

A real time multi-objective cyclists route choice model for a bike-sharing mobile application

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Abstract—The attractiveness of cycling -and in particular of bike-sharing systems- as a sustainable alternative of transportation is constantly growing, given the undeniable benefits associated with it. The aim of this paper is to present a multi-objective model based on a Fuzzy Inference System to be embedded in a mobile application that could assist cyclists in the selection of the smartest route to follow to reach their destination, in terms of travel costs (distance or time), level of air pollution and road safety. The features of the bike -sharing system (both traditional and free-floating) are considered in the generation of the final path, and also the starting and final stations to prefer (or the closest bike to pick up for the free-floating option) are provided. The proposed optimization model is dynamic, as it is synchronized with geolocated real time data regarding level of congestion and flows on the network, and availability of bikes/racks in the bike-sharing system. The mobile app gives bike users the possibility to plan, personalize and execute their trip with turn-by-turn guidance, allowing them to select the default optimal path, or to choose the desired travel time among the available route options, each of them accompanied by the related air pollution and safety. An application of the model is carried out through a test case to evaluate the proposed approach. Furthermore, a first study regarding the graphic interface of the mobile platform is presented to recommend some guidelines to follow to have a final product effective and bike users-friendly. The final goal is to improve the cycling experience, encouraging at the same time more people to elect the bike as their preferred mode of transportation.

Keywords— *Advanced Traveller Information System (ATIS); bike-sharing systems (BSS); cycle route planner; Fuzzy Inference System (FIS); mobile-app; multi-objective optimization; smart path choice model.*

I. INTRODUCTION

It is generally claimed that cycling is one of the most sustainable ways to travel [1], as it offers benefits to both individuals and environment. It is relatively fast and reliable over short distances, and it provides an affordable form of transport for most segments of the population [2]. Due to the undeniable advantages connected to cycling -it is healthy, energy efficient, quiet and compatible with the urban scale [3]- its usage is encouraged promoted more and more by transportation planning authorities [4]. Among the strategies adopted to increase the cycling population, bike-sharing systems (BSSs) need to be mentioned. The more traditional

(station-based) systems allow users to pick-up a bike in one station closer to the starting point of their trip, and cycle to the station nearby their destination, without the burdens associated with the ownership of a bike. Alternatively, with a free-floating system, there is no need for specific docking stations or kiosk machines, and every user can lock the bike to an ordinary bicycle rack [5] close to his/her destination. The management of BSSs has attracted a growing interest of the scientific community, and researchers have been focusing on different issues, like an effective planning [6], network design [7], and relocation operations [8, 9] to be applied in order to allow the scheme to function smoothly. Several tools -web and mobile mapping services- have already proved their ability in helping cyclists in the selection of the 'best' route to follow -generally coincident with the shortest or the fastest path. However, it often happens that the best path in terms of travel time is not equally capable of meeting other possible needs and preferences of cyclists, that must be understood to globally enhance the cycling experience.

To the best of our knowledge, it seems there is no route-planner able to suggest to every user a list of possible cycle-paths, weighted in accordance with a certain number of variables (e.g. the global safety associated with the trip, or the user's request to ride along less polluted routes), and at the same time connected with the BSS facilities in the network. Aiming to fill this apparent gap in the literature, we have conceived a real-time model based on a Fuzzy Inference System (FIS) that could be adopted to implement a mobile routing and navigation application for cyclists, suggesting them the starting and final bike-sharing stations to choose (or which bike is the closest to the origin of their trip if the system is free-floating), and the smartest path to follow, according to time, distance, pollution and safety. Otherwise, the user is allowed to select an alternate route with a different travel time - accompanied by its corresponding level of air pollution and safety-, in order to customize his/her choice according to the parameter that wants to prioritize.

In the following sections, our smart multi-objective model is described and proposed, and a numerical application shows its functioning in a test case study. Furthermore, the implementation of a cycle route planner mobile app is suggested, together with a starting study about the general guidelines to follow in order to develop a user-friendly tool.

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II. SMART MULTI-OBJECTIVE ROUTE CHOICE MODEL

In recent years, a variety of cyclist route choice models have been proposed, each one with distinctive features and attributes deemed worthy to consider. As a matter of fact, it has been observed that the bicycling routes actually used are often significantly longer than the shortest possible ones [10]. Hence, even in order to propose and develop more reliable cycle route planners, it is important to aim at a more comprehensive understanding of users' preferences.

Some analysis regarding the bicycle commuter routes has been done using the Geographic Information System [11], revealing that usually cyclists tend to ride along the major routes, or trying to avoid the busiest streets. On the other hand, Stinson and Bath [12] demonstrated that each factor plays a different role in the route-choice in relation to the level of cycling experience of everyone. Other studies, given the recent growing availability of GPS based observation of travelers, revealed that it turned feasible to elaborate a choice model of bicyclists estimated on the basis of a large amount of observed and collected data: the first one has been reported in Zurich [3], considering the length of each link as main cost attribute. Two years later, Broach, Gliebe, and Dill [13] developed a modified a method of route labeling, based on some experiments done with a more common choice set generation method, and on some hypotheses about the cyclists' choice set generation process. They found that a lot of factors, such as distance, turn frequency, slope, intersections, facility types, and traffic volumes influence considerably the attractiveness of a given cycle path. Furthermore, Jarjour et al. [14] studied the correlation between traffic-related air pollution and cyclist route choice, and suggest users to prefer bicycle boulevards instead of arterial streets in order to increase the potential health benefits connected with biking.

Taking into consideration what has been explained above, the goal of this paper is developing a smart multi-objective route choice model for the BSS users, to be embedded in a route planner mobile application. It has been stated that most of the currently operating cycle planners allow cyclists to set some of their preferences [4], but they have their inner principles and functioning not accessible to public or researchers. Therefore, although the main trends and basic attributes are known, it results quite hard to evaluate or compare them in terms of their link cost functions. It is worthy to mention the main published studied related to the route planners for Broward County [14], Vancouver [15], Charlottesville [16], Dublin City [17]: all of them offer different route preferences, taking cyclists safety, traffic volumes, scenery, etc. into considerations.

In this paper, we want to consider some of those parameters that proved to have the highest influence on users' route choice (time/length, air pollution, safety), building on them a cost function to be associated with every arc of a network. The proposed soft computing method, based on a Fuzzy Inference System (FIS) [18], relates the selected indicators to an overall link cost measure, taking into account the significant uncertainties and the unknown mathematical relationships existing between them. Inside the inference mechanism, the behavior of a human decision-maker can be simulated, by

using rules like [20]: "if V is X then W is Y". In the Fuzzy Logic framework, the meaning of this rule is: "the more V is X, the more W is Y", where X and Y are input variables and can assume linguistic or approximate values, i.e. fuzzy sets, defined by the means of their respective membership functions. Then, the applicability of each rule is calculated, and the center of the corresponding rule output is captured. Finally, since the result of a rule is a fuzzy set, to define a crisp (non-fuzzy) output of the FIS, it is necessary to defuzzify the output (see [19] for further details).

A. Air pollution

The air pollution output parameter embraces two indicators: the traffic flows on the network, and the presence of natural elements (such as parks, waterways, sea, etc.) in the proximity of the arc. It is important to underline that, since the suggested model is real-time (highly dynamic), it is associated with the real-time flows on the network (alternatively, if for a given time interval there are no updated/available values, historical data series or forecast models are utilized to ensure a proper functioning of the system). Being human decisions based on qualitative opinions rather than on 'numerical' judgments, the perception of the two involved indicators (more briefly called 'Flows' and 'Nature') tends to be evaluated by cyclists by means of linguistic forms as: a lot of, not much, low, high, etc. Therefore, each indicator can be easily represented by a fuzzy set, divided into a finite number of subsets (for example 'low', 'medium', 'high', see Fig. 1), so that the value of the input variable may belong or not to a defined subset (see [19] for a more detailed explanation). The shapes of each membership function (see an example of two trapezoidal and one triangular membership functions associated with the three fuzzy subsets of the input variable 'Flows', Fig. 1), and the definition of the FIS rules, at first attempt, rely on evaluations made by experts, but could be further assessed and calibrated (i.e. by means of surveys submitted to the involved groups of people). The more a value belongs to a subset, the more the degree of membership is close to 1.

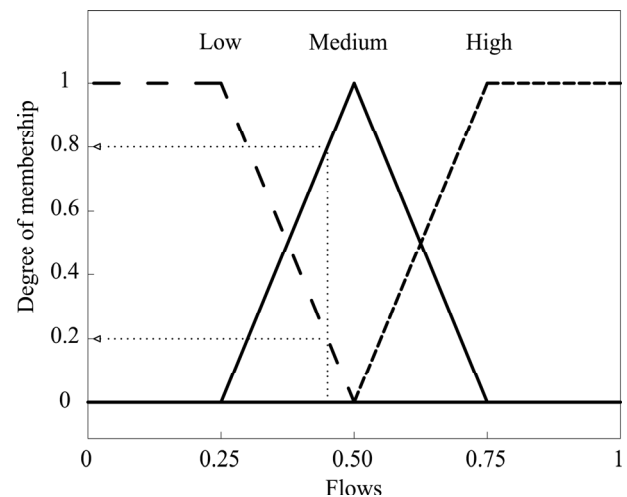


Fig. 1. Fuzzy subsets associated to 'Flows' (traffic flows on the network).

The fuzzy subsets construction can be applied both to ‘Flows’, setting on the x-axis the minimum and maximum level of traffic that could averagely be detected in each time interval on a link belonging to the context of study (normalized between 0 and 1, see Fig.1), and to the ‘Nature’ indicator, that contemplates any natural component that could be found in the vicinity of every arc of the network. In order to establish an unambiguous method to evaluate the ‘Nature’ component, we decide to take into account a certain area around each arc, computing the percentage of natural elements that fall within the buffer so identified -covering the soil surface. Therefore, the associated ‘Nature’ x-axis ranges from 0% to 100%. In the FIS framework, any specific combination of these two parameters is regulated by a set of rules. After the application of an aggregation method [18] of the results of each rule, and after the defuzzification, the final output values for the air pollution parameter can be achieved.

B. Safety

The global safety associated with a given route is linked to several (n) indicators, such as the presence and typology of bike lanes, the eventuality of some streets closed to vehicular traffic, or the number and type of crossings in the path. Each involved parameter that improves the safety of a given link has a +1 value. The final number associated with each arc constitutes its degree of safety and ranges from $[0...n]$.

C. Smartest path

The final optimal path (default path suggested by the mobile-app) starts at the origin suggested by the user. Its first part has to be covered by foot, in order to reach the closest BSS station/free-floating bike. Being a dynamic model, the estimated availability of bikes in a station is provided real-time (or, alternatively, forecasted or based on the historical usage of the system). Consequently, the suggested station is not necessarily the closest to the starting location of the bicycle trip, but the nearest one having also a disposability that satisfies the user’s request (the same applies for the destination station). On the other side, in the free-floating option, if the system allows reserving a bike, the model suggests the nearest free-floating bike actually available (not already booked by another user). The shortest path (from the origin to the bike) is then calculated by means of the Dijkstra algorithm [20], considering as link-cost the length of each arc (the same applies to the walking path between the final BSS station and the destination in the station-based option).

After that, the optimal cycle path (from the starting to the final BSS station in the station-based option, or from the bike location to the final user’s destination in the free-floating one) is then computed, once again through a FIS procedure. Input variables are length, air pollution and safety (singleton variable, that is, its subset membership functions are single-valued constants). The output variable is the overall cost associated with each link. Using these new link costs, the final cycling path (Fig. 2) is calculated (again, by means of Dijkstra [20]), together with its relative total travel time. This is the default cycle path suggested by the app, accompanied by the

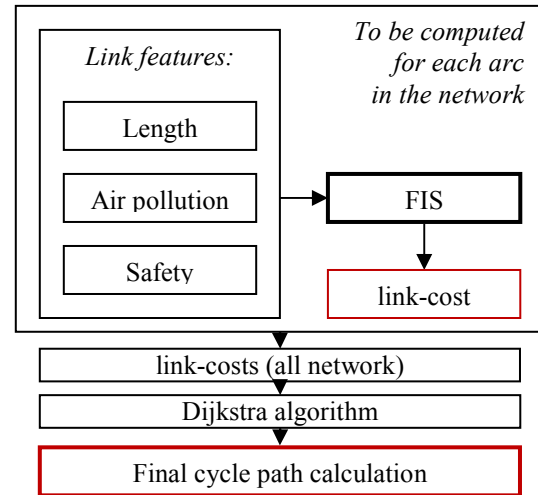


Fig. 2. Optimal/default cycle path calculation.

corresponding levels of air pollution and safety associated with this route choice. However, the model computes also 4 other alternative paths: one having as link-cost only the length (shortest path); another one having as link-cost only the air pollution (less polluted path); a further one having as link-cost only the safety (safest path); a last one, having as input of the FIS procedure only air pollution and safety (Fig.2, but without considering the length as link feature). Each one of these 5 paths (default + 4 alternatives) corresponds to its related travel times. The user, interacting with the mobile app, is allowed to change the predefined travel time (associated with the default cycle-route), selecting one of the other possible travel times associated with the alternative routes. Together with the selection of a different option, the correlated time, distance, air pollution and safety of the route-choice are displayed. In this way, the user is free to choose according to the feature that he/she wants to prioritize.

III. NUMERICAL APPLICATION

The proposed methodology, described in the previous section, has been applied to a test network. It is a grid of 3.0 km x 3.6 km, consisting of 693 nodes and 2616 arcs (see Fig 3). The idea is to closely reproduce a typical urban area of a city, with blocks (highlighted in light gray, with an alternate module of 150 m and 75 m), green areas, waterways. Some of the roads that crossed the parks are closed at the vehicular traffic (dashed gray lines), some other have a dedicated bike lane on it (dark green arcs). The signalized intersections are pointed out on the map with green-yellow-red dots. Fixed an Origin-Destination (OD) demand matrix, and thanks to a Deterministic User Equilibrium (DUE) assignment, it has been possible to obtain the vehicle flows on the network (‘Flow’), at a given time interval. In order to compute the ‘Nature’ input variable, a buffer has been considered around each arc. Namely, in this context, the nature percentage can assume a value of 0% (no natural element nearby), 50% (there are natural elements only on one side of the arc, 100% (there are natural elements on both sides of the arc, i.e. it goes through a park). Finally, in relation to the safety indicator, each arc can range from 0 to 2, according to the presence of a bicycle path

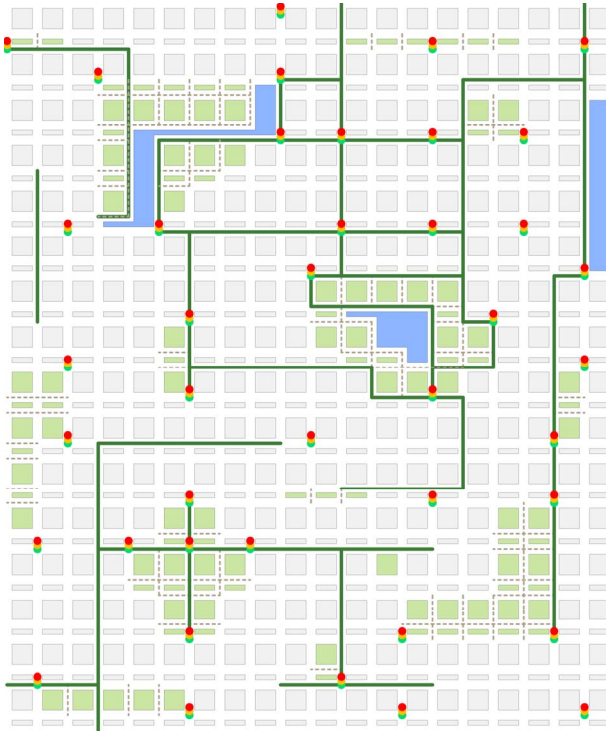


Fig. 3. Test network

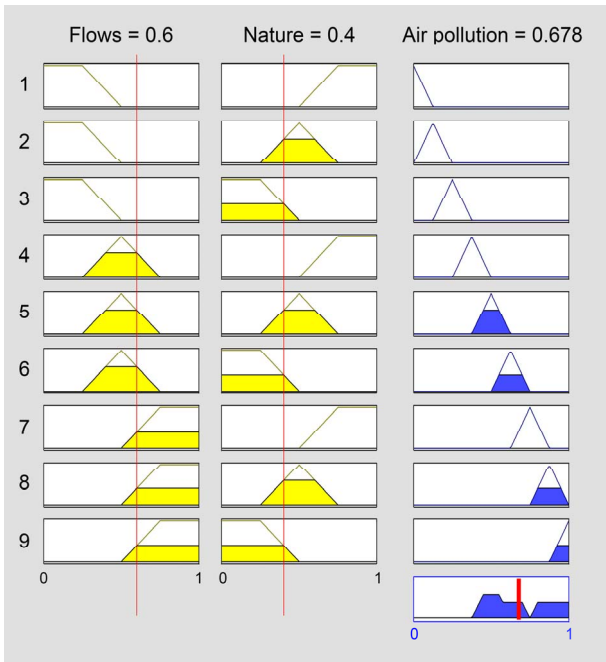


Fig. 4. FIS rule: "If 'Flows' are medium-high, and 'Nature' is medium-low, then the 'Air pollution' on the arc is moderately high."

on it (+1), a closed to traffic condition (+1) and a signalized intersection on its second/last node (+1). All these three conditions, in this case study, cannot coexist on the same arc (if a link is closed to traffic, it cannot have either a traffic light on its last node): that is, the maximum value that each arc may assume is 2 (and not 3). In the following, two numerical examples are presented, both for a station-based and a free-

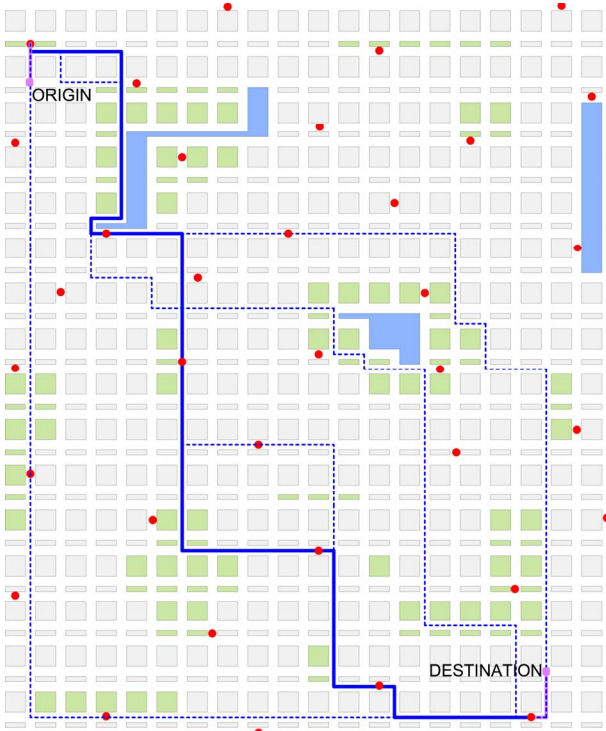


Fig. 5. Alternative routes in a station-based reality.

floating reality. The average walking speed has been assumed equal to 6 km/h, while the cycling speed has been set equal to 25 km/h and 15 km/h, respectively for arcs with and without a bike lane on them. We have considered 30 seconds of waiting time in correspondence with every signalized intersection, and 3 seconds for the remaining junctions. Then, the average speed of each path/route lowers compared with the one of the arcs. Once fixed an origin and a destination in the network, the smartest/optimal path between them, and the 4 alternative routes, have been computed. The FIS procedure has been used at first to calculate the air pollution value associated with each arc. Figure 4 shows an example of FIS rule associated with this calculation: after setting the membership functions (triangular/trapezoidal) associated with both the traffic flows and the percentage of nature, if 'Flows' is equal to 0.6, and 'Nature' = 0.4, then the corresponding crisp 'Air pollution' level (equal to 0.678) can be defined defuzzifying the fuzzy set obtained as output. Then, having as input variables length, air pollution, and safety, a further FIS has been applied in order to obtain the overall link cost to be assigned to each arc in the calculation of the default path. Figure 5 shows the two walking paths (violet), the smartest cycle path (continuous blue line) and the 4 alternative paths (dashed blue lines) that can be obtained using the proposed methodology between a given origin and destination in the test network, considering a BSS station-based system (with each station displayed with a red dot), and with a total path to cover of about 6 km. The numerical results achieved for this station-based application, and for a further one applied to a 3 km trip in a free-floating reality, are summarized in Table 1. Distance and time associated with each branch of the trip have been calculated.

TABLE I. NUMERICAL RESULTS: ALTERNATIVE PATH CALCULATION IN A STATION-BASED AND IN A FREE-FLOATING BIKE-SHARING SYSTEM.

<i>Alternative routes STATION-BASED</i>	Origin – BSS station		Cycle path						BSS station - Destination	
	Walking distance [m]	Walking time [min]	Length [m]	Air Pollution	Normalized Air pollution (0-100)	Safety	Normalized Safety (0-100)	Cycling time [min]	Walking distance [m]	Walking time [min]
<i>Length</i>	187.5	1.9	5812	10.1	71.4	12	100	26.1	300	3
<i>Air pollution</i>	187.5	1.9	6112	7.8	55.1	32	80.5	23.9	300	3
<i>Safety</i>	187.5	1.9	6262	14.2	100	56	53.7	22.9	300	3
<i>Air p. & Safety</i>	187.5	1.9	6112	9.9	69.9	45	64.6	23.3	300	3
<i>Default/Optimal</i>	187.5	1.9	6112	10.0	70.9	50	58.5	23.5	300	3

<i>Alternative routes FREE-FLOATING</i>	Origin – Free-floating bike		Cycle path						Free-floating bike - Destination	
	Walking distance [m]	Walking time [min]	Length [m]	Air Pollution	Normalized Air pollution (0-100)	Safety	Normalized Safety (0-100)	Cycling time [min]	Walking distance [m]	Walking time [min]
<i>Length</i>	150.1	1.5	2925	6.2	99.4	0	100	12.9	0	0
<i>Air pollution</i>	150.1	1.5	3075	4.3	68.9	10	83.3	12.8	0	0
<i>Safety</i>	150.1	1.5	3225	6.3	100	26	54.3	11.6	0	0
<i>Air p. & Safety</i>	150.1	1.5	3375	4.4	70.9	25	60.4	12.3	0	0
<i>Default/Optimal</i>	150.1	1.5	3225	4.5	71.7	23	60.4	12.1	0	0

The corresponding levels of air pollution and safety for each alternative route have also been presented (Table 1), together with their values normalized between 0 and 100 -the ones to be displayed to the user on the mobile app (see next section for further details).

IV. CYCLE ROUTE PLANNER MOBILE APP

In this section, we propose to embed our cycle-route choice model in a mobile platform. To be effective, efficient and satisfactory from the user's point of view, a mobile-application -in this case, a cycle route planner- needs to combine utility with aesthetics, i.e. the visual appearance of the system [21]. Hence, we develop a prototype with the basic features that we believe have to be included, in order to suggest a mobile app that could be either useful and user-friendly.

In Fig. 6, our idea of the main screen of the app is presented. We decide to show the same free-floating alternative routes proposed by the numerical example of Table 1, assuming that the user has already selected his/her preferred origin and destination. The content area displays the map of the city (test network, in this case), with a blue dot/circle corresponding to the user location, and a red map pin on the closest free-floating bike available at that moment. The bike lanes in the network are displayed in a dark green, and the suggested default/smarter path with a continuous blue line; the alternative routes are also set out with dashed light-blue lines. The distance and time related to the selected path (splitting its walking and cycling components) are located in a light gray rectangle in the upper part of the screen. The user can immediately visualize the (relative) travel time of the default path (compared with the travel times of the remaining 4 alternative options), with the corresponding associated level of air pollution (real-time) and safety. He/she has the possibility to use the slider component and select an alternative travel time, associated with another possible route. Accordingly, the

related values of distance, time, air pollution and safety will be updated. Note the use of the color bars, in the bottom area of the screen. In both cases, the 0 corresponds to the best ideal configuration that could be reached (respectively, a path without pollution, or one with the highest level of safety). The 100 in the range coincides with the worst value that can be found among the 5 possible alternatives. For example, looking at the free-floating reality (Table 1, Fig. 6): the highest level of pollution reached by the five suggested alternative routes is 6.3; then, it has been set as 100 in the range of normalized values. On the other side, the worst level of safety is the one obtained in the first free-floating path in Table 1 (based on length, i.e. shortest route); here, the level of safety is 0. This means that no arc in this path has a biked-lane, or a signalized intersection, or a closed to traffic street (according to the parameters defined at the beginning of the III section). Then, the normalized (corresponding) value of safety is 100 (dark red color looking at the color bar, low/poor level of safety). This choice has been made because, assigning similar information with similar color, the distinctions and the relationships of the information contents can be easily understood [22]: then, the dark red color is assigned to the 'negative' features, i.e. a high level of air pollution, and a low level of safety.

It is important to remark that all the values displayed in the mobile application are related to the 5 available real-time alternative routes: 'short' travel time means that the selected route is the fastest one among the possible options; the highest (100) level of air pollution means that the chosen route is the most polluted one among the 5 alternatives and so on.

V. CONCLUSIONS AND FURTHER RESEARCH

This paper outlines a multi-objective real-time methodology that can assist cyclists (in particular, the customers of a bike-sharing system, station based or free-floating) in the selection of the smartest path to follow during

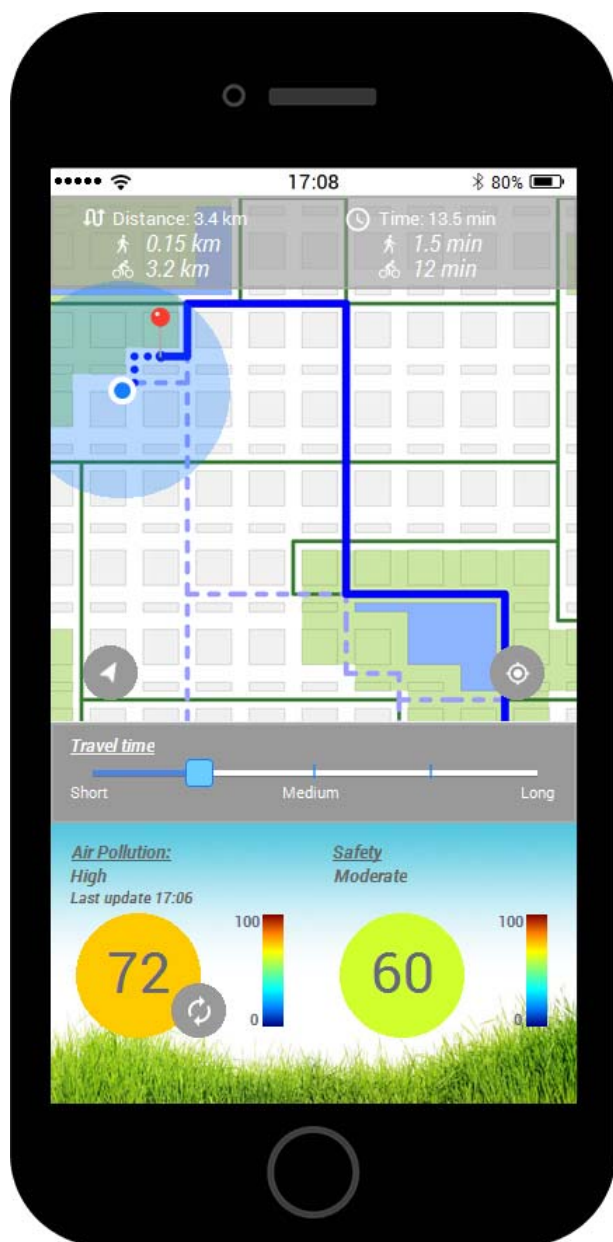


Fig. 6. Mobile-app screen: free-floating optimal and alternative paths.

their trips. It considers both the inherent characteristics of each link of the network and the cyclist personal preferences. The incorporation of the model in a mobile application is strongly recommended, and the basic features to necessarily be integrated are suggested, in order to be able to offer each user a range of 'best' options, among which he/she can freely choose satisfying his/her priorities. Further research will deal with the calibration of membership functions starting from the collection of bikers feedbacks and app usage statistics in order to improve and customize the suggested paths and remove possible correlations between the inputs of the problem.

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