SECI1013: DISCRETE STRUCTURES

SESSION 2024/2025 – SEMESTER 1

ASSIGNMENT 3 (CHAPTER 3 & 4 [Part 1])

INSTRUCTIONS:

a. This assignment must be conducted in a group. Please clearly write the group members' names & matric numbers on the front page of the submission.
b. Solutions for each question must be readable and neatly written on plain A4 paper. Every step or calculation should be properly shown. Failure to do so will result in the rejection of the submission of the assignment.
c. This assignment consists of 7 questions (85 Marks), contributing 5% of overall course marks.

STRUCTURES:

1. Chapter 3 Part 3: Pigeonhole Problem [10 Marks]
2. Chapter 3 Part 4: Probability [25 Marks]
3. Chapter 4 Part 1: Graph Theory (until Path and Cycle) [50 Marks]

Question 1 **[10 marks]**

a. How many students in a class to guarantee that at least two students received the same score on the final exam. If the exam is graded on a scale from 0 to 100 points. (5 marks)
b. what is the minimum number of students required in a Structure Discrete class so that at least six students will receive the same letter grade (A,B,C,D, or F) (5 marks)

Question 2 **[25 marks]**

The following table gives information on Mobile phone sold by a certain store:

	Percentage of Customers Purchasing	Of Those Who Purchase, Percentage Who Purchase Extended Warranty
Brand 1	70	20
Brand 2	30	40

A purchaser is randomly selected from among all those bought a mobile phone from the store.

Determine the probability that :

a. customer purchased Brand 1. (2 marks)

b. customer purchased Brand 2 (2 marks)
c. customer purchase extended warranty given that purchase brand 1. (2 marks)
d. customer who bought brand 1 and purchased extended warranty. (4 marks)
e. customer purchased brand 2 and extended warranty purchased. (5 marks)
f. extended probability purchased. (5 marks)
g. purchased brand 1 item given that also bought extended warranty (5 marks)

Question 3

Explain the given keyword using your own word and represent your understanding by drawing the graph.

a. Vertices
b. Edges
c. Adjacent Vertices
d. Incident Edge
e. Isolated Vertex
f. Loop
g. Parallel Edges

(7 Marks)

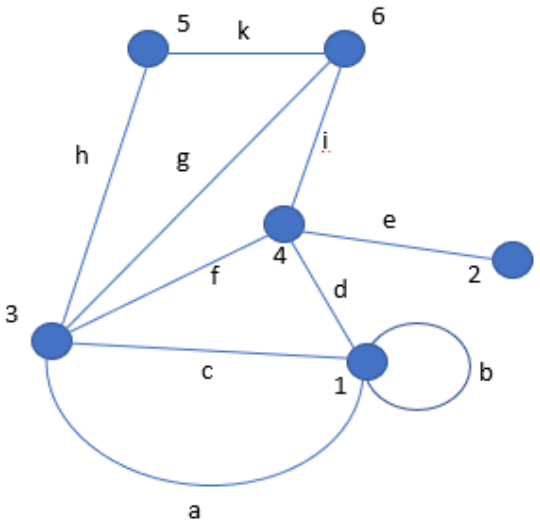
Question 4

Let G = {V, E} be a graph. An undirected graph having V ={v1, v2, v3, v4, v5} and E = {a, b, c, d, e, f}. Where *a* = (*v*1,*v*2), *b* = (*v*1,*v*3), *c* = (*v*2,*v*4), *d* = (*v*1,*v*4), *e* = (*v*3,*v*4) and *f* = (*v*3,*v*5).

Find the degree of each vertex.

(5 Marks)

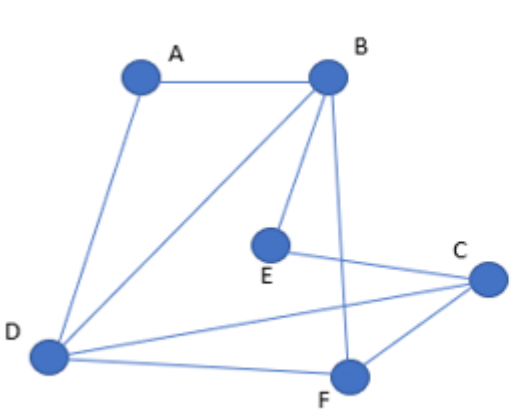
Question 5



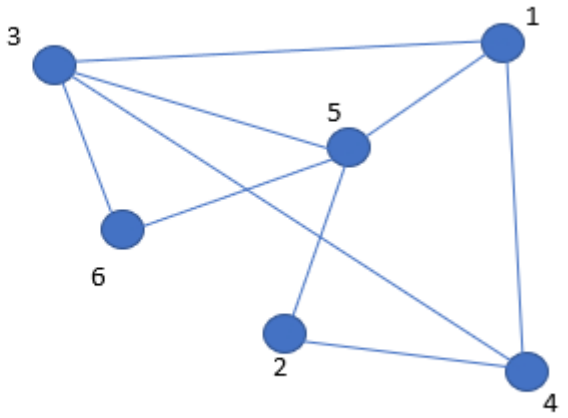
Given the graph shown above, Find:

i. Incidence Matrix (6 Marks)
ii. Adjacency Matrix (6 Marks)

Question 6



Graph Y



Graph Z

Determine whether Graph Y and Z above are isomorphic. If it is proven isomorphic, find their adjacency matrix.

(12 Marks)

Question 7

Consider an undirected graph with vertices V = {p, q, r, s, t} and edges E = {e1, e2, e3, e4, e5, e6, e7}. The edges are defined as follows:

e1 = (p, q); e2 = (q, r); e3 = (r, s); e4 = (s, t); e5 = (t, p); e6 = (q, s); e7 = (r, t)

Draw the graph and from the graph:

i. Find all possible paths from vertex p to vertex t. (5 Marks)
ii. Determine all possible trails from vertex p to vertex t. (5 Marks)
iii. Identify the shortest and longest path from vertex p to vertex t. (2.5 marks)
iv. Find the shortest and longest trail from vertex p to vertex t. (2.5 Marks)

(14 Marks)

Yeoh Keng Wei A24CS0316 Edwin Oo Ming Hao A24CS0245

Question 1 **[10 mar**

a. How many students in a class to guarantee that at least two students received the same score on the final exam. If the exam is graded on a scale from 0 to 100 points. (5 marks

1) a) 0 until 100

101 possible score 5

101 + 1 = 102

b. what is the minimum number of students required in a Structure Discrete class so that at least six students will receive the same letter grade (A,B,C,D, or F) (5 marks)

let number of student

receive same grade $= |\{x_1, x_2, x_3, x_4, x_5, x_6\}| = 6$

Number of student not receive same grade $= 5 \times 5 + 1 = n$

$n \geq x_6$

Question 2 **[25 marks]**

The following table gives information on Mobile phone sold by a certain store:

	Percentage of Customers Purchasing	Of Those Who Purchase, Percentage Who Purchase Extended Warranty
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A purchaser is randomly selected from among all those bought a mobile phone from the store.

Determine the probability that :

a. customer purchased Brand 1. (2 marks)

b. customer purchased Brand 2 (2 marks)

c. customer purchase extended warranty given that purchase brand 1. (2 marks)

d. customer who bought brand 1 and purchased extended warranty. (4 marks)

e. customer purchased brand 2 and extended warranty purchased. (5 marks)

f. extended probability purchased. (5 marks)

g. purchased brand 1 item given that also bought extended warranty (5 marks)

a) $P(\text{Brand 1}) = \frac{70}{100} = 0.7$

b) $P(\text{Brand 2}) = \frac{30}{100} = 0.3$

c) $P(\text{warranty} \mid \text{Brand 1}) = \frac{20}{100} = 0.2$

d) $P(\text{Brand 1 \& warranty}) = P(\text{Brand 1}) \times P(\text{warranty} \mid \text{Brand 1})$

$= 0.7 \times 0.2$

$= 0.14$

e) $P(\text{Brand 2 \& warranty}) = P(\text{Brand 2}) \times P(\text{warranty} \mid \text{Brand 2})$

$= 0.3 \times 0.4$

$= 0.12$

f) $P(\text{warranty}) = P(\text{Brand 1 \& warranty}) + P(\text{Brand 2 \& warranty})$

$= 0.14 + 0.12$

$= 0.26$

g) Brand 1 | warranty

$= \frac{P(\text{Brand 1 \& warranty})}{P(\text{warranty})}$

$= \frac{0.14}{0.26}$

$= 0.538$

Question 3

Explain the given keyword using your own word and represent your understanding by drawing the graph.

a. Vertices
b. Edges
c. Adjacent Vertices
d. Incident Edge
e. Isolated Vertex
f. Loop
g. Parallel Edges

(7 Marks)

a. Vertices : point / Big Dot { A , B - Z / 1 - ∞ }

b. Edges : line that connect vertices

c. Adjacent vertices = 2 vertices connected by an edge

d. Incident Edge = An Edge connected to a certain vertices

e. Isolated vertex : Alone vertex

f. Loop = An edge that start and ends at the same vertex :

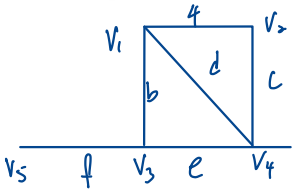
g. Parallel edge = 2 edge connected to the same pair vertex :

Question 4

Let G = {V, E} be a graph. An undirected graph having V ={v1, v2, v3, v4, v5} and E = {a, b, c, d, e, f}. Where *a* = (*v*1,*v*2), *b* = (*v*1,*v*3), *c* = (*v*2,*v*4), *d* = (*v*1,*v*4), *e* = (*v*3,*v*4) and *f* = (*v*3,*v*5).

Find the degree of each vertex.

(5 Marks)



deg(V1) = 3

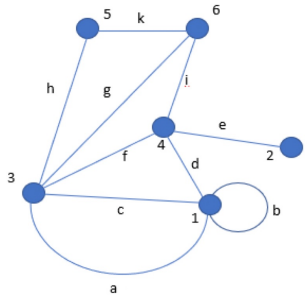
deg(V5) = 2

deg(V3) = 3

deg(V4) = 3

deg(V5) = 1

Question 5



Given the graph shown above, Find:

i. Incidence Matrix (6 Marks)
ii. Adjacency Matrix (6 Marks)

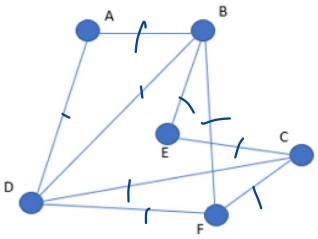
Incidence

	1	2	3	4	5	6
a	1	0	1	0	0	0
b	2	0	0	0	0	0
c	1	0	1	0	0	0
d	1	0	0	1	0	0
e	0	1	0	1	0	0
f	0	0	1	1	0	0
g	0	0	1	0	0	1
h	0	0	1	0	1	0
I	0	0	0	1	0	1
J	0	0	0	0	1	1

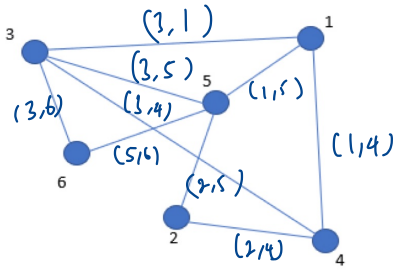
Adjacency

	1	2	3	4	5	6
1	1	0	2	1	0	0
2	0	0	0	1	0	0
3	2	0	0	1	1	1
4	1	1	1	0	0	1
5	0	0	1	0	0	1
6	0	0	1	1	1	0

Question 6



Graph Y



Graph Z

Determine whether Graph Y and Z above are isomorphic. If it is proven isomorphic, find their adjacency matrix.

(12 Marks)

Graph Y

V = A, B, C, D, E, F

E = (A, B), (A, D), (B, D), (B, E), (B, F), (C, D), (C, E), (C, F), (D, F)

Graph Z

V = 1, 2, 3, 4, 5, 6

E = (1, 5), (1, 4), (2, 5), (3, 1), (1, 5), (3, 4), (3, 6), (2, 4), (5, 6)

Both have the same vertices and same edge.

Compare degrees

Graph Y A → F [2, 4, 3, 4, 2, 3]

deg(A) = 2 deg(B) = 4 deg(C) = 3 deg(D) = 4 deg(E) = 2 deg(F) = 3

Graph Z 1 → 6 [3, 2, 4, 3, 4, 2]

deg(1) = 3 deg(2) = 2 deg(3) = 4 deg(4) = 3 deg(5) = 4 deg(6) = 2

Verify vertices: A ↔ 1 B ↔ 5 C ↔ 2 D ↔ 4 E ↔ 3 F ↔ 6

	A	B	C	D	E	F
A	0	1	0	1	0	0
B	1	0	0	1	1	1
C	0	0	0	1	1	1
D	1	1	1	0	0	1
E	0	1	1	0	0	0
F	0	1	1	1	0	0

	6	5	4	3	2	1
6	0	1	0	1	0	0
5	1	0	0	1	1	1
4	0	0	0	1	1	1
3	1	1	1	0	0	1
2	0	1	1	0	0	0
1	0	1	1	1	0	0

Question 7

Consider an undirected graph with vertices V = {p, q, r, s, t} and edges E = {e1, e2, e3, e4, e5, e6, e7}.
The edges are defined as follows:

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Draw the graph and from the graph:

i. Find all possible paths from vertex p to vertex t. (5 Marks)
ii. Determine all possible trails from vertex p to vertex t. (5 Marks)
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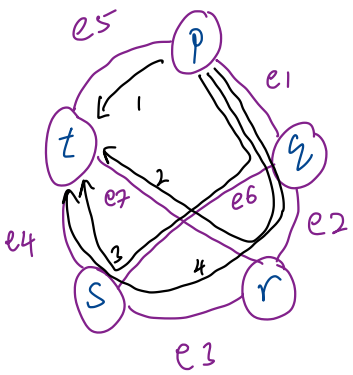
(14 Marks)



i) 1. p → e5 → t
2. p → q → r → t
3. p → q → s → t
4. p → q → r → s → t

ii) 1. p → t
2. p → q → r → t
3. p → q → s → t
4. p → q → r → s → t

iii) Shortest Path : p → t
Longest Path : p → q → r → s → t

iv) Shortest Path : p → t
Longest Path : p → q → r → s → t