Notes:

Fractional Factorial Experiments

Key Learning Points

1. Describe when to use fractional factorial designs.
2. Describe why fractional designs are used.
3. Utilize fractional factorial experiments in your improvement project.

What are Fractional Factorial Experiments?

Fractional factorials allow investigating a relatively large number of factors in a relatively small number of runs. They are sometimes referred to as "screening experiments."

As the number of factors of interest increases, so does the number of runs required to run a full 2k factorial experiment. Without repeats or replicates:

- 2 Factors, 2^2 Factorial = 4 runs
- 3 Factors, 2^3 Factorial = 8 runs
- 4 Factors, 2^4 Factorial = 16 runs
- 5 Factors, 2^5 Factorial = 32 runs, etc...

If the experimenter assumes higher-order interactions are negligible, it is possible to do a fraction of the full factorial and still get good estimates of lower-order interactions and main effects. In true screening experiments, experimenters can investigate dozens of main effects, by only looking at interactions in subsequent factorial experiments with fewer factors.

DOE Vocabulary

- Screening (Fractional) Experiments: Experiments that allow you to investigate main effects and/or lower-order interactions without having to run full-factorial experiments.
- Half Fraction: Experiments that allow you to investigate main effects and/or lower-order interactions in half the runs required by a full factorial.
- Quarter Fraction: Experiments that allow you to investigate main effects and/or lower-order interactions in one-fourth the runs required by a full factorial.
- Aliased or Confounded: The inability to determine which main effect or interaction is causing the true effect. One or more effects that cannot be unambiguously attributed to a single factor or interaction.
- Design Resolution: A Roman numeral notation which allows you to describe the ”worst case” confounding scheme associated with a design.
  - Full – All main effects and interactions are unaliased
  - III – Main effects are aliased with two-factor interactions (I + II)
  - IV – Main effects are aliased with three-factor interactions (I + III), two-factor interactions are aliased with other two-factor interactions (II + II)

Fractional Factorial Notation

$$2_R^{k-p}$$

The general notation to designate a fractional factorial design can be seen to the left.

In this notation:

- k = number of factors to be investigated
- 2^{k-p} = number of runs
- R = design resolution (III, IV,V)

Note:

- If p = 1, then half-fraction factorial
- If p = 2, then quarter-fraction factorial

Notes:

Full to Half Fraction Design

Notes:

Suppose we want to investigate four input variables in eight test runs. Consider an identity element "I", which is essentially 1. To fractionate, set "I" equal to the highest order interaction, so I=A*B*C*D. Then consider the matrix below of a three-factor experiment.

If I=ABCD, then D=ABC. So we will replace the ABC interaction with D.

When you replace the A x B x C interaction with Factor D, you say the ABC is aliased with D. ABC can no longer be estimated.

Run	A	B	C	A*B	A*C	B*C	A*B*C
1	-1	-1	-1	+1	+1	+1	-1
2	+1	-1	-1	-1	-1	+1	+1
3	-1	+1	-1	-1	+1	-1	+1
4	+1	+1	-1	+1	-1	-1	-1
5	-1	-1	+1	+1	-1	-1	+1
6	+1	-1	+1	-1	+1	-1	-1
7	-1	+1	+1	-1	-1	+1	-1
8	+1	+1	+1	+1	+1	+1	+1

The Design Generator is D = A x B x C

Why is it Called A Half Fraction?

This would be called a half fraction since a full 24 factorial design would take 16 runs to complete. In a half fraction you can estimate 4 factors in only 8 runs. But there is a cost. You lose all higher-order interactions.

Run	A	B	C	D	A*B	A*C	B*C	A*B*C
1	-1	-1	-1	-1	+1	+1	+1	-1
2	+1	-1	-1	-1	-1	-1	+1	+1
3	-1	+1	-1	-1	-1	+1	-1	+1
4	+1	+1	-1	-1	+1	-1	-1	-1
5	-1	-1	+1	-1	+1	-1	-1	+1
6	+1	-1	+1	-1	-1	+1	-1	-1
7	-1	+1	+1	-1	-1	-1	+1	-1
8	+1	+1	+1	-1	+1	+1	+1	+1

The Design Generator is D = A x B x C

Example

An improvement team at ACME Chemicals want to determine which factors in their process can increase the percent reacted of a chemical process.

Unfortunately they only have enough funds to complete 16 runs of their experiment.

Step 1: Define the Problem

The Problem:

Determine which factors in a chemical process that can increase the percent reacted from the process

Step 2: State the Hypotheses

The effect of at least one of the factors on the output is zero.

- H_o: Effect of Feed Rate = 0
- H_a: Effect of Feed Rate ≠ 0
- H_o: Effect of Catalyst= 0
- H_a: Effect of Catalyst ≠ 0
- H_o: Effect of Agitation Rate = 0
- H_a: Effect of Agitation Rate ≠ 0
- H_o: Effect of Temperature = 0
- H_a: Effect of Temperature ≠ 0
- H_o: Effect of Concentration = 0
- H_a: Effect of Concentration ≠ 0

Step 3: State the Factors and Levels of Interest

For this experiment the factors are:

- Output: % Redacted
- Inputs:
  - Feed Rate (liters/minute)
  - 10 (-1), 15 (+1)
  - B: Catalyst (%)
  - 1 (-1) , 2 (+1)
  - C: Agitation Rate (rpm)
  - 100 (-1), 120 (+1)
  - D: Temperature (°C)
  - 140 (-1) ,180 (+1)

Notes:

- E: Concentration (%)
- 3 (-1), 6 (+1)

Step 4: Create an Appropriate Experimental Data sheet

- Minitab: Stat > DOE > Factorial > Create Factorial Design
  - Number of Factors: 5
  - Designs: 1/2 Fraction (16 Runs)

↓	C1	C2	C3	C4	C5	C6	C7	C8	C9
	StdOrder	RunOrder	CenterPt	Blocks	Feed Rate	Catalyst	Agitation Rate	Temperature	Concentration
1	1	1	1	1	10	1	100	140	6
2	2	2	1	1	15	1	100	140	3
3	3	3	1	1	10	2	100	140	3
4	4	4	1	1	15	2	100	140	6
5	5	5	1	1	10	1	120	140	3
6	6	6	1	1	15	1	120	140	6
7	7	7	1	1	10	2	120	140	6
8	8	8	1	1	15	2	120	140	3
9	9	9	1	1	10	1	100	180	3
10	10	10	1	1	15	1	100	180	6
11	11	11	1	1	10	2	100	180	6
12	12	12	1	1	15	2	100	180	3
13	13	13	1	1	10	1	120	180	6
14	14	14	1	1	15	1	120	180	3
15	15	15	1	1	10	2	120	180	3
16	16	16	1	1	15	2	120	180	6

Step 5: Run the Experiment and Collect the Data

In real experiments always randomize. For this example standard order has been used.

Step 6: Construct The ANOVA Table

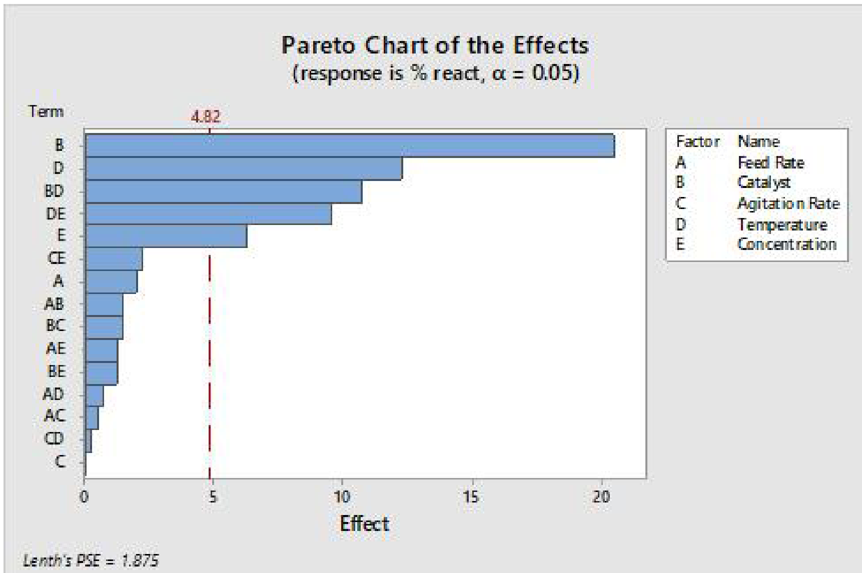
- **Minitab: Stat > DOE > Factorial > Analyze Factorial Design**
  - Response: % React
- Terms
  - Catalyst, Agitation Rate, Temperature, Concentration
- Graphs
  - Response: % React
  - Graphs: Effects Plot
  - Check off Normal and Pareto

Notes:

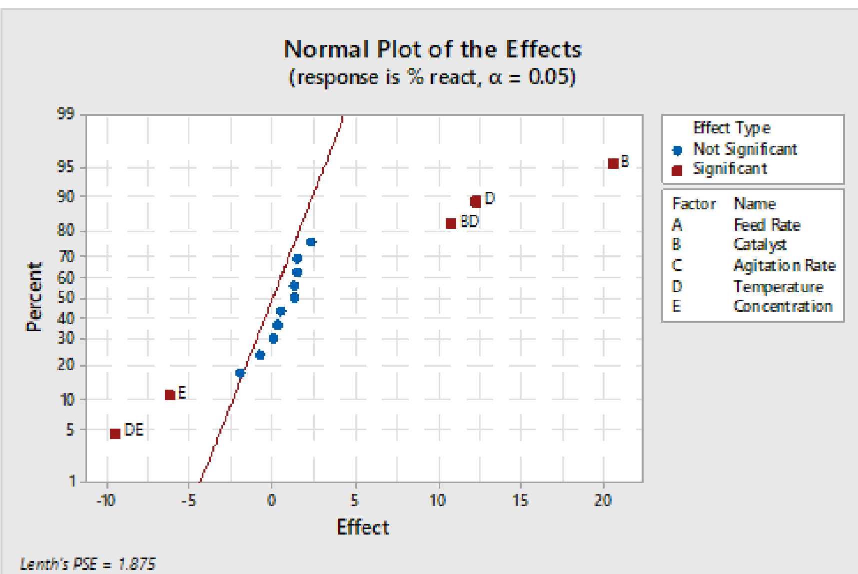
Results

Notes:

Pareto Chart of the Effects



Normal Plot of the Effects



Analysis of Variance Table

Factorial Regression: % react versus Feed Rate, Catalyst, ... ncentration

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	15	3331.00	222.07	*	*
Linear	5	2453.50	490.70	*	*
Feed Rate	1	16.00	16.00	*	*
Catalyst	1	1681.00	1681.00	*	*
Agitation Rate	1	0.00	0.00	*	*
Temperature	1	600.25	600.25	*	*
Concentration	1	156.25	156.25	*	*
2-Way Interactions	10	877.50	87.75	*	*
Feed Rate*Catalyst	1	9.00	9.00	*	*
Feed Rate*Agitation Rate	1	1.00	1.00	*	*
Feed Rate*Temperature	1	2.25	2.25	*	*
Feed Rate*Concentration	1	6.25	6.25	*	*
Catalyst*Agitation Rate	1	9.00	9.00	*	*
Catalyst*Temperature	1	462.25	462.25	*	*
Catalyst*Concentration	1	6.25	6.25	*	*
Agitation Rate*Temperature	1	0.25	0.25	*	*
Agitation Rate*Concentration	1	20.25	20.25	*	*
Temperature*Concentration	1	361.00	361.00	*	*
Error	0	*	*		
Total	15	3331.00			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
*	100.00%	*	*

Coded Coefficients

Term	Effect	Coef	SE Coef	T-Value	P-Value	VIF
Constant		65.25	*	*	*	
Feed Rate	-2.000	-1.000	*	*	*	1.00
Catalyst	20.50	10.25	*	*	*	1.00
Agitation Rate	-0.000000	-0.000000	*	*	*	1.00
Temperature	12.250	6.125	*	*	*	1.00
Concentration	-6.250	-3.125	*	*	*	1.00
Feed Rate*Catalyst	1.5000	0.7500	*	*	*	1.00
Feed Rate*Agitation Rate	0.5000	0.2500	*	*	*	1.00
Feed Rate*Temperature	-0.7500	-0.3750	*	*	*	1.00
Feed Rate*Concentration	1.2500	0.6250	*	*	*	1.00
Catalyst*Agitation Rate	1.5000	0.7500	*	*	*	1.00
Catalyst*Temperature	10.750	5.375	*	*	*	1.00
Catalyst*Concentration	1.2500	0.6250	*	*	*	1.00
Agitation Rate*Temperature	0.2500	0.1250	*	*	*	1.00
Agitation Rate*Concentration	2.250	1.125	*	*	*	1.00
Temperature*Concentration	-9.500	-4.750	*	*	*	1.00

Regression Equation in Uncoded Units

% react = 112.8 - 1.950 Feed Rate - 93.25 Catalyst - 0.7875 Agitation Rate
+ 0.2375 Temperature + 11.67 Concentration + 0.6000 Feed Rate*Catalyst
+ 0.01000 Feed Rate*Agitation Rate - 0.007500 Feed Rate*Temperature
+ 0.1667 Feed Rate*Concentration + 0.1500 Catalyst*Agitation Rate
+ 0.5375 Catalyst*Temperature + 0.8333 Catalyst*Concentration
+ 0.000625 Agitation Rate*Temperature + 0.07500 Agitation Rate*Concentration
- 0.1583 Temperature*Concentration

Alias Structure

Factor	Name
A	Feed Rate
B	Catalyst
C	Agitation Rate
D	Temperature
E	Concentration

Aliases
I + ABCDE
A + BCDE
B + ACDE
C + ABDE
D + ABCE
E + ABCD
AB + CDE
AC + BDE
AD + BCE
AE + BCD
BC + ADE
BD + ACE
BE + ACD
CD + ABE
CE + ABD
DE + ABC

Effects Plot for % react

Effects Pareto for % react

* NOTE * Could not graph the specified residual type because MSE = 0 or the degrees of freedom for error = 0.

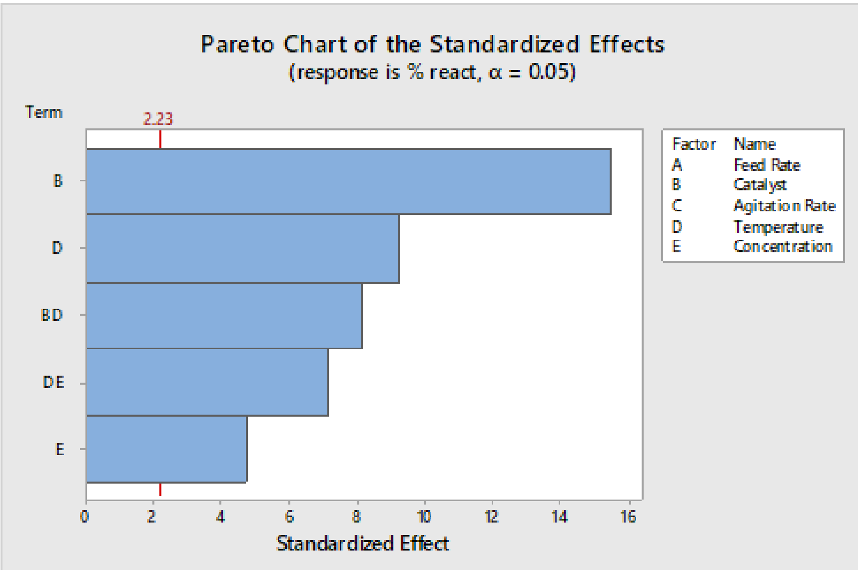
Notes:

Step 7: Rerun a Reduced Model

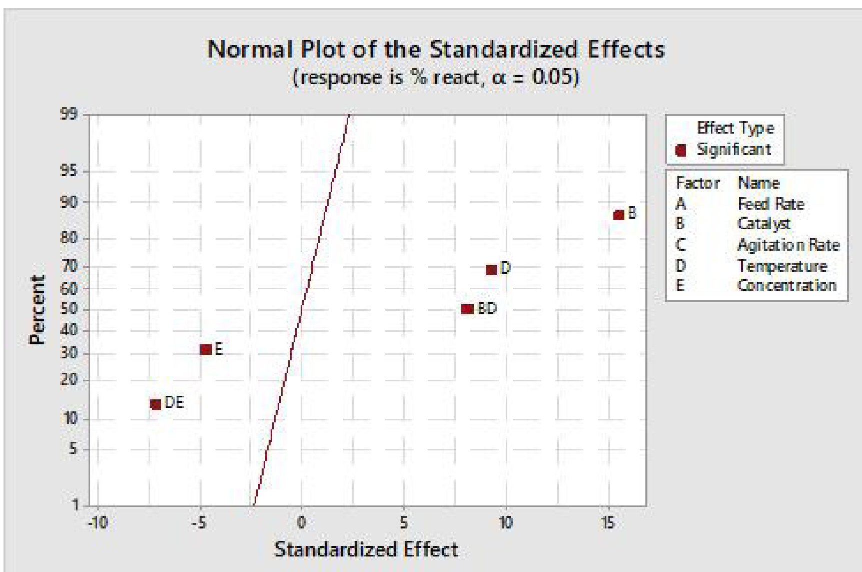
Complete the ANOVA Table again, but remove the interaction of temperature and pressure.

Reduced Results

Pareto Chart of the Effects



Normal Plot of the Effects



Notes:

Analysis of Variance Table

Notes:

Factorial Regression: % react versus Catalyst, ... erature, Concentration

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	5	3260.75	652.15	92.83	0.000
Linear	3	2437.50	812.50	115.66	0.000
Catalyst	1	1681.00	1681.00	239.29	0.000
Temperature	1	600.25	600.25	85.44	0.000
Concentration	1	156.25	156.25	22.24	0.001
2-Way Interactions	2	823.25	411.62	58.59	0.000
Catalyst*Temperature	1	462.25	462.25	65.80	0.000
Temperature*Concentration	1	361.00	361.00	51.39	0.000
Error	10	70.25	7.03		
Total	15	3331.00			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
2.65047	97.89%	96.84%	94.60%

Coded Coefficients

Term	Effect	Coef	SE Coef	T-Value	P-Value	VIF
Constant		65.250	0.663	98.47	0.000	
Catalyst	20.500	10.250	0.663	15.47	0.000	1.00
Temperature	12.250	6.125	0.663	9.24	0.000	1.00
Concentration	-6.250	-3.125	0.663	-4.72	0.001	1.00
Catalyst*Temperature	10.750	5.375	0.663	8.11	0.000	1.00
Temperature*Concentration	-9.500	-4.750	0.663	-7.17	0.000	1.00

Regression Equation in Uncoded Units

% react = 9.9 - 65.5 Catalyst + 0.212 Temperature + 23.25 Concentration
+ 0.5375 Catalyst*Temperature - 0.1583 Temperature*Concentration

Alias Structure

Factor	Name
A	Feed Rate
B	Catalyst
C	Agitation Rate
D	Temperature
E	Concentration

Aliases
I + ABCDE
B + ACDE
D + ABCE
E + ABCD
BD + ACE
DE + ABC

Fits and Diagnostics for Unusual Observations

Obs	% react	Fit	Resid	Std Resid	
9	69.00	63.63	5.38	2.57	R

R Large residual

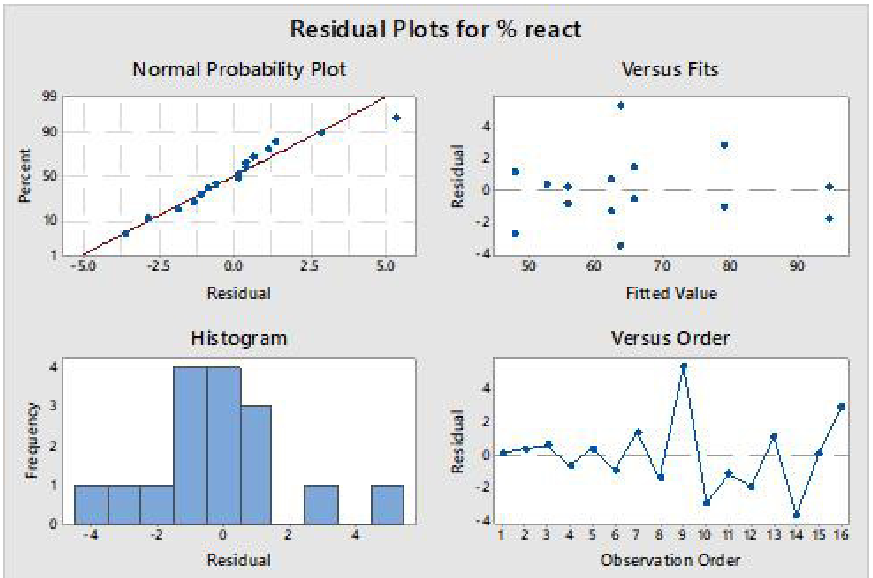
Effects Plot for % react

Effects Pareto for % react

Residual Plots for % react

Step 8: Investigate the Residual Plots to Ensure Model Fit

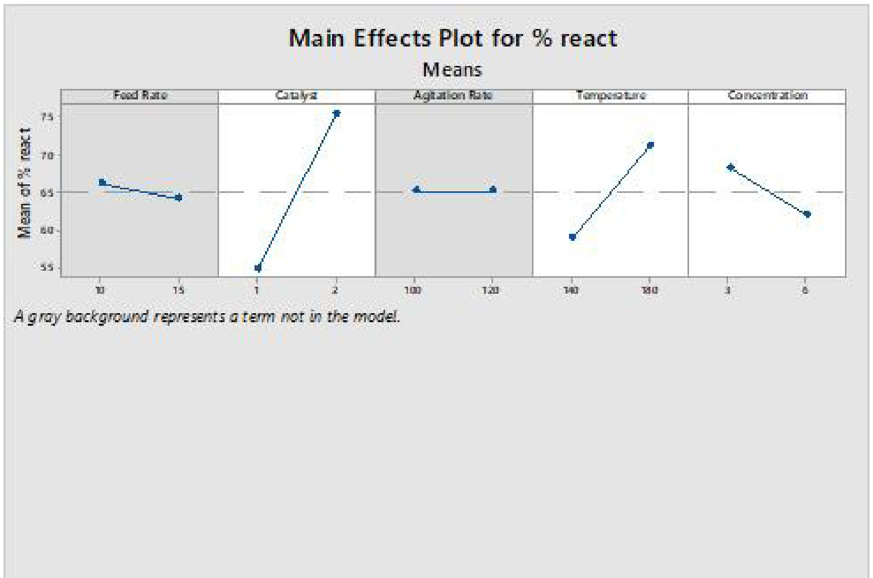
Residual Plot



Step 9: Investigate Significant Main Effects and Interactions

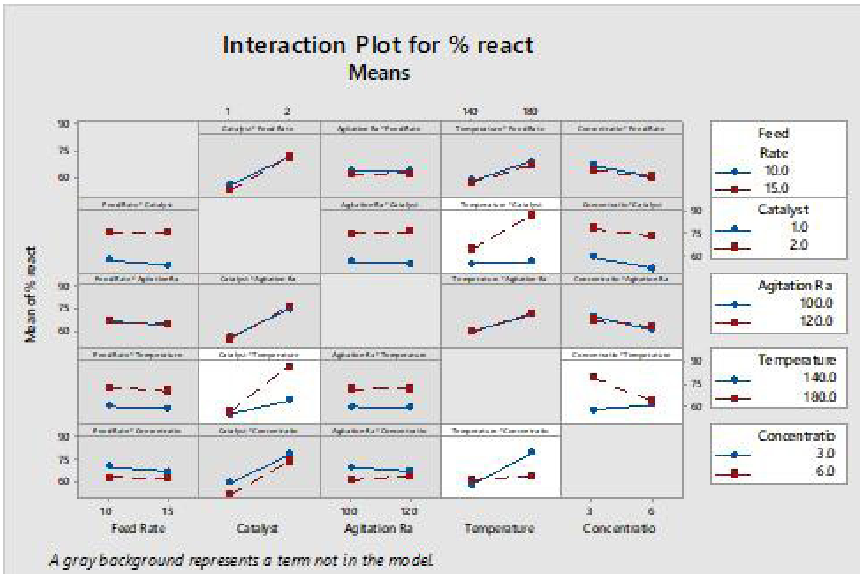
Using the ANOVA table and appropriate graphical tools, investigate the P-Values of the main effects and interactions. Assess the significance of highest order interactions first.

Main Effects Plot

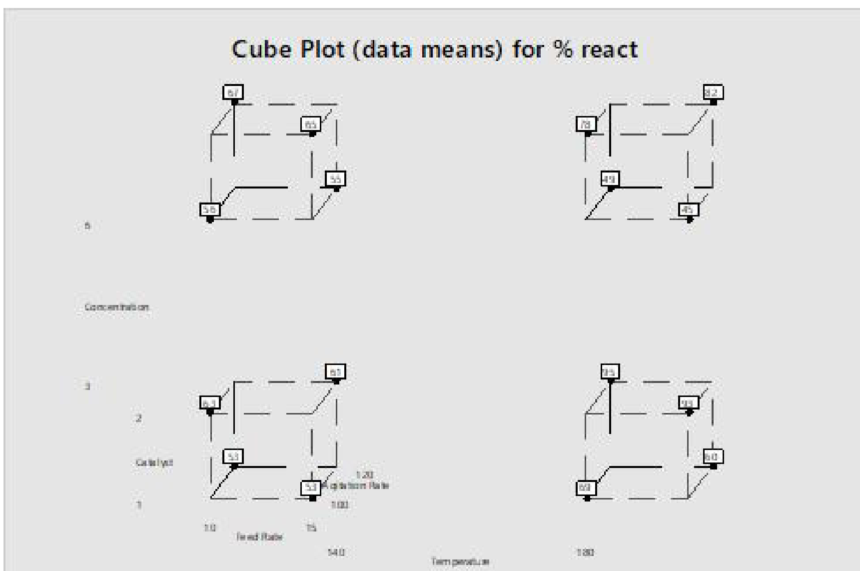


Notes:

Interaction Plot



Cube Plot



Step 10: State the Mathematical Model

Calculate the variation for the main effects and interactions left in the model. State the mathematical model obtained.

Determine the variation that is accounted for by the main effects and interactions left in the model. State the mathematical model obtained. This is quantified by the R^2 value presented in Minitab.

Mathematical Model

Use the coefficients from the analysis to derive the reduced mathematical model (Add the coefficients of the factors.)

Notes:

Notes:

Coded Coefficients

Term	Effect	Coef	SE Coef	T-Value	P-Value	VIF
Constant		65.250	0.663	98.47	0.000	
Catalyst	20.500	10.250	0.663	15.47	0.000	1.00
Temperature	12.250	6.125	0.663	9.24	0.000	1.00
Concentration	-6.250	-3.125	0.663	-4.72	0.001	1.00
Catalyst*Temperature	10.750	5.375	0.663	8.11	0.000	1.00
Temperature*Concentration	-9.500	-4.750	0.663	-7.17	0.000	1.00

Regression Equation in Uncoded Units

%React = 9.9 - 65.5(Cat) + 0.212(Temp) +23.25(Con) + 0.5375(Cat*Temp) - 0.1583(Temp*Con)

Step 11: Translate the Statistical Conclusion into Process Terms

Practical Question: To maximize % reacted you should use 2% catalyst at 180 °C and 3% concentration.

Step 12: Replicate Optimum Conditions

Plan the next experiment and/or institutionalize the change.

Achieving a Target Response

So far, scenarios in which you wish to minimize or maximize the response have been discussed. What if achieving a particular response value is desired as the optimum process result? Minitab's Response Optimizer will help achieve that objective.

Referring to the previous example, this time the experiment will be constructed using actual values for the factor levels. This will enable the Response Optimizer to be used to attain a desired yield.

When Should Fractional Factorial Experiments Be Used?

Fractional factorial DOE is used to estimate main effects and sometimes also interactions with fewer test runs than required in full factorial designs. It is perfect for screening large numbers of factors and for cases where multiple factor interactions can be considered negligible.

Recommendations

- Use the default generators in Minitab to ensure the design is balanced and to lock in the best alias pattern.

- Run the experimental runs in random order whenever possible.
- Replicate the experiment for more power. Repeated measures from the same run do not give the same estimating power as replicates.
- Perform additional experiments to better estimate main effects and interactions with fewer factors.

Notes: