrt(10)/3$$
$$x_3 = 1$$
$$x_4 = 1$$
$$x_5 = 3 - sqrt(10)$$
$$x_6 = 7 - 2 * sqrt(10)$$
$$x_7 = -2/3 + 4 * sqrt(10)/15$$
$$x_8 = 1 - sqrt(10)/5$$
$$x_9 = -4/3 + sqrt(10)/3$$
$$x_10 = -2 + 3 * sqrt(10)/5$$
$$x_11 = -4 + sqrt(10)$$
$$x_12 = -1$$
$$x_13 = -1$$
$$x_14 = 1 - 2 * sqrt(10)/5$$
$$x_15 = 8/3 - 2 * sqrt(10)/3$$
$$x_16 = 2/3 - sqrt(10)/15$$
$$x_17 = 2/3 - sqrt(10)/15$$
$$x_18 = sqrt(10)/5$$

Solution 3:

$$x_1 = 2/3 - 2 * sqrt(2) * I/3$$
$$x_2 = -1 + sqrt(2) * I$$
$$x_3 = 1$$
$$x_4 = 1$$
$$x_5 = -5/3 - sqrt(2) * I/3$$

$$x_6 = -7/3 - 2 * sqrt(2) * I/3$$
$$x_7 = 2/3 + sqrt(2) * I/3$$
$$x_8 = 1/3 + 2 * sqrt(2) * I/3$$
$$x_9 = 2/3 + 4 * sqrt(2) * I/3$$
$$x_10 = -2 * sqrt(2) * I$$
$$x_11 = 2/3 + sqrt(2) * I/3$$
$$x_12 = -1$$
$$x_13 = -1$$
$$x_14 = -1/3 + 4 * sqrt(2) * I/3$$
$$x_15 = sqrt(2) * I$$
$$x_16 = -sqrt(2) * I$$
$$x_17 = -sqrt(2) * I$$
$$x_18 = 2/3 - 2 * sqrt(2) * I/3$$

Solution 4:

$$x_1 = 2/3 + 2 * sqrt(2) * I/3$$
$$x_2 = -1 - sqrt(2) * I$$
$$x_3 = 1$$
$$x_4 = 1$$
$$x_5 = -5/3 + sqrt(2) * I/3$$
$$x_6 = -7/3 + 2 * sqrt(2) * I/3$$
$$x_7 = 2/3 - sqrt(2) * I/3$$
$$x_8 = 1/3 - 2 * sqrt(2) * I/3$$
$$x_9 = 2/3 - 4 * sqrt(2) * I/3$$
$$x_10 = 2 * sqrt(2) * I$$
$$x_11 = 2/3 - sqrt(2) * I/3$$
$$x_12 = -1$$
$$x_13 = -1$$
$$x_14 = -1/3 - 4 * sqrt(2) * I/3$$
$$x_15 = -sqrt(2) * I$$
$$x_16 = sqrt(2) * I$$
$$x_17 = sqrt(2) * I$$
$$x_18 = 2/3 + 2 * sqrt(2) * I/3$$

$\mathfrak{m}_{3B}(3,12)$

m3B312 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_2, e_7] &= e_{10} \\
[e_2, e_8] &= 3e_{11} & [e_2, e_{11}] &= e_{12} \\
[e_3, e_6] &= -e_{10} & [e_3, e_7] &= -2e_{11} \\
[e_3, e_{10}] &= \alpha_{3,10}^{12} e_{12} & [e_4, e_5] &= e_{10} \\
[e_4, e_6] &= e_{11} & [e_4, e_9] &= \alpha_{4,9}^{12} e_{12} \\
[e_5, e_8] &= \alpha_{5,8}^{12} e_{12} & [e_6, e_7] &= \alpha_{6,7}^{12} e_{12}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_{10}): &\quad -\alpha_{3,10}^{12} - 1 &&= 0 \\
(e_1, e_3, e_9): &\quad -\alpha_{3,10}^{12} - \alpha_{4,9}^{12} &&= 0 \\
(e_1, e_4, e_8): &\quad -\alpha_{4,9}^{12} - \alpha_{5,8}^{12} &&= 0 \\
(e_1, e_5, e_7): &\quad -\alpha_{5,8}^{12} - \alpha_{6,7}^{12} &&= 0 \\
(e_2, e_3, e_7): &\quad -\alpha_{3,10}^{12} - 2 &&= 0 \\
(e_2, e_4, e_6): &\quad \text{no solutions} && \\
(e_3, e_4, e_5): &\quad \alpha_{3,10}^{12} &&= 0
\end{aligned}
$$

There are no solutions.

$\mathfrak{m}_{5B}(3,12)$

m5B312 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
&[e_1, e_2] = e_3 && [e_1, e_3] = e_4 \\
&[e_1, e_4] = e_5 && [e_1, e_5] = e_6 \\
&[e_1, e_6] = e_7 && [e_1, e_7] = e_8 \\
&[e_1, e_8] = e_9 && [e_1, e_9] = e_{10} \\
&[e_1, e_{10}] = e_{11} && [e_2, e_5] = e_8 \\
&[e_2, e_6] = 2e_9 && [e_2, e_7] = \alpha_{2,7}^{10} e_{10} \\
&[e_2, e_8] = \alpha_{2,8}^{11} e_{11} && [e_2, e_{11}] = e_{12} \\
&[e_3, e_4] = -e_8 && [e_3, e_5] = -e_9 \\
&[e_3, e_6] = \alpha_{3,6}^{10} e_{10} && [e_3, e_7] = \alpha_{3,7}^{11} e_{11} \\
&[e_3, e_{10}] = \alpha_{3,10}^{12} e_{12} && [e_4, e_5] = \alpha_{4,5}^{10} e_{10} \\
&[e_4, e_6] = \alpha_{4,6}^{11} e_{11} && [e_4, e_9] = \alpha_{4,9}^{12} e_{12} \\
&[e_5, e_8] = \alpha_{5,8}^{12} e_{12} && [e_6, e_7] = \alpha_{6,7}^{12} e_{12}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_6): &\quad -\alpha_{2,7}^{10} - \alpha_{3,6}^{10} + 2 &&= 0 \\
(e_1, e_3, e_5): &\quad -\alpha_{3,6}^{10} - \alpha_{4,5}^{10} - 1 &&= 0 \\
(e_1, e_2, e_7): &\quad \alpha_{2,7}^{10} - \alpha_{2,8}^{11} - \alpha_{3,7}^{11} &&= 0 \\
(e_1, e_3, e_6): &\quad \alpha_{3,6}^{10} - \alpha_{3,7}^{11} - \alpha_{4,6}^{11} &&= 0 \\
(e_1, e_4, e_5): &\quad \alpha_{4,5}^{10} - \alpha_{4,6}^{11} &&= 0 \\
(e_2, e_3, e_4): &\quad -\alpha_{2,8}^{11} &&= 0 \\
(e_1, e_2, e_{10}): &\quad -\alpha_{3,10}^{12} - 1 &&= 0 \\
(e_1, e_3, e_9): &\quad -\alpha_{3,10}^{12} - \alpha_{4,9}^{12} &&= 0 \\
(e_1, e_4, e_8): &\quad -\alpha_{4,9}^{12} - \alpha_{5,8}^{12} &&= 0 \\
(e_1, e_5, e_7): &\quad -\alpha_{5,8}^{12} - \alpha_{6,7}^{12} &&= 0 \\
(e_2, e_3, e_7): &\quad -\alpha_{2,7}^{10}\alpha_{3,10}^{12} + \alpha_{3,7}^{11} &&= 0 \\
(e_2, e_4, e_6): &\quad \alpha_{4,6}^{11} - 2\alpha_{4,9}^{12} &&= 0 \\
(e_3, e_4, e_5): &\quad \alpha_{3,10}^{12}\alpha_{4,5}^{10} + \alpha_{4,9}^{12} - \alpha_{5,8}^{12} &&= 0
\end{aligned}
$$

No solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{4,9}^{12} \to x_1$$

$$\alpha_{6,7}^{12} \to x_2$$

$$\alpha_{3,7}^{11} \to x_3$$

$$\alpha_{2,8}^{11} \to x_4$$

$$\alpha_{5,8}^{12} \to x_5$$

$$\alpha_{2,7}^{10} \to x_6$$

$$\alpha_{3,10}^{12} \to x_7$$

$$\alpha_{4,6}^{11} \to x_8$$

$$\alpha_{4,5}^{10} \to x_9$$

$$\alpha_{3,6}^{10} \to x_{10}$$

Jacobi Tests

$$\begin{aligned}
(e_1,e_2,e_6):&\quad -x_{10}-x_6+2 &&=0\\
(e_1,e_3,e_5):&\quad -x_{10}-x_9-1 &&=0\\
(e_1,e_2,e_7):&\quad -x_3-x_4+x_6 &&=0\\
(e_1,e_3,e_6):&\quad x_{10}-x_3-x_8 &&=0\\
(e_1,e_4,e_5):&\quad -x_8+x_9 &&=0\\
(e_2,e_3,e_4):&\quad -x_4 &&=0\\
(e_1,e_2,e_{10}):&\quad -x_7-1 &&=0\\
(e_1,e_3,e_9):&\quad -x_1-x_7 &&=0\\
(e_1,e_4,e_8):&\quad -x_1-x_5 &&=0\\
(e_1,e_5,e_7):&\quad -x_2-x_5 &&=0\\
(e_2,e_3,e_7):&\quad x_3-x_6x_7 &&=0\\
(e_2,e_4,e_6):&\quad -2x_1+x_8 &&=0\\
(e_3,e_4,e_5):&\quad x_1-x_5+x_7x_9 &&=0
\end{aligned}$$

Groebner basis (10 variables, 1 linear, 0 nonlinear)

$$1=0$$

$\mathfrak{m}_{7B}(3,12)$

m7B312 (this line included for string searching purposes)

Solution 1

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4\\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6\\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8\\
[e_1,e_8] &= e_9 & [e_1,e_9] &= e_{10}\\
[e_1,e_{10}] &= e_{11} & [e_2,e_3] &= e_6\\
[e_2,e_4] &= e_7 & [e_2,e_5] &= \frac{8e_8}{5}\\
[e_2,e_6] &= \frac{11e_9}{5} & [e_2,e_7] &= 4e_{10}\\
[e_2,e_8] &= 7e_{11} & [e_2,e_{11}] &= e_{12}\\
[e_3,e_4] &= -\frac{3e_8}{5} & [e_3,e_5] &= -\frac{3e_9}{5}\\
[e_3,e_6] &= -\frac{9e_{10}}{5} & [e_3,e_7] &= -3e_{11}\\
[e_3,e_{10}] &= -e_{12} & [e_4,e_5] &= \frac{6e_{10}}{5}\\
[e_4,e_6] &= \frac{6e_{11}}{5} & [e_4,e_9] &= e_{12}\\
[e_5,e_8] &= -e_{12} & [e_6,e_7] &= e_{12}
\end{aligned}
$$

Solution 2

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4\\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6\\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8\\
[e_1,e_8] &= e_9 & [e_1,e_9] &= e_{10}\\
[e_1,e_{10}] &= e_{11} & [e_2,e_3] &= e_6\\
[e_2,e_4] &= e_7 & [e_2,e_5] &= e_8\\
[e_2,e_6] &= e_9 & [e_2,e_7] &= e_{10}\\
[e_2,e_8] &= e_{11} & [e_2,e_{11}] &= e_{12}\\
[e_3,e_4] &= 0 & [e_3,e_5] &= 0\\
[e_3,e_6] &= 0 & [e_3,e_7] &= 0\\
[e_3,e_{10}] &= -e_{12} & [e_4,e_5] &= 0\\
[e_4,e_6] &= 0 & [e_4,e_9] &= e_{12}\\
[e_5,e_8] &= -e_{12} & [e_6,e_7] &= e_{12}
\end{aligned}
$$

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_2, e_3] &= e_6 \\
[e_2, e_4] &= e_7 & [e_2, e_5] &= \alpha_{2,5}^{8} e_8 \\
[e_2, e_6] &= \alpha_{2,6}^{9} e_9 & [e_2, e_7] &= \alpha_{2,7}^{10} e_{10} \\
[e_2, e_8] &= \alpha_{2,8}^{11} e_{11} & [e_2, e_{11}] &= e_{12} \\
[e_3, e_4] &= \alpha_{3,4}^{8} e_8 & [e_3, e_5] &= \alpha_{3,5}^{9} e_9 \\
[e_3, e_6] &= \alpha_{3,6}^{10} e_{10} & [e_3, e_7] &= \alpha_{3,7}^{11} e_{11} \\
[e_3, e_{10}] &= \alpha_{3,10}^{12} e_{12} & [e_4, e_5] &= \alpha_{4,5}^{10} e_{10} \\
[e_4, e_6] &= \alpha_{4,6}^{11} e_{11} & [e_4, e_9] &= \alpha_{4,9}^{12} e_{12} \\
[e_5, e_8] &= \alpha_{5,8}^{12} e_{12} & [e_6, e_7] &= \alpha_{6,7}^{12} e_{12}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_4):&\quad -\alpha_{2,5}^{8} - \alpha_{3,4}^{8} + 1 &&= 0 \\
(e_1, e_2, e_5):&\quad \alpha_{2,5}^{8} - \alpha_{2,6}^{9} - \alpha_{3,5}^{9} &&= 0 \\
(e_1, e_3, e_4):&\quad \alpha_{3,4}^{8} - \alpha_{3,5}^{9} &&= 0 \\
(e_1, e_2, e_6):&\quad \alpha_{2,6}^{9} - \alpha_{2,7}^{10} - \alpha_{3,6}^{10} &&= 0 \\
(e_1, e_3, e_5):&\quad \alpha_{3,5}^{9} - \alpha_{3,6}^{10} - \alpha_{4,5}^{10} &&= 0 \\
(e_1, e_2, e_7):&\quad \alpha_{2,7}^{10} - \alpha_{2,8}^{11} - \alpha_{3,7}^{11} &&= 0 \\
(e_1, e_3, e_6):&\quad \alpha_{3,6}^{10} - \alpha_{3,7}^{11} - \alpha_{4,6}^{11} &&= 0 \\
(e_1, e_4, e_5):&\quad \alpha_{4,5}^{10} - \alpha_{4,6}^{11} &&= 0 \\
(e_2, e_3, e_4):&\quad \alpha_{2,8}^{11}\alpha_{3,4}^{8} - \alpha_{3,7}^{11} + \alpha_{4,6}^{11} &&= 0 \\
(e_1, e_2, e_{10}):&\quad -\alpha_{3,10}^{12} - 1 &&= 0 \\
(e_1, e_3, e_9):&\quad -\alpha_{3,10}^{12} - \alpha_{4,9}^{12} &&= 0 \\
(e_1, e_4, e_8):&\quad -\alpha_{4,9}^{12} - \alpha_{5,8}^{12} &&= 0 \\
(e_1, e_5, e_7):&\quad -\alpha_{5,8}^{12} - \alpha_{6,7}^{12} &&= 0 \\
(e_2, e_3, e_7):&\quad -\alpha_{2,7}^{10}\alpha_{3,10}^{12} + \alpha_{3,7}^{11} - \alpha_{6,7}^{12} &&= 0 \\
(e_2, e_4, e_6):&\quad -\alpha_{2,6}^{9}\alpha_{4,9}^{12} + \alpha_{4,6}^{11} + \alpha_{6,7}^{12} &&= 0 \\
(e_3, e_4, e_5):&\quad \alpha_{3,10}^{12}\alpha_{4,5}^{10} + \alpha_{3,4}^{8}\alpha_{5,8}^{12} - \alpha_{3,5}^{9}\alpha_{4,9}^{12} &&= 0
\end{aligned}
$$

Solution 1:

$$\alpha^{9}_{3,5} = -3/5$$
$$\alpha^{12}_{4,9} = 1$$
$$\alpha^{12}_{6,7} = 1$$
$$\alpha^{8}_{2,5} = 8/5$$
$$\alpha^{11}_{3,7} = -3$$
$$\alpha^{11}_{2,8} = 7$$
$$\alpha^{12}_{5,8} = -1$$
$$\alpha^{9}_{2,6} = 11/5$$
$$\alpha^{10}_{2,7} = 4$$
$$\alpha^{12}_{3,10} = -1$$
$$\alpha^{11}_{4,6} = 6/5$$
$$\alpha^{10}_{4,5} = 6/5$$
$$\alpha^{8}_{3,4} = -3/5$$
$$\alpha^{10}_{3,6} = -9/5$$

Solution 2:

$$\alpha^{9}_{3,5} = 0$$
$$\alpha^{12}_{4,9} = 1$$
$$\alpha^{12}_{6,7} = 1$$
$$\alpha^{8}_{2,5} = 1$$
$$\alpha^{11}_{3,7} = 0$$
$$\alpha^{11}_{2,8} = 1$$
$$\alpha^{12}_{5,8} = -1$$
$$\alpha^{9}_{2,6} = 1$$
$$\alpha^{10}_{2,7} = 1$$
$$\alpha^{12}_{3,10} = -1$$
$$\alpha^{11}_{4,6} = 0$$
$$\alpha^{10}_{4,5} = 0$$
$$\alpha^{8}_{3,4} = 0$$
$$\alpha^{10}_{3,6} = 0$$

How the solution(s) were or were not found:
Change variables

$$\alpha_{3,5}^{9} \to x_1$$
$$\alpha_{4,9}^{12} \to x_2$$
$$\alpha_{6,7}^{12} \to x_3$$
$$\alpha_{2,5}^{8} \to x_4$$
$$\alpha_{3,7}^{11} \to x_5$$
$$\alpha_{2,8}^{11} \to x_6$$
$$\alpha_{5,8}^{12} \to x_7$$
$$\alpha_{2,6}^{9} \to x_8$$
$$\alpha_{2,7}^{10} \to x_9$$
$$\alpha_{3,10}^{12} \to x_{10}$$
$$\alpha_{4,6}^{11} \to x_{11}$$
$$\alpha_{4,5}^{10} \to x_{12}$$
$$\alpha_{3,4}^{8} \to x_{13}$$
$$\alpha_{3,6}^{10} \to x_{14}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4):&\quad -x_{13} - x_4 + 1 &&= 0\\
(e_1, e_2, e_5):&\quad -x_1 + x_4 - x_8 &&= 0\\
(e_1, e_3, e_4):&\quad -x_1 + x_{13} &&= 0\\
(e_1, e_2, e_6):&\quad -x_{14} + x_8 - x_9 &&= 0\\
(e_1, e_3, e_5):&\quad x_1 - x_{12} - x_{14} &&= 0\\
(e_1, e_2, e_7):&\quad -x_5 - x_6 + x_9 &&= 0\\
(e_1, e_3, e_6):&\quad -x_{11} + x_{14} - x_5 &&= 0\\
(e_1, e_4, e_5):&\quad -x_{11} + x_{12} &&= 0\\
(e_2, e_3, e_4):&\quad x_{11} + x_{13}x_6 - x_5 &&= 0\\
(e_1, e_2, e_{10}):&\quad -x_{10} - 1 &&= 0\\
(e_1, e_3, e_9):&\quad -x_{10} - x_2 &&= 0\\
(e_1, e_4, e_8):&\quad -x_2 - x_7 &&= 0\\
(e_1, e_5, e_7):&\quad -x_3 - x_7 &&= 0\\
(e_2, e_3, e_7):&\quad -x_{10}x_9 - x_3 + x_5 &&= 0\\
(e_2, e_4, e_6):&\quad x_{11} - x_2x_8 + x_3 &&= 0\\
(e_3, e_4, e_5):&\quad -x_1x_2 + x_{10}x_{12} + x_{13}x_7 &&= 0
\end{aligned}$$

Groebner basis (14 variables, 13 linear, 1 nonlinear)

$$3x_1 - x_{14} = 0$$
$$x_2 - 1 = 0$$
$$x_3 - 1 = 0$$
$$x_{14} + 3x_4 - 3 = 0$$
$$-5x_{14} + 3x_5 = 0$$
$$10x_{14} + 3x_6 - 3 = 0$$
$$x_7 + 1 = 0$$
$$2x_{14} + 3x_8 - 3 = 0$$
$$5x_{14} + 3x_9 - 3 = 0$$
$$x_{10} + 1 = 0$$
$$3x_{11} + 2x_{14} = 0$$
$$3x_{12} + 2x_{14} = 0$$
$$3x_{13} - x_{14} = 0$$
$$5x_{14}^2 + 9x_{14} = 0$$

Solution 1:

$$x_1 = -3/5$$
$$x_2 = 1$$
$$x_3 = 1$$
$$x_4 = 8/5$$
$$x_5 = -3$$
$$x_6 = 7$$
$$x_7 = -1$$
$$x_8 = 11/5$$
$$x_9 = 4$$
$$x_10 = -1$$
$$x_11 = 6/5$$
$$x_12 = 6/5$$
$$x_13 = -3/5$$
$$x_14 = -9/5$$

Solution 2:

$$x_1 = 0$$

$$x_2 = 1$$
$$x_3 = 1$$
$$x_4 = 1$$
$$x_5 = 0$$
$$x_6 = 1$$
$$x_7 = -1$$
$$x_8 = 1$$
$$x_9 = 1$$
$$x_10 = -1$$
$$x_11 = 0$$
$$x_12 = 0$$
$$x_13 = 0$$
$$x_14 = 0$$

$\mathfrak{m}_{2B}(4, 12)$

m2B412 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_2, e_7] &= e_{11} \\
[e_2, e_{11}] &= e_{12} & [e_3, e_6] &= -e_{11} \\
[e_3, e_{10}] &= \alpha_{3,10}^{12} e_{12} & [e_4, e_5] &= e_{11} \\
[e_4, e_9] &= \alpha_{4,9}^{12} e_{12} & [e_5, e_8] &= \alpha_{5,8}^{12} e_{12} \\
[e_6, e_7] &= \alpha_{6,7}^{12} e_{12}
\end{aligned}$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1, e_2, e_{10}): &\quad -\alpha_{3,10}^{12} - 1 &= 0 \\
(e_1, e_3, e_9): &\quad -\alpha_{3,10}^{12} - \alpha_{4,9}^{12} &= 0 \\
(e_1, e_4, e_8): &\quad -\alpha_{4,9}^{12} - \alpha_{5,8}^{12} &= 0 \\
(e_1, e_5, e_7): &\quad -\alpha_{5,8}^{12} - \alpha_{6,7}^{12} &= 0 \\
(e_2, e_3, e_6): &\quad \text{no solutions} \\
(e_2, e_4, e_5): &\quad \text{no solutions}
\end{aligned}$$

There are no solutions.

$\mathfrak{m}_{4B}(4,12)$

m4B412 (this line included for string searching purposes)

Solution 1

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4 \\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6 \\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8 \\
[e_1,e_8] &= e_9 & [e_1,e_9] &= e_{10} \\
[e_1,e_{10}] &= e_{11} & [e_2,e_5] &= e_9 \\
[e_2,e_6] &= 2e_{10} & [e_2,e_7] &= 4e_{11} \\
[e_2,e_{11}] &= e_{12} & [e_3,e_4] &= -e_9 \\
[e_3,e_5] &= -e_{10} & [e_3,e_6] &= -2e_{11} \\
[e_3,e_{10}] &= -e_{12} & [e_4,e_5] &= e_{11} \\
[e_4,e_9] &= e_{12} & [e_5,e_8] &= -e_{12} \\
[e_6,e_7] &= e_{12}
\end{aligned}
$$

Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4 \\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6 \\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8 \\
[e_1,e_8] &= e_9 & [e_1,e_9] &= e_{10} \\
[e_1,e_{10}] &= e_{11} & [e_2,e_5] &= e_9 \\
[e_2,e_6] &= 2e_{10} & [e_2,e_7] &= \alpha_{2,7}^{11}e_{11} \\
[e_2,e_{11}] &= e_{12} & [e_3,e_4] &= -e_9 \\
[e_3,e_5] &= -e_{10} & [e_3,e_6] &= \alpha_{3,6}^{11}e_{11} \\
[e_3,e_{10}] &= \alpha_{3,10}^{12}e_{12} & [e_4,e_5] &= \alpha_{4,5}^{11}e_{11} \\
[e_4,e_9] &= \alpha_{4,9}^{12}e_{12} & [e_5,e_8] &= \alpha_{5,8}^{12}e_{12} \\
[e_6,e_7] &= \alpha_{6,7}^{12}e_{12}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1, e_2, e_6): &\quad -\alpha_{2,7}^{11} - \alpha_{3,6}^{11} + 2 &= 0\\
(e_1, e_3, e_5): &\quad -\alpha_{3,6}^{11} - \alpha_{4,5}^{11} - 1 &= 0\\
(e_1, e_2, e_{10}): &\quad -\alpha_{3,10}^{12} - 1 &= 0\\
(e_1, e_3, e_9): &\quad -\alpha_{3,10}^{12} - \alpha_{4,9}^{12} &= 0\\
(e_1, e_4, e_8): &\quad -\alpha_{4,9}^{12} - \alpha_{5,8}^{12} &= 0\\
(e_1, e_5, e_7): &\quad -\alpha_{5,8}^{12} - \alpha_{6,7}^{12} &= 0\\
(e_2, e_3, e_6): &\quad -2\alpha_{3,10}^{12} + \alpha_{3,6}^{11} &= 0\\
(e_2, e_4, e_5): &\quad \alpha_{4,5}^{11} - \alpha_{4,9}^{12} &= 0
\end{aligned}$$

Solution 1:

$$\begin{aligned}
\alpha_{2,7}^{11} &= 4\\
\alpha_{4,9}^{12} &= 1\\
\alpha_{6,7}^{12} &= 1\\
\alpha_{4,5}^{11} &= 1\\
\alpha_{5,8}^{12} &= -1\\
\alpha_{3,10}^{12} &= -1\\
\alpha_{3,6}^{11} &= -2
\end{aligned}$$

How the solution(s) were or were not found:
Change variables

$$\begin{aligned}
\alpha_{2,7}^{11} &\to x_1\\
\alpha_{4,9}^{12} &\to x_2\\
\alpha_{6,7}^{12} &\to x_3\\
\alpha_{4,5}^{11} &\to x_4\\
\alpha_{5,8}^{12} &\to x_5\\
\alpha_{3,10}^{12} &\to x_6\\
\alpha_{3,6}^{11} &\to x_7
\end{aligned}$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_6): &\quad -x_1 - x_7 + 2 &&= 0 \\
(e_1, e_3, e_5): &\quad -x_4 - x_7 - 1 &&= 0 \\
(e_1, e_2, e_{10}): &\quad -x_6 - 1 &&= 0 \\
(e_1, e_3, e_9): &\quad -x_2 - x_6 &&= 0 \\
(e_1, e_4, e_8): &\quad -x_2 - x_5 &&= 0 \\
(e_1, e_5, e_7): &\quad -x_3 - x_5 &&= 0 \\
(e_2, e_3, e_6): &\quad -2x_6 + x_7 &&= 0 \\
(e_2, e_4, e_5): &\quad -x_2 + x_4 &&= 0
\end{aligned}
$$

Groebner basis (7 variables, 7 linear, 0 nonlinear)

$$x_1 - 4 = 0$$
$$x_2 - 1 = 0$$
$$x_3 - 1 = 0$$
$$x_4 - 1 = 0$$
$$x_5 + 1 = 0$$
$$x_6 + 1 = 0$$
$$x_7 + 2 = 0$$

Solution 1:

$$x_1 = 4$$
$$x_2 = 1$$
$$x_3 = 1$$
$$x_4 = 1$$
$$x_5 = -1$$
$$x_6 = -1$$
$$x_7 = -2$$

$\mathfrak{m}_{6B}(4,12)$

m6B412 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4\\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6\\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8\\
[e_1,e_8] &= e_9 & [e_1,e_9] &= e_{10}\\
[e_1,e_{10}] &= e_{11} & [e_2,e_3] &= e_7\\
[e_2,e_4] &= e_8 & [e_2,e_5] &= \alpha_{2,5}^{9} e_9\\
[e_2,e_6] &= \alpha_{2,6}^{10} e_{10} & [e_2,e_7] &= \alpha_{2,7}^{11} e_{11}\\
[e_2,e_{11}] &= e_{12} & [e_3,e_4] &= \alpha_{3,4}^{9} e_9\\
[e_3,e_5] &= \alpha_{3,5}^{10} e_{10} & [e_3,e_6] &= \alpha_{3,6}^{11} e_{11}\\
[e_3,e_{10}] &= \alpha_{3,10}^{12} e_{12} & [e_4,e_5] &= \alpha_{4,5}^{11} e_{11}\\
[e_4,e_9] &= \alpha_{4,9}^{12} e_{12} & [e_5,e_8] &= \alpha_{5,8}^{12} e_{12}\\
[e_6,e_7] &= \alpha_{6,7}^{12} e_{12}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1,e_2,e_4):&\quad -\alpha_{2,5}^{9} - \alpha_{3,4}^{9} + 1 &= 0\\
(e_1,e_2,e_5):&\quad \alpha_{2,5}^{9} - \alpha_{2,6}^{10} - \alpha_{3,5}^{10} &= 0\\
(e_1,e_3,e_4):&\quad \alpha_{3,4}^{9} - \alpha_{3,5}^{10} &= 0\\
(e_1,e_2,e_6):&\quad \alpha_{2,6}^{10} - \alpha_{2,7}^{11} - \alpha_{3,6}^{11} &= 0\\
(e_1,e_3,e_5):&\quad \alpha_{3,5}^{10} - \alpha_{3,6}^{11} - \alpha_{4,5}^{11} &= 0\\
(e_1,e_2,e_{10}):&\quad -\alpha_{3,10}^{12} - 1 &= 0\\
(e_1,e_3,e_9):&\quad -\alpha_{3,10}^{12} - \alpha_{4,9}^{12} &= 0\\
(e_1,e_4,e_8):&\quad -\alpha_{4,9}^{12} - \alpha_{5,8}^{12} &= 0\\
(e_1,e_5,e_7):&\quad -\alpha_{5,8}^{12} - \alpha_{6,7}^{12} &= 0\\
(e_2,e_3,e_6):&\quad -\alpha_{2,6}^{10}\alpha_{3,10}^{12} + \alpha_{3,6}^{11} + \alpha_{6,7}^{12} &= 0\\
(e_2,e_4,e_5):&\quad -\alpha_{2,5}^{9}\alpha_{4,9}^{12} + \alpha_{4,5}^{11} + \alpha_{5,8}^{12} &= 0
\end{aligned}
$$

Infinite number of solutions.

How the solution(s) were or were not found:

Change variables

$$\alpha_{2,7}^{11} \to x_1$$

$$\alpha_{2,5}^{9} \to x_2$$

$$\alpha^{12}_{4,9} \to x_3$$
$$\alpha^{12}_{6,7} \to x_4$$
$$\alpha^{10}_{3,5} \to x_5$$
$$\alpha^{11}_{4,5} \to x_6$$
$$\alpha^{12}_{5,8} \to x_7$$
$$\alpha^{12}_{3,10} \to x_8$$
$$\alpha^{9}_{3,4} \to x_9$$
$$\alpha^{10}_{2,6} \to x_{10}$$
$$\alpha^{11}_{3,6} \to x_{11}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4): &\quad -x_2 - x_9 + 1 &&= 0\\
(e_1, e_2, e_5): &\quad -x_{10} + x_2 - x_5 &&= 0\\
(e_1, e_3, e_4): &\quad -x_5 + x_9 &&= 0\\
(e_1, e_2, e_6): &\quad -x_1 + x_{10} - x_{11} &&= 0\\
(e_1, e_3, e_5): &\quad -x_{11} + x_5 - x_6 &&= 0\\
(e_1, e_2, e_{10}): &\quad -x_8 - 1 &&= 0\\
(e_1, e_3, e_9): &\quad -x_3 - x_8 &&= 0\\
(e_1, e_4, e_8): &\quad -x_3 - x_7 &&= 0\\
(e_1, e_5, e_7): &\quad -x_4 - x_7 &&= 0\\
(e_2, e_3, e_6): &\quad -x_{10}x_8 + x_{11} + x_4 &&= 0\\
(e_2, e_4, e_5): &\quad -x_2x_3 + x_6 + x_7 &&= 0
\end{aligned}$$

Groebner basis (11 variables, 10 linear, 0 nonlinear)

$$x_1 + 2x_{11} + 1 = 0$$
$$x_{11} + 2x_2 = 0$$
$$x_3 - 1 = 0$$
$$x_4 - 1 = 0$$
$$-x_{11} + 2x_5 - 2 = 0$$
$$x_{11} + 2x_6 - 2 = 0$$
$$x_7 + 1 = 0$$
$$x_8 + 1 = 0$$
$$-x_{11} + 2x_9 - 2 = 0$$
$$x_{10} + x_{11} + 1 = 0$$

$\mathfrak{m}_{3B}(5,12)$

m3B512 (this line included for string searching purposes)

Solution 1

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4 \\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6 \\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8 \\
[e_1,e_8] &= e_9 & [e_1,e_9] &= e_{10} \\
[e_1,e_{10}] &= e_{11} & [e_2,e_5] &= e_{10} \\
[e_2,e_6] &= 2e_{11} & [e_2,e_{11}] &= e_{12} \\
[e_3,e_4] &= -e_{10} & [e_3,e_5] &= -e_{11} \\
[e_3,e_{10}] &= -e_{12} & [e_4,e_9] &= e_{12} \\
[e_5,e_8] &= -e_{12} & [e_6,e_7] &= e_{12}
\end{aligned}
$$

Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4 \\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6 \\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8 \\
[e_1,e_8] &= e_9 & [e_1,e_9] &= e_{10} \\
[e_1,e_{10}] &= e_{11} & [e_2,e_5] &= e_{10} \\
[e_2,e_6] &= 2e_{11} & [e_2,e_{11}] &= e_{12} \\
[e_3,e_4] &= -e_{10} & [e_3,e_5] &= -e_{11} \\
[e_3,e_{10}] &= \alpha_{3,10}^{12}e_{12} & [e_4,e_9] &= \alpha_{4,9}^{12}e_{12} \\
[e_5,e_8] &= \alpha_{5,8}^{12}e_{12} & [e_6,e_7] &= \alpha_{6,7}^{12}e_{12}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1,e_2,e_{10}):&\quad -\alpha_{3,10}^{12}-1 &= 0 \\
(e_1,e_3,e_9):&\quad -\alpha_{3,10}^{12}-\alpha_{4,9}^{12} &= 0 \\
(e_1,e_4,e_8):&\quad -\alpha_{4,9}^{12}-\alpha_{5,8}^{12} &= 0 \\
(e_1,e_5,e_7):&\quad -\alpha_{5,8}^{12}-\alpha_{6,7}^{12} &= 0 \\
(e_2,e_3,e_5):&\quad -\alpha_{3,10}^{12}-1 &= 0
\end{aligned}
$$

Solution 1:

$$\alpha^{12}_{3,10} = -1$$
$$\alpha^{12}_{4,9} = 1$$
$$\alpha^{12}_{5,8} = -1$$
$$\alpha^{12}_{6,7} = 1$$

How the solution(s) were or were not found:
Change variables

$$\alpha^{12}_{3,10} \to x_1$$
$$\alpha^{12}_{4,9} \to x_2$$
$$\alpha^{12}_{5,8} \to x_3$$
$$\alpha^{12}_{6,7} \to x_4$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_{10}):& \quad -x_1 - 1 &&= 0\\
(e_1, e_3, e_9):& \quad -x_1 - x_2 &&= 0\\
(e_1, e_4, e_8):& \quad -x_2 - x_3 &&= 0\\
(e_1, e_5, e_7):& \quad -x_3 - x_4 &&= 0\\
(e_2, e_3, e_5):& \quad -x_1 - 1 &&= 0
\end{aligned}$$

Groebner basis (4 variables, 4 linear, 0 nonlinear)

$$x_1 + 1 = 0$$
$$x_2 - 1 = 0$$
$$x_3 + 1 = 0$$
$$x_4 - 1 = 0$$

Solution 1:

$$x_1 = -1$$
$$x_2 = 1$$
$$x_3 = -1$$
$$x_4 = 1$$

$\mathfrak{m}_{5B}(5,12)$

m5B512 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4 \\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6 \\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8 \\
[e_1,e_8] &= e_9 & [e_1,e_9] &= e_{10} \\
[e_1,e_{10}] &= e_{11} & [e_2,e_3] &= e_8 \\
[e_2,e_4] &= e_9 & [e_2,e_5] &= \alpha_{2,5}^{10} e_{10} \\
[e_2,e_6] &= \alpha_{2,6}^{11} e_{11} & [e_2,e_{11}] &= e_{12} \\
[e_3,e_4] &= \alpha_{3,4}^{10} e_{10} & [e_3,e_5] &= \alpha_{3,5}^{11} e_{11} \\
[e_3,e_{10}] &= \alpha_{3,10}^{12} e_{12} & [e_4,e_9] &= \alpha_{4,9}^{12} e_{12} \\
[e_5,e_8] &= \alpha_{5,8}^{12} e_{12} & [e_6,e_7] &= \alpha_{6,7}^{12} e_{12}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1,e_2,e_4):&\quad -\alpha_{2,5}^{10} - \alpha_{3,4}^{10} + 1 &&= 0 \\
(e_1,e_2,e_5):&\quad \alpha_{2,5}^{10} - \alpha_{2,6}^{11} - \alpha_{3,5}^{11} &&= 0 \\
(e_1,e_3,e_4):&\quad \alpha_{3,4}^{10} - \alpha_{3,5}^{11} &&= 0 \\
(e_1,e_2,e_{10}):&\quad -\alpha_{3,10}^{12} - 1 &&= 0 \\
(e_1,e_3,e_9):&\quad -\alpha_{3,10}^{12} - \alpha_{4,9}^{12} &&= 0 \\
(e_1,e_4,e_8):&\quad -\alpha_{4,9}^{12} - \alpha_{5,8}^{12} &&= 0 \\
(e_1,e_5,e_7):&\quad -\alpha_{5,8}^{12} - \alpha_{6,7}^{12} &&= 0 \\
(e_2,e_3,e_5):&\quad -\alpha_{2,5}^{10}\alpha_{3,10}^{12} + \alpha_{3,5}^{11} + \alpha_{5,8}^{12} &&= 0
\end{aligned}
$$

Infinite number of solutions.

How the solution(s) were or were not found:

Change variables

$$
\begin{aligned}
\alpha_{2,6}^{11} &\to x_1 \\
\alpha_{4,9}^{12} &\to x_2 \\
\alpha_{6,7}^{12} &\to x_3 \\
\alpha_{3,5}^{11} &\to x_4 \\
\alpha_{5,8}^{12} &\to x_5
\end{aligned}
$$

$$\alpha_{3,10}^{12} \to x_6$$
$$\alpha_{3,4}^{10} \to x_7$$
$$\alpha_{2,5}^{10} \to x_8$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4): &\quad -x_7 - x_8 + 1 &= 0\\
(e_1, e_2, e_5): &\quad -x_1 - x_4 + x_8 &= 0\\
(e_1, e_3, e_4): &\quad -x_4 + x_7 &= 0\\
(e_1, e_2, e_{10}): &\quad -x_6 - 1 &= 0\\
(e_1, e_3, e_9): &\quad -x_2 - x_6 &= 0\\
(e_1, e_4, e_8): &\quad -x_2 - x_5 &= 0\\
(e_1, e_5, e_7): &\quad -x_3 - x_5 &= 0\\
(e_2, e_3, e_5): &\quad x_4 + x_5 - x_6 x_8 &= 0
\end{aligned}$$

Groebner basis (8 variables, 7 linear, 0 nonlinear)

$$x_1 - 2x_8 + 1 = 0$$
$$x_2 - 1 = 0$$
$$x_3 - 1 = 0$$
$$x_4 + x_8 - 1 = 0$$
$$x_5 + 1 = 0$$
$$x_6 + 1 = 0$$
$$x_7 + x_8 - 1 = 0$$

$\mathfrak{m}_{2B}(6, 12)$

m2B612 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4\\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6\\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8\\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10}\\
[e_1, e_{10}] &= e_{11} & [e_2, e_5] &= e_{11}\\
[e_2, e_{11}] &= e_{12} & [e_3, e_4] &= -e_{11}\\
[e_3, e_{10}] &= \alpha_{3,10}^{12} e_{12} & [e_4, e_9] &= \alpha_{4,9}^{12} e_{12}\\
[e_5, e_8] &= \alpha_{5,8}^{12} e_{12} & [e_6, e_7] &= \alpha_{6,7}^{12} e_{12}
\end{aligned}$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_{10}): &\quad -\alpha_{3,10}^{12} - 1 &&= 0\\
(e_1, e_3, e_9): &\quad -\alpha_{3,10}^{12} - \alpha_{4,9}^{12} &&= 0\\
(e_1, e_4, e_8): &\quad -\alpha_{4,9}^{12} - \alpha_{5,8}^{12} &&= 0\\
(e_1, e_5, e_7): &\quad -\alpha_{5,8}^{12} - \alpha_{6,7}^{12} &&= 0\\
(e_2, e_3, e_4): &\quad \text{no solutions}
\end{aligned}
$$

There are no solutions.

$\mathfrak{m}_{4B}(6, 12)$

m4B612 (this line included for string searching purposes)

Solution 1

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4\\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6\\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8\\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10}\\
[e_1, e_{10}] &= e_{11} & [e_2, e_3] &= e_9\\
[e_2, e_4] &= e_{10} & [e_2, e_5] &= 3e_{11}\\
[e_2, e_{11}] &= e_{12} & [e_3, e_4] &= -2e_{11}\\
[e_3, e_{10}] &= -e_{12} & [e_4, e_9] &= e_{12}\\
[e_5, e_8] &= -e_{12} & [e_6, e_7] &= e_{12}
\end{aligned}
$$

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4\\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6\\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8\\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10}\\
[e_1, e_{10}] &= e_{11} & [e_2, e_3] &= e_9\\
[e_2, e_4] &= e_{10} & [e_2, e_5] &= \alpha_{2,5}^{11} e_{11}\\
[e_2, e_{11}] &= e_{12} & [e_3, e_4] &= \alpha_{3,4}^{11} e_{11}\\
[e_3, e_{10}] &= \alpha_{3,10}^{12} e_{12} & [e_4, e_9] &= \alpha_{4,9}^{12} e_{12}\\
[e_5, e_8] &= \alpha_{5,8}^{12} e_{12} & [e_6, e_7] &= \alpha_{6,7}^{12} e_{12}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_4): &\quad -\alpha_{2,5}^{11} - \alpha_{3,4}^{11} + 1 &&= 0\\
(e_1, e_2, e_{10}): &\quad -\alpha_{3,10}^{12} - 1 &&= 0\\
(e_1, e_3, e_9): &\quad -\alpha_{3,10}^{12} - \alpha_{4,9}^{12} &&= 0\\
(e_1, e_4, e_8): &\quad -\alpha_{4,9}^{12} - \alpha_{5,8}^{12} &&= 0\\
(e_1, e_5, e_7): &\quad -\alpha_{5,8}^{12} - \alpha_{6,7}^{12} &&= 0\\
(e_2, e_3, e_4): &\quad -\alpha_{3,10}^{12} + \alpha_{3,4}^{11} + \alpha_{4,9}^{12} &&= 0
\end{aligned}
$$

Solution 1:

$$
\begin{aligned}
\alpha_{4,9}^{12} &= 1\\
\alpha_{6,7}^{12} &= 1\\
\alpha_{5,8}^{12} &= -1\\
\alpha_{2,5}^{11} &= 3\\
\alpha_{3,10}^{12} &= -1\\
\alpha_{3,4}^{11} &= -2
\end{aligned}
$$

How the solution(s) were or were not found:
Change variables

$$
\begin{aligned}
\alpha_{4,9}^{12} &\to x_1\\
\alpha_{6,7}^{12} &\to x_2\\
\alpha_{5,8}^{12} &\to x_3\\
\alpha_{2,5}^{11} &\to x_4\\
\alpha_{3,10}^{12} &\to x_5\\
\alpha_{3,4}^{11} &\to x_6
\end{aligned}
$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_4): &\quad -x_4 - x_6 + 1 &&= 0\\
(e_1, e_2, e_{10}): &\quad -x_5 - 1 &&= 0\\
(e_1, e_3, e_9): &\quad -x_1 - x_5 &&= 0\\
(e_1, e_4, e_8): &\quad -x_1 - x_3 &&= 0\\
(e_1, e_5, e_7): &\quad -x_2 - x_3 &&= 0\\
(e_2, e_3, e_4): &\quad x_1 - x_5 + x_6 &&= 0
\end{aligned}
$$

Groebner basis (6 variables, 6 linear, 0 nonlinear)

$$x_1 - 1 = 0$$
$$x_2 - 1 = 0$$
$$x_3 + 1 = 0$$
$$x_4 - 3 = 0$$
$$x_5 + 1 = 0$$
$$x_6 + 2 = 0$$

Solution 1:

$$x_1 = 1$$
$$x_2 = 1$$
$$x_3 = -1$$
$$x_4 = 3$$
$$x_5 = -1$$
$$x_6 = -2$$

$\mathfrak{m}_{3B}(7, 12)$

m3B712 (this line included for string searching purposes)

Solution 1

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_2, e_3] &= e_{10} \\
[e_2, e_4] &= e_{11} & [e_2, e_{11}] &= e_{12} \\
[e_3, e_{10}] &= -e_{12} & [e_4, e_9] &= e_{12} \\
[e_5, e_8] &= -e_{12} & [e_6, e_7] &= e_{12}
\end{aligned}$$

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_2, e_3] &= e_{10} \\
[e_2, e_4] &= e_{11} & [e_2, e_{11}] &= e_{12} \\
[e_3, e_{10}] &= \alpha_{3,10}^{12} e_{12} & [e_4, e_9] &= \alpha_{4,9}^{12} e_{12} \\
[e_5, e_8] &= \alpha_{5,8}^{12} e_{12} & [e_6, e_7] &= \alpha_{6,7}^{12} e_{12}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_{10}):& \quad -\alpha_{3,10}^{12} - 1 & &= 0 \\
(e_1, e_3, e_9):& \quad -\alpha_{3,10}^{12} - \alpha_{4,9}^{12} & &= 0 \\
(e_1, e_4, e_8):& \quad -\alpha_{4,9}^{12} - \alpha_{5,8}^{12} & &= 0 \\
(e_1, e_5, e_7):& \quad -\alpha_{5,8}^{12} - \alpha_{6,7}^{12} & &= 0
\end{aligned}
$$

Solution 1:

$$
\begin{aligned}
\alpha_{3,10}^{12} &= -1 \\
\alpha_{4,9}^{12} &= 1 \\
\alpha_{5,8}^{12} &= -1 \\
\alpha_{6,7}^{12} &= 1
\end{aligned}
$$

How the solution(s) were or were not found:
Change variables

$$
\begin{aligned}
\alpha_{3,10}^{12} &\to x_1 \\
\alpha_{4,9}^{12} &\to x_2 \\
\alpha_{5,8}^{12} &\to x_3 \\
\alpha_{6,7}^{12} &\to x_4
\end{aligned}
$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_{10}) : &\quad -x_1 - 1 &&= 0 \\
(e_1, e_3, e_9) : &\quad -x_1 - x_2 &&= 0 \\
(e_1, e_4, e_8) : &\quad -x_2 - x_3 &&= 0 \\
(e_1, e_5, e_7) : &\quad -x_3 - x_4 &&= 0
\end{aligned}
$$

Groebner basis (4 variables, 4 linear, 0 nonlinear)

$$x_1 + 1 = 0$$
$$x_2 - 1 = 0$$
$$x_3 + 1 = 0$$
$$x_4 - 1 = 0$$

Solution 1:

$$x_1 = -1$$
$$x_2 = 1$$
$$x_3 = -1$$
$$x_4 = 1$$

$\mathfrak{m}_{2B}(8, 12)$

m2B812 (this line included for string searching purposes)

Solution 1

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_2, e_3] &= e_{11} \\
[e_2, e_{11}] &= e_{12} & [e_3, e_{10}] &= -e_{12} \\
[e_4, e_9] &= e_{12} & [e_5, e_8] &= -e_{12} \\
[e_6, e_7] &= e_{12}
\end{aligned}
$$

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_2, e_3] &= e_{11} \\
[e_2, e_{11}] &= e_{12} & [e_3, e_{10}] &= \alpha_{3,10}^{12} e_{12} \\
[e_4, e_9] &= \alpha_{4,9}^{12} e_{12} & [e_5, e_8] &= \alpha_{5,8}^{12} e_{12} \\
[e_6, e_7] &= \alpha_{6,7}^{12} e_{12}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_{10}):& \quad -\alpha_{3,10}^{12} - 1 &= 0 \\
(e_1, e_3, e_9):& \quad -\alpha_{3,10}^{12} - \alpha_{4,9}^{12} &= 0 \\
(e_1, e_4, e_8):& \quad -\alpha_{4,9}^{12} - \alpha_{5,8}^{12} &= 0 \\
(e_1, e_5, e_7):& \quad -\alpha_{5,8}^{12} - \alpha_{6,7}^{12} &= 0
\end{aligned}
$$

Solution 1:

$$
\begin{aligned}
\alpha_{3,10}^{12} &= -1 \\
\alpha_{4,9}^{12} &= 1 \\
\alpha_{5,8}^{12} &= -1 \\
\alpha_{6,7}^{12} &= 1
\end{aligned}
$$

How the solution(s) were or were not found:
Change variables

$$
\begin{aligned}
\alpha_{3,10}^{12} &\to x_1 \\
\alpha_{4,9}^{12} &\to x_2 \\
\alpha_{5,8}^{12} &\to x_3 \\
\alpha_{6,7}^{12} &\to x_4
\end{aligned}
$$

Jacobi Tests

$$\begin{aligned}
(e_1,e_2,e_{10}):&\quad -x_1-1 &&=0\\
(e_1,e_3,e_9):&\quad -x_1-x_2 &&=0\\
(e_1,e_4,e_8):&\quad -x_2-x_3 &&=0\\
(e_1,e_5,e_7):&\quad -x_3-x_4 &&=0
\end{aligned}$$

Groebner basis (4 variables, 4 linear, 0 nonlinear)

$$x_1+1=0$$
$$x_2-1=0$$
$$x_3+1=0$$
$$x_4-1=0$$

Solution 1:

$$x_1=-1$$
$$x_2=1$$
$$x_3=-1$$
$$x_4=1$$

$\mathfrak{m}_{2B}(2,14)$

m2B214 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned}
[e_1,e_2]&=e_3 & [e_1,e_3]&=e_4\\
[e_1,e_4]&=e_5 & [e_1,e_5]&=e_6\\
[e_1,e_6]&=e_7 & [e_1,e_7]&=e_8\\
[e_1,e_8]&=e_9 & [e_1,e_9]&=e_{10}\\
[e_1,e_{10}]&=e_{11} & [e_1,e_{11}]&=e_{12}\\
[e_1,e_{12}]&=e_{13} & [e_2,e_{11}]&=e_{13}\\
[e_2,e_{13}]&=e_{14} & [e_3,e_{10}]&=-e_{13}\\
[e_3,e_{12}]&=\alpha_{3,12}^{14}e_{14} & [e_4,e_9]&=e_{13}\\
[e_4,e_{11}]&=\alpha_{4,11}^{14}e_{14} & [e_5,e_8]&=-e_{13}\\
[e_5,e_{10}]&=\alpha_{5,10}^{14}e_{14} & [e_6,e_7]&=e_{13}\\
[e_6,e_9]&=\alpha_{6,9}^{14}e_{14} & [e_7,e_8]&=\alpha_{7,8}^{14}e_{14}
\end{aligned}$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1, e_2, e_{12}): &\quad -\alpha^{14}_{3,12} - 1 &= 0\\
(e_1, e_3, e_{11}): &\quad -\alpha^{14}_{3,12} - \alpha^{14}_{4,11} &= 0\\
(e_1, e_4, e_{10}): &\quad -\alpha^{14}_{4,11} - \alpha^{14}_{5,10} &= 0\\
(e_1, e_5, e_9): &\quad -\alpha^{14}_{5,10} - \alpha^{14}_{6,9} &= 0\\
(e_1, e_6, e_8): &\quad -\alpha^{14}_{6,9} - \alpha^{14}_{7,8} &= 0\\
(e_2, e_3, e_{10}): &\quad \text{no solutions}\\
(e_2, e_4, e_9): &\quad \text{no solutions}\\
(e_2, e_5, e_8): &\quad \text{no solutions}\\
(e_2, e_6, e_7): &\quad \text{no solutions}
\end{aligned}$$

There are no solutions.

$\mathfrak{m}_{4B}(2, 14)$

m4B214 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4\\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6\\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8\\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10}\\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12}\\
[e_1, e_{12}] &= e_{13} & [e_2, e_9] &= e_{11}\\
[e_2, e_{10}] &= 4e_{12} & [e_2, e_{11}] &= \alpha^{13}_{2,11} e_{13}\\
[e_2, e_{13}] &= e_{14} & [e_3, e_8] &= -e_{11}\\
[e_3, e_9] &= -3e_{12} & [e_3, e_{10}] &= \alpha^{13}_{3,10} e_{13}\\
[e_3, e_{12}] &= \alpha^{14}_{3,12} e_{14} & [e_4, e_7] &= e_{11}\\
[e_4, e_8] &= 2e_{12} & [e_4, e_9] &= \alpha^{13}_{4,9} e_{13}\\
[e_4, e_{11}] &= \alpha^{14}_{4,11} e_{14} & [e_5, e_6] &= -e_{11}\\
[e_5, e_7] &= -e_{12} & [e_5, e_8] &= \alpha^{13}_{5,8} e_{13}\\
[e_5, e_{10}] &= \alpha^{14}_{5,10} e_{14} & [e_6, e_7] &= \alpha^{13}_{6,7} e_{13}\\
[e_6, e_9] &= \alpha^{14}_{6,9} e_{14} & [e_7, e_8] &= \alpha^{14}_{7,8} e_{14}
\end{aligned}$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_{10}):& \quad -\alpha_{2,11}^{13} - \alpha_{3,10}^{13} + 4 &&= 0\\
(e_1, e_3, e_9):& \quad -\alpha_{3,10}^{13} - \alpha_{4,9}^{13} - 3 &&= 0\\
(e_1, e_4, e_8):& \quad -\alpha_{4,9}^{13} - \alpha_{5,8}^{13} + 2 &&= 0\\
(e_1, e_5, e_7):& \quad -\alpha_{5,8}^{13} - \alpha_{6,7}^{13} - 1 &&= 0\\
(e_2, e_3, e_8):& \quad -\alpha_{2,11}^{13} &&= 0\\
(e_2, e_4, e_7):& \quad \alpha_{2,11}^{13} &&= 0\\
(e_2, e_5, e_6):& \quad -\alpha_{2,11}^{13} &&= 0\\
(e_1, e_2, e_{12}):& \quad -\alpha_{3,12}^{14} - 1 &&= 0\\
(e_1, e_3, e_{11}):& \quad -\alpha_{3,12}^{14} - \alpha_{4,11}^{14} &&= 0\\
(e_1, e_4, e_{10}):& \quad -\alpha_{4,11}^{14} - \alpha_{5,10}^{14} &&= 0\\
(e_1, e_5, e_9):& \quad -\alpha_{5,10}^{14} - \alpha_{6,9}^{14} &&= 0\\
(e_1, e_6, e_8):& \quad -\alpha_{6,9}^{14} - \alpha_{7,8}^{14} &&= 0\\
(e_2, e_3, e_{10}):& \quad \alpha_{3,10}^{13} - 4\alpha_{3,12}^{14} &&= 0\\
(e_2, e_4, e_9):& \quad -\alpha_{4,11}^{14} + \alpha_{4,9}^{13} &&= 0\\
(e_2, e_5, e_8):& \quad \alpha_{5,8}^{13} &&= 0\\
(e_2, e_6, e_7):& \quad \alpha_{6,7}^{13} &&= 0\\
(e_3, e_4, e_8):& \quad 2\alpha_{3,12}^{14} + \alpha_{4,11}^{14} &&= 0\\
(e_3, e_5, e_7):& \quad -\alpha_{3,12}^{14} &&= 0\\
(e_4, e_5, e_6):& \quad -\alpha_{4,11}^{14} &&= 0
\end{aligned}
$$

No solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{6,7}^{13} \to x_1$$

$$\alpha_{4,9}^{13} \to x_2$$

$$\alpha_{3,10}^{13} \to x_3$$

$$\alpha_{4,11}^{14} \to x_4$$

$$\alpha_{3,12}^{14} \to x_5$$

$$\alpha_{5,8}^{13} \to x_6$$

$$\alpha_{5,10}^{14} \to x_7$$

$$\alpha_{2,11}^{13} \to x_8$$

$$\alpha^{14}_{6,9} \to x_9$$

$$\alpha^{14}_{7,8} \to x_{10}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_{10}): \quad & -x_3 - x_8 + 4 && = 0\\
(e_1, e_3, e_9): \quad & -x_2 - x_3 - 3 && = 0\\
(e_1, e_4, e_8): \quad & -x_2 - x_6 + 2 && = 0\\
(e_1, e_5, e_7): \quad & -x_1 - x_6 - 1 && = 0\\
(e_2, e_3, e_8): \quad & -x_8 && = 0\\
(e_2, e_4, e_7): \quad & x_8 && = 0\\
(e_2, e_5, e_6): \quad & -x_8 && = 0\\
(e_1, e_2, e_{12}): \quad & -x_5 - 1 && = 0\\
(e_1, e_3, e_{11}): \quad & -x_4 - x_5 && = 0\\
(e_1, e_4, e_{10}): \quad & -x_4 - x_7 && = 0\\
(e_1, e_5, e_9): \quad & -x_7 - x_9 && = 0\\
(e_1, e_6, e_8): \quad & -x_{10} - x_9 && = 0\\
(e_2, e_3, e_{10}): \quad & x_3 - 4x_5 && = 0\\
(e_2, e_4, e_9): \quad & x_2 - x_4 && = 0\\
(e_2, e_5, e_8): \quad & x_6 && = 0\\
(e_2, e_6, e_7): \quad & x_1 && = 0\\
(e_3, e_4, e_8): \quad & x_4 + 2x_5 && = 0\\
(e_3, e_5, e_7): \quad & -x_5 && = 0\\
(e_4, e_5, e_6): \quad & -x_4 && = 0
\end{aligned}$$

Groebner basis (10 variables, 1 linear, 0 nonlinear)

$$1 = 0$$

$\mathfrak{m}_{10B}(2, 14)$

m10B214 (this line included for string searching purposes)

Original brackets:

$$
\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_1, e_9] = e_{10} \\
[e_1, e_{10}] = e_{11} & [e_1, e_{11}] = e_{12} \\
[e_1, e_{12}] = e_{13} & [e_2, e_3] = e_5 \\
[e_2, e_4] = e_6 & [e_2, e_5] = \alpha_{2,5}^{7} e_7 \\
[e_2, e_6] = \alpha_{2,6}^{8} e_8 & [e_2, e_7] = \alpha_{2,7}^{9} e_9 \\
[e_2, e_8] = \alpha_{2,8}^{10} e_{10} & [e_2, e_9] = \alpha_{2,9}^{11} e_{11} \\
[e_2, e_{10}] = \alpha_{2,10}^{12} e_{12} & [e_2, e_{11}] = \alpha_{2,11}^{13} e_{13} \\
[e_2, e_{13}] = e_{14} & [e_3, e_4] = \alpha_{3,4}^{7} e_7 \\
[e_3, e_5] = \alpha_{3,5}^{8} e_8 & [e_3, e_6] = \alpha_{3,6}^{9} e_9 \\
[e_3, e_7] = \alpha_{3,7}^{10} e_{10} & [e_3, e_8] = \alpha_{3,8}^{11} e_{11} \\
[e_3, e_9] = \alpha_{3,9}^{12} e_{12} & [e_3, e_{10}] = \alpha_{3,10}^{13} e_{13} \\
[e_3, e_{12}] = \alpha_{3,12}^{14} e_{14} & [e_4, e_5] = \alpha_{4,5}^{9} e_9 \\
[e_4, e_6] = \alpha_{4,6}^{10} e_{10} & [e_4, e_7] = \alpha_{4,7}^{11} e_{11} \\
[e_4, e_8] = \alpha_{4,8}^{12} e_{12} & [e_4, e_9] = \alpha_{4,9}^{13} e_{13} \\
[e_4, e_{11}] = \alpha_{4,11}^{14} e_{14} & [e_5, e_6] = \alpha_{5,6}^{11} e_{11} \\
[e_5, e_7] = \alpha_{5,7}^{12} e_{12} & [e_5, e_8] = \alpha_{5,8}^{13} e_{13} \\
[e_5, e_{10}] = \alpha_{5,10}^{14} e_{14} & [e_6, e_7] = \alpha_{6,7}^{13} e_{13} \\
[e_6, e_9] = \alpha_{6,9}^{14} e_{14} & [e_7, e_8] = \alpha_{7,8}^{14} e_{14}
\end{array}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1,e_2,e_4): &\quad -\alpha^7_{2,5}-\alpha^7_{3,4}+1 &&=0\\
(e_1,e_2,e_5): &\quad \alpha^7_{2,5}-\alpha^8_{2,6}-\alpha^8_{3,5} &&=0\\
(e_1,e_3,e_4): &\quad \alpha^7_{3,4}-\alpha^8_{3,5} &&=0\\
(e_1,e_2,e_6): &\quad \alpha^8_{2,6}-\alpha^9_{2,7}-\alpha^9_{3,6} &&=0\\
(e_1,e_3,e_5): &\quad \alpha^8_{3,5}-\alpha^9_{3,6}-\alpha^9_{4,5} &&=0\\
(e_2,e_3,e_4): &\quad \alpha^9_{2,7}\alpha^7_{3,4}-\alpha^9_{3,6}+\alpha^9_{4,5} &&=0\\
(e_1,e_2,e_7): &\quad \alpha^9_{2,7}-\alpha^{10}_{2,8}-\alpha^{10}_{3,7} &&=0\\
(e_1,e_3,e_6): &\quad \alpha^9_{3,6}-\alpha^{10}_{3,7}-\alpha^{10}_{4,6} &&=0\\
(e_1,e_4,e_5): &\quad \alpha^9_{4,5}-\alpha^{10}_{4,6} &&=0\\
(e_2,e_3,e_5): &\quad -\alpha^7_{2,5}\alpha^{10}_{3,7}+\alpha^{10}_{2,8}\alpha^8_{3,5} &&=0\\
(e_1,e_2,e_8): &\quad \alpha^{10}_{2,8}-\alpha^{11}_{2,9}-\alpha^{11}_{3,8} &&=0\\
(e_1,e_3,e_7): &\quad \alpha^{10}_{3,7}-\alpha^{11}_{3,8}-\alpha^{11}_{4,7} &&=0\\
(e_1,e_4,e_6): &\quad \alpha^{10}_{4,6}-\alpha^{11}_{4,7}-\alpha^{11}_{5,6} &&=0\\
(e_2,e_3,e_6): &\quad -\alpha^8_{2,6}\alpha^{11}_{3,8}+\alpha^{11}_{2,9}\alpha^9_{3,6}-\alpha^{11}_{5,6} &&=0\\
(e_2,e_4,e_5): &\quad -\alpha^7_{2,5}\alpha^{11}_{4,7}+\alpha^{11}_{2,9}\alpha^9_{4,5}+\alpha^{11}_{5,6} &&=0\\
(e_1,e_2,e_9): &\quad -\alpha^{12}_{2,10}+\alpha^{11}_{2,9}-\alpha^{12}_{3,9} &&=0\\
(e_1,e_3,e_8): &\quad \alpha^{11}_{3,8}-\alpha^{12}_{3,9}-\alpha^{12}_{4,8} &&=0\\
(e_1,e_4,e_7): &\quad \alpha^{11}_{4,7}-\alpha^{12}_{4,8}-\alpha^{12}_{5,7} &&=0\\
(e_1,e_5,e_6): &\quad \alpha^{11}_{5,6}-\alpha^{12}_{5,7} &&=0\\
(e_2,e_3,e_7): &\quad \alpha^{12}_{2,10}\alpha^{10}_{3,7}-\alpha^9_{2,7}\alpha^{12}_{3,9}-\alpha^{12}_{5,7} &&=0\\
(e_2,e_4,e_6): &\quad \alpha^{12}_{2,10}\alpha^{10}_{4,6}-\alpha^8_{2,6}\alpha^{12}_{4,8} &&=0\\
(e_3,e_4,e_5): &\quad \alpha^7_{3,4}\alpha^{12}_{5,7}-\alpha^8_{3,5}\alpha^{12}_{4,8}+\alpha^{12}_{3,9}\alpha^9_{4,5} &&=0\\
(e_1,e_2,e_{10}): &\quad \alpha^{12}_{2,10}-\alpha^{13}_{2,11}-\alpha^{13}_{3,10} &&=0\\
(e_1,e_3,e_9): &\quad -\alpha^{13}_{3,10}+\alpha^{12}_{3,9}-\alpha^{13}_{4,9} &&=0\\
(e_1,e_4,e_8): &\quad \alpha^{12}_{4,8}-\alpha^{13}_{4,9}-\alpha^{13}_{5,8} &&=0\\
(e_1,e_5,e_7): &\quad \alpha^{12}_{5,7}-\alpha^{13}_{5,8}-\alpha^{13}_{6,7} &&=0\\
(e_2,e_3,e_8): &\quad \alpha^{13}_{2,11}\alpha^{11}_{3,8}-\alpha^{10}_{2,8}\alpha^{13}_{3,10}-\alpha^{13}_{5,8} &&=0\\
(e_2,e_4,e_7): &\quad \alpha^{13}_{2,11}\alpha^{11}_{4,7}-\alpha^9_{2,7}\alpha^{13}_{4,9}-\alpha^{13}_{6,7} &&=0\\
(e_2,e_5,e_6): &\quad \alpha^{13}_{2,11}\alpha^{11}_{5,6}+\alpha^7_{2,5}\alpha^{13}_{6,7}-\alpha^8_{2,6}\alpha^{13}_{5,8} &&=0\\
(e_3,e_4,e_6): &\quad \alpha^{13}_{3,10}\alpha^{10}_{4,6}+\alpha^7_{3,4}\alpha^{13}_{6,7}-\alpha^9_{3,6}\alpha^{13}_{4,9} &&=0\\
(e_1,e_2,e_{12}): &\quad -\alpha^{14}_{3,12}-1 &&=0\\
(e_1,e_3,e_{11}): &\quad -\alpha^{14}_{3,12}-\alpha^{14}_{4,11} &&=0\\
(e_1,e_4,e_{10}): &\quad -\alpha^{14}_{4,11}-\alpha^{14}_{5,10} &&=0\\
(e_1,e_5,e_9): &\quad -\alpha^{14}_{5,10}-\alpha^{14}_{6,9} &&=0\\
(e_1,e_6,e_8): &\quad -\alpha^{14}_{6,9}-\alpha^{14}_{7,8} &&=0\\
(e_2,e_3,e_{10}): &\quad -\alpha^{12}_{2,10}\alpha^{14}_{3,12}+\alpha^{13}_{3,10}-\alpha^{14}_{5,10} &&=0\\
(e_2,e_4,e_9): &\quad -\alpha^{11}_{2,9}\alpha^{14}_{4,11}+\alpha^{13}_{4,9}-\alpha^{14}_{6,9} &&=0\\
(e_2,e_5,e_8): &\quad -\alpha^7_{2,5}\alpha^{14}_{7,8}-\alpha^{10}_{2,8}\alpha^{14}_{5,10}+\alpha^{13}_{5,8} &&=0\\
(e_2,e_6,e_7): &\quad \alpha^8_{2,6}\alpha^{14}_{7,8}-\alpha^9_{2,7}\alpha^{14}_{6,9}+\alpha^{13}_{6,7} &&=0\\
(e_3,e_4,e_8): &\quad \alpha^{14}_{3,12}\alpha^{12}_{4,8}-\alpha^7_{3,4}\alpha^{14}_{7,8}-\alpha^{11}_{3,8}\alpha^{14}_{4,11} &&=0
\end{aligned}
$$

No solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha^{13}_{6,7} \to x_1$$

$$\alpha^{9}_{3,6} \to x_2$$

$$\alpha^{7}_{2,5} \to x_3$$

$$\alpha^{13}_{5,8} \to x_4$$

$$\alpha^{11}_{4,7} \to x_5$$

$$\alpha^{10}_{4,6} \to x_6$$

$$\alpha^{12}_{3,9} \to x_7$$

$$\alpha^{11}_{3,8} \to x_8$$

$$\alpha^{9}_{2,7} \to x_9$$

$$\alpha^{12}_{2,10} \to x_{10}$$

$$\alpha^{12}_{5,7} \to x_{11}$$

$$\alpha^{11}_{2,9} \to x_{12}$$

$$\alpha^{14}_{3,12} \to x_{13}$$

$$\alpha^{14}_{7,8} \to x_{14}$$

$$\alpha^{10}_{2,8} \to x_{15}$$

$$\alpha^{10}_{3,7} \to x_{16}$$

$$\alpha^{13}_{3,10} \to x_{17}$$

$$\alpha^{13}_{4,9} \to x_{18}$$

$$\alpha^{11}_{5,6} \to x_{19}$$

$$\alpha^{8}_{2,6} \to x_{20}$$

$$\alpha^{9}_{4,5} \to x_{21}$$

$$\alpha^{7}_{3,4} \to x_{22}$$

$$\alpha^{12}_{4,8} \to x_{23}$$

$$\alpha^{14}_{4,11} \to x_{24}$$

$$\alpha^{14}_{5,10} \to x_{25}$$

$$\alpha^{13}_{2,11} \to x_{26}$$

$$\alpha_{6,9}^{14} \to x_{27}$$

$$\alpha_{3,5}^{8} \to x_{28}$$

Jacobi Tests

$$
\begin{aligned}
(e_1,e_2,e_4):&\quad -x_{22}-x_3+1 &&=0\\
(e_1,e_2,e_5):&\quad -x_{20}-x_{28}+x_3 &&=0\\
(e_1,e_3,e_4):&\quad x_{22}-x_{28} &&=0\\
(e_1,e_2,e_6):&\quad -x_2+x_{20}-x_9 &&=0\\
(e_1,e_3,e_5):&\quad -x_2-x_{21}+x_{28} &&=0\\
(e_2,e_3,e_4):&\quad -x_2+x_{21}+x_{22}x_9 &&=0\\
(e_1,e_2,e_7):&\quad -x_{15}-x_{16}+x_9 &&=0\\
(e_1,e_3,e_6):&\quad -x_{16}+x_2-x_6 &&=0\\
(e_1,e_4,e_5):&\quad x_{21}-x_6 &&=0\\
(e_2,e_3,e_5):&\quad x_{15}x_{28}-x_{16}x_3 &&=0\\
(e_1,e_2,e_8):&\quad -x_{12}+x_{15}-x_8 &&=0\\
(e_1,e_3,e_7):&\quad x_{16}-x_5-x_8 &&=0\\
(e_1,e_4,e_6):&\quad -x_{19}-x_5+x_6 &&=0\\
(e_2,e_3,e_6):&\quad x_{12}x_2-x_{19}-x_{20}x_8 &&=0\\
(e_2,e_4,e_5):&\quad x_{12}x_{21}+x_{19}-x_3x_5 &&=0\\
(e_1,e_2,e_9):&\quad -x_{10}+x_{12}-x_7 &&=0\\
(e_1,e_3,e_8):&\quad -x_{23}-x_7+x_8 &&=0\\
(e_1,e_4,e_7):&\quad -x_{11}-x_{23}+x_5 &&=0\\
(e_1,e_5,e_6):&\quad -x_{11}+x_{19} &&=0\\
(e_2,e_3,e_7):&\quad x_{10}x_{16}-x_{11}-x_7x_9 &&=0\\
(e_2,e_4,e_6):&\quad x_{10}x_6-x_{20}x_{23} &&=0\\
(e_3,e_4,e_5):&\quad x_{11}x_{22}+x_{21}x_7-x_{23}x_{28} &&=0\\
(e_1,e_2,e_{10}):&\quad x_{10}-x_{17}-x_{26} &&=0\\
(e_1,e_3,e_9):&\quad -x_{17}-x_{18}+x_7 &&=0\\
(e_1,e_4,e_8):&\quad -x_{18}+x_{23}-x_4 &&=0\\
(e_1,e_5,e_7):&\quad -x_1+x_{11}-x_4 &&=0\\
(e_2,e_3,e_8):&\quad -x_{15}x_{17}+x_{26}x_8-x_4 &&=0\\
(e_2,e_4,e_7):&\quad -x_1-x_{18}x_9+x_{26}x_5 &&=0\\
(e_2,e_5,e_6):&\quad x_1x_3+x_{19}x_{26}-x_{20}x_4 &&=0\\
(e_3,e_4,e_6):&\quad x_1x_{22}+x_{17}x_6-x_{18}x_2 &&=0\\
(e_1,e_2,e_{12}):&\quad -x_{13}-1 &&=0\\
(e_1,e_3,e_{11}):&\quad -x_{13}-x_{24} &&=0\\
(e_1,e_4,e_{10}):&\quad -x_{24}-x_{25} &&=0\\
(e_1,e_5,e_9):&\quad -x_{25}-x_{27} &&=0\\
(e_1,e_6,e_8):&\quad -x_{14}-x_{27} &&=0\\
(e_2,e_3,e_{10}):&\quad -x_{10}x_{13}+x_{17}-x_{25} &&=0\\
(e_2,e_4,e_9):&\quad -x_{12}x_{24}+x_{18}-x_{27} &&=0\\
(e_2,e_5,e_8):&\quad -x_{14}x_3-x_{15}x_{25}+x_4 &&=0\\
(e_2,e_6,e_7):&\quad x_1+x_{14}x_{20}-x_{27}x_9 &&=0\\
(e_3,e_4,e_8):&\quad x_{13}x_{23}-x_{14}x_{22}-x_{24}x_8 &&=0\\
(e_3,e_5,e_7):&\quad x_{11}x_{13}+x_{14}x_{28}-x_{16}x_{25} &&=0\\
(e_4,e_5,e_6):&\quad x_{19}x_{24}+x_{21}x_{27}-x_{25}x_6 &&=0
\end{aligned}
$$

Groebner basis (28 variables, 1 linear, 0 nonlinear)

$$1 = 0$$

$\mathfrak{m}_{3B}(3, 14)$

m3B314 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_2, e_9] &= e_{12} \\
[e_2, e_{10}] &= 4e_{13} & [e_2, e_{13}] &= e_{14} \\
[e_3, e_8] &= -e_{12} & [e_3, e_9] &= -3e_{13} \\
[e_3, e_{12}] &= \alpha_{3,12}^{14} e_{14} & [e_4, e_7] &= e_{12} \\
[e_4, e_8] &= 2e_{13} & [e_4, e_{11}] &= \alpha_{4,11}^{14} e_{14} \\
[e_5, e_6] &= -e_{12} & [e_5, e_7] &= -e_{13} \\
[e_5, e_{10}] &= \alpha_{5,10}^{14} e_{14} & [e_6, e_9] &= \alpha_{6,9}^{14} e_{14} \\
[e_7, e_8] &= \alpha_{7,8}^{14} e_{14}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_{12}) &: \quad -\alpha_{3,12}^{14} - 1 &= 0 \\
(e_1, e_3, e_{11}) &: \quad -\alpha_{3,12}^{14} - \alpha_{4,11}^{14} &= 0 \\
(e_1, e_4, e_{10}) &: \quad -\alpha_{4,11}^{14} - \alpha_{5,10}^{14} &= 0 \\
(e_1, e_5, e_9) &: \quad -\alpha_{5,10}^{14} - \alpha_{6,9}^{14} &= 0 \\
(e_1, e_6, e_8) &: \quad -\alpha_{6,9}^{14} - \alpha_{7,8}^{14} &= 0 \\
(e_2, e_3, e_9) &: \quad -\alpha_{3,12}^{14} - 3 &= 0 \\
(e_2, e_4, e_8) &: \quad \text{no solutions} \\
(e_2, e_5, e_7) &: \quad \text{no solutions} \\
(e_3, e_4, e_7) &: \quad \alpha_{3,12}^{14} &= 0 \\
(e_3, e_5, e_6) &: \quad -\alpha_{3,12}^{14} &= 0
\end{aligned}
$$

There are no solutions.

$\mathfrak{m}_{5B}(3,14)$

m5B314 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4 \\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6 \\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8 \\
[e_1,e_8] &= e_9 & [e_1,e_9] &= e_{10} \\
[e_1,e_{10}] &= e_{11} & [e_1,e_{11}] &= e_{12} \\
[e_1,e_{12}] &= e_{13} & [e_2,e_7] &= e_{10} \\
[e_2,e_8] &= 3e_{11} & [e_2,e_9] &= \alpha_{2,9}^{12}e_{12} \\
[e_2,e_{10}] &= \alpha_{2,10}^{13}e_{13} & [e_2,e_{13}] &= e_{14} \\
[e_3,e_6] &= -e_{10} & [e_3,e_7] &= -2e_{11} \\
[e_3,e_8] &= \alpha_{3,8}^{12}e_{12} & [e_3,e_9] &= \alpha_{3,9}^{13}e_{13} \\
[e_3,e_{12}] &= \alpha_{3,12}^{14}e_{14} & [e_4,e_5] &= e_{10} \\
[e_4,e_6] &= e_{11} & [e_4,e_7] &= \alpha_{4,7}^{12}e_{12} \\
[e_4,e_8] &= \alpha_{4,8}^{13}e_{13} & [e_4,e_{11}] &= \alpha_{4,11}^{14}e_{14} \\
[e_5,e_6] &= \alpha_{5,6}^{12}e_{12} & [e_5,e_7] &= \alpha_{5,7}^{13}e_{13} \\
[e_5,e_{10}] &= \alpha_{5,10}^{14}e_{14} & [e_6,e_9] &= \alpha_{6,9}^{14}e_{14} \\
[e_7,e_8] &= \alpha_{7,8}^{14}e_{14} & &
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_8): &\quad -\alpha^{12}_{2,9} - \alpha^{12}_{3,8} + 3 &= 0\\
(e_1, e_3, e_7): &\quad -\alpha^{12}_{3,8} - \alpha^{12}_{4,7} - 2 &= 0\\
(e_1, e_4, e_6): &\quad -\alpha^{12}_{4,7} - \alpha^{12}_{5,6} + 1 &= 0\\
(e_1, e_2, e_9): &\quad -\alpha^{13}_{2,10} + \alpha^{12}_{2,9} - \alpha^{13}_{3,9} &= 0\\
(e_1, e_3, e_8): &\quad \alpha^{12}_{3,8} - \alpha^{13}_{3,9} - \alpha^{13}_{4,8} &= 0\\
(e_1, e_4, e_7): &\quad \alpha^{12}_{4,7} - \alpha^{13}_{4,8} - \alpha^{13}_{5,7} &= 0\\
(e_1, e_5, e_6): &\quad \alpha^{12}_{5,6} - \alpha^{13}_{5,7} &= 0\\
(e_2, e_3, e_6): &\quad -\alpha^{13}_{2,10} &= 0\\
(e_2, e_4, e_5): &\quad \alpha^{13}_{2,10} &= 0\\
(e_1, e_2, e_{12}): &\quad -\alpha^{14}_{3,12} - 1 &= 0\\
(e_1, e_3, e_{11}): &\quad -\alpha^{14}_{3,12} - \alpha^{14}_{4,11} &= 0\\
(e_1, e_4, e_{10}): &\quad -\alpha^{14}_{4,11} - \alpha^{14}_{5,10} &= 0\\
(e_1, e_5, e_9): &\quad -\alpha^{14}_{5,10} - \alpha^{14}_{6,9} &= 0\\
(e_1, e_6, e_8): &\quad -\alpha^{14}_{6,9} - \alpha^{14}_{7,8} &= 0\\
(e_2, e_3, e_9): &\quad -\alpha^{12}_{2,9}\alpha^{14}_{3,12} + \alpha^{13}_{3,9} &= 0\\
(e_2, e_4, e_8): &\quad -3\alpha^{14}_{4,11} + \alpha^{13}_{4,8} &= 0\\
(e_2, e_5, e_7): &\quad -\alpha^{14}_{5,10} + \alpha^{13}_{5,7} &= 0\\
(e_3, e_4, e_7): &\quad \alpha^{14}_{3,12}\alpha^{12}_{4,7} + 2\alpha^{14}_{4,11} &= 0\\
(e_3, e_5, e_6): &\quad \alpha^{14}_{3,12}\alpha^{12}_{5,6} + \alpha^{14}_{5,10} &= 0
\end{aligned}
$$

No solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha^{13}_{3,9} \to x_1$$

$$\alpha^{13}_{2,10} \to x_2$$

$$\alpha^{13}_{4,8} \to x_3$$

$$\alpha^{14}_{7,8} \to x_4$$

$$\alpha^{12}_{4,7} \to x_5$$

$$\alpha^{12}_{3,8} \to x_6$$

$$\alpha^{14}_{4,11} \to x_7$$

$$\alpha^{14}_{6,9} \to x_8$$

$$\alpha_{3,12}^{14} \to x_9$$
$$\alpha_{5,10}^{14} \to x_{10}$$
$$\alpha_{5,6}^{12} \to x_{11}$$
$$\alpha_{2,9}^{12} \to x_{12}$$
$$\alpha_{5,7}^{13} \to x_{13}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_8):&\quad -x_{12} - x_6 + 3 &= 0\\
(e_1, e_3, e_7):&\quad -x_5 - x_6 - 2 &= 0\\
(e_1, e_4, e_6):&\quad -x_{11} - x_5 + 1 &= 0\\
(e_1, e_2, e_9):&\quad -x_1 + x_{12} - x_2 &= 0\\
(e_1, e_3, e_8):&\quad -x_1 - x_3 + x_6 &= 0\\
(e_1, e_4, e_7):&\quad -x_{13} - x_3 + x_5 &= 0\\
(e_1, e_5, e_6):&\quad x_{11} - x_{13} &= 0\\
(e_2, e_3, e_6):&\quad -x_2 &= 0\\
(e_2, e_4, e_5):&\quad x_2 &= 0\\
(e_1, e_2, e_{12}):&\quad -x_9 - 1 &= 0\\
(e_1, e_3, e_{11}):&\quad -x_7 - x_9 &= 0\\
(e_1, e_4, e_{10}):&\quad -x_{10} - x_7 &= 0\\
(e_1, e_5, e_9):&\quad -x_{10} - x_8 &= 0\\
(e_1, e_6, e_8):&\quad -x_4 - x_8 &= 0\\
(e_2, e_3, e_9):&\quad x_1 - x_{12}x_9 &= 0\\
(e_2, e_4, e_8):&\quad x_3 - 3x_7 &= 0\\
(e_2, e_5, e_7):&\quad -x_{10} + x_{13} &= 0\\
(e_3, e_4, e_7):&\quad x_5x_9 + 2x_7 &= 0\\
(e_3, e_5, e_6):&\quad x_{10} + x_{11}x_9 &= 0
\end{aligned}$$

Groebner basis (13 variables, 1 linear, 0 nonlinear)

$$1 = 0$$

$\mathfrak{m}_{7B}(3, 14)$

m7B314 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
&[e_1, e_2] = e_3 && [e_1, e_3] = e_4 \\
&[e_1, e_4] = e_5 && [e_1, e_5] = e_6 \\
&[e_1, e_6] = e_7 && [e_1, e_7] = e_8 \\
&[e_1, e_8] = e_9 && [e_1, e_9] = e_{10} \\
&[e_1, e_{10}] = e_{11} && [e_1, e_{11}] = e_{12} \\
&[e_1, e_{12}] = e_{13} && [e_2, e_5] = e_8 \\
&[e_2, e_6] = 2e_9 && [e_2, e_7] = \alpha_{2,7}^{10} e_{10} \\
&[e_2, e_8] = \alpha_{2,8}^{11} e_{11} && [e_2, e_9] = \alpha_{2,9}^{12} e_{12} \\
&[e_2, e_{10}] = \alpha_{2,10}^{13} e_{13} && [e_2, e_{13}] = e_{14} \\
&[e_3, e_4] = -e_8 && [e_3, e_5] = -e_9 \\
&[e_3, e_6] = \alpha_{3,6}^{10} e_{10} && [e_3, e_7] = \alpha_{3,7}^{11} e_{11} \\
&[e_3, e_8] = \alpha_{3,8}^{12} e_{12} && [e_3, e_9] = \alpha_{3,9}^{13} e_{13} \\
&[e_3, e_{12}] = \alpha_{3,12}^{14} e_{14} && [e_4, e_5] = \alpha_{4,5}^{10} e_{10} \\
&[e_4, e_6] = \alpha_{4,6}^{11} e_{11} && [e_4, e_7] = \alpha_{4,7}^{12} e_{12} \\
&[e_4, e_8] = \alpha_{4,8}^{13} e_{13} && [e_4, e_{11}] = \alpha_{4,11}^{14} e_{14} \\
&[e_5, e_6] = \alpha_{5,6}^{12} e_{12} && [e_5, e_7] = \alpha_{5,7}^{13} e_{13} \\
&[e_5, e_{10}] = \alpha_{5,10}^{14} e_{14} && [e_6, e_9] = \alpha_{6,9}^{14} e_{14} \\
&[e_7, e_8] = \alpha_{7,8}^{14} e_{14}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_6): &\quad -\alpha_{2,7}^{10} - \alpha_{3,6}^{10} + 2 &= 0\\
(e_1, e_3, e_5): &\quad -\alpha_{3,6}^{10} - \alpha_{4,5}^{10} - 1 &= 0\\
(e_1, e_2, e_7): &\quad \alpha_{2,7}^{10} - \alpha_{2,8}^{11} - \alpha_{3,7}^{11} &= 0\\
(e_1, e_3, e_6): &\quad \alpha_{3,6}^{10} - \alpha_{3,7}^{11} - \alpha_{4,6}^{11} &= 0\\
(e_1, e_4, e_5): &\quad \alpha_{4,5}^{10} - \alpha_{4,6}^{11} &= 0\\
(e_2, e_3, e_4): &\quad -\alpha_{2,8}^{11} &= 0\\
(e_1, e_2, e_8): &\quad \alpha_{2,8}^{11} - \alpha_{2,9}^{12} - \alpha_{3,8}^{12} &= 0\\
(e_1, e_3, e_7): &\quad \alpha_{3,7}^{11} - \alpha_{3,8}^{12} - \alpha_{4,7}^{12} &= 0\\
(e_1, e_4, e_6): &\quad \alpha_{4,6}^{11} - \alpha_{4,7}^{12} - \alpha_{5,6}^{12} &= 0\\
(e_2, e_3, e_5): &\quad -\alpha_{2,9}^{12} - \alpha_{3,8}^{12} &= 0\\
(e_1, e_2, e_9): &\quad -\alpha_{2,10}^{13} + \alpha_{2,9}^{12} - \alpha_{3,9}^{13} &= 0\\
(e_1, e_3, e_8): &\quad \alpha_{3,8}^{12} - \alpha_{3,9}^{13} - \alpha_{4,8}^{13} &= 0\\
(e_1, e_4, e_7): &\quad \alpha_{4,7}^{12} - \alpha_{4,8}^{13} - \alpha_{5,7}^{13} &= 0\\
(e_1, e_5, e_6): &\quad \alpha_{5,6}^{12} - \alpha_{5,7}^{13} &= 0\\
(e_2, e_3, e_6): &\quad \alpha_{2,10}^{13}\alpha_{3,6}^{10} - 2\alpha_{3,9}^{13} &= 0\\
(e_2, e_4, e_5): &\quad \alpha_{2,10}^{13}\alpha_{4,5}^{10} - \alpha_{4,8}^{13} &= 0\\
(e_1, e_2, e_{12}): &\quad -\alpha_{3,12}^{14} - 1 &= 0\\
(e_1, e_3, e_{11}): &\quad -\alpha_{3,12}^{14} - \alpha_{4,11}^{14} &= 0\\
(e_1, e_4, e_{10}): &\quad -\alpha_{4,11}^{14} - \alpha_{5,10}^{14} &= 0\\
(e_1, e_5, e_9): &\quad -\alpha_{5,10}^{14} - \alpha_{6,9}^{14} &= 0\\
(e_1, e_6, e_8): &\quad -\alpha_{6,9}^{14} - \alpha_{7,8}^{14} &= 0\\
(e_2, e_3, e_9): &\quad -\alpha_{2,9}^{12}\alpha_{3,12}^{14} + \alpha_{3,9}^{13} &= 0\\
(e_2, e_4, e_8): &\quad -\alpha_{2,8}^{11}\alpha_{4,11}^{14} + \alpha_{4,8}^{13} &= 0\\
(e_2, e_5, e_7): &\quad -\alpha_{2,7}^{10}\alpha_{5,10}^{14} + \alpha_{5,7}^{13} + \alpha_{7,8}^{14} &= 0\\
(e_3, e_4, e_7): &\quad \alpha_{3,12}^{14}\alpha_{4,7}^{12} - \alpha_{3,7}^{11}\alpha_{4,11}^{14} - \alpha_{7,8}^{14} &= 0\\
(e_3, e_5, e_6): &\quad \alpha_{3,12}^{14}\alpha_{5,6}^{12} - \alpha_{3,6}^{10}\alpha_{5,10}^{14} - \alpha_{6,9}^{14} &= 0
\end{aligned}
$$

No solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{3,9}^{13} \to x_1$$

$$\alpha_{5,7}^{13} \to x_2$$

$$\alpha^{13}_{2,10} \to x_3$$

$$\alpha^{13}_{4,8} \to x_4$$

$$\alpha^{14}_{7,8} \to x_5$$

$$\alpha^{11}_{3,7} \to x_6$$

$$\alpha^{11}_{2,8} \to x_7$$

$$\alpha^{12}_{4,7} \to x_8$$

$$\alpha^{12}_{3,8} \to x_9$$

$$\alpha^{14}_{4,11} \to x_{10}$$

$$\alpha^{10}_{2,7} \to x_{11}$$

$$\alpha^{14}_{6,9} \to x_{12}$$

$$\alpha^{14}_{3,12} \to x_{13}$$

$$\alpha^{11}_{4,6} \to x_{14}$$

$$\alpha^{14}_{5,10} \to x_{15}$$

$$\alpha^{10}_{4,5} \to x_{16}$$

$$\alpha^{12}_{5,6} \to x_{17}$$

$$\alpha^{12}_{2,9} \to x_{18}$$

$$\alpha^{10}_{3,6} \to x_{19}$$

Jacobi Tests

$$
\begin{aligned}
(e_1,e_2,e_6):&\quad -x_{11}-x_{19}+2 &&=0\\
(e_1,e_3,e_5):&\quad -x_{16}-x_{19}-1 &&=0\\
(e_1,e_2,e_7):&\quad x_{11}-x_6-x_7 &&=0\\
(e_1,e_3,e_6):&\quad -x_{14}+x_{19}-x_6 &&=0\\
(e_1,e_4,e_5):&\quad -x_{14}+x_{16} &&=0\\
(e_2,e_3,e_4):&\quad -x_7 &&=0\\
(e_1,e_2,e_8):&\quad -x_{18}+x_7-x_9 &&=0\\
(e_1,e_3,e_7):&\quad x_6-x_8-x_9 &&=0\\
(e_1,e_4,e_6):&\quad x_{14}-x_{17}-x_8 &&=0\\
(e_2,e_3,e_5):&\quad -x_{18}-x_9 &&=0\\
(e_1,e_2,e_9):&\quad -x_1+x_{18}-x_3 &&=0\\
(e_1,e_3,e_8):&\quad -x_1-x_4+x_9 &&=0\\
(e_1,e_4,e_7):&\quad -x_2-x_4+x_8 &&=0\\
(e_1,e_5,e_6):&\quad x_{17}-x_2 &&=0\\
(e_2,e_3,e_6):&\quad -2x_1+x_{19}x_3 &&=0\\
(e_2,e_4,e_5):&\quad x_{16}x_3-x_4 &&=0\\
(e_1,e_2,e_{12}):&\quad -x_{13}-1 &&=0\\
(e_1,e_3,e_{11}):&\quad -x_{10}-x_{13} &&=0\\
(e_1,e_4,e_{10}):&\quad -x_{10}-x_{15} &&=0\\
(e_1,e_5,e_9):&\quad -x_{12}-x_{15} &&=0\\
(e_1,e_6,e_8):&\quad -x_{12}-x_5 &&=0\\
(e_2,e_3,e_9):&\quad x_1-x_{13}x_{18} &&=0\\
(e_2,e_4,e_8):&\quad -x_{10}x_7+x_4 &&=0\\
(e_2,e_5,e_7):&\quad -x_{11}x_{15}+x_2+x_5 &&=0\\
(e_3,e_4,e_7):&\quad -x_{10}x_6+x_{13}x_8-x_5 &&=0\\
(e_3,e_5,e_6):&\quad -x_{12}+x_{13}x_{17}-x_{15}x_{19} &&=0
\end{aligned}
$$

Groebner basis (19 variables, 1 linear, 0 nonlinear)

$$1=0$$

$\mathfrak{m}_{9B}(3,14)$

m9B314 (this line included for string searching purposes)

Solution 1

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_1, e_{11}] = e_{12}$$
$$[e_1, e_{12}] = e_{13} \qquad [e_2, e_3] = e_6$$
$$[e_2, e_4] = e_7 \qquad [e_2, e_5] = \frac{10e_8}{7}$$
$$[e_2, e_6] = \frac{13e_9}{7} \qquad [e_2, e_7] = \frac{19e_{10}}{7}$$
$$[e_2, e_8] = 4e_{11} \qquad [e_2, e_9] = 7e_{12}$$
$$[e_2, e_{10}] = 13e_{13} \qquad [e_2, e_{13}] = e_{14}$$
$$[e_3, e_4] = -\frac{3e_8}{7} \qquad [e_3, e_5] = -\frac{3e_9}{7}$$
$$[e_3, e_6] = -\frac{6e_{10}}{7} \qquad [e_3, e_7] = -\frac{9e_{11}}{7}$$
$$[e_3, e_8] = -3e_{12} \qquad [e_3, e_9] = -6e_{13}$$
$$[e_3, e_{12}] = -e_{14} \qquad [e_4, e_5] = \frac{3e_{10}}{7}$$
$$[e_4, e_6] = \frac{3e_{11}}{7} \qquad [e_4, e_7] = \frac{12e_{12}}{7}$$
$$[e_4, e_8] = 3e_{13} \qquad [e_4, e_{11}] = e_{14}$$
$$[e_5, e_6] = -\frac{9e_{12}}{7} \qquad [e_5, e_7] = -\frac{9e_{13}}{7}$$
$$[e_5, e_{10}] = -e_{14} \qquad [e_6, e_9] = e_{14}$$
$$[e_7, e_8] = -e_{14}$$

Solution 2

$$
\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_1, e_9] = e_{10} \\
[e_1, e_{10}] = e_{11} & [e_1, e_{11}] = e_{12} \\
[e_1, e_{12}] = e_{13} & [e_2, e_3] = e_6 \\
[e_2, e_4] = e_7 & [e_2, e_5] = e_8 \\
[e_2, e_6] = e_9 & [e_2, e_7] = e_{10} \\
[e_2, e_8] = e_{11} & [e_2, e_9] = e_{12} \\
[e_2, e_{10}] = e_{13} & [e_2, e_{13}] = e_{14} \\
[e_3, e_4] = 0 & [e_3, e_5] = 0 \\
[e_3, e_6] = 0 & [e_3, e_7] = 0 \\
[e_3, e_8] = 0 & [e_3, e_9] = 0 \\
[e_3, e_{12}] = -e_{14} & [e_4, e_5] = 0 \\
[e_4, e_6] = 0 & [e_4, e_7] = 0 \\
[e_4, e_8] = 0 & [e_4, e_{11}] = e_{14} \\
[e_5, e_6] = 0 & [e_5, e_7] = 0 \\
[e_5, e_{10}] = -e_{14} & [e_6, e_9] = e_{14} \\
[e_7, e_8] = -e_{14} &
\end{array}
$$

Solution 3

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_1, e_{11}] = e_{12}$$
$$[e_1, e_{12}] = e_{13} \qquad [e_2, e_3] = e_6$$
$$[e_2, e_4] = e_7 \qquad [e_2, e_5] = \frac{e_8}{4}$$
$$[e_2, e_6] = -\frac{e_9}{2} \qquad [e_2, e_7] = -\frac{23e_{10}}{28}$$
$$[e_2, e_8] = -\frac{5e_{11}}{7} \qquad [e_2, e_9] = -\frac{5e_{12}}{4}$$
$$[e_2, e_{10}] = -\frac{7e_{13}}{2} \qquad [e_2, e_{13}] = e_{14}$$
$$[e_3, e_4] = \frac{3e_8}{4} \qquad [e_3, e_5] = \frac{3e_9}{4}$$
$$[e_3, e_6] = \frac{9e_{10}}{28} \qquad [e_3, e_7] = -\frac{3e_{11}}{28}$$
$$[e_3, e_8] = \frac{15e_{12}}{28} \qquad [e_3, e_9] = \frac{9e_{13}}{4}$$
$$[e_3, e_{12}] = -e_{14} \qquad [e_4, e_5] = \frac{3e_{10}}{7}$$
$$[e_4, e_6] = \frac{3e_{11}}{7} \qquad [e_4, e_7] = -\frac{9e_{12}}{14}$$
$$[e_4, e_8] = -\frac{12e_{13}}{7} \qquad [e_4, e_{11}] = e_{14}$$
$$[e_5, e_6] = \frac{15e_{12}}{14} \qquad [e_5, e_7] = \frac{15e_{13}}{14}$$
$$[e_5, e_{10}] = -e_{14} \qquad [e_6, e_9] = e_{14}$$
$$[e_7, e_8] = -e_{14}$$

Solution 4

$$
\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_1, e_9] = e_{10} \\
[e_1, e_{10}] = e_{11} & [e_1, e_{11}] = e_{12} \\
[e_1, e_{12}] = e_{13} & [e_2, e_3] = e_6 \\
[e_2, e_4] = e_7 & [e_2, e_5] = -2e_8 \\
[e_2, e_6] = -5e_9 & [e_2, e_7] = -5e_{10} \\
[e_2, e_8] = -2e_{11} & [e_2, e_9] = e_{12} \\
[e_2, e_{10}] = e_{13} & [e_2, e_{13}] = e_{14} \\
[e_3, e_4] = 3e_8 & [e_3, e_5] = 3e_9 \\
[e_3, e_6] = 0 & [e_3, e_7] = -3e_{11} \\
[e_3, e_8] = -3e_{12} & [e_3, e_9] = 0 \\
[e_3, e_{12}] = -e_{14} & [e_4, e_5] = 3e_{10} \\
[e_4, e_6] = 3e_{11} & [e_4, e_7] = 0 \\
[e_4, e_8] = -3e_{13} & [e_4, e_{11}] = e_{14} \\
[e_5, e_6] = 3e_{12} & [e_5, e_7] = 3e_{13} \\
[e_5, e_{10}] = -e_{14} & [e_6, e_9] = e_{14} \\
[e_7, e_8] = -e_{14} &
\end{array}
$$

Original brackets:

$$
\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_1, e_9] = e_{10} \\
[e_1, e_{10}] = e_{11} & [e_1, e_{11}] = e_{12} \\
[e_1, e_{12}] = e_{13} & [e_2, e_3] = e_6 \\
[e_2, e_4] = e_7 & [e_2, e_5] = \alpha_{2,5}^{8} e_8 \\
[e_2, e_6] = \alpha_{2,6}^{9} e_9 & [e_2, e_7] = \alpha_{2,7}^{10} e_{10} \\
[e_2, e_8] = \alpha_{2,8}^{11} e_{11} & [e_2, e_9] = \alpha_{2,9}^{12} e_{12} \\
[e_2, e_{10}] = \alpha_{2,10}^{13} e_{13} & [e_2, e_{13}] = e_{14} \\
[e_3, e_4] = \alpha_{3,4}^{8} e_8 & [e_3, e_5] = \alpha_{3,5}^{9} e_9 \\
[e_3, e_6] = \alpha_{3,6}^{10} e_{10} & [e_3, e_7] = \alpha_{3,7}^{11} e_{11} \\
[e_3, e_8] = \alpha_{3,8}^{12} e_{12} & [e_3, e_9] = \alpha_{3,9}^{13} e_{13} \\
[e_3, e_{12}] = \alpha_{3,12}^{14} e_{14} & [e_4, e_5] = \alpha_{4,5}^{10} e_{10} \\
[e_4, e_6] = \alpha_{4,6}^{11} e_{11} & [e_4, e_7] = \alpha_{4,7}^{12} e_{12} \\
[e_4, e_8] = \alpha_{4,8}^{13} e_{13} & [e_4, e_{11}] = \alpha_{4,11}^{14} e_{14} \\
[e_5, e_6] = \alpha_{5,6}^{12} e_{12} & [e_5, e_7] = \alpha_{5,7}^{13} e_{13} \\
[e_5, e_{10}] = \alpha_{5,10}^{14} e_{14} & [e_6, e_9] = \alpha_{6,9}^{14} e_{14} \\
[e_7, e_8] = \alpha_{7,8}^{14} e_{14} &
\end{array}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_4): &\quad -\alpha_{2,5}^{8} - \alpha_{3,4}^{8} + 1 &= 0\\
(e_1, e_2, e_5): &\quad \alpha_{2,5}^{8} - \alpha_{2,6}^{9} - \alpha_{3,5}^{9} &= 0\\
(e_1, e_3, e_4): &\quad \alpha_{3,4}^{8} - \alpha_{3,5}^{9} &= 0\\
(e_1, e_2, e_6): &\quad \alpha_{2,6}^{9} - \alpha_{2,7}^{10} - \alpha_{3,6}^{10} &= 0\\
(e_1, e_3, e_5): &\quad \alpha_{3,5}^{9} - \alpha_{3,6}^{10} - \alpha_{4,5}^{10} &= 0\\
(e_1, e_2, e_7): &\quad \alpha_{2,7}^{10} - \alpha_{2,8}^{11} - \alpha_{3,7}^{11} &= 0\\
(e_1, e_3, e_6): &\quad \alpha_{3,6}^{10} - \alpha_{3,7}^{11} - \alpha_{4,6}^{11} &= 0\\
(e_1, e_4, e_5): &\quad \alpha_{4,5}^{10} - \alpha_{4,6}^{11} &= 0\\
(e_2, e_3, e_4): &\quad \alpha_{2,8}^{11}\alpha_{3,4}^{8} - \alpha_{3,7}^{11} + \alpha_{4,6}^{11} &= 0\\
(e_1, e_2, e_8): &\quad \alpha_{2,8}^{11} - \alpha_{2,9}^{12} - \alpha_{3,8}^{12} &= 0\\
(e_1, e_3, e_7): &\quad \alpha_{3,7}^{11} - \alpha_{3,8}^{12} - \alpha_{4,7}^{12} &= 0\\
(e_1, e_4, e_6): &\quad \alpha_{4,6}^{11} - \alpha_{4,7}^{12} - \alpha_{5,6}^{12} &= 0\\
(e_2, e_3, e_5): &\quad -\alpha_{2,5}^{8}\alpha_{3,8}^{12} + \alpha_{2,9}^{12}\alpha_{3,5}^{9} + \alpha_{5,6}^{12} &= 0\\
(e_1, e_2, e_9): &\quad -\alpha_{2,10}^{13} + \alpha_{2,9}^{12} - \alpha_{3,9}^{13} &= 0\\
(e_1, e_3, e_8): &\quad \alpha_{3,8}^{12} - \alpha_{3,9}^{13} - \alpha_{4,8}^{13} &= 0\\
(e_1, e_4, e_7): &\quad \alpha_{4,7}^{12} - \alpha_{4,8}^{13} - \alpha_{5,7}^{13} &= 0\\
(e_1, e_5, e_6): &\quad \alpha_{5,6}^{12} - \alpha_{5,7}^{13} &= 0\\
(e_2, e_3, e_6): &\quad \alpha_{2,10}^{13}\alpha_{3,6}^{10} - \alpha_{2,6}^{9}\alpha_{3,9}^{13} &= 0\\
(e_2, e_4, e_5): &\quad \alpha_{2,10}^{13}\alpha_{4,5}^{10} - \alpha_{2,5}^{8}\alpha_{4,8}^{13} + \alpha_{5,7}^{13} &= 0\\
(e_1, e_2, e_{12}): &\quad -\alpha_{3,12}^{14} - 1 &= 0\\
(e_1, e_3, e_{11}): &\quad -\alpha_{3,12}^{14} - \alpha_{4,11}^{14} &= 0\\
(e_1, e_4, e_{10}): &\quad -\alpha_{4,11}^{14} - \alpha_{5,10}^{14} &= 0\\
(e_1, e_5, e_9): &\quad -\alpha_{5,10}^{14} - \alpha_{6,9}^{14} &= 0\\
(e_1, e_6, e_8): &\quad -\alpha_{6,9}^{14} - \alpha_{7,8}^{14} &= 0\\
(e_2, e_3, e_9): &\quad -\alpha_{2,9}^{12}\alpha_{3,12}^{14} + \alpha_{3,9}^{13} - \alpha_{6,9}^{14} &= 0\\
(e_2, e_4, e_8): &\quad -\alpha_{2,8}^{11}\alpha_{4,11}^{14} + \alpha_{4,8}^{13} - \alpha_{7,8}^{14} &= 0\\
(e_2, e_5, e_7): &\quad \alpha_{2,5}^{8}\alpha_{7,8}^{14} - \alpha_{2,7}^{10}\alpha_{5,10}^{14} + \alpha_{5,7}^{13} &= 0\\
(e_3, e_4, e_7): &\quad \alpha_{3,12}^{14}\alpha_{4,7}^{12} + \alpha_{3,4}^{8}\alpha_{7,8}^{14} - \alpha_{3,7}^{11}\alpha_{4,11}^{14} &= 0\\
(e_3, e_5, e_6): &\quad \alpha_{3,12}^{14}\alpha_{5,6}^{12} + \alpha_{3,5}^{9}\alpha_{6,9}^{14} - \alpha_{3,6}^{10}\alpha_{5,10}^{14} &= 0
\end{aligned}
$$

Solution 1:

$$\alpha^{9}_{3,5} = -3/7$$
$$\alpha^{13}_{3,9} = -6$$
$$\alpha^{13}_{2,10} = 13$$
$$\alpha^{11}_{3,7} = -9/7$$
$$\alpha^{9}_{2,6} = 13/7$$
$$\alpha^{8}_{3,4} = -3/7$$
$$\alpha^{13}_{4,8} = 3$$
$$\alpha^{14}_{6,9} = 1$$
$$\alpha^{14}_{3,12} = -1$$
$$\alpha^{11}_{4,6} = 3/7$$
$$\alpha^{12}_{5,6} = -9/7$$
$$\alpha^{13}_{5,7} = -9/7$$
$$\alpha^{10}_{4,5} = 3/7$$
$$\alpha^{14}_{7,8} = -1$$
$$\alpha^{8}_{2,5} = 10/7$$
$$\alpha^{12}_{3,8} = -3$$
$$\alpha^{12}_{2,9} = 7$$
$$\alpha^{11}_{2,8} = 4$$
$$\alpha^{14}_{4,11} = 1$$
$$\alpha^{10}_{2,7} = 19/7$$
$$\alpha^{14}_{5,10} = -1$$
$$\alpha^{12}_{4,7} = 12/7$$
$$\alpha^{10}_{3,6} = -6/7$$

Solution 2:

$$\alpha^{9}_{3,5} = 0$$
$$\alpha^{13}_{3,9} = 0$$
$$\alpha^{13}_{2,10} = 1$$
$$\alpha^{11}_{3,7} = 0$$
$$\alpha^{9}_{2,6} = 1$$
$$\alpha^{8}_{3,4} = 0$$
$$\alpha^{13}_{4,8} = 0$$
$$\alpha^{14}_{6,9} = 1$$
$$\alpha^{14}_{3,12} = -1$$
$$\alpha^{11}_{4,6} = 0$$
$$\alpha^{12}_{5,6} = 0$$
$$\alpha^{13}_{5,7} = 0$$
$$\alpha^{10}_{4,5} = 0$$
$$\alpha^{14}_{7,8} = -1$$
$$\alpha^{8}_{2,5} = 1$$
$$\alpha^{12}_{3,8} = 0$$
$$\alpha^{12}_{2,9} = 1$$
$$\alpha^{11}_{2,8} = 1$$
$$\alpha^{14}_{4,11} = 1$$
$$\alpha^{10}_{2,7} = 1$$
$$\alpha^{14}_{5,10} = -1$$
$$\alpha^{12}_{4,7} = 0$$
$$\alpha^{10}_{3,6} = 0$$

Solution 3:

$$\alpha^{9}_{3,5} = 3/4$$
$$\alpha^{13}_{3,9} = 9/4$$
$$\alpha^{13}_{2,10} = -7/2$$
$$\alpha^{11}_{3,7} = -3/28$$
$$\alpha^{9}_{2,6} = -1/2$$
$$\alpha^{8}_{3,4} = 3/4$$
$$\alpha^{13}_{4,8} = -12/7$$
$$\alpha^{14}_{6,9} = 1$$
$$\alpha^{14}_{3,12} = -1$$
$$\alpha^{11}_{4,6} = 3/7$$
$$\alpha^{12}_{5,6} = 15/14$$
$$\alpha^{13}_{5,7} = 15/14$$
$$\alpha^{10}_{4,5} = 3/7$$
$$\alpha^{14}_{7,8} = -1$$
$$\alpha^{8}_{2,5} = 1/4$$
$$\alpha^{12}_{3,8} = 15/28$$
$$\alpha^{12}_{2,9} = -5/4$$
$$\alpha^{11}_{2,8} = -5/7$$
$$\alpha^{14}_{4,11} = 1$$
$$\alpha^{10}_{2,7} = -23/28$$
$$\alpha^{14}_{5,10} = -1$$
$$\alpha^{12}_{4,7} = -9/14$$
$$\alpha^{10}_{3,6} = 9/28$$

Solution 4:

$$\alpha^{9}_{3,5} = 3$$
$$\alpha^{13}_{3,9} = 0$$
$$\alpha^{13}_{2,10} = 1$$
$$\alpha^{11}_{3,7} = -3$$
$$\alpha^{9}_{2,6} = -5$$
$$\alpha^{8}_{3,4} = 3$$
$$\alpha^{13}_{4,8} = -3$$
$$\alpha^{14}_{6,9} = 1$$
$$\alpha^{14}_{3,12} = -1$$
$$\alpha^{11}_{4,6} = 3$$
$$\alpha^{12}_{5,6} = 3$$
$$\alpha^{13}_{5,7} = 3$$
$$\alpha^{10}_{4,5} = 3$$
$$\alpha^{14}_{7,8} = -1$$
$$\alpha^{8}_{2,5} = -2$$
$$\alpha^{12}_{3,8} = -3$$
$$\alpha^{12}_{2,9} = 1$$
$$\alpha^{11}_{2,8} = -2$$
$$\alpha^{14}_{4,11} = 1$$
$$\alpha^{10}_{2,7} = -5$$
$$\alpha^{14}_{5,10} = -1$$
$$\alpha^{12}_{4,7} = 0$$
$$\alpha^{10}_{3,6} = 0$$

How the solution(s) were or were not found:
Change variables

$$\alpha^{9}_{3,5} \to x_1$$
$$\alpha^{13}_{3,9} \to x_2$$
$$\alpha^{13}_{2,10} \to x_3$$
$$\alpha^{11}_{3,7} \to x_4$$
$$\alpha^{9}_{2,6} \to x_5$$

$$\alpha_{3,4}^{8} \to x_6$$

$$\alpha_{4,8}^{13} \to x_7$$

$$\alpha_{6,9}^{14} \to x_8$$

$$\alpha_{3,12}^{14} \to x_9$$

$$\alpha_{4,6}^{11} \to x_{10}$$

$$\alpha_{5,6}^{12} \to x_{11}$$

$$\alpha_{5,7}^{13} \to x_{12}$$

$$\alpha_{4,5}^{10} \to x_{13}$$

$$\alpha_{7,8}^{14} \to x_{14}$$

$$\alpha_{2,5}^{8} \to x_{15}$$

$$\alpha_{3,8}^{12} \to x_{16}$$

$$\alpha_{2,9}^{12} \to x_{17}$$

$$\alpha_{2,8}^{11} \to x_{18}$$

$$\alpha_{4,11}^{14} \to x_{19}$$

$$\alpha_{2,7}^{10} \to x_{20}$$

$$\alpha_{5,10}^{14} \to x_{21}$$

$$\alpha_{4,7}^{12} \to x_{22}$$

$$\alpha_{3,6}^{10} \to x_{23}$$

Jacobi Tests

$$
\begin{aligned}
(e_1,e_2,e_4):&\quad -x_{15}-x_6+1 &&=0\\
(e_1,e_2,e_5):&\quad -x_1+x_{15}-x_5 &&=0\\
(e_1,e_3,e_4):&\quad -x_1+x_6 &&=0\\
(e_1,e_2,e_6):&\quad -x_{20}-x_{23}+x_5 &&=0\\
(e_1,e_3,e_5):&\quad x_1-x_{13}-x_{23} &&=0\\
(e_1,e_2,e_7):&\quad -x_{18}+x_{20}-x_4 &&=0\\
(e_1,e_3,e_6):&\quad -x_{10}+x_{23}-x_4 &&=0\\
(e_1,e_4,e_5):&\quad -x_{10}+x_{13} &&=0\\
(e_2,e_3,e_4):&\quad x_{10}+x_{18}x_6-x_4 &&=0\\
(e_1,e_2,e_8):&\quad -x_{16}-x_{17}+x_{18} &&=0\\
(e_1,e_3,e_7):&\quad -x_{16}-x_{22}+x_4 &&=0\\
(e_1,e_4,e_6):&\quad x_{10}-x_{11}-x_{22} &&=0\\
(e_2,e_3,e_5):&\quad x_1x_{17}+x_{11}-x_{15}x_{16} &&=0\\
(e_1,e_2,e_9):&\quad x_{17}-x_2-x_3 &&=0\\
(e_1,e_3,e_8):&\quad x_{16}-x_2-x_7 &&=0\\
(e_1,e_4,e_7):&\quad -x_{12}+x_{22}-x_7 &&=0\\
(e_1,e_5,e_6):&\quad x_{11}-x_{12} &&=0\\
(e_2,e_3,e_6):&\quad -x_2x_5+x_{23}x_3 &&=0\\
(e_2,e_4,e_5):&\quad x_{12}+x_{13}x_3-x_{15}x_7 &&=0\\
(e_1,e_2,e_{12}):&\quad -x_9-1 &&=0\\
(e_1,e_3,e_{11}):&\quad -x_{19}-x_9 &&=0\\
(e_1,e_4,e_{10}):&\quad -x_{19}-x_{21} &&=0\\
(e_1,e_5,e_9):&\quad -x_{21}-x_8 &&=0\\
(e_1,e_6,e_8):&\quad -x_{14}-x_8 &&=0\\
(e_2,e_3,e_9):&\quad -x_{17}x_9+x_2-x_8 &&=0\\
(e_2,e_4,e_8):&\quad -x_{14}-x_{18}x_{19}+x_7 &&=0\\
(e_2,e_5,e_7):&\quad x_{12}+x_{14}x_{15}-x_{20}x_{21} &&=0\\
(e_3,e_4,e_7):&\quad x_{14}x_6-x_{19}x_4+x_{22}x_9 &&=0\\
(e_3,e_5,e_6):&\quad x_1x_8+x_{11}x_9-x_{21}x_{23} &&=0
\end{aligned}
$$

Groebner basis (23 variables, 21 linear, 3 nonlinear)

$$2x_1+x_{20}+x_{23}-1=0$$
$$x_2-7x_{23}=0$$
$$14x_{23}+x_3-1=0$$

$$-x_{20} - 5x_{23} + 2x_4 + 1 = 0$$

$$-x_{20} - x_{23} + x_5 = 0$$

$$x_{20} + x_{23} + 2x_6 - 1 = 0$$

$$-x_{20} + 5x_{23} + 2x_7 + 1 = 0$$

$$x_8 - 1 = 0$$

$$x_9 + 1 = 0$$

$$2x_{10} + x_{20} + 3x_{23} - 1 = 0$$

$$2x_{11} + x_{20} - x_{23} - 1 = 0$$

$$2x_{12} + x_{20} - x_{23} - 1 = 0$$

$$2x_{13} + x_{20} + 3x_{23} - 1 = 0$$

$$x_{14} + 1 = 0$$

$$2x_{15} - x_{20} - x_{23} - 1 = 0$$

$$2x_{16} - x_{20} - 9x_{23} + 1 = 0$$

$$x_{17} + 7x_{23} - 1 = 0$$

$$2x_{18} - x_{20} + 5x_{23} - 1 = 0$$

$$x_{19} - 1 = 0$$

$$7x_{20}^2 + 28x_{20} + 49x_{23}^2 + 150x_{23} - 35 = 0$$

$$7x_{20}x_{23} + 21x_{23}^2 - x_{23} = 0$$

$$x_{21} + 1 = 0$$

$$x_{22} + 2x_{23} = 0$$

$$196x_{23}^3 + 105x_{23}^2 - 54x_{23} = 0$$

Solution 1:

$$x_1 = -3/7$$

$$x_2 = -6$$

$$x_3 = 13$$

$$x_4 = -9/7$$

$$x_5 = 13/7$$

$$x_6 = -3/7$$

$$x_7 = 3$$

$$x_8 = 1$$

$$x_9 = -1$$

$$x_10 = 3/7$$

$$x_1 1 = -9/7$$
$$x_1 2 = -9/7$$
$$x_1 3 = 3/7$$
$$x_1 4 = -1$$
$$x_1 5 = 10/7$$
$$x_1 6 = -3$$
$$x_1 7 = 7$$
$$x_1 8 = 4$$
$$x_1 9 = 1$$
$$x_2 0 = 19/7$$
$$x_2 1 = -1$$
$$x_2 2 = 12/7$$
$$x_2 3 = -6/7$$

Solution 2:

$$x_1 = 0$$
$$x_2 = 0$$
$$x_3 = 1$$
$$x_4 = 0$$
$$x_5 = 1$$
$$x_6 = 0$$
$$x_7 = 0$$
$$x_8 = 1$$
$$x_9 = -1$$
$$x_1 0 = 0$$
$$x_1 1 = 0$$
$$x_1 2 = 0$$
$$x_1 3 = 0$$
$$x_1 4 = -1$$
$$x_1 5 = 1$$
$$x_1 6 = 0$$
$$x_1 7 = 1$$
$$x_1 8 = 1$$

$$x_19 = 1$$
$$x_20 = 1$$
$$x_21 = -1$$
$$x_22 = 0$$
$$x_23 = 0$$

Solution 3:

$$x_1 = 3/4$$
$$x_2 = 9/4$$
$$x_3 = -7/2$$
$$x_4 = -3/28$$
$$x_5 = -1/2$$
$$x_6 = 3/4$$
$$x_7 = -12/7$$
$$x_8 = 1$$
$$x_9 = -1$$
$$x_10 = 3/7$$
$$x_11 = 15/14$$
$$x_12 = 15/14$$
$$x_13 = 3/7$$
$$x_14 = -1$$
$$x_15 = 1/4$$
$$x_16 = 15/28$$
$$x_17 = -5/4$$
$$x_18 = -5/7$$
$$x_19 = 1$$
$$x_20 = -23/28$$
$$x_21 = -1$$
$$x_22 = -9/14$$
$$x_23 = 9/28$$

Solution 4:

$$x_1 = 3$$
$$x_2 = 0$$

$$x_3 = 1$$

$$x_4 = -3$$

$$x_5 = -5$$

$$x_6 = 3$$

$$x_7 = -3$$

$$x_8 = 1$$

$$x_9 = -1$$

$$x_1 0 = 3$$

$$x_1 1 = 3$$

$$x_1 2 = 3$$

$$x_1 3 = 3$$

$$x_1 4 = -1$$

$$x_1 5 = -2$$

$$x_1 6 = -3$$

$$x_1 7 = 1$$

$$x_1 8 = -2$$

$$x_1 9 = 1$$

$$x_2 0 = -5$$

$$x_2 1 = -1$$

$$x_2 2 = 0$$

$$x_2 3 = 0$$

$\mathfrak{m}_{2B}(4,14)$

m2B414 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
&[e_1,e_2]=e_3 && [e_1,e_3]=e_4\\
&[e_1,e_4]=e_5 && [e_1,e_5]=e_6\\
&[e_1,e_6]=e_7 && [e_1,e_7]=e_8\\
&[e_1,e_8]=e_9 && [e_1,e_9]=e_{10}\\
&[e_1,e_{10}]=e_{11} && [e_1,e_{11}]=e_{12}\\
&[e_1,e_{12}]=e_{13} && [e_2,e_9]=e_{13}\\
&[e_2,e_{13}]=e_{14} && [e_3,e_8]=-e_{13}\\
&[e_3,e_{12}]=\alpha_{3,12}^{14}e_{14} && [e_4,e_7]=e_{13}\\
&[e_4,e_{11}]=\alpha_{4,11}^{14}e_{14} && [e_5,e_6]=-e_{13}\\
&[e_5,e_{10}]=\alpha_{5,10}^{14}e_{14} && [e_6,e_9]=\alpha_{6,9}^{14}e_{14}\\
&[e_7,e_8]=\alpha_{7,8}^{14}e_{14}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1,e_2,e_{12}):&\quad -\alpha_{3,12}^{14}-1 &&=0\\
(e_1,e_3,e_{11}):&\quad -\alpha_{3,12}^{14}-\alpha_{4,11}^{14} &&=0\\
(e_1,e_4,e_{10}):&\quad -\alpha_{4,11}^{14}-\alpha_{5,10}^{14} &&=0\\
(e_1,e_5,e_9):&\quad -\alpha_{5,10}^{14}-\alpha_{6,9}^{14} &&=0\\
(e_1,e_6,e_8):&\quad -\alpha_{6,9}^{14}-\alpha_{7,8}^{14} &&=0\\
(e_2,e_3,e_8):&\quad \text{no solutions}\\
(e_2,e_4,e_7):&\quad \text{no solutions}\\
(e_2,e_5,e_6):&\quad \text{no solutions}
\end{aligned}
$$

There are no solutions.

$\mathfrak{m}_{4B}(4,14)$

m4B414 (this line included for string searching purposes)

Solution 1

$$
\begin{array}{ll}
[e_1,e_2]=e_3 & [e_1,e_3]=e_4 \\
[e_1,e_4]=e_5 & [e_1,e_5]=e_6 \\
[e_1,e_6]=e_7 & [e_1,e_7]=e_8 \\
[e_1,e_8]=e_9 & [e_1,e_9]=e_{10} \\
[e_1,e_{10}]=e_{11} & [e_1,e_{11}]=e_{12} \\
[e_1,e_{12}]=e_{13} & [e_2,e_7]=e_{11} \\
[e_2,e_8]=3e_{12} & [e_2,e_9]=6e_{13} \\
[e_2,e_{13}]=e_{14} & [e_3,e_6]=-e_{11} \\
[e_3,e_7]=-2e_{12} & [e_3,e_8]=-3e_{13} \\
[e_3,e_{12}]=-e_{14} & [e_4,e_5]=e_{11} \\
[e_4,e_6]=e_{12} & [e_4,e_7]=e_{13} \\
[e_4,e_{11}]=e_{14} & [e_5,e_6]=0 \\
[e_5,e_{10}]=-e_{14} & [e_6,e_9]=e_{14} \\
[e_7,e_8]=-e_{14} &
\end{array}
$$

Original brackets:

$$
\begin{array}{ll}
[e_1,e_2]=e_3 & [e_1,e_3]=e_4 \\
[e_1,e_4]=e_5 & [e_1,e_5]=e_6 \\
[e_1,e_6]=e_7 & [e_1,e_7]=e_8 \\
[e_1,e_8]=e_9 & [e_1,e_9]=e_{10} \\
[e_1,e_{10}]=e_{11} & [e_1,e_{11}]=e_{12} \\
[e_1,e_{12}]=e_{13} & [e_2,e_7]=e_{11} \\
[e_2,e_8]=3e_{12} & [e_2,e_9]=\alpha_{2,9}^{13}e_{13} \\
[e_2,e_{13}]=e_{14} & [e_3,e_6]=-e_{11} \\
[e_3,e_7]=-2e_{12} & [e_3,e_8]=\alpha_{3,8}^{13}e_{13} \\
[e_3,e_{12}]=\alpha_{3,12}^{14}e_{14} & [e_4,e_5]=e_{11} \\
[e_4,e_6]=e_{12} & [e_4,e_7]=\alpha_{4,7}^{13}e_{13} \\
[e_4,e_{11}]=\alpha_{4,11}^{14}e_{14} & [e_5,e_6]=\alpha_{5,6}^{13}e_{13} \\
[e_5,e_{10}]=\alpha_{5,10}^{14}e_{14} & [e_6,e_9]=\alpha_{6,9}^{14}e_{14} \\
[e_7,e_8]=\alpha_{7,8}^{14}e_{14} &
\end{array}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_8): &\quad -\alpha_{2,9}^{13} - \alpha_{3,8}^{13} + 3 && = 0\\
(e_1, e_3, e_7): &\quad -\alpha_{3,8}^{13} - \alpha_{4,7}^{13} - 2 && = 0\\
(e_1, e_4, e_6): &\quad -\alpha_{4,7}^{13} - \alpha_{5,6}^{13} + 1 && = 0\\
(e_1, e_2, e_{12}): &\quad -\alpha_{3,12}^{14} - 1 && = 0\\
(e_1, e_3, e_{11}): &\quad -\alpha_{3,12}^{14} - \alpha_{4,11}^{14} && = 0\\
(e_1, e_4, e_{10}): &\quad -\alpha_{4,11}^{14} - \alpha_{5,10}^{14} && = 0\\
(e_1, e_5, e_9): &\quad -\alpha_{5,10}^{14} - \alpha_{6,9}^{14} && = 0\\
(e_1, e_6, e_8): &\quad -\alpha_{6,9}^{14} - \alpha_{7,8}^{14} && = 0\\
(e_2, e_3, e_8): &\quad -3\alpha_{3,12}^{14} + \alpha_{3,8}^{13} && = 0\\
(e_2, e_4, e_7): &\quad -\alpha_{4,11}^{14} + \alpha_{4,7}^{13} && = 0\\
(e_2, e_5, e_6): &\quad \alpha_{5,6}^{13} && = 0\\
(e_3, e_4, e_6): &\quad \alpha_{3,12}^{14} + \alpha_{4,11}^{14} && = 0
\end{aligned}
$$

Solution 1:

$$
\begin{aligned}
\alpha_{7,8}^{14} &= -1\\
\alpha_{5,6}^{13} &= 0\\
\alpha_{3,8}^{13} &= -3\\
\alpha_{4,11}^{14} &= 1\\
\alpha_{3,12}^{14} &= -1\\
\alpha_{2,9}^{13} &= 6\\
\alpha_{5,10}^{14} &= -1\\
\alpha_{6,9}^{14} &= 1\\
\alpha_{4,7}^{13} &= 1
\end{aligned}
$$

How the solution(s) were or were not found:
Change variables

$$
\begin{aligned}
\alpha_{7,8}^{14} &\to x_1\\
\alpha_{5,6}^{13} &\to x_2\\
\alpha_{3,8}^{13} &\to x_3\\
\alpha_{4,11}^{14} &\to x_4
\end{aligned}
$$

$$\alpha^{14}_{3,12} \to x_5$$
$$\alpha^{13}_{2,9} \to x_6$$
$$\alpha^{14}_{5,10} \to x_7$$
$$\alpha^{14}_{6,9} \to x_8$$
$$\alpha^{13}_{4,7} \to x_9$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_8): &\quad -x_3 - x_6 + 3 &&= 0\\
(e_1, e_3, e_7): &\quad -x_3 - x_9 - 2 &&= 0\\
(e_1, e_4, e_6): &\quad -x_2 - x_9 + 1 &&= 0\\
(e_1, e_2, e_{12}): &\quad -x_5 - 1 &&= 0\\
(e_1, e_3, e_{11}): &\quad -x_4 - x_5 &&= 0\\
(e_1, e_4, e_{10}): &\quad -x_4 - x_7 &&= 0\\
(e_1, e_5, e_9): &\quad -x_7 - x_8 &&= 0\\
(e_1, e_6, e_8): &\quad -x_1 - x_8 &&= 0\\
(e_2, e_3, e_8): &\quad x_3 - 3x_5 &&= 0\\
(e_2, e_4, e_7): &\quad -x_4 + x_9 &&= 0\\
(e_2, e_5, e_6): &\quad x_2 &&= 0\\
(e_3, e_4, e_6): &\quad x_4 + x_5 &&= 0
\end{aligned}$$

Groebner basis (9 variables, 9 linear, 0 nonlinear)

$$x_1 + 1 = 0$$
$$x_2 = 0$$
$$x_3 + 3 = 0$$
$$x_4 - 1 = 0$$
$$x_5 + 1 = 0$$
$$x_6 - 6 = 0$$
$$x_7 + 1 = 0$$
$$x_8 - 1 = 0$$
$$x_9 - 1 = 0$$

Solution 1:

$$x_1 = -1$$
$$x_2 = 0$$

$$x_3 = -3$$
$$x_4 = 1$$
$$x_5 = -1$$
$$x_6 = 6$$
$$x_7 = -1$$
$$x_8 = 1$$
$$x_9 = 1$$

$\mathfrak{m}_{6B}(4, 14)$

m6B414 (this line included for string searching purposes)

Solution 1

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_1, e_{11}] = e_{12}$$
$$[e_1, e_{12}] = e_{13} \qquad [e_2, e_5] = e_9$$
$$[e_2, e_6] = 2e_{10} \qquad [e_2, e_7] = \frac{5e_{11}}{3}$$
$$[e_2, e_8] = 0 \qquad [e_2, e_9] = 0$$
$$[e_2, e_{13}] = e_{14} \qquad [e_3, e_4] = -e_9$$
$$[e_3, e_5] = -e_{10} \qquad [e_3, e_6] = \frac{e_{11}}{3}$$
$$[e_3, e_7] = \frac{5e_{12}}{3} \qquad [e_3, e_8] = 0$$
$$[e_3, e_{12}] = -e_{14} \qquad [e_4, e_5] = -\frac{4e_{11}}{3}$$
$$[e_4, e_6] = -\frac{4e_{12}}{3} \qquad [e_4, e_7] = \frac{5e_{13}}{3}$$
$$[e_4, e_{11}] = e_{14} \qquad [e_5, e_6] = -3e_{13}$$
$$[e_5, e_{10}] = -e_{14} \qquad [e_6, e_9] = e_{14}$$
$$[e_7, e_8] = -e_{14}$$

Original brackets:

$$
\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_1, e_9] = e_{10} \\
[e_1, e_{10}] = e_{11} & [e_1, e_{11}] = e_{12} \\
[e_1, e_{12}] = e_{13} & [e_2, e_5] = e_9 \\
[e_2, e_6] = 2e_{10} & [e_2, e_7] = \alpha_{2,7}^{11} e_{11} \\
[e_2, e_8] = \alpha_{2,8}^{12} e_{12} & [e_2, e_9] = \alpha_{2,9}^{13} e_{13} \\
[e_2, e_{13}] = e_{14} & [e_3, e_4] = -e_9 \\
[e_3, e_5] = -e_{10} & [e_3, e_6] = \alpha_{3,6}^{11} e_{11} \\
[e_3, e_7] = \alpha_{3,7}^{12} e_{12} & [e_3, e_8] = \alpha_{3,8}^{13} e_{13} \\
[e_3, e_{12}] = \alpha_{3,12}^{14} e_{14} & [e_4, e_5] = \alpha_{4,5}^{11} e_{11} \\
[e_4, e_6] = \alpha_{4,6}^{12} e_{12} & [e_4, e_7] = \alpha_{4,7}^{13} e_{13} \\
[e_4, e_{11}] = \alpha_{4,11}^{14} e_{14} & [e_5, e_6] = \alpha_{5,6}^{13} e_{13} \\
[e_5, e_{10}] = \alpha_{5,10}^{14} e_{14} & [e_6, e_9] = \alpha_{6,9}^{14} e_{14} \\
[e_7, e_8] = \alpha_{7,8}^{14} e_{14} &
\end{array}
$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1, e_2, e_6): &\quad -\alpha_{2,7}^{11} - \alpha_{3,6}^{11} + 2 &= 0\\
(e_1, e_3, e_5): &\quad -\alpha_{3,6}^{11} - \alpha_{4,5}^{11} - 1 &= 0\\
(e_1, e_2, e_7): &\quad \alpha_{2,7}^{11} - \alpha_{2,8}^{12} - \alpha_{3,7}^{12} &= 0\\
(e_1, e_3, e_6): &\quad \alpha_{3,6}^{11} - \alpha_{3,7}^{12} - \alpha_{4,6}^{12} &= 0\\
(e_1, e_4, e_5): &\quad \alpha_{4,5}^{11} - \alpha_{4,6}^{12} &= 0\\
(e_1, e_2, e_8): &\quad \alpha_{2,8}^{12} - \alpha_{2,9}^{13} - \alpha_{3,8}^{13} &= 0\\
(e_1, e_3, e_7): &\quad \alpha_{3,7}^{12} - \alpha_{3,8}^{13} - \alpha_{4,7}^{13} &= 0\\
(e_1, e_4, e_6): &\quad \alpha_{4,6}^{12} - \alpha_{4,7}^{13} - \alpha_{5,6}^{13} &= 0\\
(e_2, e_3, e_4): &\quad -\alpha_{2,9}^{13} &= 0\\
(e_1, e_2, e_{12}): &\quad -\alpha_{3,12}^{14} - 1 &= 0\\
(e_1, e_3, e_{11}): &\quad -\alpha_{3,12}^{14} - \alpha_{4,11}^{14} &= 0\\
(e_1, e_4, e_{10}): &\quad -\alpha_{4,11}^{14} - \alpha_{5,10}^{14} &= 0\\
(e_1, e_5, e_9): &\quad -\alpha_{5,10}^{14} - \alpha_{6,9}^{14} &= 0\\
(e_1, e_6, e_8): &\quad -\alpha_{6,9}^{14} - \alpha_{7,8}^{14} &= 0\\
(e_2, e_3, e_8): &\quad -\alpha_{2,8}^{12}\alpha_{3,12}^{14} + \alpha_{3,8}^{13} &= 0\\
(e_2, e_4, e_7): &\quad -\alpha_{2,7}^{11}\alpha_{4,11}^{14} + \alpha_{4,7}^{13} &= 0\\
(e_2, e_5, e_6): &\quad -2\alpha_{5,10}^{14} + \alpha_{5,6}^{13} + \alpha_{6,9}^{14} &= 0\\
(e_3, e_4, e_6): &\quad \alpha_{3,12}^{14}\alpha_{4,6}^{12} - \alpha_{3,6}^{11}\alpha_{4,11}^{14} - \alpha_{6,9}^{14} &= 0
\end{aligned}$$

Solution 1:

$$\alpha^{11}_{2,7} = 5/3$$
$$\alpha^{12}_{2,8} = 0$$
$$\alpha^{12}_{4,6} = -4/3$$
$$\alpha^{13}_{5,6} = -3$$
$$\alpha^{14}_{7,8} = -1$$
$$\alpha^{11}_{4,5} = -4/3$$
$$\alpha^{13}_{3,8} = 0$$
$$\alpha^{12}_{3,7} = 5/3$$
$$\alpha^{14}_{4,11} = 1$$
$$\alpha^{14}_{3,12} = -1$$
$$\alpha^{13}_{2,9} = 0$$
$$\alpha^{14}_{5,10} = -1$$
$$\alpha^{14}_{6,9} = 1$$
$$\alpha^{13}_{4,7} = 5/3$$
$$\alpha^{11}_{3,6} = 1/3$$

How the solution(s) were or were not found:
Change variables

$$\alpha^{11}_{2,7} \to x_1$$
$$\alpha^{12}_{2,8} \to x_2$$
$$\alpha^{12}_{4,6} \to x_3$$
$$\alpha^{13}_{5,6} \to x_4$$
$$\alpha^{14}_{7,8} \to x_5$$
$$\alpha^{11}_{4,5} \to x_6$$
$$\alpha^{13}_{3,8} \to x_7$$
$$\alpha^{12}_{3,7} \to x_8$$
$$\alpha^{14}_{4,11} \to x_9$$
$$\alpha^{14}_{3,12} \to x_{10}$$
$$\alpha^{13}_{2,9} \to x_{11}$$
$$\alpha^{14}_{5,10} \to x_{12}$$

$$\alpha_{6,9}^{14} \to x_{13}$$

$$\alpha_{4,7}^{13} \to x_{14}$$

$$\alpha_{3,6}^{11} \to x_{15}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_6):&\quad -x_1 - x_{15} + 2 &&= 0\\
(e_1, e_3, e_5):&\quad -x_{15} - x_6 - 1 &&= 0\\
(e_1, e_2, e_7):&\quad x_1 - x_2 - x_8 &&= 0\\
(e_1, e_3, e_6):&\quad x_{15} - x_3 - x_8 &&= 0\\
(e_1, e_4, e_5):&\quad -x_3 + x_6 &&= 0\\
(e_1, e_2, e_8):&\quad -x_{11} + x_2 - x_7 &&= 0\\
(e_1, e_3, e_7):&\quad -x_{14} - x_7 + x_8 &&= 0\\
(e_1, e_4, e_6):&\quad -x_{14} + x_3 - x_4 &&= 0\\
(e_2, e_3, e_4):&\quad -x_{11} &&= 0\\
(e_1, e_2, e_{12}):&\quad -x_{10} - 1 &&= 0\\
(e_1, e_3, e_{11}):&\quad -x_{10} - x_9 &&= 0\\
(e_1, e_4, e_{10}):&\quad -x_{12} - x_9 &&= 0\\
(e_1, e_5, e_9):&\quad -x_{12} - x_{13} &&= 0\\
(e_1, e_6, e_8):&\quad -x_{13} - x_5 &&= 0\\
(e_2, e_3, e_8):&\quad -x_{10}x_2 + x_7 &&= 0\\
(e_2, e_4, e_7):&\quad -x_1x_9 + x_{14} &&= 0\\
(e_2, e_5, e_6):&\quad -2x_{12} + x_{13} + x_4 &&= 0\\
(e_3, e_4, e_6):&\quad x_{10}x_3 - x_{13} - x_{15}x_9 &&= 0
\end{aligned}$$

Groebner basis (15 variables, 15 linear, 0 nonlinear)

$$3x_1 - 5 = 0$$

$$x_2 = 0$$

$$3x_3 + 4 = 0$$

$$x_4 + 3 = 0$$

$$x_5 + 1 = 0$$

$$3x_6 + 4 = 0$$

$$x_7 = 0$$

$$3x_8 - 5 = 0$$

$$x_9 - 1 = 0$$
$$x_{10} + 1 = 0$$
$$x_{11} = 0$$
$$x_{12} + 1 = 0$$
$$x_{13} - 1 = 0$$
$$3x_{14} - 5 = 0$$
$$3x_{15} - 1 = 0$$

Solution 1:

$$x_1 = 5/3$$
$$x_2 = 0$$
$$x_3 = -4/3$$
$$x_4 = -3$$
$$x_5 = -1$$
$$x_6 = -4/3$$
$$x_7 = 0$$
$$x_8 = 5/3$$
$$x_9 = 1$$
$$x_1 0 = -1$$
$$x_1 1 = 0$$
$$x_1 2 = -1$$
$$x_1 3 = 1$$
$$x_1 4 = 5/3$$
$$x_1 5 = 1/3$$

$\mathfrak{m}_{8B}(4,14)$

m8B414 (this line included for string searching purposes)

Original brackets:

$$[e_1,e_2]=e_3 \qquad [e_1,e_3]=e_4$$
$$[e_1,e_4]=e_5 \qquad [e_1,e_5]=e_6$$
$$[e_1,e_6]=e_7 \qquad [e_1,e_7]=e_8$$
$$[e_1,e_8]=e_9 \qquad [e_1,e_9]=e_{10}$$
$$[e_1,e_{10}]=e_{11} \qquad [e_1,e_{11}]=e_{12}$$
$$[e_1,e_{12}]=e_{13} \qquad [e_2,e_3]=e_7$$
$$[e_2,e_4]=e_8 \qquad [e_2,e_5]=\alpha_{2,5}^{9}e_9$$
$$[e_2,e_6]=\alpha_{2,6}^{10}e_{10} \qquad [e_2,e_7]=\alpha_{2,7}^{11}e_{11}$$
$$[e_2,e_8]=\alpha_{2,8}^{12}e_{12} \qquad [e_2,e_9]=\alpha_{2,9}^{13}e_{13}$$
$$[e_2,e_{13}]=e_{14} \qquad [e_3,e_4]=\alpha_{3,4}^{9}e_9$$
$$[e_3,e_5]=\alpha_{3,5}^{10}e_{10} \qquad [e_3,e_6]=\alpha_{3,6}^{11}e_{11}$$
$$[e_3,e_7]=\alpha_{3,7}^{12}e_{12} \qquad [e_3,e_8]=\alpha_{3,8}^{13}e_{13}$$
$$[e_3,e_{12}]=\alpha_{3,12}^{14}e_{14} \qquad [e_4,e_5]=\alpha_{4,5}^{11}e_{11}$$
$$[e_4,e_6]=\alpha_{4,6}^{12}e_{12} \qquad [e_4,e_7]=\alpha_{4,7}^{13}e_{13}$$
$$[e_4,e_{11}]=\alpha_{4,11}^{14}e_{14} \qquad [e_5,e_6]=\alpha_{5,6}^{13}e_{13}$$
$$[e_5,e_{10}]=\alpha_{5,10}^{14}e_{14} \qquad [e_6,e_9]=\alpha_{6,9}^{14}e_{14}$$
$$[e_7,e_8]=\alpha_{7,8}^{14}e_{14}$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1,e_2,e_4):&\quad -\alpha_{2,5}^{9}-\alpha_{3,4}^{9}+1 &&=0\\
(e_1,e_2,e_5):&\quad \alpha_{2,5}^{9}-\alpha_{2,6}^{10}-\alpha_{3,5}^{10} &&=0\\
(e_1,e_3,e_4):&\quad \alpha_{3,4}^{9}-\alpha_{3,5}^{10} &&=0\\
(e_1,e_2,e_6):&\quad \alpha_{2,6}^{10}-\alpha_{2,7}^{11}-\alpha_{3,6}^{11} &&=0\\
(e_1,e_3,e_5):&\quad \alpha_{3,5}^{10}-\alpha_{3,6}^{11}-\alpha_{4,5}^{11} &&=0\\
(e_1,e_2,e_7):&\quad \alpha_{2,7}^{11}-\alpha_{2,8}^{12}-\alpha_{3,7}^{12} &&=0\\
(e_1,e_3,e_6):&\quad \alpha_{3,6}^{11}-\alpha_{3,7}^{12}-\alpha_{4,6}^{12} &&=0\\
(e_1,e_4,e_5):&\quad \alpha_{4,5}^{11}-\alpha_{4,6}^{12} &&=0\\
(e_1,e_2,e_8):&\quad \alpha_{2,8}^{12}-\alpha_{2,9}^{13}-\alpha_{3,8}^{13} &&=0\\
(e_1,e_3,e_7):&\quad \alpha_{3,7}^{12}-\alpha_{3,8}^{13}-\alpha_{4,7}^{13} &&=0\\
(e_1,e_4,e_6):&\quad \alpha_{4,6}^{12}-\alpha_{4,7}^{13}-\alpha_{5,6}^{13} &&=0\\
(e_2,e_3,e_4):&\quad \alpha_{2,9}^{13}\alpha_{3,4}^{9}-\alpha_{3,8}^{13}+\alpha_{4,7}^{13} &&=0\\
(e_1,e_2,e_{12}):&\quad -\alpha_{3,12}^{14}-1 &&=0\\
(e_1,e_3,e_{11}):&\quad -\alpha_{3,12}^{14}-\alpha_{4,11}^{14} &&=0\\
(e_1,e_4,e_{10}):&\quad -\alpha_{4,11}^{14}-\alpha_{5,10}^{14} &&=0\\
(e_1,e_5,e_9):&\quad -\alpha_{5,10}^{14}-\alpha_{6,9}^{14} &&=0\\
(e_1,e_6,e_8):&\quad -\alpha_{6,9}^{14}-\alpha_{7,8}^{14} &&=0\\
(e_2,e_3,e_8):&\quad -\alpha_{2,8}^{12}\alpha_{3,12}^{14}+\alpha_{3,8}^{13}-\alpha_{7,8}^{14} &&=0\\
(e_2,e_4,e_7):&\quad -\alpha_{2,7}^{11}\alpha_{4,11}^{14}+\alpha_{4,7}^{13}+\alpha_{7,8}^{14} &&=0\\
(e_2,e_5,e_6):&\quad \alpha_{2,5}^{9}\alpha_{6,9}^{14}-\alpha_{2,6}^{10}\alpha_{5,10}^{14}+\alpha_{5,6}^{13} &&=0\\
(e_3,e_4,e_6):&\quad \alpha_{3,12}^{14}\alpha_{4,6}^{12}+\alpha_{3,4}^{9}\alpha_{6,9}^{14}-\alpha_{3,6}^{11}\alpha_{4,11}^{14} &&=0
\end{aligned}
$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{2,7}^{11}\to x_1$$

$$\alpha_{2,5}^{9}\to x_2$$

$$\alpha_{3,5}^{10}\to x_3$$

$$\alpha_{2,8}^{12}\to x_4$$

$$\alpha_{4,6}^{12}\to x_5$$

$$\alpha_{5,6}^{13}\to x_6$$

$$\alpha^{14}_{7,8} \to x_7$$

$$\alpha^{11}_{4,5} \to x_8$$

$$\alpha^{13}_{3,8} \to x_9$$

$$\alpha^{12}_{3,7} \to x_{10}$$

$$\alpha^{14}_{4,11} \to x_{11}$$

$$\alpha^{14}_{6,9} \to x_{12}$$

$$\alpha^{14}_{3,12} \to x_{13}$$

$$\alpha^{9}_{3,4} \to x_{14}$$

$$\alpha^{13}_{2,9} \to x_{15}$$

$$\alpha^{14}_{5,10} \to x_{16}$$

$$\alpha^{10}_{2,6} \to x_{17}$$

$$\alpha^{13}_{4,7} \to x_{18}$$

$$\alpha^{11}_{3,6} \to x_{19}$$

Jacobi Tests

$$
\begin{aligned}
(e_1,e_2,e_4):&\quad -x_{14}-x_2+1 &&=0\\
(e_1,e_2,e_5):&\quad -x_{17}+x_2-x_3 &&=0\\
(e_1,e_3,e_4):&\quad x_{14}-x_3 &&=0\\
(e_1,e_2,e_6):&\quad -x_1+x_{17}-x_{19} &&=0\\
(e_1,e_3,e_5):&\quad -x_{19}+x_3-x_8 &&=0\\
(e_1,e_2,e_7):&\quad x_1-x_{10}-x_4 &&=0\\
(e_1,e_3,e_6):&\quad -x_{10}+x_{19}-x_5 &&=0\\
(e_1,e_4,e_5):&\quad -x_5+x_8 &&=0\\
(e_1,e_2,e_8):&\quad -x_{15}+x_4-x_9 &&=0\\
(e_1,e_3,e_7):&\quad x_{10}-x_{18}-x_9 &&=0\\
(e_1,e_4,e_6):&\quad -x_{18}+x_5-x_6 &&=0\\
(e_2,e_3,e_4):&\quad x_{14}x_{15}+x_{18}-x_9 &&=0\\
(e_1,e_2,e_{12}):&\quad -x_{13}-1 &&=0\\
(e_1,e_3,e_{11}):&\quad -x_{11}-x_{13} &&=0\\
(e_1,e_4,e_{10}):&\quad -x_{11}-x_{16} &&=0\\
(e_1,e_5,e_9):&\quad -x_{12}-x_{16} &&=0\\
(e_1,e_6,e_8):&\quad -x_{12}-x_7 &&=0\\
(e_2,e_3,e_8):&\quad -x_{13}x_4-x_7+x_9 &&=0\\
(e_2,e_4,e_7):&\quad -x_1x_{11}+x_{18}+x_7 &&=0\\
(e_2,e_5,e_6):&\quad x_{12}x_2-x_{16}x_{17}+x_6 &&=0\\
(e_3,e_4,e_6):&\quad -x_{11}x_{19}+x_{12}x_{14}+x_{13}x_5 &&=0
\end{aligned}
$$

Groebner basis (19 variables, 17 linear, 1 nonlinear)

$$x_1-x_{18}+1=0$$

$$-x_{18}-x_{19}+2x_2=0$$

$$x_{18}+x_{19}+2x_3-2=0$$

$$-x_{18}+5x_{19}+2x_4=0$$

$$x_{18}+3x_{19}+2x_5-2=0$$

$$3x_{18}+3x_{19}+2x_6-2=0$$

$$x_7+1=0$$

$$x_{18}+3x_{19}+2x_8-2=0$$

$$x_{18}-5x_{19}+2x_9+2=0$$

$$2x_{10}-x_{18}-5x_{19}+2=0$$

$$x_{11} - 1 = 0$$
$$x_{12} - 1 = 0$$
$$x_{13} + 1 = 0$$
$$2x_{14} + x_{18} + x_{19} - 2 = 0$$
$$x_{15} - x_{18} + 5x_{19} - 1 = 0$$
$$x_{16} + 1 = 0$$
$$x_{17} - x_{18} - x_{19} + 1 = 0$$
$$x_{18}^2 - 4x_{18}x_{19} - 4x_{18} - 5x_{19}^2 + 16x_{19} - 4 = 0$$

$\mathfrak{m}_{3B}(5, 14)$

m3B514 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_2, e_7] &= e_{12} \\
[e_2, e_8] &= 3e_{13} & [e_2, e_{13}] &= e_{14} \\
[e_3, e_6] &= -e_{12} & [e_3, e_7] &= -2e_{13} \\
[e_3, e_{12}] &= \alpha_{3,12}^{14} e_{14} & [e_4, e_5] &= e_{12} \\
[e_4, e_6] &= e_{13} & [e_4, e_{11}] &= \alpha_{4,11}^{14} e_{14} \\
[e_5, e_{10}] &= \alpha_{5,10}^{14} e_{14} & [e_6, e_9] &= \alpha_{6,9}^{14} e_{14} \\
[e_7, e_8] &= \alpha_{7,8}^{14} e_{14}
\end{aligned}$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1, e_2, e_{12}) &: & -\alpha_{3,12}^{14} - 1 &= 0 \\
(e_1, e_3, e_{11}) &: & -\alpha_{3,12}^{14} - \alpha_{4,11}^{14} &= 0 \\
(e_1, e_4, e_{10}) &: & -\alpha_{4,11}^{14} - \alpha_{5,10}^{14} &= 0 \\
(e_1, e_5, e_9) &: & -\alpha_{5,10}^{14} - \alpha_{6,9}^{14} &= 0 \\
(e_1, e_6, e_8) &: & -\alpha_{6,9}^{14} - \alpha_{7,8}^{14} &= 0 \\
(e_2, e_3, e_7) &: & -\alpha_{3,12}^{14} - 2 &= 0 \\
(e_2, e_4, e_6) &: & \text{no solutions} & \\
(e_3, e_4, e_5) &: & \alpha_{3,12}^{14} &= 0
\end{aligned}$$

There are no solutions.

$\mathfrak{m}_{5B}(5,14)$

m5B514 (this line included for string searching purposes)

Solution 1

$$[e_1,e_2]=e_3 \qquad [e_1,e_3]=e_4$$
$$[e_1,e_4]=e_5 \qquad [e_1,e_5]=e_6$$
$$[e_1,e_6]=e_7 \qquad [e_1,e_7]=e_8$$
$$[e_1,e_8]=e_9 \qquad [e_1,e_9]=e_{10}$$
$$[e_1,e_{10}]=e_{11} \qquad [e_1,e_{11}]=e_{12}$$
$$[e_1,e_{12}]=e_{13} \qquad [e_2,e_5]=e_{10}$$
$$[e_2,e_6]=2e_{11} \qquad [e_2,e_7]=5e_{12}$$
$$[e_2,e_8]=10e_{13} \qquad [e_2,e_{13}]=e_{14}$$
$$[e_3,e_4]=-e_{10} \qquad [e_3,e_5]=-e_{11}$$
$$[e_3,e_6]=-3e_{12} \qquad [e_3,e_7]=-5e_{13}$$
$$[e_3,e_{12}]=-e_{14} \qquad [e_4,e_5]=2e_{12}$$
$$[e_4,e_6]=2e_{13} \qquad [e_4,e_{11}]=e_{14}$$
$$[e_5,e_{10}]=-e_{14} \qquad [e_6,e_9]=e_{14}$$
$$[e_7,e_8]=-e_{14}$$

Original brackets:

$$[e_1,e_2]=e_3 \qquad [e_1,e_3]=e_4$$
$$[e_1,e_4]=e_5 \qquad [e_1,e_5]=e_6$$
$$[e_1,e_6]=e_7 \qquad [e_1,e_7]=e_8$$
$$[e_1,e_8]=e_9 \qquad [e_1,e_9]=e_{10}$$
$$[e_1,e_{10}]=e_{11} \qquad [e_1,e_{11}]=e_{12}$$
$$[e_1,e_{12}]=e_{13} \qquad [e_2,e_5]=e_{10}$$
$$[e_2,e_6]=2e_{11} \qquad [e_2,e_7]=\alpha_{2,7}^{12}e_{12}$$
$$[e_2,e_8]=\alpha_{2,8}^{13}e_{13} \qquad [e_2,e_{13}]=e_{14}$$
$$[e_3,e_4]=-e_{10} \qquad [e_3,e_5]=-e_{11}$$
$$[e_3,e_6]=\alpha_{3,6}^{12}e_{12} \qquad [e_3,e_7]=\alpha_{3,7}^{13}e_{13}$$
$$[e_3,e_{12}]=\alpha_{3,12}^{14}e_{14} \qquad [e_4,e_5]=\alpha_{4,5}^{12}e_{12}$$
$$[e_4,e_6]=\alpha_{4,6}^{13}e_{13} \qquad [e_4,e_{11}]=\alpha_{4,11}^{14}e_{14}$$
$$[e_5,e_{10}]=\alpha_{5,10}^{14}e_{14} \qquad [e_6,e_9]=\alpha_{6,9}^{14}e_{14}$$
$$[e_7,e_8]=\alpha_{7,8}^{14}e_{14}$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1,e_2,e_6):&\quad -\alpha^{12}_{2,7}-\alpha^{12}_{3,6}+2 &&=0\\
(e_1,e_3,e_5):&\quad -\alpha^{12}_{3,6}-\alpha^{12}_{4,5}-1 &&=0\\
(e_1,e_2,e_7):&\quad \alpha^{12}_{2,7}-\alpha^{13}_{2,8}-\alpha^{13}_{3,7} &&=0\\
(e_1,e_3,e_6):&\quad \alpha^{12}_{3,6}-\alpha^{13}_{3,7}-\alpha^{13}_{4,6} &&=0\\
(e_1,e_4,e_5):&\quad \alpha^{12}_{4,5}-\alpha^{13}_{4,6} &&=0\\
(e_1,e_2,e_{12}):&\quad -\alpha^{14}_{3,12}-1 &&=0\\
(e_1,e_3,e_{11}):&\quad -\alpha^{14}_{3,12}-\alpha^{14}_{4,11} &&=0\\
(e_1,e_4,e_{10}):&\quad -\alpha^{14}_{4,11}-\alpha^{14}_{5,10} &&=0\\
(e_1,e_5,e_9):&\quad -\alpha^{14}_{5,10}-\alpha^{14}_{6,9} &&=0\\
(e_1,e_6,e_8):&\quad -\alpha^{14}_{6,9}-\alpha^{14}_{7,8} &&=0\\
(e_2,e_3,e_7):&\quad -\alpha^{12}_{2,7}\alpha^{14}_{3,12}+\alpha^{13}_{3,7} &&=0\\
(e_2,e_4,e_6):&\quad -2\alpha^{14}_{4,11}+\alpha^{13}_{4,6} &&=0\\
(e_3,e_4,e_5):&\quad \alpha^{14}_{3,12}\alpha^{12}_{4,5}+\alpha^{14}_{4,11}-\alpha^{14}_{5,10} &&=0
\end{aligned}$$

Solution 1:

$$\begin{aligned}
\alpha^{13}_{3,7}&=-5\\
\alpha^{12}_{2,7}&=5\\
\alpha^{13}_{4,6}&=2\\
\alpha^{12}_{3,6}&=-3\\
\alpha^{12}_{4,5}&=2\\
\alpha^{14}_{4,11}&=1\\
\alpha^{14}_{3,12}&=-1\\
\alpha^{14}_{5,10}&=-1\\
\alpha^{14}_{6,9}&=1\\
\alpha^{14}_{7,8}&=-1\\
\alpha^{13}_{2,8}&=10
\end{aligned}$$

How the solution(s) were or were not found:
Change variables

$$\begin{aligned}
\alpha^{13}_{3,7}&\to x_1\\
\alpha^{12}_{2,7}&\to x_2
\end{aligned}$$

$$\alpha^{13}_{4,6} \to x_3$$

$$\alpha^{12}_{3,6} \to x_4$$

$$\alpha^{12}_{4,5} \to x_5$$

$$\alpha^{14}_{4,11} \to x_6$$

$$\alpha^{14}_{3,12} \to x_7$$

$$\alpha^{14}_{5,10} \to x_8$$

$$\alpha^{14}_{6,9} \to x_9$$

$$\alpha^{14}_{7,8} \to x_{10}$$

$$\alpha^{13}_{2,8} \to x_{11}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_6): \quad & -x_2 - x_4 + 2 & = 0 \\
(e_1, e_3, e_5): \quad & -x_4 - x_5 - 1 & = 0 \\
(e_1, e_2, e_7): \quad & -x_1 - x_{11} + x_2 & = 0 \\
(e_1, e_3, e_6): \quad & -x_1 - x_3 + x_4 & = 0 \\
(e_1, e_4, e_5): \quad & -x_3 + x_5 & = 0 \\
(e_1, e_2, e_{12}): \quad & -x_7 - 1 & = 0 \\
(e_1, e_3, e_{11}): \quad & -x_6 - x_7 & = 0 \\
(e_1, e_4, e_{10}): \quad & -x_6 - x_8 & = 0 \\
(e_1, e_5, e_9): \quad & -x_8 - x_9 & = 0 \\
(e_1, e_6, e_8): \quad & -x_{10} - x_9 & = 0 \\
(e_2, e_3, e_7): \quad & x_1 - x_2 x_7 & = 0 \\
(e_2, e_4, e_6): \quad & x_3 - 2x_6 & = 0 \\
(e_3, e_4, e_5): \quad & x_5 x_7 + x_6 - x_8 & = 0
\end{aligned}$$

Groebner basis (11 variables, 11 linear, 0 nonlinear)

$$x_1 + 5 = 0$$

$$x_2 - 5 = 0$$

$$x_3 - 2 = 0$$

$$x_4 + 3 = 0$$

$$x_5 - 2 = 0$$

$$x_6 - 1 = 0$$

$$x_7 + 1 = 0$$

$$x_8 + 1 = 0$$

$$x_9 - 1 = 0$$

$$x_{10} + 1 = 0$$

$$x_{11} - 10 = 0$$

Solution 1:

$$x_1 = -5$$

$$x_2 = 5$$

$$x_3 = 2$$

$$x_4 = -3$$

$$x_5 = 2$$

$$x_6 = 1$$

$$x_7 = -1$$

$$x_8 = -1$$

$$x_9 = 1$$

$$x_10 = -1$$

$$x_11 = 10$$

$\mathfrak{m}_{7B}(5, 14)$

m7B514 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
&[e_1, e_2] = e_3 && [e_1, e_3] = e_4 \\
&[e_1, e_4] = e_5 && [e_1, e_5] = e_6 \\
&[e_1, e_6] = e_7 && [e_1, e_7] = e_8 \\
&[e_1, e_8] = e_9 && [e_1, e_9] = e_{10} \\
&[e_1, e_{10}] = e_{11} && [e_1, e_{11}] = e_{12} \\
&[e_1, e_{12}] = e_{13} && [e_2, e_3] = e_8 \\
&[e_2, e_4] = e_9 && [e_2, e_5] = \alpha_{2,5}^{10} e_{10} \\
&[e_2, e_6] = \alpha_{2,6}^{11} e_{11} && [e_2, e_7] = \alpha_{2,7}^{12} e_{12} \\
&[e_2, e_8] = \alpha_{2,8}^{13} e_{13} && [e_2, e_{13}] = e_{14} \\
&[e_3, e_4] = \alpha_{3,4}^{10} e_{10} && [e_3, e_5] = \alpha_{3,5}^{11} e_{11} \\
&[e_3, e_6] = \alpha_{3,6}^{12} e_{12} && [e_3, e_7] = \alpha_{3,7}^{13} e_{13} \\
&[e_3, e_{12}] = \alpha_{3,12}^{14} e_{14} && [e_4, e_5] = \alpha_{4,5}^{12} e_{12} \\
&[e_4, e_6] = \alpha_{4,6}^{13} e_{13} && [e_4, e_{11}] = \alpha_{4,11}^{14} e_{14} \\
&[e_5, e_{10}] = \alpha_{5,10}^{14} e_{14} && [e_6, e_9] = \alpha_{6,9}^{14} e_{14} \\
&[e_7, e_8] = \alpha_{7,8}^{14} e_{14}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1, e_2, e_4): &\quad -\alpha_{2,5}^{10} - \alpha_{3,4}^{10} + 1 &= 0\\
(e_1, e_2, e_5): &\quad \alpha_{2,5}^{10} - \alpha_{2,6}^{11} - \alpha_{3,5}^{11} &= 0\\
(e_1, e_3, e_4): &\quad \alpha_{3,4}^{10} - \alpha_{3,5}^{11} &= 0\\
(e_1, e_2, e_6): &\quad \alpha_{2,6}^{11} - \alpha_{2,7}^{12} - \alpha_{3,6}^{12} &= 0\\
(e_1, e_3, e_5): &\quad \alpha_{3,5}^{11} - \alpha_{3,6}^{12} - \alpha_{4,5}^{12} &= 0\\
(e_1, e_2, e_7): &\quad \alpha_{2,7}^{12} - \alpha_{2,8}^{13} - \alpha_{3,7}^{13} &= 0\\
(e_1, e_3, e_6): &\quad \alpha_{3,6}^{12} - \alpha_{3,7}^{13} - \alpha_{4,6}^{13} &= 0\\
(e_1, e_4, e_5): &\quad \alpha_{4,5}^{12} - \alpha_{4,6}^{13} &= 0\\
(e_1, e_2, e_{12}): &\quad -\alpha_{3,12}^{14} - 1 &= 0\\
(e_1, e_3, e_{11}): &\quad -\alpha_{3,12}^{14} - \alpha_{4,11}^{14} &= 0\\
(e_1, e_4, e_{10}): &\quad -\alpha_{4,11}^{14} - \alpha_{5,10}^{14} &= 0\\
(e_1, e_5, e_9): &\quad -\alpha_{5,10}^{14} - \alpha_{6,9}^{14} &= 0\\
(e_1, e_6, e_8): &\quad -\alpha_{6,9}^{14} - \alpha_{7,8}^{14} &= 0\\
(e_2, e_3, e_7): &\quad -\alpha_{2,7}^{12}\alpha_{3,12}^{14} + \alpha_{3,7}^{13} + \alpha_{7,8}^{14} &= 0\\
(e_2, e_4, e_6): &\quad -\alpha_{2,6}^{11}\alpha_{4,11}^{14} + \alpha_{4,6}^{13} + \alpha_{6,9}^{14} &= 0\\
(e_3, e_4, e_5): &\quad \alpha_{3,12}^{14}\alpha_{4,5}^{12} + \alpha_{3,4}^{10}\alpha_{5,10}^{14} - \alpha_{3,5}^{11}\alpha_{4,11}^{14} &= 0
\end{aligned}$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{2,6}^{11} \to x_1$$

$$\alpha_{3,7}^{13} \to x_2$$

$$\alpha_{2,7}^{12} \to x_3$$

$$\alpha_{4,6}^{13} \to x_4$$

$$\alpha_{3,6}^{12} \to x_5$$

$$\alpha_{3,5}^{11} \to x_6$$

$$\alpha_{4,5}^{12} \to x_7$$

$$\alpha_{4,11}^{14} \to x_8$$

$$\alpha_{3,12}^{14} \to x_9$$

$$\alpha_{5,10}^{14} \to x_{10}$$

$$\alpha_{3,4}^{10} \to x_{11}$$

$$\alpha_{6,9}^{14} \to x_{12}$$

$$\alpha_{7,8}^{14} \to x_{13}$$

$$\alpha_{2,8}^{13} \to x_{14}$$

$$\alpha_{2,5}^{10} \to x_{15}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4): &\quad -x_{11} - x_{15} + 1 &&= 0\\
(e_1, e_2, e_5): &\quad -x_1 + x_{15} - x_6 &&= 0\\
(e_1, e_3, e_4): &\quad x_{11} - x_6 &&= 0\\
(e_1, e_2, e_6): &\quad x_1 - x_3 - x_5 &&= 0\\
(e_1, e_3, e_5): &\quad -x_5 + x_6 - x_7 &&= 0\\
(e_1, e_2, e_7): &\quad -x_{14} - x_2 + x_3 &&= 0\\
(e_1, e_3, e_6): &\quad -x_2 - x_4 + x_5 &&= 0\\
(e_1, e_4, e_5): &\quad -x_4 + x_7 &&= 0\\
(e_1, e_2, e_{12}): &\quad -x_9 - 1 &&= 0\\
(e_1, e_3, e_{11}): &\quad -x_8 - x_9 &&= 0\\
(e_1, e_4, e_{10}): &\quad -x_{10} - x_8 &&= 0\\
(e_1, e_5, e_9): &\quad -x_{10} - x_{12} &&= 0\\
(e_1, e_6, e_8): &\quad -x_{12} - x_{13} &&= 0\\
(e_2, e_3, e_7): &\quad x_{13} + x_2 - x_3 x_9 &&= 0\\
(e_2, e_4, e_6): &\quad -x_1 x_8 + x_{12} + x_4 &&= 0\\
(e_3, e_4, e_5): &\quad x_{10} x_{11} - x_6 x_8 + x_7 x_9 &&= 0
\end{aligned}$$

Groebner basis (15 variables, 14 linear, 0 nonlinear)

$$x_1 - 2x_{15} + 1 = 0$$

$$5x_{15} + x_2 - 5 = 0$$

$$-5x_{15} + x_3 + 4 = 0$$

$$-2x_{15} + x_4 + 2 = 0$$

$$3x_{15} + x_5 - 3 = 0$$

$$x_{15} + x_6 - 1 = 0$$

$$-2x_{15} + x_7 + 2 = 0$$

$$x_8 - 1 = 0$$

$$x_9 + 1 = 0$$
$$x_{10} + 1 = 0$$
$$x_{11} + x_{15} - 1 = 0$$
$$x_{12} - 1 = 0$$
$$x_{13} + 1 = 0$$
$$x_{14} - 10x_{15} + 9 = 0$$

$\mathfrak{m}_{2B}(6, 14)$

m2B614 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_2, e_7] &= e_{13} \\
[e_2, e_{13}] &= e_{14} & [e_3, e_6] &= -e_{13} \\
[e_3, e_{12}] &= \alpha_{3,12}^{14} e_{14} & [e_4, e_5] &= e_{13} \\
[e_4, e_{11}] &= \alpha_{4,11}^{14} e_{14} & [e_5, e_{10}] &= \alpha_{5,10}^{14} e_{14} \\
[e_6, e_9] &= \alpha_{6,9}^{14} e_{14} & [e_7, e_8] &= \alpha_{7,8}^{14} e_{14}
\end{aligned}$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1, e_2, e_{12}) &: \quad -\alpha_{3,12}^{14} - 1 &&= 0 \\
(e_1, e_3, e_{11}) &: \quad -\alpha_{3,12}^{14} - \alpha_{4,11}^{14} &&= 0 \\
(e_1, e_4, e_{10}) &: \quad -\alpha_{4,11}^{14} - \alpha_{5,10}^{14} &&= 0 \\
(e_1, e_5, e_9) &: \quad -\alpha_{5,10}^{14} - \alpha_{6,9}^{14} &&= 0 \\
(e_1, e_6, e_8) &: \quad -\alpha_{6,9}^{14} - \alpha_{7,8}^{14} &&= 0 \\
(e_2, e_3, e_6) &: \quad \text{no solutions} \\
(e_2, e_4, e_5) &: \quad \text{no solutions}
\end{aligned}$$

There are no solutions.

$\mathfrak{m}_{4B}(6,14)$

m4B614 (this line included for string searching purposes)

Solution 1

$$[e_1,e_2]=e_3 \qquad [e_1,e_3]=e_4$$
$$[e_1,e_4]=e_5 \qquad [e_1,e_5]=e_6$$
$$[e_1,e_6]=e_7 \qquad [e_1,e_7]=e_8$$
$$[e_1,e_8]=e_9 \qquad [e_1,e_9]=e_{10}$$
$$[e_1,e_{10}]=e_{11} \qquad [e_1,e_{11}]=e_{12}$$
$$[e_1,e_{12}]=e_{13} \qquad [e_2,e_5]=e_{11}$$
$$[e_2,e_6]=2e_{12} \qquad [e_2,e_7]=4e_{13}$$
$$[e_2,e_{13}]=e_{14} \qquad [e_3,e_4]=-e_{11}$$
$$[e_3,e_5]=-e_{12} \qquad [e_3,e_6]=-2e_{13}$$
$$[e_3,e_{12}]=-e_{14} \qquad [e_4,e_5]=e_{13}$$
$$[e_4,e_{11}]=e_{14} \qquad [e_5,e_{10}]=-e_{14}$$
$$[e_6,e_9]=e_{14} \qquad [e_7,e_8]=-e_{14}$$

Original brackets:

$$[e_1,e_2]=e_3 \qquad [e_1,e_3]=e_4$$
$$[e_1,e_4]=e_5 \qquad [e_1,e_5]=e_6$$
$$[e_1,e_6]=e_7 \qquad [e_1,e_7]=e_8$$
$$[e_1,e_8]=e_9 \qquad [e_1,e_9]=e_{10}$$
$$[e_1,e_{10}]=e_{11} \qquad [e_1,e_{11}]=e_{12}$$
$$[e_1,e_{12}]=e_{13} \qquad [e_2,e_5]=e_{11}$$
$$[e_2,e_6]=2e_{12} \qquad [e_2,e_7]=\alpha_{2,7}^{13}e_{13}$$
$$[e_2,e_{13}]=e_{14} \qquad [e_3,e_4]=-e_{11}$$
$$[e_3,e_5]=-e_{12} \qquad [e_3,e_6]=\alpha_{3,6}^{13}e_{13}$$
$$[e_3,e_{12}]=\alpha_{3,12}^{14}e_{14} \qquad [e_4,e_5]=\alpha_{4,5}^{13}e_{13}$$
$$[e_4,e_{11}]=\alpha_{4,11}^{14}e_{14} \qquad [e_5,e_{10}]=\alpha_{5,10}^{14}e_{14}$$
$$[e_6,e_9]=\alpha_{6,9}^{14}e_{14} \qquad [e_7,e_8]=\alpha_{7,8}^{14}e_{14}$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1, e_2, e_6): &\quad -\alpha_{2,7}^{13} - \alpha_{3,6}^{13} + 2 &&= 0\\
(e_1, e_3, e_5): &\quad -\alpha_{3,6}^{13} - \alpha_{4,5}^{13} - 1 &&= 0\\
(e_1, e_2, e_{12}): &\quad -\alpha_{3,12}^{14} - 1 &&= 0\\
(e_1, e_3, e_{11}): &\quad -\alpha_{3,12}^{14} - \alpha_{4,11}^{14} &&= 0\\
(e_1, e_4, e_{10}): &\quad -\alpha_{4,11}^{14} - \alpha_{5,10}^{14} &&= 0\\
(e_1, e_5, e_9): &\quad -\alpha_{5,10}^{14} - \alpha_{6,9}^{14} &&= 0\\
(e_1, e_6, e_8): &\quad -\alpha_{6,9}^{14} - \alpha_{7,8}^{14} &&= 0\\
(e_2, e_3, e_6): &\quad -2\alpha_{3,12}^{14} + \alpha_{3,6}^{13} &&= 0\\
(e_2, e_4, e_5): &\quad -\alpha_{4,11}^{14} + \alpha_{4,5}^{13} &&= 0
\end{aligned}$$

Solution 1:

$$\begin{aligned}
\alpha_{3,6}^{13} &= -2\\
\alpha_{4,5}^{13} &= 1\\
\alpha_{2,7}^{13} &= 4\\
\alpha_{4,11}^{14} &= 1\\
\alpha_{3,12}^{14} &= -1\\
\alpha_{5,10}^{14} &= -1\\
\alpha_{6,9}^{14} &= 1\\
\alpha_{7,8}^{14} &= -1
\end{aligned}$$

How the solution(s) were or were not found:
Change variables

$$\alpha_{3,6}^{13} \to x_1$$

$$\alpha_{4,5}^{13} \to x_2$$

$$\alpha_{2,7}^{13} \to x_3$$

$$\alpha_{4,11}^{14} \to x_4$$

$$\alpha_{3,12}^{14} \to x_5$$

$$\alpha_{5,10}^{14} \to x_6$$

$$\alpha_{6,9}^{14} \to x_7$$

$$\alpha_{7,8}^{14} \to x_8$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_6):&\quad -x_1 - x_3 + 2 &= 0\\
(e_1, e_3, e_5):&\quad -x_1 - x_2 - 1 &= 0\\
(e_1, e_2, e_{12}):&\quad -x_5 - 1 &= 0\\
(e_1, e_3, e_{11}):&\quad -x_4 - x_5 &= 0\\
(e_1, e_4, e_{10}):&\quad -x_4 - x_6 &= 0\\
(e_1, e_5, e_9):&\quad -x_6 - x_7 &= 0\\
(e_1, e_6, e_8):&\quad -x_7 - x_8 &= 0\\
(e_2, e_3, e_6):&\quad x_1 - 2x_5 &= 0\\
(e_2, e_4, e_5):&\quad x_2 - x_4 &= 0
\end{aligned}$$

Groebner basis (8 variables, 8 linear, 0 nonlinear)

$$x_1 + 2 = 0$$
$$x_2 - 1 = 0$$
$$x_3 - 4 = 0$$
$$x_4 - 1 = 0$$
$$x_5 + 1 = 0$$
$$x_6 + 1 = 0$$
$$x_7 - 1 = 0$$
$$x_8 + 1 = 0$$

Solution 1:

$$x_1 = -2$$
$$x_2 = 1$$
$$x_3 = 4$$
$$x_4 = 1$$
$$x_5 = -1$$
$$x_6 = -1$$
$$x_7 = 1$$
$$x_8 = -1$$

$\mathfrak{m}_{6B}(6, 14)$

m6B614 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_2, e_3] &= e_9 \\
[e_2, e_4] &= e_{10} & [e_2, e_5] &= \alpha_{2,5}^{11} e_{11} \\
[e_2, e_6] &= \alpha_{2,6}^{12} e_{12} & [e_2, e_7] &= \alpha_{2,7}^{13} e_{13} \\
[e_2, e_{13}] &= e_{14} & [e_3, e_4] &= \alpha_{3,4}^{11} e_{11} \\
[e_3, e_5] &= \alpha_{3,5}^{12} e_{12} & [e_3, e_6] &= \alpha_{3,6}^{13} e_{13} \\
[e_3, e_{12}] &= \alpha_{3,12}^{14} e_{14} & [e_4, e_5] &= \alpha_{4,5}^{13} e_{13} \\
[e_4, e_{11}] &= \alpha_{4,11}^{14} e_{14} & [e_5, e_{10}] &= \alpha_{5,10}^{14} e_{14} \\
[e_6, e_9] &= \alpha_{6,9}^{14} e_{14} & [e_7, e_8] &= \alpha_{7,8}^{14} e_{14}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_4):&\quad -\alpha_{2,5}^{11} - \alpha_{3,4}^{11} + 1 &&= 0 \\
(e_1, e_2, e_5):&\quad \alpha_{2,5}^{11} - \alpha_{2,6}^{12} - \alpha_{3,5}^{12} &&= 0 \\
(e_1, e_3, e_4):&\quad \alpha_{3,4}^{11} - \alpha_{3,5}^{12} &&= 0 \\
(e_1, e_2, e_6):&\quad \alpha_{2,6}^{12} - \alpha_{2,7}^{13} - \alpha_{3,6}^{13} &&= 0 \\
(e_1, e_3, e_5):&\quad \alpha_{3,5}^{12} - \alpha_{3,6}^{13} - \alpha_{4,5}^{13} &&= 0 \\
(e_1, e_2, e_{12}):&\quad -\alpha_{3,12}^{14} - 1 &&= 0 \\
(e_1, e_3, e_{11}):&\quad -\alpha_{3,12}^{14} - \alpha_{4,11}^{14} &&= 0 \\
(e_1, e_4, e_{10}):&\quad -\alpha_{4,11}^{14} - \alpha_{5,10}^{14} &&= 0 \\
(e_1, e_5, e_9):&\quad -\alpha_{5,10}^{14} - \alpha_{6,9}^{14} &&= 0 \\
(e_1, e_6, e_8):&\quad -\alpha_{6,9}^{14} - \alpha_{7,8}^{14} &&= 0 \\
(e_2, e_3, e_6):&\quad -\alpha_{2,6}^{12}\alpha_{3,12}^{14} + \alpha_{3,6}^{13} + \alpha_{6,9}^{14} &&= 0 \\
(e_2, e_4, e_5):&\quad -\alpha_{2,5}^{11}\alpha_{4,11}^{14} + \alpha_{4,5}^{13} + \alpha_{5,10}^{14} &&= 0
\end{aligned}
$$

Infinite number of solutions.

How the solution(s) were or were not found:

Change variables

$$\alpha^{13}_{3,6} \to x_1$$
$$\alpha^{13}_{4,5} \to x_2$$
$$\alpha^{13}_{2,7} \to x_3$$
$$\alpha^{12}_{3,5} \to x_4$$
$$\alpha^{11}_{2,5} \to x_5$$
$$\alpha^{14}_{4,11} \to x_6$$
$$\alpha^{12}_{2,6} \to x_7$$
$$\alpha^{14}_{3,12} \to x_8$$
$$\alpha^{14}_{5,10} \to x_9$$
$$\alpha^{11}_{3,4} \to x_{10}$$
$$\alpha^{14}_{6,9} \to x_{11}$$
$$\alpha^{14}_{7,8} \to x_{12}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4):&\quad -x_{10} - x_5 + 1 &&= 0\\
(e_1, e_2, e_5):&\quad -x_4 + x_5 - x_7 &&= 0\\
(e_1, e_3, e_4):&\quad x_{10} - x_4 &&= 0\\
(e_1, e_2, e_6):&\quad -x_1 - x_3 + x_7 &&= 0\\
(e_1, e_3, e_5):&\quad -x_1 - x_2 + x_4 &&= 0\\
(e_1, e_2, e_{12}):&\quad -x_8 - 1 &&= 0\\
(e_1, e_3, e_{11}):&\quad -x_6 - x_8 &&= 0\\
(e_1, e_4, e_{10}):&\quad -x_6 - x_9 &&= 0\\
(e_1, e_5, e_9):&\quad -x_{11} - x_9 &&= 0\\
(e_1, e_6, e_8):&\quad -x_{11} - x_{12} &&= 0\\
(e_2, e_3, e_6):&\quad x_1 + x_{11} - x_7 x_8 &&= 0\\
(e_2, e_4, e_5):&\quad x_2 - x_5 x_6 + x_9 &&= 0
\end{aligned}$$

Groebner basis (12 variables, 11 linear, 0 nonlinear)

$$x_1 - 2x_{10} + 2 = 0$$
$$x_{10} + x_2 - 2 = 0$$

$$4x_{10} + x_3 - 3 = 0$$
$$-x_{10} + x_4 = 0$$
$$x_{10} + x_5 - 1 = 0$$
$$x_6 - 1 = 0$$
$$2x_{10} + x_7 - 1 = 0$$
$$x_8 + 1 = 0$$
$$x_9 + 1 = 0$$
$$x_{11} - 1 = 0$$
$$x_{12} + 1 = 0$$

$\mathfrak{m}_{3B}(7, 14)$

m3B714 (this line included for string searching purposes)

Solution 1

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_2, e_5] &= e_{12} \\
[e_2, e_6] &= 2e_{13} & [e_2, e_{13}] &= e_{14} \\
[e_3, e_4] &= -e_{12} & [e_3, e_5] &= -e_{13} \\
[e_3, e_{12}] &= -e_{14} & [e_4, e_{11}] &= e_{14} \\
[e_5, e_{10}] &= -e_{14} & [e_6, e_9] &= e_{14} \\
[e_7, e_8] &= -e_{14}
\end{aligned}
$$

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_2, e_5] &= e_{12} \\
[e_2, e_6] &= 2e_{13} & [e_2, e_{13}] &= e_{14} \\
[e_3, e_4] &= -e_{12} & [e_3, e_5] &= -e_{13} \\
[e_3, e_{12}] &= \alpha_{3,12}^{14} e_{14} & [e_4, e_{11}] &= \alpha_{4,11}^{14} e_{14} \\
[e_5, e_{10}] &= \alpha_{5,10}^{14} e_{14} & [e_6, e_9] &= \alpha_{6,9}^{14} e_{14} \\
[e_7, e_8] &= \alpha_{7,8}^{14} e_{14}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_{12}): \quad & -\alpha_{3,12}^{14} - 1 &= 0 \\
(e_1, e_3, e_{11}): \quad & -\alpha_{3,12}^{14} - \alpha_{4,11}^{14} &= 0 \\
(e_1, e_4, e_{10}): \quad & -\alpha_{4,11}^{14} - \alpha_{5,10}^{14} &= 0 \\
(e_1, e_5, e_9): \quad & -\alpha_{5,10}^{14} - \alpha_{6,9}^{14} &= 0 \\
(e_1, e_6, e_8): \quad & -\alpha_{6,9}^{14} - \alpha_{7,8}^{14} &= 0 \\
(e_2, e_3, e_5): \quad & -\alpha_{3,12}^{14} - 1 &= 0
\end{aligned}
$$

Solution 1:

$$
\begin{aligned}
\alpha_{4,11}^{14} &= 1 \\
\alpha_{3,12}^{14} &= -1 \\
\alpha_{5,10}^{14} &= -1 \\
\alpha_{6,9}^{14} &= 1 \\
\alpha_{7,8}^{14} &= -1
\end{aligned}
$$

How the solution(s) were or were not found:
Change variables

$$
\begin{aligned}
\alpha_{4,11}^{14} &\to x_1 \\
\alpha_{3,12}^{14} &\to x_2
\end{aligned}
$$

$$\alpha_{5,10}^{14} \to x_3$$

$$\alpha_{6,9}^{14} \to x_4$$

$$\alpha_{7,8}^{14} \to x_5$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_{12}): \quad & -x_2 - 1 & = 0 \\
(e_1, e_3, e_{11}): \quad & -x_1 - x_2 & = 0 \\
(e_1, e_4, e_{10}): \quad & -x_1 - x_3 & = 0 \\
(e_1, e_5, e_9): \quad & -x_3 - x_4 & = 0 \\
(e_1, e_6, e_8): \quad & -x_4 - x_5 & = 0 \\
(e_2, e_3, e_5): \quad & -x_2 - 1 & = 0
\end{aligned}$$

Groebner basis (5 variables, 5 linear, 0 nonlinear)

$$x_1 - 1 = 0$$

$$x_2 + 1 = 0$$

$$x_3 + 1 = 0$$

$$x_4 - 1 = 0$$

$$x_5 + 1 = 0$$

Solution 1:

$$x_1 = 1$$

$$x_2 = -1$$

$$x_3 = -1$$

$$x_4 = 1$$

$$x_5 = -1$$

$\mathfrak{m}_{5B}(7, 14)$

m5B714 (this line included for string searching purposes)

Original brackets:

$$
\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_1, e_9] = e_{10} \\
[e_1, e_{10}] = e_{11} & [e_1, e_{11}] = e_{12} \\
[e_1, e_{12}] = e_{13} & [e_2, e_3] = e_{10} \\
[e_2, e_4] = e_{11} & [e_2, e_5] = \alpha_{2,5}^{12} e_{12} \\
[e_2, e_6] = \alpha_{2,6}^{13} e_{13} & [e_2, e_{13}] = e_{14} \\
[e_3, e_4] = \alpha_{3,4}^{12} e_{12} & [e_3, e_5] = \alpha_{3,5}^{13} e_{13} \\
[e_3, e_{12}] = \alpha_{3,12}^{14} e_{14} & [e_4, e_{11}] = \alpha_{4,11}^{14} e_{14} \\
[e_5, e_{10}] = \alpha_{5,10}^{14} e_{14} & [e_6, e_9] = \alpha_{6,9}^{14} e_{14} \\
[e_7, e_8] = \alpha_{7,8}^{14} e_{14} &
\end{array}
$$

Non-trivial Jacobi Tests:

$$
\begin{array}{lll}
(e_1, e_2, e_4): & -\alpha_{2,5}^{12} - \alpha_{3,4}^{12} + 1 & = 0 \\
(e_1, e_2, e_5): & \alpha_{2,5}^{12} - \alpha_{2,6}^{13} - \alpha_{3,5}^{13} & = 0 \\
(e_1, e_3, e_4): & \alpha_{3,4}^{12} - \alpha_{3,5}^{13} & = 0 \\
(e_1, e_2, e_{12}): & -\alpha_{3,12}^{14} - 1 & = 0 \\
(e_1, e_3, e_{11}): & -\alpha_{3,12}^{14} - \alpha_{4,11}^{14} & = 0 \\
(e_1, e_4, e_{10}): & -\alpha_{4,11}^{14} - \alpha_{5,10}^{14} & = 0 \\
(e_1, e_5, e_9): & -\alpha_{5,10}^{14} - \alpha_{6,9}^{14} & = 0 \\
(e_1, e_6, e_8): & -\alpha_{6,9}^{14} - \alpha_{7,8}^{14} & = 0 \\
(e_2, e_3, e_5): & -\alpha_{2,5}^{12}\alpha_{3,12}^{14} + \alpha_{3,5}^{13} + \alpha_{5,10}^{14} & = 0
\end{array}
$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$
\begin{gathered}
\alpha_{2,6}^{13} \to x_1 \\
\alpha_{3,5}^{13} \to x_2 \\
\alpha_{3,4}^{12} \to x_3 \\
\alpha_{2,5}^{12} \to x_4 \\
\alpha_{4,11}^{14} \to x_5
\end{gathered}
$$

$$\alpha_{3,12}^{14} \to x_6$$

$$\alpha_{5,10}^{14} \to x_7$$

$$\alpha_{6,9}^{14} \to x_8$$

$$\alpha_{7,8}^{14} \to x_9$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4): &\quad -x_3 - x_4 + 1 &= 0\\
(e_1, e_2, e_5): &\quad -x_1 - x_2 + x_4 &= 0\\
(e_1, e_3, e_4): &\quad -x_2 + x_3 &= 0\\
(e_1, e_2, e_{12}): &\quad -x_6 - 1 &= 0\\
(e_1, e_3, e_{11}): &\quad -x_5 - x_6 &= 0\\
(e_1, e_4, e_{10}): &\quad -x_5 - x_7 &= 0\\
(e_1, e_5, e_9): &\quad -x_7 - x_8 &= 0\\
(e_1, e_6, e_8): &\quad -x_8 - x_9 &= 0\\
(e_2, e_3, e_5): &\quad x_2 - x_4x_6 + x_7 &= 0
\end{aligned}$$

Groebner basis (9 variables, 8 linear, 0 nonlinear)

$$x_1 - 2x_4 + 1 = 0$$

$$x_2 + x_4 - 1 = 0$$

$$x_3 + x_4 - 1 = 0$$

$$x_5 - 1 = 0$$

$$x_6 + 1 = 0$$

$$x_7 + 1 = 0$$

$$x_8 - 1 = 0$$

$$x_9 + 1 = 0$$

$\mathfrak{m}_{2B}(8,14)$

m2B814 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4 \\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6 \\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8 \\
[e_1,e_8] &= e_9 & [e_1,e_9] &= e_{10} \\
[e_1,e_{10}] &= e_{11} & [e_1,e_{11}] &= e_{12} \\
[e_1,e_{12}] &= e_{13} & [e_2,e_5] &= e_{13} \\
[e_2,e_{13}] &= e_{14} & [e_3,e_4] &= -e_{13} \\
[e_3,e_{12}] &= \alpha_{3,12}^{14} e_{14} & [e_4,e_{11}] &= \alpha_{4,11}^{14} e_{14} \\
[e_5,e_{10}] &= \alpha_{5,10}^{14} e_{14} & [e_6,e_9] &= \alpha_{6,9}^{14} e_{14} \\
[e_7,e_8] &= \alpha_{7,8}^{14} e_{14}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1,e_2,e_{12}):& \quad -\alpha_{3,12}^{14} - 1 &&= 0 \\
(e_1,e_3,e_{11}):& \quad -\alpha_{3,12}^{14} - \alpha_{4,11}^{14} &&= 0 \\
(e_1,e_4,e_{10}):& \quad -\alpha_{4,11}^{14} - \alpha_{5,10}^{14} &&= 0 \\
(e_1,e_5,e_9):& \quad -\alpha_{5,10}^{14} - \alpha_{6,9}^{14} &&= 0 \\
(e_1,e_6,e_8):& \quad -\alpha_{6,9}^{14} - \alpha_{7,8}^{14} &&= 0 \\
(e_2,e_3,e_4):& \quad \text{no solutions}
\end{aligned}
$$

There are no solutions.

$\mathfrak{m}_{4B}(8,14)$

m4B814 (this line included for string searching purposes)

Solution 1

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_2, e_3] &= e_{11} \\
[e_2, e_4] &= e_{12} & [e_2, e_5] &= 3e_{13} \\
[e_2, e_{13}] &= e_{14} & [e_3, e_4] &= -2e_{13} \\
[e_3, e_{12}] &= -e_{14} & [e_4, e_{11}] &= e_{14} \\
[e_5, e_{10}] &= -e_{14} & [e_6, e_9] &= e_{14} \\
[e_7, e_8] &= -e_{14}
\end{aligned}
$$

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_2, e_3] &= e_{11} \\
[e_2, e_4] &= e_{12} & [e_2, e_5] &= \alpha_{2,5}^{13} e_{13} \\
[e_2, e_{13}] &= e_{14} & [e_3, e_4] &= \alpha_{3,4}^{13} e_{13} \\
[e_3, e_{12}] &= \alpha_{3,12}^{14} e_{14} & [e_4, e_{11}] &= \alpha_{4,11}^{14} e_{14} \\
[e_5, e_{10}] &= \alpha_{5,10}^{14} e_{14} & [e_6, e_9] &= \alpha_{6,9}^{14} e_{14} \\
[e_7, e_8] &= \alpha_{7,8}^{14} e_{14}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_4): &\quad -\alpha_{2,5}^{13} - \alpha_{3,4}^{13} + 1 &&= 0 \\
(e_1, e_2, e_{12}): &\quad -\alpha_{3,12}^{14} - 1 &&= 0 \\
(e_1, e_3, e_{11}): &\quad -\alpha_{3,12}^{14} - \alpha_{4,11}^{14} &&= 0 \\
(e_1, e_4, e_{10}): &\quad -\alpha_{4,11}^{14} - \alpha_{5,10}^{14} &&= 0 \\
(e_1, e_5, e_9): &\quad -\alpha_{5,10}^{14} - \alpha_{6,9}^{14} &&= 0 \\
(e_1, e_6, e_8): &\quad -\alpha_{6,9}^{14} - \alpha_{7,8}^{14} &&= 0 \\
(e_2, e_3, e_4): &\quad -\alpha_{3,12}^{14} + \alpha_{3,4}^{13} + \alpha_{4,11}^{14} &&= 0
\end{aligned}
$$

Solution 1:

$$\alpha^{13}_{2,5} = 3$$
$$\alpha^{13}_{3,4} = -2$$
$$\alpha^{14}_{4,11} = 1$$
$$\alpha^{14}_{3,12} = -1$$
$$\alpha^{14}_{5,10} = -1$$
$$\alpha^{14}_{6,9} = 1$$
$$\alpha^{14}_{7,8} = -1$$

How the solution(s) were or were not found:
Change variables

$$\alpha^{13}_{2,5} \to x_1$$
$$\alpha^{13}_{3,4} \to x_2$$
$$\alpha^{14}_{4,11} \to x_3$$
$$\alpha^{14}_{3,12} \to x_4$$
$$\alpha^{14}_{5,10} \to x_5$$
$$\alpha^{14}_{6,9} \to x_6$$
$$\alpha^{14}_{7,8} \to x_7$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4):&\quad -x_1 - x_2 + 1 &&= 0\\
(e_1, e_2, e_{12}):&\quad -x_4 - 1 &&= 0\\
(e_1, e_3, e_{11}):&\quad -x_3 - x_4 &&= 0\\
(e_1, e_4, e_{10}):&\quad -x_3 - x_5 &&= 0\\
(e_1, e_5, e_9):&\quad -x_5 - x_6 &&= 0\\
(e_1, e_6, e_8):&\quad -x_6 - x_7 &&= 0\\
(e_2, e_3, e_4):&\quad x_2 + x_3 - x_4 &&= 0
\end{aligned}$$

Groebner basis (7 variables, 7 linear, 0 nonlinear)

$$x_1 - 3 = 0$$
$$x_2 + 2 = 0$$

$$x_3 - 1 = 0$$

$$x_4 + 1 = 0$$

$$x_5 + 1 = 0$$

$$x_6 - 1 = 0$$

$$x_7 + 1 = 0$$

Solution 1:

$$x_1 = 3$$

$$x_2 = -2$$

$$x_3 = 1$$

$$x_4 = -1$$

$$x_5 = -1$$

$$x_6 = 1$$

$$x_7 = -1$$

$\mathfrak{m}_{3B}(9, 14)$

m3B914 (this line included for string searching purposes)

Solution 1

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_1, e_{11}] = e_{12}$$
$$[e_1, e_{12}] = e_{13} \qquad [e_2, e_3] = e_{12}$$
$$[e_2, e_4] = e_{13} \qquad [e_2, e_{13}] = e_{14}$$
$$[e_3, e_{12}] = -e_{14} \qquad [e_4, e_{11}] = e_{14}$$
$$[e_5, e_{10}] = -e_{14} \qquad [e_6, e_9] = e_{14}$$
$$[e_7, e_8] = -e_{14}$$

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_2, e_3] &= e_{12} \\
[e_2, e_4] &= e_{13} & [e_2, e_{13}] &= e_{14} \\
[e_3, e_{12}] &= \alpha_{3,12}^{14} e_{14} & [e_4, e_{11}] &= \alpha_{4,11}^{14} e_{14} \\
[e_5, e_{10}] &= \alpha_{5,10}^{14} e_{14} & [e_6, e_9] &= \alpha_{6,9}^{14} e_{14} \\
[e_7, e_8] &= \alpha_{7,8}^{14} e_{14}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_{12}):& \quad -\alpha_{3,12}^{14} - 1 &= 0 \\
(e_1, e_3, e_{11}):& \quad -\alpha_{3,12}^{14} - \alpha_{4,11}^{14} &= 0 \\
(e_1, e_4, e_{10}):& \quad -\alpha_{4,11}^{14} - \alpha_{5,10}^{14} &= 0 \\
(e_1, e_5, e_9):& \quad -\alpha_{5,10}^{14} - \alpha_{6,9}^{14} &= 0 \\
(e_1, e_6, e_8):& \quad -\alpha_{6,9}^{14} - \alpha_{7,8}^{14} &= 0
\end{aligned}
$$

Solution 1:

$$
\begin{aligned}
\alpha_{4,11}^{14} &= 1 \\
\alpha_{3,12}^{14} &= -1 \\
\alpha_{5,10}^{14} &= -1 \\
\alpha_{6,9}^{14} &= 1 \\
\alpha_{7,8}^{14} &= -1
\end{aligned}
$$

How the solution(s) were or were not found:
Change variables

$$
\begin{aligned}
\alpha_{4,11}^{14} &\to x_1 \\
\alpha_{3,12}^{14} &\to x_2 \\
\alpha_{5,10}^{14} &\to x_3 \\
\alpha_{6,9}^{14} &\to x_4
\end{aligned}
$$

$$\alpha_{7,8}^{14} \to x_5$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_{12}): \quad & -x_2 - 1 && = 0 \\
(e_1, e_3, e_{11}): \quad & -x_1 - x_2 && = 0 \\
(e_1, e_4, e_{10}): \quad & -x_1 - x_3 && = 0 \\
(e_1, e_5, e_9): \quad & -x_3 - x_4 && = 0 \\
(e_1, e_6, e_8): \quad & -x_4 - x_5 && = 0
\end{aligned}$$

Groebner basis (5 variables, 5 linear, 0 nonlinear)

$$x_1 - 1 = 0$$
$$x_2 + 1 = 0$$
$$x_3 + 1 = 0$$
$$x_4 - 1 = 0$$
$$x_5 + 1 = 0$$

Solution 1:

$$x_1 = 1$$
$$x_2 = -1$$
$$x_3 = -1$$
$$x_4 = 1$$
$$x_5 = -1$$

$\mathfrak{m}_{2B}(10, 14)$

m2B1014 (this line included for string searching purposes)

Solution 1

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_2, e_3] &= e_{13} \\
[e_2, e_{13}] &= e_{14} & [e_3, e_{12}] &= -e_{14} \\
[e_4, e_{11}] &= e_{14} & [e_5, e_{10}] &= -e_{14} \\
[e_6, e_9] &= e_{14} & [e_7, e_8] &= -e_{14}
\end{aligned}$$

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_2, e_3] &= e_{13} \\
[e_2, e_{13}] &= e_{14} & [e_3, e_{12}] &= \alpha_{3,12}^{14} e_{14} \\
[e_4, e_{11}] &= \alpha_{4,11}^{14} e_{14} & [e_5, e_{10}] &= \alpha_{5,10}^{14} e_{14} \\
[e_6, e_9] &= \alpha_{6,9}^{14} e_{14} & [e_7, e_8] &= \alpha_{7,8}^{14} e_{14}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_{12}): &\quad -\alpha_{3,12}^{14} - 1 &= 0 \\
(e_1, e_3, e_{11}): &\quad -\alpha_{3,12}^{14} - \alpha_{4,11}^{14} &= 0 \\
(e_1, e_4, e_{10}): &\quad -\alpha_{4,11}^{14} - \alpha_{5,10}^{14} &= 0 \\
(e_1, e_5, e_9): &\quad -\alpha_{5,10}^{14} - \alpha_{6,9}^{14} &= 0 \\
(e_1, e_6, e_8): &\quad -\alpha_{6,9}^{14} - \alpha_{7,8}^{14} &= 0
\end{aligned}
$$

Solution 1:

$$
\begin{aligned}
\alpha_{4,11}^{14} &= 1 \\
\alpha_{3,12}^{14} &= -1 \\
\alpha_{5,10}^{14} &= -1 \\
\alpha_{6,9}^{14} &= 1 \\
\alpha_{7,8}^{14} &= -1
\end{aligned}
$$

How the solution(s) were or were not found:
Change variables

$$
\begin{aligned}
\alpha_{4,11}^{14} &\to x_1 \\
\alpha_{3,12}^{14} &\to x_2 \\
\alpha_{5,10}^{14} &\to x_3 \\
\alpha_{6,9}^{14} &\to x_4
\end{aligned}
$$

$$\alpha_{7,8}^{14} \to x_5$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_{12}): &\quad -x_2 - 1 &&= 0 \\
(e_1, e_3, e_{11}): &\quad -x_1 - x_2 &&= 0 \\
(e_1, e_4, e_{10}): &\quad -x_1 - x_3 &&= 0 \\
(e_1, e_5, e_9): &\quad -x_3 - x_4 &&= 0 \\
(e_1, e_6, e_8): &\quad -x_4 - x_5 &&= 0
\end{aligned}$$

Groebner basis (5 variables, 5 linear, 0 nonlinear)

$$x_1 - 1 = 0$$

$$x_2 + 1 = 0$$

$$x_3 + 1 = 0$$

$$x_4 - 1 = 0$$

$$x_5 + 1 = 0$$

Solution 1:

$$x_1 = 1$$

$$x_2 = -1$$

$$x_3 = -1$$

$$x_4 = 1$$

$$x_5 = -1$$