02417: Time Series Analysis
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Assignment 4: Kalman Filter

We consider the following system of differential equations to describe the thermal dynamics of a room.

$$\frac{dT_i}{dt} = \frac{1}{R_{im}C_i}(T_m - T_i) + \frac{1}{R_{ia}C_i}(T_a - T_i) + \frac{1}{C_i}\Phi_h + \frac{1}{C_i}G_v$$

$$\frac{dT_m}{dt} = \frac{1}{R_{im}C_m}(T_i - T_m)$$

where

- T_i room air temperature
- T_a outdoor air temperature
- T_m interior wall temperature
- C_i thermal capacity of indoor air
- C_m thermal capacity of wall
- R_{im} thermal resistance between wall and indoor air
- R_{ia} thermal resistance between outdoor air and indoor air
- Φ_h heating power
- G_v solar radiation

Consider the indoor air and wall temperature as model states. Outdoor temperature, heating power and solar radiation serve as model inputs. Then this system can be transfered to a discrete time state space model with system equation:

$$\begin{pmatrix} T_i^t \\ T_m^t \end{pmatrix} = A \cdot \begin{pmatrix} T_i^{t-1} \\ T_m^{t-1} \end{pmatrix} + B \cdot \begin{pmatrix} T_a^{t-1} \\ \Phi_h^{t-1} \\ G_v^{t-1} \end{pmatrix} + \begin{pmatrix} \eta_1^t \\ \eta_2^t \end{pmatrix} \tag{1}$$

with $\eta^t \sim N(0, \Sigma_1)$. Assume we measure the indoor temperature but not the temperature in the wall. Therefore, the measurement equation is given by

$$Y^t = T_i^t + \varepsilon^t \tag{2}$$

with $\varepsilon^t \sim N(0, \Sigma_2)$.

In the file **building.csv** we are given a data set from a building, including the time stamp (timedate), the observed indoor temperature (yTi), the ambient temperature (T_a), the heating input (Ph) and solar radiation (Gv). The other columns in the data set are irrelevant for this assignment. Exclude the last third of the data and do not use it in the last subquestion in Questions 4.3 and 4.4. When using the last third: You are allowed to used the future input but the indoor temperature should only be used for comparisons.

Question 4.1: Discretize the above continuous time system of differential equations, so it takes the form of Equation (1). Use a constant time step Δt and approximate the derivatives by

$$\frac{dT}{dt} = \frac{T^t - T^{t-1}}{\Delta t} \tag{3}$$

Obtain the matrices A and B.

Question 4.2: Load the data and present it by making some plots. Comment on what you see.

Question 4.3: It was found that the following matrices fit the data well:

$$A = \begin{pmatrix} 0.755 & 0.24 \\ 0.1 & 0.9 \end{pmatrix}, \quad B = \begin{pmatrix} 0.005 & 0.127 & 0.335 \\ 0.0 & 0.0 & 0.0 \end{pmatrix} \tag{4}$$

For the covariance matrixes Σ_1 and Σ_2, assume:

$$\Sigma_1 = \begin{pmatrix} 0.5 & 0.0 \\ 0.0 & 0.5 \end{pmatrix}, \qquad \Sigma_2 = 0.5 \tag{5}$$

1. Use the Kalman filter to estimate the states T_i and T_m. You may implement the Kalman filter yourself, or (preferably) use a software package of your choice. In R, two options are the packages **FKF** and **dse**. Visualize the 1-step predictions of the states.
2. Predict the indoor temperature of the last third of the data. You may use the future values of the input variables. Compare the results with the measured indoor temperature using a plot. Include a 95% prediction interval. Make a table including the prediction of the first observation and the last observation (in the last third of the data).

Comment on your results

Question 4.4: Optimizing the (co-)variances

1. Instead of using the covariance matrices given above, estimate the diagonal elements in Σ_1 and Σ_2 by maximum likelihood estimation. The system matrices A and B should be kept the same as in the previous question.
2. Present the 1-step predictions as in the previous question.
3. Predict the indoor temperature of the last third of the data. You may use the future values of the input variables. Compare the results with the measured indoor temperature using a plot. Include a 95% prediction interval. Make a table including the prediction of the first observation and the last observation (in the last third of the data).

Comment on your results