Investigating particle swarm optimization

Edvard K. Karlsen

1 Introduction

1.1 Problem statement and solution overview

My task was given as follows:

1. Undersøk 'Particle Swarm Optimization' (PSO). Det beskrives i mange AI boker, på Wikipedia, osv. Ingen mangel på info.
2. Implementere det fra scratch.
3. Kjør din PSO på 2 forskjellige TYPE problem - helst noen klassiske data problemer (som travelling salesman, bare et eksempel) og minst 3 versjoner av hvert problem type.
4. Skriv en 5-10 sider rapport om arbeidet, inklusivt resultater av dine kjøringer, analyse av dine resultater, osv.

I have studied and implemented a generic PSO algorithm from scratch in Python. I discuss the underlying mathematical theory of the algorithm in Section 2. Further, I used my implementation to attack i) the Single Machine Weighted Tardiness Problem, which is a NP-hard combinatorial optimization problem, and ii) training of feed-forward neural networks for (multi-class) classification. I experimentally confirmed PSO's optimization ability for both these problems, and also did some emperical analysis of parameters to investigate optimization performance. I discuss the experiments in Section 3 and Section 4. Finally, I reflect on the work process and on my concluding impressions of PSO in Section 5.

Because there was no rigid 'project description' text for this problem, I have included much more background theory than in my previous reports, to provide necessary context for my experiments.

My main reference for this project has been Engelbrecht's excellent textbook [2].

2 Particle swarm optimization

Particle swarm optimization is an iterative, stochastic optimization algorithm inspired by the flocking of birds and schooling of fish; the main metaphor is of social creatures moving in a swarm, with each creature's behavior dependent both on personal preference and social influence. PSO was first described by Kennedy and Eberhart [3]. The algorithm sees a lot of interest from the AI research community, who among other things tries to formally investigate the optimization mechanisms of the algorithm; develop new variants increasing performance (such as hybrid methods with other bio-inspired methods); and develop special PSO incarnations to improve performance on particular problems (such as TSP [7]). PSO has been applied to a wide array of problems [5].

2.1 Theory

All PSO variants operate on an ordered set of *particles*, each having a *position*, *velocity*, and *best position* vectors, traditionally denoted $\mathbf{x}$, $\mathbf{v}$, and $\mathbf{p}$. PSO operates by 'swarming', in discrete steps, the particles through a search space.

Although there are variations in the update rules between PSO algorithms, they always include a 'social component' – which pulls each particle towards good positions known collectively in the swarm, and a 'selfish component' – which pulls each particle toward the best position it has visited.

gbest vs. lbest PSO Most texts distinguish between the *global best* and *local best* PSO variants, traditionally named *gbest* and *lbest*. The difference is that in *gbest* PSO, all particles always take the 'social step' towards the best among all seen positions by *all* members of swarm, while in *lbest*, a particle only considers the best among the members of its 'neighboorhood'. However, gbest is clearly just the special case of lbest with neighborhood size equal to the whole swarm's size, so we can focus only on the lbest PSO, without loss of generality.

Notation For each PSO problems we discuss, let g denote some function from particle space to real numbers that is to be *minimized.* Further, let $\mathbf{x}_{i,t}$ denote the position of particle i at time t, and assume similar notation for velocities. Also, let ω, ϕ_p, and ϕ_g denote real numbers expressing the *inertia*, *personal best acceleration factor*, and *neighborhood best acceleration factor* for the problem. Finally, let's abuse notation and take $\mathbf{r}_p$ and $\mathbf{r}_g$ as two vectors in particle space whose components are randomly and uniformly sampled in $[0, 1]$ each time they appear in an equation.

With this notation, we can express the PSO update step of our implementation as follows[1]:

$$\begin{aligned} \mathbf{v}_{i,t+1} &= \omega \mathbf{v}_{i,t} + \phi_p \mathbf{r}_p (\mathbf{p}_{i,t} - \mathbf{x}_{i,t}) + \phi_g \mathbf{r}_g (\mathbf{g}_{i,t} - \mathbf{x}_{i,t}) && \text{Velocity update} \\ \mathbf{x}_{i,t+1} &= \mathbf{x}_{i,t} + \mathbf{v}_{i,t} && \text{Position update} \end{aligned}$$

$$\begin{aligned} \mathbf{p}_{i,t} &= \text{argmin}\ g\ \{\ \mathbf{x}_{i,u} \mid u \in [0,\ t]\ \} && \text{Personal best position} \\ \mathbf{g}_{i,t} &= \text{argmin}\ g\ \{\ \mathbf{p}_{j,t} \mid j \in \text{neighbor-ids}(i)\ \} && \text{Neighborhood best position} \end{aligned}$$

$$\text{where neighbor-ids}(i) = \{\ i,\ i+1,\ \ldots,\ i + \textsc{neighborhood-size}\ \}$$

The update equations are the most important formality needed to describe our implementation, but, the devil is in the details and there are many other smaller aspects of the algorithm that we could have discussed in depth. Most important are *initialization*, *velocity/particle clamping*, *inertia reduction*, and *stopping criteria*. These are however more problem-specific, and we therefore discuss them in context of our experiments (Section 3 and 4).

2.2 Digression: The nature of the lbest PSO

It's interesting to philosophise about inter-neighborhood communication in the lbest PSO. Clearly, with neighborhoods comes an element of 'information lag' in the swarm. A swarm-global best position, call it p, known only in one neighborhood, may require many steps before it becomes known to a distant neighborhood; with the typical definition of neighborhood membership, a local best can propagate to at most one new particle in each

[1]Of course, our Python code is much more 'imperative-looking' than this, for example, we store best positions instead of re-computing them all the time. However, the mathematical formulation in the text helps to avoid ambiguity.

iteration. This has two important consequences, one positive and one negative: First, it enables other neighborhoods to focus on *exploring* their local bests, which may lead to even better positions than p, and hence help in avoiding premature convergence to non-global optima. Second, if p is a useful position that should be heavily *exploited*, it may take (potentially, *much*) longer time to direct the swarm's attention towards it. Ideally, PSO should be tuned so as to balance these consequences for the best results. With the *Unified PSO* [4], Parsopoulos and Vrahatis propose mitigating this problem by having the swarm gradually focus more on global information (and less on neighborhood information) during the PSO run.

2.3 Investigating the PSO

A typical evaluation of an optimization algorithm includes an empirical comparison with an alternative algorithm known from the literature, and the typical argument is that the proposed algorithm performs better, in some sense, than another method on some benchmark problems. For example, to demonstrate PSOs quality as a training algorithm for feed-forward artificial neural networks, a comparison with traditional backpropagation would be natural.

One can also 'compare it with itself', and study how PSO's performance depends on its many configuration options. Among those are:

1. The number of particles used.
2. The value of the acceleration constants ω, ϕ_p, and ϕ_g.
3. Whether to use position and/or velocity clamping, and if so, its rules.
4. Whether to use inertia dampening, and if so, the inertia dampening rules.
5. The swarm structure. (gbest/lbest, neighborhood size, alternative swarm topologies.)

In planning this project, I quickly concluded that doing a truly rigorous investigation of the PSO's workings could pose too much of a challenge; there are simply too many variables to look at. Therefore, I decided that I had to choose more modest goals, and after some pondering I settled on the following:

EXPERIMENT GOALS

1. Demonstrate PSO's optimization ability on two hard problems: (i) SMTWTP and (ii) training of feed-forward ANNs.
2. For the SMTWTP experiment: Demonstrate how a problem-specific hybrid method can improve PSO's performance.
3. For the ANN training experiment: Investigate the dependence between number of fitness evaluations and solution quality.

3 Application: Single Machine Total Weighted Tardiness

3.1 Discrete and combinatorial PSO

PSO is in its traditional form directly applicable only for real-valued problems, and no one-size-fits-all method for adapting it to discrete-valued and combinatorial problems have yet been discovered. PSO-variants that perform well on discrete/combinatorial problems often include some creative variation of the particle data structure, and alternative definitions of the position and velocity update equations.

Within the limited scope of this project, I opted to find a combinatorial optimization problems that could be solved by applying a less 'intrusive' method: Running the traditional continuous PSO, but applying some mapping from the search domain to the discrete problem domain.

One hard problem that has been convincingly attacked [6] in this way is the Single Machine Total Weighted Tardiness Problem. My solution is a basically a small-scale clone of Tasgetiren et al.'s approach.

3.2 Theory

The objective is to schedule a set of jobs given per-job *processing times*, *due times*, and *weights*, such that the weighted sum of per-job *tardiness* – a penalty measure for overdue jobs – is minimized. Two constraints apply: (i) only a single machine can be used and (ii) jobs cannot be interleaved. Let's formalise this:

SINGLE MACHINE TOTAL WEIGHTED TARDINESS PROBLEM

Let $\mathbf{P}$, $\mathbf{D}$, and $\mathbf{W}$ be three ordered sets each containing n non-negative real numbers. Call a permutation of the integers $[0, n-1]$ a *schedule*. The goal is to find a schedule minimizing the *total weighted tardiness*:

$$\text{twt}(p) = \sum_{i=1}^{n} \max \{ \mathbf{W}_i(S(p,i) + \mathbf{P}_i - \mathbf{D}_i),\ 0 \}$$

where $S(p,i)$ denotes the *starting time* of the ith job[2] with respect to permutation p, that is $\mathbf{S}_i = \sum_{j=1}^{i-1} \mathbf{P}_{p_j}$.

To map the real-valued search positions into interpretable schedules, we use a simple relation from vectors to integer permutations named the *SPV rule* [6].

SPV RULE

For a k-element vector $\mathbf{x} \in \mathbb{R}^k$, let $\text{spv}(\mathbf{x})$ denote the permutation p of the integers $[0,\ k-1]$ such that p_i is the index of ith largest element in $\mathbf{x}$. If there is ambiguity because of similar elements, always select the element with the lowest index.

For example, $\text{spv}([1,0,2]) = [1,0,2]$, and $\text{spv}([0,4,0.1,2.3]) = [0,2,3,1]$.

Augmentation: Local search In addition to implementing this 'standard' PSO, we did like Tasgetiren et al. and implemented an option to use a local search component as part of the optimization. This is a commonly seen augmentation of PSO which primarily is used to increase exploitation potential [2] (although PSO is good at finding high-quality solution spaces, it can struggle with locating the precise optimas). Specifically, we implemented functionality for client code to pass in a 'per-step manipulation function' which then is called from the PSO main loop after each position update. The specific manipulation function we used is given below, as Python-style pseudo code.

```
function local_search(p):
  in 10% of cases do:
```

```
orig_sol = p.pos
best_score, best_pos = eval_fitness(orig_sol), orig_sol
repeat fifteen times:
    v, w = random indices
    s = orig_sol
    s[v], s[w] = s[w], s[v]
    score = eval_fitness(s)
    if score < best_score:
        best_score, best_pos = score, s
p.pos = best_pos
```

This is a simple search trying fifteen random swaps of elements in the considered particles position vector, and leaves it in the best state[3]. Also, local search only runs one out of ten times.

We call the augmented PSO 'PSO+LS', and the standard PSO 'PSO'.

3.3 Experimental setup

For a k-job problem, we configured PSO as follows (see `smtwtp.py`): (The parameters were selected through trial and error, using values reported by Tasgetiren et al. as guidelines. 80 particles may seem a lot in 'traditional' PSO sense, but many-particle configurations are routinely used in high-dimensionality problems like this.)

Domains: $D = \{x \in \mathbb{R}^k | -1 \leq x_i \leq 1\}$

Initial positions: [Uniform and randomly sampled subset of D]

Optimization goal: Minimize $g = \text{twt} \circ \text{spv}$

Velocity clamping: Yes. v_i is clamped within $[-8, 8]$ before each position update.

Position clamping: No. [4]

Initial inertia: $\omega = 1$

Trust coefficients: $\phi_p = 1.5$, $\phi_g = 1.5$

Inertia decrement rule: Multiply by 0.99 before each iteration, as long as the current inertia is greater than 0.6.

Number of particles: 80

Neighborhood size: 5

We used as benchmark problems the first 20 of the 40-job SMTWTP instances in the well-known OR-Library[5]. We ran 'standard' PSO and PSO+LS eight times on each problem (for a total of $10 * 8 * 2 = 160$ evaluations), and computed the minimum, maximum, arithmetic mean, and standard deviation of the solution scores for each run-of-eight. Each algorithm was allowed to compute for forty seconds on a modern laptop.

Arguably, eight evaluations per case is *far* too low a number to allow rigorous statistical arguments, but we can use the results to do some careful analysis.[6]

3.4 Results

The results are shown in Table 3.4. The column $\mathbf{g}_{\text{opt}}$ gives the known optimum values for each instance. The rows colored red/pink are rows where at least one optimization run reached the known global minimum.

[3]My rough estimates showed that the local search found better positions than the start in approximately $\frac{1}{3}$ of all cases

[4]For most PSO-problems, position clamping is a strictly necessary to ensure solution integrity. However, when using SPV, there is no formal need to restrict the domains, and performance seems to improve greatly when we do not.

[5]`http://people.brunel.ac.uk/~mastjjb/jeb/info.html`

[6]The reason that I did not run more tests per case is simply that running the experiments took too much time, I had to get on with data analysis and writing…

#	Alg.	g_{opt}	$g_{\min}$	g_{avg}	$g_{\max}$	g_{σ}
1	PSO	913	930	949	956	11.27
	PSO+LS	913	913	932	956	15.36
2	PSO	1225	1264	1377	1535	69.33
	PSO+LS	1225	1225	1232	1263	13.11
3	PSO	537	573	573	573	0.00
	PSO+LS	537	537	568	573	11.92
4	PSO	2094	2094	2148	2307	78.20
	PSO+LS	2094	2094	2094	2094	0.00
5	PSO	990	990	990	990	0.00
	PSO+LS	990	990	990	990	0.00
6	PSO	6955	6955	7029	7484	173.17
	PSO+LS	6955	6955	6955	6955	0.00
7	PSO	6324	6324	6493	6571	105.86
	PSO+LS	6324	6324	6369	6571	84.84
8	PSO	6865	6865	6876	6927	21.26
	PSO+LS	6865	6865	6865	6865	0.00
9	PSO	16225	16225	16530	16758	180.42
	PSO+LS	16225	16225	16225	16225	0.00
10	PSO	9737	9741	9796	9903	61.85
	PSO+LS	9737	9737	9743	9771	10.44

Table 1: Experimental results from SMTWTP

3.5 Discussion

Interpreting the results can be a bit challenging; although reaching the known optimum is 'excellent' by any interpretation, it is not immediately clear how good or bad a non-optima like 956 (cf. the minimum, 913) for the first problem instance is. Reaching the solution with 5-10 % error can be very good, if the problem space is complex, and most areas contains solutions that are magnitudes larger than the optimum. To make sense of this, one option would be to compare the results with those obtained by for example a well-known greedy solution to the problem. Still, there are many interesting observations we can make from our data.

Standard PSO find the optimum at least once in eight runs for six out of ten problems. Out of these, there is one problem (# 5) that the algorithm always solves to the optimum. Problem 3, on the other hand, seems to be especially hard for standard PSO. All runs end up at 573, while the true optimum is 537. From manual analysis of the runs we believe that the 573 optimum lays in a large initially good-looking region of the search space, from which it is hard to reach the global optimum. Also, the topology of the global optimum may be such that PSO are not able to exploit it effectively. (From inspection, it seems like the swarm is not able to find the 537 global optimum once the misleading 573 optimum has been visited, even when we stretch all configuration parameters to reduce the convergence rate.)

From the minimum and maximum values and the standard deviations we see that standard PSO is quite robust on most problems, with a multiple of hundred units as the typical difference in total weighted tardiness. However, there are outliers, such as 1535 for Problem 2, where the global optimum is 1225. A lesson to draw from this is that for real-world problems several PSO runs should be used, to increase the confidence in the quality of the solution.

The results strongly suggests that adding stochastic local search to the algorithm increases robustness. First, all of the PSO+LS runs are able to reach the global optimum. Second, PSO+LS solves not one but five out of ten problems perfectly for every run. Third, the maximum solutions are usually notably better than those for standard PSO. Our conclusion is that weaving local search into the algorithm increases performance and accuracy.

Taken as a whole the results indicate that our PSO is able to optimise for the SMTWTP, but that convergence to global optima can not be guaranteed. However, helping the algorithm with a simple local search algorithm to increase exploitation power makes the algorithm more robust.

4 Application: Artificial Neural Network Training

(I must admit up-front that I have not done any programming with ANNs before this, so there might be some errors in my understanding. My goal is to show the gist of how PSO is used to train ANNs.)

Another interesting PSO application is training of artificial neural networks. Typically, the researcher proposes a network structure, and lets the PSO search in weight space, so that each particle corresponds to one specific neural network. The fitness measures is then taken as the induced network's performance on a training set. (More advanced PSO variants have been proposed to optimise both the weights and architecture of ANNs, by for instance by Carvalho and Ludermir [1].)

We investigate optimization performance for feed-forward ANNs with a single hidden layer, bias, and the traditional sigmoid activation function for the hidden and output layer nodes. We use one output node per class, and code the expected output vector for an instance with, say, correct class 2 as $[0, 0, 1]$. The network's classification of an input instance is taken as the index of the maximum value in the output vector. Such a network has $N =$ [# Inputs] $\times$ [# Hidden] + [# Hidden] $\times$ [# Outputs] weights (counting bias as an extra input), and the PSO therefore searches in $\mathbb{R}^N$.

While there of course are more intelligent error measures available[7] for multi-class ANN training, we opted to use only a very basic error measure, the percentage of misclassified instances, which for a weight configuration $\mathbf{w}$ and a data set $\mathbf{X}$ can be defined as:

$$\text{err}(\mathbf{w}, \mathbf{X}) = \frac{|\{\text{c}(\mathbf{w}, \mathbf{x}) \neq a \mid (\mathbf{x}, a) \in \mathbf{X}\}|}{|\mathbf{X}|}$$

where $\text{c}(\mathbf{w}, \mathbf{x})$ denotes the classification of instance x with respect to the weights.

[7]For binary classification problems, it's clear that coding the output with one output node, and using mean squared error as a fitness measure is straightforward. However, I did not achieve good results when testing MSE variants for my multi-class setup. This probably stems from a problem located between the chair and the computer screen. That is, my lack of experience with ANNs…

4.1 Experimental setup

For a N-coefficient ANN training problem, we configured PSO as follows (see `ann.py`):

Domains: $D = \{x \in \mathbb{R}^N | -8 \leq x_i \leq 8\}$

Initial positions: [Uniform and randomly sampled subset of D]

Optimization goal: Minimize $g = \text{err}$

Velocity clamping: No.

Position clamping: Yes.

Initial inertia: $\omega = 1$

Trust coefficients: $\phi_p = 2.4$, $\phi_g = 2.4$

Inertia decrement rule: Multiply by 0.99 before each iteration, as long as the current inertia is greater than 0.6.

Number of particles: 60

Neighborhood size: 5

These parameters were selected through trial and error (a more ambitious and 'correct' option could be to run a meta-optimiser). It's worth commenting on the clamping rules: When using position clamping, and gradually reducing inertia, it is not strictly necessary to use velocity clamping, and in fact it may often reduce search quality. Also here, 60 particles is suitable for this high-dimensionality problems.

We ran the algorithm on three classification problems from the UC Irvine data set repository:

Iris Iris is 'perhaps the best known database to be found in the pattern recognition literature', and a standard benchmark problem for any classifier. The data set contains 150 instances and encodes four attributes and three classes (with 50 instances of each). Two of the three classes are linearly separable, while one is not.

Seeds This data set contains 'measurements of geometrical properties of kernels belonging to three different varieties of wheat'. The data set consist of 210 instances with 7 attributes each, and has three classes.

Glass Another well-known data set. Consists of 214 instances, encodes 10 attributes, and has 7 classes. Glass is a notoriously hard data set for classifiers, with error rates around 30% commonly reported.

We used trial and error, with tips from the Neural Network FAQ[8], to select the number of hidden units for each problem. We initially tested 10-15 unit configurations, but in the end we settled on 3 hidden units for Iris, 5 hidden units for Seeds, and 5 hidden units for Glass. Although these sizes have been selected with care, *sub-optimal network architecture choices is a notable threat to validity of our results and the conclusions drawn from them.*

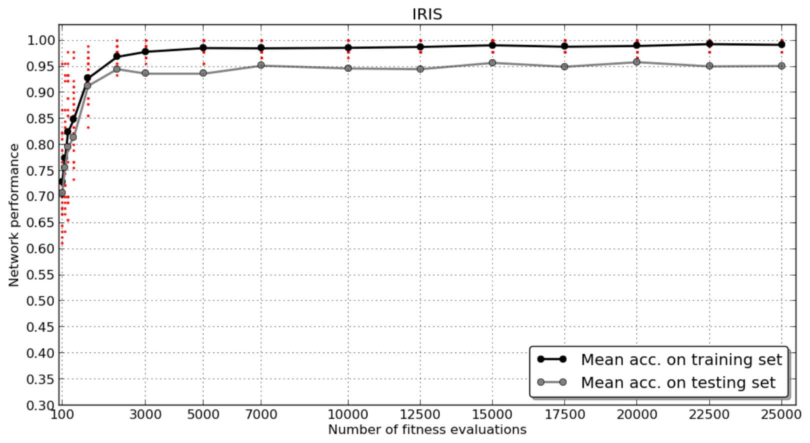
Our objective for these experiments was to show how classification performance depends on the number of fitness evaluations, that is, the number of tests of weight configurations. To do this, we evaluated training performance for each data set for various configurations of evaluation counts ranging from 100 to 30000 (see `ann.py`). For Iris and Seeds we performed 25 PSO runs for each maximum count; for Glass we performed 10 PSO runs per maximum count (probably due to the quadratic growth of the network computation requirements, computing for Glass took significantly longer than for the other sets). For each run, the data set was randomly split into training and test sets, with $\frac{3}{5}$ of instances going into the training set.

[8] `ftp://ftp.sas.com/pub/neural/FAQ.html`

4.2 Results and discussion

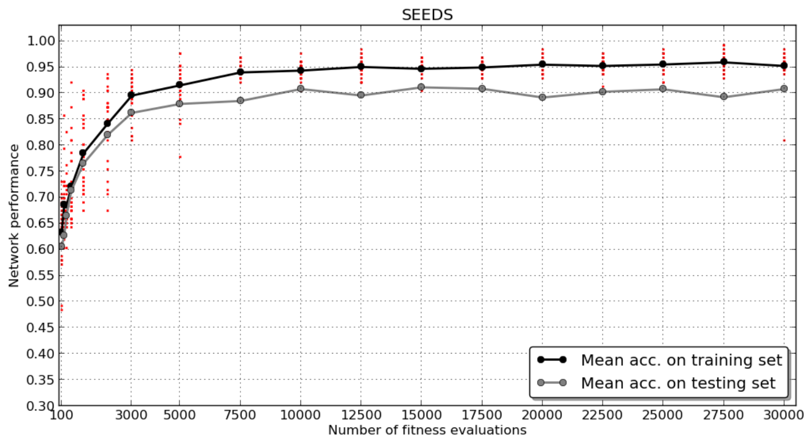
We now present and discuss the experimental results. In each figure below, we plot mean training and testing accuracy on each data set. In addition, each figure includes a scatter plot (red dots) showing *all* classification performances. The intention of the scatter plot is to give a rough indication of robustness.

4.2.1 Iris



Evidently, PSO performs very well on the Iris set. The scatter plot shows that the best runs reach 100% training accuracy with about 2000 evaluations, and that the testing accuracy always is in the upper part of the 95-100 % region as long as we use more than 3000 runs. Mean testing accuracy reaches 95% for 2000 evaluations, and is not significantly improved with more evaluations (in fact, at 3000 and 5000 evaluations, mean testing acc. is about 93%). Our conclusion is that PSO performs strongly and robustly on this data set. The robustness is evident from the scatter plot: the results of single runs tend very close to the mean.

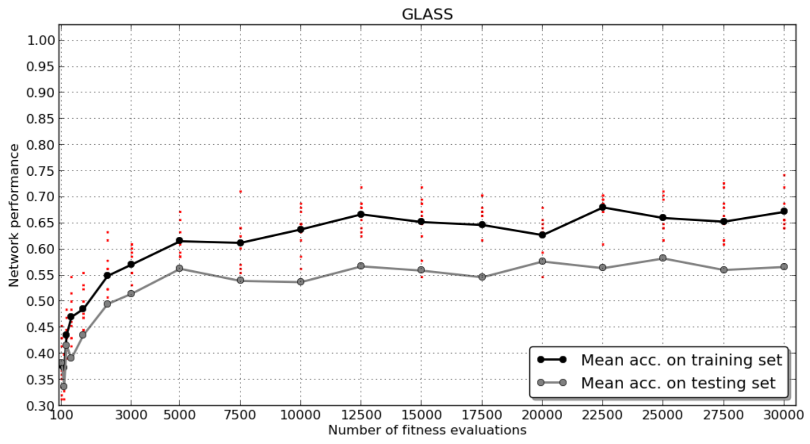
4.2.2 Seeds



In comparison with performance on Iris, PSO requires more iterations to stabilise on the Seeds data set. One probable cause is the larger search space: For Seeds we use both more input and hidden nodes than with Iris. Mean training accuracy increases steadily up to 10000 evaluations, where it levels out at about 95% level. Mean testing accuracy also levels out around 10000 evaluations, and subsequently fluctuates around the 90% point, with no clear sign of overfitting. It's evident that the biggest gains from adding fitness evaluations appears in the range up to 10000 evaluations. After that point, there are no clear changes in performance. This can either mean that the classifier has reached an optimal area of the search space, or, more probably, that PSO is not able to put the additional fitness evaluations to use. (Clearly, a search with a high bound on the number of evaluations should be configured to converge slower than a search with stricter constraints.)

The algorithm does not perform very robustly: at 12500 evaluations, we see that the span between the minimal and maximum solutions is about 10%. Also, there are a few low-performance outliers, the most extreme one being the minimal result of the 30000-evaluation runs, at about 82%. These outliers indicate a danger of early convergence to sub-optimal locations of the search space. Still, the scatter plot indicates that many runs perform strongly ($\geq$ 97%), and thus a practitioner could probably extract a very strong classifier by adding a validation set [for a (*train*, *validation*, *test*)-triple] and selecting the best of several strong-performing classifiers found by repeated runs.

4.2.3 Glass



As noted, Glass is a hard data set for most classifiers[9], and our PSO algorithm also struggles with achieving higher training accuracy than about 65%. The strongest result is the maximum training accuracy of the 30000-evaluation run, at nearly 75%. It seems that optimization performance levels out around 10000 evaluations. From the scatter plot it is evident that PSO's performance on Glass is the least robust. At 15000 evaluations the difference in training accuracy between the best and worst run is more than 15%. This can indicate early convergence to sub-optimal locations of the weight space.

Also here, using repeated runs and a validation set could help us extract powerful classifiers even though mean performance is not impressive. Already at 7500 evaluations there are runs with more than 70% training accuracy.

Given the hardness of this particular data set, we are satisfied with PSO's performance, but, it would be interesting to investigate techniques to control convergence and make the algorithm more robust for this and other hard problems.

5 Concluding remarks

In this text we have reported on my experience implementing and experimenting with particle swarm optimization. We have shown that PSO can be used effectively for optimisation

[9]We also saw this when we used it in investigate boosting for the previous AI programming project.

of two hard problems.

Doing this project has been challenging for several reasons. First, as I was the only student working on this project, I did not have the usual arena for discussing theory and gotchas with other students in the course, which has been very helpful on the previous projects... Second, as with many AI algorithms, the devil is in the detail. Unlike implementing Edmonds-Karp for the maximum flow problem – where things either works or fails hard – AI approximation algorithms comes with a big ice cream truck of variants, bugs and weirdness. Hacking up the basic PSO and applying it to a simple benchmark problem is a deceivingly easy 30-minute job, but to make the algorithm solve harder problems one need to understand the *many* parameters and algorithm variants, making things vastly more complicated. That being said, first of all this has been lots of fun; although I have found them aesthetically pleasing and read about their power, I have never before coded a bio-inspired system. Coding it up and seeing how much power it had despite of its 'simplicity' was very cool.

Doing this project has also involved an interesting literature mini-study. My impression is that there are a vast number of smart PSO variants that in various ways improves upon the original. However, it's hard to single out a clear winner; most researchers compare their variant with the original, and find that it performs better, but doesn't necessarily answer whether their variant performs better than other variations.

Personally, I'm intrigued to investigate novel ways to do inter-neighborhood communication in the swarm. Maybe one could incorporate a more probabilistic model of neighborhoods, and let social and personal influence depend stronger on personal, neighborhood, and swarm-wide performance.[10] Also, one could imagine measures to ensure that the 'smartest choice' of particles are 'called in' in the social interaction; for example, it seems better to call on 'bad-performing' particles for help, than particles that are doing fruitful exploration of another path. Also, I believe one could find inspiration by looking at 'trust hierarchies' in other biological systems, such as among a network of acquaintances in a community, were some people know each other better than other, and therefore may be more willingly to trust and share information.

References

[1] M. Carvalho and T. Ludermir. Particle swarm optimization of neural network architectures andweights. In *Hybrid Intelligent Systems, 2007. HIS 2007. 7th International Conference on*, pages 336 –339, sept. 2007.

[2] A. P. Engelbrecht. *Fundamentals of Computational Swarm Intelligence*. John Wiley & Sons, 2006.

[3] J. Kennedy and R. Eberhart. Particle swarm optimization. In *Neural Networks, 1995. Proceedings., IEEE International Conference on*, volume 4, pages 1942–1948 vol.4. IEEE, Nov. 1995.

[4] K. Parsopoulos and M. Vrahatis. Upso: A unified particle swarm optimization scheme. In *Proceedings of the International Conference of "Computational Methods in Sciences and Engineering*, 2004.

[5] R. Poli. Analysis of the publications on the applications of particle swarm optimisation. *J. Artif. Evol. App.*, 2008:4:1–4:10, Jan. 2008.

[6] M. Tasgetiren, M. Sevkli, Y.-C. Liang, and G. Gencyilmaz. Particle swarm optimization algorithm for single machine total weighted tardiness problem. In *Evolutionary Computation, 2004. CEC2004. Congress on*, volume 2, pages 1412 – 1419 Vol.2, june 2004.

[7] K.-P. Wang, L. Huang, C. guang Zhou, and W. Pang. Particle swarm optimization for traveling salesman problem. In *Machine Learning and Cybernetics, 2003 International Conference on*, volume 3, pages 1583 – 1585 Vol.3, nov. 2003.

[10] Of course, many such variants do exists. All I say is that there must be more to gain here.