Grey-box models and model selection
Models for the heat dynamics of a building

Summer school 2019 DTU - CITIES and NTNU - ZEN:

Time series analysis - with a focus on modelling and forecasting in energy systems

DTU

CITIES
Centre for IT Intelligent Energy Systems

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ZERO EMISSION NEIGHBOURHOODS IN SMART CITIES

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Installation

If you did not install the package `ctsmr` in advance, then you need to do it now. See first the web page `ctsm.info` for OS specific instructions.

Introduction

This exercise is about grey-box modelling of the heat dynamics of a building using stochastic differential equations (SDEs). Further, the properties of the PRBS signal and the use of likelihood ratio tests for model selection are introduced.

The data consists of averaged values over five-minute intervals of:

- T_i (`yTi` in data) the average of all the indoor temperatures measured (one in each room in the building). The sensors were hanging approximately in the center of each room (°C)
- Φ_h (`Ph` in data) the total heat output for all electrical heaters in the building (kW)
- T_a (`Ta` in data) the ambient temperature (°C) (notice, that in other material T_e is used as the ambient (external) temperature. In this exercise T_e is used as envelope temperature)
- G (`Ps` in the data) the global radiation (kW/m^2)
- W_s (`Ws` in data) the wind speed (m/s)

The climate variables were measured with a climate station right next to the building. See Bacher and Madsen (2010) (it is in the file `Identifying_suitable_models_for_heat_dynamics.pdf`) for more details of the experiments in which the data was recorded (it is included in the .zip file).

Q1: Fit and validate a grey-box model

Open the script "`r/q1_fit_and_validate.R`". Remember to change the path with `setwd()` (in line 5) to set the working directory to the where the script file is located (in RStudio menu "Session->Set Working Directory->To Source File Location" can be used).

The simplest applicable SDE grey-box model has one state and consists of the system equation

$$dT_{\mathrm{i}} = \left(\frac{1}{R_{\mathrm{ia}} C_{\mathrm{i}}} (T_{\mathrm{a}} - T_{\mathrm{i}}) + \frac{1}{C_{\mathrm{i}}} g_{\mathrm{A}} \Phi_{\mathrm{s}} + \frac{1}{C_{\mathrm{i}}} \Phi_{\mathrm{h}} \right) dt + \sigma_{\mathrm{i}} d\omega_{\mathrm{i}} \tag{1}$$

and the measurement equation

$$Y_k = T_{\mathrm{i},k} + \epsilon_k \tag{2}$$

where k counts the measurements from 1 to N and where the measurement error is assumed to follow a normal distribution with $\epsilon_k \sim N(0, \sigma_\epsilon^2)$.

See Bacher and Madsen (2010) (it is in the file `Bacher2011.pdf`) for a description of the different parts and parameters in the model.

The RC-diagram in Figure 1 illustrates the deterministic part of the system equation.

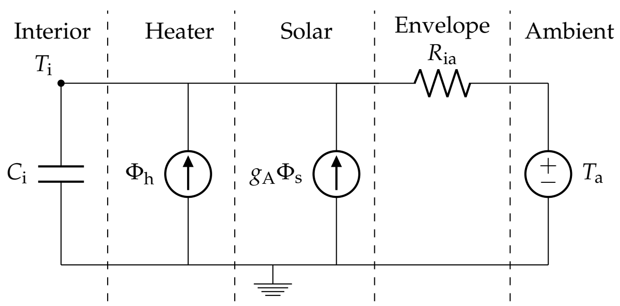

Figure 1 *RC-network of the most simple model Ti.*

Run the script line by line and read the comments in the code to see how a model and data is specified. Stop in the section where the parameter optimization is run and thereafter proceed here:

- Estimate the parameters in the simple model Ti in Equation (1) and see the estimated values with `summary(fit)`. Is the estimation successful (i.e. does the minimization of the negative log-likelihood converge)?
- Actually, the initial value and the boundary for one of the parameters are poorly set. You can see if the parameter estimates are close to one of their boundaries from the values of

  - $\frac{dF}{dPar}$ which is the partial derivative of the objective function (negative log-likelihood).
  - $\frac{dPen}{dPar}$ which is the partial derivative of the penalty function.

  If the value of $\frac{dPen}{dPar}$ is significant compared to the value of $\frac{dF}{dPar}$ for a particular parameter it indicates that a boundary should be expanded for the parameter. Correct one of the boundaries and re-estimate until the partial derivatives are all very small. Which boundary was not set appropriately?

- The one-step predictions (residuals) are estimates of the system noise (i.e. the realized values of the incremental $d\omega$ of the Wiener process) added together with the observation noise. The assumption is that the one-step predictions are white noise. Validate if this assumption is fulfilled by plotting the auto-correlation function and the accumulated periodogram for the residuals. Is the model suitable, i.e. does it describe the heat dynamics sufficiently?

- Next it is possible to gain some information about what is missing in the model, with time series plots of the residuals and the inputs. Generate the plots. Are there some systematic patterns in the residuals? If yes, do they seem to be related to the inputs? To any specific events in the inputs?

Q2: Model selection

Open "`r/q2_q3_model_selection.R`" and run the first part. As instructed in the script file go and open the file "`r/functions/Ti.R`" and see how the model Ti is defined there, it is simply wrapped in the function - simply to make the code more readable. Further, see that the model validation is also put into a function.

Now we will try to extend the simplest model, where a single part of the model is extended. For example a state variable representing the temperature in the building envelope T_e. Such a state is called "a hidden state" since it is not measured. The RC-diagram for this model is shown in Figure 2. The system equations are

$$dT_i = \left(\frac{1}{R_{ie}C_i}(T_e - T_i) + \frac{1}{C_i}g_A\Phi_s + \frac{1}{C_i}\Phi_h\right)dt + \sigma_i d\omega_i, \tag{3}$$

$$dT_e = \left(\frac{1}{R_{ie}C_e}(T_i - T_e) + \frac{1}{R_{ea}C_e}(T_a - T_e)\right)dt + \sigma_e d\omega_e \tag{4}$$

and the measurement equation is still

$$Y_k = T_{i,k} + \epsilon_k \tag{5}$$

Three other extensions of the simple model is also suggested. Their RC-diagrams are shown the figures 3, 4, 5.

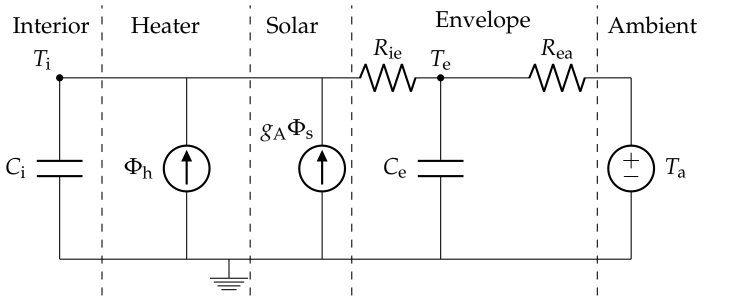

Figure 2 *RC-network of the most simple model extended with a state in the building envelope TiTe.*

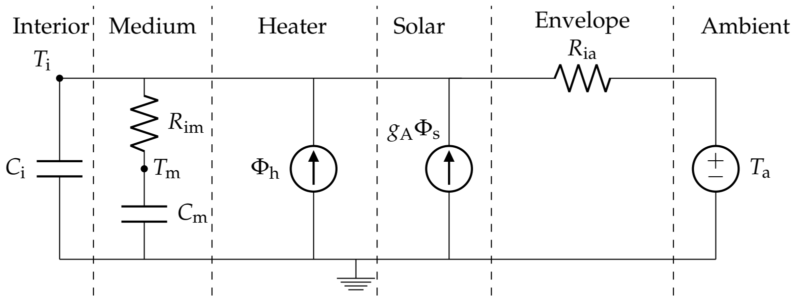

Figure 3 *RC-network of the most simple model extended with a state in which the solar radiation enters TiTm.*

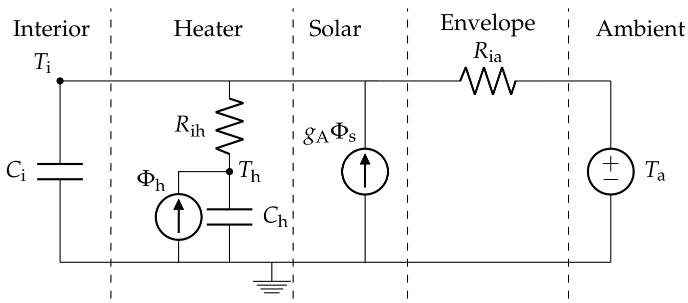

Figure 4 *RC-network of the most simple model extended with a state in the heater TiTh.*

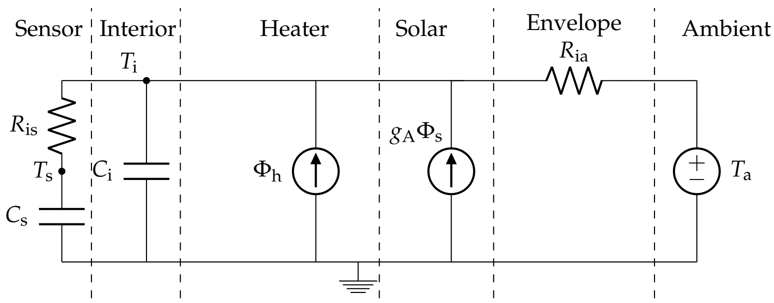

Figure 5 *RC-network of the most simple model extended with a state in the sensor TiTs.*

- Estimate the parameters in the four models (they are implemented in the functions with equivalent names).
- Are the extended models improved regarding the description of the dynamics (hint, analyse the residuals)?

Q3: Likelihood ratio test

Use the script to carry out a likelihood ratio test of the simple model to each of the extended models. If the p-values show that more than one of the extended models are a significant improvement, it is suggested to select the extended model with the highest maximum likelihood.

- Which of the four models should be selected for further extension?
- Take the selected model and extend it again once more, see the functions with 3-state models.

From this point the selection and extension procedure should be continued until no significant extension can be found, however this is beyond the scope of the exercise. In order to see what happens if you stay in that track read the article Bacher and Madsen (2011) (the .pdf is in the .zip file).

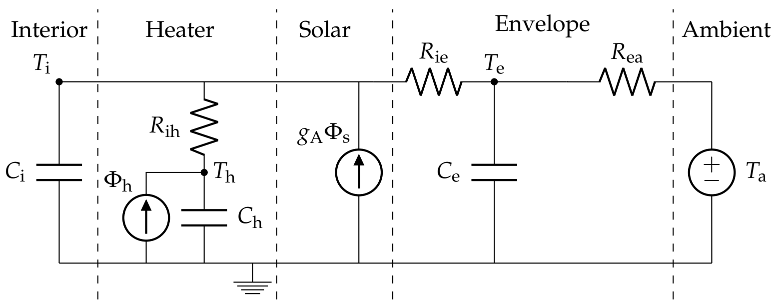

Figure 6 *RC-diagram of TiTeTh.*

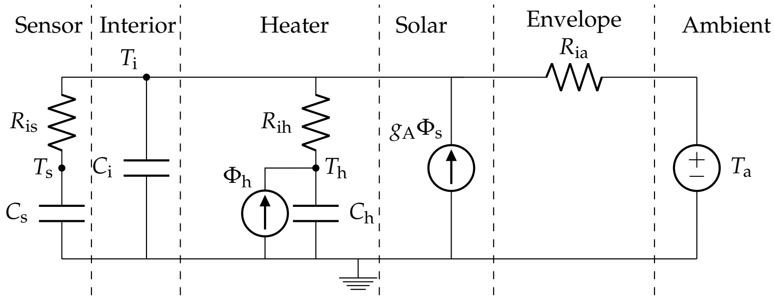

Figure 7 *RC-diagram of TiThTs.*

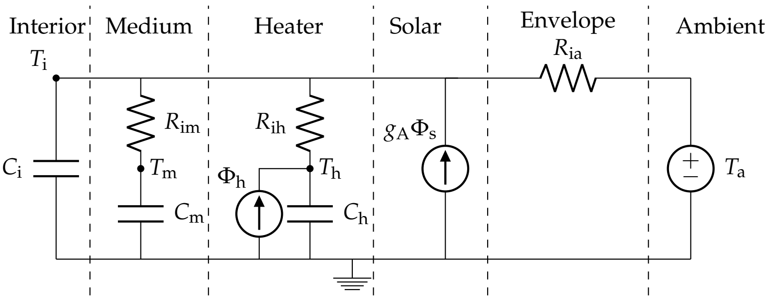

Figure 8 *RC-diagram of TiThTm.*

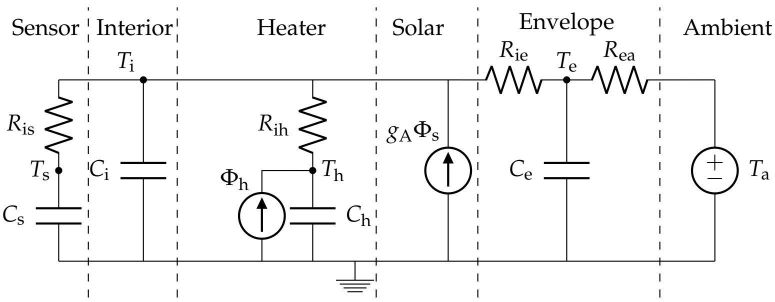

Figure 9 *RC-diagram of TiTeThTs.*

Question 4

This part deals with Pseudo Random Sequence Signals. See Godfrey (1980) for a detailed description of PRBS signals. Go and open "`r/q4_prbs.R`".

Answer the following questions:

- Do the plotting of the data. Which of the signals is a PRBS signal?
- Which of the other signals are highly dependent on the PRBS signal?
- The function `prbs()` is an implementation of the n-stage feedback registers in the paper (see the function definition in "`r/functions/prbs.R`). Generate a PRBS signal as in the script and investigate its properties with the ACF.

Optional:

- In order to generate multiple PRBS signals, they will be independent. Try generating two PRBS signals, lag one of them and plot the cross-correlation function.
- Again consider the plots of data. Can you already from this see a lot about the dynamics of the house (think about the step response)?

Finally, the R-code which was used to generate the PRBS signals used in the experiment is given, together with the code for generating three independent PRBS signals.

Questions and comments: `pbac@dtu.dk`

References

P. Bacher and H. Madsen. Experiments and data for building energy performance analysis : Financed by the danish electricity saving trust. Technical report, DTU Informatics, Building 321, Kgs. Lyngby, 2010.

P. Bacher and H. Madsen. Identifying suitable models for the heat dynamics of buildings. *Energy & Buildings*, 43(7):1511–1522, 2011. ISSN 03787788. doi: 10.1016/j.enbuild.2011.02.005.

K. Godfrey. Correlation methods. *Automatica*, 16(5):527–534, 1980. ISSN 00051098.