Notes:

2k Factorial Experiments - Center Points

Key Learning Points

1. Describe when to use center points in designed experiments.
2. Describe why center points are used in designed experiments.
3. Utilize center points when designing 2k factorial experiments in your improvement project.

What are Center Points?

In standard 2_k (2 level) designs, all prediction equations must be planar. So there is always a risk of missing a curvilinear relationship by only including two levels of the input variable. The addition of center points is an efficient way to test for curvature without adding a large number of experimental runs.

DOE Vocabulary

- k1 x k2 x k3 ... Factorial: Description of the basic design. The number of ks is the number of factors. The value of each k is the number of levels of interest for that factor. In 2k experiments, you will always have the value of k1=k2=k3=2. A 2^3 = 2 x 2 x 2, similarly a 2^2 = 2 x 2, etc… These types of basic designs indicate that all factors are investigated at only two levels.
- Repetition: Running several experimental runs consecutively using the same treatment combination.
- Replication: Replicating the entire experiment. Replication automatically implies that you do NOT run several experimental runs consecutively using the same treatment combination.

- Center Points: n repeats or replicates run at the center or midpoint of all quantitative factor levels. Center points can often represent the process parameter levels at their current operating conditions. These points allow you to check the accuracy of a linear fit.
- Curvature: The situation when the output of the process does not act linear at center levels (center points) of all factors.
- Experimental Error: The variation in the data (sum of squares) left over after all significant sources of variability have been accounted for.
- Residual Error: the variability that remains after all the main effects and interactions have been identified.
- Lack-of-Fit: an indication of the fit of a prediction model. If the lack- of- fit p-value is less than your selected a-level, evidence exists that your model does not accurately fit the data.
- Pure Error: The variation in the data (sum of squares) that can only be estimated via true repeat or replicate measurements, either via center points or via repeats/replicate sample size selection.
- Blocking Variable: A factor in an experiment that has undesired influence as a source of variability is called a "block." A block can be a batch of material or a set of conditions likely to produce experimental runs that are more homogenous within the block than between blocks. For example, parts from a single batch of material are likely to be more uniform than parts from different batches. The batch of material would be regarded as the blocking variable.
- Block: Group of homogeneous experimental runs.
- Confounding: One or more effects that cannot unambiguously be attributed to a single factor or interaction.

Adding Center Points to 2k Factorials

1. State the problem
2. State the hypotheses
3. State the factors and levels of interest
4. Create an appropriate Minitab experimental datasheet
5. Select the appropriate sample size, randomize the experimental runs and collect the data
6. Construct an ANOVA table and use the appropriate graphical tool to evaluate the data
7. Rerun a reduced model if necessary by first eliminating interaction effects with non-significant P-values, then further reduced the model with non-significant main effects

Notes:

8. Investigate the residuals plots to ensure model fit
9. Using the ANOVA table and appropriate graphical tools, investigate the P-values of the main effects and interactions
10. Calculate the variation for the main effects and interactions left in the model, and state the mathematical model
11. Translate the statistical conclusion into process terms and formulate recommendations
12. Replicate optimum conditions, and plan the next experiment or institutionalize the change

How Many Center Points Should I Have?

There is a tradeoff between the resources you have, the need for enough runs to see if there is curvature in the model, and the desire to complete the experiment as quickly as possible. As a rough guide, you should generally add approximately 3 to 5 center points to a full or fractional factorial design.

Example

A process engineer wants to improve the yield for two different die-castings. There are two inputs of interest: pressure and temperature. The engineer decides to conduct the experiment using a 2x2 design augmented with five center points to estimate experimental error and curvature.

The inputs are:

- Temp
  - Levels: 150, 155, 160
- Pressure
  - Levels: 30, 35, 40

Step 1: Define the Problem

The Problem:

A process engineer is tasked with improving yield for two different die-castings. There are two inputs of interest: pressure and temperature.

Step 2: State the Hypotheses

The effect of at least one of the factors on the output is different from zero.

- H_o: Effect of Temperature = 0
- H_a: Effect of Temperature ≠ 0
- H_o: Effect of Pressure = 0
- H_a: Effect of Pressure ≠ 0

Notes:

- H_o: The model has no curvature
- H_a: The model has curvature

Step 3: State the Factors and Levels of Interest

For this experiment the factors are:

- Temperature: 150, 155, 160
- Pressure: 30, 35, 40

Step 4: Create an Appropriate Experimental Data sheet

- **Minitab: Stat > DOE > Factorial > Create Factorial Design**
  - Type of Design: 2-Level Factorial (default generators)
  - Type of Number of factors: 2
  - Designs: Full Factorial
  - Number of center points per block: 5
  - Number of replicates per corner points: 1
  - Number of blocks: 1
  - Options: Uncheck randomized runs
  - Factors: Specify names and levels

StdOrder	RunOrder	CenterPt	Blocks	Temp	Pressure
1	1	1	1	150	30
2	2	1	1	160	30
3	3	1	1	150	40
4	4	1	1	160	40
5	5	0	1	155	35
6	6	0	1	155	35
7	7	0	1	155	35
8	8	0	1	155	35
9	9	0	1	155	35

Notes:

Step 5: Run the Experiment and Collect the Data

Notes:

In real experiments always randomize. For this example standard order has been used.

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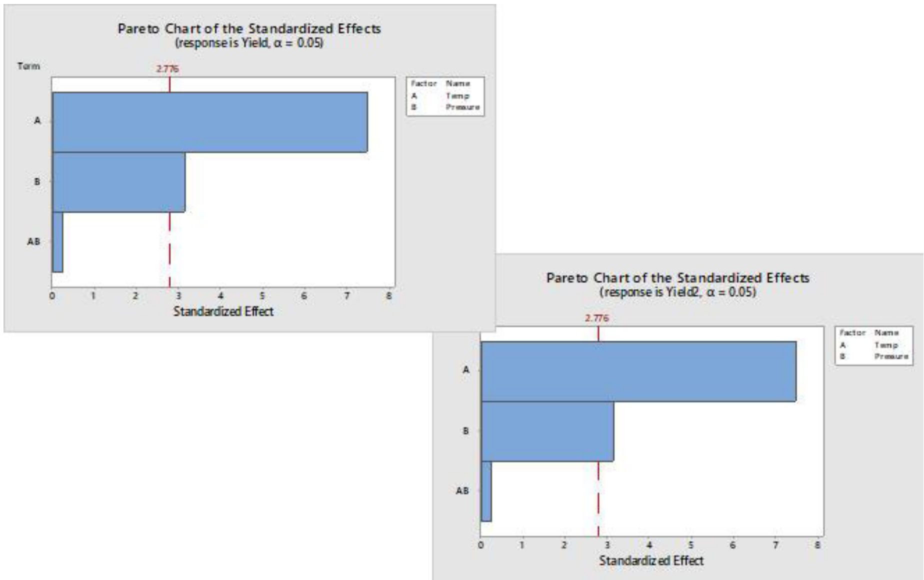
StdOrder	RunOrder	CenterPt	Blocks	Temp	Pressure	Yield	Yield2
1	1	1	1	150	30	39.3000	39.3000
2	2	1	1	160	30	40.9000	40.9000
3	3	1	1	150	40	40.0000	40.0000
4	4	1	1	160	40	41.5000	41.5000
5	5	0	1	155	35	40.3000	42.3000
6	6	0	1	155	35	40.5000	42.5000
7	7	0	1	155	35	40.7000	42.7000
8	8	0	1	155	35	40.2000	42.2000
9	9	0	1	155	35	40.6000	42.6000

Step 6: Construct The ANOVA Table

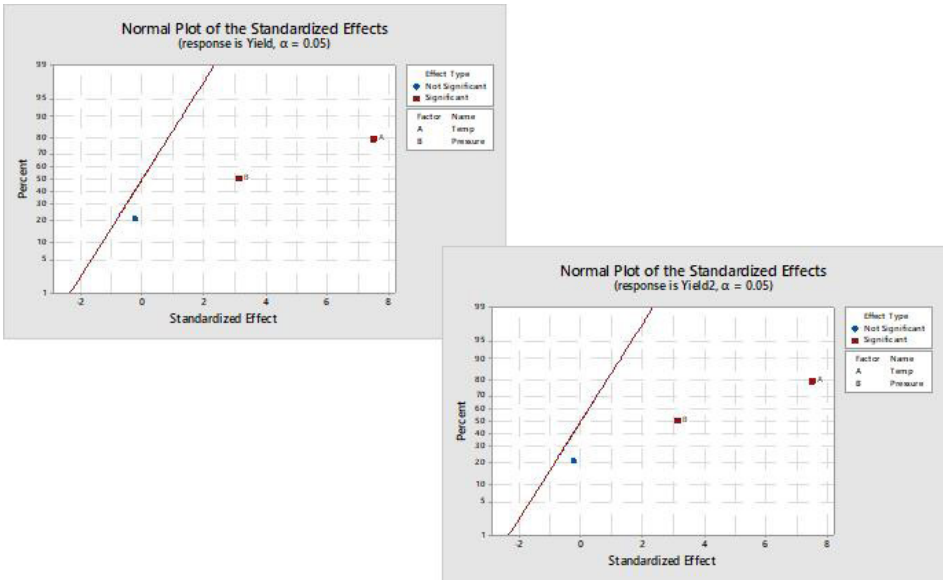
- **Minitab: Stat > DOE > Factorial > Analyze Factorial Design**
- Terms
  - Include Terms up through 2
  - Check to include center point in the model (default setting)
- Graphs
  - Response: Yield
  - Graphs: Effects Plot
  - Check off Normal and Pareto

Results

Pareto Chart of the Effects



Normal Plot of the Effects



Notes:

Analysis of Variance Table Yield 1

Factorial Regression: Yield versus Temp, Pressure, CenterPt

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	4	2.83023	0.70756	16.45	0.009
Linear	2	2.82500	1.41250	32.85	0.003
Temp	1	2.40250	2.40250	55.87	0.002
Pressure	1	0.42250	0.42250	9.83	0.035
2-Way Interactions	1	0.00250	0.00250	0.06	0.821
Temp*Pressure	1	0.00250	0.00250	0.06	0.821
Curvature	1	0.00272	0.00272	0.06	0.814
Error	4	0.17200	0.04300		
Total	8	3.00223			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
0.207364	94.27%	88.54%	*

Coded Coefficients

Term	Effect	Coef	SE Coef	T-Value	P-Value	VIF
Constant		40.425	0.104	389.89	0.000	
Temp	1.550	0.775	0.104	7.47	0.002	1.00
Pressure	0.650	0.325	0.104	3.13	0.035	1.00
Temp*Pressure	-0.050	-0.025	0.104	-0.24	0.821	1.00
Ct Pt		0.035	0.139	0.25	0.814	1.00

Regression Equation in Uncoded Units

Yield = 8.7 + 0.190 Temp + 0.220 Pressure - 0.00100 Temp*Pressure + 0.035 Ct Pt

Analysis of Variance Table Yield 2

Factorial Regression: Yield2 versus Temp, Pressure, CenterPt

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	4	12.0302	3.00756	69.94	0.001
Linear	2	2.8250	1.41250	32.85	0.003
Temp	1	2.4025	2.40250	55.87	0.002
Pressure	1	0.4225	0.42250	9.83	0.035
2-Way Interactions	1	0.0025	0.00250	0.06	0.821
Temp*Pressure	1	0.0025	0.00250	0.06	0.821
Curvature	1	9.2027	9.20272	214.02	0.000
Error	4	0.1720	0.04300		
Total	8	12.2022			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
0.207364	98.59%	97.18%	*

Coded Coefficients

Term	Effect	Coef	SE Coef	T-Value	P-Value	VIF
Constant		40.425	0.104	389.89	0.000	
Temp	1.550	0.775	0.104	7.47	0.002	1.00
Pressure	0.650	0.325	0.104	3.13	0.035	1.00
Temp*Pressure	-0.050	-0.025	0.104	-0.24	0.821	1.00
Ct Pt		2.035	0.139	14.63	0.000	1.00

Regression Equation in Uncoded Units

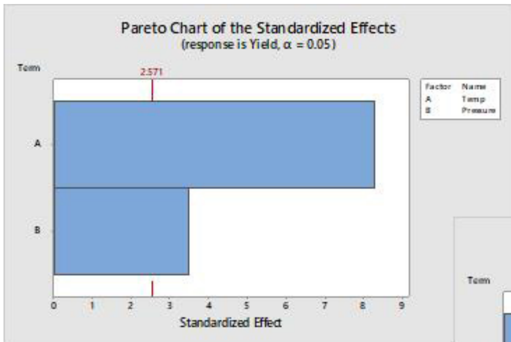
Yield2 = 8.7 + 0.190 Temp + 0.220 Pressure - 0.00100 Temp*Pressure + 2.035 Ct Pt

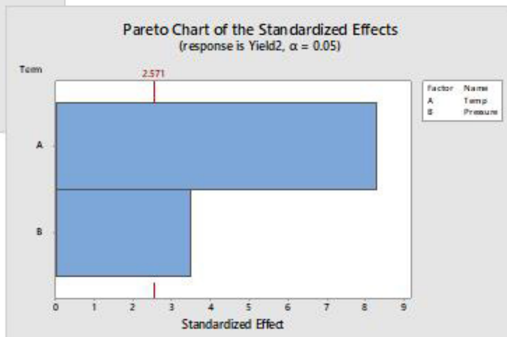
Notes:

Step 7: Rerun a Reduced Model

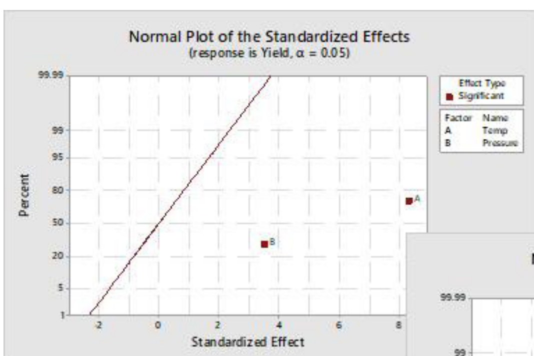
Complete the ANOVA Table again, but remove the interaction of temperature and pressure.

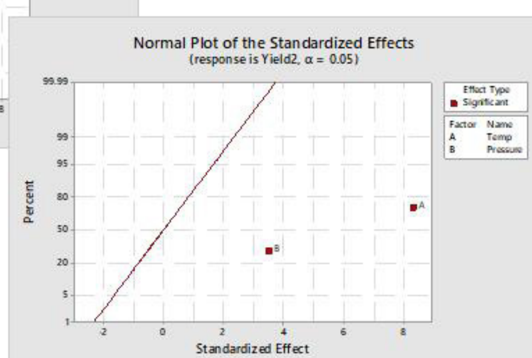
Pareto Chart of the Effects





Normal Plot of the Effects





Notes:

Analysis of Variance Table Yield 1

Factorial Regression: Yield versus Temp, Pressure

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	2	2.82500	1.41250	47.82	0.000
Linear	2	2.82500	1.41250	47.82	0.000
Temp	1	2.40250	2.40250	81.34	0.000
Pressure	1	0.42250	0.42250	14.30	0.009
Error	6	0.17722	0.02954		
Curvature	1	0.00272	0.00272	0.08	0.791
Lack-of-Fit	1	0.00250	0.00250	0.06	0.821
Pure Error	4	0.17200	0.04300		
Total	8	3.00223			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
0.171863	94.10%	92.13%	91.81%

Coded Coefficients

Term	Effect	Coef	SE Coef	T-Value	P-Value	VIF
Constant		40.4444	0.0573	705.99	0.000	
Temp	1.5500	0.7750	0.0859	9.02	0.000	1.00
Pressure	0.6500	0.3250	0.0859	3.78	0.009	1.00

Regression Equation in Uncoded Units

Yield = 14.14 + 0.1550 Temp + 0.0650 Pressure

Alias Structure

Factor	Name
A	Temp
B	Pressure

Aliases
I
A
B

Effects Pareto for Yield

Residual Plots for Yield

Analysis of Variance Table Yield 2

Factorial Regression: Yield2 versus Temp, Pressure, CenterPt

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	3	12.0277	4.00924	114.88	0.000
Linear	2	2.8250	1.41250	40.47	0.001
Temp	1	2.4025	2.40250	68.84	0.000
Pressure	1	0.4225	0.42250	12.11	0.018
Curvature	1	9.2027	9.20272	263.69	0.000
Error	5	0.1745	0.03490		
Lack-of-Fit	1	0.0025	0.00250	0.06	0.821
Pure Error	4	0.1720	0.04300		
Total	8	12.2022			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
0.186815	98.57%	97.71%	97.47%

Coded Coefficients

Term	Effect	Coef	SE Coef	T-Value	P-Value	VIF
Constant		40.4250	0.0934	432.78	0.000	
Temp	1.5500	0.7750	0.0934	8.30	0.000	1.00
Pressure	0.6500	0.3250	0.0934	3.48	0.018	1.00
Ct Pt		2.035	0.125	16.24	0.000	1.00

Regression Equation in Uncoded Units

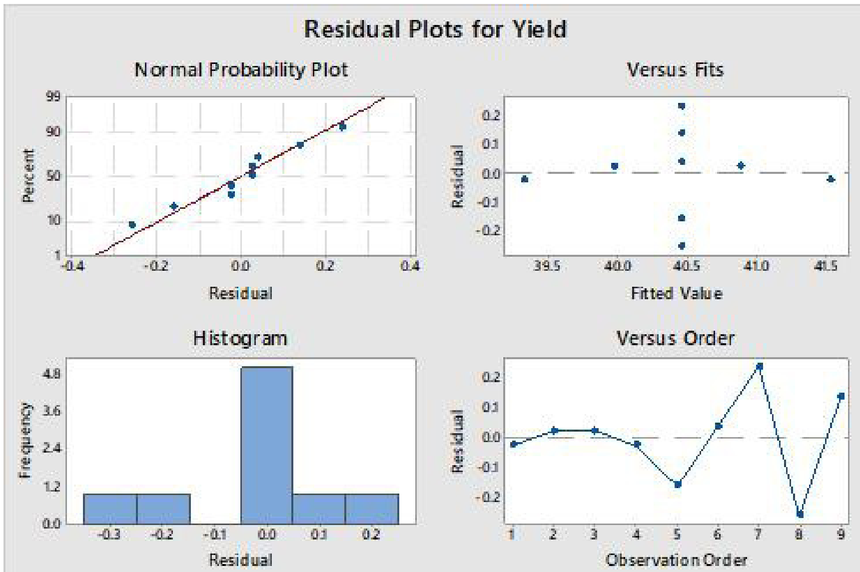
Yield2 = 14.12 + 0.1550 Temp + 0.0650 Pressure + 2.035 Ct Pt

Notes:

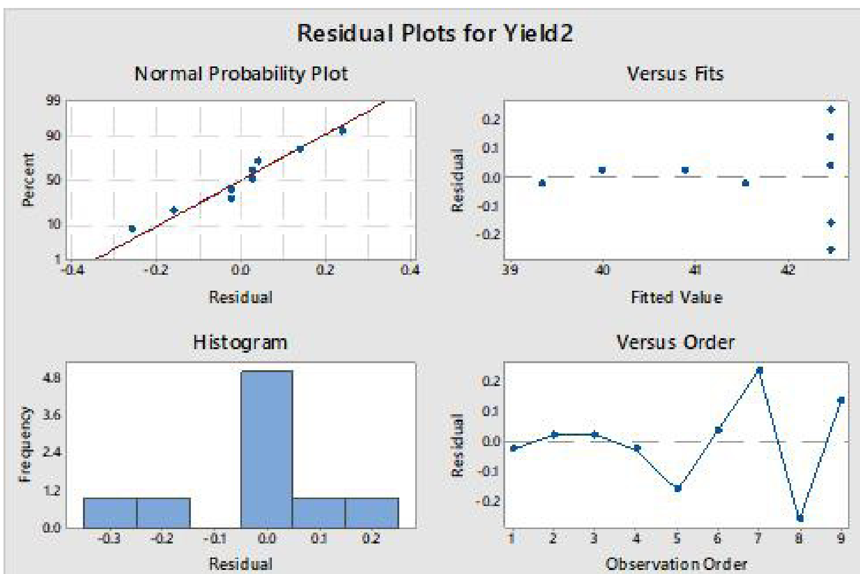
Step 8: Investigate the Residuals Plots to Ensure Model Fit

Notes:

Residual Plot for Yield



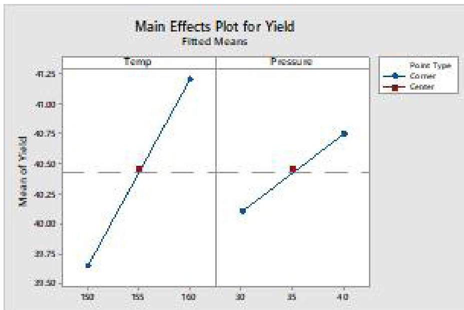
Residual Plot for Yield 2

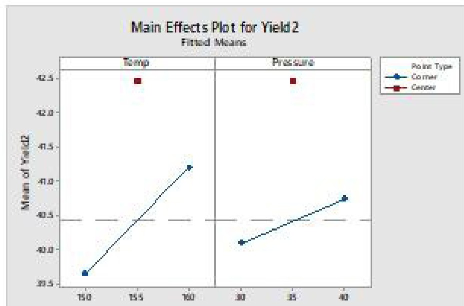


Step 9: Investigate Significant Main Effects and Interactions

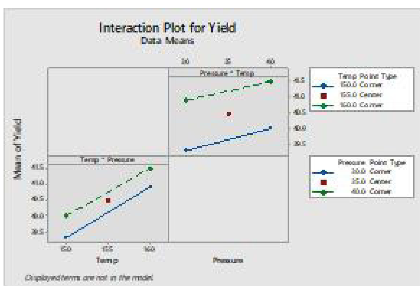
Using the ANOVA table and appropriate graphical tools, investigate the P-Values of the main effects and interactions

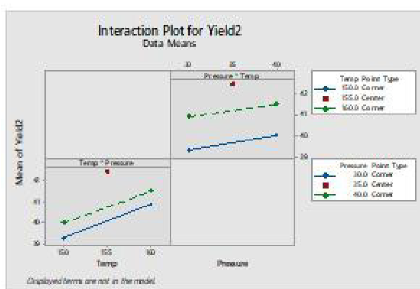
Main Effects Plots





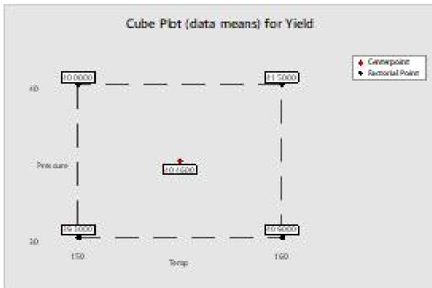
Interaction Plots

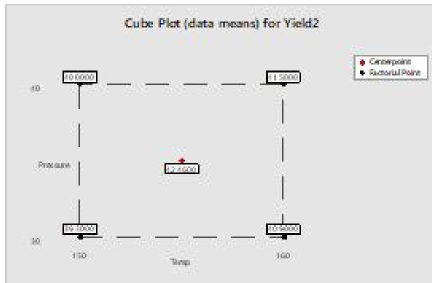




Notes:

Cube Plot





Step 10: State the Mathematical Model

Calculate the variation for the main effects and interactions left in the model. State the mathematical model obtained.

Determine the variation that is accounted for by the main effects and interactions left in the model. State the mathematical model obtained. This is quantified by the R^2 value presented in Minitab.

Mathematical Model (Yield 1)

Use the coefficients from the analysis to derive the reduced mathematical model (Add the coefficients of the factors.)

Coded Coefficients

Term	Effect	Coef	SE Coef	T-Value	P-Value	VIF
Constant		40.4444	0.0573	705.99	0.000	
Temp	1.5500	0.7750	0.0859	9.02	0.000	1.00
Pressure	0.6500	0.3250	0.0859	3.78	0.009	1.00

Yield = 14.14 + 0.1150 (Temp) + 0.0650 (Pressure)

For the Yield of Product 1 (Yield), you can reduce the model because the interaction is not significant.

Notes:

Step 11: Translate the Statistical Conclusion into Process Terms

For product 1: To increase yield set the current process to run at 160 degrees and 40 psi. It is also recommended that a follow-up investigation be done to see if yield can be improved further at factor levels not yet tested in this experiment.

For product 2: It is recommended that the current process be run at 155 degrees and 35 psi. Perform Response Surface design to discover the true optimum.

Step 12: Replicate Optimum Conditions

Plan the next experiment and/or institutionalize the change.

When Should Center Points Be Used?

Center points are used in conjunction with full and fractional factorial designs to determine if models have curvature and are not truly planar.

Pitfalls to Avoid

- Use the default generators in Minitab to ensure the design is balanced.
- Run the experimental runs in random order whenever possible.
- Replicate the experiment for more power. Repeated measures from the same run do not give the same estimating power as replicates.
- If the model has significant curvature, use a Response Surface design to determine a true optimum condition.

Notes: