

# Wavelet Secure Maps: enhancing privacy protected maps

Edwin de Jonge @edwindjonge

Statistics Netherlands Research & Development  
@edwindjonge

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# sdcsSpatial: Privacy protected maps

## Takeout message: sdcSpatial and Wavelets:

- sdcSpatial helps to assess sensitivity and create privacy protected density maps.
- protect\_wavelet novel method for protecting density maps.
- takes care of rural vs urban spatial resolution.
- can be seen as combination of protect\_smooth and protect\_quadtree.
- *work in progress, on github, not yet on CRAN*



# Who am I and why these protection methods?

- Statistical consultant, Data Scientist @cbs.nl / Statistics NL
- Statistics Netherlands is producer main official statistics in the Netherlands:
  - Stats on Demographics, economy (GDP), education, environment, agriculture, Finance etc.
  - Part of the European Statistical System, ESS.

## Motivation for sdcSpatial

- ESS has European Code of Statistical Practice (predates GDPR, European law on Data Protection):  
**no individual information may be revealed.**



# Sdc in sdcSpatial?

SDC = “Statistical Disclosure Control”

## Collection of statistical methods to:

- Check if data is safe to be published
- Protect data by slightly altering (aggregated) data
  - adding noise
  - shifting mass
- Most SDC methods operate on records.
- **sdcSpatial works upon locations.**



# Data

```
data(dwelling, package="sdcSpatial")  
nrow(dwelling)
```

```
## [1] 90603
```

```
head(dwelling) # consumption/unemployed are simulated!
```

##		x	y	consumption	unemployed
## 1		149712	470104	2049.926	FALSE
## 2		149639	469906	1814.938	FALSE
## 3		149631	469888	2074.882	FALSE
## 4		149788	469831	1927.989	FALSE
## 5		149773	469834	2164.969	FALSE
## 6		149688	469898	1987.958	FALSE



# Let's create a sdc\_raster

## Creation:

```
library(sdcSpatial)
unemployed <- sdc_raster( dwellings[c("x", "y")] # realistic locations
                        , dwellings$unemployed # simulated data!
                        , r = 500 # raster resolution of 500m
                        , min_count = 10 # min support
                        )
```

## What has been created?

```
print(unemployed)

## logical sdc_raster object:
##   resolution: 500 500 , max_risk: 0.95 , min_count: 10
##   mean sensitivity score [0,1]: 0.4249471
```

42% of the data on this map is sensitive and should be protected!



# Type of raster density maps:

(Stored in `unemployed$value`):

Density can be area-based:

- **number of people** per square (`$count`): population density.
- **(total) value** per square (`$sum`): number of unemployed per square.

Or density can population-based:

- **Mean value** per square (`$mean`): unemployment rate per square.

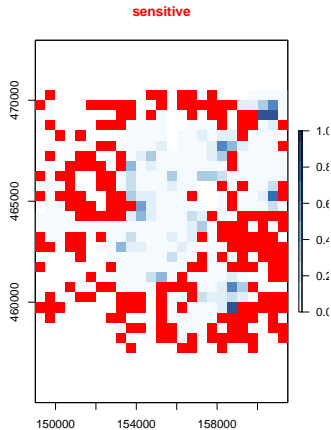
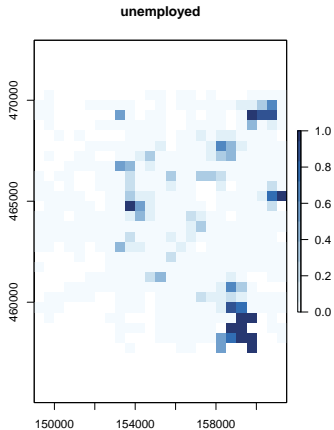
*Note: All density types are valid, but (total) value per square strongly interacts with population density.  
(e.g. <https://xkcd.com/1138>).*





# Plotting a `sdc_raster`

```
plot(unemployed, "mean")
```



# How to reduce sensitivity?

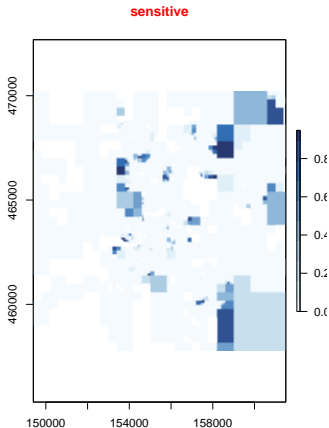
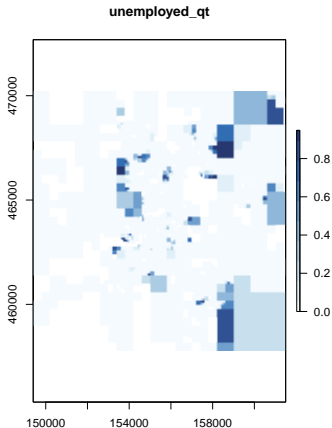
## Options:

- a) Remove sensitive locations: `remove_sensitive`.
- b) Use a coarser raster: `sdcraster`.
- c) Aggregate sensitive cells hierarchically with a quad tree until not sensitive: `protect_quadtree` (Suñé et al. 2017).
- d) Apply spatial smoothing: `protect_smooth` (Wolf and Jonge 2018; Jonge and Wolf 2016).
- e) Do a multi-resolution analysis `protect_wavelet`



# Option: protect\_quadtree

```
unemployed_100m <- sdc_raster( dwellings[c("x","y")], dwellings$unemployed  
                               , r = 100) # use a finer raster  
unemployed_qt <- protect_quadtree(unemployed_100m)  
plot(unemployed_qt)
```



# Option: protect\_quadtree

## Pro

- Adapts to data density
- Adjusts until no sensitive data is left.
- It just works. . .

## Cons

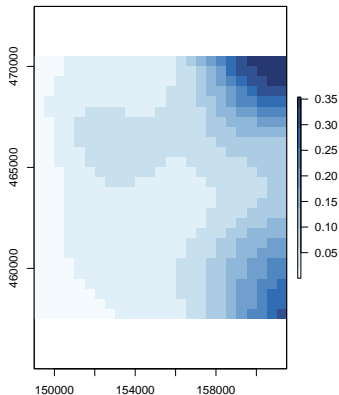
- Crude, it just works :-)
- Visually: “Blocky” / “Mondrian-like” / Minecraft-result
- Is not translation invariant (basis)
- Isn't there something smoother?



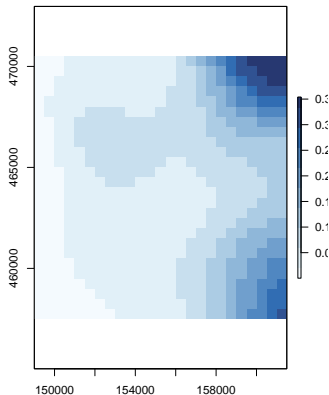
# Option: protect\_smooth

```
unemployed_smoothed <- protect_smooth(unemployed, bw = 1500)  
plot(unemployed_smoothed, "mean")
```

unemployed\_smoothed



sensitive



# Option: `protect_smooth`

## Pro's

- Often enhances spatial pattern visualization, removing spatial noise.
- Makes it a density map and used as source for e.g. contour map.

## Con's

- Does not remove all sensitive values (depends on bandwidth  $bw$ )
- A fixed band width is used for all locations: may remove detailed patterns. . .  
spatial processes often have location dependent band widths.



# Problem: smooth and adaptive

We need both a smooth and adaptive method!

## Wavelets

- Used for multi-resolution analysis (MRA), decompose signal / image at multiple resolutions.
- Used for denoising images: (“did I hear smoothing”?)

Also: - Used for lossy compression of images (e.g. JPEG!)

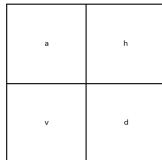
## We skip the math

- Except: wavelet decomposition can have different base functions, e.g. Haar, Daubechies etc.

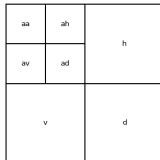


# Wavelet and images (e.g. JPEG)

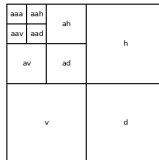
1 level  
decomposition



2 level  
decomposition



3 level  
decomposition



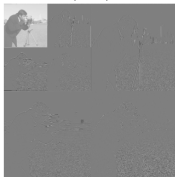
Image



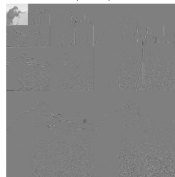
Coefficients  
(1 level)



Coefficients  
(2 level)



Coefficients  
(3 level)





# sdcSpatial and Wavelets:

- Enter: `protect_wavelet`
- Using `dwt.2d` and `waveslim::idwt.2d` of R package `waveslim` (Whitcher 2024)
- Builds a multi-resolution version of density map
- Checks sensitivity (privacy) multiple resolutions

```
unemployed_wvlt <- protect_wavelet(  
  unemployed,  
  wf = "la8", # wavelet transform / base functions  
  depth = 4, # resolution depth  
  ... # denoising parameters  
)
```



# The end

- `protect_wavelet` wip on github
- “raw version”, testing make it user friendly, in September on CRAN.

**Thank you for your attention!**

**Questions?**

**Curious?**

```
install.packages("sdcSpatial")
```

**Feedback and suggestions?**

<https://github.com/edwindj/sdcSpatial>



# References

- Jonge, Edwin de, and Peter-Paul de Wolf. 2016. "Spatial Smoothing and Statistical Disclosure Control." In *Privacy in Statistical Databases*, edited by Josep Domingo-Ferrer and Mirjana Pejić-Bach, 107–17. Springer.
- Suñé, E., C. Rovira, D. Ibáñez, and M. Farré. 2017. "Statistical Disclosure Control on Visualising Geocoded Population Data Using Quadtrees." [http://nt17.pg2.at/data/x\\_abstracts/x\\_abstract\\_286.docx](http://nt17.pg2.at/data/x_abstracts/x_abstract_286.docx).
- Whitcher, Brandon. 2024. *Waveslim: Basic Wavelet Routines for One-, Two-, and Three-Dimensional Signal Processing*. <https://CRAN.R-project.org/package=waveslim>.
- Wolf, Peter-Paul de, and Edwin de Jonge. 2018. "Spatial Smoothing and Statistical Disclosure Control." In *Privacy in Statistical Databases - PSD 2018*, edited by Josep Domingo-Ferrer and Francisco Montes Suay. Springer.

