Notes:

Introduction to Design of Experiments

Key Learning Points

1. Describe the importance of designed experiments.
2. Explain how to create an experiment to determine a mathematical model to create the best process and outputs.
3. Utilize DOE in an improvement project.

What is Design of Experiments?

Design of Experiments is a systematic method for determining the relationship of multiple factors (Xs) on a response (Y). It is the most efficient way to discover how the factors affect a process and to predict the outcome of that process.

What is DOE?

DOE identifies how factors (Xs), alone and combined, affect a process and its output (Ys).

Why use DOE?

DOE is an approach to collecting and analyzing data to determine:

- A mathematical model
- The best configuration or combination of Xs

Questions Which DOE Answers

- What are the most significant Xs? How can I narrow my list of potential

Xs to the truly vital few?

- How do the Xs affect the Ys? How can I describe the relationship quantitatively?
- Are there any interactions among the Xs that affect the Ys? What are the natures of the interactions?
- What are the best settings and tolerances for my Xs to optimize my Ys?

Illustrating the Need for DOE

Consider the case of a certain fellow who decided he wanted to investigate the causes of intoxication. As the story goes, he drank some whiskey and water on Monday and became highly inebriated.

The next day, he repeated the experiment holding all variables constant except one… he decided to replace the whiskey with vodka.

As you may guess, the result was drunkenness. On the third day, he repeated the experiment for the last time. On this trial he used gin in lieu of the whiskey and vodka. This time it took him two days just to be able to gather enough of his faculties to analyze the experimental results.

After recovering, he concluded that water causes intoxication. Why? Because it was the common variable!

Traditional Improvement Approaches

The formal plan for conducting an experiment is called the "experimental design". It includes choices in response, factors, levels, blocks, and treatments, as well as the use of certain tools called planned grouping, randomization, repetition, and/or replication.

Typically the analysis of the results of an experiment is straightforward, particularly if computer-based tools, such as MINITAB®, are available.

Because the best analysis in the world cannot rescue a poorly designed experiment, it is the way in which you conduct the experiment that is key.

One Factor At A Time

A well-known way to improve is to start out with an existing product, process, or service, and tweak it by changing individual factors, one at a time. You then look at the results of the change you made, and compare the new product, process, or service to the old one.

This process gets repeated indefinitely until you decide that you have the best results.

Notes:

Pros and Cons of One Factor At A Time

Pros

You have control over minutia

You can easily revert to pre-change specifications

Cons

If there are many factors it could take a long time to find the best result

You don't know for sure if you are getting the best possible result

Example

Suppose you are studying gas mileage in your classic car, and want to know how to get the best mileage in miles per gallon. The two variables that you can control are the cars ignition timing setting, and the type of gasoline you use. (For this example there are two types of gas to test, and two timing settings to test).

Traditionally, you might begin by holding the timing constant at T1 and testing both gas types.

Gas Type	Timing Set-Up	MPG
G1	T1	30
G2	T2	20

Gas Type 1 gave the highest miles per gallon.

Since Gas Type 1 was the best, test it with the Timing 2.

Gas Type	Timing Set-Up	MPG
G1	T1	30
G2	T2	20

Now, you see that the best results are obtained with Gas Type 1 and Timing Type 1.

What did you miss?

In the traditional approach, the combination of Gas Type 2 and Timing Type 2 was never tested.

Changing Many Things At Once

Many organizations approach change with good intentions, but without much planning. This often leads to making many changes to a product, process, or service, and hope that the change will be positive.

What this is equivalent to is throwing a bunch of ideas at a wall, and hope some will stick. While there might be positive change, there is no way to know which change actually led to the improvement.

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This may be the most commonly used method of experimentation in business today. However, these experiments are often called "solutions."

Every untested process change is actually an experiment because the results are unknown.

Pros and Cons of Changing Many Things at Once

Pros

- You can change many factors for a variety of reasons
- You don't need to think all the way through a change

Cons

- You won't know which changes are responsible for the changes in the result
- You may keep doing something that is harmful to your results
- It is impossible to understand the cost/benefit considerations for each individual change

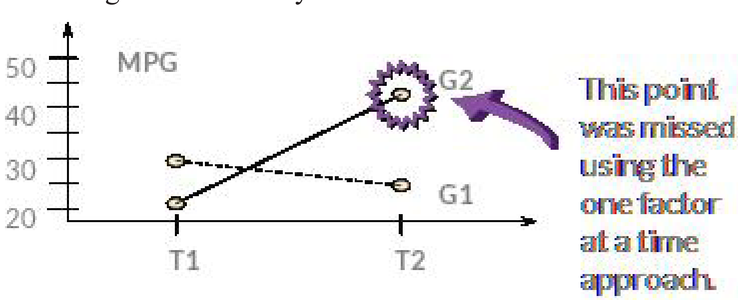
Teams often "jump to a solution" when they:

- Brainstorm a list of possible ways to improve the process
- Prioritize the ideas
- Implement as many high priority ideas as possible, all at once

Designed Experimentation

Think back to the Gas Mileage example noted earlier. In the one factor at a time approach, the combination of Gas Type 2 and Timing Type 2 was never tested. A full factorial designed experiment tests all combinations of all factors. In this case, the traditional approach would not have given the best solution.

A full-factorial designed experiment shows that Gas Type and Timing interact, and the effect of one factor depends on the level of the other factor. Without testing all factors against all levels you can miss the best result.



Gas Type	Timing Set-Up	MPG
G1	T1	30
G1	T2	25
G2	T1	20
G2	T2	45

In this example you would get the best gas mileage using Gas Type 1 and Timing Type 2.

Current State of Process Knowledge			
Low	-	-	High
Type of Design	Screening	Fractional Factorials	Full Factorials
Usual Number of Factors (Xs)	>6	4-10	2-5
Purpose ▪ Identify ▪ Estimate	▪ The most important factors (The Vital Few) ▪ Crude direction for improvement (Linear Effects)	▪ Some Interactions ▪ Some Interpolation	▪ Relationships among factors ▪ All main effects and interactions

Notes:

Types of Designed Experiments

Factorial designs are classified in three main types; Full factorial, fractional factorial, and screening designs. Factorial Designs are a family of designs that provide considerable flexibility in identification and exploration of cause-effect relationships among process factors.

- Full factorial designs provide more information, but this requires greater resources. Studying more than 4 or 5 factors makes the number of experiments required unwieldy.
- Fractional Designs provide a means of studying a higher number of factors with less experimentation. Not as much information is obtained, but the resource requirement is reduced.
- Screening Designs are simply a subset of fractional designs that provide a means of reducing the number of factors to a more manageable level. Screening experiments are used to separate the vital few X variables from possible Xs at a very low cost.
  A Nested Design is used when one (or more) factors cannot be combined

with all the levels of the other factors. For example, suppose you are evaluating a resin. The resin might come from two different suppliers. Each supplier has multiple lots. Each lot has multiple samples taken.

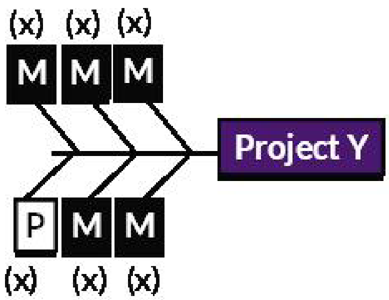
- Mixture Designs are typically used in contnuous processing operations that have constraints built into the process. One such constraint might be, the sum of the input components must equal a certain weight.

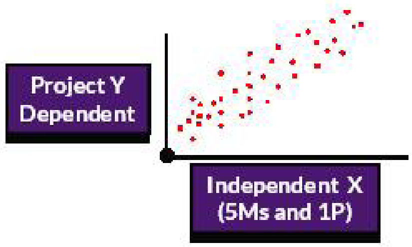
DOE Terminology

Dependent Variable

This is the Y variable. It is often called the response variable.

Dependent Variables - Ys
▪ Also called responses
▪ The impact of the solution, factor, or variable
▪ The effect

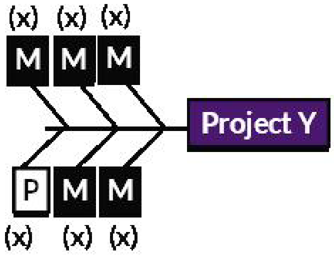


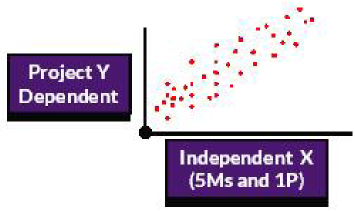


Independent Variable

This is the X variable. It is often called the factor. In all cases, a factor must be treated as a discrete variable. If the factor is continuous by nature, it must be classified into levels—a high and a low level. If the factor is discrete, factor levels naturally exist.

Independent Variables - Xs
▪ Also called factors
▪ Potential solutions or variables being studied
▪ Factors are classified into levels





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Factor

A factor (or input) is one of the controlled or uncontrolled variables whose influence on a response (output) is being studied in the experiment. A factor may be quantitative, e.g., temperature in degrees, time in seconds. A factor may also be qualitative, e.g., different machines, different operator, clean or not clean.

Level

The levels of a factor are the values of the factor being studied in the experiment. Levels should be set wide enough apart so effects on the Y variable can be detected. Levels are often referred to as "-1" and "+1." For quantitative factors, each chosen value becomes a level, e.g., if the experiment is to be conducted at two different temperatures, then the factor of temperature has two "levels." A qualitative factor such as cleanliness can have two levels as well, particularly, clean vs. not clean.

Levels
▪ The test settings for X

For example:

- Y = cycle time
- X_1 = type of application
  - Levels = new, old
- X_2 = # of associates
  - Levels = 1, 2, 3

k1 x k2 x k3 Factorial

This is a description of the basic design. The number of ks is the number of factors. The value of each k is the number of levels of interest for that factor. Example: A 2 x 3 x 3 design indicates three input variables. One input has two levels, and the other two each have three levels.

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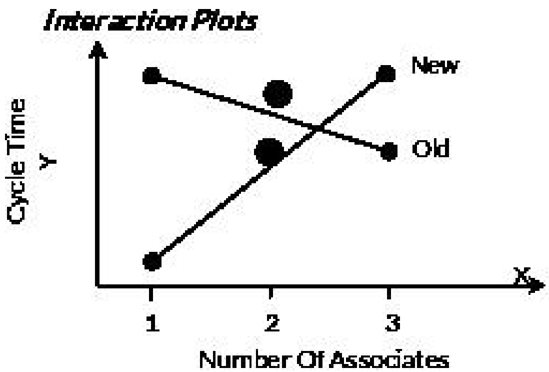
K-way Interaction

This is the interaction between different K input variables. An interaction occurs when the effect that one factor has on a response depends on the level of a second factor. In an interaction plot, significant interaction is depicted by intersecting lines.



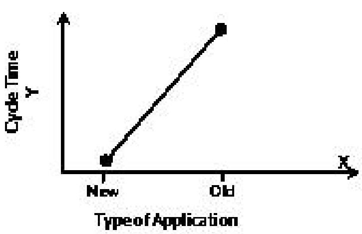
Main Effect

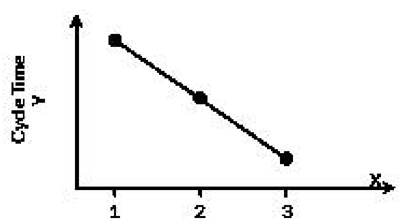
This is the change in the average response (output) observed during a change from one level to another for a single factor (input).

Main Effects
▪ Differences between each factor level

For example, is cycle time different for:

- New or old application forms?
- 1, 2, or 3 associates?





Interaction

This is the combined effect of two factors observed over and above the main (or singular) effect of each factor.

Interactions
▪ The differences between two or more factor level combinations

For example, is cycle time different when:

- new, 1 associate old, 1 associate
- new, 2 associates old, 2 associates

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- new, 3 associates old, 3 associates

Notes:

Treatment

This is a single level assigned to a single factor during an experimental run, e.g., temperature at 250 degrees.

Test Run (Experimental Run)

This is a single combination of factor levels that yields one or more observations of the output variable.

Treatment Combination

This is an experimental run using a set of the specific levels of each input variable. The number of treatment combinations in a full experiment is the product of the number of levels for each factor. In the case of a 2 x 3 x 3 design, there will be 18 possible treatment combinations in the experiment.

Repetition

This is when you run several experimental runs consecutively using the same treatment combination.

Replication

Replication automatically implies that you do NOT run several experimental runs consecutively using the same treatment combination. Replication occurs when an experimental treatment is set up and conducted more than once. If you collect two data points at each treatment, you have two replications.

In general, plan on making between two and five replications for each treatment. A replication is not two measurements of the same data point but a measurement of two data points under the same treatment conditions. This is the variation from sources other than the changes in factor levels.

Replicating an experiment allows you to estimate the residual or experimental error.

Full Factorial Table

Treatments listed in "standard order"

Factors

A	B	C	Run
-1	-1	-1	1
+1	-1	-1	2
-1	+1	-1	3
+1	+1	-1	4
-1	-1	+1	5
+1	-1	+1	6
-1	+1	+1	7
+1	+1	+1	8

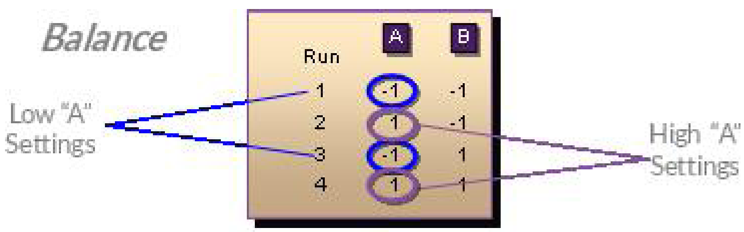
Levels

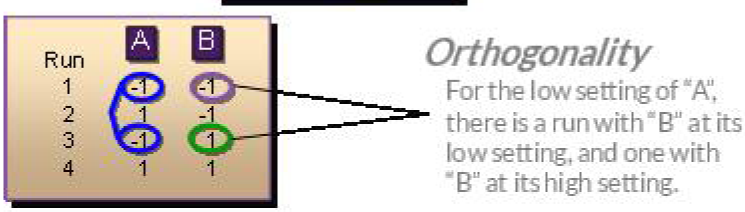
# of runs = 2^3 = (Levels)$^{\#\ OF\ FACTORS}$
2^3 = 8 Experiments

Notes:

Design Considerations

Balance and Orthogonality conditions are built into the standard designs. Use the standard designs and don't worry about these conditions!





Balanced Design Experiment

This is an experiment that has the same number of runs at the low level as at the high level for each factor. For example, in a 2 x 2 design, there are 4 runs—you would have 2 at the low setting of factor A, and 2 at the high setting of factor A. If factor "A" run 4 was not completed in the experiment, it would not be balanced. The same holds true for factor B.

Balanced Design

This is a design where each experimental level for any one factor is repeated the same number of times for all possible combinations involving the levels of the other factors.

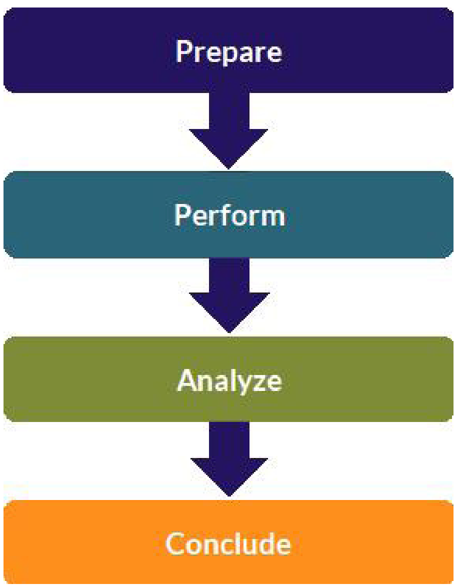
Unbalanced Design

This is a design where each experimental level for any one factor is NOT repeated the same number of times for combinations involving the levels of the other factors.

Orthogonal Design

This is a design in which one level of factor A is run with each setting for factor B. This property allows an assessment of the real impact of one variable independent of the others in the same design—this goes the same for interactions.

DOE Method



1. Define the problem.
2. State the hypothesis.
3. State the factors and levels of interest.
4. Create an appropriate Minitab experimental datasheet.
5. Run the experiment and collect the data.
    a. Select the appropriate sample size.
    b. Randomize the experimental runs.
6. Construct the ANOVA table for the full model and use the appropriate graphical tool to evaluate the data.

Notes:

7. Rerun a reduced model by eliminating:
   a. Effects with non-significant P-values.
   b. Effects plotted low on the Normal or Pareto chart.
8. Investigate the residuals plots to ensure model fit.
9. Using the ANOVA table and appropriate graphical tool, investigate significant main effects and interactions.
10. Calculate the variation for the main effects and interactions left in the model.
    a. State the mathematical model obtained.
11. Translate the statistical conclusion into process terms.
    a. Formulate conclusions and recommendations.
12. Replicate optimum conditions.
    a. Plan the next experiment or institutionalize the change.

Notes:

Revisiting The Gas Mileage Example

Let's return to the example where you are studying gas mileage in your car, and want to know how to get the best mileage in miles per gallon.

This time the Design of Experiment Method will be used.

Step 1: Define the Problem

The Problem:

Your car is not economical and gets poor gas mileage. You want to improve the car's mileage.

Step 2: State the Hypotheses

Some combination of speed, gas octane, and tire pressure will provide you with the optimum gas mileage.

Note: These factors are assumed to be "a priori" (before the fact) and are part of the decisions made by the team as most likely.

Formulas and Definitions

y = Observed Output
μ = Average Response
P = Effect Because of Tire Pressure
O = Effect Because of Octane
S = Effect Because of Speed
PO = Interaction of Tire Pressure and Octane
PS = Interaction of Tire Pressure and Speed

OS = Interaction of Octane and Speed
ε = Random Error

y_{jikl} = μ+Pi+Oj+S,+(PO)ij+(PS)ik+(OS)jk+εjikl

For each effect:
Ho: Factor effect = 0
Ha: Factor effect ≠ 0

Step 3: State the Factors and Levels of Interest

For this experiment the factors are speed, gas octane, and tire pressure.

Independent Variables (Xs) (Factors)	Level (-)	Level (+)
Tire Pressure (psi)	30	35
Octane	87	92
Speed (mph)	55	65

Project Variables (Factors)

Dependent Variable (Y) = Gas Mileage

- Y is called the dependent variable because its value depends upon the level setting for X.

Independent Variables (X)

- X is called the independent variable because its value is set independently. X will sometimes be called an experimental factor.
- Independent Variable (T) = Tire Pressure
- Independent Variable (G) = Gas Octane
- Independent Variable (S) = Speed

Levels of Interest:

- Level (-) represents the low value of the levels
- Level (+) represents the high value of the levels

Step 4: Create an Appropriate Experimental Data sheet

Before you create your data sheet, you need to determine how many trials will be run in the experiment. You can calculate trials using the number of levels, and the number of factors being studied. In this case, there are two levels, a low level and a high level, and there are three factors being studied. This is a 23 Factorial. From this 23 factorial, you can calculate that 8 trials must be run (2x2x2=8).

Runs need to be put in Yates standard order (This is when the first column is alternating as –1, +1, –1, +1, etc. The second column alternates as –1, –1, +1, +1,

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etc. The third column alternates as –1, -1, –1, –1, +1, +1, +1, +1, etc.).

Tire Pressure	Octane	Speed	Gas Mileage
-	-	-	
+	-	-	
-	+	-	
+	+	-	
-	-	+	
+	-	+	
-	+	+	
+	+	+	

Step 5: Run the Experiment and Collect the Data

Once you select the appropriate sample size, you are now ready to randomize the experimental runs, and collect data. Note that if you created your datasheet using Minitab, the randomization has been completed in the "Run Order" column.

↓	C1	C2	C3	C4	C5	C6	C7	C8	C9
	StdOrder	RunOrder	CenterPt	Blocks	Tire Pressure	Octane	Speed	Gas Mileage: Run 1	Gas Mileage: Run 2
1	4	1	1	1	35	92	55	33.6	32.1
2	1	2	1	1	30	87	55	23.2	27.0
3	8	3	1	1	35	92	65	22.3	22.5
4	6	4	1	1	35	87	65	21.1	21.4
5	5	5	1	1	30	87	65	18.2	18.8
6	2	6	1	1	35	87	55	27.5	26.0
7	3	7	1	1	30	92	55	30.2	29.9
8	7	8	1	1	30	92	65	19.4	19.6

Considerations

Before you run the experiment and collect data, be sure to consider the following:

- Develop and execute SOPs for all factors not in the study.
- Prepare a data-collection plan.
- Communicate the plan.
  - Data collectors
  - Stakeholders
- Train data collectors.
- Complete trial runs, if necessary.
  - Check understanding of the level settings.
  - Verify factor levels and produce results.
  - Ensure transitions between runs are doable in a reasonable time frame.
- Run the Experiment.

Notes:

- Note any unexpected events.

Replication

Replication is a repeat of all experimental trials to obtain additional data to increase the degree of belief in the experimental results.

Replications are used when:

- The interactions are of critical importance.
- Data are tricky to collect. Replications will supply an extra data point when data from an experimental trial is lost.
- You need to increase the degree of belief in the experiment results.
- You need to reduce the risk when implementing solutions.
- Replications enable you to estimate random error.

Results

↓	C1	C2	C3	C4	C5	C6	C7	C8
	StdOrder	RunOrder	CenterPt	Blocks	Tire Pressure	Octane	Speed	Gas Mileage
1	1	1	1	1	30	87	55	25.5
2	2	2	1	1	35	87	55	27.5
3	3	3	1	1	30	92	55	30.3
4	4	4	1	1	35	92	55	33.6
5	5	5	1	1	30	87	65	18.2
6	6	6	1	1	35	87	65	21.1
7	7	7	1	1	30	92	65	19.8
8	8	8	1	1	35	92	65	22.2
9	9	9	1	1	30	87	55	27.0
10	10	10	1	1	35	87	55	26.0
11	11	11	1	1	30	92	55	29.9
12	12	12	1	1	35	92	55	32.1
13	13	13	1	1	30	87	65	19.8
14	14	14	1	1	35	87	65	21.4
15	15	15	1	1	30	92	65	19.6
16	16	16	1	1	35	92	65	22.5

Step 6: Construct and Evaluate the ANOVA Table

- To match this analysis, make sure your data are sorted in run order and not standard order!

  Look for trends resulting from lurking variables that might have interfered with the experiment.

  If you find a trend in your data associated with time, try to uncover the source of this variation.
- If you determine its source, find a way to control or eliminate this additional source of experimental variation.

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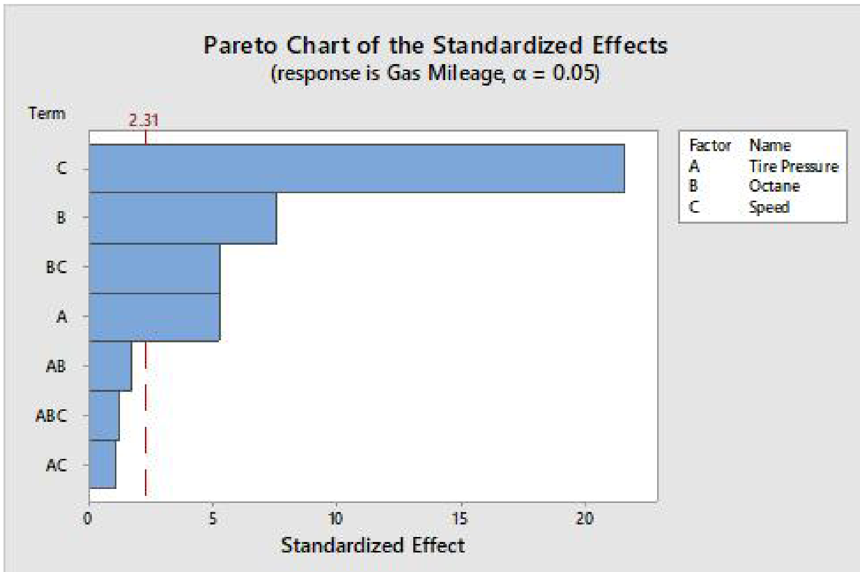
- If you cannot determine the trend's source or cannot control it, you may need to rerun the experiment in random order.

If each set of repetitions appears consistent; proceed with analysis.

Full Model



Step 7: Rerun a Reduced Model

Eliminate effects with:

- Non-Significant P-Values
- Effects plotted low on the Effects Pareto Diagram

Step 8: Investigate the Residuals Plots to Ensure Model Fit

Residuals are the difference between the actual Y value and the Y value predicted by the regression equation.

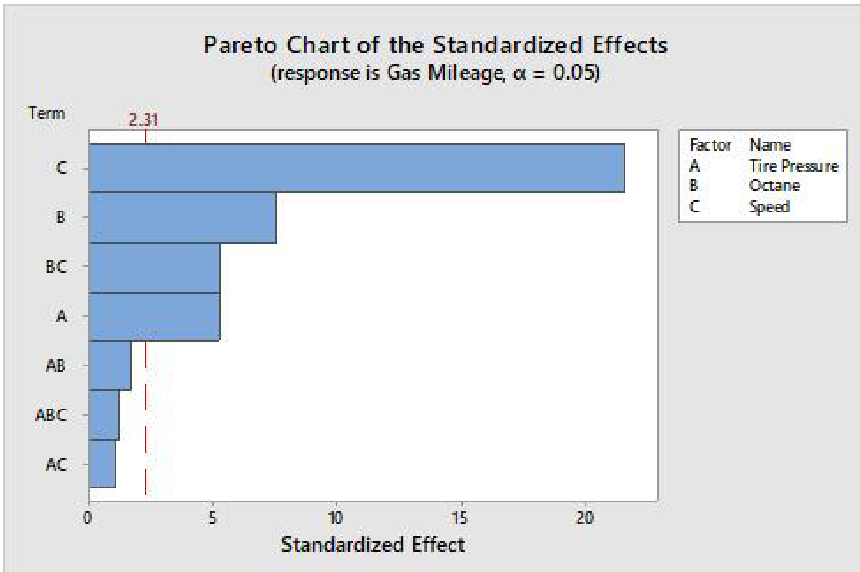
Residuals should:

- Be randomly and normally distributed about a mean of zero
- Not correlate with the predicted Y
- Not exhibit trends over time (if data is chronological)

Problems with the residuals would indicate the model is inadequate.

Notes:

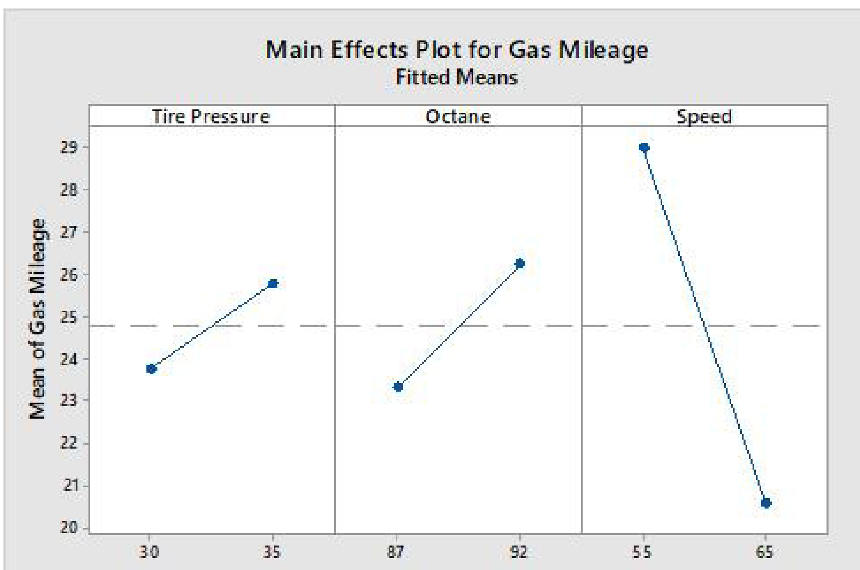
Residual Plots



Step 9: Investigate Significant Main Effects and Interactions

Using the ANOVA table and appropriate graphical tool, investigate significant main effects and interactions.

Main Effects Plot



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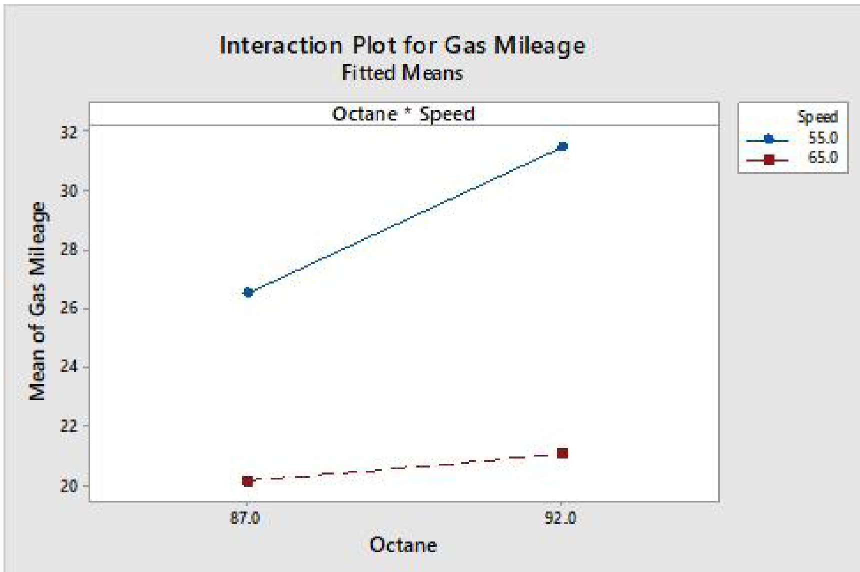
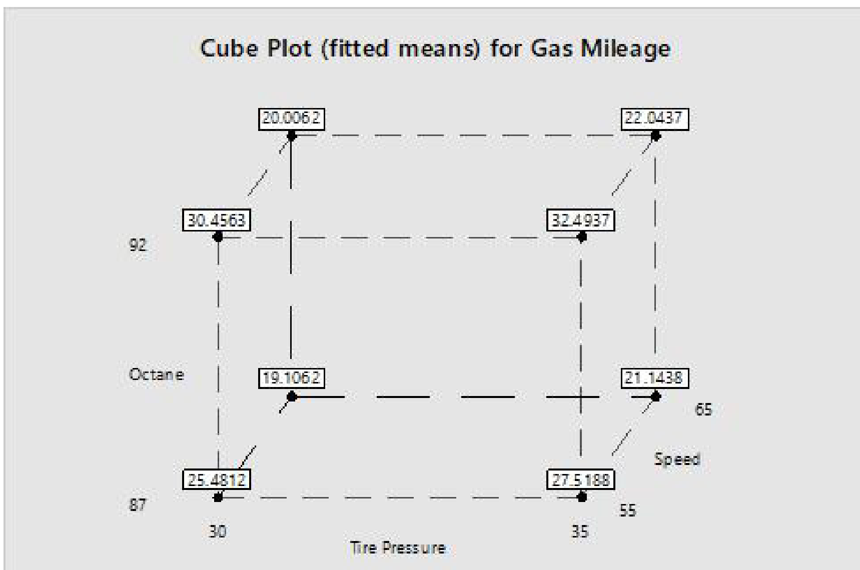
Interaction Plots



Cube Plot



Step 10: State the Mathematical Model

Calculate the variation for the main effects and interactions left in the model. State the mathematical model obtained.

Determine the variation that is accounted for by the main effects and interactions left in the model. State the mathematical model obtained. This is quantified by the R^2 value presented in Minitab.

Notes:

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Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
0.860067	97.73%	96.91%	95.20%

Coded Coefficients

Term	Effect	Coef	SE Coef	T-Value	P-Value	VIF
Constant		24.781	0.215	115.25	0.000	
Tire Pressure	2.038	1.019	0.215	4.74	0.001	1.00
Octane	2.938	1.469	0.215	6.83	0.000	1.00
Speed	-8.413	-4.206	0.215	-19.56	0.000	1.00
Octane*Speed	-2.038	-1.019	0.215	-4.74	0.001	1.00

Regression Equation in Uncoded Units

Gas Mileage = -428.2 + 0.4075 Tire Pressure + 5.48 Octane + 6.45 Speed - 0.0815 Octane*Speed

Step 11: Translate the Statistical Conclusion into Process Terms

At this point you need to formulate conclusions and recommendations.

Conclusions and Recommendations:

- Speed has a significant effect on gas mileage.
  - Drive at 55 MPH to get the best gas mileage.
- Tire Pressure has a significant effect on gas mileage.
  - Set tire pressure at 35 PSI for best gas mileage.
- Octane has a significant effect on gas mileage.
  - Buy 92 octane gasoline.

Step 12: Replicate Optimum Conditions

Plan the next experiment and/or institutionalize the change.

Final Thoughts:

- If you have 10 weeks to perform a designed experiment, take 8 weeks to PLAN it, 1 week to RUN it, and 1 week to ANALYZE it.
- You almost never run a single designed experiment, you end up running several.

When Should DOE Be Used?

Full factorial DOE is used to determine the main effects and interactions of no more than five factors. It is the perfect tool for examining the interplay of multiple factors, as the factors are guaranteed to be uncorrelated with each other.

Pitfalls to Avoid

- Use the default generators in Minitab to ensure the design is balanced and orthogonal.
- Run the experimental runs in random order whenever possible.
- Replicate the experiment for more power. Repeated measures from the same run do not give the same estimating power as replicates.
- Perform a verifying experiment to prove your best conditions.
- A true optimum may lie outside the design.

Notes: