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CSCI 170 HW 3

1. a. is equivatent to statements $\boxed{j, e, \text{ and } f}$
a: "Some students in CSCI 170 are in CS majors

Statement e: $\boxed{\exists x [F(x) \land M(x)]}$

Statement f: $\neg \forall x [\neg F(x) \lor \neg M(x)]$ (Premise)
$\exists x \neg (\neg F(x) \lor \neg M(x)]$ (De Morgan's 1)
$\boxed{\exists x (F(x) \land M(x))]}$ (De Morgan's 2)

Statement j: $\neg \forall x [F(x) \rightarrow \neg M(x)]$ (Premise)
$\neg \forall x [\neg F(x) \lor \neg M(x)]$ (Implication 1)
$\exists x \neg [\neg F(x) \lor \neg M(x)]$ (De Morgan's 2)
$\boxed{\exists x [F(x) \land M(x)]}$ (DeMorgan's 3)

We can see that statements e, f, and j are all equivalent. Translated to english, they mean, "There is a student that is in CSCI 170 and is a CS major" which is equivalent to a

b. is equivalent to statements $\boxed{b, d, \text{ and } g}$
b. "Every CS major is in CSCI 170"

Statement b: $\neg \exists x [M(x) \land \neg F(x)]$ (Premise)
$\forall x \neg [M(x) \land \neg F(x)]$ (De Morgan's 1)
$\forall x [\neg M(x) \lor F(x)]$ (De Morgan's 2)
$\boxed{\forall x [M(x) \Rightarrow F(x)]}$ (Implication 3)

Statement d: $\boxed{\forall x [M(x) \Rightarrow F(x)]}$ (Premise)

Statement g: $\forall x (\neg M(x) \lor F(x))$ (Premise)
$\forall x (M(x) \Rightarrow F(x))$ (Implication 1)

We can see that statements b, d, and g are equivalent, They come out to saying "All cs majors are in CSCI 170" which is equivalent to B.

c. is equivalent to statements $\boxed{\text{a, c, and h}}$
c: "No" cs major is in CSCI-170.

Statement a. $\forall x[M(x) \Rightarrow \neg F(x)]$ (Premise)
$\boxed{\forall x[F(x) \Rightarrow \neg M(x)]}$ (Contrapositive 1)

Statement c. $\boxed{\forall x[F(x) \Rightarrow \neg M(x)]}$ (Premise 1)

Statement h. $\forall x[\neg M(x) \vee \neg F(x)]$ (Premise)
$\forall x[\neg F(x) \vee \neg M(x)]$ (Commutative 1)
$\boxed{\forall x[F(x) \Rightarrow \neg M(x)]}$ (Implication 2)

As we can see, statements a, c, and h are all equivalent. In English they translate to "All students in CSCI 170 are not cs majors" which is equivalent to c.

2\. a. i. b= There are bears in Springfield.
r= This rock keeps tigers away.

ii. Premises: $\overset{1}{b}$, $\overset{2}{r}$
Conclusion: $\neg b \Rightarrow r$

iii.

$$\begin{array}{ll} \neg b \Rightarrow r & \text{(conclusion)} \\ b \vee r & \text{(Implication 1)} \\ F \vee T & \text{(Substitute premises)} \\ \boxed{T} & \text{(Domination 3)} \end{array}$$

✓

Here we can see that the premises are stronger statements than the conclusion. We are essentially stating that if the premises are true, then the premises are true. Therefore we have proved the conclusion correct even though it says nothing of value.

b. i. a- Aaron gives a homework assignment
b- Bob and Cindy finish the assignment
g- Bob plays video games

ii. Premises: $a \to b$, $b \to \neg g$
Conclusion: $g \to \neg a$

iii.

$$\begin{array}{ll} a \to b & \text{(Premise)} \\ b \to \neg g & \text{(Premise)} \\ a \to \neg g & \text{(Hypothetical Syllogism)} \\ \boxed{g \to \neg a} & \text{(Contrapositive 3)} \end{array}$$

✓

The conclusion is proven.

c. i. r = roses are red, v = violets are blue
s = summer is through, g = grass is green, b = robin has been seen

ii. $\neg v \to r$, $v \to s$, $s \oplus g$, $g \Leftrightarrow b$, $\neg r$ - Premises
$\neg b$ - Conclusion

iii.

$\neg v \to r$	(Premise)
$\neg r \to v$	(Contrapositive 1)
$v \to s$	(Premise)
$\neg r \to s$	(Hypothetical Syllogism)
$T \to s$	(Given by premise)
s	(To ensure premise is true)
$s \oplus g$	(Premise)
$g \oplus s$	(Commutative 7)
$g \Leftrightarrow \neg s$	(Exclusive -or 8)
$g \to \neg s \wedge \neg s \to g$	(Mutual implication 9)
$g \to F \wedge F \to g$	(Line 6)
$g \to F \wedge (T \vee g)$	(Implication 11)
$g \to F \wedge T$	(Domination 12)
$g \to F$	(Identity 13)
$\neg g \vee F$	(Implication 14)
$\neg g$	(To ensure premise is true)
$g \Leftrightarrow b$	(Premise)
$g \to b \wedge b \to g$	(Mutual implication 17)
$F \to b \wedge b \to F$	(Line 16)
$(T \vee b) \wedge b \to F$	(Implication 17)
$T \wedge b \to F$	(Domination 18)
$b \to F$	(Identity 19)
$\neg b \vee F$	(Implication 20)
$\boxed{\neg b}$	(To ensure premise is true)

The conclusion is proven.

3. p - Dr. Smith is a patient of Dr. Doe
a - Dr. Smith is younger than 55
c - Dr. Smith has an advanced care directive

Premises: p, $a \land \neg c$, $(p \land \neg a) \to c$

In this case p is true, a is true, and c is false.
Knowing this, we will see if the three premises hold true

1: p ✓

2: $a \land \neg c$ (Premise)
$T \land \neg F$ (Premise)
$T \land T$ (Logic)
T ✓

3: $(p \land \neg a) \to c$ (Premise)
$\neg(p \land \neg a) \lor c$ (Implication 1)
$(\neg p \lor a) \lor c$ (De Morgan's 2)
$(\neg T \lor T) \lor F$ (Premise)
$(F \lor T) \lor F$ ~~(Negation)~~ (Logic)
$T \lor F$ (Domination 5)
T ✓ (Domination 6)

All three premises hold true so they are logically consistent.

4. a. i. $\forall y\, L(0,y)$

ii. $\neg(\forall y\, L(0,y))$ (Premise)

$\boxed{\exists y\, \neg L(0,y)}$ (De Morgans 1)

iii. There exists an integer greater than ~~ore~~ equal to 0.

b. i. $\exists x\, I(x) \rightarrow I(0)$

ii. $\neg(\exists x\, I(x) \rightarrow I(0))$ (Premise)

$\neg(\neg(\exists x\, I(x)) \vee I(0))$ (Implication 1)

$\neg(\forall x \neg I(x) \vee I(0))$ (De Morgan's 2)

$\neg \forall x \neg I(x) \wedge \neg I(0)$ (De Morgan's 3)

$\boxed{\exists x\, I(x) \wedge \neg I(0)}$ (De Morgan's 4)

iii. There exists an interesting number and 0 is not interesting.

4. c. i. $\forall y \exists x \neg L(x,y)$

ii. $\neg(\forall y \exists x \neg L(x,y))$ (Premise)

$\exists y \neg \exists x \neg L(x,y)$ (De Morgan's 1)

$\boxed{\exists y \forall x L(x,y)}$ (De Morgan's 2)

iii. There is an integer such that all integers are less than it.

d. i. $\forall x \exists y \exists z (L(x,y) \wedge \neg L(x,z))$

ii. $\neg(\forall x \exists y \exists z (L(x,y) \wedge \neg L(x,z)))$ (Premise)

$\exists x \neg \exists y \exists z (L(x,y) \wedge \neg L(x,z))$ (De Morgan's 1)

$\exists x \forall y \neg \exists z (L(x,y) \wedge \neg L(x,z))$ (De Morgan's 2)

$\exists x \forall y \forall z \neg (L(x,y) \wedge \neg L(x,z))$ (De Morgan's 3)

$\boxed{\exists x \forall y \forall z (\neg L(x,y) \vee L(x,z))}$ (De Morgan's 4)

iii. There is an integer greater than every integer or less than every integer.

e $\forall a \neg I(a) \rightarrow (\forall y \exists x \neg L(x,y))$

$\neg(\forall a \neg I(a)) \vee \forall y \exists x \neg L(x,y)$ (Implication 1)

$\exists a I(a) \vee \forall y \exists x \neg L(x,y)$ (De Morgan's 2)

$\boxed{\exists a I(a) \vee \forall y \exists x \neg L(x,y)}$

ii. $\neg(\exists a I(a) \vee \forall y \exists x \neg L(x,y))$ (Premise

$\neg \exists a I(a) \wedge \neg \forall y \exists x \neg L(x,y)$ (De Morgan's 1)

$\forall a \neg I(a) \wedge \neg \forall y \exists x \neg L(x,y)$ (De Morgan's 2)

$\forall a \neg I(a) \wedge \exists y \neg \exists x \neg L(x,y)$ (De Morgan's 3)

$\boxed{\forall a \neg I(a) \wedge \exists y \forall x L(x,y)}$ (De Morgan's 4)

iii. No integers are interesting and there is an integer such that all integers are less than it.

5. $\boxed{\text{a is false and b is true}}$

Disproving a

a. • $\exists_x \in S : [P(x) \land Q(x)] \Leftrightarrow [\exists_x \in S : P(x) \land \exists_x \in S : Q(x)]$ (Premise)

• $[\exists_x \in S : [P(x) \land Q(x)] \rightarrow [\exists_x \in S : P(x) \land \exists_x \in S : Q(x)]] \land$
$[\exists_x \in S : P(x) \land \exists_x \in S : Q(x)] \rightarrow \exists_x \in S : [P(x) \land Q(x)]$ (Mutual Implication 1)

Let us focus on the left side of this "and" statement
$\exists_x \in S : [P(x) \land Q(x)] \rightarrow [\exists_x \in S : P(x) \land \exists_x \in S : Q(x)]$

Let us suppose that there is a Set $S = \{a, b\}$.
Let us suppose $P(a)$ is true, $P(b)$ is false, $Q(a)$ is false and $Q(b)$ is true.

Then when we examine the right side of this statement, we see:

$\exists x \in S : P(x)$ is true as shown by $P(a)$.
$\exists x \in S : Q(x)$ is true as shown by $Q(b)$.

So we have
$$\exists x \in S : [P(x) \land Q(x)] \rightarrow T$$

Now we look at the left side. This evaluates to false as there is no value in S that results in $P(x)$ and $Q(x)$ evaluating to true. So we have:

$$T \rightarrow F$$

Which is

$$F:$$

So we have:

$F \quad \wedge ([\exists x \in S: P(x)] \wedge [\exists x \in S: Q(x)]) \rightarrow \exists_x \in S: [P(x) \wedge Q(x)]$

which is:

F ✓

Thus we have dis proven a.

Proving b:

$\forall_x \in S: [P(x) \wedge Q(x)] \Leftrightarrow [\forall_x \in S: P(x)] \wedge [\forall x \in S: Q(x)]$ (Premise)

$(\forall x \in S: [P(x) \wedge Q(x)] \rightarrow [\forall x \in S: P(x)] \wedge [\forall x \in S: Q(x)]) \wedge$
$([\forall x \in S: P(x)] \wedge [\forall x \in S: Q(x)]) \rightarrow [\forall x \in S: [P(x) \wedge Q(x)]])$ (Mutual & Implication 1)

We have to prove both implications. Lets start with the one on the left of the "and" statement.

$\forall x \in S: [P(x) \wedge Q(x)] \rightarrow [\forall x \in S: P(x)] \wedge [\forall x \in S: Q(x)]$

Let's do a proof by contradiction

$\forall x \in S: [P(x) \wedge Q(x)] \rightarrow \neg([\forall x \in S: P(x)] \wedge [\forall x \in S: Q(x)])$ (Premise)
$\forall x \in S: [P(x) \wedge Q(x)] \rightarrow [\neg \forall x \in S: P(x)] \vee [\neg \forall x \in S: Q(x)]$ (De Morgan's 1)
$\forall x \in S: [P(x) \wedge Q(x)] \rightarrow [\exists x \in S: \neg P(x)] \vee [\exists x \in S: \neg Q(x)]$ (De Morgan's 2)

Assuming that the left side of the statement is true, the right side must also be true if the overall statement is true. The right side translates to "There is a value in S such that P(x) ~~isn't~~ true or there is a value in S such

Q(x) isn't true. For the right-side to evaluate to true, either one or both of these statements must be true. Regardless of which one or how many of the statements is true, there is a contradiction.

The left side of the statement essentially states that for every member of S, both P(x) and Q(x) evaluate to true. However this contradicts the right side of the statement which states that there is at least one value in S for which P(x) or Q(x) evaluates to false. Thus, finding this contradiction, we have proven that

$$\forall x \in S: [P(x) \land Q(x)] \rightarrow [\forall x \in S: P(x)] \land [\forall x \in S: Q(x)]$$

is true.

Now we will look at the right side of the "and" statement

$$[\forall x \in S: P(x)] \land [\forall x \in S: Q(x)] \rightarrow [\forall x \in S: [P(x) \land Q(x)]]$$

Once again, we will do a proof by contradiction.

$[\forall x \in S: P(x)] \land [\forall x \in S: Q(x)] \rightarrow \neg[\forall x \in S: [P(x) \land Q(x)]]$ (Premise)
$[\forall x \in S: P(x)] \land [\forall x \in S: Q(x)] \rightarrow \exists x \in S: \neg[P(x) \land Q(x)]$ (De Morgan's 1)
$[\forall x \in S: P(x)] \land [\forall x \in S: Q(x)] \rightarrow \exists x \in S: [\neg P(x) \lor \neg Q(x)]$ (De Morgan's 2)

Here we have found a contradiction. The right side of the statement states that there is a value ~~for~~ in S for which either P(x), Q(x) or both evaluate to false. Regardless of which possibility is true, this contradicts the left-side which states that ~~for~~ every value of S P(x) evaluates to true and for every value of S, Q(x) evaluates to true. Thus, finding this contradiction, we have proven that:

$[\forall x \in S: P(x)] \wedge [\forall x \in S: Q(x)] \rightarrow [\forall x \in S: [P(x) \wedge Q(x)]]$

is true.

Since we have proven both implications derived from the original statement true, the original statement

$\forall x \in S: [P(x) \wedge Q(x)] \Leftrightarrow [\forall x \in S: P(x)] \wedge [\forall x \in S: Q(x)]$

has been proven true.