Notes:

2 Sample T-Test

Key Learning Points

1. Describe the importance of 2 sample t-tests.
2. Explain how to compare the means of 2 samples.
3. Utilize 2 sample t-tests in improvement projects.

What is a 2 Sample T-Test?

A 2 sample t-test compares the means of two independent populations. When comparing populations, you rarely know the true population parameters. Statistics computed from samples are used to determine how likely it is that the samples came from the same population. With this test, we can conclude, with a degree of statistical confidence, whether these parameters came from one population or if the parameters are statistically different.

t-Distribution

2 sample t-tests require the t-distribution and variable data.

Population(s)

This statistic can be used to test if samples are taken from one population, or from two distinct populations.

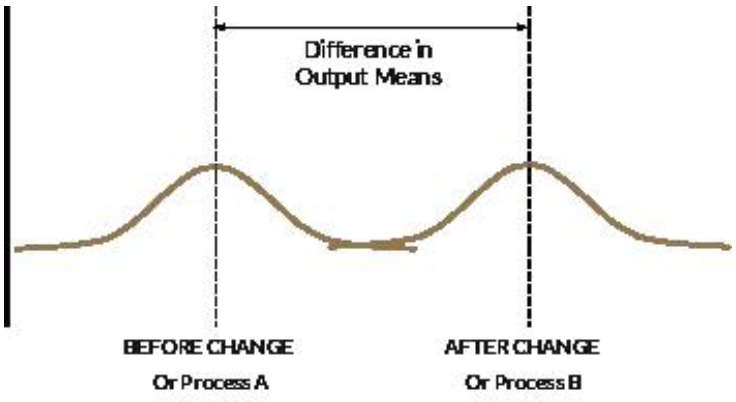
Comparing 2 Means

Comparing two population means requires the t-distribution and continuous data. The t-statistic can be used to test if the population means are significantly different

from each other.

A 2 sample t-test answers the question: Is there a significant different in the output means?

In statistical terms, is there a difference between μ_{pop1} and μ_{pop2}?



Formulas

The formula for the "2-Sample t-Test" statistic is:

- Data Sets With Equal Variances (degrees of freedom = $n_1 + n_2 - 2$):
  - $t(calc) = (xbar_{pop\ 1} - xbar_{pop\ 2}) / \sqrt{[s^2 pooled * \{(1 / n_1) + (1/n_2)\}]}$
- Data Sets With Unequal Variances (degrees of freedom = $n_1 + n_2$ - 2):
  - $t(calc) = (xbar_{pop\ 1} - xbar_{pop\ 2}) / \sqrt{[(s^2\ n_1) + (s2^2 / n_2)]}$

where:

- n = sample size
- s = sample standard deviation

Comparing 2 Populations

A remedial Summer School wants to evaluate two alternative programs. They randomly assigned eleven kids to each program. Four dropped out of program A. They will use a 2 sample t test to determine if the test scores from these sample classes indicate that the population of test scores would be higher for one of the programs.

Step 1: State the Practical Problem

The Practical Problem:

Does one educational program deliver superior results?

In statistical terms, do the two programs have similar populations?

Notes:

Step 2: Establish the Hypotheses

The hypotheses of interest involve whether the population mean scores of one program equal the population mean scores of the other program:

H_o: $\mu_A = \mu_B$

H_a: $\mu_A \neq \mu_B$

Step 3: Decide on Appropriate Statistical Test

A 2 sample t-test is used to compare two continuous population means against each other.

Step 4: Set the Alpha Level

We will use 95% Confidence for this test.

$\alpha = 0.05$

Steps 5&6: Set the Power and Sample Size, and Collect the Data

For this test, the evaluators used the entire group registered for summer session. A larger sample was desirable, but not possible.

Step 7: Use the Appropriate Graphical Tool To Explore the Data

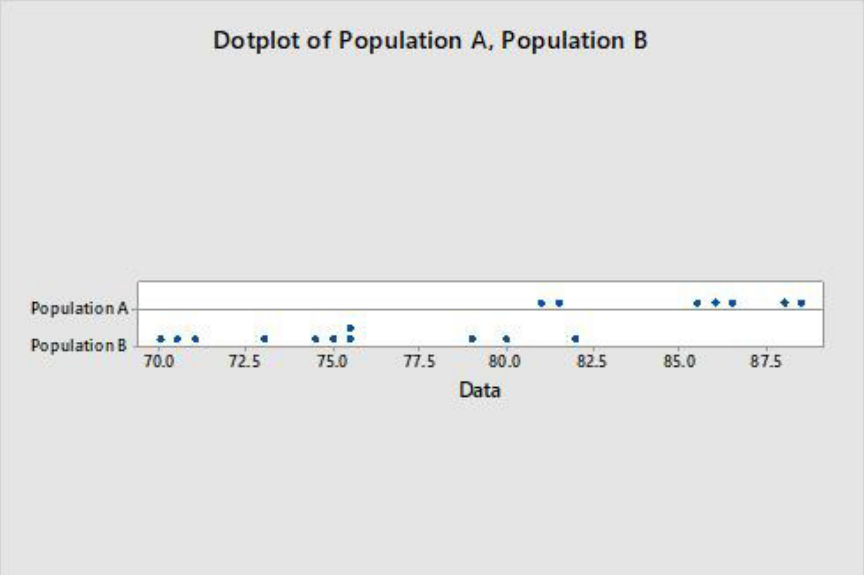
The following graphs can be used to explore the data.

Notes:

Dotplot

- **Minitab: Graph > Dotplot > Multiple Ys > Simple**
  - Graph Variables: Population A and Population B

Dotplot Results

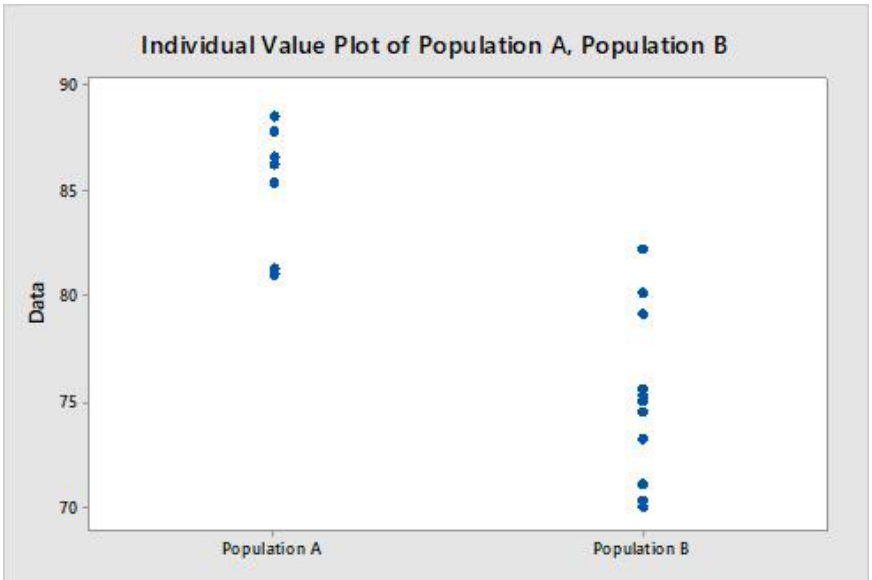


Notes:

Individual Value Plot

- **Minitab: Graph > Individual Value Plot > Multiple Ys > Simple**
  - Graph Variables: Population A and Population B

Individual Value Plot Results

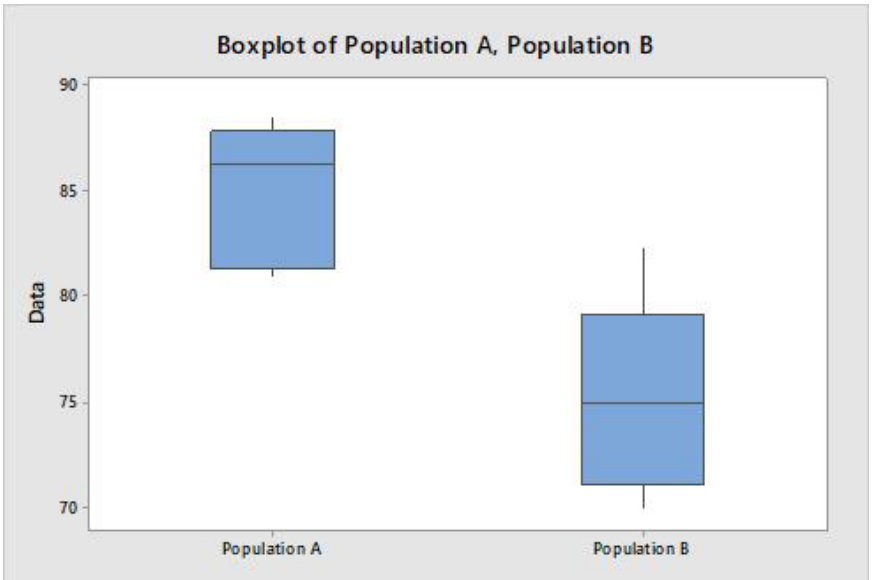


Notes:

Box Plots

- **Minitab: Graph > Box Plots > Multiple Ys > Simple**
  - Graph Variables: Population A and Population B

Box Plot Results



Notes:

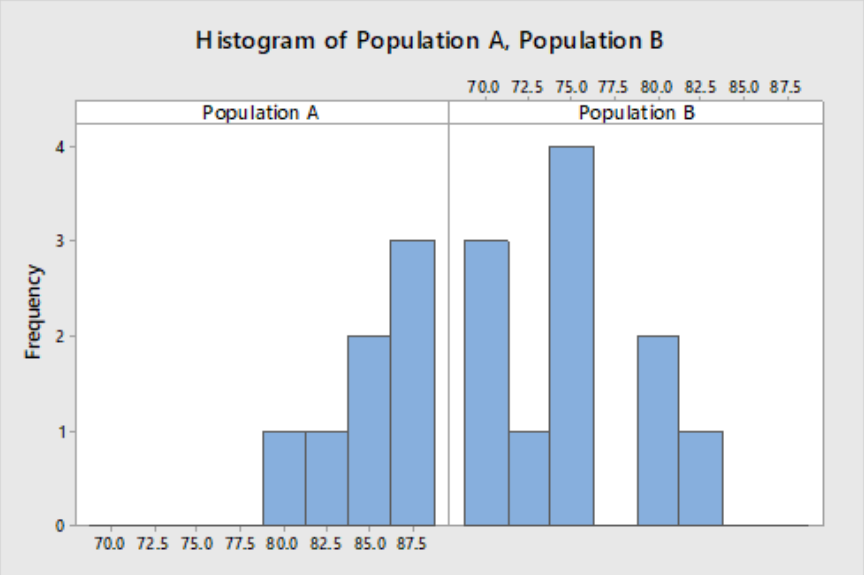
Histograms

- **Minitab: Graph >Histogram > Simple**
  - Graph Variables: Population A and Population B
  - Multiple graphs: In separate panels of the same graph

or

  - Multiple graphs: Overlaid on same graphs

Histogram Results

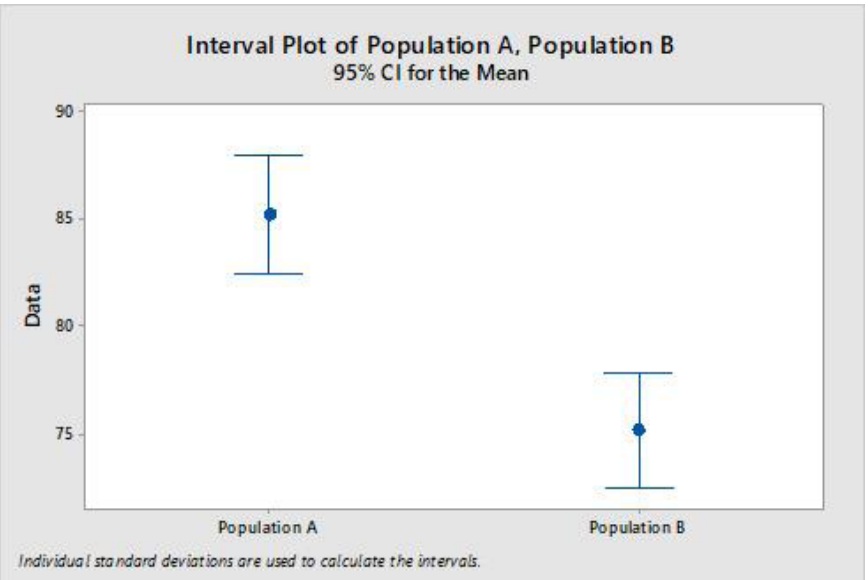


Interval Plot

- Minitab: Graph > Interval Plot > Multiple Ys > Simple
  - Graph Variables: Population A and Population B

Notes:

Interval Plot Results



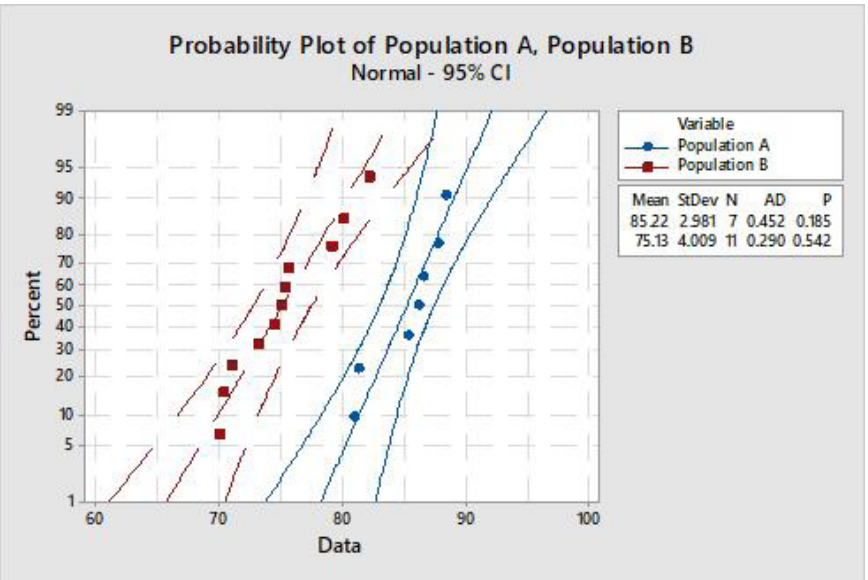
Step 8: Check Data Assumptions

Following the Hypothesis Testing Roadmap, we next check for normality and equal variances.

Normality Test

- **Minitab: Stat > Basic Statistics > Normality Test**
  - Variable: Population
  - Tests for Normality: Anderson Darling

Normality Results

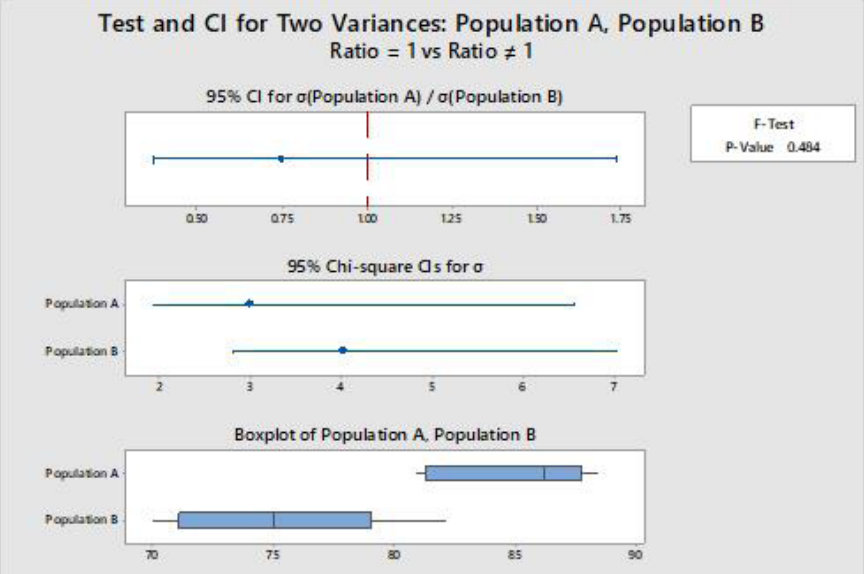


Notes:

Test of Equal Variances

- **Minitab: Stat > Basic Statistics > 2 Variances**
  - Each sample in its own column
  - Options: Use test and confidence intervals based on normal distribution

Test of Equal Variances Results



Notes:

Step 9: Run the Statistical Test

Harold was now ready to run the 2 Sample t-Test.

- **Minitab: Stat > Basic Statistics > 2-Sample t**
  - Samples in different columns
  - First: Manufacturer A
  - Second: Manufacturer B
  - Leave Assume Equal Variances Unchecked

Results

Two-Sample T-Test and CI: Population A, Population B

Method

μ_1: mean of Population A
μ_2: mean of Population B
Difference: $\mu_1 - \mu_2$

Equal variances are assumed for this analysis.

Descriptive Statistics

Sample	N	Mean	StDev	SE Mean
Population A	7	85.22	2.98	1.1
Population B	11	75.13	4.01	1.2

Estimation for Difference

Difference	Pooled StDev	95% CI for Difference
10.09	3.66	(6.34, 13.84)

Test

Null hypothesis $H_0: \mu_1 - \mu_2 = 0$
Alternative hypothesis $H_1: \mu_1 - \mu_2 \neq 0$

T-Value	DF	P-Value
5.71	16	0.000

Statistical Conclusion

Since the p value (0.000) < 0.05, reject the null hypothesis at 95% confidence.

$H_a: \mu_A \neq \mu_B$

There is a statistically significant difference between μ_A and μ_B.

Step 10: Translate the Statistical Conclusion Into a Practical Conclusion

Program A produced higher test scores than Program B.

When Should 2 Sample t-Tests Be Used?

A 2 Sample t-test is appropriate to determine if population means of two normally distributed continuous data sets are significantly different from each other.

Notes:

Pitfalls to Avoid

- Data must be normally distributed for the 2 Sample t-test.
- For increased statistical power, where power is the ability to detect a true difference (1-b), use the pooled variance option when variances are proven statistically equal.
- Be sure to test for power if you fail to reject your null hypothesis.

Notes: