

# Metsa-Oy Forest and Supply Chain 1: Static

## 1, Introduction

## 2, Definition of Mathematical Expressions

Symbol	Definition	Expression
$T^1$	type of timber 1	{MAT, KUT, KOT}
$T^2$	type of timber 2	{MAK, KUK, KOK}
$T$	type of timber	$T^1 \cup T^2$
$I_{t=\text{MAT}}^{T^1}$	outputs of wood production corresponding to input MAT	{MAS}
$I_{t=\text{KUT}}^{T^1}$	outputs of wood production corresponding to input KUT	{KUS, KUV}
$I_{t=\text{KOT}}^{T^1}$	outputs of wood production corresponding to input KOT	{KOS, KOV}
$I_{t=\text{MAK}}^{T^2}$	outputs of wood production corresponding to output MAK	{MAS}
$I_{t=\text{KUK}}^{T^2}$	outputs of wood production corresponding to output KUK	{KUS, KUV}
$I_{t=\text{KOK}}^{T^2}$	outputs of wood production corresponding to output KOK	{KOS, KOV}
$I^{\text{saw}}$	outputs of wood production in saw mill	{MAS, KUS, KOS}
$I^{\text{plywood}}$	outputs of wood production in plywood mill	{KUV, KOV}
$I$	outputs of wood production	$I^{\text{saw}} \cup I^{\text{plywood}}$
$J$	outputs of pulp production	{HSEL, LSEL}
$J_{t=\text{MAK}}^{T^2}$	outputs of pulp production corresponding to input MAK	{HSEL}
$J_{t=\text{KUK}}^{T^2}$	outputs of pulp production corresponding to input KUK	$\emptyset$
$J_{t=\text{KOK}}^{T^2}$	outputs of pulp production corresponding to input KOK	{LSEL}
$K$	regions to sell products	{EU, IE, PA, KI}

Table 1, summary of sets

Symbol	Definition	Type	Unit	Set
$h_t$	purchasing amount of timber $t$	integer	$1000m^3$	$T$
$y_i^I$	production amount of wood $i$	linear	$1000m^3$	$I$
$y_j^J$	production amount of pulp $j$	linear	$1000m^3$	$J$
$y^{\text{paper}}$	production amount of paper	linear	$1000m^3$	—
$z_{i,k}^I$	selling amount of wood $i$ in region $k$	linear	$1000m^3$	$I, K$
$z_{j,k}^J$	selling amount of pulp $j$ in region $k$	linear	$1000m^3$	$J, K$
$z_k^{\text{paper}}$	selling amount of paper in region $k$	linear	$1000m^3$	$K$

Table 2, summary of decision variables

Symbol	Definition	Unit	Set
$\alpha_t$	fixed cost factor of purchasing wood $t$	euro/(1000m <sup>3</sup> )	$T$
$\beta_t$	unit cost factor of purchasing wood $t$	euro/(1000m <sup>6</sup> )	$T$
$a_i^I$	relation of timber input and output in wood production $i$	—	$I$
$b_i^I$	relation of timber output and output in wood production $i$	—	$I$
$e_i^I$	relation of fuel output and output in wood production $i$	—	$I$
$c_i^I$	cost of wood production $i$	euro/(1000m <sup>3</sup> )	$I$
$r_{\text{saw}}$	capacity of saw mill in wood production $i$	1000m <sup>3</sup> /year	—
$r_{\text{plywood}}$	capacity of plywood mill in wood production $i$	1000m <sup>3</sup> /year	—
$a_j^J$	input and output relation of pulp production $j$	—	$J$
$c_j^J$	cost of pulp production $j$	euro/(1000ton)	$J$
$r_j^J$	capacity of pulp production $j$	1000ton/year	$J$
$a_t^{\text{paper}}$	relation of timber inputs in paper production	—	$T$
$b_j^{\text{paper}}$	relation of pulp inputs in paper production	—	$J$
$c^{\text{paper}}$	cost of paper production	euro/(1000ton)	—
$r^{\text{paper}}$	capacity of paper production	1000ton/year	—
$\gamma_{i,k}^I$	fixed price factor of wood $i$ in region $k$	euro/(1000m <sup>3</sup> )	$I, K$
$\delta_{i,k}^I$	unit price factor of wood $i$ in region $k$	euro/(10 <sup>6</sup> m <sup>6</sup> )	$I, K$
$\gamma_{j,k}^J$	fixed price factor of pulp $j$ in region $k$	euro/(1000ton)	$J, K$
$\delta_{j,k}^J$	unit price factor of pulp $j$ in region $k$	euro/(10 <sup>6</sup> ton <sup>2</sup> )	$J, K$
$\gamma_k^{\text{paper}}$	fixed price factor of paper in region $k$	euro/(1000ton)	$K$
$\delta_k^{\text{paper}}$	unit price factor of paper in region $k$	euro/(10 <sup>6</sup> ton <sup>2</sup> )	$K$
$p^{\text{fuel}}$	price of fuel wood	euro/(1000m <sup>3</sup> )	—

Table 3, summary of constants

### 3, Objective Function

The Objective function composes of seven parts:

$$f^{\text{timber}} + f^{\text{wood}} + f^{\text{pp}} + g^{\text{timber}} + g^{\text{fuel}} + g^{\text{wood}} + g^{\text{pulp}} + g^{\text{paper}}$$

1. cost of timber procurement:  $f^{\text{timber}} = - \sum_{t \in T} h_t (\alpha_t + \beta_t h_t)$
2. cost of wood production:  $f^{\text{wood}} = - \sum_{i \in I} c_i^I y_i^I$
3. cost of pulp and paper production:  $f^{\text{pp}} = - \sum_{j \in J} c_j^J y_j^J - c^{\text{paper}} y^{\text{paper}}$
4. profit of left timbers selling:

$$g^{\text{timber}} = \sum_{t \in T^1} \alpha_t \left( h_t - \sum_{i \in I_t^{T_1}} a_i^I y_i^I \right) + \sum_{t \in T^2} \alpha_t \left( h_t + \sum_{i \in I_t^{T_2}} b_i^I y_i^I - \sum_{j \in J_t^{T_2}} a_j^J y_j^J - a_t^{\text{paper}} y^{\text{paper}} \right)$$

5. profit of fuel wood selling:  $g^{\text{fuel}} = \sum_{i \in I} p^{\text{fuel}} e_i^I y_i^I$

6. profit of wood selling:  $g^{\text{wood}} = \sum_{i \in I} \sum_{k \in K} z_{i,k}^I (\gamma_{i,k}^I - \delta_{i,k}^I z_{i,k}^I)$
7. profit of pulp selling:  $g^{\text{pulp}} = \sum_{j \in J} \sum_{k \in K} z_{j,k}^J (\gamma_{j,k}^J - \delta_{j,k}^J z_{j,k}^J)$
8. profit of paper selling:  $g^{\text{paper}} = \sum_{k \in K} z_k^{\text{paper}} (\gamma_k^{\text{paper}} - \delta_k^{\text{paper}} z_k^{\text{paper}})$

## 4, Constraints

Besides the constraints that all variables are non-negative, there are ten sets of constraints:

1. limit of timber amount in wood production:

$$h_t \geq \sum_{i \in I_t^{T1}} a_i^I y_i^I \quad \forall t \in T^1$$

2. limit of timber amount in pulp and paper production:

$$\left( h_t + \sum_{i \in I_t^{T2}} b_i^I y_i^I \right) \geq \sum_{j \in J_t^{T2}} a_j^J y_j^J + a_t^{\text{paper}} y^{\text{paper}} \quad \forall t \in T^2$$

3. limit of pulp amount in paper production:

$$y_j^J \geq b_j^{\text{paper}} y^{\text{paper}} \quad \forall j \in J$$

4. limit of wood amount in selling:

$$y_i^I \geq \sum_{k \in K} z_{i,k}^I \quad \forall i \in I$$

5. limit of pulp amount in selling:

$$y_j^J - b_j^{\text{paper}} y^{\text{paper}} \geq \sum_{k \in K} z_{j,k}^J \quad \forall j \in J$$

6. limit of paper amount in selling:

$$y^{\text{paper}} \geq \sum_{k \in K} z_k^{\text{paper}}$$

7. limit of capacity in saw mill:

$$\sum_{i \in I^{\text{saw}}} y_i^I \leq r^{\text{saw}}$$

8. limit of capacity in plywood mill:

$$\sum_{i \in I^{\text{plywood}}} y_i^I \leq r^{\text{plywood}}$$

9. limit of capacity in pulp production:

$$y_j^J \leq r_j^J \quad \forall j \in J$$

10. limit of capacity in paper production:

$$y^{\text{paper}} \leq r^{\text{paper}}$$

## 5, Result

obj = 5.9649e6

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result_h_t = [0.0, -0.0, -0.0, 77.0, 80.0, 68.0]
result_y_i = [0.0, 0.0, 0.0, 0.0, 0.0]
result_y_j = [16.0, 16.0]
result_y_paper = 80.0
result_z_ik = [
    0.0 130.0 58.33 50.0;
    0.0 60.0 54.17 46.67;
    0.0 35.0 31.25 32.0;
    0.0 190.0 150.0 97.22;
    537.5 292.86 162.5 126.67
]
result_z_jk = [
    0.0 312.5 230.0 216.67;
    0.0 700.0 191.67 178.57142857142858
]
result_z_paper_k = [24.85, 27.39, 6.16, 21.60]
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# Metsa-Oy Production and Supply Chain 2: Dynamic

## 1, Introduction

## 2, Definition of Mathematical Expressions

Symbol	Definition	Expression
$T^1$	type of timber 1	{MAT, KUT, KOT}
$T^2$	type of timber 2	{MAK, KUK, KOK}
$T$	type of timber	$T^1 \cup T^2$
$I_{t=\text{MAT}}^{T^1}$	outputs of wood production corresponding to input MAT	{MAS}
$I_{t=\text{KUT}}^{T^1}$	outputs of wood production corresponding to input KUT	{KUS, KUV}
$I_{t=\text{KOT}}^{T^1}$	outputs of wood production corresponding to input KOT	{KOS, KOV}
$I_{t=\text{MAK}}^{T^2}$	outputs of wood production corresponding to output MAK	{MAS}
$I_{t=\text{KUK}}^{T^2}$	outputs of wood production corresponding to output KUK	{KUS, KUV}
$I_{t=\text{KOK}}^{T^2}$	outputs of wood production corresponding to output KOK	{KOS, KOV}
$I^{\text{saw}}$	outputs of wood production in saw mill	{MAS, KUS, KOS}
$I^{\text{plywood}}$	outputs of wood production in plywood mill	{KUV, KOV}
$I$	outputs of wood production	$I^{\text{saw}} \cup I^{\text{plywood}}$
$J$	outputs of pulp production	{HSEL, LSEL}
$J_{t=\text{MAK}}^{T^2}$	outputs of pulp production corresponding to input MAK	{HSEL}
$J_{t=\text{KUK}}^{T^2}$	outputs of pulp production corresponding to input KUK	$\emptyset$
$J_{t=\text{KOK}}^{T^2}$	outputs of pulp production corresponding to input KOK	{LSEL}
$K$	regions to sell products	{EU, IE, PA, KI}
$M$	three years in the planning horizon	{1, 2, 3}

Table 1, summary of sets

Symbol	Definition	Type	Unit	Set
$h_{m,t}$	purchasing amount of timber $t$ in the year $m$	integer	$1000m^3$	$M, T$
$y_{m,i}^I$	production amount of wood $i$ in the year $m$	linear	$1000m^3$	$M, I$
$y_{m,j}^J$	production amount of pulp $j$ in the year $m$	linear	$1000m^3$	$M, J$
$y_m^{\text{paper}}$	production amount of paper in the year $m$	linear	$1000m^3$	$M$
$z_{m,i,k}^I$	selling amount of wood in the region $k$ in the year $m$	linear	$1000m^3$	$M, I, K$
$z_{m,j,k}^J$	selling amount of pulp in the region $k$ in the year $m$	linear	$1000m^3$	$M, J, K$
$z_{m,k}^{\text{paper}}$	selling amount of paper in the region $k$ in the year $m$	linear	$1000m^3$	$M, K$
$x_m^{\text{saw}}$	capacity of saw mill	linear	—	$M^2$
$x_m^{\text{plywood}}$	capacity of plywood mill	linear	—	$M^2$
$x_{m,j}^J$	capacity of pulp production line	linear	—	$M^2, J$
$x_m^{\text{paper}}$	capacity of paper production line	linear	—	$M^2$

Table 2, summary of decision variables

Symbol	Definition	Unit	Set
$p^{\text{fuel}}$	price of fuel wood	euro/(1000m <sup>3</sup> )	—
$\sigma$	annual discounting factor	—	—
$\omega_i^I$	demand growth coefficient of wood $i$	—	$I$
$\omega_j^J$	demand growth coefficient of pulp $j$	—	$J$
$\omega^{\text{paper}}$	demand growth coefficient of paper	—	—
$\alpha_t$	fixed cost factor of purchasing timber $t$	euro/(1000m <sup>3</sup> )	$T$
$\beta_t$	unit cost factor of purchasing timber $t$	euro/(1000m <sup>6</sup> )	$T$
$a_i^I$	relation of timber input and output in wood production $i$	—	$I$
$b_i^I$	relation of timber output and output in wood production $i$	—	$I$
$e_i^I$	relation of fuel output and output in wood production $i$	—	$I$
$c_i^I$	cost of wood production $i$	euro/(1000m <sup>3</sup> )	$I$
$r^{\text{saw}}$	original capacity of saw mill	1000m <sup>3</sup> /year	—
$r^{\text{plywood}}$	original capacity of plywood mill	1000m <sup>3</sup> /year	—
$o^{\text{saw}}$	capacity expansion cost of saw mill	euro/(1000m <sup>3</sup> /year)	—
$o^{\text{plywood}}$	capacity expansion cost of plywood mill	euro/(1000m <sup>3</sup> /year)	—
$\nu^{\text{saw}}$	max capacity expansion factor of saw mill	—	—
$\nu^{\text{plywood}}$	max capacity expansion factor of saw mill	—	—
$a_j^J$	input and output relation of pulp production $j$	—	$J$
$c_j^J$	cost of pulp production $j$	euro/(1000ton)	$J$
$r_j^J$	original capacity of pulp production $j$	1000ton/year	$J$
$o_j^J$	capacity expansion cost of pulp production $j$	euro/(1000m <sup>3</sup> /year)	—
$\nu_j^J$	max capacity expansion factor of pulp production $j$	—	$J$
$a_t^{\text{paper}}$	relation of timber inputs in paper production	—	$T$
$b_j^{\text{paper}}$	relation of pulp inputs in paper production	—	$J$
$c^{\text{paper}}$	cost of paper production	euro/(1000ton)	—
$r^{\text{paper}}$	original capacity of paper production	1000ton/year	—
$o^{\text{paper}}$	capacity expansion cost of paper production	euro/(1000m <sup>3</sup> /year)	—
$\nu^{\text{paper}}$	max capacity expansion factor of paper production	—	—
$\gamma_{m,i,k}^I$	fixed price factor of wood $i$ in the region $k$ in the year $m$	euro/(1000m <sup>3</sup> )	$M, I, K$
$\delta_{m,i,k}^I$	unit price factor of wood $i$ in the region $k$ in the year $m$	euro/(10 <sup>6</sup> m <sup>6</sup> )	$M, I, K$
$\gamma_{m,j,k}^J$	fixed price factor of pulp $j$ in the region $k$ in the year $m$	euro/(1000ton)	$M, J, K$
$\delta_{m,j,k}^J$	unit price factor of pulp $j$ in the region $k$ in the year $m$	euro/(10 <sup>6</sup> ton <sup>2</sup> )	$M, J, K$
$\gamma_{m,k}^{\text{paper}}$	fixed price factor of paper in the region $k$ in the year $m$	euro/(1000ton)	$M, K$
$\delta_{m,k}^{\text{paper}}$	unit price factor of paper in the region $k$ in the year $m$	euro/(10 <sup>6</sup> ton <sup>2</sup> )	$M, K$

Table 3, summary of constants

### 3, Objective Function

The Objective function composes of seven parts:

$$\sum_{m \in M} \sigma^{m-1} \left[ f_m^{\text{timber}} + f_m^{\text{wood}} + f_m^{\text{pp}} + g_m^{\text{timber}} + g_m^{\text{fuel}} + g_m^{\text{wood}} + g_m^{\text{pulp}} + g_m^{\text{paper}} \right] + f^{\text{cap}}$$

1. cost of timber procurement:  $f_m^{\text{timber}} = - \sum_{t \in T} h_{m,t} (\alpha_t + \beta_t h_{m,t})$
2. cost of wood production:  $f_m^{\text{wood}} = - \sum_{i \in I} c_i^I y_{m,i}^I$
3. cost of pulp and paper production:  $f_m^{\text{pp}} = - \sum_{j \in J} c_j^J y_{m,j}^J - \sum_{m \in M} c^{\text{paper}} y_m^{\text{paper}}$

4. profit of left timbers selling:

$$g_m^{\text{timber}} = \sum_{t \in T^1} \alpha_t \left( h_{m,t} - \sum_{i \in I_t^{T^1}} a_i^I y_{m,i}^I \right) + \sum_{t \in T^2} \alpha_t \left( h_{m,t} + \sum_{i \in I_t^{T^2}} b_i^I y_{m,i}^I - \sum_{j \in J_t^{T^2}} a_j^J y_{m,j}^J - a_t^{\text{paper}} y_m^{\text{paper}} \right)$$

5. profit of fuel wood selling:  $g_m^{\text{fuel}} = \sum_{i \in I} p^{\text{fuel}} e_i^I y_{m,i}^I$

6. profit of wood selling:  $g_m^{\text{wood}} = \sum_{i \in I} \sum_{k \in K} (\omega_i^I)^{m-1} z_{m,i,k}^I (\gamma_{m,i,k}^I - \delta_{m,i,k}^I z_{m,i,k}^I)$

7. profit of pulp selling:  $g_m^{\text{pulp}} = \sum_{j \in J} \sum_{k \in K} (\omega_j^J)^{m-1} z_{m,j,k}^J (\gamma_{m,j,k}^J - \delta_{m,j,k}^J z_{m,j,k}^J)$

8. profit of paper selling:  $g_m^{\text{paper}} = \sum_{k \in K} (\omega^{\text{paper}})^{m-1} z_{m,k}^{\text{paper}} (\gamma_{m,k}^{\text{paper}} - \delta_{m,k}^{\text{paper}} z_{m,k}^{\text{paper}})$

9. cost of capacity expansion:

$$f^{\text{cap}} = \sum_{m=2}^3 \sigma^{m-2} \left[ o^{\text{saw}} (x_m^{\text{saw}} - x_{m-1}^{\text{saw}}) + o^{\text{plywood}} (x_m^{\text{plywood}} - x_{m-1}^{\text{plywood}}) + \sum_{j \in J} o_j^J (x_{m,j}^J - x_{m-1,j}^J) + o^{\text{paper}} (x_m^{\text{paper}} - x_{m-1}^{\text{paper}}) \right]$$

## 4, Constraints

Besides the constraints that all variables are non-negative, there are ten sets of constraints:

1. limit of timber amount in wood production:

$$h_{m,t} \geq \sum_{i \in I_t^{T^1}} a_i^I y_{m,i}^I \quad \forall t \in T^1, m \in M$$

2. limit of timber amount in pulp and paper production:

$$\left( h_{m,t} + \sum_{i \in I_t^{T^2}} b_i^I y_{m,i}^I \right) \geq \sum_{j \in J_t^{T^2}} a_j^J y_{m,j}^J + a_{m,t}^{\text{paper}} y_m^{\text{paper}} \quad \forall t \in T^2, m \in M$$

3. limit of pulp amount in paper production:

$$y_{m,j}^J \geq b_j^{\text{paper}} y_m^{\text{paper}} \quad \forall j \in J, m \in M$$

4. limit of wood amount in selling:

$$y_{m,i}^I \geq \sum_{k \in K} z_{m,i,k}^I \quad \forall i \in I, m \in M$$

5. limit of pulp amount in selling:

$$y_{m,j}^J - b_j^{\text{paper}} y_m^{\text{paper}} \geq \sum_{k \in K} z_{m,j,k}^J \quad \forall j \in J, m \in M$$

6. limit of paper amount in selling:

$$y_m^{\text{paper}} \geq \sum_{k \in K} z_{m,k}^{\text{paper}} \quad \forall m \in M$$

7. limit of capacity in saw mill:

$$\sum_{i \in I^{\text{saw}}} y_{m,i}^I \leq x_m^{\text{saw}} \quad \forall m \in M$$

8. limit of capacity in plywood mill:

$$\sum_{i \in I^{\text{plywood}}} y_{m,i}^I \leq x_m^{\text{plywood}} \quad \forall m \in M$$

9. limit of capacity in pulp production:

$$y_{m,j}^J \leq x_{m,j}^J \quad \forall j \in J, m \in M$$

10. limit of capacity in paper production:

$$y_m^{\text{paper}} \leq x_m^{\text{paper}} \quad \forall m \in M$$

11. relation between capacity expansion factors

$$\begin{aligned}x_1 &= r \quad \forall x, r \\x_2 &\geq x_1 \quad \forall x \\x_3 &\geq x_2 \quad \forall x \\x_m &\leq \nu r \quad \forall x, m \in M\end{aligned}$$

# Metsa-Oy Production and Supply Chain 3: Stochastic Dynamic

## 1, Introduction

## 2, Definition of Mathematical Expressions

Symbol	Definition	Expression
$T^1$	type of timber 1	{MAT, KUT, KOT}
$T^2$	type of timber 2	{MAK, KUK, KOK}
$T$	type of timber	$T^1 \cup T^2$
$I_{t=\text{MAT}}^{T^1}$	outputs of wood production corresponding to input MAT	{MAS}
$I_{t=\text{KUT}}^{T^1}$	outputs of wood production corresponding to input KUT	{KUS, KUV}
$I_{t=\text{KOT}}^{T^1}$	outputs of wood production corresponding to input KOT	{KOS, KOV}
$I_{t=\text{MAK}}^{T^2}$	outputs of wood production corresponding to output MAK	{MAS}
$I_{t=\text{KUK}}^{T^2}$	outputs of wood production corresponding to output KUK	{KUS, KUV}
$I_{t=\text{KOK}}^{T^2}$	outputs of wood production corresponding to output KOK	{KOS, KOV}
$I^{\text{saw}}$	outputs of wood production in saw mill	{MAS, KUS, KOS}
$I^{\text{plywood}}$	outputs of wood production in plywood mill	{KUV, KOV}
$I$	outputs of wood production	$I^{\text{saw}} \cup I^{\text{plywood}}$
$J$	outputs of pulp production	{HSEL, LSEL}
$J_{t=\text{MAK}}^{T^2}$	outputs of pulp production corresponding to input MAK	{HSEL}
$J_{t=\text{KUK}}^{T^2}$	outputs of pulp production corresponding to input KUK	$\emptyset$
$J_{t=\text{KOK}}^{T^2}$	outputs of pulp production corresponding to input KOK	{LSEL}
$K$	regions to sell products	{EU, IE, PA, KI}
$M$	three years in the planning horizon	{1, 2, 3}
$N$	four scenarios	{1, 2, 3, 4}

Table 1, summary of sets

Symbol	Definition	Type	Unit	Set
$h_{m,t}$	purchasing amount of timber $t$ in the year $m$	integer	$1000m^3$	$M, T$
$y_{m,i}^I$	production amount of wood $i$ in the year $m$	linear	$1000m^3$	$M, I$
$y_{m,j}^J$	production amount of pulp $j$ in the year $m$	linear	$1000m^3$	$M, J$
$y_m^{\text{paper}}$	production amount of paper in the year $m$	linear	$1000m^3$	$M$
$z_{m,i,k}^I$	selling amount of wood in the region $k$ in the year $m$	linear	$1000m^3$	$M, I, K$
$z_{m,j,k}^J$	selling amount of pulp in the region $k$ in the year $m$	linear	$1000m^3$	$M, J, K$
$z_{m,k}^{\text{paper}}$	selling amount of paper in the region $k$ in the year $m$	linear	$1000m^3$	$M, K$
$x_m^{\text{saw}}$	capacity of saw mill	linear	—	$M$
$x_m^{\text{plywood}}$	capacity of plywood mill	linear	—	$M$
$x_{m,j}^J$	capacity of pulp production line	linear	—	$M, J$
$x_m^{\text{paper}}$	capacity of paper production line	linear	—	$M$
$x_{n,3}^{\text{saw}}$	capacity of saw mill in the year 3 decide in the year 2	linear	—	$N$
$x_{n,3}^{\text{plywood}}$	capacity of plywood mill in the year 3 decide in the year 2	linear	—	$N$
$x_{n,3,j}^J$	capacity of pulp production line in the year 3 decide in the year 2	linear	—	$N, J$
$x_{n,3}^{\text{paper}}$	capacity of paper production line in the year 3 decide in the year 2	linear	—	$N$

Table 2, summary of decision variables

Symbol	Definition	Unit	Set
$\rho_{n,m}$	price coefficient of the scenario $n$ in the year $m$	—	$N, M$
$\omega_i^I$	demand growth coefficient of wood $i$	—	$I$
$\omega_j^J$	demand growth coefficient of pulp $j$	—	$J$
$\omega^{\text{paper}}$	demand growth coefficient of paper	—	—
$p^{\text{fuel}}$	price of fuel wood	euro/(1000m <sup>3</sup> )	—
$\sigma$	annual discounting factor	—	—
$\pi_n$	probability of scenario $n$	—	$N$
$\alpha_t$	fixed cost factor of purchasing timber $t$	euro/(1000m <sup>3</sup> )	$T$
$\beta_t$	unit cost factor of purchasing timber $t$	euro/(1000m <sup>6</sup> )	$T$
$a_i^I$	relation of timber input and output in wood production $i$	—	$I$
$b_i^I$	relation of timber output and output in wood production $i$	—	$I$
$e_i^I$	relation of fuel output and output in wood production $i$	—	$I$
$c_i^I$	cost of wood production $i$	euro/(1000m <sup>3</sup> )	$I$
$r^{\text{saw}}$	original capacity of saw mill	1000m <sup>3</sup> /year	—
$r^{\text{plywood}}$	original capacity of plywood mill	1000m <sup>3</sup> /year	—
$o^{\text{saw}}$	capacity expansion cost of saw mill	euro/(1000m <sup>3</sup> /year)	—
$o^{\text{plywood}}$	capacity expansion cost of plywood mill	euro/(1000m <sup>3</sup> /year)	—
$\nu^{\text{saw}}$	max capacity expansion factor of saw mill	—	—
$\nu^{\text{plywood}}$	max capacity expansion factor of saw mill	—	—
$a_j^J$	input and output relation of pulp production $j$	—	$J$
$c_j^J$	cost of pulp production $j$	euro/(1000ton)	$J$
$r_j^J$	original capacity of pulp production $j$	1000ton/year	$J$
$o_j^J$	capacity expansion cost of pulp production $j$	euro/(1000m <sup>3</sup> /year)	—
$\nu_j^J$	max capacity expansion factor of pulp production $j$	—	$J$
$a_t^{\text{paper}}$	relation of timber inputs in paper production	—	$T$
$b_j^{\text{paper}}$	relation of pulp inputs in paper production	—	$J$
$c^{\text{paper}}$	cost of paper production	euro/(1000ton)	—
$r^{\text{paper}}$	original capacity of paper production	1000ton/year	—
$o^{\text{paper}}$	capacity expansion cost of paper production	euro/(1000m <sup>3</sup> /year)	—
$\nu^{\text{paper}}$	max capacity expansion factor of paper production	—	—
$\gamma_{m,i,k}^I$	fixed price factor of wood $i$ in the region $k$ in the year $m$	euro/(1000m <sup>3</sup> )	$M, I, K$
$\delta_{m,i,k}^I$	unit price factor of wood $i$ in the region $k$ in the year $m$	euro/(10 <sup>6</sup> m <sup>6</sup> )	$M, I, K$
$\gamma_{m,j,k}^J$	fixed price factor of pulp $j$ in the region $k$ in the year $m$	euro/(1000ton)	$M, J, K$
$\delta_{m,j,k}^J$	unit price factor of pulp $j$ in the region $k$ in the year $m$	euro/(10 <sup>6</sup> ton <sup>2</sup> )	$M, J, K$
$\gamma_{m,k}^{\text{paper}}$	fixed price factor of paper in the region $k$ in the year $m$	euro/(1000ton)	$M, K$
$\delta_{m,k}^{\text{paper}}$	unit price factor of paper in the region $k$ in the year $m$	euro/(10 <sup>6</sup> ton <sup>2</sup> )	$M, K$

Table 3, summary of constants

### 3, Objective Function

The Objective function composes of many parts:

$$f_1^{\text{timber}} + f_1^{\text{wood}} + f_1^{\text{pp}} + g_1^{\text{timber}} + g_1^{\text{fuel}} + g_1^{\text{wood}} + g_1^{\text{pulp}} + g_1^{\text{paper}} + f_2^{\text{cap}} + \sum_{n \in N} \pi_n \left\{ \sum_{m=2}^3 \sigma^{m-1} \left[ f_m^{\text{timber}} + f_m^{\text{wood}} + f_m^{\text{pp}} + g_m^{\text{timber}} + g_m^{\text{fuel}} + g_m^{\text{wood}} + g_m^{\text{pulp}} + g_m^{\text{paper}} \right] + \sigma f_3^{\text{cap}} \right\}$$

1. cost of timber procurement:  $f_m^{\text{timber}} = - \sum_{t \in T} h_{m,t} (\alpha_t + \beta_t h_{m,t})$
2. cost of wood production:  $f_m^{\text{wood}} = - \sum_{i \in I} c_i^I y_{m,i}^I$
3. cost of pulp and paper production:  $f_m^{\text{pp}} = - \sum_{j \in J} c_j^J y_{m,j}^J - \sum_{m \in M} c^{\text{paper}} y_m^{\text{paper}}$
4. profit of left timbers selling:

$$g_m^{\text{timber}} = \sum_{t \in T^1} \alpha_t \left( h_{m,t} - \sum_{i \in I_t^{T^1}} a_i^I y_{m,i}^I \right) + \sum_{t \in T^2} \alpha_t \left( h_{m,t} + \sum_{i \in I_t^{T^2}} b_i^I y_{m,i}^I - \sum_{j \in J_t^{T^2}} a_j^J y_{m,j}^J - a_t^{\text{paper}} y_m^{\text{paper}} \right)$$

5. profit of fuel wood selling:  $g_m^{\text{fuel}} = \sum_{i \in I} p^{\text{fuel}} e_i^I y_{m,i}^I$
6. profit of wood selling:  $g_m^{\text{wood}} = \sum_{i \in I} \sum_{k \in K} \rho_{n,m} (\omega_i^I)^{m-1} z_{m,i,k}^I (\gamma_{m,i,k}^I - \delta_{m,i,k}^I z_{m,i,k}^I)$
7. profit of pulp selling:  $g_m^{\text{pulp}} = \sum_{j \in J} \sum_{k \in K} \rho_{n,m} (\omega_j^J)^{m-1} z_{m,j,k}^J (\gamma_{m,j,k}^J - \delta_{m,j,k}^J z_{m,j,k}^J)$
8. profit of paper selling:  $g_m^{\text{paper}} = \sum_{k \in K} \rho_{n,m} (\omega^{\text{paper}})^{m-1} z_{m,k}^{\text{paper}} (\gamma_{m,k}^{\text{paper}} - \delta_{m,k}^{\text{paper}} z_{m,k}^{\text{paper}})$
9. cost of capacity expansion:

$$f_2^{\text{cap}} = o^{\text{saw}} (x_2^{\text{saw}} - x_1^{\text{saw}}) + o^{\text{plywood}} (x_2^{\text{plywood}} - x_1^{\text{plywood}}) + \sum_{j \in J} o_j^J (x_{2,j}^J - x_{1,j}^J) + o^{\text{paper}} (x_2^{\text{paper}} - x_1^{\text{paper}})$$

$$f_3^{\text{cap}} = o^{\text{saw}} (x_3^{\text{saw}} - x_2^{\text{saw}}) + o^{\text{plywood}} (x_3^{\text{plywood}} - x_2^{\text{plywood}}) + \sum_{j \in J} o_j^J (x_{3,j}^J - x_{2,j}^J) + o^{\text{paper}} (x_3^{\text{paper}} - x_2^{\text{paper}})$$

## 4, Constraints

Besides the constraints that all variables are non-negative, there many ten sets of constraints:

1. limit of timber amount in wood production:

$$h_{m,t} \geq \sum_{i \in I_t^{T^1}} a_i^I y_{m,i}^I \quad \forall t \in T^1, m \in M$$

2. limit of timber amount in pulp and paper production:

$$\left( h_{m,t} + \sum_{i \in I_t^{T^2}} b_i^I y_{m,i}^I \right) \geq \sum_{j \in J_t^{T^2}} a_j^J y_{m,j}^J + a_{m,t}^{\text{paper}} y_m^{\text{paper}} \quad \forall t \in T^2, m \in M$$

3. limit of pulp amount in paper production:

$$y_{m,j}^J \geq b_j^{\text{paper}} y_m^{\text{paper}} \quad \forall j \in J, m \in M$$

4. limit of wood amount in selling:

$$y_{m,i}^I \geq \sum_{k \in K} z_{m,i,k}^I \quad \forall i \in I, m \in M$$

5. limit of pulp amount in selling:

$$y_{m,j}^J - b_j^{\text{paper}} y_m^{\text{paper}} \geq \sum_{k \in K} z_{m,j,k}^J \quad \forall j \in J, m \in M$$

6. limit of paper amount in selling:

$$y_m^{\text{paper}} \geq \sum_{k \in K} z_{m,k}^{\text{paper}} \quad \forall m \in M$$

7. limit of capacity in saw mill:

$$\sum_{i \in I^{\text{saw}}} y_{m,i}^I \leq x_m^{\text{saw}} \quad \forall m \in M$$

8. limit of capacity in plywood mill:

$$\sum_{i \in I^{\text{plywood}}} y_{m,i}^I \leq x_m^{\text{plywood}} \quad \forall m \in M$$

9. limit of capacity in pulp production:

$$y_{m,j}^J \leq x_{m,j}^J \quad \forall j \in J, m \in M$$

10. limit of capacity in paper production:

$$y_m^{\text{paper}} \leq x_m^{\text{paper}} \quad \forall m \in M$$

11. relation between capacity expansion factors

$$\begin{aligned} x_1 &= r & \forall x, r \\ x_2 &\geq x_1 & \forall x \\ x_3 &\geq x_2 & \forall x \\ x_m &\leq \nu r & \forall x, m \in M \end{aligned}$$