Historia de la Recombinación del Universo

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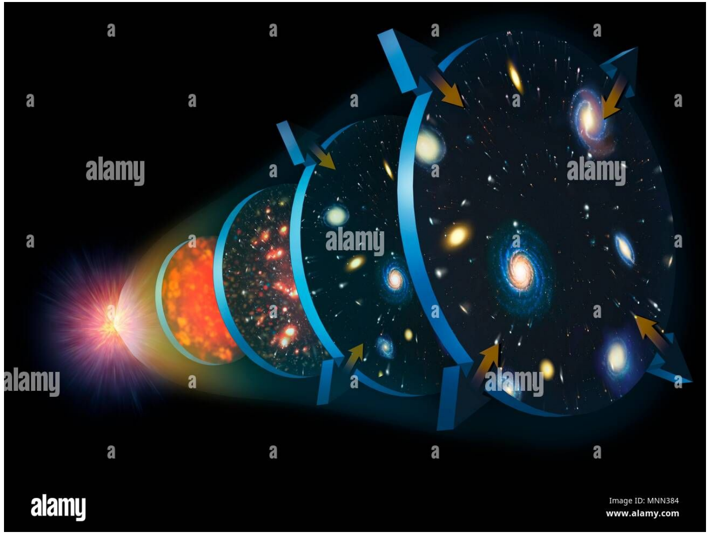
Based on: AST5220 (Master). University of Oslo
https://cmb.wintherscoming.no/milestone2.php

El universo: un sistema termodinámico

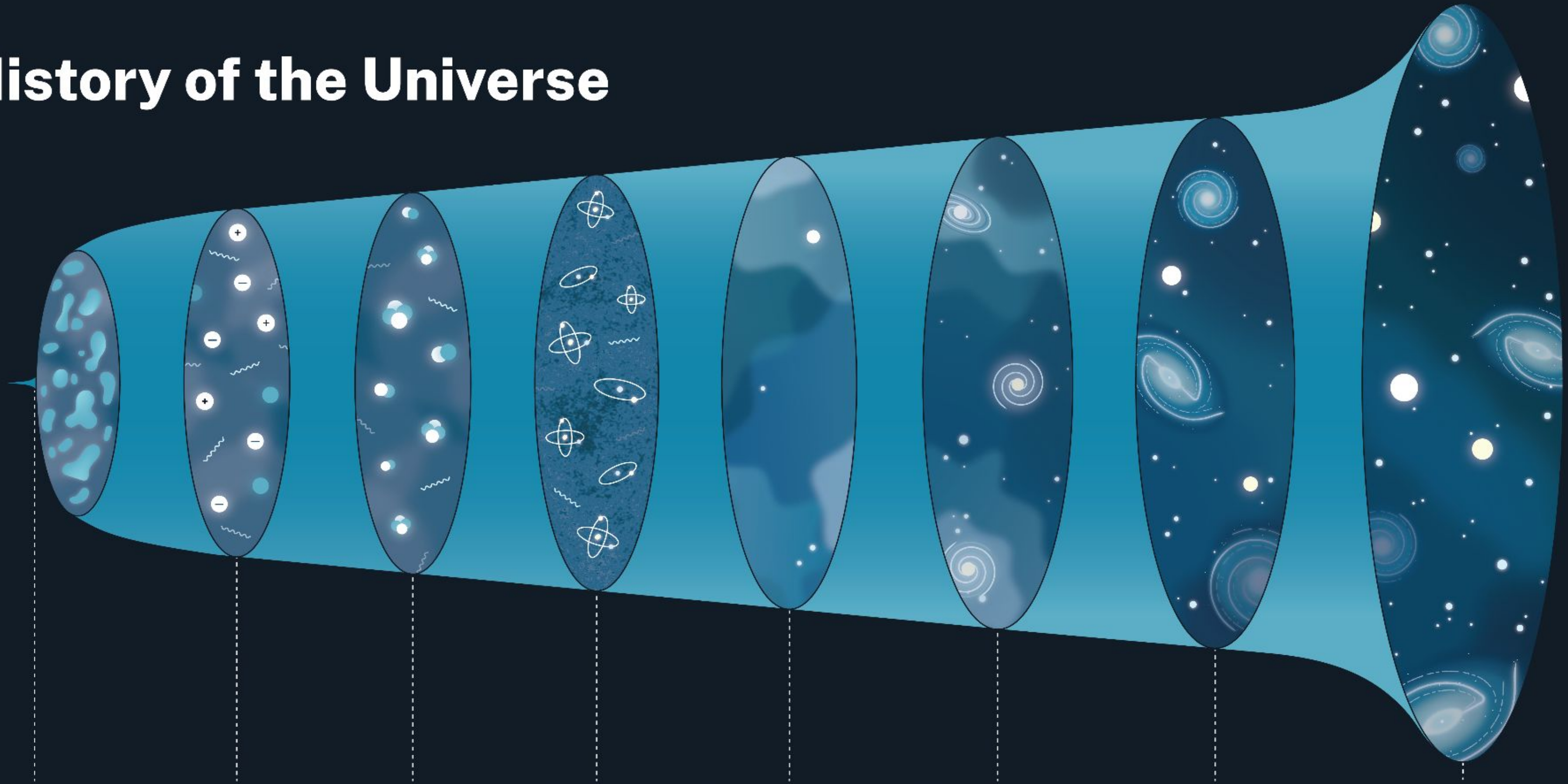


$$n = \frac{g}{(2\pi)^3} \int f d^3p$$

$$\rho = \frac{g}{(2\pi)^3} \int E f d^3p \qquad f = \left(e^{\frac{E-\mu}{T}} \pm 1\right)^{-1}$$

$$P = \frac{g}{(2\pi)^3} \int \frac{p^2}{3E} f d^3p$$

History of the Universe



10^{-32} seconds	1 microsecond	3 minutes	380,000 years	200 million years	400 million years	10 billion years	13.8 billion years
Inflation	**First Particles**	**First Nuclei**	**First Light**	**First Stars**	**Galaxies & Dark Matter**	**Dark Energy**	**Today**
Initial expansion	Neutrons, protons, and electrons form	Helium and hydrogen form	The first atoms form	Gas and dust condense into stars	Galaxies form in dark matter cradles	Expansion accelerates	Humans observe the universe

Historia del Universo

- Inflation and reheating (we'll go through this later in the course)
- Electroweak phase transition
  $T \sim 100$ GeV, $z \sim 10^{15}$, $x = \log a \sim -35$, $t \sim 10^{-12}$ sec
- Quark hadron phase transition
  $T \sim 150$ MeV, $z \sim 10^{12}$, $x = \log a \sim -28$, $t \sim 10^{-5}$ sec
- Neutrino decoupling
  $T \sim 1$ MeV, $z \sim 10^{10}$, $x = \log a \sim -23$, $t \sim 1$ sec
- Electron-positron annihilation
  $T \sim 0.5$ MeV, $z \sim 10^{9}$, $x = \log a \sim -21$, $t \sim 5$ sec
- Big Bang Nucleosyntesis (BBN)
  $T \sim 0.1$ MeV, $z \sim 10^{8}$, $x = \log a \sim -18$, $t \sim 180$ sec
- Recombination and the release of the CMB
  $T \sim 0.4$ eV, $z \sim 1200$, $x = \log a \sim -7$, $t \sim 380.000$ years

$$H = H_0 \sqrt{(\Omega_{b0} + \Omega_{\mathrm{CDM}0})a^{-3} + (\Omega_{\gamma 0} + \Omega_{\nu 0})a^{-4} + \Omega_{k0}a^{-2} + \Omega_{\Lambda 0}},$$

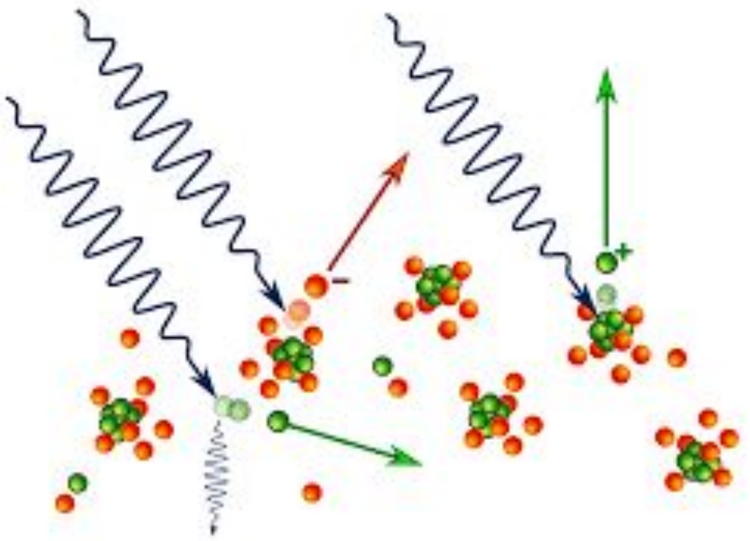
Interacciónes: Boltzmann Equation

$$e^- + p^+ \rightleftharpoons e^- + p^+$$

$$e^- + \gamma \rightleftharpoons e^- + \gamma$$

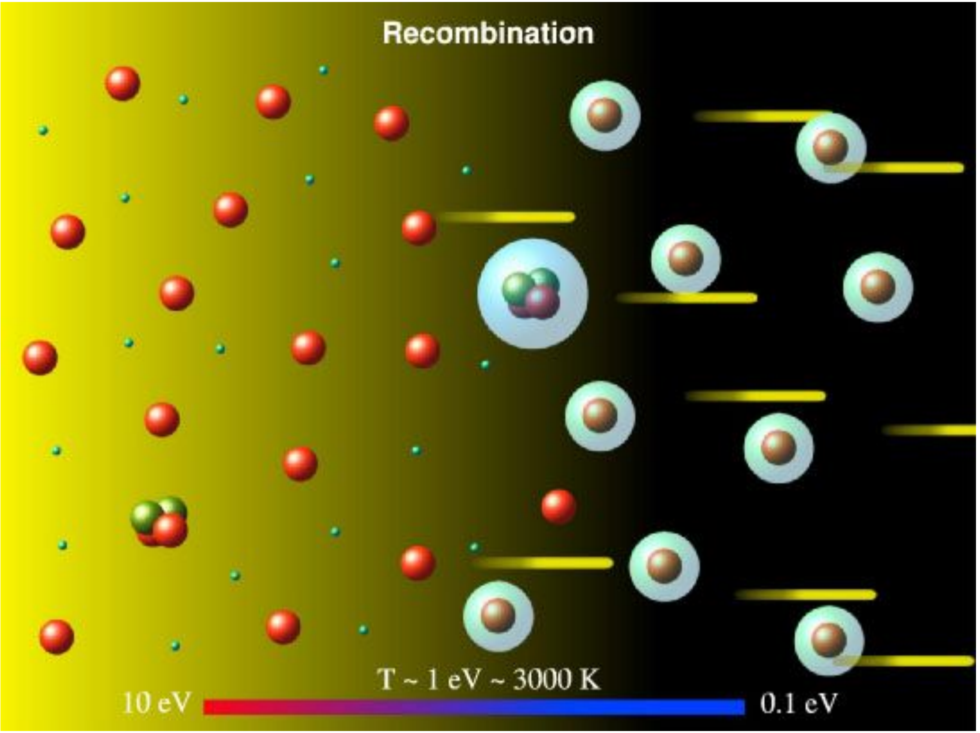
$$e^- + p^+ \rightleftharpoons H + \gamma$$

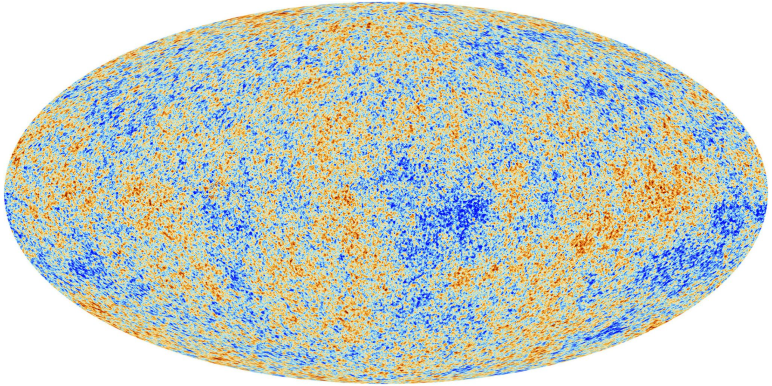
$$\frac{1}{n_1 a^3}\frac{d(n_1 a^3)}{dx} = -\frac{\Gamma_1}{H}\left(1 - \frac{n_3 n_4}{n_1 n_2}\left(\frac{n_1 n_2}{n_3 n_4}\right)_{\mathrm{eq}}\right)$$



Saha Approximation: $$\frac{n_1 n_2}{n_3 n_4} \approx \left(\frac{n_1 n_2}{n_3 n_4}\right)_{\mathrm{eq}}$$

Recombinación

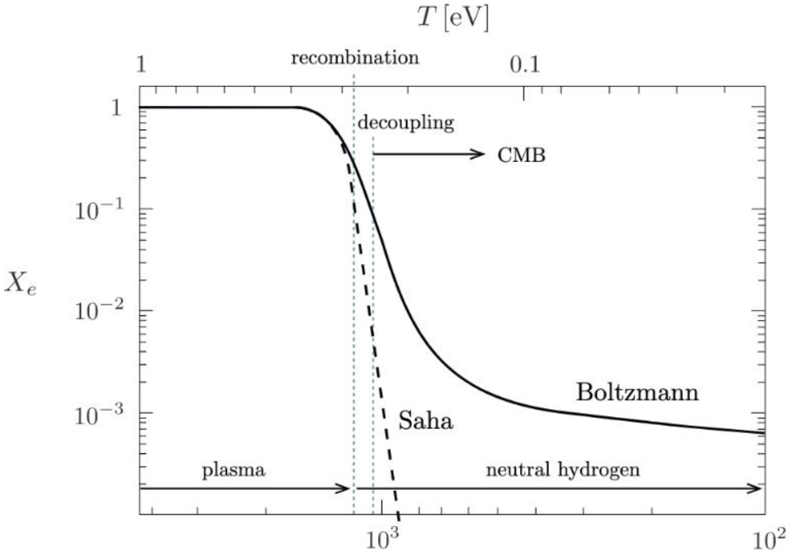




- $\Gamma \ll H$
- Thompson Scattering ineficiente.
- Formación libre de átomos de Hidrógeno.
- Fotones en propagación libre

Electron Fraction Equations

$$X_e = \frac{n_e}{n_b}$$



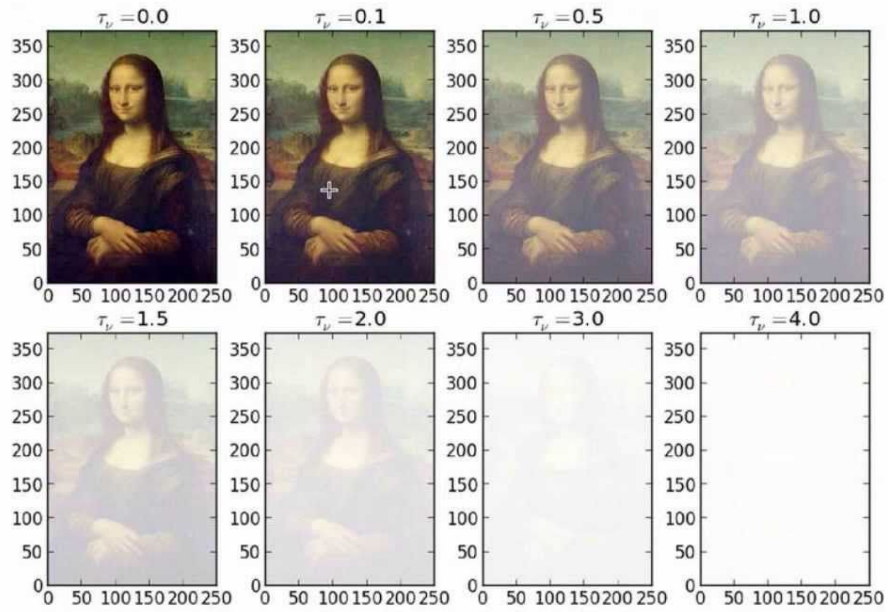
Saha Equation:
(Equilibrium)

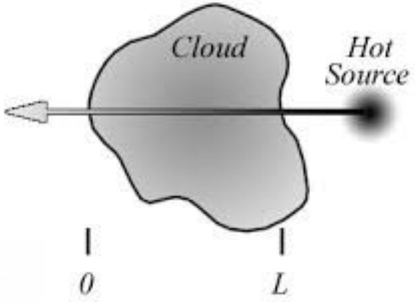
$$\frac{X_e^2}{(1 - X_e)} = \frac{1}{n_b}\left(\frac{k_b T m_e}{2\pi\hbar^2}\right)^{3/2} e^{-\frac{\epsilon_0}{k_b T}}$$

Peebles Equation:
(Recombination)

$$\frac{dX_e}{dx} = \frac{C_r(T_b)}{H}\left[\beta(T_b)(1 - X_e) - n_H \alpha^{(2)}(T_b) X_e^2\right]$$

Optical Depth



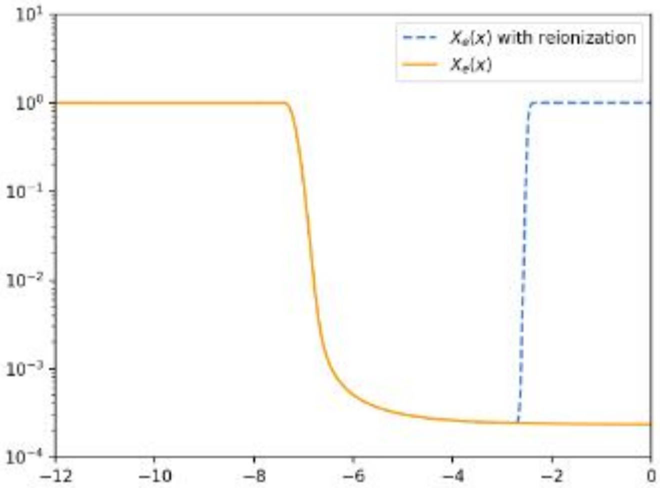


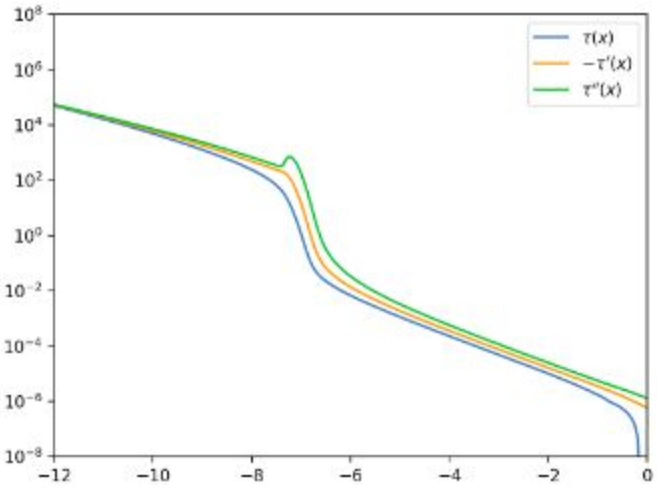
$$I(r) = I_0 e^{-\tau(r)}.$$

$$\tau(\eta) = \int_{\eta}^{\eta_0} n_e \sigma_T a d\eta'$$

$$\frac{d\tau}{dx} = -\frac{c n_e \sigma_T}{H}$$

Electron Fraction and Optical Depth

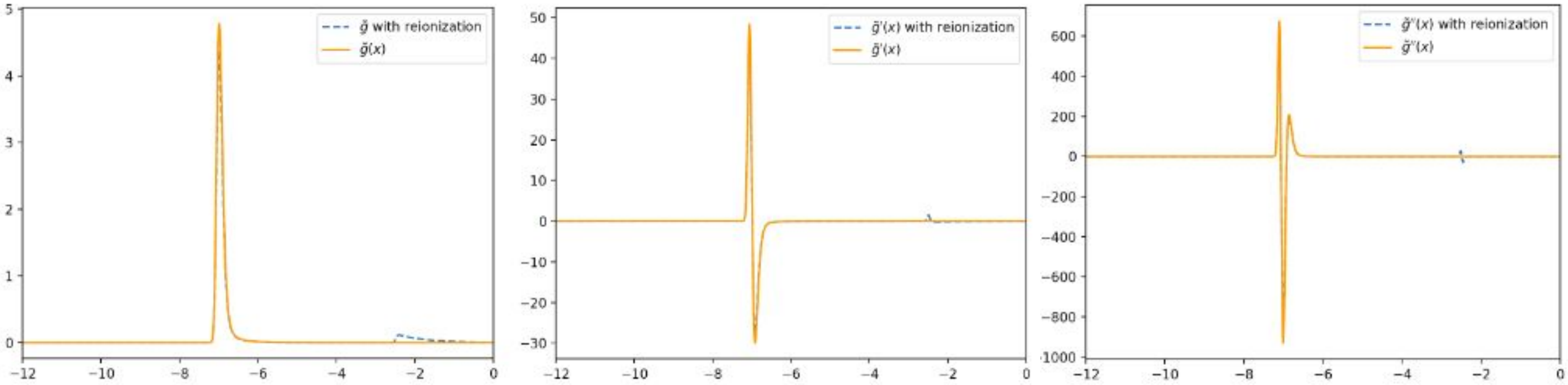




Resultados de: https://cmb.wintherscoming.no/milestone2.php

Visibility Function

$$\tilde{g}(x) = \frac{d}{dx}e^{-\tau} = -\frac{d\tau}{dx}e^{-\tau}, \quad \int_{-\infty}^{0} \tilde{g}(x)dx = 1.$$



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Ejecutemos los programas…

$$C_r(T_b) = \frac{\Lambda_{2s\to1s} + \Lambda_\alpha}{\Lambda_{2s\to1s} + \Lambda_\alpha + \beta^{(2)}(T_b)}, \text{ (dimensionless)}$$
$$\Lambda_{2s\to1s} = 8.227\mathrm{s}^{-1},$$
$$\Lambda_\alpha = H\frac{(3\epsilon_0)^3}{(8\pi)^2c^3\hbar^3n_{1s}}, \text{ (dimension 1/s)}$$
$$n_{1s} = (1 - X_e)n_H, \text{ (dimension 1/m}^3\text{)}$$
$$n_H = (1 - Y_p)n_b, \text{ (dimension 1/m}^3\text{)}$$
$$n_b = (1 - Y_p)\frac{3H_0^2\Omega_{b0}}{8\pi G m_H a^3}, \text{ (dimension 1/m}^3\text{)}$$
$$\beta^{(2)}(T_b) = \beta(T_b)e^{\frac{3\epsilon_0}{4k_bT_b}}, \text{ (dimension 1/s)}$$
$$\beta(T_b) = \alpha^{(2)}(T_b)\left(\frac{m_ek_bT_b}{2\pi\hbar^2}\right)^{3/2}e^{-\frac{\epsilon_0}{k_bT_b}}, \text{ (dimension 1/s)}$$
$$\alpha^{(2)}(T_b) = \frac{8}{\sqrt{3\pi}}c\sigma_T\sqrt{\frac{\epsilon_0}{k_bT_b}}\phi_2(T_b), \text{ (dimension m}^3\text{/s)}$$
$$\phi_2(T_b) = 0.448\ln\left(\frac{\epsilon_0}{k_bT_b}\right), \text{ (dimensionless)}$$

$$h = 0.67,$$
$$T_{\mathrm{CMB0}} = 2.7255\,K,$$
$$N_{\mathrm{eff}} = 3.046,$$
$$\Omega_{\mathrm{b0}} = 0.05,$$
$$\Omega_{\mathrm{CDM0}} = 0.267,$$
$$\Omega_{k0} = 0,$$
$$\Omega_{\nu0} = N_{\mathrm{eff}}\cdot\frac{7}{8}\left(\frac{4}{11}\right)^{4/3}\Omega_{\gamma0},$$
$$\Omega_{\Lambda0} = 1 - (\Omega_{k0} + \Omega_{b0} + \Omega_{\mathrm{CDM0}} + \Omega_{\gamma0} + \Omega_{\nu0}),$$
$$\Omega_{\gamma0} = 2\cdot\frac{\pi^2}{30}\frac{(k_bT_{\mathrm{CMB0}})^4}{\hbar^3c^5}\cdot\frac{8\pi G}{3H_0^2}$$