search	algebra	Jac	sol
m1A25	$\mathfrak{m}_{1A}(2,5)$	$\surd$	1
m2A26	$\mathfrak{m}_{2A}(2,6)$	$\surd$	1
m1A36	$\mathfrak{m}_{1A}(3,6)$	$\surd$	1
m1A27	$\mathfrak{m}_{1A}(2,7)$	$\surd$	1
m3A27	$\mathfrak{m}_{3A}(2,7)$	$\surd$	∞
m2A37	$\mathfrak{m}_{2A}(3,7)$	$\surd$	1
m1A47	$\mathfrak{m}_{1A}(4,7)$	$\surd$	1
m2A28	$\mathfrak{m}_{2A}(2,8)$	$\surd$	1
m4A28	$\mathfrak{m}_{4A}(2,8)$	$\surd$	∞
m1A38	$\mathfrak{m}_{1A}(3,8)$	$\surd$	1
m3A38	$\mathfrak{m}_{3A}(3,8)$	$\surd$	∞
m2A48	$\mathfrak{m}_{2A}(4,8)$	$\surd$	1
m1A58	$\mathfrak{m}_{1A}(5,8)$	$\surd$	1
m1A29	$\mathfrak{m}_{1A}(2,9)$	$\surd$	1
m3A29	$\mathfrak{m}_{3A}(2,9)$	$\surd$	1
m5A29	$\mathfrak{m}_{5A}(2,9)$	$\surd$	∞
m2A39	$\mathfrak{m}_{2A}(3,9)$	$\surd$	1
m4A39	$\mathfrak{m}_{4A}(3,9)$	$\surd$	∞
m1A49	$\mathfrak{m}_{1A}(4,9)$	$\surd$	1
m3A49	$\mathfrak{m}_{3A}(4,9)$	$\surd$	∞
m2A59	$\mathfrak{m}_{2A}(5,9)$	$\surd$	1
m1A69	$\mathfrak{m}_{1A}(6,9)$	$\surd$	1
m2A210	$\mathfrak{m}_{2A}(2,10)$	$\surd$	1
m4A210	$\mathfrak{m}_{4A}(2,10)$	$\surd$	1
m6A210	$\mathfrak{m}_{6A}(2,10)$	$\surd$	∞
m1A310	$\mathfrak{m}_{1A}(3,10)$	$\surd$	1
m3A310	$\mathfrak{m}_{3A}(3,10)$	$\surd$	∞
m5A310	$\mathfrak{m}_{5A}(3,10)$	$\surd$	∞
m2A410	$\mathfrak{m}_{2A}(4,10)$	$\surd$	1
m4A410	$\mathfrak{m}_{4A}(4,10)$	$\surd$	∞
m1A510	$\mathfrak{m}_{1A}(5,10)$	$\surd$	1
m3A510	$\mathfrak{m}_{3A}(5,10)$	$\surd$	∞
m2A610	$\mathfrak{m}_{2A}(6,10)$	$\surd$	1
m1A710	$\mathfrak{m}_{1A}(7,10)$	$\surd$	1
m1A211	$\mathfrak{m}_{1A}(2,11)$	$\surd$	1
m3A211	$\mathfrak{m}_{3A}(2,11)$	$\surd$	1
m5A211	$\mathfrak{m}_{5A}(2,11)$	$\surd$	1
m7A211	$\mathfrak{m}_{7A}(2,11)$	$\surd$	∞
m2A311	$\mathfrak{m}_{2A}(3,11)$	$\surd$	1
m4A311	$\mathfrak{m}_{4A}(3,11)$	$\surd$	1
m6A311	$\mathfrak{m}_{6A}(3,11)$	$\surd$	∞
m1A411	$\mathfrak{m}_{1A}(4,11)$	$\surd$	1

search	algebra	Jac	sol
m3A411	$\mathfrak{m}_{3A}(4,11)$	$\surd$	∞
m5A411	$\mathfrak{m}_{5A}(4,11)$	$\surd$	∞
m2A511	$\mathfrak{m}_{2A}(5,11)$	$\surd$	1
m4A511	$\mathfrak{m}_{4A}(5,11)$	$\surd$	∞
m1A611	$\mathfrak{m}_{1A}(6,11)$	$\surd$	1
m3A611	$\mathfrak{m}_{3A}(6,11)$	$\surd$	∞
m2A711	$\mathfrak{m}_{2A}(7,11)$	$\surd$	1
m1A811	$\mathfrak{m}_{1A}(8,11)$	$\surd$	1
m2A212	$\mathfrak{m}_{2A}(2,12)$	$\surd$	1
m4A212	$\mathfrak{m}_{4A}(2,12)$		0
m6A212	$\mathfrak{m}_{6A}(2,12)$		0
m8A212	$\mathfrak{m}_{8A}(2,12)$	$\surd$	2
m1A312	$\mathfrak{m}_{1A}(3,12)$	$\surd$	1
m3A312	$\mathfrak{m}_{3A}(3,12)$	$\surd$	∞
m5A312	$\mathfrak{m}_{5A}(3,12)$	$\surd$	∞
m7A312	$\mathfrak{m}_{7A}(3,12)$	$\surd$	∞
m2A412	$\mathfrak{m}_{2A}(4,12)$	$\surd$	1
m4A412	$\mathfrak{m}_{4A}(4,12)$	$\surd$	∞
m6A412	$\mathfrak{m}_{6A}(4,12)$	$\surd$	∞
m1A512	$\mathfrak{m}_{1A}(5,12)$	$\surd$	1
m3A512	$\mathfrak{m}_{3A}(5,12)$	$\surd$	∞
m5A512	$\mathfrak{m}_{5A}(5,12)$	$\surd$	∞
m2A612	$\mathfrak{m}_{2A}(6,12)$	$\surd$	1
m4A612	$\mathfrak{m}_{4A}(6,12)$	$\surd$	∞
m1A712	$\mathfrak{m}_{1A}(7,12)$	$\surd$	1
m3A712	$\mathfrak{m}_{3A}(7,12)$	$\surd$	∞
m2A812	$\mathfrak{m}_{2A}(8,12)$	$\surd$	1
m1A912	$\mathfrak{m}_{1A}(9,12)$	$\surd$	1
m2B26	$\mathfrak{m}_{2B}(2,6)$	$\surd$	1
m2B28	$\mathfrak{m}_{2B}(2,8)$		0
m4B28	$\mathfrak{m}_{4B}(2,8)$	$\surd$	1
m3B38	$\mathfrak{m}_{3B}(3,8)$	$\surd$	1
m2B48	$\mathfrak{m}_{2B}(4,8)$	$\surd$	1
m2B210	$\mathfrak{m}_{2B}(2,10)$		0
m4B210	$\mathfrak{m}_{4B}(2,10)$		0
m6B210	$\mathfrak{m}_{6B}(2,10)$	$\surd$	2
m3B310	$\mathfrak{m}_{3B}(3,10)$	$\surd$	1
m5B310	$\mathfrak{m}_{5B}(3,10)$	$\surd$	∞
m2B410	$\mathfrak{m}_{2B}(4,10)$		0
m4B410	$\mathfrak{m}_{4B}(4,10)$	$\surd$	1
m3B510	$\mathfrak{m}_{3B}(5,10)$	$\surd$	1
m2B610	$\mathfrak{m}_{2B}(6,10)$	$\surd$	1

search	algebra	Jac	sol
m2B212	$\mathfrak{m}_{2B}(2,12)$		0
m4B212	$\mathfrak{m}_{4B}(2,12)$		0
m6B212	$\mathfrak{m}_{6B}(2,12)$		0
m8B212	$\mathfrak{m}_{8B}(2,12)$	$\surd$	4
m3B312	$\mathfrak{m}_{3B}(3,12)$		0
m5B312	$\mathfrak{m}_{5B}(3,12)$		0
m7B312	$\mathfrak{m}_{7B}(3,12)$	$\surd$	2
m2B412	$\mathfrak{m}_{2B}(4,12)$		0
m4B412	$\mathfrak{m}_{4B}(4,12)$	$\surd$	1
m6B412	$\mathfrak{m}_{6B}(4,12)$	$\surd$	∞
m3B512	$\mathfrak{m}_{3B}(5,12)$	$\surd$	1
m5B512	$\mathfrak{m}_{5B}(5,12)$	$\surd$	∞
m2B612	$\mathfrak{m}_{2B}(6,12)$		0
m4B612	$\mathfrak{m}_{4B}(6,12)$	$\surd$	1
m3B712	$\mathfrak{m}_{3B}(7,12)$	$\surd$	1
m2B812	$\mathfrak{m}_{2B}(8,12)$	$\surd$	1

Jac = Jacobi tests are consistent
lin = Equations in Groebner basis are linear
sol = Found solution

$\mathfrak{m}_{1A}(2,5)$

m1A25 (this line included for string searching purposes)

Original brackets:

$$[e_1,e_2]=e_3 \qquad [e_1,e_3]=e_4$$
$$[e_1,e_4]=e_5 \qquad [e_2,e_3]=e_5$$

Non-trivial Jacobi Tests:

$\mathfrak{m}_{2A}(2,6)$

m2A26 (this line included for string searching purposes)

Original brackets:

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_2, e_3] = e_5 \qquad [e_2, e_4] = e_6$$

Non-trivial Jacobi Tests:

$\mathfrak{m}_{1A}(3,6)$

m1A36 (this line included for string searching purposes)

Original brackets:

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_2, e_3] = e_6$$

Non-trivial Jacobi Tests:

$\mathfrak{m}_{1A}(2,7)$

m1A27 (this line included for string searching purposes)

Original brackets:

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_2, e_5] = e_7$$
$$[e_3, e_4] = -e_7$$

Non-trivial Jacobi Tests:

$\mathfrak{m}_{3A}(2,7)$

m3A27 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
&[e_1, e_2] = e_3 && [e_1, e_3] = e_4 \\
&[e_1, e_4] = e_5 && [e_1, e_5] = e_6 \\
&[e_1, e_6] = e_7 && [e_2, e_3] = e_5 \\
&[e_2, e_4] = e_6 && [e_2, e_5] = \alpha_{2,5}^{7} e_7 \\
&[e_3, e_4] = \alpha_{3,4}^{7} e_7
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$(e_1, e_2, e_4): \quad -\alpha_{2,5}^{7} - \alpha_{3,4}^{7} + 1 = 0$$

Infinite number of solutions: only zero-dimensional systems supported (finite number of solutions).
How the solution(s) were or were not found:
Change variables

$$\alpha_{2,5}^{7} \to x_1$$
$$\alpha_{3,4}^{7} \to x_2$$

Jacobi Tests

$$(e_1, e_2, e_4): \quad -x_1 - x_2 + 1 = 0$$

Groebner basis (2 variables, 1 linear, 0 nonlinear)

$$x_1 + x_2 - 1 = 0$$

$\mathfrak{m}_{2A}(3,7)$

m2A37 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_2, e_3] &= e_6 \\
[e_2, e_4] &= e_7
\end{aligned}
$$

Non-trivial Jacobi Tests:

$\mathfrak{m}_{1A}(4,7)$

m1A47 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_2, e_3] &= e_7
\end{aligned}
$$

Non-trivial Jacobi Tests:

$\mathfrak{m}_{2A}(2,8)$

m2A28 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_2, e_5] &= e_7 & [e_2, e_6] &= 2e_8 \\
[e_3, e_4] &= -e_7 & [e_3, e_5] &= -e_8
\end{aligned}
$$

Non-trivial Jacobi Tests:

$\mathfrak{m}_{4A}(2,8)$

m4A28 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
&[e_1,e_2]=e_3 && [e_1,e_3]=e_4\\
&[e_1,e_4]=e_5 && [e_1,e_5]=e_6\\
&[e_1,e_6]=e_7 && [e_1,e_7]=e_8\\
&[e_2,e_3]=e_5 && [e_2,e_4]=e_6\\
&[e_2,e_5]=\alpha_{2,5}^{7}e_7 && [e_2,e_6]=\alpha_{2,6}^{8}e_8\\
&[e_3,e_4]=\alpha_{3,4}^{7}e_7 && [e_3,e_5]=\alpha_{3,5}^{8}e_8
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1,e_2,e_4):&\quad -\alpha_{2,5}^{7}-\alpha_{3,4}^{7}+1 &&=0\\
(e_1,e_2,e_5):&\quad \alpha_{2,5}^{7}-\alpha_{2,6}^{8}-\alpha_{3,5}^{8} &&=0\\
(e_1,e_3,e_4):&\quad \alpha_{3,4}^{7}-\alpha_{3,5}^{8} &&=0
\end{aligned}
$$

Infinite number of solutions: only zero-dimensional systems supported (finite number of solutions).
How the solution(s) were or were not found:
Change variables

$$\alpha_{2,5}^{7}\to x_1$$

$$\alpha_{2,6}^{8}\to x_2$$

$$\alpha_{3,4}^{7}\to x_3$$

$$\alpha_{3,5}^{8}\to x_4$$

Jacobi Tests

$$
\begin{aligned}
(e_1,e_2,e_4):&\quad -x_1-x_3+1 &&=0\\
(e_1,e_2,e_5):&\quad x_1-x_2-x_4 &&=0\\
(e_1,e_3,e_4):&\quad x_3-x_4 &&=0
\end{aligned}
$$

Groebner basis (4 variables, 3 linear, 0 nonlinear)

$$x_1 + x_4 - 1 = 0$$
$$x_2 + 2x_4 - 1 = 0$$
$$x_3 - x_4 = 0$$

$\mathfrak{m}_{1A}(3, 8)$

m1A38 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned} [e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\ [e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\ [e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\ [e_2, e_5] &= e_8 & [e_3, e_4] &= -e_8 \end{aligned}$$

Non-trivial Jacobi Tests:

$\mathfrak{m}_{3A}(3, 8)$

m3A38 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned} [e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\ [e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\ [e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\ [e_2, e_3] &= e_6 & [e_2, e_4] &= e_7 \\ [e_2, e_5] &= \alpha_{2,5}^8 e_8 & [e_3, e_4] &= \alpha_{3,4}^8 e_8 \end{aligned}$$

Non-trivial Jacobi Tests:

$$(e_1, e_2, e_4): \quad -\alpha_{2,5}^8 - \alpha_{3,4}^8 + 1 = 0$$

Infinite number of solutions: only zero-dimensional systems supported (finite number of solutions).
How the solution(s) were or were not found:
Change variables

$$\alpha^8_{3,4} \to x_1$$
$$\alpha^8_{2,5} \to x_2$$

Jacobi Tests

$$(e_1, e_2, e_4): \quad -x_1 - x_2 + 1 \qquad = 0$$

Groebner basis (2 variables, 1 linear, 0 nonlinear)

$$x_1 + x_2 - 1 = 0$$

$\mathfrak{m}_{2A}(4,8)$

m2A48 (this line included for string searching purposes)

Original brackets:

$$\begin{array}{ll} [e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\ [e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\ [e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\ [e_2, e_3] = e_7 & [e_2, e_4] = e_8 \end{array}$$

Non-trivial Jacobi Tests:

$\mathfrak{m}_{1A}(5,8)$

m1A58 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_2, e_3] &= e_8
\end{aligned}
$$

Non-trivial Jacobi Tests:

$\mathfrak{m}_{1A}(2,9)$

m1A29 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_2, e_7] &= e_9 \\
[e_3, e_6] &= -e_9 & [e_4, e_5] &= e_9
\end{aligned}
$$

Non-trivial Jacobi Tests:

$\mathfrak{m}_{3A}(2,9)$

m3A29 (this line included for string searching purposes)
Solution 1

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_2, e_5] &= e_7 \\
[e_2, e_6] &= 2e_8 & [e_2, e_7] &= 0 \\
[e_3, e_4] &= -e_7 & [e_3, e_5] &= -e_8 \\
[e_3, e_6] &= 2e_9 & [e_4, e_5] &= -3e_9
\end{aligned}
$$

Original brackets:

$$
\begin{aligned}
&[e_1, e_2] = e_3 && [e_1, e_3] = e_4 \\
&[e_1, e_4] = e_5 && [e_1, e_5] = e_6 \\
&[e_1, e_6] = e_7 && [e_1, e_7] = e_8 \\
&[e_1, e_8] = e_9 && [e_2, e_5] = e_7 \\
&[e_2, e_6] = 2e_8 && [e_2, e_7] = \alpha_{2,7}^9 e_9 \\
&[e_3, e_4] = -e_7 && [e_3, e_5] = -e_8 \\
&[e_3, e_6] = \alpha_{3,6}^9 e_9 && [e_4, e_5] = \alpha_{4,5}^9 e_9
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_6): &\quad -\alpha_{2,7}^9 - \alpha_{3,6}^9 + 2 &&= 0 \\
(e_1, e_3, e_5): &\quad -\alpha_{3,6}^9 - \alpha_{4,5}^9 - 1 &&= 0 \\
(e_2, e_3, e_4): &\quad -\alpha_{2,7}^9 &&= 0
\end{aligned}
$$

Solution 1:

$$
\begin{aligned}
\alpha_{4,5}^9 &= -3 \\
\alpha_{3,6}^9 &= 2 \\
\alpha_{2,7}^9 &= 0
\end{aligned}
$$

How the solution(s) were or were not found:
Change variables

$$
\begin{aligned}
\alpha_{4,5}^9 &\to x_1 \\
\alpha_{3,6}^9 &\to x_2 \\
\alpha_{2,7}^9 &\to x_3
\end{aligned}
$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_6): &\quad -x_2 - x_3 + 2 &&= 0 \\
(e_1, e_3, e_5): &\quad -x_1 - x_2 - 1 &&= 0 \\
(e_2, e_3, e_4): &\quad -x_3 &&= 0
\end{aligned}
$$

Groebner basis (3 variables, 3 linear, 0 nonlinear)

$$x_1 + 3 = 0$$
$$x_2 - 2 = 0$$
$$x_3 = 0$$

Solution 1:

$$x_1 = -3$$
$$x_2 = 2$$
$$x_3 = 0$$

$\mathfrak{m}_{5A}(2, 9)$

m5A29 (this line included for string searching purposes)

Original brackets:

$$\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_2, e_3] = e_5 \\
[e_2, e_4] = e_6 & [e_2, e_5] = \alpha_{2,5}^{7} e_7 \\
[e_2, e_6] = \alpha_{2,6}^{8} e_8 & [e_2, e_7] = \alpha_{2,7}^{9} e_9 \\
[e_3, e_4] = \alpha_{3,4}^{7} e_7 & [e_3, e_5] = \alpha_{3,5}^{8} e_8 \\
[e_3, e_6] = \alpha_{3,6}^{9} e_9 & [e_4, e_5] = \alpha_{4,5}^{9} e_9
\end{array}$$

Non-trivial Jacobi Tests:

$$\begin{array}{lll}
(e_1, e_2, e_4): & -\alpha_{2,5}^{7} - \alpha_{3,4}^{7} + 1 & = 0 \\
(e_1, e_2, e_5): & \alpha_{2,5}^{7} - \alpha_{2,6}^{8} - \alpha_{3,5}^{8} & = 0 \\
(e_1, e_3, e_4): & \alpha_{3,4}^{7} - \alpha_{3,5}^{8} & = 0 \\
(e_1, e_2, e_6): & \alpha_{2,6}^{8} - \alpha_{2,7}^{9} - \alpha_{3,6}^{9} & = 0 \\
(e_1, e_3, e_5): & \alpha_{3,5}^{8} - \alpha_{3,6}^{9} - \alpha_{4,5}^{9} & = 0 \\
(e_2, e_3, e_4): & \alpha_{2,7}^{9}\alpha_{3,4}^{7} - \alpha_{3,6}^{9} + \alpha_{4,5}^{9} & = 0
\end{array}$$

Infinite number of solutions: only zero-dimensional systems supported (finite number of solutions).

How the solution(s) were or were not found:

Change variables

$$\alpha^7_{2,5} \to x_1$$

$$\alpha^8_{2,6} \to x_2$$

$$\alpha^8_{3,5} \to x_3$$

$$\alpha^7_{3,4} \to x_4$$

$$\alpha^9_{4,5} \to x_5$$

$$\alpha^9_{2,7} \to x_6$$

$$\alpha^9_{3,6} \to x_7$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4): &\quad -x_1 - x_4 + 1 && = 0\\
(e_1, e_2, e_5): &\quad x_1 - x_2 - x_3 && = 0\\
(e_1, e_3, e_4): &\quad -x_3 + x_4 && = 0\\
(e_1, e_2, e_6): &\quad x_2 - x_6 - x_7 && = 0\\
(e_1, e_3, e_5): &\quad x_3 - x_5 - x_7 && = 0\\
(e_2, e_3, e_4): &\quad x_4x_6 + x_5 - x_7 && = 0
\end{aligned}$$

Groebner basis (7 variables, 5 linear, 1 nonlinear)

$$2x_1 - x_6 - x_7 - 1 = 0$$

$$x_2 - x_6 - x_7 = 0$$

$$2x_3 + x_6 + x_7 - 1 = 0$$

$$2x_4 + x_6 + x_7 - 1 = 0$$

$$2x_5 + x_6 + 3x_7 - 1 = 0$$

$$x_6^2 + x_6x_7 + 5x_7 - 1 = 0$$

$\mathfrak{m}_{2A}(3,9)$

m2A39 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
&[e_1,e_2]=e_3 && [e_1,e_3]=e_4\\
&[e_1,e_4]=e_5 && [e_1,e_5]=e_6\\
&[e_1,e_6]=e_7 && [e_1,e_7]=e_8\\
&[e_1,e_8]=e_9 && [e_2,e_5]=e_8\\
&[e_2,e_6]=2e_9 && [e_3,e_4]=-e_8\\
&[e_3,e_5]=-e_9 &&
\end{aligned}
$$

Non-trivial Jacobi Tests:

$\mathfrak{m}_{4A}(3,9)$

m4A39 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
&[e_1,e_2]=e_3 && [e_1,e_3]=e_4\\
&[e_1,e_4]=e_5 && [e_1,e_5]=e_6\\
&[e_1,e_6]=e_7 && [e_1,e_7]=e_8\\
&[e_1,e_8]=e_9 && [e_2,e_3]=e_6\\
&[e_2,e_4]=e_7 && [e_2,e_5]=\alpha_{2,5}^{8}e_8\\
&[e_2,e_6]=\alpha_{2,6}^{9}e_9 && [e_3,e_4]=\alpha_{3,4}^{8}e_8\\
&[e_3,e_5]=\alpha_{3,5}^{9}e_9 &&
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
&(e_1,e_2,e_4): && -\alpha_{2,5}^{8}-\alpha_{3,4}^{8}+1 && =0\\
&(e_1,e_2,e_5): && \alpha_{2,5}^{8}-\alpha_{2,6}^{9}-\alpha_{3,5}^{9} && =0\\
&(e_1,e_3,e_4): && \alpha_{3,4}^{8}-\alpha_{3,5}^{9} && =0
\end{aligned}
$$

Infinite number of solutions: only zero-dimensional systems supported (finite number of solutions).
How the solution(s) were or were not found:
Change variables

$$\alpha_{3,5}^{9} \to x_1$$

$$\alpha_{3,4}^{8} \to x_2$$

$$\alpha_{2,6}^{9} \to x_3$$

$$\alpha_{2,5}^{8} \to x_4$$

Jacobi Tests

$$\begin{array}{lll} (e_1, e_2, e_4): & -x_2 - x_4 + 1 & = 0 \\ (e_1, e_2, e_5): & -x_1 - x_3 + x_4 & = 0 \\ (e_1, e_3, e_4): & -x_1 + x_2 & = 0 \end{array}$$

Groebner basis (4 variables, 3 linear, 0 nonlinear)

$$x_1 + x_4 - 1 = 0$$

$$x_2 + x_4 - 1 = 0$$

$$x_3 - 2x_4 + 1 = 0$$

$\mathfrak{m}_{1A}(4, 9)$

m1A49 (this line included for string searching purposes)

Original brackets:

$$\begin{array}{ll} [e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\ [e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\ [e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\ [e_1, e_8] = e_9 & [e_2, e_5] = e_9 \\ [e_3, e_4] = -e_9 & \end{array}$$

Non-trivial Jacobi Tests:

$\mathfrak{m}_{3A}(4,9)$

m3A49 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
&[e_1,e_2]=e_3 && [e_1,e_3]=e_4\\
&[e_1,e_4]=e_5 && [e_1,e_5]=e_6\\
&[e_1,e_6]=e_7 && [e_1,e_7]=e_8\\
&[e_1,e_8]=e_9 && [e_2,e_3]=e_7\\
&[e_2,e_4]=e_8 && [e_2,e_5]=\alpha_{2,5}^{9}e_9\\
&[e_3,e_4]=\alpha_{3,4}^{9}e_9
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$(e_1,e_2,e_4):\quad -\alpha_{2,5}^{9}-\alpha_{3,4}^{9}+1 \qquad = 0$$

Infinite number of solutions: only zero-dimensional systems supported (finite number of solutions).
How the solution(s) were or were not found:
Change variables

$$\alpha_{3,4}^{9}\to x_1$$
$$\alpha_{2,5}^{9}\to x_2$$

Jacobi Tests

$$(e_1,e_2,e_4):\quad -x_1-x_2+1 \qquad = 0$$

Groebner basis (2 variables, 1 linear, 0 nonlinear)

$$x_1+x_2-1=0$$

$\mathfrak{m}_{2A}(5,9)$

m2A59 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_2, e_3] &= e_8 \\
[e_2, e_4] &= e_9
\end{aligned}
$$

Non-trivial Jacobi Tests:

$\mathfrak{m}_{1A}(6, 9)$

m1A69 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_2, e_3] &= e_9
\end{aligned}
$$

Non-trivial Jacobi Tests:

$\mathfrak{m}_{2A}(2, 10)$

m2A210 (this line included for string searching purposes)

Original brackets:

$[e_1, e_2] = e_3$	$[e_1, e_3] = e_4$
$[e_1, e_4] = e_5$	$[e_1, e_5] = e_6$
$[e_1, e_6] = e_7$	$[e_1, e_7] = e_8$
$[e_1, e_8] = e_9$	$[e_1, e_9] = e_{10}$
$[e_2, e_7] = e_9$	$[e_2, e_8] = 3e_{10}$
$[e_3, e_6] = -e_9$	$[e_3, e_7] = -2e_{10}$
$[e_4, e_5] = e_9$	$[e_4, e_6] = e_{10}$

Non-trivial Jacobi Tests:

$\mathfrak{m}_{4A}(2, 10)$

m4A210 (this line included for string searching purposes)
Solution 1

$[e_1, e_2] = e_3$	$[e_1, e_3] = e_4$
$[e_1, e_4] = e_5$	$[e_1, e_5] = e_6$
$[e_1, e_6] = e_7$	$[e_1, e_7] = e_8$
$[e_1, e_8] = e_9$	$[e_1, e_9] = e_{10}$
$[e_2, e_5] = e_7$	$[e_2, e_6] = 2e_8$
$[e_2, e_7] = 0$	$[e_2, e_8] = -5e_{10}$
$[e_3, e_4] = -e_7$	$[e_3, e_5] = -e_8$
$[e_3, e_6] = 2e_9$	$[e_3, e_7] = 5e_{10}$
$[e_4, e_5] = -3e_9$	$[e_4, e_6] = -3e_{10}$

Original brackets:

$$
\begin{aligned}
&[e_1, e_2] = e_3 && [e_1, e_3] = e_4 \\
&[e_1, e_4] = e_5 && [e_1, e_5] = e_6 \\
&[e_1, e_6] = e_7 && [e_1, e_7] = e_8 \\
&[e_1, e_8] = e_9 && [e_1, e_9] = e_{10} \\
&[e_2, e_5] = e_7 && [e_2, e_6] = 2e_8 \\
&[e_2, e_7] = \alpha_{2,7}^{9} e_9 && [e_2, e_8] = \alpha_{2,8}^{10} e_{10} \\
&[e_3, e_4] = -e_7 && [e_3, e_5] = -e_8 \\
&[e_3, e_6] = \alpha_{3,6}^{9} e_9 && [e_3, e_7] = \alpha_{3,7}^{10} e_{10} \\
&[e_4, e_5] = \alpha_{4,5}^{9} e_9 && [e_4, e_6] = \alpha_{4,6}^{10} e_{10}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_6) :&\quad -\alpha_{2,7}^{9} - \alpha_{3,6}^{9} + 2 &&= 0 \\
(e_1, e_3, e_5) :&\quad -\alpha_{3,6}^{9} - \alpha_{4,5}^{9} - 1 &&= 0 \\
(e_2, e_3, e_4) :&\quad -\alpha_{2,7}^{9} &&= 0 \\
(e_1, e_2, e_7) :&\quad \alpha_{2,7}^{9} - \alpha_{2,8}^{10} - \alpha_{3,7}^{10} &&= 0 \\
(e_1, e_3, e_6) :&\quad \alpha_{3,6}^{9} - \alpha_{3,7}^{10} - \alpha_{4,6}^{10} &&= 0 \\
(e_1, e_4, e_5) :&\quad \alpha_{4,5}^{9} - \alpha_{4,6}^{10} &&= 0 \\
(e_2, e_3, e_5) :&\quad -\alpha_{2,8}^{10} - \alpha_{3,7}^{10} &&= 0
\end{aligned}
$$

Solution 1:

$$
\begin{aligned}
\alpha_{2,8}^{10} &= -5 \\
\alpha_{3,7}^{10} &= 5 \\
\alpha_{4,5}^{9} &= -3 \\
\alpha_{2,7}^{9} &= 0 \\
\alpha_{3,6}^{9} &= 2 \\
\alpha_{4,6}^{10} &= -3
\end{aligned}
$$

How the solution(s) were or were not found:
Change variables

$$\alpha_{2,8}^{10} \to x_1$$

$$\alpha_{3,7}^{10} \to x_2$$

$$\alpha_{4,5}^{9} \to x_3$$

$$\alpha_{2,7}^{9} \to x_4$$

$$\alpha_{3,6}^{9} \to x_5$$

$$\alpha_{4,6}^{10} \to x_6$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_6): &\quad -x_4 - x_5 + 2 &&= 0\\
(e_1, e_3, e_5): &\quad -x_3 - x_5 - 1 &&= 0\\
(e_2, e_3, e_4): &\quad -x_4 &&= 0\\
(e_1, e_2, e_7): &\quad -x_1 - x_2 + x_4 &&= 0\\
(e_1, e_3, e_6): &\quad -x_2 + x_5 - x_6 &&= 0\\
(e_1, e_4, e_5): &\quad x_3 - x_6 &&= 0\\
(e_2, e_3, e_5): &\quad -x_1 - x_2 &&= 0
\end{aligned}$$

Groebner basis (6 variables, 6 linear, 0 nonlinear)

$$x_1 + 5 = 0$$

$$x_2 - 5 = 0$$

$$x_3 + 3 = 0$$

$$x_4 = 0$$

$$x_5 - 2 = 0$$

$$x_6 + 3 = 0$$

Solution 1:

$$x_1 = -5$$

$$x_2 = 5$$

$$x_3 = -3$$

$$x_4 = 0$$

$$x_5 = 2$$

$$x_6 = -3$$

$\mathfrak{m}_{6A}(2,10)$

m6A210 (this line included for string searching purposes)

Original brackets:

$$
\begin{array}{ll}
[e_1,e_2]=e_3 & [e_1,e_3]=e_4 \\
[e_1,e_4]=e_5 & [e_1,e_5]=e_6 \\
[e_1,e_6]=e_7 & [e_1,e_7]=e_8 \\
[e_1,e_8]=e_9 & [e_1,e_9]=e_{10} \\
[e_2,e_3]=e_5 & [e_2,e_4]=e_6 \\
[e_2,e_5]=\alpha_{2,5}^{7}e_7 & [e_2,e_6]=\alpha_{2,6}^{8}e_8 \\
[e_2,e_7]=\alpha_{2,7}^{9}e_9 & [e_2,e_8]=\alpha_{2,8}^{10}e_{10} \\
[e_3,e_4]=\alpha_{3,4}^{7}e_7 & [e_3,e_5]=\alpha_{3,5}^{8}e_8 \\
[e_3,e_6]=\alpha_{3,6}^{9}e_9 & [e_3,e_7]=\alpha_{3,7}^{10}e_{10} \\
[e_4,e_5]=\alpha_{4,5}^{9}e_9 & [e_4,e_6]=\alpha_{4,6}^{10}e_{10}
\end{array}
$$

Non-trivial Jacobi Tests:

$$
\begin{array}{lll}
(e_1,e_2,e_4): & -\alpha_{2,5}^{7}-\alpha_{3,4}^{7}+1 & =0 \\
(e_1,e_2,e_5): & \alpha_{2,5}^{7}-\alpha_{2,6}^{8}-\alpha_{3,5}^{8} & =0 \\
(e_1,e_3,e_4): & \alpha_{3,4}^{7}-\alpha_{3,5}^{8} & =0 \\
(e_1,e_2,e_6): & \alpha_{2,6}^{8}-\alpha_{2,7}^{9}-\alpha_{3,6}^{9} & =0 \\
(e_1,e_3,e_5): & \alpha_{3,5}^{8}-\alpha_{3,6}^{9}-\alpha_{4,5}^{9} & =0 \\
(e_2,e_3,e_4): & \alpha_{2,7}^{9}\alpha_{3,4}^{7}-\alpha_{3,6}^{9}+\alpha_{4,5}^{9} & =0 \\
(e_1,e_2,e_7): & \alpha_{2,7}^{9}-\alpha_{2,8}^{10}-\alpha_{3,7}^{10} & =0 \\
(e_1,e_3,e_6): & \alpha_{3,6}^{9}-\alpha_{3,7}^{10}-\alpha_{4,6}^{10} & =0 \\
(e_1,e_4,e_5): & \alpha_{4,5}^{9}-\alpha_{4,6}^{10} & =0 \\
(e_2,e_3,e_5): & -\alpha_{2,5}^{7}\alpha_{3,7}^{10}+\alpha_{2,8}^{10}\alpha_{3,5}^{8} & =0
\end{array}
$$

Infinite number of solutions: only zero-dimensional systems supported (finite number of solutions).
How the solution(s) were or were not found:
Change variables

$$\alpha_{2,5}^{7}\to x_1$$

$$\alpha^8_{2,6} \to x_2$$
$$\alpha^8_{3,5} \to x_3$$
$$\alpha^{10}_{2,8} \to x_4$$
$$\alpha^{10}_{3,7} \to x_5$$
$$\alpha^7_{3,4} \to x_6$$
$$\alpha^9_{4,5} \to x_7$$
$$\alpha^9_{2,7} \to x_8$$
$$\alpha^9_{3,6} \to x_9$$
$$\alpha^{10}_{4,6} \to x_{10}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4): &\quad -x_1 - x_6 + 1 &&= 0\\
(e_1, e_2, e_5): &\quad x_1 - x_2 - x_3 &&= 0\\
(e_1, e_3, e_4): &\quad -x_3 + x_6 &&= 0\\
(e_1, e_2, e_6): &\quad x_2 - x_8 - x_9 &&= 0\\
(e_1, e_3, e_5): &\quad x_3 - x_7 - x_9 &&= 0\\
(e_2, e_3, e_4): &\quad x_6x_8 + x_7 - x_9 &&= 0\\
(e_1, e_2, e_7): &\quad -x_4 - x_5 + x_8 &&= 0\\
(e_1, e_3, e_6): &\quad -x_{10} - x_5 + x_9 &&= 0\\
(e_1, e_4, e_5): &\quad -x_{10} + x_7 &&= 0\\
(e_2, e_3, e_5): &\quad -x_1x_5 + x_3x_4 &&= 0
\end{aligned}$$

Groebner basis (10 variables, 8 linear, 1 nonlinear)

$$x_1 + x_{10} + x_9 - 1 = 0$$
$$2x_{10} + x_2 + 2x_9 - 1 = 0$$
$$-x_{10} + x_3 - x_9 = 0$$
$$x_{10} + x_4 + 4x_9 - 1 = 0$$
$$x_{10} + x_5 - x_9 = 0$$
$$-x_{10} + x_6 - x_9 = 0$$
$$-x_{10} + x_7 = 0$$
$$2x_{10} + x_8 + 3x_9 - 1 = 0$$
$$2x_{10}^2 + 5x_{10}x_9 - 2x_{10} + 3x_9^2 = 0$$

$\mathfrak{m}_{1A}(3, 10)$

m1A310 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
&[e_1, e_2] = e_3 && [e_1, e_3] = e_4 \\
&[e_1, e_4] = e_5 && [e_1, e_5] = e_6 \\
&[e_1, e_6] = e_7 && [e_1, e_7] = e_8 \\
&[e_1, e_8] = e_9 && [e_1, e_9] = e_{10} \\
&[e_2, e_7] = e_{10} && [e_3, e_6] = -e_{10} \\
&[e_4, e_5] = e_{10}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$\mathfrak{m}_{3A}(3, 10)$

m3A310 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
&[e_1, e_2] = e_3 && [e_1, e_3] = e_4 \\
&[e_1, e_4] = e_5 && [e_1, e_5] = e_6 \\
&[e_1, e_6] = e_7 && [e_1, e_7] = e_8 \\
&[e_1, e_8] = e_9 && [e_1, e_9] = e_{10} \\
&[e_2, e_5] = e_8 && [e_2, e_6] = 2e_9 \\
&[e_2, e_7] = \alpha_{2,7}^{10} e_{10} && [e_3, e_4] = -e_8 \\
&[e_3, e_5] = -e_9 && [e_3, e_6] = \alpha_{3,6}^{10} e_{10} \\
&[e_4, e_5] = \alpha_{4,5}^{10} e_{10}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_6): &\quad -\alpha_{2,7}^{10} - \alpha_{3,6}^{10} + 2 &&= 0 \\
(e_1, e_3, e_5): &\quad -\alpha_{3,6}^{10} - \alpha_{4,5}^{10} - 1 &&= 0
\end{aligned}
$$

Infinite number of solutions: only zero-dimensional systems supported (finite number of solutions).

How the solution(s) were or were not found:
Change variables

$$\alpha_{2,7}^{10} \to x_1$$
$$\alpha_{4,5}^{10} \to x_2$$
$$\alpha_{3,6}^{10} \to x_3$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_6): \quad & -x_1 - x_3 + 2 && = 0 \\
(e_1, e_3, e_5): \quad & -x_2 - x_3 - 1 && = 0
\end{aligned}$$

Groebner basis (3 variables, 2 linear, 0 nonlinear)

$$x_1 + x_3 - 2 = 0$$
$$x_2 + x_3 + 1 = 0$$

$\mathfrak{m}_{5A}(3, 10)$

m5A310 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_2, e_3] &= e_6 & [e_2, e_4] &= e_7 \\
[e_2, e_5] &= \alpha_{2,5}^{8} e_8 & [e_2, e_6] &= \alpha_{2,6}^{9} e_9 \\
[e_2, e_7] &= \alpha_{2,7}^{10} e_{10} & [e_3, e_4] &= \alpha_{3,4}^{8} e_8 \\
[e_3, e_5] &= \alpha_{3,5}^{9} e_9 & [e_3, e_6] &= \alpha_{3,6}^{10} e_{10} \\
[e_4, e_5] &= \alpha_{4,5}^{10} e_{10}
\end{aligned}$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1,e_2,e_4): &\quad -\alpha^8_{2,5}-\alpha^8_{3,4}+1 &&=0\\
(e_1,e_2,e_5): &\quad \alpha^8_{2,5}-\alpha^9_{2,6}-\alpha^9_{3,5} &&=0\\
(e_1,e_3,e_4): &\quad \alpha^8_{3,4}-\alpha^9_{3,5} &&=0\\
(e_1,e_2,e_6): &\quad \alpha^9_{2,6}-\alpha^{10}_{2,7}-\alpha^{10}_{3,6} &&=0\\
(e_1,e_3,e_5): &\quad \alpha^9_{3,5}-\alpha^{10}_{3,6}-\alpha^{10}_{4,5} &&=0
\end{aligned}
$$

Infinite number of solutions: only zero-dimensional systems supported (finite number of solutions).
How the solution(s) were or were not found:
Change variables

$$\alpha^8_{2,5}\to x_1$$

$$\alpha^{10}_{3,6}\to x_2$$

$$\alpha^9_{3,5}\to x_3$$

$$\alpha^8_{3,4}\to x_4$$

$$\alpha^9_{2,6}\to x_5$$

$$\alpha^{10}_{4,5}\to x_6$$

$$\alpha^{10}_{2,7}\to x_7$$

Jacobi Tests

$$
\begin{aligned}
(e_1,e_2,e_4): &\quad -x_1-x_4+1 &&=0\\
(e_1,e_2,e_5): &\quad x_1-x_3-x_5 &&=0\\
(e_1,e_3,e_4): &\quad -x_3+x_4 &&=0\\
(e_1,e_2,e_6): &\quad -x_2+x_5-x_7 &&=0\\
(e_1,e_3,e_5): &\quad -x_2+x_3-x_6 &&=0
\end{aligned}
$$

Groebner basis (7 variables, 5 linear, 0 nonlinear)

$$3x_1+x_6-x_7-2=0$$

$$3x_2+2x_6+x_7-1=0$$

$$3x_3-x_6+x_7-1=0$$

$$3x_4-x_6+x_7-1=0$$

$$3x_5+2x_6-2x_7-1=0$$

$\mathfrak{m}_{2A}(4, 10)$

m2A410 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_2, e_5] &= e_9 & [e_2, e_6] &= 2e_{10} \\
[e_3, e_4] &= -e_9 & [e_3, e_5] &= -e_{10}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$\mathfrak{m}_{4A}(4, 10)$

m4A410 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_2, e_3] &= e_7 & [e_2, e_4] &= e_8 \\
[e_2, e_5] &= \alpha_{2,5}^{9} e_9 & [e_2, e_6] &= \alpha_{2,6}^{10} e_{10} \\
[e_3, e_4] &= \alpha_{3,4}^{9} e_9 & [e_3, e_5] &= \alpha_{3,5}^{10} e_{10}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_4): &\quad -\alpha_{2,5}^{9} - \alpha_{3,4}^{9} + 1 &&= 0 \\
(e_1, e_2, e_5): &\quad \alpha_{2,5}^{9} - \alpha_{2,6}^{10} - \alpha_{3,5}^{10} &&= 0 \\
(e_1, e_3, e_4): &\quad \alpha_{3,4}^{9} - \alpha_{3,5}^{10} &&= 0
\end{aligned}
$$

Infinite number of solutions: only zero-dimensional systems supported (finite number of solutions).
How the solution(s) were or were not found:
Change variables

$$\alpha_{3,5}^{10} \to x_1$$
$$\alpha_{3,4}^{9} \to x_2$$
$$\alpha_{2,5}^{9} \to x_3$$
$$\alpha_{2,6}^{10} \to x_4$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4): &\quad -x_2 - x_3 + 1 &&= 0\\
(e_1, e_2, e_5): &\quad -x_1 + x_3 - x_4 &&= 0\\
(e_1, e_3, e_4): &\quad -x_1 + x_2 &&= 0
\end{aligned}$$

Groebner basis (4 variables, 3 linear, 0 nonlinear)

$$2x_1 + x_4 - 1 = 0$$
$$2x_2 + x_4 - 1 = 0$$
$$2x_3 - x_4 - 1 = 0$$

$\mathfrak{m}_{1A}(5, 10)$

m1A510 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4\\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6\\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8\\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10}\\
[e_2, e_5] &= e_{10} & [e_3, e_4] &= -e_{10}
\end{aligned}$$

Non-trivial Jacobi Tests:

$\mathfrak{m}_{3A}(5, 10)$

m3A510 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
&[e_1, e_2] = e_3 && [e_1, e_3] = e_4 \\
&[e_1, e_4] = e_5 && [e_1, e_5] = e_6 \\
&[e_1, e_6] = e_7 && [e_1, e_7] = e_8 \\
&[e_1, e_8] = e_9 && [e_1, e_9] = e_{10} \\
&[e_2, e_3] = e_8 && [e_2, e_4] = e_9 \\
&[e_2, e_5] = \alpha_{2,5}^{10} e_{10} && [e_3, e_4] = \alpha_{3,4}^{10} e_{10}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$(e_1, e_2, e_4): \quad -\alpha_{2,5}^{10} - \alpha_{3,4}^{10} + 1 = 0$$

Infinite number of solutions: only zero-dimensional systems supported (finite number of solutions).
How the solution(s) were or were not found:
Change variables

$$\alpha_{2,5}^{10} \to x_1$$
$$\alpha_{3,4}^{10} \to x_2$$

Jacobi Tests

$$(e_1, e_2, e_4): \quad -x_1 - x_2 + 1 = 0$$

Groebner basis (2 variables, 1 linear, 0 nonlinear)

$$x_1 + x_2 - 1 = 0$$

$\mathfrak{m}_{2A}(6, 10)$

m2A610 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
&[e_1, e_2] = e_3 && [e_1, e_3] = e_4 \\
&[e_1, e_4] = e_5 && [e_1, e_5] = e_6 \\
&[e_1, e_6] = e_7 && [e_1, e_7] = e_8 \\
&[e_1, e_8] = e_9 && [e_1, e_9] = e_{10} \\
&[e_2, e_3] = e_9 && [e_2, e_4] = e_{10}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$\mathfrak{m}_{1A}(7, 10)$

m1A710 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
&[e_1, e_2] = e_3 && [e_1, e_3] = e_4 \\
&[e_1, e_4] = e_5 && [e_1, e_5] = e_6 \\
&[e_1, e_6] = e_7 && [e_1, e_7] = e_8 \\
&[e_1, e_8] = e_9 && [e_1, e_9] = e_{10} \\
&[e_2, e_3] = e_{10}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$\mathfrak{m}_{1A}(2, 11)$

m1A211 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_2, e_9] &= e_{11} \\
[e_3, e_8] &= -e_{11} & [e_4, e_7] &= e_{11} \\
[e_5, e_6] &= -e_{11}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$\mathfrak{m}_{3A}(2, 11)$

m3A211 (this line included for string searching purposes)
Solution 1

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_2, e_7] &= e_9 \\
[e_2, e_8] &= 3e_{10} & [e_2, e_9] &= 0 \\
[e_3, e_6] &= -e_9 & [e_3, e_7] &= -2e_{10} \\
[e_3, e_8] &= 3e_{11} & [e_4, e_5] &= e_9 \\
[e_4, e_6] &= e_{10} & [e_4, e_7] &= -5e_{11} \\
[e_5, e_6] &= 6e_{11}
\end{aligned}
$$

Original brackets:

$$
\begin{aligned}
&[e_1,e_2]=e_3 && [e_1,e_3]=e_4\\
&[e_1,e_4]=e_5 && [e_1,e_5]=e_6\\
&[e_1,e_6]=e_7 && [e_1,e_7]=e_8\\
&[e_1,e_8]=e_9 && [e_1,e_9]=e_{10}\\
&[e_1,e_{10}]=e_{11} && [e_2,e_7]=e_9\\
&[e_2,e_8]=3e_{10} && [e_2,e_9]=\alpha_{2,9}^{11}e_{11}\\
&[e_3,e_6]=-e_9 && [e_3,e_7]=-2e_{10}\\
&[e_3,e_8]=\alpha_{3,8}^{11}e_{11} && [e_4,e_5]=e_9\\
&[e_4,e_6]=e_{10} && [e_4,e_7]=\alpha_{4,7}^{11}e_{11}\\
&[e_5,e_6]=\alpha_{5,6}^{11}e_{11}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1,e_2,e_8): &\quad -\alpha_{2,9}^{11}-\alpha_{3,8}^{11}+3 &&=0\\
(e_1,e_3,e_7): &\quad -\alpha_{3,8}^{11}-\alpha_{4,7}^{11}-2 &&=0\\
(e_1,e_4,e_6): &\quad -\alpha_{4,7}^{11}-\alpha_{5,6}^{11}+1 &&=0\\
(e_2,e_3,e_6): &\quad -\alpha_{2,9}^{11} &&=0\\
(e_2,e_4,e_5): &\quad \alpha_{2,9}^{11} &&=0
\end{aligned}
$$

Solution 1:

$$
\begin{aligned}
\alpha_{2,9}^{11}&=0\\
\alpha_{4,7}^{11}&=-5\\
\alpha_{3,8}^{11}&=3\\
\alpha_{5,6}^{11}&=6
\end{aligned}
$$

How the solution(s) were or were not found:
Change variables

$$
\begin{aligned}
\alpha_{2,9}^{11}&\to x_1\\
\alpha_{4,7}^{11}&\to x_2\\
\alpha_{3,8}^{11}&\to x_3\\
\alpha_{5,6}^{11}&\to x_4
\end{aligned}
$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_8):\quad & -x_1 - x_3 + 3 && = 0 \\
(e_1, e_3, e_7):\quad & -x_2 - x_3 - 2 && = 0 \\
(e_1, e_4, e_6):\quad & -x_2 - x_4 + 1 && = 0 \\
(e_2, e_3, e_6):\quad & -x_1 && = 0 \\
(e_2, e_4, e_5):\quad & x_1 && = 0
\end{aligned}
$$

Groebner basis (4 variables, 4 linear, 0 nonlinear)

$$x_1 = 0$$

$$x_2 + 5 = 0$$

$$x_3 - 3 = 0$$

$$x_4 - 6 = 0$$

Solution 1:

$$x_1 = 0$$

$$x_2 = -5$$

$$x_3 = 3$$

$$x_4 = 6$$

$\mathfrak{m}_{5A}(2, 11)$

m5A211 (this line included for string searching purposes)
Solution 1

$$
\begin{aligned}
&[e_1, e_2] = e_3 && [e_1, e_3] = e_4 \\
&[e_1, e_4] = e_5 && [e_1, e_5] = e_6 \\
&[e_1, e_6] = e_7 && [e_1, e_7] = e_8 \\
&[e_1, e_8] = e_9 && [e_1, e_9] = e_{10} \\
&[e_1, e_{10}] = e_{11} && [e_2, e_5] = e_7 \\
&[e_2, e_6] = 2e_8 && [e_2, e_7] = 0 \\
&[e_2, e_8] = -5e_{10} && [e_2, e_9] = -\frac{5e_{11}}{2} \\
&[e_3, e_4] = -e_7 && [e_3, e_5] = -e_8 \\
&[e_3, e_6] = 2e_9 && [e_3, e_7] = 5e_{10} \\
&[e_3, e_8] = -\frac{5e_{11}}{2} && [e_4, e_5] = -3e_9 \\
&[e_4, e_6] = -3e_{10} && [e_4, e_7] = \frac{15e_{11}}{2} \\
&[e_5, e_6] = -\frac{21e_{11}}{2}
\end{aligned}
$$

Original brackets:

$$
\begin{aligned}
&[e_1, e_2] = e_3 && [e_1, e_3] = e_4 \\
&[e_1, e_4] = e_5 && [e_1, e_5] = e_6 \\
&[e_1, e_6] = e_7 && [e_1, e_7] = e_8 \\
&[e_1, e_8] = e_9 && [e_1, e_9] = e_{10} \\
&[e_1, e_{10}] = e_{11} && [e_2, e_5] = e_7 \\
&[e_2, e_6] = 2e_8 && [e_2, e_7] = \alpha_{2,7}^{9} e_9 \\
&[e_2, e_8] = \alpha_{2,8}^{10} e_{10} && [e_2, e_9] = \alpha_{2,9}^{11} e_{11} \\
&[e_3, e_4] = -e_7 && [e_3, e_5] = -e_8 \\
&[e_3, e_6] = \alpha_{3,6}^{9} e_9 && [e_3, e_7] = \alpha_{3,7}^{10} e_{10} \\
&[e_3, e_8] = \alpha_{3,8}^{11} e_{11} && [e_4, e_5] = \alpha_{4,5}^{9} e_9 \\
&[e_4, e_6] = \alpha_{4,6}^{10} e_{10} && [e_4, e_7] = \alpha_{4,7}^{11} e_{11} \\
&[e_5, e_6] = \alpha_{5,6}^{11} e_{11}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_6):&\quad -\alpha_{2,7}^{9} - \alpha_{3,6}^{9} + 2 &&= 0\\
(e_1, e_3, e_5):&\quad -\alpha_{3,6}^{9} - \alpha_{4,5}^{9} - 1 &&= 0\\
(e_2, e_3, e_4):&\quad -\alpha_{2,7}^{9} &&= 0\\
(e_1, e_2, e_7):&\quad \alpha_{2,7}^{9} - \alpha_{2,8}^{10} - \alpha_{3,7}^{10} &&= 0\\
(e_1, e_3, e_6):&\quad \alpha_{3,6}^{9} - \alpha_{3,7}^{10} - \alpha_{4,6}^{10} &&= 0\\
(e_1, e_4, e_5):&\quad \alpha_{4,5}^{9} - \alpha_{4,6}^{10} &&= 0\\
(e_2, e_3, e_5):&\quad -\alpha_{2,8}^{10} - \alpha_{3,7}^{10} &&= 0\\
(e_1, e_2, e_8):&\quad \alpha_{2,8}^{10} - \alpha_{2,9}^{11} - \alpha_{3,8}^{11} &&= 0\\
(e_1, e_3, e_7):&\quad \alpha_{3,7}^{10} - \alpha_{3,8}^{11} - \alpha_{4,7}^{11} &&= 0\\
(e_1, e_4, e_6):&\quad \alpha_{4,6}^{10} - \alpha_{4,7}^{11} - \alpha_{5,6}^{11} &&= 0\\
(e_2, e_3, e_6):&\quad \alpha_{2,9}^{11}\alpha_{3,6}^{9} - 2\alpha_{3,8}^{11} &&= 0\\
(e_2, e_4, e_5):&\quad \alpha_{2,9}^{11}\alpha_{4,5}^{9} - \alpha_{4,7}^{11} &&= 0
\end{aligned}
$$

Solution 1:

$$
\begin{aligned}
\alpha_{2,9}^{11} &= -5/2\\
\alpha_{2,8}^{10} &= -5\\
\alpha_{3,7}^{10} &= 5\\
\alpha_{5,6}^{11} &= -21/2\\
\alpha_{3,8}^{11} &= -5/2\\
\alpha_{4,5}^{9} &= -3\\
\alpha_{2,7}^{9} &= 0\\
\alpha_{4,7}^{11} &= 15/2\\
\alpha_{3,6}^{9} &= 2\\
\alpha_{4,6}^{10} &= -3
\end{aligned}
$$

How the solution(s) were or were not found:
Change variables

$$\alpha_{2,9}^{11} \to x_1$$

$$\alpha_{2,8}^{10} \to x_2$$

$$\alpha_{3,7}^{10} \to x_3$$

$$\alpha^{11}_{5,6} \to x_4$$

$$\alpha^{11}_{3,8} \to x_5$$

$$\alpha^{9}_{4,5} \to x_6$$

$$\alpha^{9}_{2,7} \to x_7$$

$$\alpha^{11}_{4,7} \to x_8$$

$$\alpha^{9}_{3,6} \to x_9$$

$$\alpha^{10}_{4,6} \to x_{10}$$

Jacobi Tests

$$\begin{aligned}
(e_1,e_2,e_6):&\quad -x_7-x_9+2 &&=0\\
(e_1,e_3,e_5):&\quad -x_6-x_9-1 &&=0\\
(e_2,e_3,e_4):&\quad -x_7 &&=0\\
(e_1,e_2,e_7):&\quad -x_2-x_3+x_7 &&=0\\
(e_1,e_3,e_6):&\quad -x_{10}-x_3+x_9 &&=0\\
(e_1,e_4,e_5):&\quad -x_{10}+x_6 &&=0\\
(e_2,e_3,e_5):&\quad -x_2-x_3 &&=0\\
(e_1,e_2,e_8):&\quad -x_1+x_2-x_5 &&=0\\
(e_1,e_3,e_7):&\quad x_3-x_5-x_8 &&=0\\
(e_1,e_4,e_6):&\quad x_{10}-x_4-x_8 &&=0\\
(e_2,e_3,e_6):&\quad x_1x_9-2x_5 &&=0\\
(e_2,e_4,e_5):&\quad x_1x_6-x_8 &&=0
\end{aligned}$$

Groebner basis (10 variables, 10 linear, 0 nonlinear)

$$2x_1+5=0$$

$$x_2+5=0$$

$$x_3-5=0$$

$$2x_4+21=0$$

$$2x_5+5=0$$

$$x_6+3=0$$

$$x_7=0$$

$$2x_8-15=0$$

$$x_9-2=0$$

$$x_{10} + 3 = 0$$

Solution 1:

$$x_1 = -5/2$$
$$x_2 = -5$$
$$x_3 = 5$$
$$x_4 = -21/2$$
$$x_5 = -5/2$$
$$x_6 = -3$$
$$x_7 = 0$$
$$x_8 = 15/2$$
$$x_9 = 2$$
$$x_10 = -3$$

$\mathfrak{m}_{7A}(2, 11)$

m7A211 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_2, e_3] &= e_5 \\
[e_2, e_4] &= e_6 & [e_2, e_5] &= \alpha_{2,5}^{7} e_7 \\
[e_2, e_6] &= \alpha_{2,6}^{8} e_8 & [e_2, e_7] &= \alpha_{2,7}^{9} e_9 \\
[e_2, e_8] &= \alpha_{2,8}^{10} e_{10} & [e_2, e_9] &= \alpha_{2,9}^{11} e_{11} \\
[e_3, e_4] &= \alpha_{3,4}^{7} e_7 & [e_3, e_5] &= \alpha_{3,5}^{8} e_8 \\
[e_3, e_6] &= \alpha_{3,6}^{9} e_9 & [e_3, e_7] &= \alpha_{3,7}^{10} e_{10} \\
[e_3, e_8] &= \alpha_{3,8}^{11} e_{11} & [e_4, e_5] &= \alpha_{4,5}^{9} e_9 \\
[e_4, e_6] &= \alpha_{4,6}^{10} e_{10} & [e_4, e_7] &= \alpha_{4,7}^{11} e_{11} \\
[e_5, e_6] &= \alpha_{5,6}^{11} e_{11}
\end{aligned}$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1,e_2,e_4):&\quad -\alpha^{7}_{2,5}-\alpha^{7}_{3,4}+1 &&=0\\
(e_1,e_2,e_5):&\quad \alpha^{7}_{2,5}-\alpha^{8}_{2,6}-\alpha^{8}_{3,5} &&=0\\
(e_1,e_3,e_4):&\quad \alpha^{7}_{3,4}-\alpha^{8}_{3,5} &&=0\\
(e_1,e_2,e_6):&\quad \alpha^{8}_{2,6}-\alpha^{9}_{2,7}-\alpha^{9}_{3,6} &&=0\\
(e_1,e_3,e_5):&\quad \alpha^{8}_{3,5}-\alpha^{9}_{3,6}-\alpha^{9}_{4,5} &&=0\\
(e_2,e_3,e_4):&\quad \alpha^{9}_{2,7}\alpha^{7}_{3,4}-\alpha^{9}_{3,6}+\alpha^{9}_{4,5} &&=0\\
(e_1,e_2,e_7):&\quad \alpha^{9}_{2,7}-\alpha^{10}_{2,8}-\alpha^{10}_{3,7} &&=0\\
(e_1,e_3,e_6):&\quad \alpha^{9}_{3,6}-\alpha^{10}_{3,7}-\alpha^{10}_{4,6} &&=0\\
(e_1,e_4,e_5):&\quad \alpha^{9}_{4,5}-\alpha^{10}_{4,6} &&=0\\
(e_2,e_3,e_5):&\quad -\alpha^{7}_{2,5}\alpha^{10}_{3,7}+\alpha^{10}_{2,8}\alpha^{8}_{3,5} &&=0\\
(e_1,e_2,e_8):&\quad \alpha^{10}_{2,8}-\alpha^{11}_{2,9}-\alpha^{11}_{3,8} &&=0\\
(e_1,e_3,e_7):&\quad \alpha^{10}_{3,7}-\alpha^{11}_{3,8}-\alpha^{11}_{4,7} &&=0\\
(e_1,e_4,e_6):&\quad \alpha^{10}_{4,6}-\alpha^{11}_{4,7}-\alpha^{11}_{5,6} &&=0\\
(e_2,e_3,e_6):&\quad -\alpha^{8}_{2,6}\alpha^{11}_{3,8}+\alpha^{11}_{2,9}\alpha^{9}_{3,6}-\alpha^{11}_{5,6} &&=0\\
(e_2,e_4,e_5):&\quad -\alpha^{7}_{2,5}\alpha^{11}_{4,7}+\alpha^{11}_{2,9}\alpha^{9}_{4,5}+\alpha^{11}_{5,6} &&=0
\end{aligned}
$$

Infinite number of solutions: only zero-dimensional systems supported (finite number of solutions).
How the solution(s) were or were not found:
Change variables

$$
\begin{gathered}
\alpha^{11}_{2,9}\to x_1\\
\alpha^{7}_{2,5}\to x_2\\
\alpha^{8}_{2,6}\to x_3\\
\alpha^{8}_{3,5}\to x_4\\
\alpha^{10}_{2,8}\to x_5\\
\alpha^{10}_{3,7}\to x_6\\
\alpha^{11}_{5,6}\to x_7\\
\alpha^{11}_{3,8}\to x_8\\
\alpha^{7}_{3,4}\to x_9\\
\alpha^{9}_{4,5}\to x_{10}\\
\alpha^{9}_{2,7}\to x_{11}
\end{gathered}
$$

$$\alpha_{4,7}^{11} \to x_{12}$$
$$\alpha_{3,6}^{9} \to x_{13}$$
$$\alpha_{4,6}^{10} \to x_{14}$$

Jacobi Tests

$$\begin{aligned}
(e_1,e_2,e_4):&\quad -x_2 - x_9 + 1 &&= 0\\
(e_1,e_2,e_5):&\quad x_2 - x_3 - x_4 &&= 0\\
(e_1,e_3,e_4):&\quad -x_4 + x_9 &&= 0\\
(e_1,e_2,e_6):&\quad -x_{11} - x_{13} + x_3 &&= 0\\
(e_1,e_3,e_5):&\quad -x_{10} - x_{13} + x_4 &&= 0\\
(e_2,e_3,e_4):&\quad x_{10} + x_{11}x_9 - x_{13} &&= 0\\
(e_1,e_2,e_7):&\quad x_{11} - x_5 - x_6 &&= 0\\
(e_1,e_3,e_6):&\quad x_{13} - x_{14} - x_6 &&= 0\\
(e_1,e_4,e_5):&\quad x_{10} - x_{14} &&= 0\\
(e_2,e_3,e_5):&\quad -x_2x_6 + x_4x_5 &&= 0\\
(e_1,e_2,e_8):&\quad -x_1 + x_5 - x_8 &&= 0\\
(e_1,e_3,e_7):&\quad -x_{12} + x_6 - x_8 &&= 0\\
(e_1,e_4,e_6):&\quad -x_{12} + x_{14} - x_7 &&= 0\\
(e_2,e_3,e_6):&\quad x_1x_{13} - x_3x_8 - x_7 &&= 0\\
(e_2,e_4,e_5):&\quad x_1x_{10} - x_{12}x_2 + x_7 &&= 0
\end{aligned}$$

Groebner basis (14 variables, 11 linear, 3 nonlinear)

$$x_1 - x_{12} + 5x_{13} - 1 = 0$$
$$x_{13} + x_{14} + x_2 - 1 = 0$$
$$2x_{13} + 2x_{14} + x_3 - 1 = 0$$
$$-x_{13} - x_{14} + x_4 = 0$$
$$4x_{13} + x_{14} + x_5 - 1 = 0$$
$$-x_{13} + x_{14} + x_6 = 0$$
$$x_{12} - x_{14} + x_7 = 0$$
$$x_{12} - x_{13} + x_{14} + x_8 = 0$$
$$-x_{13} - x_{14} + x_9 = 0$$
$$x_{10} - x_{14} = 0$$
$$x_{11} + 3x_{13} + 2x_{14} - 1 = 0$$
$$x_{12}x_{13} + 2x_{12}x_{14} - 2x_{12} - 5x_{13}x_{14} + 2x_{14} = 0$$
$$2x_{12}x_{14}^2 - 8x_{12}x_{14} + 6x_{12} - 15x_{13}x_{14}^2 + 12x_{13}x_{14} - 5x_{14}^3 + 6x_{14}^2 - 6x_{14} = 0$$
$$3x_{13}^2 + 5x_{13}x_{14} + 2x_{14}^2 - 2x_{14} = 0$$

$\mathfrak{m}_{2A}(3, 11)$

m2A311 (this line included for string searching purposes)

Original brackets:

$$
\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_1, e_9] = e_{10} \\
[e_1, e_{10}] = e_{11} & [e_2, e_7] = e_{10} \\
[e_2, e_8] = 3e_{11} & [e_3, e_6] = -e_{10} \\
[e_3, e_7] = -2e_{11} & [e_4, e_5] = e_{10} \\
[e_4, e_6] = e_{11} &
\end{array}
$$

Non-trivial Jacobi Tests:

$\mathfrak{m}_{4A}(3, 11)$

m4A311 (this line included for string searching purposes)
Solution 1

$$
\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_1, e_9] = e_{10} \\
[e_1, e_{10}] = e_{11} & [e_2, e_5] = e_8 \\
[e_2, e_6] = 2e_9 & [e_2, e_7] = \frac{5e_{10}}{3} \\
[e_2, e_8] = 0 & [e_3, e_4] = -e_8 \\
[e_3, e_5] = -e_9 & [e_3, e_6] = \frac{e_{10}}{3} \\
[e_3, e_7] = \frac{5e_{11}}{3} & [e_4, e_5] = -\frac{4e_{10}}{3} \\
[e_4, e_6] = -\frac{4e_{11}}{3} &
\end{array}
$$

Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4\\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6\\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8\\
[e_1,e_8] &= e_9 & [e_1,e_9] &= e_{10}\\
[e_1,e_{10}] &= e_{11} & [e_2,e_5] &= e_8\\
[e_2,e_6] &= 2e_9 & [e_2,e_7] &= \alpha_{2,7}^{10}e_{10}\\
[e_2,e_8] &= \alpha_{2,8}^{11}e_{11} & [e_3,e_4] &= -e_8\\
[e_3,e_5] &= -e_9 & [e_3,e_6] &= \alpha_{3,6}^{10}e_{10}\\
[e_3,e_7] &= \alpha_{3,7}^{11}e_{11} & [e_4,e_5] &= \alpha_{4,5}^{10}e_{10}\\
[e_4,e_6] &= \alpha_{4,6}^{11}e_{11}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1,e_2,e_6):&\quad -\alpha_{2,7}^{10}-\alpha_{3,6}^{10}+2 &&=0\\
(e_1,e_3,e_5):&\quad -\alpha_{3,6}^{10}-\alpha_{4,5}^{10}-1 &&=0\\
(e_1,e_2,e_7):&\quad \alpha_{2,7}^{10}-\alpha_{2,8}^{11}-\alpha_{3,7}^{11} &&=0\\
(e_1,e_3,e_6):&\quad \alpha_{3,6}^{10}-\alpha_{3,7}^{11}-\alpha_{4,6}^{11} &&=0\\
(e_1,e_4,e_5):&\quad \alpha_{4,5}^{10}-\alpha_{4,6}^{11} &&=0\\
(e_2,e_3,e_4):&\quad -\alpha_{2,8}^{11} &&=0
\end{aligned}
$$

Solution 1:

$$
\begin{aligned}
\alpha_{3,6}^{10} &= 1/3\\
\alpha_{4,5}^{10} &= -4/3\\
\alpha_{4,6}^{11} &= -4/3\\
\alpha_{3,7}^{11} &= 5/3\\
\alpha_{2,7}^{10} &= 5/3\\
\alpha_{2,8}^{11} &= 0
\end{aligned}
$$

How the solution(s) were or were not found:
Change variables

$$
\begin{aligned}
\alpha_{3,6}^{10} &\to x_1\\
\alpha_{4,5}^{10} &\to x_2
\end{aligned}
$$

$$\alpha^{11}_{4,6} \to x_3$$

$$\alpha^{11}_{3,7} \to x_4$$

$$\alpha^{10}_{2,7} \to x_5$$

$$\alpha^{11}_{2,8} \to x_6$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_6):&\quad -x_1 - x_5 + 2 &&= 0\\
(e_1, e_3, e_5):&\quad -x_1 - x_2 - 1 &&= 0\\
(e_1, e_2, e_7):&\quad -x_4 + x_5 - x_6 &&= 0\\
(e_1, e_3, e_6):&\quad x_1 - x_3 - x_4 &&= 0\\
(e_1, e_4, e_5):&\quad x_2 - x_3 &&= 0\\
(e_2, e_3, e_4):&\quad -x_6 &&= 0
\end{aligned}$$

Groebner basis (6 variables, 6 linear, 0 nonlinear)

$$3x_1 - 1 = 0$$

$$3x_2 + 4 = 0$$

$$3x_3 + 4 = 0$$

$$3x_4 - 5 = 0$$

$$3x_5 - 5 = 0$$

$$x_6 = 0$$

Solution 1:

$$x_1 = 1/3$$

$$x_2 = -4/3$$

$$x_3 = -4/3$$

$$x_4 = 5/3$$

$$x_5 = 5/3$$

$$x_6 = 0$$

$\mathfrak{m}_{6A}(3, 11)$

m6A311 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_2, e_3] &= e_6 \\
[e_2, e_4] &= e_7 & [e_2, e_5] &= \alpha_{2,5}^{8} e_8 \\
[e_2, e_6] &= \alpha_{2,6}^{9} e_9 & [e_2, e_7] &= \alpha_{2,7}^{10} e_{10} \\
[e_2, e_8] &= \alpha_{2,8}^{11} e_{11} & [e_3, e_4] &= \alpha_{3,4}^{8} e_8 \\
[e_3, e_5] &= \alpha_{3,5}^{9} e_9 & [e_3, e_6] &= \alpha_{3,6}^{10} e_{10} \\
[e_3, e_7] &= \alpha_{3,7}^{11} e_{11} & [e_4, e_5] &= \alpha_{4,5}^{10} e_{10} \\
[e_4, e_6] &= \alpha_{4,6}^{11} e_{11}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_4): &\quad -\alpha_{2,5}^{8} - \alpha_{3,4}^{8} + 1 &&= 0 \\
(e_1, e_2, e_5): &\quad \alpha_{2,5}^{8} - \alpha_{2,6}^{9} - \alpha_{3,5}^{9} &&= 0 \\
(e_1, e_3, e_4): &\quad \alpha_{3,4}^{8} - \alpha_{3,5}^{9} &&= 0 \\
(e_1, e_2, e_6): &\quad \alpha_{2,6}^{9} - \alpha_{2,7}^{10} - \alpha_{3,6}^{10} &&= 0 \\
(e_1, e_3, e_5): &\quad \alpha_{3,5}^{9} - \alpha_{3,6}^{10} - \alpha_{4,5}^{10} &&= 0 \\
(e_1, e_2, e_7): &\quad \alpha_{2,7}^{10} - \alpha_{2,8}^{11} - \alpha_{3,7}^{11} &&= 0 \\
(e_1, e_3, e_6): &\quad \alpha_{3,6}^{10} - \alpha_{3,7}^{11} - \alpha_{4,6}^{11} &&= 0 \\
(e_1, e_4, e_5): &\quad \alpha_{4,5}^{10} - \alpha_{4,6}^{11} &&= 0 \\
(e_2, e_3, e_4): &\quad \alpha_{2,8}^{11}\alpha_{3,4}^{8} - \alpha_{3,7}^{11} + \alpha_{4,6}^{11} &&= 0
\end{aligned}
$$

Infinite number of solutions: only zero-dimensional systems supported (finite number of solutions).
How the solution(s) were or were not found:
Change variables

$$\alpha_{2,5}^{8} \to x_1$$

$$\alpha_{3,6}^{10} \to x_2$$

$$\alpha^9_{3,5} \to x_3$$

$$\alpha^8_{3,4} \to x_4$$

$$\alpha^9_{2,6} \to x_5$$

$$\alpha^{10}_{4,5} \to x_6$$

$$\alpha^{11}_{4,6} \to x_7$$

$$\alpha^{11}_{3,7} \to x_8$$

$$\alpha^{10}_{2,7} \to x_9$$

$$\alpha^{11}_{2,8} \to x_{10}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4) : &\quad - x_1 - x_4 + 1 &&= 0\\
(e_1, e_2, e_5) : &\quad x_1 - x_3 - x_5 &&= 0\\
(e_1, e_3, e_4) : &\quad - x_3 + x_4 &&= 0\\
(e_1, e_2, e_6) : &\quad - x_2 + x_5 - x_9 &&= 0\\
(e_1, e_3, e_5) : &\quad - x_2 + x_3 - x_6 &&= 0\\
(e_1, e_2, e_7) : &\quad - x_{10} - x_8 + x_9 &&= 0\\
(e_1, e_3, e_6) : &\quad x_2 - x_7 - x_8 &&= 0\\
(e_1, e_4, e_5) : &\quad x_6 - x_7 &&= 0\\
(e_2, e_3, e_4) : &\quad x_{10}x_4 + x_7 - x_8 &&= 0
\end{aligned}$$

Groebner basis (10 variables, 8 linear, 1 nonlinear)

$$5x_1 + x_{10} - 3x_9 - 3 = 0$$

$$2x_{10} + 5x_2 - x_9 - 1 = 0$$

$$-x_{10} + 5x_3 + 3x_9 - 2 = 0$$

$$-x_{10} + 5x_4 + 3x_9 - 2 = 0$$

$$2x_{10} + 5x_5 - 6x_9 - 1 = 0$$

$$-3x_{10} + 5x_6 + 4x_9 - 1 = 0$$

$$-3x_{10} + 5x_7 + 4x_9 - 1 = 0$$

$$x_{10} + x_8 - x_9 = 0$$

$$-x_{10}^2 + 3x_{10}x_9 - 10x_{10} + 9x_9 - 1 = 0$$

$\mathfrak{m}_{1A}(4, 11)$

m1A411 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_2, e_7] &= e_{11} \\
[e_3, e_6] &= -e_{11} & [e_4, e_5] &= e_{11}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$\mathfrak{m}_{3A}(4, 11)$

m3A411 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_2, e_5] &= e_9 \\
[e_2, e_6] &= 2e_{10} & [e_2, e_7] &= \alpha_{2,7}^{11} e_{11} \\
[e_3, e_4] &= -e_9 & [e_3, e_5] &= -e_{10} \\
[e_3, e_6] &= \alpha_{3,6}^{11} e_{11} & [e_4, e_5] &= \alpha_{4,5}^{11} e_{11}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_6): &\quad -\alpha_{2,7}^{11} - \alpha_{3,6}^{11} + 2 &&= 0 \\
(e_1, e_3, e_5): &\quad -\alpha_{3,6}^{11} - \alpha_{4,5}^{11} - 1 &&= 0
\end{aligned}
$$

Infinite number of solutions: only zero-dimensional systems supported (finite number of solutions).
How the solution(s) were or were not found:
Change variables

$$\alpha_{2,7}^{11} \to x_1$$

$$\alpha_{4,5}^{11} \to x_2$$

$$\alpha_{3,6}^{11} \to x_3$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_6): \quad & -x_1 - x_3 + 2 && = 0 \\
(e_1, e_3, e_5): \quad & -x_2 - x_3 - 1 && = 0
\end{aligned}$$

Groebner basis (3 variables, 2 linear, 0 nonlinear)

$$x_1 + x_3 - 2 = 0$$

$$x_2 + x_3 + 1 = 0$$

$\mathfrak{m}_{5A}(4, 11)$

m5A411 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_2, e_3] &= e_7 \\
[e_2, e_4] &= e_8 & [e_2, e_5] &= \alpha_{2,5}^{9} e_9 \\
[e_2, e_6] &= \alpha_{2,6}^{10} e_{10} & [e_2, e_7] &= \alpha_{2,7}^{11} e_{11} \\
[e_3, e_4] &= \alpha_{3,4}^{9} e_9 & [e_3, e_5] &= \alpha_{3,5}^{10} e_{10} \\
[e_3, e_6] &= \alpha_{3,6}^{11} e_{11} & [e_4, e_5] &= \alpha_{4,5}^{11} e_{11}
\end{aligned}$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1,e_2,e_4):&\quad -\alpha_{2,5}^{9}-\alpha_{3,4}^{9}+1 &&=0\\
(e_1,e_2,e_5):&\quad \alpha_{2,5}^{9}-\alpha_{2,6}^{10}-\alpha_{3,5}^{10} &&=0\\
(e_1,e_3,e_4):&\quad \alpha_{3,4}^{9}-\alpha_{3,5}^{10} &&=0\\
(e_1,e_2,e_6):&\quad \alpha_{2,6}^{10}-\alpha_{2,7}^{11}-\alpha_{3,6}^{11} &&=0\\
(e_1,e_3,e_5):&\quad \alpha_{3,5}^{10}-\alpha_{3,6}^{11}-\alpha_{4,5}^{11} &&=0
\end{aligned}
$$

Infinite number of solutions: only zero-dimensional systems supported (finite number of solutions).
How the solution(s) were or were not found:
Change variables

$$
\begin{gathered}
\alpha_{2,5}^{9}\to x_1\\
\alpha_{2,7}^{11}\to x_2\\
\alpha_{4,5}^{11}\to x_3\\
\alpha_{3,6}^{11}\to x_4\\
\alpha_{2,6}^{10}\to x_5\\
\alpha_{3,5}^{10}\to x_6\\
\alpha_{3,4}^{9}\to x_7
\end{gathered}
$$

Jacobi Tests

$$
\begin{aligned}
(e_1,e_2,e_4):&\quad -x_1-x_7+1 &&=0\\
(e_1,e_2,e_5):&\quad x_1-x_5-x_6 &&=0\\
(e_1,e_3,e_4):&\quad -x_6+x_7 &&=0\\
(e_1,e_2,e_6):&\quad -x_2-x_4+x_5 &&=0\\
(e_1,e_3,e_5):&\quad -x_3-x_4+x_6 &&=0
\end{aligned}
$$

Groebner basis (7 variables, 5 linear, 0 nonlinear)

$$
\begin{gathered}
x_1+x_7-1=0\\
x_2+x_4+2x_7-1=0\\
x_3+x_4-x_7=0\\
x_5+2x_7-1=0\\
x_6-x_7=0
\end{gathered}
$$

$\mathfrak{m}_{2A}(5, 11)$

m2A511 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_2, e_5] &= e_{10} \\
[e_2, e_6] &= 2e_{11} & [e_3, e_4] &= -e_{10} \\
[e_3, e_5] &= -e_{11}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$\mathfrak{m}_{4A}(5, 11)$

m4A511 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_2, e_3] &= e_8 \\
[e_2, e_4] &= e_9 & [e_2, e_5] &= \alpha_{2,5}^{10} e_{10} \\
[e_2, e_6] &= \alpha_{2,6}^{11} e_{11} & [e_3, e_4] &= \alpha_{3,4}^{10} e_{10} \\
[e_3, e_5] &= \alpha_{3,5}^{11} e_{11}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_4):& \quad -\alpha_{2,5}^{10} - \alpha_{3,4}^{10} + 1 &&= 0 \\
(e_1, e_2, e_5):& \quad \alpha_{2,5}^{10} - \alpha_{2,6}^{11} - \alpha_{3,5}^{11} &&= 0 \\
(e_1, e_3, e_4):& \quad \alpha_{3,4}^{10} - \alpha_{3,5}^{11} &&= 0
\end{aligned}
$$

Infinite number of solutions: only zero-dimensional systems supported (finite number of solutions).
How the solution(s) were or were not found:
Change variables

$$\alpha_{3,5}^{11} \to x_1$$
$$\alpha_{2,5}^{10} \to x_2$$
$$\alpha_{3,4}^{10} \to x_3$$
$$\alpha_{2,6}^{11} \to x_4$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4) : \quad & -x_2 - x_3 + 1 && = 0 \\
(e_1, e_2, e_5) : \quad & -x_1 + x_2 - x_4 && = 0 \\
(e_1, e_3, e_4) : \quad & -x_1 + x_3 && = 0
\end{aligned}$$

Groebner basis (4 variables, 3 linear, 0 nonlinear)

$$2x_1 + x_4 - 1 = 0$$
$$2x_2 - x_4 - 1 = 0$$
$$2x_3 + x_4 - 1 = 0$$

$\mathfrak{m}_{1A}(6, 11)$

m1A611 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_2, e_5] &= e_{11} \\
[e_3, e_4] &= -e_{11}
\end{aligned}$$

Non-trivial Jacobi Tests:

$\mathfrak{m}_{3A}(6,11)$

m3A611 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4 \\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6 \\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8 \\
[e_1,e_8] &= e_9 & [e_1,e_9] &= e_{10} \\
[e_1,e_{10}] &= e_{11} & [e_2,e_3] &= e_9 \\
[e_2,e_4] &= e_{10} & [e_2,e_5] &= \alpha_{2,5}^{11} e_{11} \\
[e_3,e_4] &= \alpha_{3,4}^{11} e_{11}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$(e_1,e_2,e_4): \quad -\alpha_{2,5}^{11} - \alpha_{3,4}^{11} + 1 = 0$$

Infinite number of solutions: only zero-dimensional systems supported (finite number of solutions).
How the solution(s) were or were not found:
Change variables

$$\alpha_{2,5}^{11} \to x_1$$
$$\alpha_{3,4}^{11} \to x_2$$

Jacobi Tests

$$(e_1,e_2,e_4): \quad -x_1 - x_2 + 1 = 0$$

Groebner basis (2 variables, 1 linear, 0 nonlinear)

$$x_1 + x_2 - 1 = 0$$

$\mathfrak{m}_{2A}(7,11)$

m2A711 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_2, e_3] &= e_{10} \\
[e_2, e_4] &= e_{11}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$\mathfrak{m}_{1A}(8, 11)$

m1A811 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_2, e_3] &= e_{11}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$\mathfrak{m}_{2A}(2, 12)$

m2A212 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1,e_2]&=e_3 & [e_1,e_3]&=e_4\\
[e_1,e_4]&=e_5 & [e_1,e_5]&=e_6\\
[e_1,e_6]&=e_7 & [e_1,e_7]&=e_8\\
[e_1,e_8]&=e_9 & [e_1,e_9]&=e_{10}\\
[e_1,e_{10}]&=e_{11} & [e_1,e_{11}]&=e_{12}\\
[e_2,e_9]&=e_{11} & [e_2,e_{10}]&=4e_{12}\\
[e_3,e_8]&=-e_{11} & [e_3,e_9]&=-3e_{12}\\
[e_4,e_7]&=e_{11} & [e_4,e_8]&=2e_{12}\\
[e_5,e_6]&=-e_{11} & [e_5,e_7]&=-e_{12}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$\mathfrak{m}_{4A}(2,12)$

m4A212 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1,e_2]&=e_3 & [e_1,e_3]&=e_4\\
[e_1,e_4]&=e_5 & [e_1,e_5]&=e_6\\
[e_1,e_6]&=e_7 & [e_1,e_7]&=e_8\\
[e_1,e_8]&=e_9 & [e_1,e_9]&=e_{10}\\
[e_1,e_{10}]&=e_{11} & [e_1,e_{11}]&=e_{12}\\
[e_2,e_7]&=e_9 & [e_2,e_8]&=3e_{10}\\
[e_2,e_9]&=\alpha_{2,9}^{11}e_{11} & [e_2,e_{10}]&=\alpha_{2,10}^{12}e_{12}\\
[e_3,e_6]&=-e_9 & [e_3,e_7]&=-2e_{10}\\
[e_3,e_8]&=\alpha_{3,8}^{11}e_{11} & [e_3,e_9]&=\alpha_{3,9}^{12}e_{12}\\
[e_4,e_5]&=e_9 & [e_4,e_6]&=e_{10}\\
[e_4,e_7]&=\alpha_{4,7}^{11}e_{11} & [e_4,e_8]&=\alpha_{4,8}^{12}e_{12}\\
[e_5,e_6]&=\alpha_{5,6}^{11}e_{11} & [e_5,e_7]&=\alpha_{5,7}^{12}e_{12}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_8): &\quad -\alpha^{11}_{2,9} - \alpha^{11}_{3,8} + 3 &&= 0\\
(e_1, e_3, e_7): &\quad -\alpha^{11}_{3,8} - \alpha^{11}_{4,7} - 2 &&= 0\\
(e_1, e_4, e_6): &\quad -\alpha^{11}_{4,7} - \alpha^{11}_{5,6} + 1 &&= 0\\
(e_2, e_3, e_6): &\quad -\alpha^{11}_{2,9} &&= 0\\
(e_2, e_4, e_5): &\quad \alpha^{11}_{2,9} &&= 0\\
(e_1, e_2, e_9): &\quad -\alpha^{12}_{2,10} + \alpha^{11}_{2,9} - \alpha^{12}_{3,9} &&= 0\\
(e_1, e_3, e_8): &\quad \alpha^{11}_{3,8} - \alpha^{12}_{3,9} - \alpha^{12}_{4,8} &&= 0\\
(e_1, e_4, e_7): &\quad \alpha^{11}_{4,7} - \alpha^{12}_{4,8} - \alpha^{12}_{5,7} &&= 0\\
(e_1, e_5, e_6): &\quad \alpha^{11}_{5,6} - \alpha^{12}_{5,7} &&= 0\\
(e_2, e_3, e_7): &\quad -2\alpha^{12}_{2,10} - \alpha^{12}_{3,9} &&= 0\\
(e_2, e_4, e_6): &\quad \alpha^{12}_{2,10} &&= 0\\
(e_3, e_4, e_5): &\quad \alpha^{12}_{3,9} &&= 0
\end{aligned}
$$

No solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha^{11}_{2,9} \to x_1$$
$$\alpha^{12}_{4,8} \to x_2$$
$$\alpha^{12}_{5,7} \to x_3$$
$$\alpha^{11}_{3,8} \to x_4$$
$$\alpha^{12}_{3,9} \to x_5$$
$$\alpha^{12}_{2,10} \to x_6$$
$$\alpha^{11}_{4,7} \to x_7$$
$$\alpha^{11}_{5,6} \to x_8$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_8): &\quad -x_1 - x_4 + 3 &= 0\\
(e_1, e_3, e_7): &\quad -x_4 - x_7 - 2 &= 0\\
(e_1, e_4, e_6): &\quad -x_7 - x_8 + 1 &= 0\\
(e_2, e_3, e_6): &\quad -x_1 &= 0\\
(e_2, e_4, e_5): &\quad x_1 &= 0\\
(e_1, e_2, e_9): &\quad x_1 - x_5 - x_6 &= 0\\
(e_1, e_3, e_8): &\quad -x_2 + x_4 - x_5 &= 0\\
(e_1, e_4, e_7): &\quad -x_2 - x_3 + x_7 &= 0\\
(e_1, e_5, e_6): &\quad -x_3 + x_8 &= 0\\
(e_2, e_3, e_7): &\quad -x_5 - 2x_6 &= 0\\
(e_2, e_4, e_6): &\quad x_6 &= 0\\
(e_3, e_4, e_5): &\quad x_5 &= 0
\end{aligned}
$$

Groebner basis (8 variables, 1 linear, 0 nonlinear)

$$1 = 0$$

$\mathfrak{m}_{6A}(2, 12)$

m6A212 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_2, e_5] &= e_7 & [e_2, e_6] &= 2e_8 \\
[e_2, e_7] &= \alpha_{2,7}^{9} e_9 & [e_2, e_8] &= \alpha_{2,8}^{10} e_{10} \\
[e_2, e_9] &= \alpha_{2,9}^{11} e_{11} & [e_2, e_{10}] &= \alpha_{2,10}^{12} e_{12} \\
[e_3, e_4] &= -e_7 & [e_3, e_5] &= -e_8 \\
[e_3, e_6] &= \alpha_{3,6}^{9} e_9 & [e_3, e_7] &= \alpha_{3,7}^{10} e_{10} \\
[e_3, e_8] &= \alpha_{3,8}^{11} e_{11} & [e_3, e_9] &= \alpha_{3,9}^{12} e_{12} \\
[e_4, e_5] &= \alpha_{4,5}^{9} e_9 & [e_4, e_6] &= \alpha_{4,6}^{10} e_{10} \\
[e_4, e_7] &= \alpha_{4,7}^{11} e_{11} & [e_4, e_8] &= \alpha_{4,8}^{12} e_{12} \\
[e_5, e_6] &= \alpha_{5,6}^{11} e_{11} & [e_5, e_7] &= \alpha_{5,7}^{12} e_{12}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1,e_2,e_6):&\quad -\alpha_{2,7}^{9}-\alpha_{3,6}^{9}+2 &&=0\\
(e_1,e_3,e_5):&\quad -\alpha_{3,6}^{9}-\alpha_{4,5}^{9}-1 &&=0\\
(e_2,e_3,e_4):&\quad -\alpha_{2,7}^{9} &&=0\\
(e_1,e_2,e_7):&\quad \alpha_{2,7}^{9}-\alpha_{2,8}^{10}-\alpha_{3,7}^{10} &&=0\\
(e_1,e_3,e_6):&\quad \alpha_{3,6}^{9}-\alpha_{3,7}^{10}-\alpha_{4,6}^{10} &&=0\\
(e_1,e_4,e_5):&\quad \alpha_{4,5}^{9}-\alpha_{4,6}^{10} &&=0\\
(e_2,e_3,e_5):&\quad -\alpha_{2,8}^{10}-\alpha_{3,7}^{10} &&=0\\
(e_1,e_2,e_8):&\quad \alpha_{2,8}^{10}-\alpha_{2,9}^{11}-\alpha_{3,8}^{11} &&=0\\
(e_1,e_3,e_7):&\quad \alpha_{3,7}^{10}-\alpha_{3,8}^{11}-\alpha_{4,7}^{11} &&=0\\
(e_1,e_4,e_6):&\quad \alpha_{4,6}^{10}-\alpha_{4,7}^{11}-\alpha_{5,6}^{11} &&=0\\
(e_2,e_3,e_6):&\quad \alpha_{2,9}^{11}\alpha_{3,6}^{9}-2\alpha_{3,8}^{11} &&=0\\
(e_2,e_4,e_5):&\quad \alpha_{2,9}^{11}\alpha_{4,5}^{9}-\alpha_{4,7}^{11} &&=0\\
(e_1,e_2,e_9):&\quad -\alpha_{2,10}^{12}+\alpha_{2,9}^{11}-\alpha_{3,9}^{12} &&=0\\
(e_1,e_3,e_8):&\quad \alpha_{3,8}^{11}-\alpha_{3,9}^{12}-\alpha_{4,8}^{12} &&=0\\
(e_1,e_4,e_7):&\quad \alpha_{4,7}^{11}-\alpha_{4,8}^{12}-\alpha_{5,7}^{12} &&=0\\
(e_1,e_5,e_6):&\quad \alpha_{5,6}^{11}-\alpha_{5,7}^{12} &&=0\\
(e_2,e_3,e_7):&\quad \alpha_{2,10}^{12}\alpha_{3,7}^{10}-\alpha_{2,7}^{9}\alpha_{3,9}^{12} &&=0\\
(e_2,e_4,e_6):&\quad \alpha_{2,10}^{12}\alpha_{4,6}^{10}-2\alpha_{4,8}^{12} &&=0\\
(e_3,e_4,e_5):&\quad \alpha_{3,9}^{12}\alpha_{4,5}^{9}+\alpha_{4,8}^{12}-\alpha_{5,7}^{12} &&=0
\end{aligned}
$$

No solutions.
How the solution(s) were or were not found:
Change variables

$$
\begin{aligned}
\alpha_{2,9}^{11}&\to x_1\\
\alpha_{4,8}^{12}&\to x_2\\
\alpha_{2,8}^{10}&\to x_3\\
\alpha_{3,7}^{10}&\to x_4\\
\alpha_{5,7}^{12}&\to x_5\\
\alpha_{5,6}^{11}&\to x_6\\
\alpha_{3,8}^{11}&\to x_7\\
\alpha_{2,10}^{12}&\to x_8
\end{aligned}
$$

$$\alpha^{12}_{3,9} \to x_9$$

$$\alpha^{9}_{4,5} \to x_{10}$$

$$\alpha^{9}_{2,7} \to x_{11}$$

$$\alpha^{11}_{4,7} \to x_{12}$$

$$\alpha^{9}_{3,6} \to x_{13}$$

$$\alpha^{10}_{4,6} \to x_{14}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_6): &\quad -x_{11} - x_{13} + 2 &= 0\\
(e_1, e_3, e_5): &\quad -x_{10} - x_{13} - 1 &= 0\\
(e_2, e_3, e_4): &\quad -x_{11} &= 0\\
(e_1, e_2, e_7): &\quad x_{11} - x_3 - x_4 &= 0\\
(e_1, e_3, e_6): &\quad x_{13} - x_{14} - x_4 &= 0\\
(e_1, e_4, e_5): &\quad x_{10} - x_{14} &= 0\\
(e_2, e_3, e_5): &\quad -x_3 - x_4 &= 0\\
(e_1, e_2, e_8): &\quad -x_1 + x_3 - x_7 &= 0\\
(e_1, e_3, e_7): &\quad -x_{12} + x_4 - x_7 &= 0\\
(e_1, e_4, e_6): &\quad -x_{12} + x_{14} - x_6 &= 0\\
(e_2, e_3, e_6): &\quad x_1x_{13} - 2x_7 &= 0\\
(e_2, e_4, e_5): &\quad x_1x_{10} - x_{12} &= 0\\
(e_1, e_2, e_9): &\quad x_1 - x_8 - x_9 &= 0\\
(e_1, e_3, e_8): &\quad -x_2 + x_7 - x_9 &= 0\\
(e_1, e_4, e_7): &\quad x_{12} - x_2 - x_5 &= 0\\
(e_1, e_5, e_6): &\quad -x_5 + x_6 &= 0\\
(e_2, e_3, e_7): &\quad -x_{11}x_9 + x_4x_8 &= 0\\
(e_2, e_4, e_6): &\quad x_{14}x_8 - 2x_2 &= 0\\
(e_3, e_4, e_5): &\quad x_{10}x_9 + x_2 - x_5 &= 0
\end{aligned}$$

Groebner basis (14 variables, 1 linear, 0 nonlinear)

$$1 = 0$$

$\mathfrak{m}_{8A}(2,12)$

m8A212 (this line included for string searching purposes)
Solution 1

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_1, e_{11}] = e_{12}$$
$$[e_2, e_3] = e_5 \qquad [e_2, e_4] = e_6$$
$$[e_2, e_5] = \frac{9e_7}{10} \qquad [e_2, e_6] = \frac{4e_8}{5}$$
$$[e_2, e_7] = \frac{5e_9}{7} \qquad [e_2, e_8] = \frac{9e_{10}}{14}$$
$$[e_2, e_9] = \frac{7e_{11}}{12} \qquad [e_2, e_{10}] = \frac{8e_{12}}{15}$$
$$[e_3, e_4] = \frac{e_7}{10} \qquad [e_3, e_5] = \frac{e_8}{10}$$
$$[e_3, e_6] = \frac{3e_9}{35} \qquad [e_3, e_7] = \frac{e_{10}}{14}$$
$$[e_3, e_8] = \frac{5e_{11}}{84} \qquad [e_3, e_9] = \frac{e_{12}}{20}$$
$$[e_4, e_5] = \frac{e_9}{70} \qquad [e_4, e_6] = \frac{e_{10}}{70}$$
$$[e_4, e_7] = \frac{e_{11}}{84} \qquad [e_4, e_8] = \frac{e_{12}}{105}$$
$$[e_5, e_6] = \frac{e_{11}}{420} \qquad [e_5, e_7] = \frac{e_{12}}{420}$$

Solution 2

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_2, e_3] &= e_5 & [e_2, e_4] &= e_6 \\
[e_2, e_5] &= e_7 & [e_2, e_6] &= e_8 \\
[e_2, e_7] &= e_9 & [e_2, e_8] &= e_{10} \\
[e_2, e_9] &= e_{11} & [e_2, e_{10}] &= e_{12} \\
[e_3, e_4] &= 0 & [e_3, e_5] &= 0 \\
[e_3, e_6] &= 0 & [e_3, e_7] &= 0 \\
[e_3, e_8] &= 0 & [e_3, e_9] &= 0 \\
[e_4, e_5] &= 0 & [e_4, e_6] &= 0 \\
[e_4, e_7] &= 0 & [e_4, e_8] &= 0 \\
[e_5, e_6] &= 0 & [e_5, e_7] &= 0
\end{aligned}
$$

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_2, e_3] &= e_5 & [e_2, e_4] &= e_6 \\
[e_2, e_5] &= \alpha_{2,5}^{7} e_7 & [e_2, e_6] &= \alpha_{2,6}^{8} e_8 \\
[e_2, e_7] &= \alpha_{2,7}^{9} e_9 & [e_2, e_8] &= \alpha_{2,8}^{10} e_{10} \\
[e_2, e_9] &= \alpha_{2,9}^{11} e_{11} & [e_2, e_{10}] &= \alpha_{2,10}^{12} e_{12} \\
[e_3, e_4] &= \alpha_{3,4}^{7} e_7 & [e_3, e_5] &= \alpha_{3,5}^{8} e_8 \\
[e_3, e_6] &= \alpha_{3,6}^{9} e_9 & [e_3, e_7] &= \alpha_{3,7}^{10} e_{10} \\
[e_3, e_8] &= \alpha_{3,8}^{11} e_{11} & [e_3, e_9] &= \alpha_{3,9}^{12} e_{12} \\
[e_4, e_5] &= \alpha_{4,5}^{9} e_9 & [e_4, e_6] &= \alpha_{4,6}^{10} e_{10} \\
[e_4, e_7] &= \alpha_{4,7}^{11} e_{11} & [e_4, e_8] &= \alpha_{4,8}^{12} e_{12} \\
[e_5, e_6] &= \alpha_{5,6}^{11} e_{11} & [e_5, e_7] &= \alpha_{5,7}^{12} e_{12}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_4): &\quad -\alpha_{2,5}^{7} - \alpha_{3,4}^{7} + 1 &&= 0\\
(e_1, e_2, e_5): &\quad \alpha_{2,5}^{7} - \alpha_{2,6}^{8} - \alpha_{3,5}^{8} &&= 0\\
(e_1, e_3, e_4): &\quad \alpha_{3,4}^{7} - \alpha_{3,5}^{8} &&= 0\\
(e_1, e_2, e_6): &\quad \alpha_{2,6}^{8} - \alpha_{2,7}^{9} - \alpha_{3,6}^{9} &&= 0\\
(e_1, e_3, e_5): &\quad \alpha_{3,5}^{8} - \alpha_{3,6}^{9} - \alpha_{4,5}^{9} &&= 0\\
(e_2, e_3, e_4): &\quad \alpha_{2,7}^{9}\alpha_{3,4}^{7} - \alpha_{3,6}^{9} + \alpha_{4,5}^{9} &&= 0\\
(e_1, e_2, e_7): &\quad \alpha_{2,7}^{9} - \alpha_{2,8}^{10} - \alpha_{3,7}^{10} &&= 0\\
(e_1, e_3, e_6): &\quad \alpha_{3,6}^{9} - \alpha_{3,7}^{10} - \alpha_{4,6}^{10} &&= 0\\
(e_1, e_4, e_5): &\quad \alpha_{4,5}^{9} - \alpha_{4,6}^{10} &&= 0\\
(e_2, e_3, e_5): &\quad -\alpha_{2,5}^{7}\alpha_{3,7}^{10} + \alpha_{2,8}^{10}\alpha_{3,5}^{8} &&= 0\\
(e_1, e_2, e_8): &\quad \alpha_{2,8}^{10} - \alpha_{2,9}^{11} - \alpha_{3,8}^{11} &&= 0\\
(e_1, e_3, e_7): &\quad \alpha_{3,7}^{10} - \alpha_{3,8}^{11} - \alpha_{4,7}^{11} &&= 0\\
(e_1, e_4, e_6): &\quad \alpha_{4,6}^{10} - \alpha_{4,7}^{11} - \alpha_{5,6}^{11} &&= 0\\
(e_2, e_3, e_6): &\quad -\alpha_{2,6}^{8}\alpha_{3,8}^{11} + \alpha_{2,9}^{11}\alpha_{3,6}^{9} - \alpha_{5,6}^{11} &&= 0\\
(e_2, e_4, e_5): &\quad -\alpha_{2,5}^{7}\alpha_{4,7}^{11} + \alpha_{2,9}^{11}\alpha_{4,5}^{9} + \alpha_{5,6}^{11} &&= 0\\
(e_1, e_2, e_9): &\quad -\alpha_{2,10}^{12} + \alpha_{2,9}^{11} - \alpha_{3,9}^{12} &&= 0\\
(e_1, e_3, e_8): &\quad \alpha_{3,8}^{11} - \alpha_{3,9}^{12} - \alpha_{4,8}^{12} &&= 0\\
(e_1, e_4, e_7): &\quad \alpha_{4,7}^{11} - \alpha_{4,8}^{12} - \alpha_{5,7}^{12} &&= 0\\
(e_1, e_5, e_6): &\quad \alpha_{5,6}^{11} - \alpha_{5,7}^{12} &&= 0\\
(e_2, e_3, e_7): &\quad \alpha_{2,10}^{12}\alpha_{3,7}^{10} - \alpha_{2,7}^{9}\alpha_{3,9}^{12} - \alpha_{5,7}^{12} &&= 0\\
(e_2, e_4, e_6): &\quad \alpha_{2,10}^{12}\alpha_{4,6}^{10} - \alpha_{2,6}^{8}\alpha_{4,8}^{12} &&= 0\\
(e_3, e_4, e_5): &\quad \alpha_{3,4}^{7}\alpha_{5,7}^{12} - \alpha_{3,5}^{8}\alpha_{4,8}^{12} + \alpha_{3,9}^{12}\alpha_{4,5}^{9} &&= 0
\end{aligned}
$$

Solution 1:

$$\alpha_{2,9}^{11} = 7/12$$
$$\alpha_{2,5}^{7} = 9/10$$
$$\alpha_{2,6}^{8} = 4/5$$
$$\alpha_{3,5}^{8} = 1/10$$
$$\alpha_{2,8}^{10} = 9/14$$
$$\alpha_{4,8}^{12} = 1/105$$
$$\alpha_{3,7}^{10} = 1/14$$
$$\alpha_{5,7}^{12} = 1/420$$
$$\alpha_{5,6}^{11} = 1/420$$
$$\alpha_{3,8}^{11} = 5/84$$
$$\alpha_{2,10}^{12} = 8/15$$
$$\alpha_{3,9}^{12} = 1/20$$
$$\alpha_{3,4}^{7} = 1/10$$
$$\alpha_{4,5}^{9} = 1/70$$
$$\alpha_{2,7}^{9} = 5/7$$
$$\alpha_{4,7}^{11} = 1/84$$
$$\alpha_{3,6}^{9} = 3/35$$
$$\alpha_{4,6}^{10} = 1/70$$

Solution 2:

$$\alpha_{2,9}^{11} = 1$$
$$\alpha_{2,5}^{7} = 1$$
$$\alpha_{2,6}^{8} = 1$$
$$\alpha_{3,5}^{8} = 0$$
$$\alpha_{2,8}^{10} = 1$$
$$\alpha_{4,8}^{12} = 0$$
$$\alpha_{3,7}^{10} = 0$$
$$\alpha_{5,7}^{12} = 0$$
$$\alpha_{5,6}^{11} = 0$$
$$\alpha_{3,8}^{11} = 0$$
$$\alpha_{2,10}^{12} = 1$$
$$\alpha_{3,9}^{12} = 0$$
$$\alpha_{3,4}^{7} = 0$$
$$\alpha_{4,5}^{9} = 0$$
$$\alpha_{2,7}^{9} = 1$$
$$\alpha_{4,7}^{11} = 0$$
$$\alpha_{3,6}^{9} = 0$$
$$\alpha_{4,6}^{10} = 0$$

How the solution(s) were or were not found:
Change variables

$$\alpha_{2,9}^{11} \to x_1$$
$$\alpha_{2,5}^{7} \to x_2$$
$$\alpha_{2,6}^{8} \to x_3$$
$$\alpha_{3,5}^{8} \to x_4$$
$$\alpha_{2,8}^{10} \to x_5$$
$$\alpha_{4,8}^{12} \to x_6$$
$$\alpha_{3,7}^{10} \to x_7$$
$$\alpha_{5,7}^{12} \to x_8$$
$$\alpha_{5,6}^{11} \to x_9$$

$$\alpha_{3,8}^{11} \to x_{10}$$

$$\alpha_{2,10}^{12} \to x_{11}$$

$$\alpha_{3,9}^{12} \to x_{12}$$

$$\alpha_{3,4}^{7} \to x_{13}$$

$$\alpha_{4,5}^{9} \to x_{14}$$

$$\alpha_{2,7}^{9} \to x_{15}$$

$$\alpha_{4,7}^{11} \to x_{16}$$

$$\alpha_{3,6}^{9} \to x_{17}$$

$$\alpha_{4,6}^{10} \to x_{18}$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_4): &\quad -x_{13} - x_2 + 1 && = 0\\
(e_1, e_2, e_5): &\quad x_2 - x_3 - x_4 && = 0\\
(e_1, e_3, e_4): &\quad x_{13} - x_4 && = 0\\
(e_1, e_2, e_6): &\quad -x_{15} - x_{17} + x_3 && = 0\\
(e_1, e_3, e_5): &\quad -x_{14} - x_{17} + x_4 && = 0\\
(e_2, e_3, e_4): &\quad x_{13}x_{15} + x_{14} - x_{17} && = 0\\
(e_1, e_2, e_7): &\quad x_{15} - x_5 - x_7 && = 0\\
(e_1, e_3, e_6): &\quad x_{17} - x_{18} - x_7 && = 0\\
(e_1, e_4, e_5): &\quad x_{14} - x_{18} && = 0\\
(e_2, e_3, e_5): &\quad -x_2x_7 + x_4x_5 && = 0\\
(e_1, e_2, e_8): &\quad -x_1 - x_{10} + x_5 && = 0\\
(e_1, e_3, e_7): &\quad -x_{10} - x_{16} + x_7 && = 0\\
(e_1, e_4, e_6): &\quad -x_{16} + x_{18} - x_9 && = 0\\
(e_2, e_3, e_6): &\quad x_1x_{17} - x_{10}x_3 - x_9 && = 0\\
(e_2, e_4, e_5): &\quad x_1x_{14} - x_{16}x_2 + x_9 && = 0\\
(e_1, e_2, e_9): &\quad x_1 - x_{11} - x_{12} && = 0\\
(e_1, e_3, e_8): &\quad x_{10} - x_{12} - x_6 && = 0\\
(e_1, e_4, e_7): &\quad x_{16} - x_6 - x_8 && = 0\\
(e_1, e_5, e_6): &\quad -x_8 + x_9 && = 0\\
(e_2, e_3, e_7): &\quad x_{11}x_7 - x_{12}x_{15} - x_8 && = 0\\
(e_2, e_4, e_6): &\quad x_{11}x_{18} - x_3x_6 && = 0\\
(e_3, e_4, e_5): &\quad x_{12}x_{14} + x_{13}x_8 - x_4x_6 && = 0
\end{aligned}
$$

Groebner basis (18 variables, 8 linear, 11 nonlinear)

$$3x_1 + 6x_{17}x_{18} + 15x_{17} - x_{18}^2 - 3x_{18} - 3 = 0$$
$$x_{17} + x_{18} + x_2 - 1 = 0$$
$$2x_{17} + 2x_{18} + x_3 - 1 = 0$$
$$-x_{17} - x_{18} + x_4 = 0$$
$$4x_{17} + x_{18} + x_5 - 1 = 0$$
$$12x_{17}x_{18} - 2x_{18}^2 - 3x_{18} + 3x_6 = 0$$
$$-x_{17} + x_{18} + x_7 = 0$$
$$-6x_{17}x_{18} + x_{18}^2 + 3x_8 = 0$$
$$-6x_{17}x_{18} + x_{18}^2 + 3x_9 = 0$$
$$3x_{10} - 6x_{17}x_{18} - 3x_{17} + x_{18}^2 + 6x_{18} = 0$$
$$3x_{11} + 24x_{17}x_{18} + 18x_{17} - 4x_{18}^2 - 12x_{18} - 3 = 0$$
$$x_{12} - 6x_{17}x_{18} - x_{17} + x_{18}^2 + 3x_{18} = 0$$
$$x_{13} - x_{17} - x_{18} = 0$$
$$x_{14} - x_{18} = 0$$
$$x_{15} + 3x_{17} + 2x_{18} - 1 = 0$$
$$3x_{16} + 6x_{17}x_{18} - x_{18}^2 - 3x_{18} = 0$$
$$3x_{17}^2 + 5x_{17}x_{18} + 2x_{18}^2 - 2x_{18} = 0$$
$$x_{17}x_{18}^2 - 6x_{18}^3 = 0$$
$$70x_{18}^4 - x_{18}^3 = 0$$

Solution 1:

$$x_1 = 7/12$$
$$x_2 = 9/10$$
$$x_3 = 4/5$$
$$x_4 = 1/10$$
$$x_5 = 9/14$$
$$x_6 = 1/105$$
$$x_7 = 1/14$$
$$x_8 = 1/420$$
$$x_9 = 1/420$$
$$x_10 = 5/84$$

$$x_11 = 8/15$$
$$x_12 = 1/20$$
$$x_13 = 1/10$$
$$x_14 = 1/70$$
$$x_15 = 5/7$$
$$x_16 = 1/84$$
$$x_17 = 3/35$$
$$x_18 = 1/70$$

Solution 2:

$$x_1 = 1$$
$$x_2 = 1$$
$$x_3 = 1$$
$$x_4 = 0$$
$$x_5 = 1$$
$$x_6 = 0$$
$$x_7 = 0$$
$$x_8 = 0$$
$$x_9 = 0$$
$$x_10 = 0$$
$$x_11 = 1$$
$$x_12 = 0$$
$$x_13 = 0$$
$$x_14 = 0$$
$$x_15 = 1$$
$$x_16 = 0$$
$$x_17 = 0$$
$$x_18 = 0$$

$\mathfrak{m}_{1A}(3, 12)$

m1A312 (this line included for string searching purposes)

Original brackets:

$$
\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_1, e_9] = e_{10} \\
[e_1, e_{10}] = e_{11} & [e_1, e_{11}] = e_{12} \\
[e_2, e_9] = e_{12} & [e_3, e_8] = -e_{12} \\
[e_4, e_7] = e_{12} & [e_5, e_6] = -e_{12}
\end{array}
$$

Non-trivial Jacobi Tests:

$\mathfrak{m}_{3A}(3, 12)$

m3A312 (this line included for string searching purposes)

Original brackets:

$$
\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_1, e_9] = e_{10} \\
[e_1, e_{10}] = e_{11} & [e_1, e_{11}] = e_{12} \\
[e_2, e_7] = e_{10} & [e_2, e_8] = 3e_{11} \\
[e_2, e_9] = \alpha_{2,9}^{12} e_{12} & [e_3, e_6] = -e_{10} \\
[e_3, e_7] = -2e_{11} & [e_3, e_8] = \alpha_{3,8}^{12} e_{12} \\
[e_4, e_5] = e_{10} & [e_4, e_6] = e_{11} \\
[e_4, e_7] = \alpha_{4,7}^{12} e_{12} & [e_5, e_6] = \alpha_{5,6}^{12} e_{12}
\end{array}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_8):& \quad -\alpha_{2,9}^{12} - \alpha_{3,8}^{12} + 3 &= 0\\
(e_1, e_3, e_7):& \quad -\alpha_{3,8}^{12} - \alpha_{4,7}^{12} - 2 &= 0\\
(e_1, e_4, e_6):& \quad -\alpha_{4,7}^{12} - \alpha_{5,6}^{12} + 1 &= 0
\end{aligned}
$$

Infinite number of solutions: only zero-dimensional systems supported (finite number of solutions).
How the solution(s) were or were not found:
Change variables

$$\alpha_{5,6}^{12} \to x_1$$

$$\alpha_{2,9}^{12} \to x_2$$

$$\alpha_{4,7}^{12} \to x_3$$

$$\alpha_{3,8}^{12} \to x_4$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_8):& \quad -x_2 - x_4 + 3 &= 0\\
(e_1, e_3, e_7):& \quad -x_3 - x_4 - 2 &= 0\\
(e_1, e_4, e_6):& \quad -x_1 - x_3 + 1 &= 0
\end{aligned}
$$

Groebner basis (4 variables, 3 linear, 0 nonlinear)

$$x_1 - x_4 - 3 = 0$$

$$x_2 + x_4 - 3 = 0$$

$$x_3 + x_4 + 2 = 0$$

$\mathfrak{m}_{5A}(3, 12)$

m5A312 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
&[e_1,e_2]=e_3 && [e_1,e_3]=e_4\\
&[e_1,e_4]=e_5 && [e_1,e_5]=e_6\\
&[e_1,e_6]=e_7 && [e_1,e_7]=e_8\\
&[e_1,e_8]=e_9 && [e_1,e_9]=e_{10}\\
&[e_1,e_{10}]=e_{11} && [e_1,e_{11}]=e_{12}\\
&[e_2,e_5]=e_8 && [e_2,e_6]=2e_9\\
&[e_2,e_7]=\alpha_{2,7}^{10}e_{10} && [e_2,e_8]=\alpha_{2,8}^{11}e_{11}\\
&[e_2,e_9]=\alpha_{2,9}^{12}e_{12} && [e_3,e_4]=-e_8\\
&[e_3,e_5]=-e_9 && [e_3,e_6]=\alpha_{3,6}^{10}e_{10}\\
&[e_3,e_7]=\alpha_{3,7}^{11}e_{11} && [e_3,e_8]=\alpha_{3,8}^{12}e_{12}\\
&[e_4,e_5]=\alpha_{4,5}^{10}e_{10} && [e_4,e_6]=\alpha_{4,6}^{11}e_{11}\\
&[e_4,e_7]=\alpha_{4,7}^{12}e_{12} && [e_5,e_6]=\alpha_{5,6}^{12}e_{12}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
&(e_1,e_2,e_6): && -\alpha_{2,7}^{10}-\alpha_{3,6}^{10}+2 &&=0\\
&(e_1,e_3,e_5): && -\alpha_{3,6}^{10}-\alpha_{4,5}^{10}-1 &&=0\\
&(e_1,e_2,e_7): && \alpha_{2,7}^{10}-\alpha_{2,8}^{11}-\alpha_{3,7}^{11} &&=0\\
&(e_1,e_3,e_6): && \alpha_{3,6}^{10}-\alpha_{3,7}^{11}-\alpha_{4,6}^{11} &&=0\\
&(e_1,e_4,e_5): && \alpha_{4,5}^{10}-\alpha_{4,6}^{11} &&=0\\
&(e_2,e_3,e_4): && -\alpha_{2,8}^{11} &&=0\\
&(e_1,e_2,e_8): && \alpha_{2,8}^{11}-\alpha_{2,9}^{12}-\alpha_{3,8}^{12} &&=0\\
&(e_1,e_3,e_7): && \alpha_{3,7}^{11}-\alpha_{3,8}^{12}-\alpha_{4,7}^{12} &&=0\\
&(e_1,e_4,e_6): && \alpha_{4,6}^{11}-\alpha_{4,7}^{12}-\alpha_{5,6}^{12} &&=0\\
&(e_2,e_3,e_5): && -\alpha_{2,9}^{12}-\alpha_{3,8}^{12} &&=0
\end{aligned}
$$

Infinite number of solutions: only zero-dimensional systems supported (finite number of solutions).
How the solution(s) were or were not found:
Change variables

$$
\begin{aligned}
\alpha_{3,8}^{12}&\to x_1\\
\alpha_{5,6}^{12}&\to x_2
\end{aligned}
$$

$$\alpha_{4,7}^{12} \to x_3$$

$$\alpha_{3,6}^{10} \to x_4$$

$$\alpha_{4,5}^{10} \to x_5$$

$$\alpha_{4,6}^{11} \to x_6$$

$$\alpha_{2,9}^{12} \to x_7$$

$$\alpha_{3,7}^{11} \to x_8$$

$$\alpha_{2,7}^{10} \to x_9$$

$$\alpha_{2,8}^{11} \to x_{10}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_6) :&\quad -x_4 - x_9 + 2 &= 0\\
(e_1, e_3, e_5) :&\quad -x_4 - x_5 - 1 &= 0\\
(e_1, e_2, e_7) :&\quad -x_{10} - x_8 + x_9 &= 0\\
(e_1, e_3, e_6) :&\quad x_4 - x_6 - x_8 &= 0\\
(e_1, e_4, e_5) :&\quad x_5 - x_6 &= 0\\
(e_2, e_3, e_4) :&\quad -x_{10} &= 0\\
(e_1, e_2, e_8) :&\quad -x_1 + x_{10} - x_7 &= 0\\
(e_1, e_3, e_7) :&\quad -x_1 - x_3 + x_8 &= 0\\
(e_1, e_4, e_6) :&\quad -x_2 - x_3 + x_6 &= 0\\
(e_2, e_3, e_5) :&\quad -x_1 - x_7 &= 0
\end{aligned}$$

Groebner basis (10 variables, 9 linear, 0 nonlinear)

$$x_1 + x_7 = 0$$

$$x_2 + x_7 + 3 = 0$$

$$3x_3 - 3x_7 - 5 = 0$$

$$3x_4 - 1 = 0$$

$$3x_5 + 4 = 0$$

$$3x_6 + 4 = 0$$

$$3x_8 - 5 = 0$$

$$3x_9 - 5 = 0$$

$$x_{10} = 0$$

$\mathfrak{m}_{7A}(3, 12)$

m7A312 (this line included for string searching purposes)

Original brackets:

$$
\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_1, e_9] = e_{10} \\
[e_1, e_{10}] = e_{11} & [e_1, e_{11}] = e_{12} \\
[e_2, e_3] = e_6 & [e_2, e_4] = e_7 \\
[e_2, e_5] = \alpha_{2,5}^{8} e_8 & [e_2, e_6] = \alpha_{2,6}^{9} e_9 \\
[e_2, e_7] = \alpha_{2,7}^{10} e_{10} & [e_2, e_8] = \alpha_{2,8}^{11} e_{11} \\
[e_2, e_9] = \alpha_{2,9}^{12} e_{12} & [e_3, e_4] = \alpha_{3,4}^{8} e_8 \\
[e_3, e_5] = \alpha_{3,5}^{9} e_9 & [e_3, e_6] = \alpha_{3,6}^{10} e_{10} \\
[e_3, e_7] = \alpha_{3,7}^{11} e_{11} & [e_3, e_8] = \alpha_{3,8}^{12} e_{12} \\
[e_4, e_5] = \alpha_{4,5}^{10} e_{10} & [e_4, e_6] = \alpha_{4,6}^{11} e_{11} \\
[e_4, e_7] = \alpha_{4,7}^{12} e_{12} & [e_5, e_6] = \alpha_{5,6}^{12} e_{12}
\end{array}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_4): &\quad -\alpha^{8}_{2,5} - \alpha^{8}_{3,4} + 1 &&= 0 \\
(e_1, e_2, e_5): &\quad \alpha^{8}_{2,5} - \alpha^{9}_{2,6} - \alpha^{9}_{3,5} &&= 0 \\
(e_1, e_3, e_4): &\quad \alpha^{8}_{3,4} - \alpha^{9}_{3,5} &&= 0 \\
(e_1, e_2, e_6): &\quad \alpha^{9}_{2,6} - \alpha^{10}_{2,7} - \alpha^{10}_{3,6} &&= 0 \\
(e_1, e_3, e_5): &\quad \alpha^{9}_{3,5} - \alpha^{10}_{3,6} - \alpha^{10}_{4,5} &&= 0 \\
(e_1, e_2, e_7): &\quad \alpha^{10}_{2,7} - \alpha^{11}_{2,8} - \alpha^{11}_{3,7} &&= 0 \\
(e_1, e_3, e_6): &\quad \alpha^{10}_{3,6} - \alpha^{11}_{3,7} - \alpha^{11}_{4,6} &&= 0 \\
(e_1, e_4, e_5): &\quad \alpha^{10}_{4,5} - \alpha^{11}_{4,6} &&= 0 \\
(e_2, e_3, e_4): &\quad \alpha^{11}_{2,8}\alpha^{8}_{3,4} - \alpha^{11}_{3,7} + \alpha^{11}_{4,6} &&= 0 \\
(e_1, e_2, e_8): &\quad \alpha^{11}_{2,8} - \alpha^{12}_{2,9} - \alpha^{12}_{3,8} &&= 0 \\
(e_1, e_3, e_7): &\quad \alpha^{11}_{3,7} - \alpha^{12}_{3,8} - \alpha^{12}_{4,7} &&= 0 \\
(e_1, e_4, e_6): &\quad \alpha^{11}_{4,6} - \alpha^{12}_{4,7} - \alpha^{12}_{5,6} &&= 0 \\
(e_2, e_3, e_5): &\quad -\alpha^{8}_{2,5}\alpha^{12}_{3,8} + \alpha^{12}_{2,9}\alpha^{9}_{3,5} + \alpha^{12}_{5,6} &&= 0
\end{aligned}
$$

Infinite number of solutions: only zero-dimensional systems supported (finite number of solutions).
How the solution(s) were or were not found:
Change variables

$$
\begin{gathered}
\alpha^{8}_{2,5} \to x_1 \\
\alpha^{12}_{3,8} \to x_2 \\
\alpha^{12}_{5,6} \to x_3 \\
\alpha^{12}_{4,7} \to x_4 \\
\alpha^{10}_{3,6} \to x_5 \\
\alpha^{9}_{3,5} \to x_6 \\
\alpha^{8}_{3,4} \to x_7 \\
\alpha^{9}_{2,6} \to x_8 \\
\alpha^{10}_{4,5} \to x_9 \\
\alpha^{11}_{4,6} \to x_{10} \\
\alpha^{12}_{2,9} \to x_{11} \\
\alpha^{11}_{3,7} \to x_{12}
\end{gathered}
$$

$$\alpha_{2,7}^{10} \to x_{13}$$

$$\alpha_{2,8}^{11} \to x_{14}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4): &\quad -x_1 - x_7 + 1 &&= 0\\
(e_1, e_2, e_5): &\quad x_1 - x_6 - x_8 &&= 0\\
(e_1, e_3, e_4): &\quad -x_6 + x_7 &&= 0\\
(e_1, e_2, e_6): &\quad -x_{13} - x_5 + x_8 &&= 0\\
(e_1, e_3, e_5): &\quad -x_5 + x_6 - x_9 &&= 0\\
(e_1, e_2, e_7): &\quad -x_{12} + x_{13} - x_{14} &&= 0\\
(e_1, e_3, e_6): &\quad -x_{10} - x_{12} + x_5 &&= 0\\
(e_1, e_4, e_5): &\quad -x_{10} + x_9 &&= 0\\
(e_2, e_3, e_4): &\quad x_{10} - x_{12} + x_{14}x_7 &&= 0\\
(e_1, e_2, e_8): &\quad -x_{11} + x_{14} - x_2 &&= 0\\
(e_1, e_3, e_7): &\quad x_{12} - x_2 - x_4 &&= 0\\
(e_1, e_4, e_6): &\quad x_{10} - x_3 - x_4 &&= 0\\
(e_2, e_3, e_5): &\quad -x_1x_2 + x_{11}x_6 + x_3 &&= 0
\end{aligned}$$

Groebner basis (14 variables, 11 linear, 1 nonlinear)

$$5x_1 - 3x_{13} + x_{14} - 3 = 0$$

$$x_{11} - x_{14} + x_2 = 0$$

$$5x_{11} + 9x_{13} - 13x_{14} + 5x_3 - 1 = 0$$

$$-x_{11} - x_{13} + 2x_{14} + x_4 = 0$$

$$-x_{13} + 2x_{14} + 5x_5 - 1 = 0$$

$$3x_{13} - x_{14} + 5x_6 - 2 = 0$$

$$3x_{13} - x_{14} + 5x_7 - 2 = 0$$

$$-6x_{13} + 2x_{14} + 5x_8 - 1 = 0$$

$$4x_{13} - 3x_{14} + 5x_9 - 1 = 0$$

$$5x_{10} + 4x_{13} - 3x_{14} - 1 = 0$$

$$x_{12} - x_{13} + x_{14} = 0$$

$$3x_{13}x_{14} + 9x_{13} - x_{14}^2 - 10x_{14} - 1 = 0$$

$\mathfrak{m}_{2A}(4, 12)$

m2A412 (this line included for string searching purposes)

Original brackets:

$$
\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_1, e_9] = e_{10} \\
[e_1, e_{10}] = e_{11} & [e_1, e_{11}] = e_{12} \\
[e_2, e_7] = e_{11} & [e_2, e_8] = 3e_{12} \\
[e_3, e_6] = -e_{11} & [e_3, e_7] = -2e_{12} \\
[e_4, e_5] = e_{11} & [e_4, e_6] = e_{12}
\end{array}
$$

Non-trivial Jacobi Tests:

$\mathfrak{m}_{4A}(4, 12)$

m4A412 (this line included for string searching purposes)

Original brackets:

$$
\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_1, e_9] = e_{10} \\
[e_1, e_{10}] = e_{11} & [e_1, e_{11}] = e_{12} \\
[e_2, e_5] = e_9 & [e_2, e_6] = 2e_{10} \\
[e_2, e_7] = \alpha_{2,7}^{11} e_{11} & [e_2, e_8] = \alpha_{2,8}^{12} e_{12} \\
[e_3, e_4] = -e_9 & [e_3, e_5] = -e_{10} \\
[e_3, e_6] = \alpha_{3,6}^{11} e_{11} & [e_3, e_7] = \alpha_{3,7}^{12} e_{12} \\
[e_4, e_5] = \alpha_{4,5}^{11} e_{11} & [e_4, e_6] = \alpha_{4,6}^{12} e_{12}
\end{array}
$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1,e_2,e_6): &\quad -\alpha_{2,7}^{11}-\alpha_{3,6}^{11}+2 &&=0\\
(e_1,e_3,e_5): &\quad -\alpha_{3,6}^{11}-\alpha_{4,5}^{11}-1 &&=0\\
(e_1,e_2,e_7): &\quad \alpha_{2,7}^{11}-\alpha_{2,8}^{12}-\alpha_{3,7}^{12} &&=0\\
(e_1,e_3,e_6): &\quad \alpha_{3,6}^{11}-\alpha_{3,7}^{12}-\alpha_{4,6}^{12} &&=0\\
(e_1,e_4,e_5): &\quad \alpha_{4,5}^{11}-\alpha_{4,6}^{12} &&=0
\end{aligned}$$

Infinite number of solutions: only zero-dimensional systems supported (finite number of solutions).

How the solution(s) were or were not found:

Change variables

$$\alpha_{3,7}^{12}\to x_1$$

$$\alpha_{2,8}^{12}\to x_2$$

$$\alpha_{4,6}^{12}\to x_3$$

$$\alpha_{2,7}^{11}\to x_4$$

$$\alpha_{4,5}^{11}\to x_5$$

$$\alpha_{3,6}^{11}\to x_6$$

Jacobi Tests

$$\begin{aligned}
(e_1,e_2,e_6): &\quad -x_4-x_6+2 &&=0\\
(e_1,e_3,e_5): &\quad -x_5-x_6-1 &&=0\\
(e_1,e_2,e_7): &\quad -x_1-x_2+x_4 &&=0\\
(e_1,e_3,e_6): &\quad -x_1-x_3+x_6 &&=0\\
(e_1,e_4,e_5): &\quad -x_3+x_5 &&=0
\end{aligned}$$

Groebner basis (6 variables, 5 linear, 0 nonlinear)

$$x_1-2x_6-1=0$$

$$x_2+3x_6-1=0$$

$$x_3+x_6+1=0$$

$$x_4+x_6-2=0$$

$$x_5+x_6+1=0$$

$\mathfrak{m}_{6A}(4,12)$

m6A412 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4 \\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6 \\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8 \\
[e_1,e_8] &= e_9 & [e_1,e_9] &= e_{10} \\
[e_1,e_{10}] &= e_{11} & [e_1,e_{11}] &= e_{12} \\
[e_2,e_3] &= e_7 & [e_2,e_4] &= e_8 \\
[e_2,e_5] &= \alpha_{2,5}^{9} e_9 & [e_2,e_6] &= \alpha_{2,6}^{10} e_{10} \\
[e_2,e_7] &= \alpha_{2,7}^{11} e_{11} & [e_2,e_8] &= \alpha_{2,8}^{12} e_{12} \\
[e_3,e_4] &= \alpha_{3,4}^{9} e_9 & [e_3,e_5] &= \alpha_{3,5}^{10} e_{10} \\
[e_3,e_6] &= \alpha_{3,6}^{11} e_{11} & [e_3,e_7] &= \alpha_{3,7}^{12} e_{12} \\
[e_4,e_5] &= \alpha_{4,5}^{11} e_{11} & [e_4,e_6] &= \alpha_{4,6}^{12} e_{12}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1,e_2,e_4):&\quad -\alpha_{2,5}^{9} - \alpha_{3,4}^{9} + 1 &= 0 \\
(e_1,e_2,e_5):&\quad \alpha_{2,5}^{9} - \alpha_{2,6}^{10} - \alpha_{3,5}^{10} &= 0 \\
(e_1,e_3,e_4):&\quad \alpha_{3,4}^{9} - \alpha_{3,5}^{10} &= 0 \\
(e_1,e_2,e_6):&\quad \alpha_{2,6}^{10} - \alpha_{2,7}^{11} - \alpha_{3,6}^{11} &= 0 \\
(e_1,e_3,e_5):&\quad \alpha_{3,5}^{10} - \alpha_{3,6}^{11} - \alpha_{4,5}^{11} &= 0 \\
(e_1,e_2,e_7):&\quad \alpha_{2,7}^{11} - \alpha_{2,8}^{12} - \alpha_{3,7}^{12} &= 0 \\
(e_1,e_3,e_6):&\quad \alpha_{3,6}^{11} - \alpha_{3,7}^{12} - \alpha_{4,6}^{12} &= 0 \\
(e_1,e_4,e_5):&\quad \alpha_{4,5}^{11} - \alpha_{4,6}^{12} &= 0
\end{aligned}
$$

Infinite number of solutions: only zero-dimensional systems supported (finite number of solutions).
How the solution(s) were or were not found:
Change variables

$$\alpha_{2,5}^{9} \to x_1$$

$$\alpha_{3,7}^{12} \to x_2$$

$$\alpha^{12}_{2,8} \to x_3$$

$$\alpha^{12}_{4,6} \to x_4$$

$$\alpha^{11}_{2,7} \to x_5$$

$$\alpha^{11}_{4,5} \to x_6$$

$$\alpha^{11}_{3,6} \to x_7$$

$$\alpha^{10}_{2,6} \to x_8$$

$$\alpha^{10}_{3,5} \to x_9$$

$$\alpha^{9}_{3,4} \to x_{10}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4):&\quad -x_1 - x_{10} + 1 &&= 0\\
(e_1, e_2, e_5):&\quad x_1 - x_8 - x_9 &&= 0\\
(e_1, e_3, e_4):&\quad x_{10} - x_9 &&= 0\\
(e_1, e_2, e_6):&\quad -x_5 - x_7 + x_8 &&= 0\\
(e_1, e_3, e_5):&\quad -x_6 - x_7 + x_9 &&= 0\\
(e_1, e_2, e_7):&\quad -x_2 - x_3 + x_5 &&= 0\\
(e_1, e_3, e_6):&\quad -x_2 - x_4 + x_7 &&= 0\\
(e_1, e_4, e_5):&\quad -x_4 + x_6 &&= 0
\end{aligned}$$

Groebner basis (10 variables, 8 linear, 0 nonlinear)

$$x_1 + x_{10} - 1 = 0$$

$$x_{10} + x_2 - 2x_7 = 0$$

$$x_{10} + x_3 + 3x_7 - 1 = 0$$

$$-x_{10} + x_4 + x_7 = 0$$

$$2x_{10} + x_5 + x_7 - 1 = 0$$

$$-x_{10} + x_6 + x_7 = 0$$

$$2x_{10} + x_8 - 1 = 0$$

$$-x_{10} + x_9 = 0$$

$\mathfrak{m}_{1A}(5, 12)$

m1A512 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_2, e_7] &= e_{12} & [e_3, e_6] &= -e_{12} \\
[e_4, e_5] &= e_{12}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$\mathfrak{m}_{3A}(5, 12)$

m3A512 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_2, e_5] &= e_{10} & [e_2, e_6] &= 2e_{11} \\
[e_2, e_7] &= \alpha_{2,7}^{12} e_{12} & [e_3, e_4] &= -e_{10} \\
[e_3, e_5] &= -e_{11} & [e_3, e_6] &= \alpha_{3,6}^{12} e_{12} \\
[e_4, e_5] &= \alpha_{4,5}^{12} e_{12}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_6): &\quad -\alpha_{2,7}^{12} - \alpha_{3,6}^{12} + 2 &= 0 \\
(e_1, e_3, e_5): &\quad -\alpha_{3,6}^{12} - \alpha_{4,5}^{12} - 1 &= 0
\end{aligned}
$$

Infinite number of solutions: only zero-dimensional systems supported (finite number of solutions).
How the solution(s) were or were not found:
Change variables

$$\alpha^{12}_{2,7} \to x_1$$
$$\alpha^{12}_{3,6} \to x_2$$
$$\alpha^{12}_{4,5} \to x_3$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_6): \quad & -x_1 - x_2 + 2 && = 0 \\
(e_1, e_3, e_5): \quad & -x_2 - x_3 - 1 && = 0
\end{aligned}$$

Groebner basis (3 variables, 2 linear, 0 nonlinear)

$$x_1 - x_3 - 3 = 0$$
$$x_2 + x_3 + 1 = 0$$

$\mathfrak{m}_{5A}(5, 12)$

m5A512 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_2, e_3] &= e_8 & [e_2, e_4] &= e_9 \\
[e_2, e_5] &= \alpha^{10}_{2,5} e_{10} & [e_2, e_6] &= \alpha^{11}_{2,6} e_{11} \\
[e_2, e_7] &= \alpha^{12}_{2,7} e_{12} & [e_3, e_4] &= \alpha^{10}_{3,4} e_{10} \\
[e_3, e_5] &= \alpha^{11}_{3,5} e_{11} & [e_3, e_6] &= \alpha^{12}_{3,6} e_{12} \\
[e_4, e_5] &= \alpha^{12}_{4,5} e_{12}
\end{aligned}$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_4): &\quad -\alpha_{2,5}^{10} - \alpha_{3,4}^{10} + 1 &= 0\\
(e_1, e_2, e_5): &\quad \alpha_{2,5}^{10} - \alpha_{2,6}^{11} - \alpha_{3,5}^{11} &= 0\\
(e_1, e_3, e_4): &\quad \alpha_{3,4}^{10} - \alpha_{3,5}^{11} &= 0\\
(e_1, e_2, e_6): &\quad \alpha_{2,6}^{11} - \alpha_{2,7}^{12} - \alpha_{3,6}^{12} &= 0\\
(e_1, e_3, e_5): &\quad \alpha_{3,5}^{11} - \alpha_{3,6}^{12} - \alpha_{4,5}^{12} &= 0
\end{aligned}
$$

Infinite number of solutions: only zero-dimensional systems supported (finite number of solutions).
How the solution(s) were or were not found:
Change variables

$$\alpha_{4,5}^{12} \to x_1$$

$$\alpha_{2,6}^{11} \to x_2$$

$$\alpha_{3,5}^{11} \to x_3$$

$$\alpha_{2,5}^{10} \to x_4$$

$$\alpha_{3,6}^{12} \to x_5$$

$$\alpha_{3,4}^{10} \to x_6$$

$$\alpha_{2,7}^{12} \to x_7$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_4): &\quad -x_4 - x_6 + 1 &= 0\\
(e_1, e_2, e_5): &\quad -x_2 - x_3 + x_4 &= 0\\
(e_1, e_3, e_4): &\quad -x_3 + x_6 &= 0\\
(e_1, e_2, e_6): &\quad x_2 - x_5 - x_7 &= 0\\
(e_1, e_3, e_5): &\quad -x_1 + x_3 - x_5 &= 0
\end{aligned}
$$

Groebner basis (7 variables, 5 linear, 0 nonlinear)

$$x_1 - 3x_6 - x_7 + 1 = 0$$

$$x_2 + 2x_6 - 1 = 0$$

$$x_3 - x_6 = 0$$

$$x_4 + x_6 - 1 = 0$$

$$x_5 + 2x_6 + x_7 - 1 = 0$$

$\mathfrak{m}_{2A}(6,12)$

m2A612 (this line included for string searching purposes)

Original brackets:

$$
\begin{array}{ll}
[e_1,e_2]=e_3 & [e_1,e_3]=e_4 \\
[e_1,e_4]=e_5 & [e_1,e_5]=e_6 \\
[e_1,e_6]=e_7 & [e_1,e_7]=e_8 \\
[e_1,e_8]=e_9 & [e_1,e_9]=e_{10} \\
[e_1,e_{10}]=e_{11} & [e_1,e_{11}]=e_{12} \\
[e_2,e_5]=e_{11} & [e_2,e_6]=2e_{12} \\
[e_3,e_4]=-e_{11} & [e_3,e_5]=-e_{12}
\end{array}
$$

Non-trivial Jacobi Tests:

$\mathfrak{m}_{4A}(6,12)$

m4A612 (this line included for string searching purposes)

Original brackets:

$$
\begin{array}{ll}
[e_1,e_2]=e_3 & [e_1,e_3]=e_4 \\
[e_1,e_4]=e_5 & [e_1,e_5]=e_6 \\
[e_1,e_6]=e_7 & [e_1,e_7]=e_8 \\
[e_1,e_8]=e_9 & [e_1,e_9]=e_{10} \\
[e_1,e_{10}]=e_{11} & [e_1,e_{11}]=e_{12} \\
[e_2,e_3]=e_9 & [e_2,e_4]=e_{10} \\
[e_2,e_5]=\alpha_{2,5}^{11}e_{11} & [e_2,e_6]=\alpha_{2,6}^{12}e_{12} \\
[e_3,e_4]=\alpha_{3,4}^{11}e_{11} & [e_3,e_5]=\alpha_{3,5}^{12}e_{12}
\end{array}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_4): &\quad -\alpha_{2,5}^{11} - \alpha_{3,4}^{11} + 1 &= 0\\
(e_1, e_2, e_5): &\quad \alpha_{2,5}^{11} - \alpha_{2,6}^{12} - \alpha_{3,5}^{12} &= 0\\
(e_1, e_3, e_4): &\quad \alpha_{3,4}^{11} - \alpha_{3,5}^{12} &= 0
\end{aligned}
$$

Infinite number of solutions: only zero-dimensional systems supported (finite number of solutions).

How the solution(s) were or were not found:

Change variables

$$\alpha_{2,6}^{12} \to x_1$$

$$\alpha_{3,5}^{12} \to x_2$$

$$\alpha_{2,5}^{11} \to x_3$$

$$\alpha_{3,4}^{11} \to x_4$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_4): &\quad -x_3 - x_4 + 1 &= 0\\
(e_1, e_2, e_5): &\quad -x_1 - x_2 + x_3 &= 0\\
(e_1, e_3, e_4): &\quad -x_2 + x_4 &= 0
\end{aligned}
$$

Groebner basis (4 variables, 3 linear, 0 nonlinear)

$$x_1 + 2x_4 - 1 = 0$$

$$x_2 - x_4 = 0$$

$$x_3 + x_4 - 1 = 0$$

$\mathfrak{m}_{1A}(7, 12)$

m1A712 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_2, e_5] &= e_{12} & [e_3, e_4] &= -e_{12}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$\mathfrak{m}_{3A}(7, 12)$

m3A712 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_2, e_3] &= e_{10} & [e_2, e_4] &= e_{11} \\
[e_2, e_5] &= \alpha_{2,5}^{12} e_{12} & [e_3, e_4] &= \alpha_{3,4}^{12} e_{12}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
(e_1, e_2, e_4): \quad -\alpha_{2,5}^{12} - \alpha_{3,4}^{12} + 1 \qquad = 0
$$

Infinite number of solutions: only zero-dimensional systems supported (finite number of solutions).

How the solution(s) were or were not found:

Change variables

$$
\alpha_{3,4}^{12} \to x_1
$$

$$\alpha_{2,5}^{12} \to x_2$$

Jacobi Tests

$$(e_1, e_2, e_4): \quad -x_1 - x_2 + 1 \qquad = 0$$

Groebner basis (2 variables, 1 linear, 0 nonlinear)

$$x_1 + x_2 - 1 = 0$$

$\mathfrak{m}_{2A}(8, 12)$

m2A812 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned} [e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\ [e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\ [e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\ [e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\ [e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\ [e_2, e_3] &= e_{11} & [e_2, e_4] &= e_{12} \end{aligned}$$

Non-trivial Jacobi Tests:

$\mathfrak{m}_{1A}(9, 12)$

m1A912 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned} [e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\ [e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\ [e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\ [e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\ [e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\ [e_2, e_3] &= e_{12} \end{aligned}$$

Non-trivial Jacobi Tests:

$\mathfrak{m}_{2B}(2,6)$

m2B26 (this line included for string searching purposes)
Solution 1

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4\\
[e_1,e_4] &= e_5 & [e_2,e_3] &= e_5\\
[e_2,e_5] &= e_6 & [e_3,e_4] &= -e_6
\end{aligned}
$$

Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4\\
[e_1,e_4] &= e_5 & [e_2,e_3] &= e_5\\
[e_2,e_5] &= e_6 & [e_3,e_4] &= \alpha_{3,4}^6 e_6
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$(e_1,e_2,e_4): \quad -\alpha_{3,4}^6 - 1 \qquad = 0$$

Solution 1:

$$\alpha_{3,4}^6 = -1$$

How the solution(s) were or were not found:
Change variables

$$\alpha_{3,4}^6 \to x_1$$

Jacobi Tests

$$(e_1,e_2,e_4): \quad -x_1 - 1 \qquad = 0$$

Groebner basis (1 variables, 1 linear, 0 nonlinear)

$$x_1 + 1 = 0$$

Solution 1:

$$x_1 = -1$$

$\mathfrak{m}_{2B}(2,8)$

m2B28 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned}
&[e_1, e_2] = e_3 && [e_1, e_3] = e_4 \\
&[e_1, e_4] = e_5 && [e_1, e_5] = e_6 \\
&[e_1, e_6] = e_7 && [e_2, e_5] = e_7 \\
&[e_2, e_7] = e_8 && [e_3, e_4] = -e_7 \\
&[e_3, e_6] = \alpha_{3,6}^8 e_8 && [e_4, e_5] = \alpha_{4,5}^8 e_8
\end{aligned}$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
&(e_1, e_2, e_6): && -\alpha_{3,6}^8 - 1 && = 0 \\
&(e_1, e_3, e_5): && -\alpha_{3,6}^8 - \alpha_{4,5}^8 && = 0 \\
&(e_2, e_3, e_4): && \text{no solutions}
\end{aligned}$$

There are no solutions.

$\mathfrak{m}_{4B}(2,8)$

m4B28 (this line included for string searching purposes)
Solution 1

$$\begin{aligned}
&[e_1, e_2] = e_3 && [e_1, e_3] = e_4 \\
&[e_1, e_4] = e_5 && [e_1, e_5] = e_6 \\
&[e_1, e_6] = e_7 && [e_2, e_3] = e_5 \\
&[e_2, e_4] = e_6 && [e_2, e_5] = 3e_7 \\
&[e_2, e_7] = e_8 && [e_3, e_4] = -2e_7 \\
&[e_3, e_6] = -e_8 && [e_4, e_5] = e_8
\end{aligned}$$

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_2, e_3] &= e_5 \\
[e_2, e_4] &= e_6 & [e_2, e_5] &= \alpha_{2,5}^7 e_7 \\
[e_2, e_7] &= e_8 & [e_3, e_4] &= \alpha_{3,4}^7 e_7 \\
[e_3, e_6] &= \alpha_{3,6}^8 e_8 & [e_4, e_5] &= \alpha_{4,5}^8 e_8
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_4): &\quad -\alpha_{2,5}^7 - \alpha_{3,4}^7 + 1 &&= 0 \\
(e_1, e_2, e_6): &\quad -\alpha_{3,6}^8 - 1 &&= 0 \\
(e_1, e_3, e_5): &\quad -\alpha_{3,6}^8 - \alpha_{4,5}^8 &&= 0 \\
(e_2, e_3, e_4): &\quad \alpha_{3,4}^7 - \alpha_{3,6}^8 + \alpha_{4,5}^8 &&= 0
\end{aligned}
$$

Solution 1:

$$
\begin{aligned}
\alpha_{3,6}^8 &= -1 \\
\alpha_{2,5}^7 &= 3 \\
\alpha_{3,4}^7 &= -2 \\
\alpha_{4,5}^8 &= 1
\end{aligned}
$$

How the solution(s) were or were not found:
Change variables

$$
\begin{aligned}
\alpha_{3,6}^8 &\to x_1 \\
\alpha_{2,5}^7 &\to x_2 \\
\alpha_{3,4}^7 &\to x_3 \\
\alpha_{4,5}^8 &\to x_4
\end{aligned}
$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_4):\quad & -x_2 - x_3 + 1 && = 0\\
(e_1, e_2, e_6):\quad & -x_1 - 1 && = 0\\
(e_1, e_3, e_5):\quad & -x_1 - x_4 && = 0\\
(e_2, e_3, e_4):\quad & -x_1 + x_3 + x_4 && = 0
\end{aligned}
$$

Groebner basis (4 variables, 4 linear, 0 nonlinear)

$$x_1 + 1 = 0$$
$$x_2 - 3 = 0$$
$$x_3 + 2 = 0$$
$$x_4 - 1 = 0$$

Solution 1:

$$x_1 = -1$$
$$x_2 = 3$$
$$x_3 = -2$$
$$x_4 = 1$$

$\mathfrak{m}_{3B}(3, 8)$

m3B38 (this line included for string searching purposes)
Solution 1

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4\\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6\\
[e_1, e_6] &= e_7 & [e_2, e_3] &= e_6\\
[e_2, e_4] &= e_7 & [e_2, e_7] &= e_8\\
[e_3, e_6] &= -e_8 & [e_4, e_5] &= e_8
\end{aligned}
$$

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4\\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6\\
[e_1, e_6] &= e_7 & [e_2, e_3] &= e_6\\
[e_2, e_4] &= e_7 & [e_2, e_7] &= e_8\\
[e_3, e_6] &= \alpha_{3,6}^{8} e_8 & [e_4, e_5] &= \alpha_{4,5}^{8} e_8
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1, e_2, e_6): &\quad -\alpha_{3,6}^{8} - 1 && = 0 \\
(e_1, e_3, e_5): &\quad -\alpha_{3,6}^{8} - \alpha_{4,5}^{8} && = 0
\end{aligned}$$

Solution 1:

$$\alpha_{3,6}^{8} = -1$$
$$\alpha_{4,5}^{8} = 1$$

How the solution(s) were or were not found:
Change variables

$$\alpha_{3,6}^{8} \to x_1$$
$$\alpha_{4,5}^{8} \to x_2$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_6): &\quad -x_1 - 1 && = 0 \\
(e_1, e_3, e_5): &\quad -x_1 - x_2 && = 0
\end{aligned}$$

Groebner basis (2 variables, 2 linear, 0 nonlinear)

$$x_1 + 1 = 0$$
$$x_2 - 1 = 0$$

Solution 1:

$$x_1 = -1$$
$$x_2 = 1$$

$\mathfrak{m}_{2B}(4, 8)$

m2B48 (this line included for string searching purposes)
Solution 1

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_2, e_3] &= e_7 \\
[e_2, e_7] &= e_8 & [e_3, e_6] &= -e_8 \\
[e_4, e_5] &= e_8 & &
\end{aligned}$$

Original brackets:

$$
\begin{aligned}
&[e_1, e_2] = e_3 && [e_1, e_3] = e_4 \\
&[e_1, e_4] = e_5 && [e_1, e_5] = e_6 \\
&[e_1, e_6] = e_7 && [e_2, e_3] = e_7 \\
&[e_2, e_7] = e_8 && [e_3, e_6] = \alpha_{3,6}^{8} e_8 \\
&[e_4, e_5] = \alpha_{4,5}^{8} e_8
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_6):& \quad -\alpha_{3,6}^{8} - 1 &&= 0 \\
(e_1, e_3, e_5):& \quad -\alpha_{3,6}^{8} - \alpha_{4,5}^{8} &&= 0
\end{aligned}
$$

Solution 1:

$$
\begin{aligned}
\alpha_{3,6}^{8} &= -1 \\
\alpha_{4,5}^{8} &= 1
\end{aligned}
$$

How the solution(s) were or were not found:
Change variables

$$\alpha_{3,6}^{8} \to x_1$$

$$\alpha_{4,5}^{8} \to x_2$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_6):& \quad -x_1 - 1 &&= 0 \\
(e_1, e_3, e_5):& \quad -x_1 - x_2 &&= 0
\end{aligned}
$$

Groebner basis (2 variables, 2 linear, 0 nonlinear)

$$x_1 + 1 = 0$$

$$x_2 - 1 = 0$$

Solution 1:

$$x_1 = -1$$

$$x_2 = 1$$

$\mathfrak{m}_{2B}(2,10)$

m2B210 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
&[e_1,e_2]=e_3 && [e_1,e_3]=e_4\\
&[e_1,e_4]=e_5 && [e_1,e_5]=e_6\\
&[e_1,e_6]=e_7 && [e_1,e_7]=e_8\\
&[e_1,e_8]=e_9 && [e_2,e_7]=e_9\\
&[e_2,e_9]=e_{10} && [e_3,e_6]=-e_9\\
&[e_3,e_8]=\alpha_{3,8}^{10}e_{10} && [e_4,e_5]=e_9\\
&[e_4,e_7]=\alpha_{4,7}^{10}e_{10} && [e_5,e_6]=\alpha_{5,6}^{10}e_{10}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1,e_2,e_8):&\quad -\alpha_{3,8}^{10}-1 &&=0\\
(e_1,e_3,e_7):&\quad -\alpha_{3,8}^{10}-\alpha_{4,7}^{10} &&=0\\
(e_1,e_4,e_6):&\quad -\alpha_{4,7}^{10}-\alpha_{5,6}^{10} &&=0\\
(e_2,e_3,e_6):&\quad \text{no solutions}\\
(e_2,e_4,e_5):&\quad \text{no solutions}
\end{aligned}
$$

There are no solutions.

$\mathfrak{m}_{4B}(2,10)$

m4B210 (this line included for string searching purposes)

Original brackets:

$$
\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_2, e_5] = e_7 \\
[e_2, e_6] = 2e_8 & [e_2, e_7] = \alpha_{2,7}^{9} e_9 \\
[e_2, e_9] = e_{10} & [e_3, e_4] = -e_7 \\
[e_3, e_5] = -e_8 & [e_3, e_6] = \alpha_{3,6}^{9} e_9 \\
[e_3, e_8] = \alpha_{3,8}^{10} e_{10} & [e_4, e_5] = \alpha_{4,5}^{9} e_9 \\
[e_4, e_7] = \alpha_{4,7}^{10} e_{10} & [e_5, e_6] = \alpha_{5,6}^{10} e_{10}
\end{array}
$$

Non-trivial Jacobi Tests:

$$
\begin{array}{lll}
(e_1, e_2, e_6): & -\alpha_{2,7}^{9} - \alpha_{3,6}^{9} + 2 & = 0 \\
(e_1, e_3, e_5): & -\alpha_{3,6}^{9} - \alpha_{4,5}^{9} - 1 & = 0 \\
(e_2, e_3, e_4): & -\alpha_{2,7}^{9} & = 0 \\
(e_1, e_2, e_8): & -\alpha_{3,8}^{10} - 1 & = 0 \\
(e_1, e_3, e_7): & -\alpha_{3,8}^{10} - \alpha_{4,7}^{10} & = 0 \\
(e_1, e_4, e_6): & -\alpha_{4,7}^{10} - \alpha_{5,6}^{10} & = 0 \\
(e_2, e_3, e_6): & \alpha_{3,6}^{9} - 2\alpha_{3,8}^{10} & = 0 \\
(e_2, e_4, e_5): & \alpha_{4,5}^{9} - \alpha_{4,7}^{10} & = 0
\end{array}
$$

No solutions.
How the solution(s) were or were not found:
Change variables

$$
\begin{array}{c}
\alpha_{4,7}^{10} \to x_1 \\
\alpha_{4,5}^{9} \to x_2 \\
\alpha_{3,8}^{10} \to x_3 \\
\alpha_{2,7}^{9} \to x_4 \\
\alpha_{5,6}^{10} \to x_5 \\
\alpha_{3,6}^{9} \to x_6
\end{array}
$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_6) &: \quad -x_4 - x_6 + 2 &&= 0\\
(e_1, e_3, e_5) &: \quad -x_2 - x_6 - 1 &&= 0\\
(e_2, e_3, e_4) &: \quad -x_4 &&= 0\\
(e_1, e_2, e_8) &: \quad -x_3 - 1 &&= 0\\
(e_1, e_3, e_7) &: \quad -x_1 - x_3 &&= 0\\
(e_1, e_4, e_6) &: \quad -x_1 - x_5 &&= 0\\
(e_2, e_3, e_6) &: \quad -2x_3 + x_6 &&= 0\\
(e_2, e_4, e_5) &: \quad -x_1 + x_2 &&= 0
\end{aligned}$$

Groebner basis (6 variables, 1 linear, 0 nonlinear)

$$1 = 0$$

$\mathfrak{m}_{6B}(2, 10)$

m6B210 (this line included for string searching purposes)
Solution 1

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4\\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6\\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8\\
[e_1, e_8] &= e_9 & [e_2, e_3] &= e_5\\
[e_2, e_4] &= e_6 & [e_2, e_5] &= 0\\
[e_2, e_6] &= -e_8 & [e_2, e_7] &= -e_9\\
[e_2, e_9] &= e_{10} & [e_3, e_4] &= e_7\\
[e_3, e_5] &= e_8 & [e_3, e_6] &= 0\\
[e_3, e_8] &= -e_{10} & [e_4, e_5] &= e_9\\
[e_4, e_7] &= e_{10} & [e_5, e_6] &= -e_{10}
\end{aligned}$$

Solution 2

$$
\begin{aligned}
&[e_1, e_2] = e_3 && [e_1, e_3] = e_4 \\
&[e_1, e_4] = e_5 && [e_1, e_5] = e_6 \\
&[e_1, e_6] = e_7 && [e_1, e_7] = e_8 \\
&[e_1, e_8] = e_9 && [e_2, e_3] = e_5 \\
&[e_2, e_4] = e_6 && [e_2, e_5] = 2e_7 \\
&[e_2, e_6] = 3e_8 && [e_2, e_7] = 7e_9 \\
&[e_2, e_9] = e_{10} && [e_3, e_4] = -e_7 \\
&[e_3, e_5] = -e_8 && [e_3, e_6] = -4e_9 \\
&[e_3, e_8] = -e_{10} && [e_4, e_5] = 3e_9 \\
&[e_4, e_7] = e_{10} && [e_5, e_6] = -e_{10}
\end{aligned}
$$

Original brackets:

$$
\begin{aligned}
&[e_1, e_2] = e_3 && [e_1, e_3] = e_4 \\
&[e_1, e_4] = e_5 && [e_1, e_5] = e_6 \\
&[e_1, e_6] = e_7 && [e_1, e_7] = e_8 \\
&[e_1, e_8] = e_9 && [e_2, e_3] = e_5 \\
&[e_2, e_4] = e_6 && [e_2, e_5] = \alpha_{2,5}^{7} e_7 \\
&[e_2, e_6] = \alpha_{2,6}^{8} e_8 && [e_2, e_7] = \alpha_{2,7}^{9} e_9 \\
&[e_2, e_9] = e_{10} && [e_3, e_4] = \alpha_{3,4}^{7} e_7 \\
&[e_3, e_5] = \alpha_{3,5}^{8} e_8 && [e_3, e_6] = \alpha_{3,6}^{9} e_9 \\
&[e_3, e_8] = \alpha_{3,8}^{10} e_{10} && [e_4, e_5] = \alpha_{4,5}^{9} e_9 \\
&[e_4, e_7] = \alpha_{4,7}^{10} e_{10} && [e_5, e_6] = \alpha_{5,6}^{10} e_{10}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_4) : &\quad -\alpha_{2,5}^{7} - \alpha_{3,4}^{7} + 1 &&= 0\\
(e_1, e_2, e_5) : &\quad \alpha_{2,5}^{7} - \alpha_{2,6}^{8} - \alpha_{3,5}^{8} &&= 0\\
(e_1, e_3, e_4) : &\quad \alpha_{3,4}^{7} - \alpha_{3,5}^{8} &&= 0\\
(e_1, e_2, e_6) : &\quad \alpha_{2,6}^{8} - \alpha_{2,7}^{9} - \alpha_{3,6}^{9} &&= 0\\
(e_1, e_3, e_5) : &\quad \alpha_{3,5}^{8} - \alpha_{3,6}^{9} - \alpha_{4,5}^{9} &&= 0\\
(e_2, e_3, e_4) : &\quad \alpha_{2,7}^{9}\alpha_{3,4}^{7} - \alpha_{3,6}^{9} + \alpha_{4,5}^{9} &&= 0\\
(e_1, e_2, e_8) : &\quad -\alpha_{3,8}^{10} - 1 &&= 0\\
(e_1, e_3, e_7) : &\quad -\alpha_{3,8}^{10} - \alpha_{4,7}^{10} &&= 0\\
(e_1, e_4, e_6) : &\quad -\alpha_{4,7}^{10} - \alpha_{5,6}^{10} &&= 0\\
(e_2, e_3, e_6) : &\quad -\alpha_{2,6}^{8}\alpha_{3,8}^{10} + \alpha_{3,6}^{9} - \alpha_{5,6}^{10} &&= 0\\
(e_2, e_4, e_5) : &\quad -\alpha_{2,5}^{7}\alpha_{4,7}^{10} + \alpha_{4,5}^{9} + \alpha_{5,6}^{10} &&= 0
\end{aligned}
$$

Solution 1:

$$
\begin{aligned}
\alpha_{2,5}^{7} &= 0\\
\alpha_{2,6}^{8} &= -1\\
\alpha_{3,5}^{8} &= 1\\
\alpha_{4,7}^{10} &= 1\\
\alpha_{3,4}^{7} &= 1\\
\alpha_{4,5}^{9} &= 1\\
\alpha_{3,8}^{10} &= -1\\
\alpha_{2,7}^{9} &= -1\\
\alpha_{5,6}^{10} &= -1\\
\alpha_{3,6}^{9} &= 0
\end{aligned}
$$

Solution 2:

$$\alpha^{7}_{2,5} = 2$$
$$\alpha^{8}_{2,6} = 3$$
$$\alpha^{8}_{3,5} = -1$$
$$\alpha^{10}_{4,7} = 1$$
$$\alpha^{7}_{3,4} = -1$$
$$\alpha^{9}_{4,5} = 3$$
$$\alpha^{10}_{3,8} = -1$$
$$\alpha^{9}_{2,7} = 7$$
$$\alpha^{10}_{5,6} = -1$$
$$\alpha^{9}_{3,6} = -4$$

How the solution(s) were or were not found:
Change variables

$$\alpha^{7}_{2,5} \to x_1$$
$$\alpha^{8}_{2,6} \to x_2$$
$$\alpha^{8}_{3,5} \to x_3$$
$$\alpha^{10}_{4,7} \to x_4$$
$$\alpha^{7}_{3,4} \to x_5$$
$$\alpha^{9}_{4,5} \to x_6$$
$$\alpha^{10}_{3,8} \to x_7$$
$$\alpha^{9}_{2,7} \to x_8$$
$$\alpha^{10}_{5,6} \to x_9$$
$$\alpha^{9}_{3,6} \to x_{10}$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_4):&\quad -x_1 - x_5 + 1 &&= 0\\
(e_1, e_2, e_5):&\quad x_1 - x_2 - x_3 &&= 0\\
(e_1, e_3, e_4):&\quad -x_3 + x_5 &&= 0\\
(e_1, e_2, e_6):&\quad -x_{10} + x_2 - x_8 &&= 0\\
(e_1, e_3, e_5):&\quad -x_{10} + x_3 - x_6 &&= 0\\
(e_2, e_3, e_4):&\quad -x_{10} + x_5 x_8 + x_6 &&= 0\\
(e_1, e_2, e_8):&\quad -x_7 - 1 &&= 0\\
(e_1, e_3, e_7):&\quad -x_4 - x_7 &&= 0\\
(e_1, e_4, e_6):&\quad -x_4 - x_9 &&= 0\\
(e_2, e_3, e_6):&\quad x_{10} - x_2 x_7 - x_9 &&= 0\\
(e_2, e_4, e_5):&\quad -x_1 x_4 + x_6 + x_9 &&= 0
\end{aligned}
$$

Groebner basis (10 variables, 9 linear, 1 nonlinear)

$$2x_1 + x_{10} = 0$$
$$x_{10} + x_2 + 1 = 0$$
$$-x_{10} + 2x_3 - 2 = 0$$
$$x_4 - 1 = 0$$
$$-x_{10} + 2x_5 - 2 = 0$$
$$x_{10} + 2x_6 - 2 = 0$$
$$x_7 + 1 = 0$$
$$2x_{10} + x_8 + 1 = 0$$
$$x_9 + 1 = 0$$
$$x_{10}^2 + 4x_{10} = 0$$

Solution 1:

$$x_1 = 0$$
$$x_2 = -1$$
$$x_3 = 1$$
$$x_4 = 1$$
$$x_5 = 1$$
$$x_6 = 1$$
$$x_7 = -1$$

$$x_8 = -1$$
$$x_9 = -1$$
$$x_10 = 0$$

Solution 2:

$$x_1 = 2$$
$$x_2 = 3$$
$$x_3 = -1$$
$$x_4 = 1$$
$$x_5 = -1$$
$$x_6 = 3$$
$$x_7 = -1$$
$$x_8 = 7$$
$$x_9 = -1$$
$$x_10 = -4$$

$\mathfrak{m}_{3B}(3, 10)$

m3B310 (this line included for string searching purposes)
Solution 1

$$
\begin{aligned}
&[e_1, e_2] = e_3 && [e_1, e_3] = e_4 \\
&[e_1, e_4] = e_5 && [e_1, e_5] = e_6 \\
&[e_1, e_6] = e_7 && [e_1, e_7] = e_8 \\
&[e_1, e_8] = e_9 && [e_2, e_5] = e_8 \\
&[e_2, e_6] = 2e_9 && [e_2, e_9] = e_{10} \\
&[e_3, e_4] = -e_8 && [e_3, e_5] = -e_9 \\
&[e_3, e_8] = -e_{10} && [e_4, e_7] = e_{10} \\
&[e_5, e_6] = -e_{10}
\end{aligned}
$$

Original brackets:

$$
\begin{aligned}
&[e_1, e_2] = e_3 && [e_1, e_3] = e_4 \\
&[e_1, e_4] = e_5 && [e_1, e_5] = e_6 \\
&[e_1, e_6] = e_7 && [e_1, e_7] = e_8 \\
&[e_1, e_8] = e_9 && [e_2, e_5] = e_8 \\
&[e_2, e_6] = 2e_9 && [e_2, e_9] = e_{10} \\
&[e_3, e_4] = -e_8 && [e_3, e_5] = -e_9 \\
&[e_3, e_8] = \alpha_{3,8}^{10} e_{10} && [e_4, e_7] = \alpha_{4,7}^{10} e_{10} \\
&[e_5, e_6] = \alpha_{5,6}^{10} e_{10}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_8): &\quad -\alpha_{3,8}^{10} - 1 &= 0\\
(e_1, e_3, e_7): &\quad -\alpha_{3,8}^{10} - \alpha_{4,7}^{10} &= 0\\
(e_1, e_4, e_6): &\quad -\alpha_{4,7}^{10} - \alpha_{5,6}^{10} &= 0\\
(e_2, e_3, e_5): &\quad -\alpha_{3,8}^{10} - 1 &= 0
\end{aligned}
$$

Solution 1:

$$
\begin{aligned}
\alpha_{5,6}^{10} &= -1\\
\alpha_{3,8}^{10} &= -1\\
\alpha_{4,7}^{10} &= 1
\end{aligned}
$$

How the solution(s) were or were not found:
Change variables

$$
\begin{aligned}
\alpha_{5,6}^{10} &\to x_1\\
\alpha_{3,8}^{10} &\to x_2\\
\alpha_{4,7}^{10} &\to x_3
\end{aligned}
$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_8): &\quad -x_2 - 1 &= 0\\
(e_1, e_3, e_7): &\quad -x_2 - x_3 &= 0\\
(e_1, e_4, e_6): &\quad -x_1 - x_3 &= 0\\
(e_2, e_3, e_5): &\quad -x_2 - 1 &= 0
\end{aligned}
$$

Groebner basis (3 variables, 3 linear, 0 nonlinear)

$$
\begin{aligned}
x_1 + 1 &= 0\\
x_2 + 1 &= 0\\
x_3 - 1 &= 0
\end{aligned}
$$

Solution 1:

$$
\begin{aligned}
x_1 &= -1\\
x_2 &= -1\\
x_3 &= 1
\end{aligned}
$$

$\mathfrak{m}_{5B}(3,10)$

m5B310 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
&[e_1,e_2]=e_3 && [e_1,e_3]=e_4\\
&[e_1,e_4]=e_5 && [e_1,e_5]=e_6\\
&[e_1,e_6]=e_7 && [e_1,e_7]=e_8\\
&[e_1,e_8]=e_9 && [e_2,e_3]=e_6\\
&[e_2,e_4]=e_7 && [e_2,e_5]=\alpha_{2,5}^{8}e_8\\
&[e_2,e_6]=\alpha_{2,6}^{9}e_9 && [e_2,e_9]=e_{10}\\
&[e_3,e_4]=\alpha_{3,4}^{8}e_8 && [e_3,e_5]=\alpha_{3,5}^{9}e_9\\
&[e_3,e_8]=\alpha_{3,8}^{10}e_{10} && [e_4,e_7]=\alpha_{4,7}^{10}e_{10}\\
&[e_5,e_6]=\alpha_{5,6}^{10}e_{10}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
&(e_1,e_2,e_4): && -\alpha_{2,5}^{8}-\alpha_{3,4}^{8}+1 && =0\\
&(e_1,e_2,e_5): && \alpha_{2,5}^{8}-\alpha_{2,6}^{9}-\alpha_{3,5}^{9} && =0\\
&(e_1,e_3,e_4): && \alpha_{3,4}^{8}-\alpha_{3,5}^{9} && =0\\
&(e_1,e_2,e_8): && -\alpha_{3,8}^{10}-1 && =0\\
&(e_1,e_3,e_7): && -\alpha_{3,8}^{10}-\alpha_{4,7}^{10} && =0\\
&(e_1,e_4,e_6): && -\alpha_{4,7}^{10}-\alpha_{5,6}^{10} && =0\\
&(e_2,e_3,e_5): && -\alpha_{2,5}^{8}\alpha_{3,8}^{10}+\alpha_{3,5}^{9}+\alpha_{5,6}^{10} && =0
\end{aligned}
$$

Infinite number of solutions: only zero-dimensional systems supported (finite number of solutions).

How the solution(s) were or were not found:

Change variables

$$\alpha_{2,5}^{8}\to x_1$$

$$\alpha_{4,7}^{10}\to x_2$$

$$\alpha_{3,5}^{9}\to x_3$$

$$\alpha_{3,4}^{8}\to x_4$$

$$\alpha_{2,6}^{9}\to x_5$$

$$\alpha_{3,8}^{10} \to x_6$$

$$\alpha_{5,6}^{10} \to x_7$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4): &\quad -x_1 - x_4 + 1 &= 0\\
(e_1, e_2, e_5): &\quad x_1 - x_3 - x_5 &= 0\\
(e_1, e_3, e_4): &\quad -x_3 + x_4 &= 0\\
(e_1, e_2, e_8): &\quad -x_6 - 1 &= 0\\
(e_1, e_3, e_7): &\quad -x_2 - x_6 &= 0\\
(e_1, e_4, e_6): &\quad -x_2 - x_7 &= 0\\
(e_2, e_3, e_5): &\quad -x_1x_6 + x_3 + x_7 &= 0
\end{aligned}$$

Groebner basis (7 variables, 6 linear, 0 nonlinear)

$$2x_1 - x_5 - 1 = 0$$

$$x_2 - 1 = 0$$

$$2x_3 + x_5 - 1 = 0$$

$$2x_4 + x_5 - 1 = 0$$

$$x_6 + 1 = 0$$

$$x_7 + 1 = 0$$

$\mathfrak{m}_{2B}(4, 10)$

m2B410 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4\\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6\\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8\\
[e_1, e_8] &= e_9 & [e_2, e_5] &= e_9\\
[e_2, e_9] &= e_{10} & [e_3, e_4] &= -e_9\\
[e_3, e_8] &= \alpha_{3,8}^{10} e_{10} & [e_4, e_7] &= \alpha_{4,7}^{10} e_{10}\\
[e_5, e_6] &= \alpha_{5,6}^{10} e_{10}
\end{aligned}$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_8): &\quad -\alpha_{3,8}^{10} - 1 &&= 0\\
(e_1, e_3, e_7): &\quad -\alpha_{3,8}^{10} - \alpha_{4,7}^{10} &&= 0\\
(e_1, e_4, e_6): &\quad -\alpha_{4,7}^{10} - \alpha_{5,6}^{10} &&= 0\\
(e_2, e_3, e_4): &\quad \text{no solutions}
\end{aligned}
$$

There are no solutions.

$\mathfrak{m}_{4B}(4, 10)$

m4B410 (this line included for string searching purposes)
Solution 1

$$
\begin{aligned}
&[e_1, e_2] = e_3 && [e_1, e_3] = e_4\\
&[e_1, e_4] = e_5 && [e_1, e_5] = e_6\\
&[e_1, e_6] = e_7 && [e_1, e_7] = e_8\\
&[e_1, e_8] = e_9 && [e_2, e_3] = e_7\\
&[e_2, e_4] = e_8 && [e_2, e_5] = 3e_9\\
&[e_2, e_9] = e_{10} && [e_3, e_4] = -2e_9\\
&[e_3, e_8] = -e_{10} && [e_4, e_7] = e_{10}\\
&[e_5, e_6] = -e_{10}
\end{aligned}
$$

Original brackets:

$$
\begin{aligned}
&[e_1, e_2] = e_3 && [e_1, e_3] = e_4\\
&[e_1, e_4] = e_5 && [e_1, e_5] = e_6\\
&[e_1, e_6] = e_7 && [e_1, e_7] = e_8\\
&[e_1, e_8] = e_9 && [e_2, e_3] = e_7\\
&[e_2, e_4] = e_8 && [e_2, e_5] = \alpha_{2,5}^{9} e_9\\
&[e_2, e_9] = e_{10} && [e_3, e_4] = \alpha_{3,4}^{9} e_9\\
&[e_3, e_8] = \alpha_{3,8}^{10} e_{10} && [e_4, e_7] = \alpha_{4,7}^{10} e_{10}\\
&[e_5, e_6] = \alpha_{5,6}^{10} e_{10}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1,e_2,e_4):&\quad -\alpha^9_{2,5}-\alpha^9_{3,4}+1 &&=0\\
(e_1,e_2,e_8):&\quad -\alpha^{10}_{3,8}-1 &&=0\\
(e_1,e_3,e_7):&\quad -\alpha^{10}_{3,8}-\alpha^{10}_{4,7} &&=0\\
(e_1,e_4,e_6):&\quad -\alpha^{10}_{4,7}-\alpha^{10}_{5,6} &&=0\\
(e_2,e_3,e_4):&\quad \alpha^9_{3,4}-\alpha^{10}_{3,8}+\alpha^{10}_{4,7} &&=0
\end{aligned}
$$

Solution 1:

$$
\begin{aligned}
\alpha^{10}_{4,7}&=1\\
\alpha^9_{2,5}&=3\\
\alpha^{10}_{3,8}&=-1\\
\alpha^{10}_{5,6}&=-1\\
\alpha^9_{3,4}&=-2
\end{aligned}
$$

How the solution(s) were or were not found:
Change variables

$$
\begin{aligned}
\alpha^{10}_{4,7}&\to x_1\\
\alpha^9_{2,5}&\to x_2\\
\alpha^{10}_{3,8}&\to x_3\\
\alpha^{10}_{5,6}&\to x_4\\
\alpha^9_{3,4}&\to x_5
\end{aligned}
$$

Jacobi Tests

$$
\begin{aligned}
(e_1,e_2,e_4):&\quad -x_2-x_5+1 &&=0\\
(e_1,e_2,e_8):&\quad -x_3-1 &&=0\\
(e_1,e_3,e_7):&\quad -x_1-x_3 &&=0\\
(e_1,e_4,e_6):&\quad -x_1-x_4 &&=0\\
(e_2,e_3,e_4):&\quad x_1-x_3+x_5 &&=0
\end{aligned}
$$

Groebner basis (5 variables, 5 linear, 0 nonlinear)

$$x_1-1=0$$

$$x_2 - 3 = 0$$
$$x_3 + 1 = 0$$
$$x_4 + 1 = 0$$
$$x_5 + 2 = 0$$

Solution 1:

$$x_1 = 1$$
$$x_2 = 3$$
$$x_3 = -1$$
$$x_4 = -1$$
$$x_5 = -2$$

$\mathfrak{m}_{3B}(5, 10)$

m3B510 (this line included for string searching purposes)
Solution 1

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_2, e_3] &= e_8 \\
[e_2, e_4] &= e_9 & [e_2, e_9] &= e_{10} \\
[e_3, e_8] &= -e_{10} & [e_4, e_7] &= e_{10} \\
[e_5, e_6] &= -e_{10}
\end{aligned}$$

Original brackets:

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_2, e_3] &= e_8 \\
[e_2, e_4] &= e_9 & [e_2, e_9] &= e_{10} \\
[e_3, e_8] &= \alpha_{3,8}^{10} e_{10} & [e_4, e_7] &= \alpha_{4,7}^{10} e_{10} \\
[e_5, e_6] &= \alpha_{5,6}^{10} e_{10}
\end{aligned}$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_8): &\quad -\alpha^{10}_{3,8} - 1 &&= 0\\
(e_1, e_3, e_7): &\quad -\alpha^{10}_{3,8} - \alpha^{10}_{4,7} &&= 0\\
(e_1, e_4, e_6): &\quad -\alpha^{10}_{4,7} - \alpha^{10}_{5,6} &&= 0
\end{aligned}
$$

Solution 1:

$$
\begin{aligned}
\alpha^{10}_{5,6} &= -1\\
\alpha^{10}_{3,8} &= -1\\
\alpha^{10}_{4,7} &= 1
\end{aligned}
$$

How the solution(s) were or were not found:
Change variables

$$
\begin{aligned}
\alpha^{10}_{5,6} &\to x_1\\
\alpha^{10}_{3,8} &\to x_2\\
\alpha^{10}_{4,7} &\to x_3
\end{aligned}
$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_8): &\quad -x_2 - 1 &&= 0\\
(e_1, e_3, e_7): &\quad -x_2 - x_3 &&= 0\\
(e_1, e_4, e_6): &\quad -x_1 - x_3 &&= 0
\end{aligned}
$$

Groebner basis (3 variables, 3 linear, 0 nonlinear)

$$
\begin{aligned}
x_1 + 1 &= 0\\
x_2 + 1 &= 0\\
x_3 - 1 &= 0
\end{aligned}
$$

Solution 1:

$$
\begin{aligned}
x_1 &= -1\\
x_2 &= -1\\
x_3 &= 1
\end{aligned}
$$

$\mathfrak{m}_{2B}(6,10)$

m2B610 (this line included for string searching purposes)
Solution 1

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4 \\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6 \\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8 \\
[e_1,e_8] &= e_9 & [e_2,e_3] &= e_9 \\
[e_2,e_9] &= e_{10} & [e_3,e_8] &= -e_{10} \\
[e_4,e_7] &= e_{10} & [e_5,e_6] &= -e_{10}
\end{aligned}
$$

Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4 \\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6 \\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8 \\
[e_1,e_8] &= e_9 & [e_2,e_3] &= e_9 \\
[e_2,e_9] &= e_{10} & [e_3,e_8] &= \alpha_{3,8}^{10}e_{10} \\
[e_4,e_7] &= \alpha_{4,7}^{10}e_{10} & [e_5,e_6] &= \alpha_{5,6}^{10}e_{10}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1,e_2,e_8): &\quad -\alpha_{3,8}^{10}-1 &&= 0 \\
(e_1,e_3,e_7): &\quad -\alpha_{3,8}^{10}-\alpha_{4,7}^{10} &&= 0 \\
(e_1,e_4,e_6): &\quad -\alpha_{4,7}^{10}-\alpha_{5,6}^{10} &&= 0
\end{aligned}
$$

Solution 1:

$$
\begin{aligned}
\alpha_{5,6}^{10} &= -1 \\
\alpha_{3,8}^{10} &= -1 \\
\alpha_{4,7}^{10} &= 1
\end{aligned}
$$

How the solution(s) were or were not found:
Change variables

$$\alpha_{5,6}^{10} \to x_1$$

$$\alpha_{3,8}^{10} \to x_2$$

$$\alpha_{4,7}^{10} \to x_3$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_8):&\quad -x_2 - 1 &= 0\\
(e_1, e_3, e_7):&\quad -x_2 - x_3 &= 0\\
(e_1, e_4, e_6):&\quad -x_1 - x_3 &= 0
\end{aligned}$$

Groebner basis (3 variables, 3 linear, 0 nonlinear)

$$x_1 + 1 = 0$$

$$x_2 + 1 = 0$$

$$x_3 - 1 = 0$$

Solution 1:

$$x_1 = -1$$

$$x_2 = -1$$

$$x_3 = 1$$

$\mathfrak{m}_{2B}(2, 12)$

m2B212 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4\\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6\\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8\\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10}\\
[e_1, e_{10}] &= e_{11} & [e_2, e_9] &= e_{11}\\
[e_2, e_{11}] &= e_{12} & [e_3, e_8] &= -e_{11}\\
[e_3, e_{10}] &= \alpha_{3,10}^{12} e_{12} & [e_4, e_7] &= e_{11}\\
[e_4, e_9] &= \alpha_{4,9}^{12} e_{12} & [e_5, e_6] &= -e_{11}\\
[e_5, e_8] &= \alpha_{5,8}^{12} e_{12} & [e_6, e_7] &= \alpha_{6,7}^{12} e_{12}
\end{aligned}$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1,e_2,e_{10}): &\quad -\alpha^{12}_{3,10}-1 &= 0\\
(e_1,e_3,e_9): &\quad -\alpha^{12}_{3,10}-\alpha^{12}_{4,9} &= 0\\
(e_1,e_4,e_8): &\quad -\alpha^{12}_{4,9}-\alpha^{12}_{5,8} &= 0\\
(e_1,e_5,e_7): &\quad -\alpha^{12}_{5,8}-\alpha^{12}_{6,7} &= 0\\
(e_2,e_3,e_8): &\quad \text{no solutions}\\
(e_2,e_4,e_7): &\quad \text{no solutions}\\
(e_2,e_5,e_6): &\quad \text{no solutions}
\end{aligned}
$$

There are no solutions.

$\mathfrak{m}_{4B}(2,12)$

m4B212 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4\\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6\\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8\\
[e_1,e_8] &= e_9 & [e_1,e_9] &= e_{10}\\
[e_1,e_{10}] &= e_{11} & [e_2,e_7] &= e_9\\
[e_2,e_8] &= 3e_{10} & [e_2,e_9] &= \alpha^{11}_{2,9}e_{11}\\
[e_2,e_{11}] &= e_{12} & [e_3,e_6] &= -e_9\\
[e_3,e_7] &= -2e_{10} & [e_3,e_8] &= \alpha^{11}_{3,8}e_{11}\\
[e_3,e_{10}] &= \alpha^{12}_{3,10}e_{12} & [e_4,e_5] &= e_9\\
[e_4,e_6] &= e_{10} & [e_4,e_7] &= \alpha^{11}_{4,7}e_{11}\\
[e_4,e_9] &= \alpha^{12}_{4,9}e_{12} & [e_5,e_6] &= \alpha^{11}_{5,6}e_{11}\\
[e_5,e_8] &= \alpha^{12}_{5,8}e_{12} & [e_6,e_7] &= \alpha^{12}_{6,7}e_{12}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1,e_2,e_8):&\quad -\alpha^{11}_{2,9}-\alpha^{11}_{3,8}+3 &&=0\\
(e_1,e_3,e_7):&\quad -\alpha^{11}_{3,8}-\alpha^{11}_{4,7}-2 &&=0\\
(e_1,e_4,e_6):&\quad -\alpha^{11}_{4,7}-\alpha^{11}_{5,6}+1 &&=0\\
(e_2,e_3,e_6):&\quad -\alpha^{11}_{2,9} &&=0\\
(e_2,e_4,e_5):&\quad \alpha^{11}_{2,9} &&=0\\
(e_1,e_2,e_{10}):&\quad -\alpha^{12}_{3,10}-1 &&=0\\
(e_1,e_3,e_9):&\quad -\alpha^{12}_{3,10}-\alpha^{12}_{4,9} &&=0\\
(e_1,e_4,e_8):&\quad -\alpha^{12}_{4,9}-\alpha^{12}_{5,8} &&=0\\
(e_1,e_5,e_7):&\quad -\alpha^{12}_{5,8}-\alpha^{12}_{6,7} &&=0\\
(e_2,e_3,e_8):&\quad -3\alpha^{12}_{3,10}+\alpha^{11}_{3,8} &&=0\\
(e_2,e_4,e_7):&\quad \alpha^{11}_{4,7}-\alpha^{12}_{4,9} &&=0\\
(e_2,e_5,e_6):&\quad \alpha^{11}_{5,6} &&=0\\
(e_3,e_4,e_6):&\quad \alpha^{12}_{3,10}+\alpha^{12}_{4,9} &&=0
\end{aligned}
$$

No solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha^{11}_{2,9}\to x_1$$
$$\alpha^{12}_{3,10}\to x_2$$
$$\alpha^{12}_{6,7}\to x_3$$
$$\alpha^{12}_{4,9}\to x_4$$
$$\alpha^{11}_{3,8}\to x_5$$
$$\alpha^{12}_{5,8}\to x_6$$
$$\alpha^{11}_{4,7}\to x_7$$
$$\alpha^{11}_{5,6}\to x_8$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_8): &\quad -x_1 - x_5 + 3 &&= 0\\
(e_1, e_3, e_7): &\quad -x_5 - x_7 - 2 &&= 0\\
(e_1, e_4, e_6): &\quad -x_7 - x_8 + 1 &&= 0\\
(e_2, e_3, e_6): &\quad -x_1 &&= 0\\
(e_2, e_4, e_5): &\quad x_1 &&= 0\\
(e_1, e_2, e_{10}): &\quad -x_2 - 1 &&= 0\\
(e_1, e_3, e_9): &\quad -x_2 - x_4 &&= 0\\
(e_1, e_4, e_8): &\quad -x_4 - x_6 &&= 0\\
(e_1, e_5, e_7): &\quad -x_3 - x_6 &&= 0\\
(e_2, e_3, e_8): &\quad -3x_2 + x_5 &&= 0\\
(e_2, e_4, e_7): &\quad -x_4 + x_7 &&= 0\\
(e_2, e_5, e_6): &\quad x_8 &&= 0\\
(e_3, e_4, e_6): &\quad x_2 + x_4 &&= 0
\end{aligned}
$$

Groebner basis (8 variables, 1 linear, 0 nonlinear)

$$1 = 0$$

$\mathfrak{m}_{6B}(2, 12)$

m6B212 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_2, e_5] &= e_7 \\
[e_2, e_6] &= 2e_8 & [e_2, e_7] &= \alpha_{2,7}^{9} e_9 \\
[e_2, e_8] &= \alpha_{2,8}^{10} e_{10} & [e_2, e_9] &= \alpha_{2,9}^{11} e_{11} \\
[e_2, e_{11}] &= e_{12} & [e_3, e_4] &= -e_7 \\
[e_3, e_5] &= -e_8 & [e_3, e_6] &= \alpha_{3,6}^{9} e_9 \\
[e_3, e_7] &= \alpha_{3,7}^{10} e_{10} & [e_3, e_8] &= \alpha_{3,8}^{11} e_{11} \\
[e_3, e_{10}] &= \alpha_{3,10}^{12} e_{12} & [e_4, e_5] &= \alpha_{4,5}^{9} e_9 \\
[e_4, e_6] &= \alpha_{4,6}^{10} e_{10} & [e_4, e_7] &= \alpha_{4,7}^{11} e_{11} \\
[e_4, e_9] &= \alpha_{4,9}^{12} e_{12} & [e_5, e_6] &= \alpha_{5,6}^{11} e_{11} \\
[e_5, e_8] &= \alpha_{5,8}^{12} e_{12} & [e_6, e_7] &= \alpha_{6,7}^{12} e_{12}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_6): &\quad -\alpha_{2,7}^{9} - \alpha_{3,6}^{9} + 2 &&= 0\\
(e_1, e_3, e_5): &\quad -\alpha_{3,6}^{9} - \alpha_{4,5}^{9} - 1 &&= 0\\
(e_2, e_3, e_4): &\quad -\alpha_{2,7}^{9} &&= 0\\
(e_1, e_2, e_7): &\quad \alpha_{2,7}^{9} - \alpha_{2,8}^{10} - \alpha_{3,7}^{10} &&= 0\\
(e_1, e_3, e_6): &\quad \alpha_{3,6}^{9} - \alpha_{3,7}^{10} - \alpha_{4,6}^{10} &&= 0\\
(e_1, e_4, e_5): &\quad \alpha_{4,5}^{9} - \alpha_{4,6}^{10} &&= 0\\
(e_2, e_3, e_5): &\quad -\alpha_{2,8}^{10} - \alpha_{3,7}^{10} &&= 0\\
(e_1, e_2, e_8): &\quad \alpha_{2,8}^{10} - \alpha_{2,9}^{11} - \alpha_{3,8}^{11} &&= 0\\
(e_1, e_3, e_7): &\quad \alpha_{3,7}^{10} - \alpha_{3,8}^{11} - \alpha_{4,7}^{11} &&= 0\\
(e_1, e_4, e_6): &\quad \alpha_{4,6}^{10} - \alpha_{4,7}^{11} - \alpha_{5,6}^{11} &&= 0\\
(e_2, e_3, e_6): &\quad \alpha_{2,9}^{11}\alpha_{3,6}^{9} - 2\alpha_{3,8}^{11} &&= 0\\
(e_2, e_4, e_5): &\quad \alpha_{2,9}^{11}\alpha_{4,5}^{9} - \alpha_{4,7}^{11} &&= 0\\
(e_1, e_2, e_{10}): &\quad -\alpha_{3,10}^{12} - 1 &&= 0\\
(e_1, e_3, e_9): &\quad -\alpha_{3,10}^{12} - \alpha_{4,9}^{12} &&= 0\\
(e_1, e_4, e_8): &\quad -\alpha_{4,9}^{12} - \alpha_{5,8}^{12} &&= 0\\
(e_1, e_5, e_7): &\quad -\alpha_{5,8}^{12} - \alpha_{6,7}^{12} &&= 0\\
(e_2, e_3, e_8): &\quad -\alpha_{2,8}^{10}\alpha_{3,10}^{12} + \alpha_{3,8}^{11} &&= 0\\
(e_2, e_4, e_7): &\quad -\alpha_{2,7}^{9}\alpha_{4,9}^{12} + \alpha_{4,7}^{11} &&= 0\\
(e_2, e_5, e_6): &\quad \alpha_{5,6}^{11} - 2\alpha_{5,8}^{12} + \alpha_{6,7}^{12} &&= 0\\
(e_3, e_4, e_6): &\quad \alpha_{3,10}^{12}\alpha_{4,6}^{10} - \alpha_{3,6}^{9}\alpha_{4,9}^{12} - \alpha_{6,7}^{12} &&= 0
\end{aligned}
$$

No solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{2,9}^{11} \to x_1$$

$$\alpha_{3,10}^{12} \to x_2$$

$$\alpha_{2,8}^{10} \to x_3$$

$$\alpha_{3,7}^{10} \to x_4$$

$$\alpha_{6,7}^{12} \to x_5$$

$$\alpha_{4,9}^{12} \to x_6$$

$$\alpha_{5,6}^{11} \to x_7$$

$$\alpha_{3,8}^{11} \to x_8$$

$$\alpha_{4,5}^{9} \to x_9$$

$$\alpha_{5,8}^{12} \to x_{10}$$

$$\alpha_{2,7}^{9} \to x_{11}$$

$$\alpha_{4,7}^{11} \to x_{12}$$

$$\alpha_{3,6}^{9} \to x_{13}$$

$$\alpha_{4,6}^{10} \to x_{14}$$

Jacobi Tests

$$
\begin{aligned}
(e_1,e_2,e_6):&\quad -x_{11}-x_{13}+2 &&=0\\
(e_1,e_3,e_5):&\quad -x_{13}-x_9-1 &&=0\\
(e_2,e_3,e_4):&\quad -x_{11} &&=0\\
(e_1,e_2,e_7):&\quad x_{11}-x_3-x_4 &&=0\\
(e_1,e_3,e_6):&\quad x_{13}-x_{14}-x_4 &&=0\\
(e_1,e_4,e_5):&\quad -x_{14}+x_9 &&=0\\
(e_2,e_3,e_5):&\quad -x_3-x_4 &&=0\\
(e_1,e_2,e_8):&\quad -x_1+x_3-x_8 &&=0\\
(e_1,e_3,e_7):&\quad -x_{12}+x_4-x_8 &&=0\\
(e_1,e_4,e_6):&\quad -x_{12}+x_{14}-x_7 &&=0\\
(e_2,e_3,e_6):&\quad x_1x_{13}-2x_8 &&=0\\
(e_2,e_4,e_5):&\quad x_1x_9-x_{12} &&=0\\
(e_1,e_2,e_{10}):&\quad -x_2-1 &&=0\\
(e_1,e_3,e_9):&\quad -x_2-x_6 &&=0\\
(e_1,e_4,e_8):&\quad -x_{10}-x_6 &&=0\\
(e_1,e_5,e_7):&\quad -x_{10}-x_5 &&=0\\
(e_2,e_3,e_8):&\quad -x_2x_3+x_8 &&=0\\
(e_2,e_4,e_7):&\quad -x_{11}x_6+x_{12} &&=0\\
(e_2,e_5,e_6):&\quad -2x_{10}+x_5+x_7 &&=0\\
(e_3,e_4,e_6):&\quad -x_{13}x_6+x_{14}x_2-x_5 &&=0
\end{aligned}
$$

Groebner basis (14 variables, 1 linear, 0 nonlinear)

$$1=0$$

$\mathfrak{m}_{8B}(2, 12)$

m8B212 (this line included for string searching purposes)
Solution 1

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_2, e_3] = e_5$$
$$[e_2, e_4] = e_6 \qquad [e_2, e_5] = e_7\left(\frac{1}{3} + \frac{2\sqrt{2}i}{3}\right)$$
$$[e_2, e_6] = e_8\left(-\frac{1}{3} + \frac{4\sqrt{2}i}{3}\right) \qquad [e_2, e_7] = e_9\left(-1 + \sqrt{2}i\right)$$
$$[e_2, e_8] = e_{10}\left(-\frac{5}{3} - \frac{\sqrt{2}i}{3}\right) \qquad [e_2, e_9] = e_{11}\left(-\frac{7}{3} - \frac{2\sqrt{2}i}{3}\right)$$
$$[e_2, e_{11}] = e_{12} \qquad [e_3, e_4] = e_7\left(\frac{2}{3} - \frac{2\sqrt{2}i}{3}\right)$$
$$[e_3, e_5] = e_8\left(\frac{2}{3} - \frac{2\sqrt{2}i}{3}\right) \qquad [e_3, e_6] = e_9\left(\frac{2}{3} + \frac{\sqrt{2}i}{3}\right)$$
$$[e_3, e_7] = e_{10}\left(\frac{2}{3} + \frac{4\sqrt{2}i}{3}\right) \qquad [e_3, e_8] = e_{11}\left(\frac{2}{3} + \frac{\sqrt{2}i}{3}\right)$$
$$[e_3, e_{10}] = -e_{12} \qquad [e_4, e_5] = -\sqrt{2}ie_9$$
$$[e_4, e_6] = -\sqrt{2}ie_{10} \qquad [e_4, e_7] = \sqrt{2}ie_{11}$$
$$[e_4, e_9] = e_{12} \qquad [e_5, e_6] = -2\sqrt{2}ie_{11}$$
$$[e_5, e_8] = -e_{12} \qquad [e_6, e_7] = e_{12}$$

Solution 2

$$
\begin{aligned}
&[e_1, e_2] = e_3 && [e_1, e_3] = e_4 \\
&[e_1, e_4] = e_5 && [e_1, e_5] = e_6 \\
&[e_1, e_6] = e_7 && [e_1, e_7] = e_8 \\
&[e_1, e_8] = e_9 && [e_1, e_9] = e_{10} \\
&[e_1, e_{10}] = e_{11} && [e_2, e_3] = e_5 \\
&[e_2, e_4] = e_6 && [e_2, e_5] = e_7 \left(\frac{1}{3} - \frac{2\sqrt{2}i}{3}\right) \\
&[e_2, e_6] = e_8 \left(-\frac{1}{3} - \frac{4\sqrt{2}i}{3}\right) && [e_2, e_7] = e_9 \left(-1 - \sqrt{2}i\right) \\
&[e_2, e_8] = e_{10} \left(-\frac{5}{3} + \frac{\sqrt{2}i}{3}\right) && [e_2, e_9] = e_{11} \left(-\frac{7}{3} + \frac{2\sqrt{2}i}{3}\right) \\
&[e_2, e_{11}] = e_{12} && [e_3, e_4] = e_7 \left(\frac{2}{3} + \frac{2\sqrt{2}i}{3}\right) \\
&[e_3, e_5] = e_8 \left(\frac{2}{3} + \frac{2\sqrt{2}i}{3}\right) && [e_3, e_6] = e_9 \left(\frac{2}{3} - \frac{\sqrt{2}i}{3}\right) \\
&[e_3, e_7] = e_{10} \left(\frac{2}{3} - \frac{4\sqrt{2}i}{3}\right) && [e_3, e_8] = e_{11} \left(\frac{2}{3} - \frac{\sqrt{2}i}{3}\right) \\
&[e_3, e_{10}] = -e_{12} && [e_4, e_5] = \sqrt{2}ie_9 \\
&[e_4, e_6] = \sqrt{2}ie_{10} && [e_4, e_7] = -\sqrt{2}ie_{11} \\
&[e_4, e_9] = e_{12} && [e_5, e_6] = 2\sqrt{2}ie_{11} \\
&[e_5, e_8] = -e_{12} && [e_6, e_7] = e_{12}
\end{aligned}
$$

Solution 3

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_2, e_3] = e_5$$
$$[e_2, e_4] = e_6 \qquad [e_2, e_5] = e_7 \left(1 - \frac{\sqrt{10}}{5}\right)$$
$$[e_2, e_6] = e_8 \left(1 - \frac{2\sqrt{10}}{5}\right) \qquad [e_2, e_7] = e_9 \left(\frac{5}{3} - \frac{2\sqrt{10}}{3}\right)$$
$$[e_2, e_8] = e_{10} \left(3 - \sqrt{10}\right) \qquad [e_2, e_9] = e_{11} \left(7 - 2\sqrt{10}\right)$$
$$[e_2, e_{11}] = e_{12} \qquad [e_3, e_4] = \frac{\sqrt{10}e_7}{5}$$
$$[e_3, e_5] = \frac{\sqrt{10}e_8}{5} \qquad [e_3, e_6] = e_9 \left(-\frac{2}{3} + \frac{4\sqrt{10}}{15}\right)$$
$$[e_3, e_7] = e_{10} \left(-\frac{4}{3} + \frac{\sqrt{10}}{3}\right) \qquad [e_3, e_8] = e_{11} \left(-4 + \sqrt{10}\right)$$
$$[e_3, e_{10}] = -e_{12} \qquad [e_4, e_5] = e_9 \left(\frac{2}{3} - \frac{\sqrt{10}}{15}\right)$$
$$[e_4, e_6] = e_{10} \left(\frac{2}{3} - \frac{\sqrt{10}}{15}\right) \qquad [e_4, e_7] = e_{11} \left(\frac{8}{3} - \frac{2\sqrt{10}}{3}\right)$$
$$[e_4, e_9] = e_{12} \qquad [e_5, e_6] = e_{11} \left(-2 + \frac{3\sqrt{10}}{5}\right)$$
$$[e_5, e_8] = -e_{12} \qquad [e_6, e_7] = e_{12}$$

Solution 4

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_2, e_3] = e_5$$
$$[e_2, e_4] = e_6 \qquad [e_2, e_5] = e_7 \left(\frac{\sqrt{10}}{5} + 1\right)$$
$$[e_2, e_6] = e_8 \left(1 + \frac{2\sqrt{10}}{5}\right) \qquad [e_2, e_7] = e_9 \left(\frac{5}{3} + \frac{2\sqrt{10}}{3}\right)$$
$$[e_2, e_8] = e_{10} \left(3 + \sqrt{10}\right) \qquad [e_2, e_9] = e_{11} \left(2\sqrt{10} + 7\right)$$
$$[e_2, e_{11}] = e_{12} \qquad [e_3, e_4] = -\frac{\sqrt{10}e_7}{5}$$
$$[e_3, e_5] = -\frac{\sqrt{10}e_8}{5} \qquad [e_3, e_6] = e_9 \left(-\frac{4\sqrt{10}}{15} - \frac{2}{3}\right)$$
$$[e_3, e_7] = e_{10} \left(-\frac{4}{3} - \frac{\sqrt{10}}{3}\right) \qquad [e_3, e_8] = e_{11} \left(-4 - \sqrt{10}\right)$$
$$[e_3, e_{10}] = -e_{12} \qquad [e_4, e_5] = e_9 \left(\frac{\sqrt{10}}{15} + \frac{2}{3}\right)$$
$$[e_4, e_6] = e_{10} \left(\frac{\sqrt{10}}{15} + \frac{2}{3}\right) \qquad [e_4, e_7] = e_{11} \left(\frac{2\sqrt{10}}{3} + \frac{8}{3}\right)$$
$$[e_4, e_9] = e_{12} \qquad [e_5, e_6] = e_{11} \left(-2 - \frac{3\sqrt{10}}{5}\right)$$
$$[e_5, e_8] = -e_{12} \qquad [e_6, e_7] = e_{12}$$

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_2, e_3] &= e_5 \\
[e_2, e_4] &= e_6 & [e_2, e_5] &= \alpha_{2,5}^{7} e_7 \\
[e_2, e_6] &= \alpha_{2,6}^{8} e_8 & [e_2, e_7] &= \alpha_{2,7}^{9} e_9 \\
[e_2, e_8] &= \alpha_{2,8}^{10} e_{10} & [e_2, e_9] &= \alpha_{2,9}^{11} e_{11} \\
[e_2, e_{11}] &= e_{12} & [e_3, e_4] &= \alpha_{3,4}^{7} e_7 \\
[e_3, e_5] &= \alpha_{3,5}^{8} e_8 & [e_3, e_6] &= \alpha_{3,6}^{9} e_9 \\
[e_3, e_7] &= \alpha_{3,7}^{10} e_{10} & [e_3, e_8] &= \alpha_{3,8}^{11} e_{11} \\
[e_3, e_{10}] &= \alpha_{3,10}^{12} e_{12} & [e_4, e_5] &= \alpha_{4,5}^{9} e_9 \\
[e_4, e_6] &= \alpha_{4,6}^{10} e_{10} & [e_4, e_7] &= \alpha_{4,7}^{11} e_{11} \\
[e_4, e_9] &= \alpha_{4,9}^{12} e_{12} & [e_5, e_6] &= \alpha_{5,6}^{11} e_{11} \\
[e_5, e_8] &= \alpha_{5,8}^{12} e_{12} & [e_6, e_7] &= \alpha_{6,7}^{12} e_{12}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1,e_2,e_4): &\quad -\alpha^{7}_{2,5}-\alpha^{7}_{3,4}+1 &&=0\\
(e_1,e_2,e_5): &\quad \alpha^{7}_{2,5}-\alpha^{8}_{2,6}-\alpha^{8}_{3,5} &&=0\\
(e_1,e_3,e_4): &\quad \alpha^{7}_{3,4}-\alpha^{8}_{3,5} &&=0\\
(e_1,e_2,e_6): &\quad \alpha^{8}_{2,6}-\alpha^{9}_{2,7}-\alpha^{9}_{3,6} &&=0\\
(e_1,e_3,e_5): &\quad \alpha^{8}_{3,5}-\alpha^{9}_{3,6}-\alpha^{9}_{4,5} &&=0\\
(e_2,e_3,e_4): &\quad \alpha^{9}_{2,7}\alpha^{7}_{3,4}-\alpha^{9}_{3,6}+\alpha^{9}_{4,5} &&=0\\
(e_1,e_2,e_7): &\quad \alpha^{9}_{2,7}-\alpha^{10}_{2,8}-\alpha^{10}_{3,7} &&=0\\
(e_1,e_3,e_6): &\quad \alpha^{9}_{3,6}-\alpha^{10}_{3,7}-\alpha^{10}_{4,6} &&=0\\
(e_1,e_4,e_5): &\quad \alpha^{9}_{4,5}-\alpha^{10}_{4,6} &&=0\\
(e_2,e_3,e_5): &\quad -\alpha^{7}_{2,5}\alpha^{10}_{3,7}+\alpha^{10}_{2,8}\alpha^{8}_{3,5} &&=0\\
(e_1,e_2,e_8): &\quad \alpha^{10}_{2,8}-\alpha^{11}_{2,9}-\alpha^{11}_{3,8} &&=0\\
(e_1,e_3,e_7): &\quad \alpha^{10}_{3,7}-\alpha^{11}_{3,8}-\alpha^{11}_{4,7} &&=0\\
(e_1,e_4,e_6): &\quad \alpha^{10}_{4,6}-\alpha^{11}_{4,7}-\alpha^{11}_{5,6} &&=0\\
(e_2,e_3,e_6): &\quad -\alpha^{8}_{2,6}\alpha^{11}_{3,8}+\alpha^{11}_{2,9}\alpha^{9}_{3,6}-\alpha^{11}_{5,6} &&=0\\
(e_2,e_4,e_5): &\quad -\alpha^{7}_{2,5}\alpha^{11}_{4,7}+\alpha^{11}_{2,9}\alpha^{9}_{4,5}+\alpha^{11}_{5,6} &&=0\\
(e_1,e_2,e_{10}): &\quad -\alpha^{12}_{3,10}-1 &&=0\\
(e_1,e_3,e_9): &\quad -\alpha^{12}_{3,10}-\alpha^{12}_{4,9} &&=0\\
(e_1,e_4,e_8): &\quad -\alpha^{12}_{4,9}-\alpha^{12}_{5,8} &&=0\\
(e_1,e_5,e_7): &\quad -\alpha^{12}_{5,8}-\alpha^{12}_{6,7} &&=0\\
(e_2,e_3,e_8): &\quad -\alpha^{10}_{2,8}\alpha^{12}_{3,10}+\alpha^{11}_{3,8}-\alpha^{12}_{5,8} &&=0\\
(e_2,e_4,e_7): &\quad -\alpha^{9}_{2,7}\alpha^{12}_{4,9}+\alpha^{11}_{4,7}-\alpha^{12}_{6,7} &&=0\\
(e_2,e_5,e_6): &\quad \alpha^{7}_{2,5}\alpha^{12}_{6,7}-\alpha^{8}_{2,6}\alpha^{12}_{5,8}+\alpha^{11}_{5,6} &&=0\\
(e_3,e_4,e_6): &\quad \alpha^{12}_{3,10}\alpha^{10}_{4,6}+\alpha^{7}_{3,4}\alpha^{12}_{6,7}-\alpha^{9}_{3,6}\alpha^{12}_{4,9} &&=0
\end{aligned}
$$

Solution 1:

$$\alpha_{2,9}^{11} = -7/3 - 2 * sqrt(2) * I/3$$
$$\alpha_{2,5}^{7} = 1/3 + 2 * sqrt(2) * I/3$$
$$\alpha_{2,6}^{8} = -1/3 + 4 * sqrt(2) * I/3$$
$$\alpha_{3,5}^{8} = 2/3 - 2 * sqrt(2) * I/3$$
$$\alpha_{2,8}^{10} = -5/3 - sqrt(2) * I/3$$
$$\alpha_{3,10}^{12} = -1$$
$$\alpha_{3,7}^{10} = 2/3 + 4 * sqrt(2) * I/3$$
$$\alpha_{6,7}^{12} = 1$$
$$\alpha_{4,9}^{12} = 1$$
$$\alpha_{5,6}^{11} = -2 * sqrt(2) * I$$
$$\alpha_{3,8}^{11} = 2/3 + sqrt(2) * I/3$$
$$\alpha_{3,4}^{7} = 2/3 - 2 * sqrt(2) * I/3$$
$$\alpha_{4,5}^{9} = -sqrt(2) * I$$
$$\alpha_{5,8}^{12} = -1$$
$$\alpha_{2,7}^{9} = -1 + sqrt(2) * I$$
$$\alpha_{4,7}^{11} = sqrt(2) * I$$
$$\alpha_{3,6}^{9} = 2/3 + sqrt(2) * I/3$$
$$\alpha_{4,6}^{10} = -sqrt(2) * I$$

Solution 2:

$$\alpha^{11}_{2,9} = -7/3 + 2 * sqrt(2) * I/3$$
$$\alpha^{7}_{2,5} = 1/3 - 2 * sqrt(2) * I/3$$
$$\alpha^{8}_{2,6} = -1/3 - 4 * sqrt(2) * I/3$$
$$\alpha^{8}_{3,5} = 2/3 + 2 * sqrt(2) * I/3$$
$$\alpha^{10}_{2,8} = -5/3 + sqrt(2) * I/3$$
$$\alpha^{12}_{3,10} = -1$$
$$\alpha^{10}_{3,7} = 2/3 - 4 * sqrt(2) * I/3$$
$$\alpha^{12}_{6,7} = 1$$
$$\alpha^{12}_{4,9} = 1$$
$$\alpha^{11}_{5,6} = 2 * sqrt(2) * I$$
$$\alpha^{11}_{3,8} = 2/3 - sqrt(2) * I/3$$
$$\alpha^{7}_{3,4} = 2/3 + 2 * sqrt(2) * I/3$$
$$\alpha^{9}_{4,5} = sqrt(2) * I$$
$$\alpha^{12}_{5,8} = -1$$
$$\alpha^{9}_{2,7} = -1 - sqrt(2) * I$$
$$\alpha^{11}_{4,7} = -sqrt(2) * I$$
$$\alpha^{9}_{3,6} = 2/3 - sqrt(2) * I/3$$
$$\alpha^{10}_{4,6} = sqrt(2) * I$$

Solution 3:

$$\alpha_{2,9}^{11} = 7 - 2 * sqrt(10)$$
$$\alpha_{2,5}^{7} = 1 - sqrt(10)/5$$
$$\alpha_{2,6}^{8} = 1 - 2 * sqrt(10)/5$$
$$\alpha_{3,5}^{8} = sqrt(10)/5$$
$$\alpha_{2,8}^{10} = 3 - sqrt(10)$$
$$\alpha_{3,10}^{12} = -1$$
$$\alpha_{3,7}^{10} = -4/3 + sqrt(10)/3$$
$$\alpha_{6,7}^{12} = 1$$
$$\alpha_{4,9}^{12} = 1$$
$$\alpha_{5,6}^{11} = -2 + 3 * sqrt(10)/5$$
$$\alpha_{3,8}^{11} = -4 + sqrt(10)$$
$$\alpha_{3,4}^{7} = sqrt(10)/5$$
$$\alpha_{4,5}^{9} = 2/3 - sqrt(10)/15$$
$$\alpha_{5,8}^{12} = -1$$
$$\alpha_{2,7}^{9} = 5/3 - 2 * sqrt(10)/3$$
$$\alpha_{4,7}^{11} = 8/3 - 2 * sqrt(10)/3$$
$$\alpha_{3,6}^{9} = -2/3 + 4 * sqrt(10)/15$$
$$\alpha_{4,6}^{10} = 2/3 - sqrt(10)/15$$

Solution 4:

$$\alpha^{11}_{2,9} = 2 * sqrt(10) + 7$$
$$\alpha^{7}_{2,5} = sqrt(10)/5 + 1$$
$$\alpha^{8}_{2,6} = 1 + 2 * sqrt(10)/5$$
$$\alpha^{8}_{3,5} = -sqrt(10)/5$$
$$\alpha^{10}_{2,8} = 3 + sqrt(10)$$
$$\alpha^{12}_{3,10} = -1$$
$$\alpha^{10}_{3,7} = -4/3 - sqrt(10)/3$$
$$\alpha^{12}_{6,7} = 1$$
$$\alpha^{12}_{4,9} = 1$$
$$\alpha^{11}_{5,6} = -2 - 3 * sqrt(10)/5$$
$$\alpha^{11}_{3,8} = -4 - sqrt(10)$$
$$\alpha^{7}_{3,4} = -sqrt(10)/5$$
$$\alpha^{9}_{4,5} = sqrt(10)/15 + 2/3$$
$$\alpha^{12}_{5,8} = -1$$
$$\alpha^{9}_{2,7} = 5/3 + 2 * sqrt(10)/3$$
$$\alpha^{11}_{4,7} = 2 * sqrt(10)/3 + 8/3$$
$$\alpha^{9}_{3,6} = -4 * sqrt(10)/15 - 2/3$$
$$\alpha^{10}_{4,6} = sqrt(10)/15 + 2/3$$

How the solution(s) were or were not found:
Change variables

$$\alpha^{11}_{2,9} \to x_1$$
$$\alpha^{7}_{2,5} \to x_2$$
$$\alpha^{8}_{2,6} \to x_3$$
$$\alpha^{8}_{3,5} \to x_4$$
$$\alpha^{10}_{2,8} \to x_5$$
$$\alpha^{12}_{3,10} \to x_6$$
$$\alpha^{10}_{3,7} \to x_7$$
$$\alpha^{12}_{6,7} \to x_8$$
$$\alpha^{12}_{4,9} \to x_9$$

$$\alpha_{5,6}^{11} \to x_{10}$$
$$\alpha_{3,8}^{11} \to x_{11}$$
$$\alpha_{3,4}^{7} \to x_{12}$$
$$\alpha_{4,5}^{9} \to x_{13}$$
$$\alpha_{5,8}^{12} \to x_{14}$$
$$\alpha_{2,7}^{9} \to x_{15}$$
$$\alpha_{4,7}^{11} \to x_{16}$$
$$\alpha_{3,6}^{9} \to x_{17}$$
$$\alpha_{4,6}^{10} \to x_{18}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4): &\quad -x_{12} - x_2 + 1 &&= 0 \\
(e_1, e_2, e_5): &\quad x_2 - x_3 - x_4 &&= 0 \\
(e_1, e_3, e_4): &\quad x_{12} - x_4 &&= 0 \\
(e_1, e_2, e_6): &\quad -x_{15} - x_{17} + x_3 &&= 0 \\
(e_1, e_3, e_5): &\quad -x_{13} - x_{17} + x_4 &&= 0 \\
(e_2, e_3, e_4): &\quad x_{12}x_{15} + x_{13} - x_{17} &&= 0 \\
(e_1, e_2, e_7): &\quad x_{15} - x_5 - x_7 &&= 0 \\
(e_1, e_3, e_6): &\quad x_{17} - x_{18} - x_7 &&= 0 \\
(e_1, e_4, e_5): &\quad x_{13} - x_{18} &&= 0 \\
(e_2, e_3, e_5): &\quad -x_2x_7 + x_4x_5 &&= 0 \\
(e_1, e_2, e_8): &\quad -x_1 - x_{11} + x_5 &&= 0 \\
(e_1, e_3, e_7): &\quad -x_{11} - x_{16} + x_7 &&= 0 \\
(e_1, e_4, e_6): &\quad -x_{10} - x_{16} + x_{18} &&= 0 \\
(e_2, e_3, e_6): &\quad x_1x_{17} - x_{10} - x_{11}x_3 &&= 0 \\
(e_2, e_4, e_5): &\quad x_1x_{13} + x_{10} - x_{16}x_2 &&= 0 \\
(e_1, e_2, e_{10}): &\quad -x_6 - 1 &&= 0 \\
(e_1, e_3, e_9): &\quad -x_6 - x_9 &&= 0 \\
(e_1, e_4, e_8): &\quad -x_{14} - x_9 &&= 0 \\
(e_1, e_5, e_7): &\quad -x_{14} - x_8 &&= 0 \\
(e_2, e_3, e_8): &\quad x_{11} - x_{14} - x_5x_6 &&= 0 \\
(e_2, e_4, e_7): &\quad -x_{15}x_9 + x_{16} - x_8 &&= 0 \\
(e_2, e_5, e_6): &\quad x_{10} - x_{14}x_3 + x_2x_8 &&= 0 \\
(e_3, e_4, e_6): &\quad x_{12}x_8 - x_{17}x_9 + x_{18}x_6 &&= 0
\end{aligned}$$

Groebner basis (18 variables, 5 linear, 13 nonlinear)

$$3x_1 - 30x_{18}^3 + 10x_{18}^2 - 62x_{18} + 27 = 0$$
$$-15x_{18}^3 + 5x_{18}^2 - 22x_{18} + 12x_2 + 6 = 0$$
$$-15x_{18}^3 + 5x_{18}^2 - 22x_{18} + 6x_3 + 12 = 0$$
$$15x_{18}^3 - 5x_{18}^2 + 22x_{18} + 12x_4 - 18 = 0$$
$$-15x_{18}^3 + 5x_{18}^2 - 31x_{18} + 3x_5 + 15 = 0$$
$$x_6 + 1 = 0$$
$$15x_{18}^3 - 5x_{18}^2 + 46x_{18} + 12x_7 - 18 = 0$$
$$x_8 - 1 = 0$$
$$x_9 - 1 = 0$$
$$4x_{10} + 15x_{18}^3 - 5x_{18}^2 + 22x_{18} - 10 = 0$$
$$3x_{11} + 15x_{18}^3 - 5x_{18}^2 + 31x_{18} - 12 = 0$$
$$12x_{12} + 15x_{18}^3 - 5x_{18}^2 + 22x_{18} - 18 = 0$$
$$x_{13} - x_{18} = 0$$
$$x_{14} + 1 = 0$$
$$4x_{15} - 15x_{18}^3 + 5x_{18}^2 - 26x_{18} + 14 = 0$$
$$4x_{16} - 15x_{18}^3 + 5x_{18}^2 - 26x_{18} + 10 = 0$$
$$12x_{17} + 15x_{18}^3 - 5x_{18}^2 + 34x_{18} - 18 = 0$$
$$15x_{18}^4 - 20x_{18}^3 + 36x_{18}^2 - 40x_{18} + 12 = 0$$

Solution 1:

$$x_1 = -7/3 - 2 * sqrt(2) * I/3$$
$$x_2 = 1/3 + 2 * sqrt(2) * I/3$$
$$x_3 = -1/3 + 4 * sqrt(2) * I/3$$
$$x_4 = 2/3 - 2 * sqrt(2) * I/3$$
$$x_5 = -5/3 - sqrt(2) * I/3$$
$$x_6 = -1$$
$$x_7 = 2/3 + 4 * sqrt(2) * I/3$$
$$x_8 = 1$$
$$x_9 = 1$$
$$x_10 = -2 * sqrt(2) * I$$
$$x_11 = 2/3 + sqrt(2) * I/3$$

$$x_12 = 2/3 - 2 * sqrt(2) * I/3$$

$$x_13 = -sqrt(2) * I$$

$$x_14 = -1$$

$$x_15 = -1 + sqrt(2) * I$$

$$x_16 = sqrt(2) * I$$

$$x_17 = 2/3 + sqrt(2) * I/3$$

$$x_18 = -sqrt(2) * I$$

Solution 2:

$$x_1 = -7/3 + 2 * sqrt(2) * I/3$$

$$x_2 = 1/3 - 2 * sqrt(2) * I/3$$

$$x_3 = -1/3 - 4 * sqrt(2) * I/3$$

$$x_4 = 2/3 + 2 * sqrt(2) * I/3$$

$$x_5 = -5/3 + sqrt(2) * I/3$$

$$x_6 = -1$$

$$x_7 = 2/3 - 4 * sqrt(2) * I/3$$

$$x_8 = 1$$

$$x_9 = 1$$

$$x_10 = 2 * sqrt(2) * I$$

$$x_11 = 2/3 - sqrt(2) * I/3$$

$$x_12 = 2/3 + 2 * sqrt(2) * I/3$$

$$x_13 = sqrt(2) * I$$

$$x_14 = -1$$

$$x_15 = -1 - sqrt(2) * I$$

$$x_16 = -sqrt(2) * I$$

$$x_17 = 2/3 - sqrt(2) * I/3$$

$$x_18 = sqrt(2) * I$$

Solution 3:

$$x_1 = 7 - 2 * sqrt(10)$$

$$x_2 = 1 - sqrt(10)/5$$

$$x_3 = 1 - 2 * sqrt(10)/5$$

$$x_4 = sqrt(10)/5$$

$$x_5 = 3 - sqrt(10)$$

$$x_6 = -1$$

$$x_7 = -4/3 + sqrt(10)/3$$

$$x_8 = 1$$

$$x_9 = 1$$

$$x_10 = -2 + 3 * sqrt(10)/5$$

$$x_11 = -4 + sqrt(10)$$

$$x_12 = sqrt(10)/5$$

$$x_13 = 2/3 - sqrt(10)/15$$

$$x_14 = -1$$

$$x_15 = 5/3 - 2 * sqrt(10)/3$$

$$x_16 = 8/3 - 2 * sqrt(10)/3$$

$$x_17 = -2/3 + 4 * sqrt(10)/15$$

$$x_18 = 2/3 - sqrt(10)/15$$

Solution 4:

$$x_1 = 2 * sqrt(10) + 7$$

$$x_2 = sqrt(10)/5 + 1$$

$$x_3 = 1 + 2 * sqrt(10)/5$$

$$x_4 = -sqrt(10)/5$$

$$x_5 = 3 + sqrt(10)$$

$$x_6 = -1$$

$$x_7 = -4/3 - sqrt(10)/3$$

$$x_8 = 1$$

$$x_9 = 1$$

$$x_10 = -2 - 3 * sqrt(10)/5$$

$$x_11 = -4 - sqrt(10)$$

$$x_12 = -sqrt(10)/5$$

$$x_13 = sqrt(10)/15 + 2/3$$

$$x_14 = -1$$

$$x_15 = 5/3 + 2 * sqrt(10)/3$$

$$x_16 = 2 * sqrt(10)/3 + 8/3$$

$$x_17 = -4 * sqrt(10)/15 - 2/3$$

$$x_18 = sqrt(10)/15 + 2/3$$

$\mathfrak{m}_{3B}(3,12)$

m3B312 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4 \\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6 \\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8 \\
[e_1,e_8] &= e_9 & [e_1,e_9] &= e_{10} \\
[e_1,e_{10}] &= e_{11} & [e_2,e_7] &= e_{10} \\
[e_2,e_8] &= 3e_{11} & [e_2,e_{11}] &= e_{12} \\
[e_3,e_6] &= -e_{10} & [e_3,e_7] &= -2e_{11} \\
[e_3,e_{10}] &= \alpha_{3,10}^{12} e_{12} & [e_4,e_5] &= e_{10} \\
[e_4,e_6] &= e_{11} & [e_4,e_9] &= \alpha_{4,9}^{12} e_{12} \\
[e_5,e_8] &= \alpha_{5,8}^{12} e_{12} & [e_6,e_7] &= \alpha_{6,7}^{12} e_{12}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1,e_2,e_{10}):& \quad -\alpha_{3,10}^{12}-1 &&= 0 \\
(e_1,e_3,e_9):& \quad -\alpha_{3,10}^{12}-\alpha_{4,9}^{12} &&= 0 \\
(e_1,e_4,e_8):& \quad -\alpha_{4,9}^{12}-\alpha_{5,8}^{12} &&= 0 \\
(e_1,e_5,e_7):& \quad -\alpha_{5,8}^{12}-\alpha_{6,7}^{12} &&= 0 \\
(e_2,e_3,e_7):& \quad -\alpha_{3,10}^{12}-2 &&= 0 \\
(e_2,e_4,e_6):& \quad \text{no solutions} && \\
(e_3,e_4,e_5):& \quad \alpha_{3,10}^{12} &&= 0
\end{aligned}
$$

There are no solutions.

$\mathfrak{m}_{5B}(3,12)$

m5B312 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_2, e_5] &= e_8 \\
[e_2, e_6] &= 2e_9 & [e_2, e_7] &= \alpha_{2,7}^{10} e_{10} \\
[e_2, e_8] &= \alpha_{2,8}^{11} e_{11} & [e_2, e_{11}] &= e_{12} \\
[e_3, e_4] &= -e_8 & [e_3, e_5] &= -e_9 \\
[e_3, e_6] &= \alpha_{3,6}^{10} e_{10} & [e_3, e_7] &= \alpha_{3,7}^{11} e_{11} \\
[e_3, e_{10}] &= \alpha_{3,10}^{12} e_{12} & [e_4, e_5] &= \alpha_{4,5}^{10} e_{10} \\
[e_4, e_6] &= \alpha_{4,6}^{11} e_{11} & [e_4, e_9] &= \alpha_{4,9}^{12} e_{12} \\
[e_5, e_8] &= \alpha_{5,8}^{12} e_{12} & [e_6, e_7] &= \alpha_{6,7}^{12} e_{12}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_6): &\quad -\alpha_{2,7}^{10} - \alpha_{3,6}^{10} + 2 &&= 0 \\
(e_1, e_3, e_5): &\quad -\alpha_{3,6}^{10} - \alpha_{4,5}^{10} - 1 &&= 0 \\
(e_1, e_2, e_7): &\quad \alpha_{2,7}^{10} - \alpha_{2,8}^{11} - \alpha_{3,7}^{11} &&= 0 \\
(e_1, e_3, e_6): &\quad \alpha_{3,6}^{10} - \alpha_{3,7}^{11} - \alpha_{4,6}^{11} &&= 0 \\
(e_1, e_4, e_5): &\quad \alpha_{4,5}^{10} - \alpha_{4,6}^{11} &&= 0 \\
(e_2, e_3, e_4): &\quad -\alpha_{2,8}^{11} &&= 0 \\
(e_1, e_2, e_{10}): &\quad -\alpha_{3,10}^{12} - 1 &&= 0 \\
(e_1, e_3, e_9): &\quad -\alpha_{3,10}^{12} - \alpha_{4,9}^{12} &&= 0 \\
(e_1, e_4, e_8): &\quad -\alpha_{4,9}^{12} - \alpha_{5,8}^{12} &&= 0 \\
(e_1, e_5, e_7): &\quad -\alpha_{5,8}^{12} - \alpha_{6,7}^{12} &&= 0 \\
(e_2, e_3, e_7): &\quad -\alpha_{2,7}^{10}\alpha_{3,10}^{12} + \alpha_{3,7}^{11} &&= 0 \\
(e_2, e_4, e_6): &\quad \alpha_{4,6}^{11} - 2\alpha_{4,9}^{12} &&= 0 \\
(e_3, e_4, e_5): &\quad \alpha_{3,10}^{12}\alpha_{4,5}^{10} + \alpha_{4,9}^{12} - \alpha_{5,8}^{12} &&= 0
\end{aligned}
$$

No solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{3,10}^{12} \to x_1$$

$$\alpha_{6,7}^{12} \to x_2$$

$$\alpha_{4,9}^{12} \to x_3$$

$$\alpha_{3,6}^{10} \to x_4$$

$$\alpha_{4,5}^{10} \to x_5$$

$$\alpha_{4,6}^{11} \to x_6$$

$$\alpha_{5,8}^{12} \to x_7$$

$$\alpha_{3,7}^{11} \to x_8$$

$$\alpha_{2,7}^{10} \to x_9$$

$$\alpha_{2,8}^{11} \to x_{10}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_6): &\quad -x_4 - x_9 + 2 &&= 0\\
(e_1, e_3, e_5): &\quad -x_4 - x_5 - 1 &&= 0\\
(e_1, e_2, e_7): &\quad -x_{10} - x_8 + x_9 &&= 0\\
(e_1, e_3, e_6): &\quad x_4 - x_6 - x_8 &&= 0\\
(e_1, e_4, e_5): &\quad x_5 - x_6 &&= 0\\
(e_2, e_3, e_4): &\quad -x_{10} &&= 0\\
(e_1, e_2, e_{10}): &\quad -x_1 - 1 &&= 0\\
(e_1, e_3, e_9): &\quad -x_1 - x_3 &&= 0\\
(e_1, e_4, e_8): &\quad -x_3 - x_7 &&= 0\\
(e_1, e_5, e_7): &\quad -x_2 - x_7 &&= 0\\
(e_2, e_3, e_7): &\quad -x_1x_9 + x_8 &&= 0\\
(e_2, e_4, e_6): &\quad -2x_3 + x_6 &&= 0\\
(e_3, e_4, e_5): &\quad x_1x_5 + x_3 - x_7 &&= 0
\end{aligned}$$

Groebner basis (10 variables, 1 linear, 0 nonlinear)

$$1 = 0$$

$\mathfrak{m}_{7B}(3, 12)$

m7B312 (this line included for string searching purposes)
Solution 1

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_2, e_3] &= e_6 \\
[e_2, e_4] &= e_7 & [e_2, e_5] &= e_8 \\
[e_2, e_6] &= e_9 & [e_2, e_7] &= e_{10} \\
[e_2, e_8] &= e_{11} & [e_2, e_{11}] &= e_{12} \\
[e_3, e_4] &= 0 & [e_3, e_5] &= 0 \\
[e_3, e_6] &= 0 & [e_3, e_7] &= 0 \\
[e_3, e_{10}] &= -e_{12} & [e_4, e_5] &= 0 \\
[e_4, e_6] &= 0 & [e_4, e_9] &= e_{12} \\
[e_5, e_8] &= -e_{12} & [e_6, e_7] &= e_{12}
\end{aligned}
$$

Solution 2

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_2, e_3] &= e_6 \\
[e_2, e_4] &= e_7 & [e_2, e_5] &= \frac{8e_8}{5} \\
[e_2, e_6] &= \frac{11e_9}{5} & [e_2, e_7] &= 4e_{10} \\
[e_2, e_8] &= 7e_{11} & [e_2, e_{11}] &= e_{12} \\
[e_3, e_4] &= -\frac{3e_8}{5} & [e_3, e_5] &= -\frac{3e_9}{5} \\
[e_3, e_6] &= -\frac{9e_{10}}{5} & [e_3, e_7] &= -3e_{11} \\
[e_3, e_{10}] &= -e_{12} & [e_4, e_5] &= \frac{6e_{10}}{5} \\
[e_4, e_6] &= \frac{6e_{11}}{5} & [e_4, e_9] &= e_{12} \\
[e_5, e_8] &= -e_{12} & [e_6, e_7] &= e_{12}
\end{aligned}
$$

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_2, e_3] &= e_6 \\
[e_2, e_4] &= e_7 & [e_2, e_5] &= \alpha_{2,5}^{8} e_8 \\
[e_2, e_6] &= \alpha_{2,6}^{9} e_9 & [e_2, e_7] &= \alpha_{2,7}^{10} e_{10} \\
[e_2, e_8] &= \alpha_{2,8}^{11} e_{11} & [e_2, e_{11}] &= e_{12} \\
[e_3, e_4] &= \alpha_{3,4}^{8} e_8 & [e_3, e_5] &= \alpha_{3,5}^{9} e_9 \\
[e_3, e_6] &= \alpha_{3,6}^{10} e_{10} & [e_3, e_7] &= \alpha_{3,7}^{11} e_{11} \\
[e_3, e_{10}] &= \alpha_{3,10}^{12} e_{12} & [e_4, e_5] &= \alpha_{4,5}^{10} e_{10} \\
[e_4, e_6] &= \alpha_{4,6}^{11} e_{11} & [e_4, e_9] &= \alpha_{4,9}^{12} e_{12} \\
[e_5, e_8] &= \alpha_{5,8}^{12} e_{12} & [e_6, e_7] &= \alpha_{6,7}^{12} e_{12}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_4):&\quad -\alpha_{2,5}^{8} - \alpha_{3,4}^{8} + 1 &&= 0\\
(e_1, e_2, e_5):&\quad \alpha_{2,5}^{8} - \alpha_{2,6}^{9} - \alpha_{3,5}^{9} &&= 0\\
(e_1, e_3, e_4):&\quad \alpha_{3,4}^{8} - \alpha_{3,5}^{9} &&= 0\\
(e_1, e_2, e_6):&\quad \alpha_{2,6}^{9} - \alpha_{2,7}^{10} - \alpha_{3,6}^{10} &&= 0\\
(e_1, e_3, e_5):&\quad \alpha_{3,5}^{9} - \alpha_{3,6}^{10} - \alpha_{4,5}^{10} &&= 0\\
(e_1, e_2, e_7):&\quad \alpha_{2,7}^{10} - \alpha_{2,8}^{11} - \alpha_{3,7}^{11} &&= 0\\
(e_1, e_3, e_6):&\quad \alpha_{3,6}^{10} - \alpha_{3,7}^{11} - \alpha_{4,6}^{11} &&= 0\\
(e_1, e_4, e_5):&\quad \alpha_{4,5}^{10} - \alpha_{4,6}^{11} &&= 0\\
(e_2, e_3, e_4):&\quad \alpha_{2,8}^{11}\alpha_{3,4}^{8} - \alpha_{3,7}^{11} + \alpha_{4,6}^{11} &&= 0\\
(e_1, e_2, e_{10}):&\quad -\alpha_{3,10}^{12} - 1 &&= 0\\
(e_1, e_3, e_9):&\quad -\alpha_{3,10}^{12} - \alpha_{4,9}^{12} &&= 0\\
(e_1, e_4, e_8):&\quad -\alpha_{4,9}^{12} - \alpha_{5,8}^{12} &&= 0\\
(e_1, e_5, e_7):&\quad -\alpha_{5,8}^{12} - \alpha_{6,7}^{12} &&= 0\\
(e_2, e_3, e_7):&\quad -\alpha_{2,7}^{10}\alpha_{3,10}^{12} + \alpha_{3,7}^{11} - \alpha_{6,7}^{12} &&= 0\\
(e_2, e_4, e_6):&\quad -\alpha_{2,6}^{9}\alpha_{4,9}^{12} + \alpha_{4,6}^{11} + \alpha_{6,7}^{12} &&= 0\\
(e_3, e_4, e_5):&\quad \alpha_{3,10}^{12}\alpha_{4,5}^{10} + \alpha_{3,4}^{8}\alpha_{5,8}^{12} - \alpha_{3,5}^{9}\alpha_{4,9}^{12} &&= 0
\end{aligned}
$$

Solution 1:

$$\alpha^{12}_{3,10} = -1$$
$$\alpha^{8}_{2,5} = 1$$
$$\alpha^{12}_{6,7} = 1$$
$$\alpha^{12}_{4,9} = 1$$
$$\alpha^{10}_{3,6} = 0$$
$$\alpha^{9}_{3,5} = 0$$
$$\alpha^{8}_{3,4} = 0$$
$$\alpha^{9}_{2,6} = 1$$
$$\alpha^{10}_{4,5} = 0$$
$$\alpha^{11}_{4,6} = 0$$
$$\alpha^{12}_{5,8} = -1$$
$$\alpha^{11}_{3,7} = 0$$
$$\alpha^{10}_{2,7} = 1$$
$$\alpha^{11}_{2,8} = 1$$

Solution 2:

$$\alpha^{12}_{3,10} = -1$$
$$\alpha^{8}_{2,5} = 8/5$$
$$\alpha^{12}_{6,7} = 1$$
$$\alpha^{12}_{4,9} = 1$$
$$\alpha^{10}_{3,6} = -9/5$$
$$\alpha^{9}_{3,5} = -3/5$$
$$\alpha^{8}_{3,4} = -3/5$$
$$\alpha^{9}_{2,6} = 11/5$$
$$\alpha^{10}_{4,5} = 6/5$$
$$\alpha^{11}_{4,6} = 6/5$$
$$\alpha^{12}_{5,8} = -1$$
$$\alpha^{11}_{3,7} = -3$$
$$\alpha^{10}_{2,7} = 4$$
$$\alpha^{11}_{2,8} = 7$$

How the solution(s) were or were not found:
Change variables

$$\alpha^{12}_{3,10} \to x_1$$
$$\alpha^{8}_{2,5} \to x_2$$
$$\alpha^{12}_{6,7} \to x_3$$
$$\alpha^{12}_{4,9} \to x_4$$
$$\alpha^{10}_{3,6} \to x_5$$
$$\alpha^{9}_{3,5} \to x_6$$
$$\alpha^{8}_{3,4} \to x_7$$
$$\alpha^{9}_{2,6} \to x_8$$
$$\alpha^{10}_{4,5} \to x_9$$
$$\alpha^{11}_{4,6} \to x_{10}$$
$$\alpha^{12}_{5,8} \to x_{11}$$
$$\alpha^{11}_{3,7} \to x_{12}$$
$$\alpha^{10}_{2,7} \to x_{13}$$
$$\alpha^{11}_{2,8} \to x_{14}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4): &\quad -x_2 - x_7 + 1 &&= 0\\
(e_1, e_2, e_5): &\quad x_2 - x_6 - x_8 &&= 0\\
(e_1, e_3, e_4): &\quad -x_6 + x_7 &&= 0\\
(e_1, e_2, e_6): &\quad -x_{13} - x_5 + x_8 &&= 0\\
(e_1, e_3, e_5): &\quad -x_5 + x_6 - x_9 &&= 0\\
(e_1, e_2, e_7): &\quad -x_{12} + x_{13} - x_{14} &&= 0\\
(e_1, e_3, e_6): &\quad -x_{10} - x_{12} + x_5 &&= 0\\
(e_1, e_4, e_5): &\quad -x_{10} + x_9 &&= 0\\
(e_2, e_3, e_4): &\quad x_{10} - x_{12} + x_{14}x_7 &&= 0\\
(e_1, e_2, e_{10}): &\quad -x_1 - 1 &&= 0\\
(e_1, e_3, e_9): &\quad -x_1 - x_4 &&= 0\\
(e_1, e_4, e_8): &\quad -x_{11} - x_4 &&= 0\\
(e_1, e_5, e_7): &\quad -x_{11} - x_3 &&= 0\\
(e_2, e_3, e_7): &\quad -x_1x_{13} + x_{12} - x_3 &&= 0\\
(e_2, e_4, e_6): &\quad x_{10} + x_3 - x_4x_8 &&= 0\\
(e_3, e_4, e_5): &\quad x_1x_9 + x_{11}x_7 - x_4x_6 &&= 0
\end{aligned}$$

Groebner basis (14 variables, 13 linear, 1 nonlinear)

$$x_1 + 1 = 0$$
$$-x_{14} + 10x_2 - 9 = 0$$
$$x_3 - 1 = 0$$
$$x_4 - 1 = 0$$
$$3x_{14} + 10x_5 - 3 = 0$$
$$x_{14} + 10x_6 - 1 = 0$$
$$x_{14} + 10x_7 - 1 = 0$$
$$-x_{14} + 5x_8 - 4 = 0$$
$$-x_{14} + 5x_9 + 1 = 0$$
$$5x_{10} - x_{14} + 1 = 0$$
$$x_{11} + 1 = 0$$
$$2x_{12} + x_{14} - 1 = 0$$
$$2x_{13} - x_{14} - 1 = 0$$
$$x_{14}^2 - 8x_{14} + 7 = 0$$

Solution 1:

$$x_1 = -1$$
$$x_2 = 1$$
$$x_3 = 1$$
$$x_4 = 1$$
$$x_5 = 0$$
$$x_6 = 0$$
$$x_7 = 0$$
$$x_8 = 1$$
$$x_9 = 0$$
$$x_1 0 = 0$$
$$x_1 1 = -1$$
$$x_1 2 = 0$$
$$x_1 3 = 1$$
$$x_1 4 = 1$$

Solution 2:

$$x_1 = -1$$

$$x_2 = 8/5$$
$$x_3 = 1$$
$$x_4 = 1$$
$$x_5 = -9/5$$
$$x_6 = -3/5$$
$$x_7 = -3/5$$
$$x_8 = 11/5$$
$$x_9 = 6/5$$
$$x_10 = 6/5$$
$$x_11 = -1$$
$$x_12 = -3$$
$$x_13 = 4$$
$$x_14 = 7$$

$\mathfrak{m}_{2B}(4, 12)$

m2B412 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_2, e_7] &= e_{11} \\
[e_2, e_{11}] &= e_{12} & [e_3, e_6] &= -e_{11} \\
[e_3, e_{10}] &= \alpha_{3,10}^{12} e_{12} & [e_4, e_5] &= e_{11} \\
[e_4, e_9] &= \alpha_{4,9}^{12} e_{12} & [e_5, e_8] &= \alpha_{5,8}^{12} e_{12} \\
[e_6, e_7] &= \alpha_{6,7}^{12} e_{12}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_{10}):\quad & -\alpha_{3,10}^{12} - 1 && = 0\\
(e_1, e_3, e_9):\quad & -\alpha_{3,10}^{12} - \alpha_{4,9}^{12} && = 0\\
(e_1, e_4, e_8):\quad & -\alpha_{4,9}^{12} - \alpha_{5,8}^{12} && = 0\\
(e_1, e_5, e_7):\quad & -\alpha_{5,8}^{12} - \alpha_{6,7}^{12} && = 0\\
(e_2, e_3, e_6):\quad & \text{no solutions}\\
(e_2, e_4, e_5):\quad & \text{no solutions}
\end{aligned}
$$

There are no solutions.

$\mathfrak{m}_{4B}(4, 12)$

m4B412 (this line included for string searching purposes)
Solution 1

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4\\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6\\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8\\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10}\\
[e_1, e_{10}] &= e_{11} & [e_2, e_5] &= e_9\\
[e_2, e_6] &= 2e_{10} & [e_2, e_7] &= 4e_{11}\\
[e_2, e_{11}] &= e_{12} & [e_3, e_4] &= -e_9\\
[e_3, e_5] &= -e_{10} & [e_3, e_6] &= -2e_{11}\\
[e_3, e_{10}] &= -e_{12} & [e_4, e_5] &= e_{11}\\
[e_4, e_9] &= e_{12} & [e_5, e_8] &= -e_{12}\\
[e_6, e_7] &= e_{12}
\end{aligned}
$$

Original brackets:

$$
\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_1, e_9] = e_{10} \\
[e_1, e_{10}] = e_{11} & [e_2, e_5] = e_9 \\
[e_2, e_6] = 2e_{10} & [e_2, e_7] = \alpha_{2,7}^{11} e_{11} \\
[e_2, e_{11}] = e_{12} & [e_3, e_4] = -e_9 \\
[e_3, e_5] = -e_{10} & [e_3, e_6] = \alpha_{3,6}^{11} e_{11} \\
[e_3, e_{10}] = \alpha_{3,10}^{12} e_{12} & [e_4, e_5] = \alpha_{4,5}^{11} e_{11} \\
[e_4, e_9] = \alpha_{4,9}^{12} e_{12} & [e_5, e_8] = \alpha_{5,8}^{12} e_{12} \\
[e_6, e_7] = \alpha_{6,7}^{12} e_{12} &
\end{array}
$$

Non-trivial Jacobi Tests:

$$
\begin{array}{rll}
(e_1, e_2, e_6): & -\alpha_{2,7}^{11} - \alpha_{3,6}^{11} + 2 & = 0 \\
(e_1, e_3, e_5): & -\alpha_{3,6}^{11} - \alpha_{4,5}^{11} - 1 & = 0 \\
(e_1, e_2, e_{10}): & -\alpha_{3,10}^{12} - 1 & = 0 \\
(e_1, e_3, e_9): & -\alpha_{3,10}^{12} - \alpha_{4,9}^{12} & = 0 \\
(e_1, e_4, e_8): & -\alpha_{4,9}^{12} - \alpha_{5,8}^{12} & = 0 \\
(e_1, e_5, e_7): & -\alpha_{5,8}^{12} - \alpha_{6,7}^{12} & = 0 \\
(e_2, e_3, e_6): & -2\alpha_{3,10}^{12} + \alpha_{3,6}^{11} & = 0 \\
(e_2, e_4, e_5): & \alpha_{4,5}^{11} - \alpha_{4,9}^{12} & = 0
\end{array}
$$

Solution 1:

$$
\begin{array}{r}
\alpha_{3,10}^{12} = -1 \\
\alpha_{6,7}^{12} = 1 \\
\alpha_{2,7}^{11} = 4 \\
\alpha_{4,5}^{11} = 1 \\
\alpha_{4,9}^{12} = 1 \\
\alpha_{3,6}^{11} = -2 \\
\alpha_{5,8}^{12} = -1
\end{array}
$$

How the solution(s) were or were not found:
Change variables

$$\alpha^{12}_{3,10} \to x_1$$

$$\alpha^{12}_{6,7} \to x_2$$

$$\alpha^{11}_{2,7} \to x_3$$

$$\alpha^{11}_{4,5} \to x_4$$

$$\alpha^{12}_{4,9} \to x_5$$

$$\alpha^{11}_{3,6} \to x_6$$

$$\alpha^{12}_{5,8} \to x_7$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_6): &\quad -x_3 - x_6 + 2 &&= 0\\
(e_1, e_3, e_5): &\quad -x_4 - x_6 - 1 &&= 0\\
(e_1, e_2, e_{10}): &\quad -x_1 - 1 &&= 0\\
(e_1, e_3, e_9): &\quad -x_1 - x_5 &&= 0\\
(e_1, e_4, e_8): &\quad -x_5 - x_7 &&= 0\\
(e_1, e_5, e_7): &\quad -x_2 - x_7 &&= 0\\
(e_2, e_3, e_6): &\quad -2x_1 + x_6 &&= 0\\
(e_2, e_4, e_5): &\quad x_4 - x_5 &&= 0
\end{aligned}$$

Groebner basis (7 variables, 7 linear, 0 nonlinear)

$$x_1 + 1 = 0$$

$$x_2 - 1 = 0$$

$$x_3 - 4 = 0$$

$$x_4 - 1 = 0$$

$$x_5 - 1 = 0$$

$$x_6 + 2 = 0$$

$$x_7 + 1 = 0$$

Solution 1:

$$x_1 = -1$$

$$x_2 = 1$$

$$x_3 = 4$$

$$x_4 = 1$$
$$x_5 = 1$$
$$x_6 = -2$$
$$x_7 = -1$$

$\mathfrak{m}_{6B}(4,12)$

m6B412 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4 \\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6 \\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8 \\
[e_1,e_8] &= e_9 & [e_1,e_9] &= e_{10} \\
[e_1,e_{10}] &= e_{11} & [e_2,e_3] &= e_7 \\
[e_2,e_4] &= e_8 & [e_2,e_5] &= \alpha_{2,5}^{9} e_9 \\
[e_2,e_6] &= \alpha_{2,6}^{10} e_{10} & [e_2,e_7] &= \alpha_{2,7}^{11} e_{11} \\
[e_2,e_{11}] &= e_{12} & [e_3,e_4] &= \alpha_{3,4}^{9} e_9 \\
[e_3,e_5] &= \alpha_{3,5}^{10} e_{10} & [e_3,e_6] &= \alpha_{3,6}^{11} e_{11} \\
[e_3,e_{10}] &= \alpha_{3,10}^{12} e_{12} & [e_4,e_5] &= \alpha_{4,5}^{11} e_{11} \\
[e_4,e_9] &= \alpha_{4,9}^{12} e_{12} & [e_5,e_8] &= \alpha_{5,8}^{12} e_{12} \\
[e_6,e_7] &= \alpha_{6,7}^{12} e_{12}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1,e_2,e_4):&\quad -\alpha_{2,5}^{9}-\alpha_{3,4}^{9}+1 &&=0\\
(e_1,e_2,e_5):&\quad \alpha_{2,5}^{9}-\alpha_{2,6}^{10}-\alpha_{3,5}^{10} &&=0\\
(e_1,e_3,e_4):&\quad \alpha_{3,4}^{9}-\alpha_{3,5}^{10} &&=0\\
(e_1,e_2,e_6):&\quad \alpha_{2,6}^{10}-\alpha_{2,7}^{11}-\alpha_{3,6}^{11} &&=0\\
(e_1,e_3,e_5):&\quad \alpha_{3,5}^{10}-\alpha_{3,6}^{11}-\alpha_{4,5}^{11} &&=0\\
(e_1,e_2,e_{10}):&\quad -\alpha_{3,10}^{12}-1 &&=0\\
(e_1,e_3,e_9):&\quad -\alpha_{3,10}^{12}-\alpha_{4,9}^{12} &&=0\\
(e_1,e_4,e_8):&\quad -\alpha_{4,9}^{12}-\alpha_{5,8}^{12} &&=0\\
(e_1,e_5,e_7):&\quad -\alpha_{5,8}^{12}-\alpha_{6,7}^{12} &&=0\\
(e_2,e_3,e_6):&\quad -\alpha_{2,6}^{10}\alpha_{3,10}^{12}+\alpha_{3,6}^{11}+\alpha_{6,7}^{12} &&=0\\
(e_2,e_4,e_5):&\quad -\alpha_{2,5}^{9}\alpha_{4,9}^{12}+\alpha_{4,5}^{11}+\alpha_{5,8}^{12} &&=0
\end{aligned}
$$

Infinite number of solutions: only zero-dimensional systems supported (finite number of solutions).
How the solution(s) were or were not found:
Change variables

$$
\begin{gathered}
\alpha_{3,10}^{12}\to x_1\\
\alpha_{2,5}^{9}\to x_2\\
\alpha_{6,7}^{12}\to x_3\\
\alpha_{2,7}^{11}\to x_4\\
\alpha_{4,5}^{11}\to x_5\\
\alpha_{4,9}^{12}\to x_6\\
\alpha_{3,6}^{11}\to x_7\\
\alpha_{5,8}^{12}\to x_8\\
\alpha_{2,6}^{10}\to x_9\\
\alpha_{3,5}^{10}\to x_{10}\\
\alpha_{3,4}^{9}\to x_{11}
\end{gathered}
$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4):&\quad -x_{11} - x_2 + 1 && = 0\\
(e_1, e_2, e_5):&\quad -x_{10} + x_2 - x_9 && = 0\\
(e_1, e_3, e_4):&\quad -x_{10} + x_{11} && = 0\\
(e_1, e_2, e_6):&\quad -x_4 - x_7 + x_9 && = 0\\
(e_1, e_3, e_5):&\quad x_{10} - x_5 - x_7 && = 0\\
(e_1, e_2, e_{10}):&\quad -x_1 - 1 && = 0\\
(e_1, e_3, e_9):&\quad -x_1 - x_6 && = 0\\
(e_1, e_4, e_8):&\quad -x_6 - x_8 && = 0\\
(e_1, e_5, e_7):&\quad -x_3 - x_8 && = 0\\
(e_2, e_3, e_6):&\quad -x_1x_9 + x_3 + x_7 && = 0\\
(e_2, e_4, e_5):&\quad -x_2x_6 + x_5 + x_8 && = 0
\end{aligned}$$

Groebner basis (11 variables, 10 linear, 0 nonlinear)

$$x_1 + 1 = 0$$

$$x_{11} + x_2 - 1 = 0$$

$$x_3 - 1 = 0$$

$$4x_{11} + x_4 - 3 = 0$$

$$x_{11} + x_5 - 2 = 0$$

$$x_6 - 1 = 0$$

$$-2x_{11} + x_7 + 2 = 0$$

$$x_8 + 1 = 0$$

$$2x_{11} + x_9 - 1 = 0$$

$$x_{10} - x_{11} = 0$$

$\mathfrak{m}_{3B}(5,12)$

m3B512 (this line included for string searching purposes)
Solution 1

$$\begin{array}{rlrl}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4 \\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6 \\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8 \\
[e_1,e_8] &= e_9 & [e_1,e_9] &= e_{10} \\
[e_1,e_{10}] &= e_{11} & [e_2,e_5] &= e_{10} \\
[e_2,e_6] &= 2e_{11} & [e_2,e_{11}] &= e_{12} \\
[e_3,e_4] &= -e_{10} & [e_3,e_5] &= -e_{11} \\
[e_3,e_{10}] &= -e_{12} & [e_4,e_9] &= e_{12} \\
[e_5,e_8] &= -e_{12} & [e_6,e_7] &= e_{12}
\end{array}$$

Original brackets:

$$\begin{array}{rlrl}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4 \\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6 \\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8 \\
[e_1,e_8] &= e_9 & [e_1,e_9] &= e_{10} \\
[e_1,e_{10}] &= e_{11} & [e_2,e_5] &= e_{10} \\
[e_2,e_6] &= 2e_{11} & [e_2,e_{11}] &= e_{12} \\
[e_3,e_4] &= -e_{10} & [e_3,e_5] &= -e_{11} \\
[e_3,e_{10}] &= \alpha_{3,10}^{12}e_{12} & [e_4,e_9] &= \alpha_{4,9}^{12}e_{12} \\
[e_5,e_8] &= \alpha_{5,8}^{12}e_{12} & [e_6,e_7] &= \alpha_{6,7}^{12}e_{12}
\end{array}$$

Non-trivial Jacobi Tests:

$$\begin{array}{rll}
(e_1,e_2,e_{10}): & -\alpha_{3,10}^{12}-1 & =0 \\
(e_1,e_3,e_9): & -\alpha_{3,10}^{12}-\alpha_{4,9}^{12} & =0 \\
(e_1,e_4,e_8): & -\alpha_{4,9}^{12}-\alpha_{5,8}^{12} & =0 \\
(e_1,e_5,e_7): & -\alpha_{5,8}^{12}-\alpha_{6,7}^{12} & =0 \\
(e_2,e_3,e_5): & -\alpha_{3,10}^{12}-1 & =0
\end{array}$$

Solution 1:

$$\alpha^{12}_{3,10} = -1$$
$$\alpha^{12}_{6,7} = 1$$
$$\alpha^{12}_{4,9} = 1$$
$$\alpha^{12}_{5,8} = -1$$

How the solution(s) were or were not found:
Change variables

$$\alpha^{12}_{3,10} \to x_1$$
$$\alpha^{12}_{6,7} \to x_2$$
$$\alpha^{12}_{4,9} \to x_3$$
$$\alpha^{12}_{5,8} \to x_4$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_{10}): &\quad -x_1 - 1 &&= 0\\
(e_1, e_3, e_9): &\quad -x_1 - x_3 &&= 0\\
(e_1, e_4, e_8): &\quad -x_3 - x_4 &&= 0\\
(e_1, e_5, e_7): &\quad -x_2 - x_4 &&= 0\\
(e_2, e_3, e_5): &\quad -x_1 - 1 &&= 0
\end{aligned}$$

Groebner basis (4 variables, 4 linear, 0 nonlinear)

$$x_1 + 1 = 0$$
$$x_2 - 1 = 0$$
$$x_3 - 1 = 0$$
$$x_4 + 1 = 0$$

Solution 1:

$$x_1 = -1$$
$$x_2 = 1$$
$$x_3 = 1$$
$$x_4 = -1$$

$\mathfrak{m}_{5B}(5, 12)$

m5B512 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_2, e_3] &= e_8 \\
[e_2, e_4] &= e_9 & [e_2, e_5] &= \alpha_{2,5}^{10} e_{10} \\
[e_2, e_6] &= \alpha_{2,6}^{11} e_{11} & [e_2, e_{11}] &= e_{12} \\
[e_3, e_4] &= \alpha_{3,4}^{10} e_{10} & [e_3, e_5] &= \alpha_{3,5}^{11} e_{11} \\
[e_3, e_{10}] &= \alpha_{3,10}^{12} e_{12} & [e_4, e_9] &= \alpha_{4,9}^{12} e_{12} \\
[e_5, e_8] &= \alpha_{5,8}^{12} e_{12} & [e_6, e_7] &= \alpha_{6,7}^{12} e_{12}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_4): &\quad -\alpha_{2,5}^{10} - \alpha_{3,4}^{10} + 1 &= 0 \\
(e_1, e_2, e_5): &\quad \alpha_{2,5}^{10} - \alpha_{2,6}^{11} - \alpha_{3,5}^{11} &= 0 \\
(e_1, e_3, e_4): &\quad \alpha_{3,4}^{10} - \alpha_{3,5}^{11} &= 0 \\
(e_1, e_2, e_{10}): &\quad -\alpha_{3,10}^{12} - 1 &= 0 \\
(e_1, e_3, e_9): &\quad -\alpha_{3,10}^{12} - \alpha_{4,9}^{12} &= 0 \\
(e_1, e_4, e_8): &\quad -\alpha_{4,9}^{12} - \alpha_{5,8}^{12} &= 0 \\
(e_1, e_5, e_7): &\quad -\alpha_{5,8}^{12} - \alpha_{6,7}^{12} &= 0 \\
(e_2, e_3, e_5): &\quad -\alpha_{2,5}^{10}\alpha_{3,10}^{12} + \alpha_{3,5}^{11} + \alpha_{5,8}^{12} &= 0
\end{aligned}
$$

Infinite number of solutions: only zero-dimensional systems supported (finite number of solutions).
How the solution(s) were or were not found:
Change variables

$$
\begin{aligned}
\alpha_{3,10}^{12} &\to x_1 \\
\alpha_{2,6}^{11} &\to x_2 \\
\alpha_{3,5}^{11} &\to x_3
\end{aligned}
$$

$$\alpha_{6,7}^{12} \to x_4$$

$$\alpha_{2,5}^{10} \to x_5$$

$$\alpha_{4,9}^{12} \to x_6$$

$$\alpha_{3,4}^{10} \to x_7$$

$$\alpha_{5,8}^{12} \to x_8$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4): &\quad -x_5 - x_7 + 1 &&= 0\\
(e_1, e_2, e_5): &\quad -x_2 - x_3 + x_5 &&= 0\\
(e_1, e_3, e_4): &\quad -x_3 + x_7 &&= 0\\
(e_1, e_2, e_{10}): &\quad -x_1 - 1 &&= 0\\
(e_1, e_3, e_9): &\quad -x_1 - x_6 &&= 0\\
(e_1, e_4, e_8): &\quad -x_6 - x_8 &&= 0\\
(e_1, e_5, e_7): &\quad -x_4 - x_8 &&= 0\\
(e_2, e_3, e_5): &\quad -x_1 x_5 + x_3 + x_8 &&= 0
\end{aligned}$$

Groebner basis (8 variables, 7 linear, 0 nonlinear)

$$x_1 + 1 = 0$$

$$x_2 + 2x_7 - 1 = 0$$

$$x_3 - x_7 = 0$$

$$x_4 - 1 = 0$$

$$x_5 + x_7 - 1 = 0$$

$$x_6 - 1 = 0$$

$$x_8 + 1 = 0$$

$\mathfrak{m}_{2B}(6, 12)$

m2B612 (this line included for string searching purposes)

Original brackets:

$$
\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_1, e_9] = e_{10} \\
[e_1, e_{10}] = e_{11} & [e_2, e_5] = e_{11} \\
[e_2, e_{11}] = e_{12} & [e_3, e_4] = -e_{11} \\
[e_3, e_{10}] = \alpha_{3,10}^{12} e_{12} & [e_4, e_9] = \alpha_{4,9}^{12} e_{12} \\
[e_5, e_8] = \alpha_{5,8}^{12} e_{12} & [e_6, e_7] = \alpha_{6,7}^{12} e_{12}
\end{array}
$$

Non-trivial Jacobi Tests:

$$
\begin{array}{rll}
(e_1, e_2, e_{10}): & -\alpha_{3,10}^{12} - 1 & = 0 \\
(e_1, e_3, e_9): & -\alpha_{3,10}^{12} - \alpha_{4,9}^{12} & = 0 \\
(e_1, e_4, e_8): & -\alpha_{4,9}^{12} - \alpha_{5,8}^{12} & = 0 \\
(e_1, e_5, e_7): & -\alpha_{5,8}^{12} - \alpha_{6,7}^{12} & = 0 \\
(e_2, e_3, e_4): & \text{no solutions} &
\end{array}
$$

There are no solutions.

$\mathfrak{m}_{4B}(6, 12)$

m4B612 (this line included for string searching purposes)
Solution 1

$$
\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_1, e_9] = e_{10} \\
[e_1, e_{10}] = e_{11} & [e_2, e_3] = e_9 \\
[e_2, e_4] = e_{10} & [e_2, e_5] = 3e_{11} \\
[e_2, e_{11}] = e_{12} & [e_3, e_4] = -2e_{11} \\
[e_3, e_{10}] = -e_{12} & [e_4, e_9] = e_{12} \\
[e_5, e_8] = -e_{12} & [e_6, e_7] = e_{12}
\end{array}
$$

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_2, e_3] &= e_9 \\
[e_2, e_4] &= e_{10} & [e_2, e_5] &= \alpha_{2,5}^{11} e_{11} \\
[e_2, e_{11}] &= e_{12} & [e_3, e_4] &= \alpha_{3,4}^{11} e_{11} \\
[e_3, e_{10}] &= \alpha_{3,10}^{12} e_{12} & [e_4, e_9] &= \alpha_{4,9}^{12} e_{12} \\
[e_5, e_8] &= \alpha_{5,8}^{12} e_{12} & [e_6, e_7] &= \alpha_{6,7}^{12} e_{12}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_4): &\quad -\alpha_{2,5}^{11} - \alpha_{3,4}^{11} + 1 &= 0 \\
(e_1, e_2, e_{10}): &\quad -\alpha_{3,10}^{12} - 1 &= 0 \\
(e_1, e_3, e_9): &\quad -\alpha_{3,10}^{12} - \alpha_{4,9}^{12} &= 0 \\
(e_1, e_4, e_8): &\quad -\alpha_{4,9}^{12} - \alpha_{5,8}^{12} &= 0 \\
(e_1, e_5, e_7): &\quad -\alpha_{5,8}^{12} - \alpha_{6,7}^{12} &= 0 \\
(e_2, e_3, e_4): &\quad -\alpha_{3,10}^{12} + \alpha_{3,4}^{11} + \alpha_{4,9}^{12} &= 0
\end{aligned}
$$

Solution 1:

$$
\begin{aligned}
\alpha_{3,10}^{12} &= -1 \\
\alpha_{6,7}^{12} &= 1 \\
\alpha_{2,5}^{11} &= 3 \\
\alpha_{4,9}^{12} &= 1 \\
\alpha_{5,8}^{12} &= -1 \\
\alpha_{3,4}^{11} &= -2
\end{aligned}
$$

How the solution(s) were or were not found:
Change variables

$$\alpha_{3,10}^{12} \to x_1$$

$$\alpha_{6,7}^{12} \to x_2$$
$$\alpha_{2,5}^{11} \to x_3$$
$$\alpha_{4,9}^{12} \to x_4$$
$$\alpha_{5,8}^{12} \to x_5$$
$$\alpha_{3,4}^{11} \to x_6$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4): \quad & -x_3 - x_6 + 1 && = 0 \\
(e_1, e_2, e_{10}): \quad & -x_1 - 1 && = 0 \\
(e_1, e_3, e_9): \quad & -x_1 - x_4 && = 0 \\
(e_1, e_4, e_8): \quad & -x_4 - x_5 && = 0 \\
(e_1, e_5, e_7): \quad & -x_2 - x_5 && = 0 \\
(e_2, e_3, e_4): \quad & -x_1 + x_4 + x_6 && = 0
\end{aligned}$$

Groebner basis (6 variables, 6 linear, 0 nonlinear)

$$x_1 + 1 = 0$$
$$x_2 - 1 = 0$$
$$x_3 - 3 = 0$$
$$x_4 - 1 = 0$$
$$x_5 + 1 = 0$$
$$x_6 + 2 = 0$$

Solution 1:

$$x_1 = -1$$
$$x_2 = 1$$
$$x_3 = 3$$
$$x_4 = 1$$
$$x_5 = -1$$
$$x_6 = -2$$

$\mathfrak{m}_{3B}(7,12)$

m3B712 (this line included for string searching purposes)
Solution 1

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4\\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6\\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8\\
[e_1,e_8] &= e_9 & [e_1,e_9] &= e_{10}\\
[e_1,e_{10}] &= e_{11} & [e_2,e_3] &= e_{10}\\
[e_2,e_4] &= e_{11} & [e_2,e_{11}] &= e_{12}\\
[e_3,e_{10}] &= -e_{12} & [e_4,e_9] &= e_{12}\\
[e_5,e_8] &= -e_{12} & [e_6,e_7] &= e_{12}
\end{aligned}
$$

Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4\\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6\\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8\\
[e_1,e_8] &= e_9 & [e_1,e_9] &= e_{10}\\
[e_1,e_{10}] &= e_{11} & [e_2,e_3] &= e_{10}\\
[e_2,e_4] &= e_{11} & [e_2,e_{11}] &= e_{12}\\
[e_3,e_{10}] &= \alpha_{3,10}^{12}e_{12} & [e_4,e_9] &= \alpha_{4,9}^{12}e_{12}\\
[e_5,e_8] &= \alpha_{5,8}^{12}e_{12} & [e_6,e_7] &= \alpha_{6,7}^{12}e_{12}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1,e_2,e_{10}): &\quad -\alpha_{3,10}^{12}-1 &= 0\\
(e_1,e_3,e_9): &\quad -\alpha_{3,10}^{12}-\alpha_{4,9}^{12} &= 0\\
(e_1,e_4,e_8): &\quad -\alpha_{4,9}^{12}-\alpha_{5,8}^{12} &= 0\\
(e_1,e_5,e_7): &\quad -\alpha_{5,8}^{12}-\alpha_{6,7}^{12} &= 0
\end{aligned}
$$

Solution 1:

$$\alpha^{12}_{3,10} = -1$$
$$\alpha^{12}_{6,7} = 1$$
$$\alpha^{12}_{4,9} = 1$$
$$\alpha^{12}_{5,8} = -1$$

How the solution(s) were or were not found:
Change variables

$$\alpha^{12}_{3,10} \to x_1$$
$$\alpha^{12}_{6,7} \to x_2$$
$$\alpha^{12}_{4,9} \to x_3$$
$$\alpha^{12}_{5,8} \to x_4$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_{10}): &\quad -x_1 - 1 && = 0\\
(e_1, e_3, e_9): &\quad -x_1 - x_3 && = 0\\
(e_1, e_4, e_8): &\quad -x_3 - x_4 && = 0\\
(e_1, e_5, e_7): &\quad -x_2 - x_4 && = 0
\end{aligned}$$

Groebner basis (4 variables, 4 linear, 0 nonlinear)

$$x_1 + 1 = 0$$
$$x_2 - 1 = 0$$
$$x_3 - 1 = 0$$
$$x_4 + 1 = 0$$

Solution 1:

$$x_1 = -1$$
$$x_2 = 1$$
$$x_3 = 1$$
$$x_4 = -1$$

$\mathfrak{m}_{2B}(8,12)$

m2B812 (this line included for string searching purposes)
Solution 1

$$
\begin{aligned}
[e_1,e_2]&=e_3 & [e_1,e_3]&=e_4\\
[e_1,e_4]&=e_5 & [e_1,e_5]&=e_6\\
[e_1,e_6]&=e_7 & [e_1,e_7]&=e_8\\
[e_1,e_8]&=e_9 & [e_1,e_9]&=e_{10}\\
[e_1,e_{10}]&=e_{11} & [e_2,e_3]&=e_{11}\\
[e_2,e_{11}]&=e_{12} & [e_3,e_{10}]&=-e_{12}\\
[e_4,e_9]&=e_{12} & [e_5,e_8]&=-e_{12}\\
[e_6,e_7]&=e_{12}
\end{aligned}
$$

Original brackets:

$$
\begin{aligned}
[e_1,e_2]&=e_3 & [e_1,e_3]&=e_4\\
[e_1,e_4]&=e_5 & [e_1,e_5]&=e_6\\
[e_1,e_6]&=e_7 & [e_1,e_7]&=e_8\\
[e_1,e_8]&=e_9 & [e_1,e_9]&=e_{10}\\
[e_1,e_{10}]&=e_{11} & [e_2,e_3]&=e_{11}\\
[e_2,e_{11}]&=e_{12} & [e_3,e_{10}]&=\alpha_{3,10}^{12}e_{12}\\
[e_4,e_9]&=\alpha_{4,9}^{12}e_{12} & [e_5,e_8]&=\alpha_{5,8}^{12}e_{12}\\
[e_6,e_7]&=\alpha_{6,7}^{12}e_{12}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1,e_2,e_{10}):&\quad -\alpha_{3,10}^{12}-1 &&=0\\
(e_1,e_3,e_9):&\quad -\alpha_{3,10}^{12}-\alpha_{4,9}^{12} &&=0\\
(e_1,e_4,e_8):&\quad -\alpha_{4,9}^{12}-\alpha_{5,8}^{12} &&=0\\
(e_1,e_5,e_7):&\quad -\alpha_{5,8}^{12}-\alpha_{6,7}^{12} &&=0
\end{aligned}
$$

Solution 1:

$$
\begin{aligned}
\alpha_{3,10}^{12}&=-1\\
\alpha_{6,7}^{12}&=1\\
\alpha_{4,9}^{12}&=1\\
\alpha_{5,8}^{12}&=-1
\end{aligned}
$$

How the solution(s) were or were not found:
Change variables

$$\alpha^{12}_{3,10} \to x_1$$

$$\alpha^{12}_{6,7} \to x_2$$

$$\alpha^{12}_{4,9} \to x_3$$

$$\alpha^{12}_{5,8} \to x_4$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_{10}):&\quad -x_1 - 1 &&= 0\\
(e_1, e_3, e_9):&\quad -x_1 - x_3 &&= 0\\
(e_1, e_4, e_8):&\quad -x_3 - x_4 &&= 0\\
(e_1, e_5, e_7):&\quad -x_2 - x_4 &&= 0
\end{aligned}$$

Groebner basis (4 variables, 4 linear, 0 nonlinear)

$$x_1 + 1 = 0$$

$$x_2 - 1 = 0$$

$$x_3 - 1 = 0$$

$$x_4 + 1 = 0$$

Solution 1:

$$x_1 = -1$$

$$x_2 = 1$$

$$x_3 = 1$$

$$x_4 = -1$$