A *job* is a list of m versions of uniformly-sized rectangular printed pieces, represented as an m-tuple of positive integers:

$$V = (v_1, \ldots, v_m)$$

An n-up *form* is an arrangement of n uniformly-sized rectangular images on a single printed press sheet that can be cut into individual pieces after printing. A form is represented as an m-tuple of non-negative integers: $(f_1, \ldots, f_m)$, where each f_i is the number of copies of version v_i on the press sheet and $\sum f_i = n$. Then the set of all possible n-up forms for m versions is given by

$$F_{m,n} = \{(f_1, \ldots, f_m) \mid \sum_{i=1}^{m} f_i = n\}$$

Note that the number of such forms is the number of weak compositions[1] of n in m terms, giving

$$|F_{m,n}| = \binom{m+n-1}{n-1}$$

An N-tuple $(x_1, \ldots, x_N)$, where $N = |F_{m,n}|$, is a *solution* for job V if

$$\sum_{i=1}^{N} x_i f_j \geq v_j, \text{ for all } 1 \leq j \leq m$$

Conceptually, x_i is the quantity of the press run of the ith form.

Let the cost of a press run of quantity $x > 0$ be defined as $Ax + B$, where A is the cost per sheet and B is the fixed set up cost of a press run. Then the cost function of a solution is

$$c(x_1, \ldots, x_N) = \begin{cases} Ax_i + B & x > 0 \\ 0 & x = 0 \end{cases}$$

Question. *Given a job V run n-up with cost per sheet A and set up cost B per press run, what solution $(x_1, \ldots, x_N)$ has the minimum cost?*

[1] see `https://en.wikipedia.org/wiki/Composition_(combinatorics)`.