Embedding Words as Distributions with a Bayesian Skip-gram Model

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Abstract

We introduce a method for embedding words as probability densities in a low-dimensional space. Rather than assuming that a word embedding is fixed across the entire text collection, as in standard word embedding methods, in our Bayesian model we generate it from a word-specific prior density for each occurrence of a given word. Intuitively, for each word, the prior density encodes the distribution of its potential 'meanings'. These prior densities are conceptually similar to Gaussian embeddings of Vilnis and McCallum (2014). Interestingly, unlike the Gaussian embeddings, we can also obtain context-specific densities: they encode uncertainty about the sense of a word given its context and correspond to posterior distributions within our model. The context-dependent densities have many potential applications: for example, we show that they can be directly used in the lexical substitution task. We describe an effective estimation method based on the variational autoencoding framework. We also demonstrate that our embeddings achieve competitive results on standard benchmarks.

1 Introduction

Distributed representations of words induced from large unlabeled text collections have had a large impact on many natural language processing (NLP) applications, providing an effective and simple way of dealing with data sparsity. Word embedding methods typically represent words as vectors in a low-dimensional space (Deerwester et al., 1990; Collobert et al., 2011; Mikolov et al., 2013; Pennington et al., 2014). In contrast, we encode them as probability densities (see an illustration in Figure 1). Intuitively, the densities represent the distributions over possible 'meanings' of a word. Representing a word as a distribution provides many potential benefits. For example, such embeddings let us encode generality of terms (e.g., 'kakapo' is a hypernym of 'bird'), characterize uncertainty about semantic properties of the corresponding referent (e.g., a proper noun, such as 'John', encodes little about the person it refers to) or represent polysemy (e.g., 'kiwi' may refer to a fruit, a bird or a New Zealander).

The main inspiration for this work is Gaussian embeddings (*word2gauss*, W2G) introduced by Vilnis and McCallum (2014). They represent words as Gaussian distributions and directly optimize an objective expressed in terms of divergences (e.g., the Kullback-Liebler divergence) between the distributions. In contrast, we approach the problem from the generative Bayesian perspective. Though, as in W2G, context-agnostic densities are present in our model (they correspond to data-dependent priors), unlike W2G, we can also perform posterior inference and obtain context-specific densities. These posterior densities encode semantic properties of a word in a given context. For example, in Figure 1, when 'kiwi' appears in a context suggesting the 'bird' sense, the posterior (represented by the shaded ellipsoid) becomes more 'peaky' and moves towards the representation of word 'bird'. We use a lexical substitution task (McCarthy and Navigli, 2007) to demonstrate that the posterior densities are effective

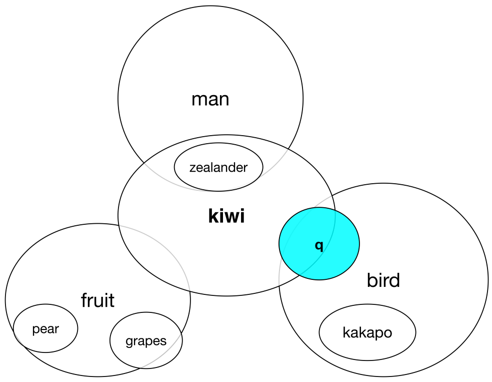


Figure 1: An idealized illustration of density embeddings. Unshaded ellipsoids encode prior densities of Gaussians. The shaded ellipsoid corresponds to the posterior for the word 'kiwi' when it appears in a context indicating that 'kiwi' refers to a bird.

in predicting potential replacements of a word given a context. Importantly, though the Gaussian assumption for context-agnostic embeddings is questionable (e.g., polysemous words would need multimodal distributions to properly represent their meanings), the same assumption for the posterior context-specific densities is more reasonable: the word is likely to be disambiguated by the provided context and hence does not require complex families of distributions.

In principle, using densities to represent words provides a natural way of encoding entailment: the decision regarding entailment relation can be made by testing the level sets of the distributions for 'soft inclusion'. For example, in Figure 1, the ellipse for 'kakapo' lies within the ellipse for 'bird'. Vilnis and McCallum (2014) proposed to use the KL divergence to detect entailment. In our analysis, we observe that, though the covariances indeed encode information relevant to entailment, their direct use is somewhat problematic both with W2G and with our model.

We are not the first to propose a Bayesian version of word embedding methods (Zhang et al., 2014; Sakaya, 2015; Barkan, 2016). However, our approach is fundamentally different from the previous work. The previous methods can be regarded as applications of Bayesian matrix factorization (BMF) (Salakhutdinov and Mnih, 2008) to word co-occurrence matrices. Hence they assume that every word has a fixed (but unknown) real-valued embeddings which are shared across the entire text collection. Instead, in our approach, we acknowledge that word meaning inherently depends on a context and do not assume that any fixed vector exists at type level: we draw it at token level (i.e. for each word occurrence) from a parameterized word-specific prior distribution. Consequently, unlike us and also unlike densities in word2gauss, the posteriors in previous Bayesian embeddings models encode uncertainty about the embedding parameters; they will converge to a delta distribution as the amount of data increases. In contrast, our densities encode the distribution of senses and, as confirmed in our experiments, do not follow this trend.

More formally, our model can be regarded as a form of coupled matrix factorization where each sliding window is factorized individually, but priors for words are shared across all the windows. We describe an effective estimation method based on the variational autoencoding (VAE) framework (Kingma and Welling, 2013). The context-specific densities are provided by the encoder component of VAE (i.e. the inference network).

Our main contributions can be summarized as follows:

- we proposed a Bayesian model for embedding words as probability densities;
- we derived a computationally-efficient inference algorithm, which, as a by-product, yields context-sensitives densities;
- we demonstrate their effectiveness, including on the lexical substitution task.

2 Bayesian Skip-gram

To motivate our Bayesian extension of the Skip-gram model, consider the polysemous word 'kiwi' (Figure 1). Its meaning changes depending on the context: for example, when it appears in 'I like apples, kiwi, and bananas', we can deduce that 'kiwi' refers to fruit. Polysemy cannot be effectively captured with a single representation induced by standard word embedding methods (e.g., skip-gram (Mikolov et al., 2013)) or even with a single distribution, as in W2G. We also do not want to assume that there is a finite set of discrete senses, as made in multi-sense mixture models (Li and Jurafsky, 2015). Sense distinctions are often gradual and discrete senses are only clusters that approximate underlying meaning distributions (Kilgarriff, 1997; Erk and McCarthy, 2009). Our Bayesian Skip-gram (BSG) model addresses this problem. It predicts a distribution of 'meanings' given a context. Intuitively, if the context is very discriminative, this density becomes very peaky, assigning the entire probability mass to the predicted 'meaning' (in the limit, encoding the word as a point vector). If the context is less informative, the distribution will remain flat, representing uncertainty about the word sense.

2.1 Generative model

The skip-gram (SG) model aims at maximizing the probability of words in a context window given its central word. The probability of each context word is assumed proportional to the dot product of its representation and that of the central word (see the graphical model in Figure 2a). In contrast, BSG assumes that the choice of context words is dependent on the context-specific (latent) meaning of the central word (see Figure 2b).

The generative story of BSG is the following: first, draw a word $w \sim p(w)$ (e.g. 'kiwi'), then sample its latent meaning $\mathbf{z} \sim p_{\boldsymbol{\theta}}(\mathbf{z}|w)$ (e.g., a vector that encodes the meaning 'bird'), finally draw context words $c \sim p_{\boldsymbol{\theta}}(c|\mathbf{z})$ (e.g., 'flightless', 'forest', and 'feather').

2.2 Model definition

We maximize the log-likelihood, which is the sum of the following terms, one per each window:

$$\log p_{\boldsymbol{\theta}}(\mathbf{c}|w) = \log \int \prod_{j=1}^{C} p_{\boldsymbol{\theta}}(c_j|\mathbf{z}) p_{\boldsymbol{\theta}}(\mathbf{z}|w) \mathbf{d}\boldsymbol{z}, \quad (1)$$

where C is the size of the context window, c_j is the context word for the central word w, and θ are model parameters. Unfortunately, due to the integration over the latent space, the marginal log-likelihood and its derivatives cannot be efficiently computed. Instead, we rely on variational inference, specifically the variational auto-encoding framework (Kingma and Welling, 2013). We optimize the variational lower bound of the marginal likelihood:

$$\begin{aligned} \log p_{\boldsymbol{\theta}}(\mathbf{c}|w) \geq \sum_{j=1}^{C} \mathbb{E}_{q_{\boldsymbol{\phi}}(\mathbf{z}|\mathbf{c},w)} \left[\log p_{\boldsymbol{\theta}}(c_j|\mathbf{z})\right] - \\ \mathbb{D}_{KL}\left[q_{\boldsymbol{\phi}}(\mathbf{z}|\mathbf{c},w) \| p_{\boldsymbol{\theta}}(\mathbf{z}|w)\right] \end{aligned} \quad (2)$$

Here, $\mathbb{D}_{KL}$ denotes Kullback–Leibler divergence, and $q_{\boldsymbol{\phi}}(\mathbf{z}|\mathbf{c},w)$ is the approximate posterior distribution, which we can be used to infer the representation ('meaning') of the central word w in the context $\mathbf{c}$. For now, we will assume that the approximate posterior is a Gaussian distribution with a diagonal covariance matrix, namely $q_{\boldsymbol{\phi}}(\mathbf{z}|\mathbf{c},w) = \mathcal{N}(\boldsymbol{z}|\boldsymbol{\mu}_q, \boldsymbol{\Sigma}_q)$. The covariance matrix of the approximate posterior encodes uncertainty or generality of meaning of the central word w in the context $\boldsymbol{c}$. Intuitively, if the meaning is concrete, we would like to get small variance values and large, otherwise.

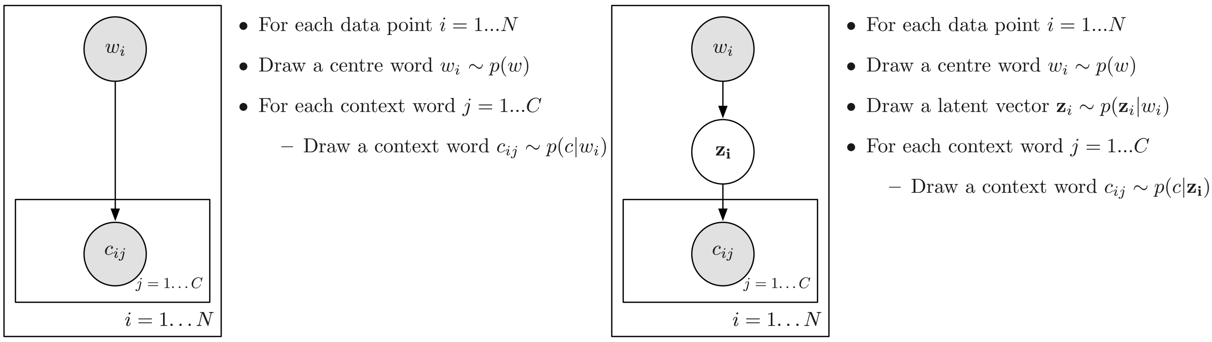
Another interesting component of our model is $p_{\boldsymbol{\theta}}(\mathbf{z}|w)$, to which we will refer as a *prior*, and we will assume that it is also a Gaussian distribution with a diagonal covariance and individual parameters for each central word $p_{\boldsymbol{\theta}}(\mathbf{z}|w) = \mathcal{N}(\boldsymbol{z}|\boldsymbol{\mu}_w, \boldsymbol{\Sigma}_w)$. Fortunately, the KL divergence term between two normal distributions can be expressed in closed form.

2.3 Reconstruction error

The most troublesome component of the lower-bound is the expected reconstruction term $\mathbb{E}_{q_{\boldsymbol{\phi}}(\mathbf{z}|\mathbf{c},w)}\left[\log p_{\boldsymbol{\theta}}(c_j|\mathbf{z})\right]$ which can be decomposed as shown in Eq. (3):

$$\begin{aligned} &\mathbb{E}_{q_{\boldsymbol{\phi}}(\mathbf{z}|\mathbf{c},w)}\left[\log p_{\boldsymbol{\theta}}(c_j|\mathbf{z})\right] = \\ &\mathbb{E}_{q_{\boldsymbol{\phi}}(\mathbf{z}|\mathbf{c},w)}\left[\log \frac{\exp(f_{\boldsymbol{\theta}}(\mathbf{z}, c_j))}{\sum_{k=1}^{|V|}\exp(f_{\boldsymbol{\theta}}(\mathbf{z}, c_k))}\right] \end{aligned} \quad (3)$$

where $f_{\boldsymbol{\theta}}(\mathbf{z}, c_j)$ is a function that models relationship between latent vector $\mathbf{z}$ and context word c_j. In the original skip-gram model it was the dot product. Intuitively, the dot product is based on the angle between two vectors, which makes a lot of sense with point estimates. However, it is problematic when we use normal distributions. Consider the fixed angle cone and two densities in Figure 3. Both densities encode the same uncertainty about the angle (and the angle is all that matters for the dot product model), whereas their variances are very different. In other



(a) SG

(b) BSG

Figure 2: Bayesian networks corresponding to Skip-gram and Bayesian skip-gram.

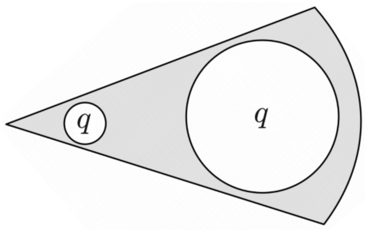


Figure 3: Shaded cone is a fixed angle, and ellipses are approximate posterior Gaussian distributions. The corner of the cone is at the origin.

words, the model has too many degrees of freedom. For example, uncertainty in the dot product can be increased either by moving the density towards the origin or by increasing the variance.

To address this problem, we propose to use a different form of the function f: $f_{\boldsymbol{\theta}}(\mathbf{z}, c_j) = \log\left(\mathcal{N}(\mathbf{z}; \boldsymbol{\mu}_{c_j}, \boldsymbol{\Sigma}_{c_j}) \times p(c_j)\right)$ as shown in Eq. (4). Intuitively, scaling with $p(c_j)$ (e.g., the empirical unigram probability) encodes that frequent words are more likely to appear in any context. The Gaussian density evaluates how well a word fits context given the meaning $\mathbf{z}$ of the central word and is sensitive to the variance, hence addressing the above-mentioned problem.

$$\mathbb{E}_{q_{\boldsymbol{\phi}}(\mathbf{z}|\mathbf{c},w)}\left[\log \frac{\exp(f_{\boldsymbol{\theta}}(\mathbf{z}, c_j))}{\sum_{k=1}^{|V|}\exp(f_{\boldsymbol{\theta}}(\mathbf{z}, c_k))}\right] = \mathbb{E}_{q_{\boldsymbol{\phi}}(\mathbf{z}|\mathbf{c},w)}\Bigg[\log\left(\mathcal{N}(\mathbf{z}; \boldsymbol{\mu}_{c_j}, \boldsymbol{\Sigma}_{c_j})p(c_j)\right) - \log\sum_{k=1}^{|V|}\mathcal{N}(\mathbf{z}; \boldsymbol{\mu}_{c_k}, \boldsymbol{\Sigma}_{c_k})p(c_k)\Bigg], \tag{4}$$

2.4 Encoder

A very important component of our model is the inference network $q_{\boldsymbol{\phi}}(\mathbf{z}|\mathbf{c}, w)$ (also known as an encoder), which approximates the posterior distribution $p_\theta(\mathbf{z}|\mathbf{c}, w)$. As we discussed, we make the Gaussian assumption for q.

Using the same principle as in (Kingma and Welling, 2013), we will use a simple feed-forward neural network to compute variance and mean parameters of the distribution by inputting context words and a central word. Its architecture is shown in Figure 4.

$$\mathbf{h} = \sum_{j=1}^{C} relu\left(\mathbf{M}\begin{bmatrix}\mathbf{R}_{c_j}\\ \mathbf{R}_w\end{bmatrix}\right) \tag{5}$$

$$\boldsymbol{\mu}_q = \mathbf{U}\mathbf{h} + \mathbf{b}_1 \tag{6}$$

$$\log\sigma_q^2 = \mathbf{W}\mathbf{h} + b_2 \tag{7}$$

We obtain the hidden layer as in Eq. (5) by concatenating context and central words representations and performing linear projection with matrix $\mathbf{M}$, followed by relu non-linearity. Finally, we sum up the

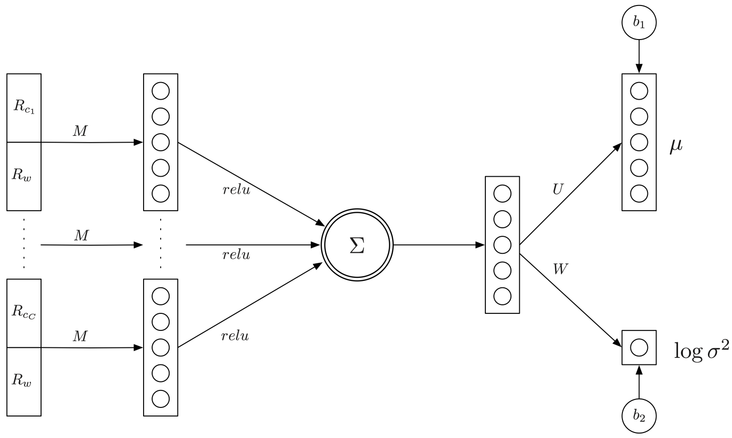


Figure 4: Encoder architecture.

transformed representations into one hidden layer that is used to generate $\boldsymbol{\mu}_q$ and $\log\sigma^2$ vectors as shown in Eq. (6) and (7). Using logarithm is necessary to ensure that the matrix is positive definite.

The encoder can be understood as a parametrized function that produces distributions corresponding to meanings of central words in different contexts. Technically, using this function, we can model infinitely many meanings of words without explicitly storing their representations.

2.5 Approximation of the log-partition function

Despite the fact that one can obtain a low-variance estimate of the first part of the expectation from the Eq. (4) using the re-parameterization trick (Kingma and Welling, 2013), the expected log-partition function is still very computationally demanding, as it involves summation over all words in the vocabulary.

In order to make it scalable to large vocabularies, we compute the lower bound of the log-partition function and use a Monte Carlo (MC) approximation to obtain an unbiased estimate of the expectation over all words as shown in Eq. (8).

$$\begin{aligned}&\mathbb{E}_{q_{\boldsymbol{\phi}}(\mathbf{z}|\mathbf{c},w)}\left[\log \mathbb{E}_{p(\tilde{c})}\left[\mathcal{N}(\mathbf{z};\boldsymbol{\mu}_{\tilde{c}},\boldsymbol{\Sigma}_{\tilde{c}})\right]\right] \geq \\ &\mathbb{E}_{q_{\boldsymbol{\phi}}(\mathbf{z}|\mathbf{c},w)}\left[\mathbb{E}_{p(\tilde{c})}\left[\log \mathcal{N}(\mathbf{z};\boldsymbol{\mu}_{\tilde{c}},\boldsymbol{\Sigma}_{\tilde{c}})\right]\right] \approx \\ &\mathbb{E}_{q_{\boldsymbol{\phi}}(\mathbf{z}|\mathbf{c},w)}\left[\log \mathcal{N}(\mathbf{z};\boldsymbol{\mu}_{\tilde{c}_j},\boldsymbol{\Sigma}_{\tilde{c}_j})\right]\end{aligned} \tag{8}$$

The estimate involves only one word $\tilde{c}_j$ which can be easily sampled from $p(\tilde{c})$. By replacing the log-partition function from Eq. (4) with the unbiased estimate of its lower bound derived in Eq. (8), we can get the approximate lower-bound of marginal log-likelihood[1] shown in Eq. (9). The sum $\sum_{(j,k)}$ in the equation is now over pairs of positive c_j and negative $\tilde{c_k}$ context words. Furthermore, we can transform the difference of negative cross-entropies into the difference of Kullback-Leibler divergences by adding and subtracting entropies of $q_\phi(\mathbf{z}|\mathbf{c},w)$.

$$\begin{aligned}&\hat{\mathcal{L}}(\boldsymbol{\phi},\boldsymbol{\theta};\mathbf{c},w) = -\mathbb{D}_{KL}\left[q_{\boldsymbol{\phi}}(\mathbf{z}|\mathbf{c},w)\|p_{\boldsymbol{\theta}}(\mathbf{z}|w)\right] \\ &+\sum_{(j,k)}\Big(\mathbb{E}_{q_{\boldsymbol{\phi}}(\mathbf{z}|\mathbf{c},w)}\left[\log\mathcal{N}(\mathbf{z};\boldsymbol{\mu}_{c_j},\boldsymbol{\Sigma}_{c_j})\right] - \\ &\qquad\mathbb{E}_{q_{\boldsymbol{\phi}}(\mathbf{z}|\mathbf{c},w)}\left[\log\mathcal{N}(\mathbf{z};\boldsymbol{\mu}_{\tilde{c_k}},\boldsymbol{\Sigma}_{\tilde{c_k}})\right]\Big) = \\ &\sum_{(j,k)}\left(\mathbb{D}_{KL}\left[q_{\boldsymbol{\phi}}(\mathbf{z}|\mathbf{c},w)\|\mathcal{N}(\mathbf{z};\boldsymbol{\mu}_{\tilde{c_k}},\boldsymbol{\Sigma}_{\tilde{c_k}})\right] - \right. \\ &\qquad\left.\mathbb{D}_{KL}\left[q_{\boldsymbol{\phi}}(\mathbf{z}|\mathbf{c},w)\|\mathcal{N}(\mathbf{z};\boldsymbol{\mu}_{c_j},\boldsymbol{\Sigma}_{c_j})\right]\right) - \\ &\qquad\mathbb{D}_{KL}\left[q_{\boldsymbol{\phi}}(\mathbf{z}|\mathbf{c},w)\|p_{\boldsymbol{\theta}}(\mathbf{z}|w)\right]\end{aligned} \tag{9}$$

First term in the above equation suggests maximization of the margin between divergences that involve negative and positive words. We transform it into the hard-margin form that involves the hinge loss as shown in Eq. (10). This is the final objective function that we maximize when training the BSG model.

[1] Notice that $p(c_j)$ is omitted because it factors out as a constant and does not change the optimum.

$$\widetilde{\mathcal{L}}_m(\boldsymbol{\phi}, \boldsymbol{\theta}; \mathbf{c}, w, \tilde{c}) = \mathbb{D}_{KL}\left[q_{\boldsymbol{\phi}}(\mathbf{z}) \| p_{\boldsymbol{\theta}}(\mathbf{z}|w)\right] + \sum_{(j,k)} \max\Big(0,\ \mathbb{D}_{KL}\left[q_{\boldsymbol{\phi}}(\mathbf{z}) \| \mathcal{N}(\mathbf{z}; \boldsymbol{\mu}_{c_j}, \boldsymbol{\Sigma}_{c_j})\right] - \mathbb{D}_{KL}\left[q_{\phi}(\mathbf{z}) \| \mathcal{N}(\mathbf{z}; \boldsymbol{\mu}_{\tilde{c}_k}, \boldsymbol{\Sigma}_{\tilde{c}_k})\right] + m\Big) \tag{10}$$

This transformation of the objective is necessary because the KL terms are unconstrained, and during optimization, the model tends to assign extreme values for the frequent negative words and this has detrimental effect on the overall performance. The hinge loss solves this problem by setting the gain to zero when the KL term of a negative context word is larger than the KL term of a positive one by a margin.

The intuition behind the objective is the following: once we generated the posterior distribution $q_{\boldsymbol{\phi}}(\mathbf{z}|\mathbf{c}, w)$, we optimize parameters to make $\mathbb{D}_{KL}\left[q_{\boldsymbol{\phi}}(\mathbf{z}|\mathbf{c}, w) \| p_{\boldsymbol{\theta}}(\mathbf{z}|w)\right]$ small, which can be intuitively understood as a regularization preventing $q_{\boldsymbol{\phi}}(\mathbf{z}|\mathbf{c}, w)$ ellipsoid from diverging from $p_{\boldsymbol{\theta}}(\mathbf{z}|w)$ ellipsoid. In addition, we discriminate positive context words from negative ones by comparing their divergences from $q_{\boldsymbol{\phi}}(\mathbf{z}|\mathbf{c}, w)$.

However, because the expected value of the log-partition function in Eq. (3) is with the negative sign, and we approximate its lower-bound, one could argue that resulting objective function in Eq. (9) is not a lower bound on the likelihood anymore. However, we still can use gradients of the resulting objective function to optimize the original one. Equations (11) and (12) show gradients of the expectation of the log-partition function and its lower bound with respect to $\boldsymbol{\theta}_i = \{\boldsymbol{\mu}_{c_i}, \boldsymbol{\Sigma}_{c_i}\}$, respectively.

$$\nabla_{\boldsymbol{\theta}_i} \mathbb{E}_{q_{\boldsymbol{\phi}}(\mathbf{z}|\mathbf{c},w)}\left[\log \mathbb{E}_{p(c)}\left[\mathcal{N}(\mathbf{z}; \boldsymbol{\mu}_c, \boldsymbol{\Sigma}_c)\right]\right] = \mathbb{E}_{q_{\boldsymbol{\phi}}(\mathbf{z}|\mathbf{c},w)}\left[\frac{p(c_i)\nabla_{\boldsymbol{\theta}_i}\mathcal{N}(\mathbf{z}; \boldsymbol{\mu}_{c_i}, \boldsymbol{\Sigma}_{c_i})}{\mathbb{E}_{p(c)}\left[\mathcal{N}(\mathbf{z}; \boldsymbol{\mu}_c, \boldsymbol{\Sigma}_c)\right]}\right] \tag{11}$$

$$\nabla_{\boldsymbol{\theta}_i} \mathbb{E}_{q_{\boldsymbol{\phi}}(\mathbf{z}|\mathbf{c},w)}\left[\mathbb{E}_{p(c)}\left[\log \mathcal{N}(\mathbf{z}; \boldsymbol{\mu}_c, \boldsymbol{\Sigma}_c)\right]\right] = \mathbb{E}_{q_{\boldsymbol{\phi}}(\mathbf{z}|\mathbf{c},w)}\left[\frac{p(c_i)\nabla_{\boldsymbol{\theta}_i}\mathcal{N}(\mathbf{z}; \boldsymbol{\mu}_{c_i}, \boldsymbol{\Sigma}_{c_i})}{\mathcal{N}(\mathbf{z}; \boldsymbol{\mu}_{c_i}, \boldsymbol{\Sigma}_{c_i})}\right] \tag{12}$$

We observe that the direction of the MC estimates of the gradients is the same: the only difference is the positive multiplicative constant. Therefore, optimizing the lower bound is closely related to optimizing the true expectation of the partition function.

3 Experiments

In this section we empirically evaluate our approach. First, in subsections 3.2 - 3.4, we use standard benchmarks to evaluate and better understand properties of our context-agnostic embeddings (i.e. the prior distributions $\mathcal{N}(\mathbf{z}; \boldsymbol{\mu}_{c_j}, \boldsymbol{\Sigma}_{c_j})$). In these experiments, we can directly compare BSG to W2G and the vanilla version of skip-gram. In subsection 3.5, we also demonstrate that the context-sensitive posterior distributions $q_{\boldsymbol{\phi}}(\mathbf{z}|\mathbf{c}, w)$, estimated by the VAE inference network, can be used to select potential substitutes of a word in a given context.

3.1 Experimental settings

We used a concatenation of ukWaC and WaCkypedia (Baroni et al., 2009) corpora, resulting in approximately 3 billion tokens. We restricted the vocabulary size to 280,000 most frequent word types and used the sub-sampling procedure introduced in Mikolov et al. (2013) with $t = 10^{-4}$. The window size was set to 5 words on each side, the dimensionality of the embeddings and of the hidden layer of the encoder were both set to 100. The number of negative samples was equal to the number of positive ones (i.e. 10). For BSG, we used spherical covariances both for the posterior and prior densities, as they resulted in better performance in preliminary experiments: this is consistent with results reported for W2G in Vilnis and McCallum (2014). As an optimizer we used Adam (Kingma and Ba, 2014). We used large batches of 22,000 prediction tasks each.

In order to enable fair comparison, we used our own implementation of W2G and SG, which shared the above hyperparameters with BSG.[2] The implementations of our model and the baselines will be publicly released should the paper get accepted. The

[2]The original implementation of W2G is not publicly available. Our re-implementation yields stronger results on similarity benchmarks but weaker on the entailment dataset of Baroni et al. (2012). Our implementation is also stronger across the board (including entailment) than the third party implementation used in Vulić et al. (2016) (`https://github.com/seomoz/word2gauss`). Athiwaratkun and Wilson (2017) also report W2G numbers very similar to ours.

initial learning rates were tuned for all models individually and set to 0.00055, 0.0065, 0.0015 and 0.0015 for BSG, W2G with the spherical covariances (WG(S)), W2G with the diagonal covariance matrix (WG(D)) and SG, respectively. All results presented in this section were obtained from one model of each type (i.e. no tuning was performed for individual benchmarks).

3.2 Word similarity

Datasets	BSG	WG(S)	WG(D)	SG
MC-30	0.71	0.69	0.70	**0.72**
MEN-TR-3k	**0.73**	0.72	0.71	0.72
MTurk-287	0.70	0.70	0.69	**0.70**
MTurk-771	**0.67**	0.65	0.64	0.65
RG-65	0.70	0.69	0.71	**0.72**
RW-STNFRD	0.43	0.43	0.42	**0.44**
SIMLEX-999	**0.35**	0.34	0.34	0.34
VERB-143	0.32	**0.38**	0.29	0.36
WS-353-ALL	**0.72**	0.68	0.67	0.69
WS-353-REL	**0.68**	0.66	0.65	0.65
WS-353-SIM	**0.75**	0.70	0.68	0.71
YP-130	**0.50**	0.46	0.46	0.45
Sum	**7.26**	7.10	6.95	7.15

Table 1: Evaluation results on word similarity datasets.

In Table 1 we present similarity results computed using the online evaluation tool of Faruqui and Dyer (2014). In these experiments, we used only the mean vectors from the prior and ignored the covariance information, both for BSG and W2Gs. First, we observe that BSG has a slight edge over both the original SG and also over W2G versions. Second, contrary to observations in (Vilnis and McCallum, 2014), W2G does not reach a performance of SG in our experiments. As their implementation is not available, it is hard to pinpoint the reason for the discrepancy. Note that their SG baseline is considerably weaker than ours, so it may be easier to beat. Our SG baselines outperforms their reported results on all similarity datasets (they considered only 8 out of 12 in Table 1) and also in average on these datasets (0.6248 vs. 0.5724).[3] These results suggest that prior means induced by BSG are indeed effective in capturing semantic properties of words.

3.3 Entailment recognition

In this section, we consider the lexical entailment task (i.e. essentially hyponymy detection). Note that, as with word similarity, word context is not provided, and, thus, we cannot showcase the main advantage of our approach: its disambiguation capabilities.

Given an ordered pair of words, the task is to predict if the first word (w_1) entails the second one (w_2). We use two entailment measures: the negated KL divergence $-D_{KL}\left[\mathcal{N}(\mathbf{z};\boldsymbol{\mu}_{w_1},\boldsymbol{\Sigma}_{w_1})\|\mathcal{N}(\mathbf{z};\boldsymbol{\mu}_{w_2},\boldsymbol{\Sigma}_{w_2})\right]$ and the cosine similarity between the means. We predict that w_1 entails w_2 if the corresponding score is above a certain threshold. As in (Vilnis and McCallum, 2014), the scores are optimistic, as the threshold is chosen on the test set. The thresholds are set individually for each method (including the baselines) and, hence, the comparison is fair.

Intuitively, KL should be a good choice: it would favor word pairs such that, not only their means are similar, but also such that the region, where the density function for w_1 is non-negligible, lies within the area where the density of w_2 is also high enough. Roughly speaking, level sets for w_1 should lie within level sets for w_2 (as with ('pear', 'fruit') in Figure 1).

First, we consider the entailment benchmark of Baroni et al. (2012), we will refer to it as BBDS (Table 2, left part). The results are generally consistent with ones reported by Vilnis and McCallum. The best W2G(S) model outperforms SG. They also have the edge over BSG. We also observe that, unlike W2G, covariance information appears not to be particularly beneficial for BSG (i.e. cosine performs essentially as well as KL).

Second, we turn to the BLESS dataset (Baroni and Lenci, 2011).[4] All the considered methods perform badly on BLESS. This is even more evident from Figure 5, where we plot the histogram of the KL divergence for the entailing and not entailing pairs. Clearly, the two classes cannot be separated based on KL alone, neither for BSG, nor W2G.

[3]The popular Gensim SG implementation is even slightly stronger than ours (0.6365), though not really comparable because of major differences in optimization.

[4]The version we are using is from Levy et al. (2015)

	BBDS		BLESS	
Model	KL	Cos	KL	Cos
BSG	76.2	**75.9**	20.0	20.8
W2G(S)	**77.0**	75.7	18.9	20.4
W2G(D)	76.5	74.9	18.7	20.3
SG	-	75.7	-	20.3

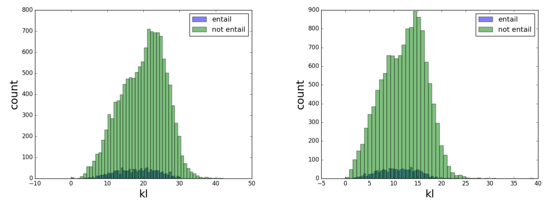
Table 2: F1 scores (%) on entailment recognition.

We hypothesize that the reason for these, seemingly inconsistent results, is that all considered methods struggle with distinguishing hypernymy ('dog' and 'pet') from co-hyponymy ('dog' and 'cat'), as well as from other types of semantic relatedness. Making such distinction is a challenging for unsupervised method (Weeds et al., 2014). Indeed, the BLESS dataset is harder than BBDS: it is unbalanced and contains about 10 times more negatives examples than positive ones, and the negative examples are often co-hyponyms.

So, why does KL not work as well as we expected? Intuitively, KL can be regarded as balancing two trade-offs: penalizing for divergences between means and also penalizing for the 'wrong' type of 'inclusion' between the distributions. It is very easy to see this for the one-dimensional case, where KL can be written as:

$$\log \frac{\sigma_2}{\sigma_1} + \frac{\sigma_1^2 + (\mu_1 - \mu_2)^2}{2\sigma_2^2} - \frac{1}{2},$$

but holds for the multivariate case as well. If the variances are sufficiently similar, then the distance between means is all that matters. It does seem that, for the Gaussian priors and for both methods, covariances end up playing a relatively minor role. Indeed, when we checked for correlations between KL and cosine similarity, we observed that it is very strong, confirming our hypothesis. For example, dog entails animal, but dog does not entail cat, while scores in Table 3 indicate the opposite. Generally, the Pearson correlation coefficient between cosine similarity and KL equals -0.652 and -0.703, for BLESS and BBDS, respectively, indicated a pretty strong (linear) relation between the two. Here, we considered W2G(S), as it seemed the most promising on the basis of BBDS results, but the trend is the same for BSG and W2G(D).



(a) BSG (b) W2G(S)

Figure 5: Histograms of KL scores on BLESS.

Though we see that KL on its own does not seem to be sufficient for detecting entailment, we still hypothesize that it does capture information directly relevant to this task, namely, represents a degree of 'generality' of a term. In order to see that this is the case, in next subsection, we consider a modification of the entailment task. It will also hint at why W2G performs better than BSG on BBDS, while, as we will see, BSG appears to capture generality more accurately.

word 1	word 2	KL	cosine sim.
dog	cat	15.47	0.71
dog	pet	18.52	0.70
dog	hound	21.20	0.64
dog	animal	27.69	0.52
cappuccino	espresso	12.59	0.76
cappuccino	latte	13.39	0.7
cappuccino	coffee	22.54	0.69
cappuccino	drink	30.81	0.54
microsoft	windows	24.41	0.65
microsoft	google	24.44	0.60
microsoft	corporation	39.40	0.29
microsoft	company	46.05	0.19

Table 3: KL and cosine similarity of selected words. We used $D_{KL}[p(w_1)||p(w_2)]$, where $p(w)$ is the prior density for word w.

3.4 Entailment directionality prediction

In these experiments, given a pair of words, the system needs to predict if the first word entails the second one, or the other way around. In other words, it is known that entailment holds for the pair but its directionality needs to be predicted. As we would like to get extra insights into results obtained in the preceding section, we extract these pairs from the same datasets, BBDS and BLESS. As the symmetric cosine similarity would be useless here, we use only KL. As a baseline, we consider the following heuristic: we compare frequencies of the two words in our corpus and predict that the less frequent word (e.g., 'kiwi') entails the more frequent one (e.g., 'fruit').

Model	BBDS	BLESS
BSG	78.23	**67.34**
W2G(S)	78.41	57.50
W2S(D)	78.05	54.58
Freq. Baseline	**78.84**	55.26

Table 4: Entailment directionality detection.

Results are presented in Table 4. First, we observe that the situations are very different for the two datasets. On BBDS, the scores are much higher for all the approaches. However, depressingly, the frequency baseline appears the strongest: neither model manages to beat it. In contrast, on BLESS, the frequency baseline is weak (only 5% higher than chance), and both W2G versions achieve results very similar to the baseline. However, BSG outperforms them by a substantial margin (10 %).

These results hint at possibility that the main reason for slightly stronger results of W2Gs in the original set-up on BBDS (Table 2) is that covariances in W2Gs capture information about token frequencies. We verify this by plotting log-determinants of covariance matrices (representing the spread of the 'ellipsoids'), as a function of token frequencies (see Figure 6). The plots confirm our hypothesis: the covariances for W2Gs are growing with the frequency, and hence KL with W2Gs will prefer to label frequent words as hypernyms. This is not the case at all for BSG.

Note that BSG achieves similar results in directionality detection on BBDS without directly capturing frequency information (and hence 'mimicking' the frequency baseline). This, together with strong results on directionality detection for BLESS, suggests that covariances of BSG are better at capturing genuine information about generality of a term.

3.5 Lexical substitution

We argued that the posterior densities in our model encode semantic properties of a word given its context. In other words, the posteriors can be used to disambiguate the word. In order to show that this is indeed the case, we consider the *lexical substitution* task, where the goal is to choose a suitable replacement for a word given its context. For example, the word 'bright' in an expression 'bright child' can be substituted with 'smart' or 'gifted' rather than 'shining'.

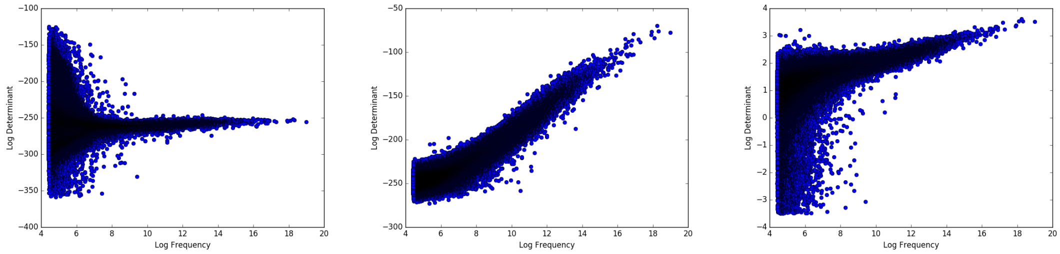
We used the SemEval-2007 task 10 dataset (McCarthy and Navigli, 2007), which has a list of ranked replacement candidates based on the number of labeler votes for each sentence with a highlighted target word. We followed the set-up of Melamud et al. (2015) but kept the model the same as in the preceding sections. Namely, we did not use syntactic dependency information (shown highly beneficial in (Melamud et al., 2015)) but rather relied on a bag-of-word representation of a window of 5 words on each side. We also used 100 dimensional embeddings instead of 600 in (Melamud et al., 2015)).

For BSG, we ranked candidates s from the set of candidates S by the KL divergence: $\mathbb{D}_{KL}\left[q_{\phi}(\mathbf{z}|\mathbf{c},w)||p_{\theta}(\mathbf{z}|s)\right]$, where $q_{\phi}(\mathbf{z}|\mathbf{c},w)$ is the approximate posterior computed by the encoder, and $p_{\theta}(\mathbf{z}|s)$ is the prior distribution for the candidate word. As baselines, we used the two best performing heuristics from (Melamud et al., 2015), namely *Add* and *Mult*. We used these heuristics on top of embeddings produced by SG and mean vectors for W2Gs and BSG. Note that they do not use covariance information.

Model	GAP
BSG (Encoder)	**0.461**
BSG (Add)	0.437
BSG (Mult)	0.439
W2G(S) (Add)	0.431
W2G(S) (Mult)	0.432
W2G(D) (Add)	0.427
W2G(D) (Mult)	0.427
SG (Add)	0.426
SG (Mult)	0.428

Table 5: Average precision on SemEval-2007 task 10.

The results (*generalized average precision*, GAP) are shown in Table 5. The encoder indeed appears



(a) BSG $\rho = -0.0706$. (b) W2G(D) $\rho = 0.9098$. (c) W2G(S) $\rho = 0.5915$.

Figure 6: Log determinant vs. log frequency; ρ is the Pearson correlation coefficient.

substantially more effective in predicting word substitutes than the alternative methods. This shows that the posterior densities are effective at disambiguating words. In Table 6, we show top 3 substitutes proposed by the BSG encoder in several example sentences.

4 Additional Related Work

Our model bears some relation to topic models: for example, one can interpret the latent meaning vector z of the central word as encoding topics of the window. Topic models relying on word embeddings has been considered in the past (Das et al., 2015; Li et al., 2016). However, their modeling approaches, inference methods and applications have been quite different. Methods for inducing context-sensitive representations relying on syntactic dependency tree (using forward-backward-style algorithms) have also been studied in the past (Grave et al., 2014). Their approach is unlikely to be scalable. Our method and W2G are specifically designed to induce information about the generality of a word and capture entailment. Previous work has shown that entailment decision can also been made by 'post-processing' representations produced by standard embedding methods (e.g., (Weeds et al., 2014; Henderson and Popa, 2016)). They only deal with fixed rather than context-specific embeddings.

5 Conclusion

We introduced a method for embeddings words as probabilities densities. Our method produces two types of embeddings: (1) 'prior' / static embeddings representing a word type and collapsing all word senses; (2) 'posterior' / dynamic embeddings encoding a representation of a word given its context. The Gaussian embeddings of Vilnis and McCallum (2014) have been shown effective in a range of applications (e.g., modeling knowledge graphs (He et al., 2015) and cross-modal transfer from language to vision (Mukherjee and Hospedales, 2016)), our framework, providing more flexible context-sensitive alternatives, is likely to be beneficial in these applications. Though fixed embeddings are widely used and, in principle, can be disambiguated when used within application-specific compositional models (e.g., in an LSTM encoder), explicit disambiguation have been shown beneficial (e.g., (Choi et al., 2017)). Moreover, the context-specific densities can also be used to regularize models: generating noisy versions of word embeddings relying on these posteriors is likely to be effective, providing an adaptive alternative to dropout.

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Excerpts	Top 3 Substitutes
at that size it would have a **mass** of about the same as an average galaxy	conglomeration, magnitude, bulk
few people parallels the growing poverty of the **masses**	multitude, proletariat, throng
he was **really** he always wanted to be a politician	genuinely, very, definitely
it is his passion and to be denied that is **really** hard	absolutely, very, truly
between shutdown and power up when the latchup **cross** section is evaluated	sequence, segment, transverse
and hurting like the torments of hell for being so close to the **cross**	crucifix, sequence, angry
something more sedate there are quieter sophisticated **bars** in the hotels	pub, saloon, lounge
put granola **bars** in bowl	snack, pub, biscuit
i never expected to be **touched** by a weird global media event personally	affect, feel, stir
these stripes are continuous and do not **touch** each other	contact ,finger, stroke

Table 6: Substitutes proposed by the BSG encoder on SemEval-2007.

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