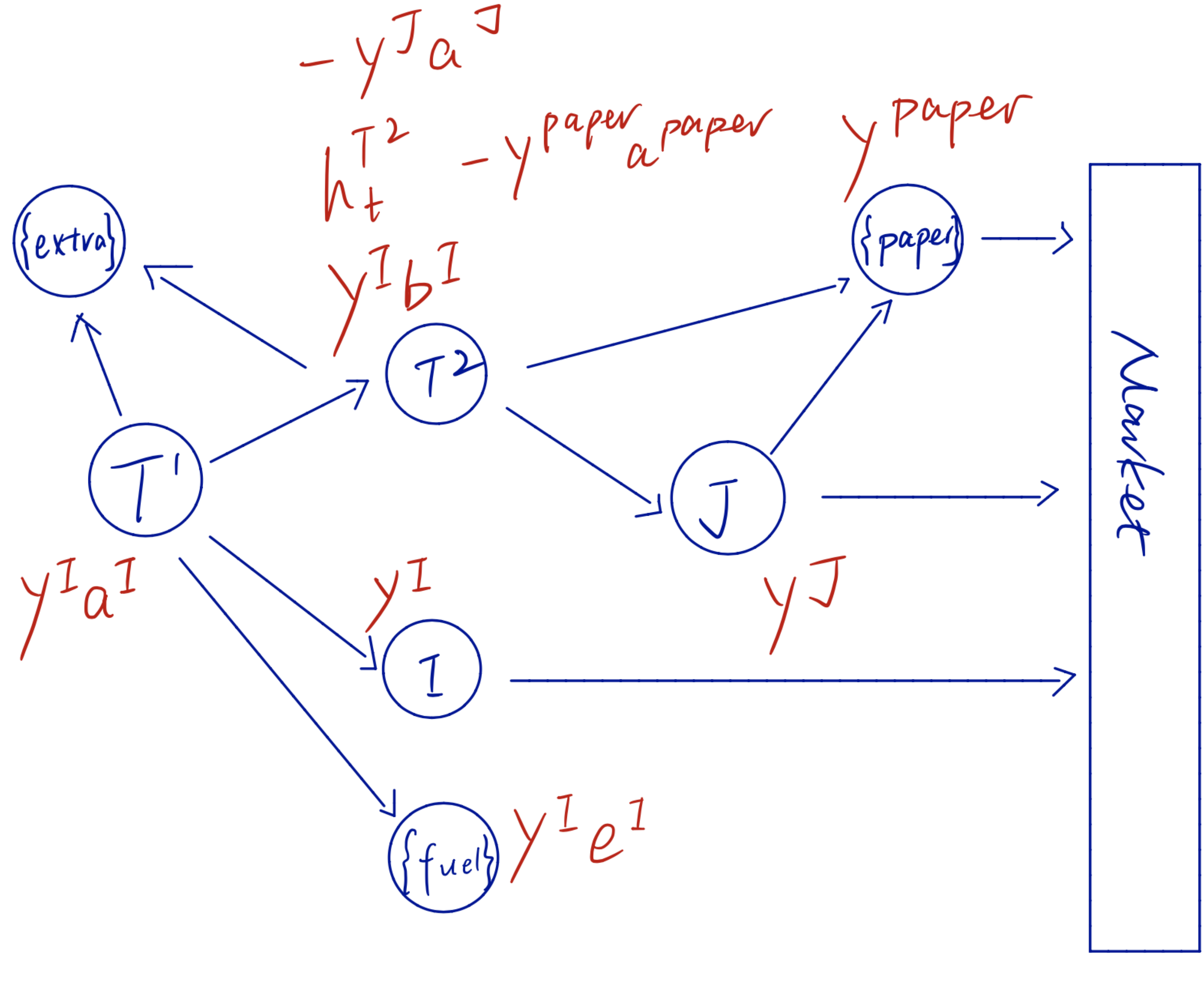
Metsa-Oy Forest and Supply Chain 1: Static

1, Introduction

The supply chain is illustrated using the following figure:



2, Definition of Mathematical Expressions

Symbol	Definition	Expression
T^1	type of timber 1	$\{\mathrm{MAT}, \mathrm{KUT}, \mathrm{KOT}\}$
T^2	type of timber 2	$\{\mathrm{MAK}, \mathrm{KUK}, \mathrm{KOK}\}$
T	type of timber	$T^1 \cup T^2$
$I^{T1}_{t=\mathrm{MAT}}$	outputs of wood production corresponding to input MAT	$\{\mathrm{MAS}\}$
$I^{T1}_{t=\mathrm{KUT}}$	outputs of wood production corresponding to input KUT	$\{\mathrm{KUS}, \mathrm{KUV}\}$
$I^{T1}_{t=\mathrm{KOT}}$	outputs of wood production corresponding to input KOT	$\{\mathrm{KOS}, \mathrm{KOV}\}$
$I^{T2}_{t=\mathrm{MAK}}$	outputs of wood production corresponding to output MAK	$\{\mathrm{MAS}\}$
$I^{T2}_{t=\mathrm{KUK}}$	outputs of wood production corresponding to output KUK	$\{\mathrm{KUS}, \mathrm{KUV}\}$
$I^{T2}_{t=\mathrm{KOK}}$	outputs of wood production corresponding to output KOK	$\{\mathrm{KOS}, \mathrm{KOV}\}$
I^{saw}	outputs of wood production in saw mill	$\{\mathrm{MAS}, \mathrm{KUS}, \mathrm{KOS}\}$
I^{plywood}	outputs of wood production in plywood mill	$\{\mathrm{KUV}, \mathrm{KOV}\}$
I	outputs of wood production	$I^{\mathrm{saw}} \cup I^{\mathrm{plywood}}$
J	outputs of pulp production	$\{\mathrm{HSEL}, \mathrm{LSEL}\}$
$J^{T2}_{t=\mathrm{MAK}}$	outputs of pulp production corresponding to input MAK	$\{\mathrm{HSEL}\}$
$J^{T2}_{t=\mathrm{KUK}}$	outputs of pulp production corresponding to input KUK	$\varnothing$
$J^{T2}_{t=\mathrm{KOK}}$	outputs of pulp production corresponding to input KOK	$\{\mathrm{LSEL}\}$
K	regions to sell products	$\{\mathrm{EU}, \mathrm{IE}, \mathrm{PA}, \mathrm{KI}\}$

Table 1, summary of sets

Symbol	Definition	Type	Unit	Set
h_t	purchasing amount of timber t	integer	$1000m^3$	T
y^I_i	production amount of wood i	linear	$1000m^3$	I
y^J_j	production amount of pulp j	linear	$1000m^3$	J
y^{paper}	production amount of paper	linear	$1000m^3$	–
$z^I_{i,k}$	selling amount of wood i in region k	linear	$1000m^3$	I, K
$z^J_{j,k}$	selling amount of pulp j in region k	linear	$1000m^3$	J, K
z^{paper}_k	selling amount of paper in region k	linear	$1000m^3$	K

Table 2, summary of decision variables

Symbol	Definition	Unit	Set
α_t	fixed cost factor of purchasing wood t	euro/$(1000m^3)$	T
β_t	unit cost factor of purchasing wood t	euro/$(1000m^6)$	T
a_i^I	relation of timber input and output in wood production i	–	I
b_i^I	relation of timber output and output in wood production i	–	I
e_i^I	relation of fuel output and output in wood production i	–	I
c_i^I	cost of wood production i	euro/$(1000m^3)$	I
r^{saw}	capacity of saw mill in wood production i	$1000m^3$/year	–
r^{plywood}	capacity of plywood mill in wood production i	$1000m^3$/year	–
a_j^J	input and output relation of pulp production j	–	J
c_j^J	cost of pulp production j	euro/(1000ton)	J
r_j^J	capacity of pulp production j	1000ton/year	J
a_t^{paper}	relation of timber inputs in paper production	–	T
b_j^{paper}	relation of pulp inputs in paper production	–	J
c^{paper}	cost of paper production	euro/(1000ton)	–
r^{paper}	capacity of paper production	1000ton/year	–
$\gamma_{i,k}^I$	fixed price factor of wood i in region k	euro/$(1000m^3)$	I, K
$\delta_{i,k}^I$	unit price factor of wood i in region k	euro/(10^6m^6)	I, K
$\gamma_{j,k}^J$	fixed price factor of pulp j in region k	euro/(1000ton)	J, K
$\delta_{j,k}^J$	unit price factor of pulp j in region k	euro/(10^6ton^2)	J, K
γ_k^{paper}	fixed price factor of paper in region k	euro/(1000ton)	K
δ_k^{paper}	unit price factor of paper in region k	euro/(10^6ton^2)	K
p^{fuel}	price of fuel wood	euro/$(1000m^3)$	–

Table 3, summary of constants

3, Objective Function

The Objective function composes of seven parts:

$$f^{\text{timber}} + f^{\text{wood}} + f^{\text{pp}} + g^{\text{timber}} + g^{\text{fuel}} + g^{\text{wood}} + g^{\text{pulp}} + g^{\text{paper}}$$

1. cost of timber procurement: $f^{\text{timber}} = -\sum_{t\in T} h_t(\alpha_t + \beta_t h_t)$
2. cost of wood production: $f^{\text{wood}} = -\sum_{i\in I} c_i^I y_i^I$
3. cost of pulp and paper production: $f^{\text{pp}} = -\sum_{j\in J} c_j^J y_j^J - c^{\text{paper}} y^{\text{paper}}$
4. profit of left timbers selling:

$$g^{\text{timber}} = \sum_{t\in T^1} \alpha_t \left(h_t - \sum_{i\in I_t^{T1}} a_i^I y_i^I \right) + \sum_{t\in T^2} \alpha_t \left(h_t + \sum_{i\in I_t^{T2}} b_i^I y_i^I - \sum_{j\in J_t^{T2}} a_j^J y_j^J - a_t^{\text{paper}} y^{\text{paper}} \right)$$

5. profit of fuel wood selling: $g^{\text{fuel}} = \sum_{i\in I} p^{\text{fuel}} e_i^I y_i^I$

6. profit of wood selling: $g^{\mathrm{wood}} = \sum_{i \in I} \sum_{k \in K} z_{i,k}^I (\gamma_{i,k}^I - \delta_{i,k}^I z_{i,k}^I)$
7. profit of pulp selling: $g^{\mathrm{pulp}} = \sum_{j \in J} \sum_{k \in K} z_{j,k}^J (\gamma_{j,k}^J - \delta_{j,k}^J z_{j,k}^J)$
8. profit of paper selling: $g^{\mathrm{paper}} = \sum_{k \in K} z_k^{\mathrm{paper}} (\gamma_k^{\mathrm{paper}} - \delta_k^{\mathrm{paper}} z_k^{\mathrm{paper}})$

4, Constraints

Besides the constraints that all variables are non-negative, there are ten sets of constraints:

1. limit of timber amount in wood production:

$$h_t \geq \sum_{i \in I_t^{T1}} a_i^I y_i^I \quad \forall t \in T^1$$

2. limit of timber amount in pulp and paper production:

$$\left(h_t + \sum_{i \in I_t^{T2}} b_i^I y_i^I \right) \geq \sum_{j \in J_t^{T2}} a_j^J y_j^J + a_t^{\mathrm{paper}} y^{\mathrm{paper}} \quad \forall t \in T^2$$

3. limit of pulp amount in paper production:

$$y_j^J \geq b_j^{\mathrm{paper}} y^{\mathrm{paper}} \quad \forall j \in J$$

4. limit of wood amount in selling:

$$y_i^I \geq \sum_{k \in K} z_{i,k}^I \quad \forall i \in I$$

5. limit of pulp amount in selling:

$$y_j^J - b_j^{\mathrm{paper}} y^{\mathrm{paper}} \geq \sum_{k \in K} z_{j,k}^J \quad \forall j \in J$$

6. limit of paper amount in selling:

$$y^{\mathrm{paper}} \geq \sum_{k \in K} z_k^{\mathrm{paper}}$$

7. limit of capacity in saw mill:

$$\sum_{i \in I^{\mathrm{saw}}} y_i^I \leq r^{\mathrm{saw}}$$

8. limit of capacity in plywood mill:

$$\sum_{i \in I^{\mathrm{plywood}}} y_i^I \leq r^{\mathrm{plywood}}$$

9. limit of capacity in pulp production:

$$y_j^J \leq r_j^J \quad \forall j \in J$$

10. limit of capacity in paper production:

$$y^{\text{paper}} \leq r^{\text{paper}}$$

5, Result

obj = $\underline{\underline{0.3940e6}}$

Produced quantity of final products {MAS, KUS, KOS, KUV, KOV, HSEL, LSEL, PAPER} = [0, 0, 0, 0, 0, 16, 16, 80]. The units of first five quantities are $1000m^3/\text{year}$, and the units of last three are $1000\text{ton}/\text{year}$.

```
result_h_t = [0, -0, -0, 77, 80, 68]
result_y_i = [0, 0, 0, 0, 0]
result_y_j = [16, 16]
result_y_paper = 80
result_z_ik = [
        0.0 130.0 58.33 50.0;
        0.0 60.0 54.17 46.67;
        0.0 35.0 31.25 32.0;
        0.0 190.0 150.0 97.22;
        537.5 292.86 162.5 126.67
        ]
result_z_jk = [
        0.0 312.5 230.0 216.67;
        0.0 700.0 191.67 178.57142857142858
        ]
result_z_paper_k = [24.85, 27.39, 6.16, 21.60]
```

Metsa-Oy Production and Supply Chain 2: Dynamic

1, Introduction

It is assumed that all the products produced are sold in the current year. For example, wood products produced in the first year are sold in the first year only.

2, Definition of Mathematical Expressions

Symbol	Definition	Expression
T^1	type of timber 1	$\{\mathrm{MAT}, \mathrm{KUT}, \mathrm{KOT}\}$
T^2	type of timber 2	$\{\mathrm{MAK}, \mathrm{KUK}, \mathrm{KOK}\}$
T	type of timber	$T^1 \cup T^2$
$I^{T1}_{t=\mathrm{MAT}}$	outputs of wood production corresponding to input MAT	$\{\mathrm{MAS}\}$
$I^{T1}_{t=\mathrm{KUT}}$	outputs of wood production corresponding to input KUT	$\{\mathrm{KUS}, \mathrm{KUV}\}$
$I^{T1}_{t=\mathrm{KOT}}$	outputs of wood production corresponding to input KOT	$\{\mathrm{KOS}, \mathrm{KOV}\}$
$I^{T2}_{t=\mathrm{MAK}}$	outputs of wood production corresponding to output MAK	$\{\mathrm{MAS}\}$
$I^{T2}_{t=\mathrm{KUK}}$	outputs of wood production corresponding to output KUK	$\{\mathrm{KUS}, \mathrm{KUV}\}$
$I^{T2}_{t=\mathrm{KOK}}$	outputs of wood production corresponding to output KOK	$\{\mathrm{KOS}, \mathrm{KOV}\}$
I^{saw}	outputs of wood production in saw mill	$\{\mathrm{MAS}, \mathrm{KUS}, \mathrm{KOS}\}$
I^{plywood}	outputs of wood production in plywood mill	$\{\mathrm{KUV}, \mathrm{KOV}\}$
I	outputs of wood production	$I^{\mathrm{saw}} \cup I^{\mathrm{plywood}}$
J	outputs of pulp production	$\{\mathrm{HSEL}, \mathrm{LSEL}\}$
$J^{T2}_{t=\mathrm{MAK}}$	outputs of pulp production corresponding to input MAK	$\{\mathrm{HSEL}\}$
$J^{T2}_{t=\mathrm{KUK}}$	outputs of pulp production corresponding to input KUK	$\varnothing$
$J^{T2}_{t=\mathrm{KOK}}$	outputs of pulp production corresponding to input KOK	$\{\mathrm{LSEL}\}$
K	regions to sell products	$\{\mathrm{EU}, \mathrm{IE}, \mathrm{PA}, \mathrm{KI}\}$
M	three years in the planning horizon	$\{1, 2, 3\}$

Table 1, summary of sets

Symbol	Definition	Type	Unit	Set
$h_{m,t}$	purchasing amount of timber t in the year m	integer	$1000m^3$	M, T
$y^I_{m,i}$	production amount of wood i in the year m	linear	$1000m^3$	M, I
$y^J_{m,j}$	production amount of pulp j in the year m	linear	$1000m^3$	M, J
y^{paper}_m	production amount of paper in the year m	linear	$1000m^3$	M
$z^I_{m,i,k}$	selling amount of wood in the region k in the year m	linear	$1000m^3$	M, I, K
$z^J_{m,j,k}$	selling amount of pulp in the region k in the year m	linear	$1000m^3$	M, J, K
$z^{\mathrm{paper}}_{m,k}$	selling amount of paper in the region k in the year m	linear	$1000m^3$	M, K
x^{saw}_m	capacity of saw mill	linear	$1000m^3/\mathrm{year}$	M^2
x^{plywood}_m	capacity of plywood mill	linear	$1000m^3/\mathrm{year}$	M^2
$x^J_{m,j}$	capacity of pulp production line	linear	$1000\mathrm{ton}/\mathrm{year}$	M^2, J
x^{paper}_m	capacity of paper production line	linear	$1000\mathrm{ton}/\mathrm{year}$	M^2

Table 2, summary of decision variables

Symbol	Definition	Unit	Set
p^{fuel}	price of fuel wood	euro/(1000m^3)	–
α_t	fixed cost factor of purchasing timber t	euro/(1000m^3)	T
β_t	unit cost factor of purchasing timber t	euro/(1000m^6)	T
a_i^I	relation of timber input and output in wood production i	–	I
b_i^I	relation of timber output and output in wood production i i	–	I
e_i^I	relation of fuel output and output in wood production i	–	I
c_i^I	cost of wood production i	euro/(1000m^3)	I
r^{saw}	original capacity of saw mill	1000m^3/year	–
r^{plywood}	original capacity of plywood mill	1000m^3/year	–
a_j^J	input and output relation of pulp production j	–	J
c_j^J	cost of pulp production j	euro/(1000ton)	J
r_j^J	original capacity of pulp production j	1000ton/year	J
a_t^{paper}	relation of timber inputs in paper production	–	T
b_j^{paper}	relation of pulp inputs in paper production	–	J
c^{paper}	cost of paper production	euro/(1000ton)	–
r^{paper}	original capacity of paper production	1000ton/year	–
$\gamma_{m,i,k}^I$	fixed price factor of wood i in the region k in the year m	euro/(1000m^3)	M, I, K
$\delta_{m,i,k}^I$	unit price factor of wood i in the region k in the year m	euro/($10^6 m^6$)	M, I, K
$\gamma_{m,j,k}^J$	fixed price factor of pulp j in the region k in the year m	euro/(1000ton)	M, J, K
$\delta_{m,j,k}^J$	unit price factor of pulp j in the region k in the year m	euro/(10^6ton^2)	M, J, K
$\gamma_{m,k}^{\text{paper}}$	fixed price factor of paper in the region k in the year m	euro/(1000ton)	M, K
$\delta_{m,k}^{\text{paper}}$	unit price factor of paper in the region k in the year m	euro/(10^6ton^2)	M, K
σ	annual discounting factor	–	–
ω_i^I	demand growth coefficient of wood i	–	I
ω_j^J	demand growth coefficient of pulp j	–	J
ω^{paper}	demand growth coefficient of paper	–	–
o^{saw}	capacity expansion cost of saw mill	euro/(1000m^3/year)	–
o^{plywood}	capacity expansion cost of plywood mill	euro/(1000m^3/year)	–
o_j^J	capacity expansion cost of pulp production j	euro/(1000m^3/year)	–
o^{paper}	capacity expansion cost of paper production	euro/(1000m^3/year)	–
ν^{saw}	max capacity expansion factor of saw mill	–	–
ν^{plywood}	max capacity expansion factor of saw mill	–	–
ν_j^J	max capacity expansion factor of pulp production j	–	J
ν^{paper}	max capacity expansion factor of paper production	–	–

Table 3, summary of constants

3, Objective Function

The Objective function composes of seven parts:

$$\sum_{m \in M} \sigma^{m-1} \left[f_m^{\text{timber}} + f_m^{\text{wood}} + f_m^{\text{pp}} + g_m^{\text{timber}} + g_m^{\text{fuel}} + g_m^{\text{wood}} + g_m^{\text{pulp}} + g_m^{\text{paper}} \right] + f^{\text{cap}}$$

1. cost of timber procurement: $f_m^{\text{timber}} = -\sum_{t \in T} h_{m,t}(\alpha_t + \beta_t h_{m,t})$
2. cost of wood production: $f_m^{\text{wood}} = -\sum_{i \in I} c_i^I y_{m,i}^I$
3. cost of pulp and paper production: $f_m^{\text{pp}} = -\sum_{j \in J} c_j^J y_{m,j}^J - \sum_{m \in M} c^{\text{paper}} y_m^{\text{paper}}$

4. profit of left timbers selling:

$$g_m^{\text{timber}} = \sum_{t \in T^1} \alpha_t \left(h_{m,t} - \sum_{i \in I_t^{T1}} a_i^I y_{m,i}^I \right) + \sum_{t \in T^2} \alpha_t \left(h_{m,t} + \sum_{i \in I_t^{T2}} b_i^I y_{m,i}^I - \sum_{j \in J_t^{T2}} a_j^J y_{m,j}^J - a_t^{\text{paper}} y_m^{\text{paper}} \right)$$

5. profit of fuel wood selling: $g_m^{\text{fuel}} = \sum_{i \in I} p^{\text{fuel}} e_i^I y_{m,i}^I$
6. profit of wood selling: $g_m^{\text{wood}} = \sum_{i \in I} \sum_{k \in K} (\omega_i^I)^{m-1} z_{m,i,k}^I (\gamma_{i,k}^I - \delta_{i,k}^I z_{m,i,k}^I)$
7. profit of pulp selling: $g_m^{\text{pulp}} = \sum_{j \in J} \sum_{k \in K} (\omega_j^J)^{m-1} z_{m,j,k}^J (\gamma_{j,k}^J - \delta_{j,k}^J z_{m,j,k}^J)$
8. profit of paper selling: $g_m^{\text{paper}} = \sum_{k \in K} (\omega^{\text{paper}})^{m-1} z_{m,k}^{\text{paper}} (\gamma_k^{\text{paper}} - \delta_k^{\text{paper}} z_{m,k}^{\text{paper}})$
9. cost of capacity expansion:

$$f^{\text{cap}} = \sum_{m=2}^{3} \sigma^{m-2} \left[o^{\text{saw}} (x_m^{\text{saw}} - x_{m-1}^{\text{saw}}) + o^{\text{plywood}} (x_m^{\text{plywood}} - x_{m-1}^{\text{plywood}}) + \sum_{j \in J} o_j^J (x_{m,j}^J - x_{m-1,j}^J) + o^{\text{paper}} (x_m^{\text{paper}} - x_{m-1}^{\text{paper}}) \right]$$

4, Constraints

Besides the constraints that all variables are non-negative, there are ten sets of constraints:

1. limit of timber amount in wood production:

$$h_{m,t} \geq \sum_{i \in I_t^{T1}} a_i^I y_{m,i}^I \quad \forall t \in T^1, m \in M$$

2. limit of timber amount in pulp and paper production:

$$\left(h_{m,t} + \sum_{i \in I_t^{T2}} b_i^I y_{m,i}^I \right) \geq \sum_{j \in J_t^{T2}} a_j^J y_{m,j}^J + a_t^{\text{paper}} y_m^{\text{paper}} \quad \forall t \in T^2, m \in M$$

3. limit of pulp amount in paper production:

$$y_{m,j}^J \geq b_j^{\text{paper}} y_m^{\text{paper}} \quad \forall j \in J, m \in M$$

4. limit of wood amount in selling:

$$y_{m,i}^I \geq \sum_{k \in K} z_{m,i,k}^I \quad \forall i \in I, m \in M$$

5. limit of pulp amount in selling:

$$y_{m,j}^J - b_j^{\text{paper}} y_m^{\text{paper}} \geq \sum_{k \in K} z_{m,j,k}^J \quad \forall j \in J, m \in M$$

6. limit of paper amount in selling:

$$y_m^{\text{paper}} \geq \sum_{k \in K} z_{m,k}^{\text{paper}} \quad \forall m \in M$$

7. limit of capacity in saw mill:

$$\sum_{i \in I^{\text{saw}}} y_{m,i}^I \leq x_m^{\text{saw}} \quad \forall m \in M$$

8. limit of capacity in plywood mill:

$$\sum_{i \in I^{\text{plywood}}} y_{m,i}^I \leq x_m^{\text{plywood}} \quad \forall m \in M$$

9. limit of capacity in pulp production:

$$y_{m,j}^J \leq x_{m,j}^J \quad \forall j \in J, m \in M$$

10. limit of capacity in paper production:

$$y_m^{\text{paper}} \leq x_m^{\text{paper}} \quad \forall m \in M$$

11. relation between capacities

$$\begin{aligned} &x_1 = r \quad \forall x, r \\ &x_2 \geq x_1 \quad \forall x \\ &x_3 \geq x_2 \quad \forall x \\ &x_m \leq \nu r \quad \forall x, m \in M \end{aligned}$$

4, Result

obj = $\underline{\underline{1.3751e6}}$;

profit in the first year = $\underline{\underline{324658.3857}}$;
discounted profit in the second year = $\underline{\underline{516294.0822}}$;
discounted profit in the third year = $\underline{\underline{534189.434}}$;

The following printed results are the market shares of different products in EU, IE, PA and KI markets in different years and the corresponding discounted profits:

```
m = 1 ;
MAS: [66.65, 10.15, 10.23, 12.98]
KUS: [83.55, 0.0, 5.31, 11.14]
KOS: [0.0, 9.47, 33.6, 56.93]
KUV: [100.0, 0.0, 0.0, 0.0]
KOV: [100.0, 0.0, 0.0, 0.0]
HSEL: [11.15, 43.77, 2.23, 42.84]
LSEL: [14.48, 79.32, 0.0, 6.2]
PAP: [31.06, 34.24, 7.7, 26.99]

m = 2 ;
MAS: [65.95, 10.8, 10.33, 12.92]
KUS: [88.24, 0.0, 0.0, 11.76]
KOS: [0.0, 0.0, 0.0, 100.0]
KUV: [93.09, 6.91, 0.0, 0.0]
KOV: [89.28, 10.72, 0.0, 0.0]
HSEL: [3.32, 48.15, 0.66, 47.87]
LSEL: [14.15, 79.78, 0.0, 6.07]
PAP: [33.71, 29.85, 14.46, 21.98]

m = 3 ;
MAS: [65.29, 11.41, 10.42, 12.87]
KUS: [88.24, 0.0, 0.0, 11.76]
KOS: [0.0, 0.0, 0.0, 100.0]
KUV: [93.31, 6.69, 0.0, 0.0]
KOV: [88.59, 11.41, 0.0, 0.0]
HSEL: [9.06, 44.94, 1.81, 44.18]
LSEL: [15.36, 78.05, 0.0, 6.58]
PAP: [33.71, 29.85, 14.46, 21.98]
```

The following printed results are the capacities of different factories in different years. There is no upgrade of capacities in the second year.

```
m = 1 ;
x_saw_m[m] = 200.0 ;
x_plywood_m[m] = 90.0 ;
x_mj[m, :] = [220.0, 180.0] ;
x_paper_m[m] = 80.0 ;

m = 2 ;
x_saw_m[m] = 200.0 ;
x_plywood_m[m] = 134.38 ;
x_mj[m, :] = [220.0, 180.0] ;
x_paper_m[m] = 160.0 ;

m = 3 ;
x_saw_m[m] = 200.0 ;
x_plywood_m[m] = 134.38 ;
x_mj[m, :] = [220.0, 180.0] ;
x_paper_m[m] = 160.0 ;
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Metsa-Oy Production and Supply Chain 3: Stochastic Dynamic

1, Introduction

It is assumed that all the products produced are sold in the current year. For example, wood products produced in the first year are sold in the first year only.

2, Definition of Mathematical Expressions

Symbol	Definition	Expression
T^1	type of timber 1	$\{\mathrm{MAT}, \mathrm{KUT}, \mathrm{KOT}\}$
T^2	type of timber 2	$\{\mathrm{MAK}, \mathrm{KUK}, \mathrm{KOK}\}$
T	type of timber	$T^1 \cup T^2$
$I^{T1}_{t=\mathrm{MAT}}$	outputs of wood production corresponding to input MAT	$\{\mathrm{MAS}\}$
$I^{T1}_{t=\mathrm{KUT}}$	outputs of wood production corresponding to input KUT	$\{\mathrm{KUS}, \mathrm{KUV}\}$
$I^{T1}_{t=\mathrm{KOT}}$	outputs of wood production corresponding to input KOT	$\{\mathrm{KOS}, \mathrm{KOV}\}$
$I^{T2}_{t=\mathrm{MAK}}$	outputs of wood production corresponding to output MAK	$\{\mathrm{MAS}\}$
$I^{T2}_{t=\mathrm{KUK}}$	outputs of wood production corresponding to output KUK	$\{\mathrm{KUS}, \mathrm{KUV}\}$
$I^{T2}_{t=\mathrm{KOK}}$	outputs of wood production corresponding to output KOK	$\{\mathrm{KOS}, \mathrm{KOV}\}$
I^{saw}	outputs of wood production in saw mill	$\{\mathrm{MAS}, \mathrm{KUS}, \mathrm{KOS}\}$
I^{plywood}	outputs of wood production in plywood mill	$\{\mathrm{KUV}, \mathrm{KOV}\}$
I	outputs of wood production	$I^{\mathrm{saw}} \cup I^{\mathrm{plywood}}$
J	outputs of pulp production	$\{\mathrm{HSEL}, \mathrm{LSEL}\}$
$J^{T2}_{t=\mathrm{MAK}}$	outputs of pulp production corresponding to input MAK	$\{\mathrm{HSEL}\}$
$J^{T2}_{t=\mathrm{KUK}}$	outputs of pulp production corresponding to input KUK	$\varnothing$
$J^{T2}_{t=\mathrm{KOK}}$	outputs of pulp production corresponding to input KOK	$\{\mathrm{LSEL}\}$
K	regions to sell products	$\{\mathrm{EU}, \mathrm{IE}, \mathrm{PA}, \mathrm{KI}\}$
M	three years in the planning horizon	$\{1, 2, 3\}$
N	four scenarios	$\{1, 2, 3, 4\}$

Table 1, summary of sets

Symbol	Definition	Type	Unit	Set
h_t	purchasing amount of timber t in the first year	integer	$1000m^3$	T
y_i^I	production amount of wood i in the first year	linear	$1000m^3$	I
y_j^J	production amount of pulp j in the first year	linear	$1000m^3$	J
y^{paper}	production amount of paper in the first year	linear	$1000m^3$	–
$z_{i,k}^I$	selling amount of wood i in region k in the first year	linear	$1000m^3$	I, K
$z_{j,k}^J$	selling amount of pulp j in region k in the first year	linear	$1000m^3$	J, K
z_k^{paper}	selling amount of paper in region k in the first year	linear	$1000m^3$	K
$h_{m,t}$	purchasing amount of timber t in the year m	integer	$1000m^3$	$M/\{1\}, T$
$y_{m,i}^I$	production amount of wood i in the year m	linear	$1000m^3$	$M/\{1\}, I$
$y_{m,j}^J$	production amount of pulp j in the year m	linear	$1000m^3$	$M/\{1\}, J$
y_m^{paper}	production amount of paper in the year m	linear	$1000m^3$	$M/\{1\}$
$z_{m,i,k}^I$	selling amount of wood in the region k in the year m	linear	$1000m^3$	$M/\{1\}, I, K$
$z_{m,j,k}^J$	selling amount of pulp in the region k in the year m	linear	$1000m^3$	$M/\{1\}, J, K$
$z_{m,k}^{\text{paper}}$	selling amount of paper in the region k in the year m	linear	$1000m^3$	$M/\{1\}, K$
x_m^{saw}	capacity of saw mill	linear	$1000m^3$/year	$M/\{3\}$
x_m^{plywood}	capacity of plywood mill	linear	$1000m^3$/year	$M/\{3\}$
$x_{m,j}^J$	capacity of pulp production line	linear	1000ton/year	$M/\{3\}, J$
x_m^{paper}	capacity of paper production line	linear	1000ton/year	$M/\{3\}$
$x_{n,3}^{\text{saw}}$	capacity of saw mill in the year 3 decide in the year 2	linear	$1000m^3$/year	N
$x_{n,3}^{\text{plywood}}$	capacity of plywood mill in the year 3 decide in the year 2	linear	$1000m^3$/year	N
$x_{n,3,j}^J$	capacity of pulp production line in the year 3 decide in the year 2	linear	1000ton/year	N, J
$x_{n,3}^{\text{paper}}$	capacity of paper production line in the year 3 decide in the year 2	linear	1000ton/year	N

Table 2, summary of decision variables

Symbol	Definition	Unit	Set
p^{fuel}	price of fuel wood	euro/(1000m^3)	–
α_t	fixed cost factor of purchasing timber t	euro/(1000m^3)	T
β_t	unit cost factor of purchasing timber t	euro/(1000m^6)	T
a_i^I	relation of timber input and output in wood production i	–	I
b_i^I	relation of timber output and output in wood production i i	–	I
e_i^I	relation of fuel output and output in wood production i	–	I
c_i^I	cost of wood production i	euro/(1000m^3)	I
r^{saw}	original capacity of saw mill	1000m^3/year	–
r^{plywood}	original capacity of plywood mill	1000m^3/year	–
a_j^J	input and output relation of pulp production j	–	J
c_j^J	cost of pulp production j	euro/(1000ton)	J
r_j^J	original capacity of pulp production j	1000ton/year	J
a_t^{paper}	relation of timber inputs in paper production	–	T
b_j^{paper}	relation of pulp inputs in paper production	–	J
c^{paper}	cost of paper production	euro/(1000ton)	–
r^{paper}	original capacity of paper production	1000ton/year	–
$\gamma_{m,i,k}^I$	fixed price factor of wood i in the region k in the year m	euro/(1000m^3)	M, I, K
$\delta_{m,i,k}^I$	unit price factor of wood i in the region k in the year m	euro/($10^6 m^6$)	M, I, K
$\gamma_{m,j,k}^J$	fixed price factor of pulp j in the region k in the year m	euro/(1000ton)	M, J, K
$\delta_{m,j,k}^J$	unit price factor of pulp j in the region k in the year m	euro/(10^6ton^2)	M, J, K
$\gamma_{m,k}^{\text{paper}}$	fixed price factor of paper in the region k in the year m	euro/(1000ton)	M, K
$\delta_{m,k}^{\text{paper}}$	unit price factor of paper in the region k in the year m	euro/(10^6ton^2)	M, K
σ	annual discounting factor	–	–
ω_i^I	demand growth coefficient of wood i	–	I
ω_j^J	demand growth coefficient of pulp j	–	J
ω^{paper}	demand growth coefficient of paper	–	–
o^{saw}	capacity expansion cost of saw mill	euro/(1000m^3/year)	–
o^{plywood}	capacity expansion cost of plywood mill	euro/(1000m^3/year)	–
o_j^J	capacity expansion cost of pulp production j	euro/(1000m^3/year)	–
o^{paper}	capacity expansion cost of paper production	euro/(1000m^3/year)	–
ν^{saw}	max capacity expansion factor of saw mill	–	–
ν^{plywood}	max capacity expansion factor of saw mill	–	–
ν_j^J	max capacity expansion factor of pulp production j	–	J
ν^{paper}	max capacity expansion factor of paper production	–	–
$\rho_{n,m}$	price coefficient of the scenario n in the year m	–	N, M
π_n	probability of scenario n	–	N

Table 3, summary of constants

3, Objective Function

The Objective function composes of many parts:

$$f_1^{\text{timber}} + f_1^{\text{wood}} + f_1^{\text{pp}} + g_1^{\text{timber}} + g_1^{\text{fuel}} + g_1^{\text{wood}} + g_1^{\text{pulp}} + g_1^{\text{paper}} + f_2^{\text{cap}} +$$
$$\sum_{n \in N} \pi_n \left\{ \sum_{m=2}^{3} \sigma^{m-1} \left[f_{n,m}^{\text{timber}} + f_{n,m}^{\text{wood}} + f_{n,m}^{\text{pp}} + g_{n,m}^{\text{timber}} + g_{n,m}^{\text{fuel}} + g_{n,m}^{\text{wood}} + g_{n,m}^{\text{pulp}} + g_{n,m}^{\text{paper}} \right] + \sigma f_{n,3}^{\text{cap}} \right\}$$

- When m = 1, the variables are here-and-now decisions variables, and this part of the objective function is the same as those in the static model:

1. cost of timber procurement: $f_1^{\text{timber}} = -\sum_{t \in T} h_t(\alpha_t + \beta_t h_t)$
2. cost of wood production: $f_1^{\text{wood}} = -\sum_{i \in I} c_i^I y_i^I$
3. cost of pulp and paper production: $f_1^{\text{pp}} = -\sum_{j \in J} c_j^J y_j^J - c^{\text{paper}} y^{\text{paper}}$
4. profit of left timbers selling:

$$g_1^{\text{timber}} = \sum_{t \in T^1} \alpha_t \left(h_t - \sum_{i \in I_t^{T1}} a_i^I y_i^I \right) + \sum_{t \in T^2} \alpha_t \left(h_t + \sum_{i \in I_t^{T2}} b_i^I y_i^I - \sum_{j \in J_t^{T2}} a_j^J y_j^J - a_t^{\text{paper}} y^{\text{paper}} \right)$$

5. profit of fuel wood selling: $g_1^{\text{fuel}} = \sum_{i \in I} p^{\text{fuel}} e_i^I y_i^I$
6. profit of wood selling: $g_1^{\text{wood}} = \sum_{i \in I} \sum_{k \in K} z_{i,k}^I (\gamma_{i,k}^I - \delta_{i,k}^I z_{i,k}^I)$
7. profit of pulp selling: $g_1^{\text{pulp}} = \sum_{j \in J} \sum_{k \in K} z_{j,k}^J (\gamma_{j,k}^J - \delta_{j,k}^J z_{j,k}^J)$
8. profit of paper selling: $g_1^{\text{paper}} = \sum_{k \in K} z_k^{\text{paper}} (\gamma_k^{\text{paper}} - \delta_k^{\text{paper}} z_k^{\text{paper}})$

- When m = 2 or 3, the variables are wait-and-see decisions variables:

1. cost of timber procurement: $f_{n,m}^{\text{timber}} = -\sum_{t \in T} h_{n,m,t}(\alpha_t + \beta_t h_{n,m,t})$
2. cost of wood production: $f_{n,m}^{\text{wood}} = -\sum_{i \in I} c_i^I y_{n,m,i}^I$
3. cost of pulp and paper production: $f_{n,m}^{\text{pp}} = -\sum_{j \in J} c_j^J y_{n,m,j}^J - \sum_{m \in M} c^{\text{paper}} y_{n,m}^{\text{paper}}$
4. profit of left timbers selling:

$$g_{n,m}^{\text{timber}} = \sum_{t \in T^1} \alpha_t \left(h_{n,m,t} - \sum_{i \in I_t^{T1}} a_i^I y_{n,m,i}^I \right) + \sum_{t \in T^2} \alpha_t \left(h_{n,m,t} + \sum_{i \in I_t^{T2}} b_i^I y_{n,m,i}^I - \sum_{j \in J_t^{T2}} a_j^J y_{n,m,j}^J - a_t^{\text{paper}} y_{n,m}^{\text{paper}} \right)$$

5. profit of fuel wood selling: $g_{n,m}^{\text{fuel}} = \sum_{i \in I} p^{\text{fuel}} e_i^I y_{n,m,i}^I$
6. profit of wood selling: $g_{n,m}^{\text{wood}} = \sum_{i \in I} \sum_{k \in K} \rho_{n,m} (\omega_i^I)^{m-1} z_{n,m,i,k}^I (\gamma_{i,k}^I - \delta_{i,k}^I z_{n,m,i,k}^I)$
7. profit of pulp selling: $g_{n,m}^{\text{pulp}} = \sum_{j \in J} \sum_{k \in K} \rho_{n,m} (\omega_j^J)^{m-1} z_{n,m,j,k}^J (\gamma_{j,k}^J - \delta_{j,k}^J z_{n,m,j,k}^J)$
8. profit of paper selling: $g_{n,m}^{\text{paper}} = \sum_{k \in K} \rho_{n,m} (\omega^{\text{paper}})^{m-1} z_{n,m,k}^{\text{paper}} (\gamma_k^{\text{paper}} - \delta_k^{\text{paper}} z_{n,m,k}^{\text{paper}})$

- Cost of capacity expansion:

$$f_2^{\text{cap}} = o^{\text{saw}} (x_2^{\text{saw}} - x_1^{\text{saw}}) + o^{\text{plywood}} (x_2^{\text{plywood}} - x_1^{\text{plywood}}) + \sum_{j \in J} o_j^J (x_{2,j}^J - x_{1,j}^J) + o^{\text{paper}} (x_2^{\text{paper}} - x_1^{\text{paper}})$$
$$f_{n,3}^{\text{cap}} = o^{\text{saw}} (x_{n,3}^{\text{saw}} - x_2^{\text{saw}}) + o^{\text{plywood}} (x_{n,3}^{\text{plywood}} - x_2^{\text{plywood}}) + \sum_{j \in J} o_j^J (x_{n,3,j}^J - x_{2,j}^J) + o^{\text{paper}} (x_{n,3}^{\text{paper}} - x_2^{\text{paper}})$$

where variables regarding capacities in the first year, like x_1^{saw}, are fixed value variables, those in the second year are here-and-now decision variables, and those in the third year are wait-and-see variables. Those here-and-now decision variables are forced to be equal for different scenarios by constraints 7-10 in the following section.

4, Constraints

When $m = 1$, the constraints are the same as those in dynamic model, which are neglected here. When $m = 2 \text{ or } 3$, besides the constraints that all variables are non-negative, there many ten sets of constraints:

1. limit of timber amount in wood production:

$$h_{n,m,t} \geq \sum_{i \in I_t^{T1}} a_i^I y_{n,m,i}^I \quad \forall t \in T^1, m \in \{2,3\}, n \in N$$

2. limit of timber amount in pulp and paper production:

$$\left(h_{n,m,t} + \sum_{i \in I_t^{T2}} b_i^I y_{n,m,i}^I \right) \geq \sum_{j \in J_t^{T2}} a_j^J y_{n,m,j}^J + a_t^{\text{paper}} y_{n,m}^{\text{paper}} \quad \forall t \in T^2, m \in \{2,3\}, n \in N$$

3. limit of pulp amount in paper production:

$$y^J_{n,m,j} \geq b^{\text{paper}}_j y^{\text{paper}}_{n,m} \quad \forall j \in J, m \in \{2,3\}, n \in N$$

4. limit of wood amount in selling:

$$y^I_{n,m,i} \geq \sum_{k \in K} z^I_{n,m,i,k} \quad \forall i \in I, m \in \{2,3\}, n \in N$$

5. limit of pulp amount in selling:

$$y^J_{n,m,j} - b^{\text{paper}}_j y^{\text{paper}}_{n,m} \geq \sum_{k \in K} z^J_{n,m,j,k} \quad \forall j \in J, m \in \{2,3\}, n \in N$$

6. limit of paper amount in selling:

$$y^{\text{paper}}_{n,m} \geq \sum_{k \in K} z^{\text{paper}}_{n,m,k} \quad \forall m \in \{2,3\}, n \in N$$

7. limit of capacity in saw mill:

$$\sum_{i \in I^{\text{saw}}} y^I_{n,m,i} \leq x^{\text{saw}}_{n,m} \quad \forall m \in \{2,3\}, n \in N$$
$$x^{\text{saw}}_{1,2} = x^{\text{saw}}_{n,2} \quad \forall n \in \{2,3,4\}$$

8. limit of capacity in plywood mill:

$$\sum_{i \in I^{\text{plywood}}} y^I_{m,i} \leq x^{\text{plywood}}_{n,m} \quad \forall m \in \{2,3\}, n \in N$$
$$x^{\text{plywood}}_{1,2} = x^{\text{plywood}}_{n,2} \quad \forall n \in \{2,3,4\}$$

9. limit of capacity in pulp production:

$$y^J_{n,m,j} \leq x^J_{n,m,j} \quad \forall j \in J, m \in \{2,3\}, n \in N$$
$$x^J_{1,2,j} = x^J_{n,2,j} \quad \forall n \in \{2,3,4\}, j \in J$$

10. limit of capacity in paper production:

$$y^{\text{paper}}_{n,m} \leq x^{\text{paper}}_{n,m} \quad \forall m \in \{2,3\}, n \in N$$
$$x^{\text{paper}}_{1,2} = x^{\text{paper}}_{n,2} \quad \forall n \in \{2,3,4\}$$

11. relation between capacity expansion factors

$$x_1 = r \quad \forall x, r$$
$$x_2 \geq x_1 \quad \forall x$$
$$x_3 \geq x_2 \quad \forall x$$
$$x_m \leq \nu r \quad \forall x, m \in M$$

4, Results

obj = $\underline{\underline{1.3801e6}}$

profit in the first year = $\underline{\underline{325372.4171}}$;
discounted average profit in the second year = $\underline{\underline{517107.8649}}$;
discounted average profit in the third year = $\underline{\underline{537152.0540}}$;