LAB # 7

Applying Kepler's Second and Third Laws

OVERVIEW

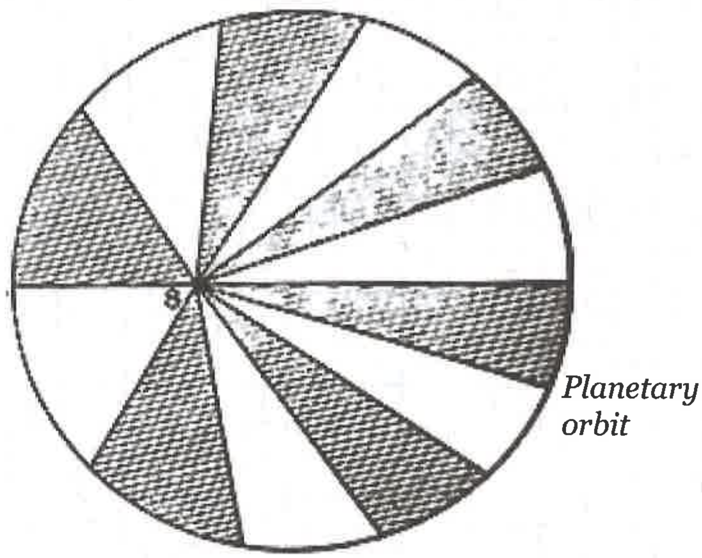
In the preceding lab you examined the elliptical nature of planetary orbits. Here you will look at two more of Kepler's contributions to our knowledge of planetary motion.

Part 1: The Law of Equal Areas

Ancient astronomers noticed that the planets do not appear to move uniformly in their orbits. Elaborate models were contrived to account for the phenomena. Aiming to explain the varying speeds, Kepler recognized that a planet's speed was related to how far it was from the Sun. This relationship is stated in his second law:

Law #2: Planets sweep out equal areas of orbit in equal time intervals.

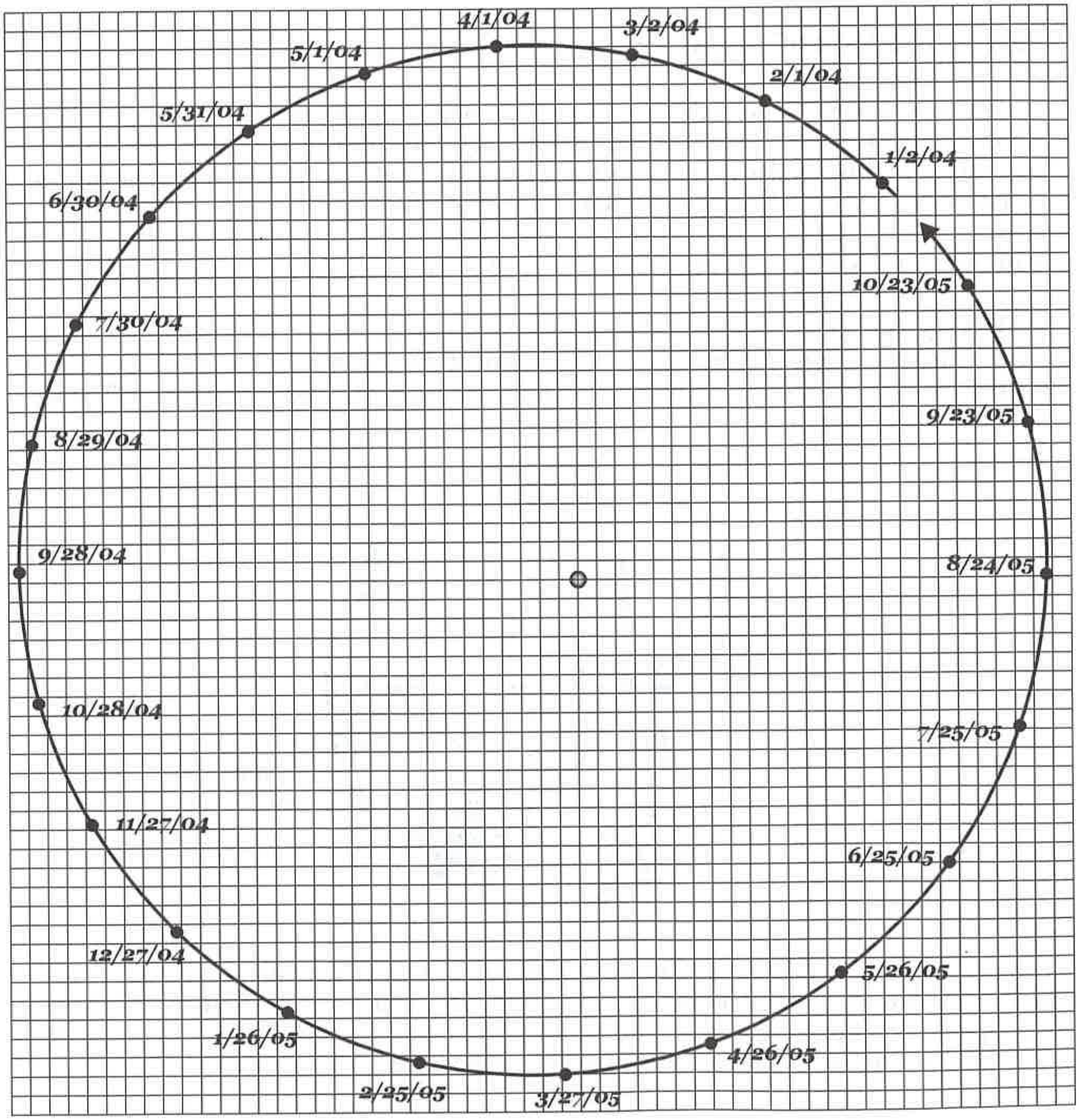
In the diagram below, an elliptical orbit of a hypothetical planet is divided into equal areas. The outer edges of each slice are the paths that the planet travels in a specified time interval, such as one month. If the time intervals are equal, the area of any slice will equal the area of any other slice.



One consequence of this law is that when a planet is close to the Sun it must travel at a faster speed than when it is far from the Sun. Did you know that in the northern hemisphere the winter season is shorter than the summer season? That's because the Earth is closer to the Sun in the winter than it is in the summer.

PROCEDURE

The orbit of the planet Mars for the years 2004-5 has been plotted in the diagram below. The small circle to the right of center represents the Sun's location at one focus. Check Kepler's second law by comparing the areas of three slices covered in equal time intervals. Choose three ***widely separated*** slices covering equal time intervals and draw straight lines from the Sun to the positions of Mars on the dates that define those slices. Label each slice on the outer edge of the orbit: 1, 2 and 3.



• Count the number of whole boxes that each slice encloses. *To make counting the boxes easier, you might wish to make a mark in each box as you count it.* Record those values in the spaces below. Then estimate the area of the partial boxes by adding up their fractional areas. Don't be too concerned if you think you are overestimating or underestimating the area. Simply try to be consistent.

Slice #1

Area of whole boxes = ______ boxes

Area of partial boxes = ______ boxes

Total area of slice = _______ boxes

Slice #2

Area of whole boxes = ______ boxes

Area of partial boxes = ______ boxes

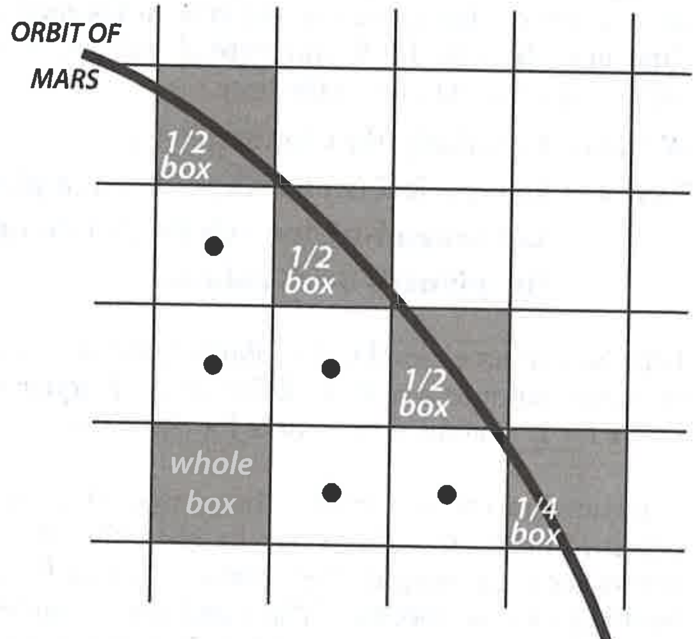
Total area of slice = _______ boxes

Slice #3

Area of whole boxes = ______ boxes

Area of partial boxes = ______ boxes

Total area of slice = _______ boxes



When finding the area of the partial boxes, add the fractional areas as you examine each box. In the example above, the areas of the four boxes would sum as follows:

1/2 + 1/2 + 1/2 + 1/4 = 1 3/4 boxes.

• Find the average area of the slices:

$$\text{Average area} = \frac{(\text{Area of Slice \#1} + \text{Area of Slice \#2} + \text{Area of Slice \#3})}{3}$$

= ____ boxes

• Find the percent difference between the areas of each slice and the average area:

Percent difference = (|(Area of slice - Average area)| / Average area) x 100

Slice #1: ___________ %   Slice #2: ___________ %   Slice #3: ___________ %

Do you think Kepler's second law is supported by your results? Explain briefly.

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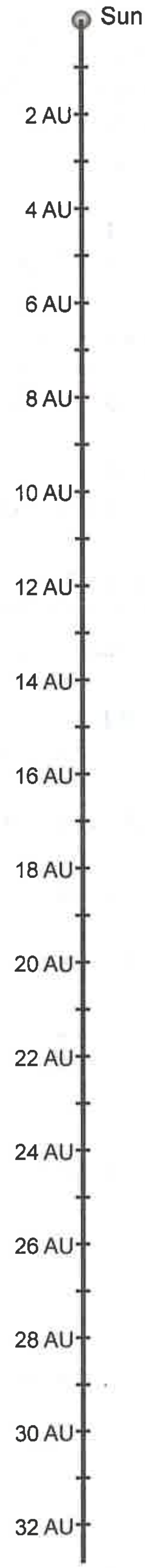
Lab #7: Applying Kepler's Laws II & III (continued) *Page 4 of 8*

Part 2: The Period - Semi-Major Axis Relationship



Kepler discovered a mathematical relationship between the **period (P)** or the amount of time it takes a planet to revolve around the Sun once and the length of the **semi-major axis (a)** of the planet's elliptical orbit. He had been searching for geometric patterns in the motions of the planets for some time and the one he found agreed well with the observed periods and semi-major axes known at the time.

Written out concisely, the relationship is:

Law #3: The period (in Earth years) of a planet squared is equal to the semi-major axis (in Astronomical Units or AU) of the planet's orbit cubed.

Let's look at an example. The Earth takes one year to complete its orbit and the semi-major axis of its orbit is 1 AU. **P** squared is 1 x 1 = 1 and **a** cubed is 1 x 1 x 1 = 1. The third law works for these values.

Astronomers can determine the period of a planet or any other celestial object in orbit around the Sun by observing it carefully. Kepler's third law allows them to compute the semi-major axis from the periods they observe. Nothing else is needed. The third law established the scale of the solar system before astronomers had an independent way of determining the true dimensions of the orbits of the planets.

For the planets and asteroid in the table below, compute the semi-major axes using Kepler's third law. You will need to find the cube root of the period squared. Then plot and label the objects on the figure at right. This will illustrate their approximate relative distances from the Sun.

Object	P (yr)	P^2	$\sqrt[3]{(P^2)}$ = **a** (AU)
Mercury	0.24		
Venus	0.62		
Earth	1.0		
Mars	1.88		
Asteroid Ceres	4.6		
Jupiter	11.86		
Saturn	29.5		
Uranus	84		
Neptune	165		

Did you know that you can use Google to find the cube root of a number?
In the search box, type **cube root(number)**. *Then press the Enter key.*
For example, to find the cube root of 64, you would type **cube root(64)**.

Computer analysis with *Microsoft Excel*

In order to enhance your understanding of Kepler's Third Law, your instructor may ask you to make a chart of $\mathbf{P^2}$ versus $\mathbf{a^3}$ using a spreadsheet program such as Microsoft ***Excel***.

On the desktop of one of the computers in the lab room, double-click on a search engine icon and perform a search for the *accepted* values of the semi-major axes in AU of the planets and asteroid in the table on page 4. In addition, find the periods and semi-major axes of Pluto and Halley's Comet. Use key words such as planet, Halley's Comet and semi-major axis in your search. Enter the values you find in the table below. Calculate the squares of the periods and the cubes of the semi-major axes.

Object	Period (yr)	Semi-major axis (AU) from web	P^2(calculate)	a^3(calculate)
Me	0.24			
Ve	0.62			
Ea	1.0			
Ma	1.88			
Ce	4.6			
Ju	11.86			
Sa	29.5			
Ur	84			
Ne	165			
Pl				
Ha				

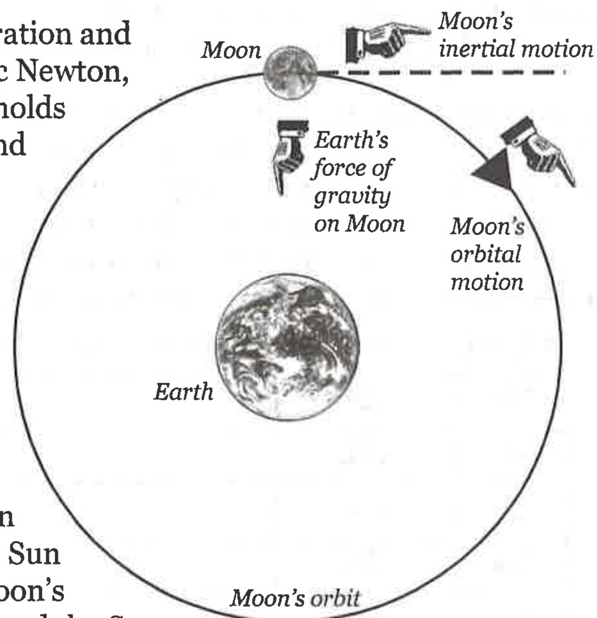
Close the search engine program and double-click on the icon labeled Microsoft ***Excel***. Select a blank spreadsheet. In the first column, enter the first two letters of the names of the objects listed above. In the second column, enter the squares of the periods. In the third column, enter the cubes of the semi-major axes.

Select all of the squared periods and cubed semi-major axes in columns two and three. From the **Insert** menu, select **Chart** and choose **X-Y scatter**.
From the **Chart** menu, select **Add Trendline**. For **Type**, choose **Linear**. For **Options**, choose **Display equation on chart**.
An equation of the form $\boldsymbol{y = m\,x + b}$ will appear near a straight black line that the computer program has drawn near the points on the chart. Write down the slope of this line in the space at right. Slope = ______________

Question: *Is the slope of the line approximately one?* _____ *Why should it be nearly one?*

____

Part 3: Finding masses using Kepler's and Newton's Laws.



Using the concepts of inertia, centripetal acceleration and universal gravitation, the English scientist, Isaac Newton, succeeded in explaining why Kepler's third law holds true. He understood that the masses, periods and semi-major axes of gravitationally interacting objects were related. A knowledge of the mass of an object can help astronomers make a model of that object.

Let's look at how an astronomer would determine the mass of the Sun using Newton's ideas. To determine an unknown quantity from a set of related, known quantities, astronomers set up a *proportion*. The proportion shown below can be used to find the mass of the Sun using the periods and semi-major axes of the Moon's orbit around the Earth and the Earth's orbit around the Sun.

First, examine the following two statements. They follow from Kepler's and Newton's laws.

- **a** (Earth's orbit) cubed / **P** (Earth's orbit) squared is related to the mass of the Sun.
- **a** (Moon's orbit) cubed / **P** (Moon's orbit) squared is related to the mass of the Earth.

Data (Earth's orbit): **a** = 150,000,000 km = 1 AU **P** = 6366 hours = 1 year

Data (Moon's orbit): **a** = 384,000 km = 0.0026 AU **P** = 655 hours = 0.075 year

Second, set up the proportion that will provide the ratio between the mass of the Sun and the mass of the Earth. Then, substitute actual values for **a** and **P**.

$$\frac{\mathbf{a}\text{ (Earth's orbit) cubed } / \mathbf{P}\text{ (Earth's orbit) squared}}{\mathbf{a}\text{ (Moon's orbit) cubed } / \mathbf{P}\text{ (Moon's orbit) squared}} = \frac{\text{Mass of the Sun}}{\text{Mass of the Earth}}$$

$$\frac{(1\text{ AU})^3 \;/\; (1\text{ yr})^2}{(0.0026\text{ AU})^3 / (0.075\text{ yr})^2} = \frac{1}{0.0000031} = \frac{\text{Mass of the Sun}}{\text{Mass of the Earth}}$$

From this proportion, we see that the Sun is about **320,000** times more massive than the Earth. If the mass of the Earth is known in grams, the mass of the Sun can be computed in grams. ***Would you agree that this is a valuable tool?*** The following exercises will demonstrate its utility.

EXERCISES

Find the masses of the following three objects by obtaining the necessary data and substituting it in the proportion in the previous example.

1) The mass of Jupiter. Use the period and semi-major axis of the orbit of Jupiter's moon, Io. See the chart of Io's orbit on the next page to get the data.
Remember to convert kilometers into AU and days into years.
(There are 150,000,000 km in one AU and 365.25 days in one year.)

Data: **a** = ____________ km = ______________ AU **P** = _____ days = ______yr

$$\frac{(\_\_\_\_\text{AU})^3 / (\_\_\_\_\text{yr})^2}{(0.0026\ \text{AU})^3 / (0.075\ \text{yr})^2} = \_\_\_\_\_\_\_\_\_\_ = \frac{\text{Mass of Jupiter}}{\text{Mass of the Earth}}$$

How many Earth masses is Jupiter's mass? ____________________

How close is your value to the accepted value of Jupiter's mass, 318 Earth masses? ____%

2) The mass of Neptune. Use the period and semi-major axis of Neptune's moon, Triton. Use the chart of Triton's orbit on the next page to obtain the data.

Data: **a** = ____________ km = ______________ AU **P** = _____ days = ______yr

$$\frac{(\_\_\_\_\text{AU})^3 / (\_\_\_\_\text{yr})^2}{(0.0026\ \text{AU})^3 / (0.075\ \text{yr})^2} = \_\_\_\_\_\_\_\_\_\_ = \frac{\text{Mass of Neptune}}{\text{Mass of the Earth}}$$

How many Earth masses is Neptune's mass? ____________________

How close is your value to the accepted value of Neptune's mass, 17 Earth masses? ____%

3) The mass of a mystery star. Use the period and semi-major axis of the orbit of a recently discovered planet revolving around its star.

Data: **a** = 400,000,000 km = ______________ AU **P** = 4.35 yr

$$\frac{(\_\_\_\_\text{AU})^3 / (\_\_\_\_\text{yr})^2}{(0.0026\ \text{AU})^3 / (0.075\ \text{yr})^2} = \_\_\_\_\_\_\_\_\_\_ = \frac{\text{Mass of mystery star}}{\text{Mass of the Earth}}$$

How many Earth masses is the mystery star's mass? ____________________

What familiar star do you think the mystery star resembles? ______________

Relative positions of Io and Triton

In the diagrams below are the positions of Io relative to Jupiter and Triton relative to Neptune from June 14 - 27, 2004. Draw smooth curves through the positions of these moons. Use the horizontal distance scale to find the semi-major axes in km. Use the vertical date scale to find the periods of these two moons in days.