02417: Time Series Analysis

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Assignment 2: ARMA Processes and Seasonal Processes

NOTE that the model parametrizations in the assignment and in your favorite software package may not be identical. You will study how the choice of coefficients (of the operator polynomials in ARMA processes) affects the structure of the process, through simulated data and empirical autocorrelation functions. The assignment is to some extent specified in terms of R commands. Users of other software packages should find similar functions to answer the questions.

Question 2.1: Stability Let the process X_t be given by

$$X_t - 0.8X_{t-1} - 0.1X_{t-2} = \epsilon_t + 0.8\epsilon_{t-1}$$

where ϵ_t is a white noise process with $\sigma = 0.2$.

1. Is the process stationary and invertible?
2. Investigate analytically the second order moment representation of the process.
3. Simulate 10 realisations with 200 observations from the process and plot them (Preferably in the same plot).
4. Estimate the ACF for each realisation and plot those (Again, preferably in the same plot). Comment on the results.
5. Repeat for the PACF of the same realisations.
6. Calculate the variance of each of the realisations.

Compare and discuss the analytical and numerical results.

Question 2.2: Predicting the Carbondioxide level in an office The data below is measured the logarithm of the CO2 level in parts per million in an office, measured every three hours.

Based on historical data the following model has been identified:

$$(1 - 0.547B)(1 - 0.860B^8)(log(Y_t) - \mu) = \epsilon_t$$

where ϵ_t is a white-noise process with variance σ_ϵ^2. Based on six days of data, it is found that $\sigma_\epsilon^2 = 0.065^2$. Furthermore, μ was estimated to 6.24. The table below shows the last 16 observations of $log(Y_t)$:

Date	02/05	02/05	02/05	02/05	02/06	02/06	02/06	02/06
Time	12:00	15:00	18:00	21:00	00:00	03:00	06:00	09:00
log(CO2)	6.153	6.308	6.327	6.344	6.266	6.121	6.096	6.081
Date	02/06	02/06	02/06	02/06	02/07	02/07	02/07	02/07
Time	12:00	15:00	18:00	21:00	00:00	03:00	06:00	09:00
log(CO2)	6.178	6.411	6.442	6.505	6.195	6.085	6.086	6.079

Predict the values of Y_t corresponding to t = 02/07 12:00 and 02/07 15:00, together with 95% prediction intervals for the predictions. Why is a seasonality of 8 steps reasonable in this model? Plot the observations along with the two predictions and their 95% prediction intervals.

Question 2.3: Simulating seasonal processes A process Y_t is said to follow a multiplicative $(p, d, q) \times (P, D, Q)_s$ seasonal model if

$$\phi(B)\Phi(B^s)\nabla^d\nabla_s^D Y_t = \theta(B)\Theta(B^s)\epsilon_t$$

where (ϵ_t) is a white noise process, and $\phi(B)$ and $\theta(B)$ are polynomials of order p and q, respectively. Furthermore, $\Phi(B^s)$ and $\Theta(B^s)$ are polynomials in B^s. All according to definition 5.22 in the textbook.

Simulate the following models (where monthly data are assumed). Plot the simulations and the associated autocorrelation functions (ACF and PACF).

1. A $(1, 0, 0) \times (0, 0, 0)_{12}$ model with the parameter $\phi_1 = 0.8$.
2. A $(0, 0, 0) \times (1, 0, 0)_{12}$ model with the parameter $\Phi_1 = -0.85$.
3. A $(1, 0, 0) \times (0, 0, 1)_{12}$ model with the parameters $\phi_1 = -0.7$ and $\Theta_1 = 0.8$.
4. A $(1, 0, 0) \times (1, 0, 0)_{12}$ model with the parameters $\phi_1 = -0.8$ and $\Phi_1 = 0.7$.
5. A $(2, 0, 0) \times (1, 0, 0)_{12}$ model with the parameters $\phi_1 = 0.6$, $\phi_2 = 0.5$, and $\Phi_1 = 0.9$.
6. A $(0, 0, 1) \times (0, 0, 1)_{12}$ model with the parameters $\theta_1 = -0.9$ and $\Theta_1 = 0.7$.

Are all models seasonal? And stationary? Can you identify the model order based on the ACF and PACF? Summarize your observations on the processes and the autocorrelation functions. Which conclusions can you draw on the general behavior of the autocorrelation function for seasonal processes?

Note: `arima.sim` does not have a seasonal module, so model formulations as standard ARIMA processes have to be made.

Question 2.4: Consider the following ARMA(1,1) process

$$X_t - \phi X_{t-1} = \epsilon_t + 0.4\epsilon_{t-1}$$

1. Consider the four variations of the process with $\phi \in \{0.9, 0.995\}$ and $\sigma^2 \in \{0.1^2, 5^2\}$. Simulate 300 observations of each of the four processes 100 times. Subsequently, estimate the model parameters based on the simulated sequences. For each process, make a histogram plot of the estimates parameter ϕ and indicate the 95% quantiles. How do different values of ϕ affect the variance of the estimated ϕ? How do different values of σ affect the variance of the estimated ϕ?
2. Try different numbers of observations (e.g. 100 and 1000) in the simulation (with $\phi = 0.9$ and $\sigma = 1$) and investigate how it affects the variance of the estimates.