**Homework 5: Probability and Counting**

**Instructions:** Please do not use R to answer the following questions. You should be able to answer these by hand / analytically.You are allowed to use a calculator and only allowed to use R when calculating choose(n,k). Below is a reproduction of Bayes Theorem that you will utilize through the course of this homework assignment. Please submit your assignment with screen shots attached of your work.

***P(A∣B)= P(B∣A)⋅P(B) / P(A)***

*P(A∣B) is the conditional probability of event A given event B.*

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*P(A) is the probability of event A occurring.*

*P(B) is the probability of event B occurring.*

**Problem 1: Conditional Probability**

Suppose a deck of cards contains 4 red cards and 6 blue cards. Two cards are drawn sequentially without replacement. Calculate the probability that the second card drawn is red, given that the first card drawn was blue.

P(Red on second | Blue on first) = P(Red on second AND Blue on first) / P(Blue on first)

**Problem 2: Disease Testing**

A certain disease affects 2% of the population. A diagnostic test for the disease is known to produce a false positive result 5% of the time and a false negative result 3% of the time. If a randomly selected person tests positive for the disease, what is the probability that they actually have the disease?

**Problem 3: Dice Rolling**

Consider two fair six-sided dice, one red and one green. The red die has the numbers 1 through 6, and the green die has the numbers 2 through 7. What is the probability that the sum of the two dice is 8, given that the red die shows an even number?

**Problem 4: Card Game**

In a standard deck of 52 playing cards, there are 26 red cards and 13 face cards (jacks, queens, and kings). If you draw one card at random, what is the probability that it is a red face card?

**Problem 5: Product Defects**

A factory produces electronic games. 5% of the games are defective. Quality control tests catch 90% of the defective games but also incorrectly label 5% of the non-defective games as defective. If a game is chosen at random and fails a quality control test, what is the probability that it is actually defective?

**Problem 6: Combinations and Conditional Probability (Extra Credit)**

You are organizing a raffle with 20 tickets, numbered from 1 to 20. Five of these tickets are winners. If you select 3 tickets at random without replacement, what is the probability that exactly 2 of the selected tickets are winners?