Notes:

Paired T-Test

Key Learning Points

1. Describe the importance of paired t-tests.
2. Explain how to compare 2 paired samples.
3. Utilize paired t-tests in improvement projects.

What is a Paired T-Test?

A paired t-test calculates the difference between paired measurements (for example before and after samples or naturally paired data) and determines the mean difference. The paired t-test then determines if the mean difference between the paired measurements is significantly different from a target or reference value. A 1 sample t-test on a column of differences is equivalent to a paired t-test.

Normality

The paired t-test works well when the assumption of normality is violated, but only if the underlying distribution is symmetric, unimodal, and continuous. If the values are highly skewed, it might be appropriate to use a nonparametric procedure, such as a 1-sample sign test.

Equal Variance

A paired t-test does not require both samples to have equal variance.

Paired Comparisons

"In many instances problems arise in which two random samples are available

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but they are not independent; rather, each observation in one sample is naturally or by design paired with an observation in the other."1

Each data point has a mate; measurements of the same thing twice. The pairing is the multiple measurement.

"Blocking" is used to block out excess variability caused by the dependency of the samples.

"Block what you can, randomize what you cannot."2

With two samples, the raw data are not used in the analysis. Rather, the average differences between paired values in the two sets of data is compared to zero. Thus, if the population curve of the differences has a mean that is not statistically different from 0, then conclude there is no difference between the samples.

Sources

Milton, Janet Susan & Jesse C Arnold. Introduction to Probability and Statistics: Principles and Applications for Engineering and the Computing Sciences 3rd Ed. (New York: 1995)

Box, George EP, William G. Hunter & J Stuart Hunter. Statistics for Experimenters An Introduction to Design, Data Analysis, and Model Building. (New York; John Wiley and Sons, Inc: 1978)

Shoes Example

This example is adapted from Box, Hunter & Hunter, "Statistics for Experimenters."

Ten boys are randomly selected to test two types of shoe material. Each boy wears one shoe made from each material. The shoes are randomly assigned to the right or the left foot. Because both materials cost the same amount, the quality group wants to produce shoes with the material that will wear the best.

Step 1: State the Practical Problem

The Practical Problem:

Which material should the shoe company use to make shoes?

Step 2: Establish the Hypotheses

The company is looking for a difference between the two materials.

For $\Delta = m1 - m2$

$H_o: \Delta = 0$

$\mathbf{H_a: \Delta \neq 0}$

Step 3: Decide on Appropriate Statistical Test

Since the data are naturally paired, this is a paired t-test.

Formulas

The formula for the Paired T-Test is:

$$H_o : \Delta = 0$$

$$H_a : \Delta \neq 0$$

Where $$\bar{d} = \frac{\sum_{i=1}^{n}(\Delta_{1i} - \Delta_{2i})}{n}$$

where:

- n = sample size in each group
- 1 and 2 refer to group 1 and group 2
- i refers to the paired values from each group

Step 4: Set the Alpha Level

We will use 95% confidence .

$\alpha = 0.05$

Steps 5&6: Set the Power and Sample Size, and Collect the Data

After a set time, the company collected 20 shoes from the 10 boys, 10 of material A, and 10 of Material B. They then measured the wear of each shoe based on their proprietary method.

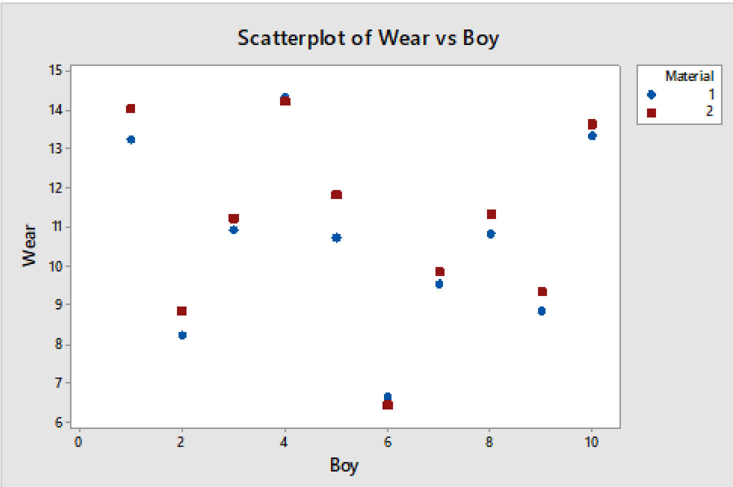
Step 7: Use the Appropriate Graphical Tool To Explore the Data

The following graph was created to explore the data:

Notes:

Scatterplot of Wear vs Boy

- **Minitab: Graph > Scatterplot > With Groups**
  - Y Variable: Wear, X Variable: Boy, Categorical Variable: Material

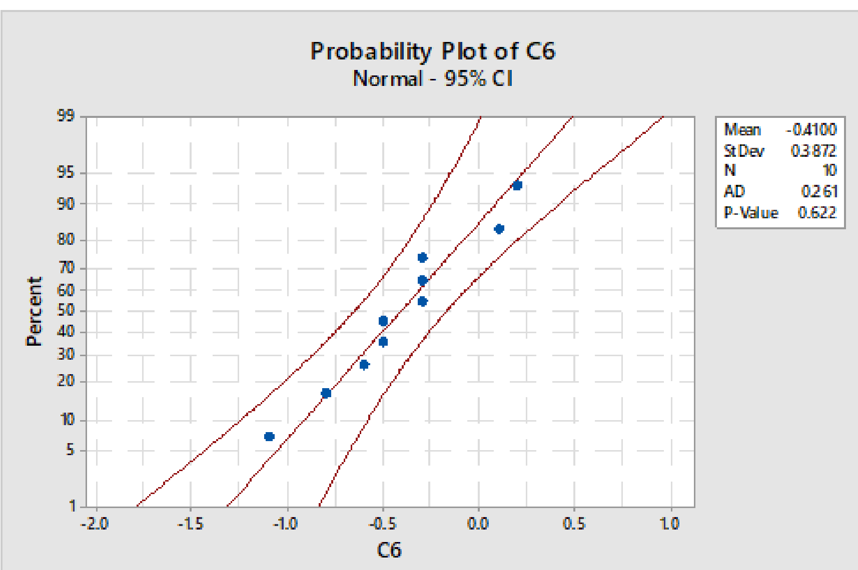


Step 8: Check Data Assumptions

The primary assumption is that the data being used are paired. The column of differences should also be normally distributed.

Normality Test

- **Minitab: Graph > Probability Plot**
  - Variables: Mat A – Mat B



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Step 9: Run the Statistical Test

The company was now ready to run the paired t-test.

- **Minitab: Stat > Basic Statistics > Paired t**
  - Samples in different columns
  - First Sample: "Mat A"
  - Second Sample: "Mat B"

Results

Paired T-Test and CI: MAT A, MAT B

Descriptive Statistics

Sample	N	Mean	StDev	SE Mean
MAT A	10	10.630	2.451	0.775
MAT B	10	11.040	2.518	0.796

Estimation for Paired Difference

Mean	StDev	SE Mean	95% CI for µ_difference
-0.410	0.387	0.122	(-0.687, -0.133)

µ_difference: mean of (MAT A - MAT B)

Test

Null hypothesis H_0: µ_difference = 0
Alternative hypothesis H_1: µ_difference ≠ 0

T-Value	P-Value
-3.35	0.009

Statistical Conclusion

Since the p value (0.009) < 0.05, the company concluded that they must reject the null hypothesis.

$H_a: \Delta \neq 0$

There is a statistically significant difference between Material A and Material B.

Step 10: Translate the Statistical Conclusion Into a Practical Conclusion

The company can conclude that material A wears significantly better than material B.

When Should Paired t-Tests Be Used?

A paired t-test is appropriate to determine if the difference between paired measurements are significantly different.

Notes:

Pitfalls to Avoid

- All measurements must be paired. Every data point in one column must have a mate in the second column.
- Data must be normally distributed for the paired t-test.
- Be sure to test for power if you fail to reject your null hypothesis.

Notes: