Matrix 행렬

2×3 matrix

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

- Rectangular array of numbers, 2D array
- Dimension of matrix : # of rows × # of columns

$$A = \begin{bmatrix} 1402 & 191 \\ 1371 & 821 \\ 949 & 1437 \\ 147 & 1448 \end{bmatrix}$$

A_{ij} = "i,j entry" in the i^{th} row, j^{th} col.

ex. $A_{32} = 1437$

⇒ 정리 & 인덱싱 용이

Vector 벡터

- An $n \times 1$ matrix
- 행렬인데 n행 1열
- 1-indexed vs 0-indexed

$$y = \begin{bmatrix} 460 \\ 232 \\ 315 \\ 178 \end{bmatrix}$$

4차원 벡터 ($\mathbb{R}^4$)

y^i = i^{th} elemen

$y^1 = 460$, $y^2 = 232$ ···

보통 행렬에 대문자(ABCX), 벡터에 소문자(abcx) 사용

Matrix Addition 행렬 덧셈

$$\begin{bmatrix} 1 & 0 \\ 2 & 5 \\ 3 & 1 \end{bmatrix} + \begin{bmatrix} 4 & 0.5 \\ 2 & 5 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 0.5 \\ 4 & 10 \\ 3 & 2 \end{bmatrix}$$

3×2 matrix

※ 같은 차원 행렬끼리만 셈 가능

Scalar Multiplication 스칼라 곱셈

$$3 \times \begin{bmatrix} 1 & 0 \\ 2 & 5 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 6 & 15 \\ 9 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 0 \\ 6 & 3 \end{bmatrix} / 4 = \begin{bmatrix} 1 & 0 \\ 1.5 & 0.75 \end{bmatrix}$$

$$3 \times \begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix} - \begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix} / 3 = \begin{bmatrix} 3 \\ 12 \\ 6 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ \frac{2}{3} \end{bmatrix} = \begin{bmatrix} 2 \\ 12 \\ 10\frac{1}{3} \end{bmatrix}$$

Matrix Multiplication 행렬곱셈 – Mat · Vec

$$\begin{bmatrix} 1 & 3 \\ 4 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 5 \end{bmatrix} = \begin{bmatrix} 16 \\ 4 \\ 7 \end{bmatrix}$$

$1 \times 1 + 3 \times 5 = 16$

$4 \times 1 + 0 \times 5 = 4$

$2 \times 1 + 1 \times 5 = 7$

$$A \times x = y$$

$$\begin{bmatrix} \quad & \quad \\ & \end{bmatrix} \begin{bmatrix} \; \\ \; \end{bmatrix} = \begin{bmatrix} \; \\ \; \end{bmatrix}$$

m×n mat n×1 mat m-dim vector

To get y_i, multiply A's i^{th} row with elements of vector x, and add them up.

ex.

$$\begin{bmatrix} 1 & 2 & 1 & 5 \\ 0 & 3 & 0 & 4 \\ -1 & -2 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1\times1 + 2\times3 + 1\times2 + 5\times1 \\ 0\times1 + 3\times3 + 0\times2 + 4\times1 \\ -1\times1 + (-2)\times3 + 0\times2 + 0\times1 \end{bmatrix} = \begin{bmatrix} 14 \\ 13 \\ -7 \end{bmatrix}$$

hypothesis 에 적용.

House sizes : 2104, 1416, 1534, 852

$h_\theta(x) = -40 + 0.25x$

$$\begin{bmatrix} 1 & 2104 \\ 1 & 1416 \\ 1 & 1534 \\ 1 & 852 \end{bmatrix} \begin{bmatrix} -40 \\ 0.25 \end{bmatrix} = \begin{bmatrix} \; \\ \; \\ \; \\ \; \end{bmatrix}$$

$\therefore$ prediction = DataMatrix × Parameters

Matrix Multiplication 행렬 곱셈 - Mat × Mat

$$\underset{2\times3}{\begin{bmatrix} 1 & 3 & 2 \\ 4 & 0 & 1 \end{bmatrix}} \underset{3\times2}{\begin{bmatrix} 1 & 3 \\ 0 & 1 \\ 5 & 2 \end{bmatrix}} = \underset{2\times2}{\begin{bmatrix} 11 & 10 \\ 9 & 14 \end{bmatrix}}$$

$A \times B = C$

n×1 벡터를 이어 붙인다는 개념으로 접근 OK

The i^{th} col of the matrix C is obtained by multiplying A with the i^{th} col of B.

ex.
$$\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 9 & 7 \\ 15 & 12 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} 0 \\ 3 \end{bmatrix} = \begin{bmatrix} 1\times0 + 3\times3 \\ 2\times0 + 5\times3 \end{bmatrix} = \begin{bmatrix} 9 \\ 15 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1\times1 + 3\times2 \\ 2\times1 + 5\times2 \end{bmatrix} = \begin{bmatrix} 7 \\ 12 \end{bmatrix}$$

hypothesis 에 적용.

House sizes : 2104, 1416, 1534, 852

3 competitive hypotheses

1. $h_\theta(x) = -40 + 0.25x$
2. $h_\theta(x) = 200 + 0.1x$
3. $h_\theta(x) = -150 + 0.4x$

$$\underset{4\times 3}{\begin{bmatrix} 1 & 2104 \\ 1 & 1416 \\ 1 & 1534 \\ 1 & 852 \end{bmatrix}} \underset{2\times 3}{\begin{bmatrix} -40 & 200 & -150 \\ 0.25 & 0.1 & 0.4 \end{bmatrix}} = \underset{4\times 3}{\begin{bmatrix} \bigcirc & & \\ & & \\ & & \\ & & \end{bmatrix}}$$

"Commutative" 3x5 = 5x3

Matmul은 교환법칙이 성립하지 않음

$$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\neq$$

$$\begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 2 & 2 \end{bmatrix}$$

$$\underset{m\times n}{A} \times \underset{n\times m}{B} = \underset{m\times m}{C}$$

$$\neq$$

$$B \times A = \underset{n\times n}{D}$$

But!

$$A\times B\times C \Rightarrow (A\times B)\times C = A\times(B\times C)$$

결합법칙은 성립 "Associative"

Identity Matrix 단위행렬 I

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdots$$

대각선 성분들이 '1'

$$\begin{bmatrix} 1 & & 0 \\ & 1 & \\ & 1 & \\ 0 & & \ddots \quad 1 \end{bmatrix}$$

for any matrix A, $A \cdot I = I \cdot A = A$ * $AB \neq BA$

Matrix Inverse 역행렬 A^{-1}

If A is an m×m $\underbrace{\text{(square matrix)}}_{\text{정방행렬}}$ and if it has an inverse,

$$AA^{-1} = A^{-1}A = I$$

역행렬이 없는 행렬을 singular matrix / degeneactive matrix 라고 함. ex. $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Matrix Transpose 전치행렬 A^T

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 5 & 9 \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & 3 \\ 2 & 5 \\ 0 & 9 \end{bmatrix} \quad A^T_{ij} = A_{ji} = 3$$

A^T_{12}

A_{21}