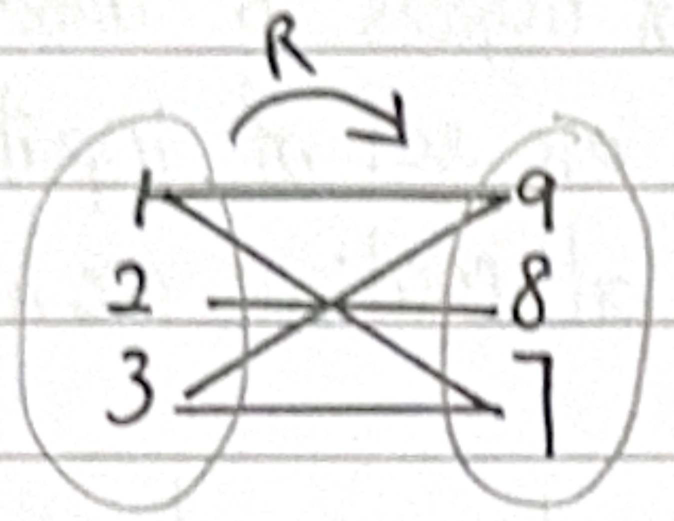
No.:

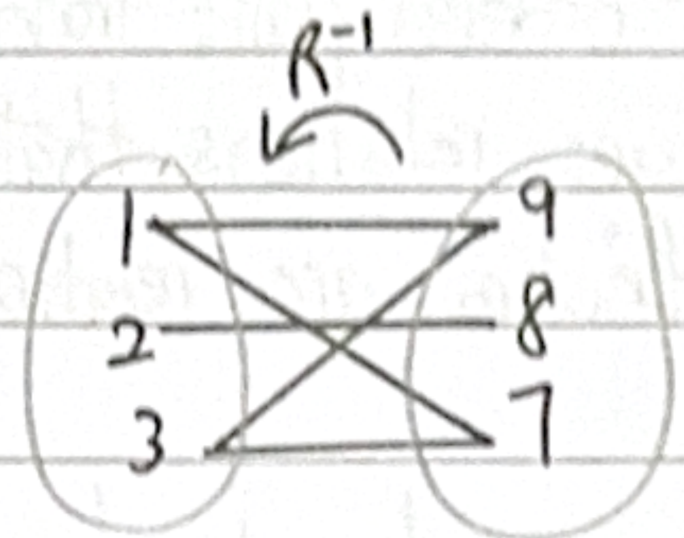
Assignment 2

1. $R = \{(2,2),(3,3),(4,4),(5,5),(6,6),(7,7),(8,8),(8,5),(8,2),(7,4),(6,3),$
$(5,2),(5,8),(2,8),(4,7),(3,6),(2,5)\}$

2(a) $R = \{(1,9),(1,7),(2,8),(3,9),(3,7)\}$
$R^{-1} = \{(9,1),(7,1),(8,2),(9,3),(7,3)\}$

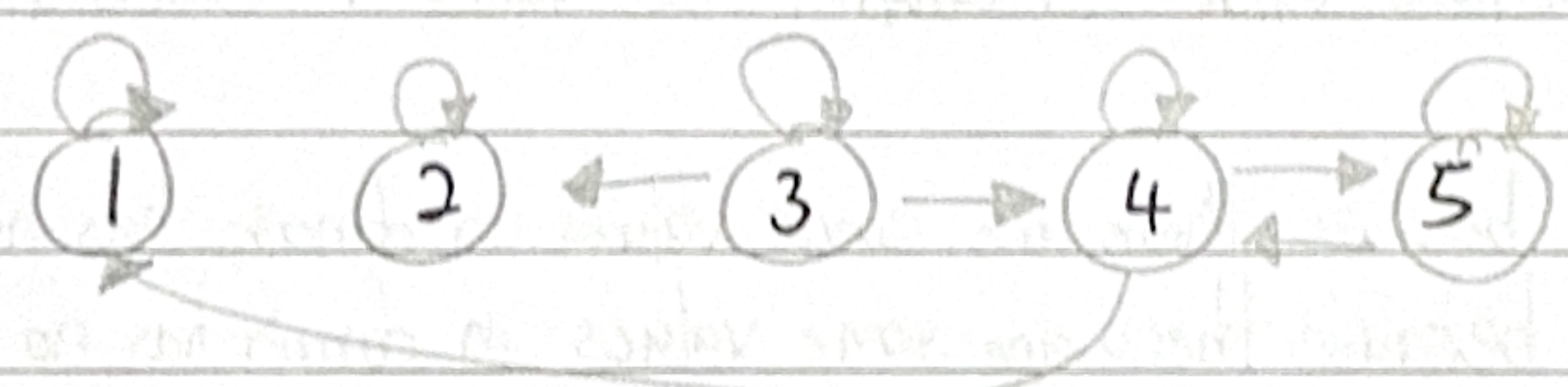
(b)





(c) For all $(b,a) \in B \times A$, $(b,a) \in R^{-1} \Leftrightarrow b+a$ is an even number

3



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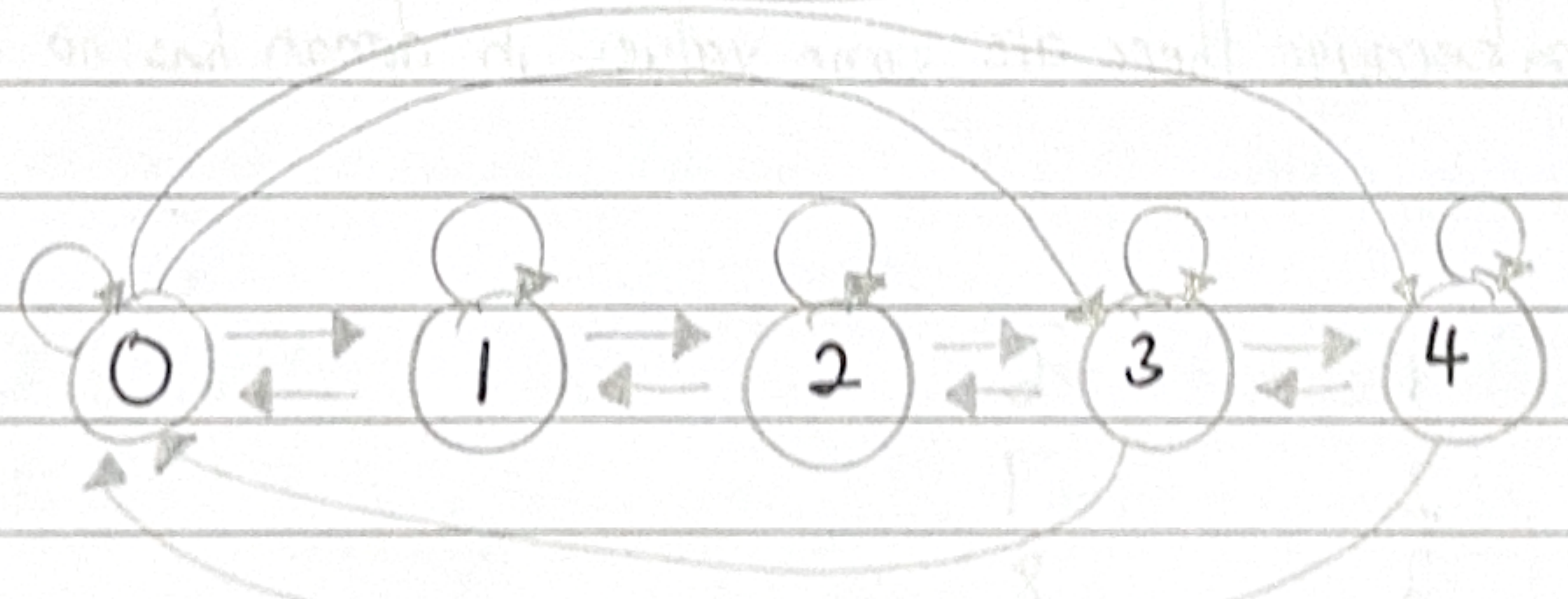


Reflexive: $\forall x \in A, (x,x) \in R$
Symmetric: $\forall x,y \in A$, if $(x,y) \in R$ then $(y,x) \in R$
Not transitive: $(0,1),(1,2) \in R$ but $(0,2) \notin R$

5 $R = \{(1,3),(2,6),(3,9),(4,12)\}$

$$R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(a) Reflexive: The relation is reflexive because
(i) The diagonal value is 1
(ii) $\forall x \in A, (x,x) \in R$

(b) Symmetric: The relation is symmetric because
(i) all values except at the diagonal are 0
(ii) $\forall x,y \in A$, if $(x,y) \in R$ then $(y,x) \in R$

(c) Transitive: The relation is transitive because
(i) all values except at the diagonal are 0

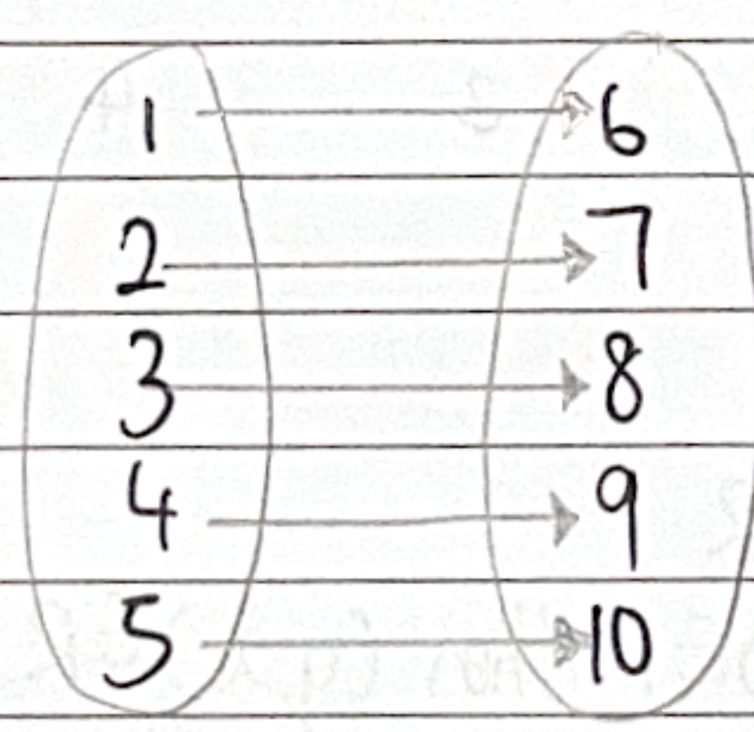
No.:

6(a) $RS = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$ (b) $SR = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

7 Relations are a group of ordered pairs from one set of objects to another set of objects, while functions are relations that connect one set of inputs to another set of outputs. So, all functions are relations while all relations are not functions.

8(i) This is a function because value in domain has value in codomain.
One-to-one function
(ii) This is a function because values in domain has value in codomain
Not one to one
(iii) This is not a function because there are some values in domain has no value in codomain
(iv) This is not a function because there are some values in domain has no value in codomain.
(v) This is not a function because there are some values in domain has no value in codomain.

9. $x = \{1,2,3,4,5\}$
$y = x+5$
$x=1, y=6$
$x=2, y=7$
$x=3, y=8$
$x=4$ $y=9$
$x=5$, y 10



10(i) $f(x_1) = f(x_2)$
$f(x) = 1-2x_1$, $f(x_2) = 1-2x_2$
$1-2x_1 = 1-2x_2$
$1-1-2x_1 = -2x_2$
$x_1 = \frac{-2x_2}{-2}$
$x_1 = x_2$
$x_1 - x_2 = 0$
One to one function

$y = 1-2x$ $x = \frac{1-y}{2}$
$f(\frac{1-y}{2}) = 1-2(\frac{1-y}{2})$
$f(\frac{1-y}{2}) = y$
For each y in codomain R, there exists $\frac{1-y}{2}$, such that $f(\frac{1-y}{2}) = 1-2(\frac{1-y}{2}) = y$, $\therefore$ the function is onto
$\therefore$ The function is bijective because function is both one-one function and onto function

No.:

(ii) if $f(x_1)=f(x_2)$; $x_1=x_2$

$f(x_1)=5x_1^2-1$; $f(x_2)=5x_2^2-1$

$f(x_1)=f(x_2)$

$5x_1^2-1=5x_2^2-1$

$x_1^2=x_2^2$

$x_1^2-x_2^2=0$

$x_1^2-x_2^2=(x_1-x_2)(x_1+x_2)$

$(x_1-x_2)=0$ or $(x_1+x_2)=0$

$x_1=x_2$ or $x_1=x_2$

∴ Not one to one function

$y=5x^2-1$ $x=\sqrt{\frac{y+1}{5}}$

$f\left(\sqrt{\frac{y+1}{5}}\right)=5\left(\sqrt{\frac{y+1}{5}}\right)^2-1=y$

$f\left(\frac{1-y}{2}\right)=y$

∴ The function is onto

∴ The function is not bijective because the function is not one to one although it is onto function.

(iii) if $f(x_1)=f(x_2)$; $x_1=x_2$

$f(x_1)=x_1^4$; $f(x_2)=x_2^4$

$f(x_1)=f(x_2)$

$x_1^4=x_2^4$

$(x_1^2)^2=(x_2^2)^2$

$(x_1^2)^2-(x_2^2)^2=0$

$(x_1^2-x_2^2)(x_1^2+x_2^2)=0$

$x_1\neq x_2$

∴ the function is not one to one

if $y=x^4$; $x=\sqrt[4]{y}=y^{\frac{1}{4}}$

$f(y^{\frac{1}{4}})=(y^{\frac{1}{4}})^4=y$

For each y in codomain R, there exist $y^{\frac{1}{4}}$, such that $f(y^{\frac{1}{4}})=(y^{\frac{1}{4}})^4=y$;

∴ the function is onto

∴ The function is not bijective because the function is not one to one function although it is onto function

(iv) if $f(x_1)=f(x_2)$; $x_1=x_2$

$f(x_1)=\left(\frac{x_1-2}{x_1-3}\right)$; $f(x_2)=\left(\frac{x_2-2}{x_2-3}\right)$

$f(x_1)=f(x_2)$

$\left(\frac{x_1-2}{x_1-3}\right)=\left(\frac{x_2-2}{x_2-3}\right)$

$(x_1-2)(x_2-3)=(x_2-2)(x_1-3)$

$(x_1)(x_2)-2x_2+6=(x_2)(x_1)-2x_1+6$

$(x_1)(x_2)-2x_2+6=(x_2)(x_1)-2x_1+6$

$2x_2=2x_1$

$x_2=x_1$

the function is one to one

if $y=\left(\frac{x-2}{x-3}\right)$; $x=\left(\frac{3y-2}{y-1}\right)$

$f\left(\frac{3y-2}{y-1}\right)=\left(\frac{\left(\frac{3y-2}{y-1}\right)-2}{\left(\frac{3y-2}{y-1}\right)-3}\right)=y$

$f\left(\frac{3y-2}{y-1}\right)=\left(\frac{\frac{(3y-2)-2(y-1)}{y-1}}{\frac{(3y-2)-3(y-1)}{y-1}}\right)=y$

$f\left(\frac{3y-2}{y-1}\right)=\frac{(3y-2)-2(y-1)}{(3y-2)-3(y-1)}=y$

$f\left(\frac{3y-2}{y-1}\right)=\frac{y}{1}=y$

For each y in codomain R, there exist $\frac{3y-2}{y-1}$, such that $f\left(\frac{3y-2}{y-1}\right)=y$;

∴ the function is onto

∴ The function is bijective because the function is both one to one function and onto function

No.:

11(i) $f(x)=3x-1; g(x)=x^2-1$

$f(g(x))=3(x^2-1)-1$

$x=0, f(g(0))=3(0^2-1)-1=-4$

$x=1, f(g(1))=3(1^2-1)-1=-1$

$x=2, f(g(2))=3(2^2-1)-1=8$

$x=3, f(g(3))=3(3^2-1)-1=23$

(ii) $f(x)=x^2; g(x)=5x-6$

$f(g(x))=(5x-6)^2$

$x=0, f(g(0))=(5(0)-6)^2=36$

$x=1, f(g(1))=(5(1)-6)^2=1$

$x=2, f(g(2))=(5(2)-6)^2=16$

$x=3, f(g(3))=(5(3)-6)^2=81$

(iii) $f(x)=x-1; g(x)=x^3+1$

$f(g(x))=(x^3+1)-1=x^3$

$x=0, f(g(0))=0^3=0$

$x=1, f(g(1))=1^3=1$

$x=2, f(g(2))=2^3=8$

$x=3, f(g(3))=3^3=27$

12(i) $a_n=6a_{n-1}-9a_{n-2}$ $a_0=1, a_1=6$

$a_2=6a_{2-1}-9a_{2-2}=6(6)-9(1)=27$

$a_3=6a_{3-1}-9a_{3-2}=6(27)-9(6)=108$

$a_4=6a_{4-1}-9a_{4-2}=6(108)-9(27)=405$

The sequence = 1, 6, 27, 108, 405, ...

12(ii) $a_n=6a_{n-1}-11a_{n-2}+6a_{n-3}$;

$a_0=2, a_1=5, a_2=15$

$a_3=6a_{3-1}-11a_{3-2}+6a_{3-3}=6(15)-11(5)+6(2)=47$

$a_4=6a_{4-1}-11a_{4-2}+6a_{4-3}=6(47)-11(15)+6(5)=147$

$a_5=6a_{5-1}-11a_{5-2}+6a_{5-3}=6(147)-11(47)+6(15)=455$

The sequence = 2, 5, 15, 47, 147, 455, ...

(iii) $a_n=-3a_{n-1}-3a_{n-2}+a_{n-3}$

$a_0=1, a_1=2, a_2=-1$

$a_3=-3a_{3-1}-3a_{3-2}+a_{3-3}=-3(-1)-3(-2)+1=10$

$a_4=-3a_{4-1}-3a_{4-2}+a_{4-3}=-3(10)-3(-1)+(-2)=-29$

$a_5=-3a_{5-1}-3a_{5-2}+a_{5-3}=-3(-29)-3(10)+(-1)=56$

The sequence = 1, -2, -1, 10, -29, 56, ...

13(i) $n=1; a_{1+1}=a_2=5a_1-3=5(k)-3=5k-3$

$n=2, a_{2+1}=a_3=5a_2-3=5(5k-3)-3=25k-15-3=25k-18$

$n=3, a_{3+1}=a_4=5a_3-3=5(25k-18)-3=125k-90-3=125k-93$

$n=4, a_{4+1}=a_5=5a_4-3=5(125k-93)-3=625k-465-3=625k-468$

(ii) $a_4=7$

$a_4=125k-93$

$7=125k-93$

$k=\frac{4}{5}$