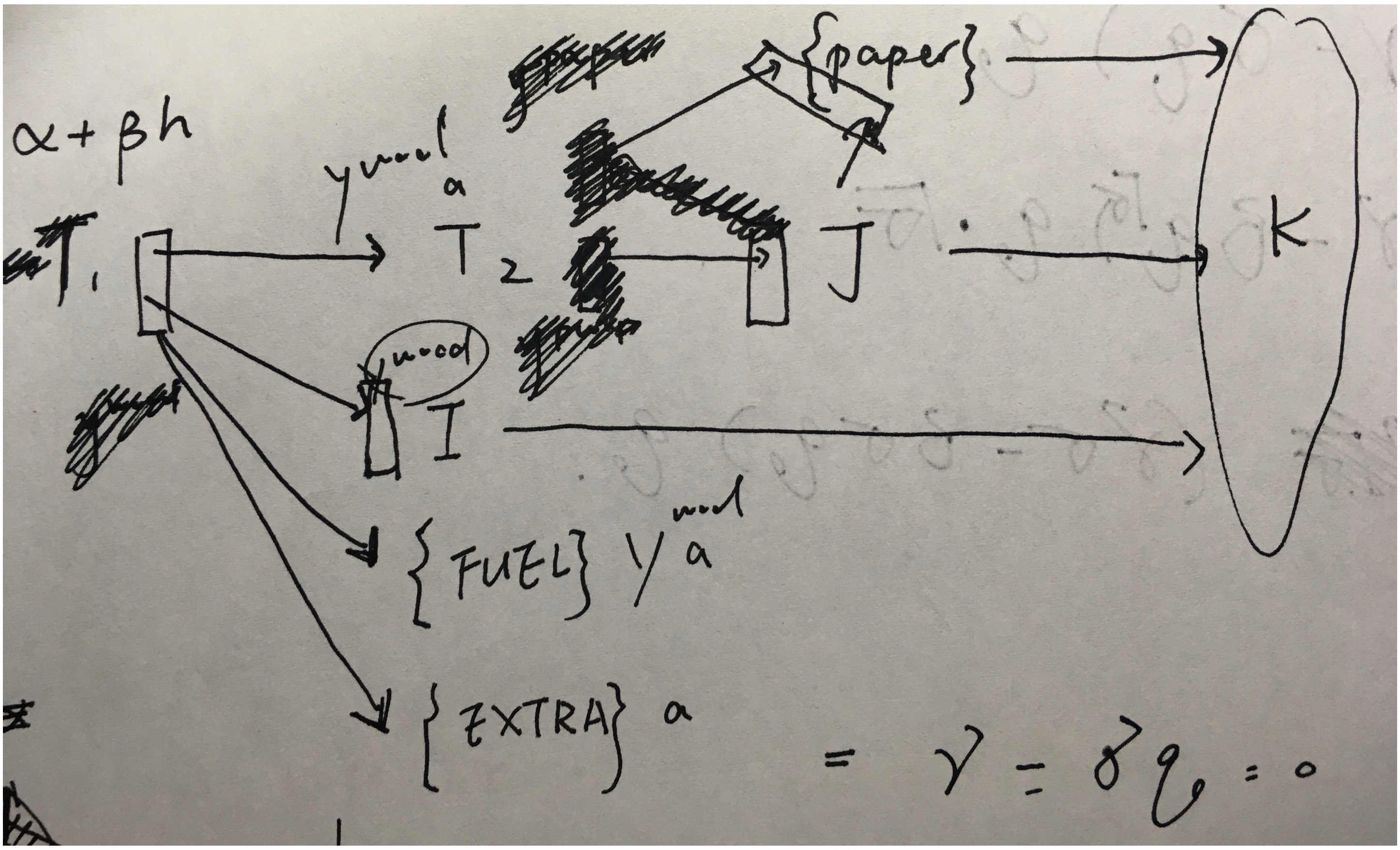
Metsa-Oy Forest and Supply Chain 1: Static

1, Introduction

The supply chain is illustrated using the following figure:



2, Definition of Mathematical Expressions

Symbol	Definition	Expression
T^1	type of timber 1	$\{\mathrm{MAT}, \mathrm{KUT}, \mathrm{KOT}\}$
T^2	type of timber 2	$\{\mathrm{MAK}, \mathrm{KUK}, \mathrm{KOK}\}$
T	type of timber	$T^1 \cup T^2$
$I^{T1}_{t=\mathrm{MAT}}$	outputs of wood production corresponding to input MAT	$\{\mathrm{MAS}\}$
$I^{T1}_{t=\mathrm{KUT}}$	outputs of wood production corresponding to input KUT	$\{\mathrm{KUS}, \mathrm{KUV}\}$
$I^{T1}_{t=\mathrm{KOT}}$	outputs of wood production corresponding to input KOT	$\{\mathrm{KOS}, \mathrm{KOV}\}$
$I^{T2}_{t=\mathrm{MAK}}$	outputs of wood production corresponding to output MAK	$\{\mathrm{MAS}\}$
$I^{T2}_{t=\mathrm{KUK}}$	outputs of wood production corresponding to output KUK	$\{\mathrm{KUS}, \mathrm{KUV}\}$
$I^{T2}_{t=\mathrm{KOK}}$	outputs of wood production corresponding to output KOK	$\{\mathrm{KOS}, \mathrm{KOV}\}$
I^{saw}	outputs of wood production in saw mill	$\{\mathrm{MAS}, \mathrm{KUS}, \mathrm{KOS}\}$
I^{plywood}	outputs of wood production in plywood mill	$\{\mathrm{KUV}, \mathrm{KOV}\}$
I	outputs of wood production	$I^{\mathrm{saw}} \cup I^{\mathrm{plywood}}$
J	outputs of pulp production	$\{\mathrm{HSEL}, \mathrm{LSEL}\}$
$J^{T2}_{t=\mathrm{MAK}}$	outputs of pulp production corresponding to input MAK	$\{\mathrm{HSEL}\}$
$J^{T2}_{t=\mathrm{KUK}}$	outputs of pulp production corresponding to input KUK	$\varnothing$
$J^{T2}_{t=\mathrm{KOK}}$	outputs of pulp production corresponding to input KOK	$\{\mathrm{LSEL}\}$
K	regions to sell products	$\{\mathrm{EU}, \mathrm{IE}, \mathrm{PA}, \mathrm{KI}\}$

Table 1, summary of sets

Symbol	Definition	Type	Unit	Set
h_t	purchasing amount of timber t	integer	$1000m^3$	T
y^I_i	production amount of wood i	linear	$1000m^3$	I
y^J_j	production amount of pulp j	linear	$1000m^3$	J
y^{paper}	production amount of paper	linear	$1000m^3$	–
$z^I_{i,k}$	selling amount of wood i in region k	linear	$1000m^3$	I, K
$z^J_{j,k}$	selling amount of pulp j in region k	linear	$1000m^3$	J, K
z^{paper}_k	selling amount of paper in region k	linear	$1000m^3$	K

Table 2, summary of decision variables

Symbol	Definition	Unit	Set
α_t	fixed cost factor of purchasing wood t	$\text{euro}/(1000m^3)$	T
β_t	unit cost factor of purchasing wood t	$\text{euro}/(1000m^6)$	T
a_i^I	relation of timber input and output in wood production i	–	I
b_i^I	relation of timber output and output in wood production i	–	I
e_i^I	relation of fuel output and output in wood production i	–	I
c_i^I	cost of wood production i	$\text{euro}/(1000m^3)$	I
r^{saw}	capacity of saw mill in wood production i	$1000m^3/\text{year}$	–
r^{plywood}	capacity of plywood mill in wood production i	$1000m^3/\text{year}$	–
a_j^J	input and output relation of pulp production j	–	J
c_j^J	cost of pulp production j	$\text{euro}/(1000\text{ton})$	J
r_j^J	capacity of pulp production j	$1000\text{ton}/\text{year}$	J
a_t^{paper}	relation of timber inputs in paper production	–	T
b_j^{paper}	relation of pulp inputs in paper production	–	J
c^{paper}	cost of paper production	$\text{euro}/(1000\text{ton})$	–
r^{paper}	capacity of paper production	$1000\text{ton}/\text{year}$	–
$\gamma_{i,k}^I$	fixed price factor of wood i in region k	$\text{euro}/(1000m^3)$	I, K
$\delta_{i,k}^I$	unit price factor of wood i in region k	$\text{euro}/(10^6m^6)$	I, K
$\gamma_{j,k}^J$	fixed price factor of pulp j in region k	$\text{euro}/(1000\text{ton})$	J, K
$\delta_{j,k}^J$	unit price factor of pulp j in region k	$\text{euro}/(10^6\text{ton}^2)$	J, K
γ_k^{paper}	fixed price factor of paper in region k	$\text{euro}/(1000\text{ton})$	K
δ_k^{paper}	unit price factor of paper in region k	$\text{euro}/(10^6\text{ton}^2)$	K
p^{fuel}	price of fuel wood	$\text{euro}/(1000m^3)$	–

Table 3, summary of constants

3, Objective Function

The Objective function composes of seven parts:

$$f^{\text{timber}} + f^{\text{wood}} + f^{\text{pp}} + g^{\text{timber}} + g^{\text{fuel}} + g^{\text{wood}} + g^{\text{pulp}} + g^{\text{paper}}$$

1. cost of timber procurement: $f^{\text{timber}} = -\sum_{t\in T} h_t(\alpha_t + \beta_t h_t)$
2. cost of wood production: $f^{\text{wood}} = -\sum_{i\in I} c_i^I y_i^I$
3. cost of pulp and paper production: $f^{\text{pp}} = -\sum_{j\in J} c_j^J y_j^J - c^{\text{paper}} y^{\text{paper}}$
4. profit of left timbers selling:

$$g^{\text{timber}} = \sum_{t\in T^1} \alpha_t \left(h_t - \sum_{i\in I_t^{T1}} a_i^I y_i^I \right) + \sum_{t\in T^2} \alpha_t \left(h_t + \sum_{i\in I_t^{T2}} b_i^I y_i^I - \sum_{j\in J_t^{T2}} a_j^J y_j^J - a_t^{\text{paper}} y^{\text{paper}} \right)$$

5. profit of fuel wood selling: $g^{\text{fuel}} = \sum_{i\in I} p^{\text{fuel}} e_i^I y_i^I$

6. profit of wood selling: $g^{\text{wood}} = \sum_{i\in I}\sum_{k\in K} z^I_{i,k}(\gamma^I_{i,k} - \delta^I_{i,k} z^I_{i,k})$
7. profit of pulp selling: $g^{\text{pulp}} = \sum_{j\in J}\sum_{k\in K} z^J_{j,k}(\gamma^J_{j,k} - \delta^J_{j,k} z^J_{j,k})$
8. profit of paper selling: $g^{\text{paper}} = \sum_{k\in K} z^{\text{paper}}_k(\gamma^{\text{paper}}_k - \delta^{\text{paper}}_k z^{\text{paper}}_k)$

4, Constraints

Besides the constraints that all variables are non-negative, there are ten sets of constraints:

1. limit of timber amount in wood production:

$$h_t \geq \sum_{i\in I^{T1}_t} a^I_i y^I_i \quad \forall t \in T^1$$

2. limit of timber amount in pulp and paper production:

$$\left(h_t + \sum_{i\in I^{T2}_t} b^I_i y^I_i\right) \geq \sum_{j\in J^{T2}_t} a^J_j y^J_j + a^{\text{paper}}_t y^{\text{paper}} \quad \forall t \in T^2$$

3. limit of pulp amount in paper production:

$$y^J_j \geq b^{\text{paper}}_j y^{\text{paper}} \quad \forall j \in J$$

4. limit of wood amount in selling:

$$y^I_i \geq \sum_{k\in K} z^I_{i,k} \quad \forall i \in I$$

5. limit of pulp amount in selling:

$$y^J_j - b^{\text{paper}}_j y^{\text{paper}} \geq \sum_{k\in K} z^J_{j,k} \quad \forall j \in J$$

6. limit of paper amount in selling:

$$y^{\text{paper}} \geq \sum_{k\in K} z^{\text{paper}}_k$$

7. limit of capacity in saw mill:

$$\sum_{i\in I^{\text{saw}}} y^I_i \leq r^{\text{saw}}$$

8. limit of capacity in plywood mill:

$$\sum_{i\in I^{\text{plywood}}} y^I_i \leq r^{\text{plywood}}$$

9. limit of capacity in pulp production:

$$y^J_j \leq r^J_j \quad \forall j \in J$$

10. limit of capacity in paper production:

$$y^{\mathrm{paper}} \leq r^{\mathrm{paper}}$$

5, Result

obj = $\underline{\underline{0.3940e6}}$

Produced quantity of final products {MAS, KUS, KOS, KUV, KOV, HSEL, LSEL, PAPER} = [0, 0, 0, 0, 0, 16, 16, 80]. The units of first five quantities are $1000m^3/\mathrm{year}$, and the units of last three are $1000\mathrm{ton}/\mathrm{year}$.

```
result_h_t = [0, -0, -0, 77, 80, 68]
result_y_i = [0, 0, 0, 0, 0]
result_y_j = [16, 16]
result_y_paper = 80
result_z_ik = [
        0.0 130.0 58.33 50.0;
        0.0 60.0 54.17 46.67;
        0.0 35.0 31.25 32.0;
        0.0 190.0 150.0 97.22;
        537.5 292.86 162.5 126.67
        ]
result_z_jk = [
        0.0 312.5 230.0 216.67;
        0.0 700.0 191.67 178.57142857142858
        ]
result_z_paper_k = [24.85, 27.39, 6.16, 21.60]
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Metsa-Oy Production and Supply Chain 2: Dynamic

1, Introduction

2, Definition of Mathematical Expressions

Symbol	Definition	Expression
T^1	type of timber 1	$\{\mathrm{MAT}, \mathrm{KUT}, \mathrm{KOT}\}$
T^2	type of timber 2	$\{\mathrm{MAK}, \mathrm{KUK}, \mathrm{KOK}\}$
T	type of timber	$T^1 \cup T^2$
$I^{T1}_{t=\mathrm{MAT}}$	outputs of wood production corresponding to input MAT	$\{\mathrm{MAS}\}$
$I^{T1}_{t=\mathrm{KUT}}$	outputs of wood production corresponding to input KUT	$\{\mathrm{KUS}, \mathrm{KUV}\}$
$I^{T1}_{t=\mathrm{KOT}}$	outputs of wood production corresponding to input KOT	$\{\mathrm{KOS}, \mathrm{KOV}\}$
$I^{T2}_{t=\mathrm{MAK}}$	outputs of wood production corresponding to output MAK	$\{\mathrm{MAS}\}$
$I^{T2}_{t=\mathrm{KUK}}$	outputs of wood production corresponding to output KUK	$\{\mathrm{KUS}, \mathrm{KUV}\}$
$I^{T2}_{t=\mathrm{KOK}}$	outputs of wood production corresponding to output KOK	$\{\mathrm{KOS}, \mathrm{KOV}\}$
I^{saw}	outputs of wood production in saw mill	$\{\mathrm{MAS}, \mathrm{KUS}, \mathrm{KOS}\}$
I^{plywood}	outputs of wood production in plywood mill	$\{\mathrm{KUV}, \mathrm{KOV}\}$
I	outputs of wood production	$I^{\mathrm{saw}} \cup I^{\mathrm{plywood}}$
J	outputs of pulp production	$\{\mathrm{HSEL}, \mathrm{LSEL}\}$
$J^{T2}_{t=\mathrm{MAK}}$	outputs of pulp production corresponding to input MAK	$\{\mathrm{HSEL}\}$
$J^{T2}_{t=\mathrm{KUK}}$	outputs of pulp production corresponding to input KUK	$\varnothing$
$J^{T2}_{t=\mathrm{KOK}}$	outputs of pulp production corresponding to input KOK	$\{\mathrm{LSEL}\}$
K	regions to sell products	$\{\mathrm{EU}, \mathrm{IE}, \mathrm{PA}, \mathrm{KI}\}$
M	three years in the planning horizon	$\{1, 2, 3\}$

Table 1, summary of sets

Symbol	Definition	Type	Unit	Set
$h_{m,t}$	purchasing amount of timber t in the year m	integer	$1000m^3$	M, T
$y^I_{m,i}$	production amount of wood i in the year m	linear	$1000m^3$	M, I
$y^J_{m,j}$	production amount of pulp j in the year m	linear	$1000m^3$	M, J
y^{paper}_m	production amount of paper in the year m	linear	$1000m^3$	M
$z^I_{m,i,k}$	selling amount of wood in the region k in the year m	linear	$1000m^3$	M, I, K
$z^J_{m,j,k}$	selling amount of pulp in the region k in the year m	linear	$1000m^3$	M, J, K
$z^{\mathrm{paper}}_{m,k}$	selling amount of paper in the region k in the year m	linear	$1000m^3$	M, K
x^{saw}_m	capacity of saw mill	linear	$1000m^3/\mathrm{year}$	M^2
x^{plywood}_m	capacity of plywood mill	linear	$1000m^3/\mathrm{year}$	M^2
$x^J_{m,j}$	capacity of pulp production line	linear	$1000\mathrm{ton}/\mathrm{year}$	M^2, J
x^{paper}_m	capacity of paper production line	linear	$1000\mathrm{ton}/\mathrm{year}$	M^2

Table 2, summary of decision variables

Symbol	Definition	Unit	Set
p^{fuel}	price of fuel wood	euro/$(1000m^3)$	–
σ	annual discounting factor	–	–
ω_i^I	demand growth coefficient of wood i	–	I
ω_j^J	demand growth coefficient of pulp j	–	J
ω^{paper}	demand growth coefficient of paper	–	–
α_t	fixed cost factor of purchasing timber t	euro/$(1000m^3)$	T
β_t	unit cost factor of purchasing timber t	euro/$(1000m^6)$	T
a_i^I	relation of timber input and output in wood production i	–	I
b_i^I	relation of timber output and output in wood production i i	–	I
e_i^I	relation of fuel output and output in wood production i	–	I
c_i^I	cost of wood production i	euro/$(1000m^3)$	I
r^{saw}	original capacity of saw mill	$1000m^3$/year	–
r^{plywood}	original capacity of plywood mill	$1000m^3$/year	–
o^{saw}	capacity expansion cost of saw mill	euro/$(1000m^3$/year)	–
o^{plywood}	capacity expansion cost of plywood mill	euro/$(1000m^3$/year)	–
ν^{saw}	max capacity expansion factor of saw mill	–	–
ν^{plywood}	max capacity expansion factor of saw mill	–	–
a_j^J	input and output relation of pulp production j	–	J
c_j^J	cost of pulp production j	euro/(1000ton)	J
r_j^J	original capacity of pulp production j	1000ton/year	J
o_j^J	capacity expansion cost of pulp production j	euro/$(1000m^3$/year)	–
ν_j^J	max capacity expansion factor of pulp production j	–	J
a_t^{paper}	relation of timber inputs in paper production	–	T
b_j^{paper}	relation of pulp inputs in paper production	–	J
c^{paper}	cost of paper production	euro/(1000ton)	–
r^{paper}	original capacity of paper production	1000ton/year	–
o^{paper}	capacity expansion cost of paper production	euro/$(1000m^3$/year)	–
ν^{paper}	max capacity expansion factor of paper production	–	–
$\gamma_{m,i,k}^I$	fixed price factor of wood i in the region k in the year m	euro/$(1000m^3)$	M, I, K
$\delta_{m,i,k}^I$	unit price factor of wood i in the region k in the year m	euro/(10^6m^6)	M, I, K
$\gamma_{m,j,k}^J$	fixed price factor of pulp j in the region k in the year m	euro/(1000ton)	M, J, K
$\delta_{m,j,k}^J$	unit price factor of pulp j in the region k in the year m	euro/(10^6ton^2)	M, J, K
$\gamma_{m,k}^{\text{paper}}$	fixed price factor of paper in the region k in the year m	euro/(1000ton)	M, K
$\delta_{m,k}^{\text{paper}}$	unit price factor of paper in the region k in the year m	euro/(10^6ton^2)	M, K

Table 3, summary of constants

3, Objective Function

The Objective function composes of seven parts:

$$\sum_{m\in M} \sigma^{m-1}\left[f_m^{\text{timber}} + f_m^{\text{wood}} + f_m^{\text{pp}} + g_m^{\text{timber}} + g_m^{\text{fuel}} + g_m^{\text{wood}} + g_m^{\text{pulp}} + g_m^{\text{paper}}\right] + f^{\text{cap}}$$

1. cost of timber procurement: $f_m^{\text{timber}} = -\sum_{t\in T} h_{m,t}(\alpha_t + \beta_t h_{m,t})$
2. cost of wood production: $f_m^{\text{wood}} = -\sum_{i\in I} c_i^I y_{m,i}^I$
3. cost of pulp and paper production: $f_m^{\text{pp}} = -\sum_{j\in J} c_j^J y_{m,j}^J - \sum_{m\in M} c^{\text{paper}} y_m^{\text{paper}}$

4. profit of left timbers selling:

$$g_m^{\text{timber}} = \sum_{t \in T^1} \alpha_t \left(h_{m,t} - \sum_{i \in I_t^{T1}} a_i^I y_{m,i}^I \right) + \sum_{t \in T^2} \alpha_t \left(h_{m,t} + \sum_{i \in I_t^{T2}} b_i^I y_{m,i}^I - \sum_{j \in J_t^{T2}} a_j^J y_{m,j}^J - a_t^{\text{paper}} y_m^{\text{paper}} \right)$$

5. profit of fuel wood selling: $g_m^{\text{fuel}} = \sum_{i \in I} p^{\text{fuel}} e_i^I y_{m,i}^I$
6. profit of wood selling: $g_m^{\text{wood}} = \sum_{i \in I} \sum_{k \in K} (\omega_i^I)^{m-1} z_{m,i,k}^I (\gamma_{i,k}^I - \delta_{i,k}^I z_{m,i,k}^I)$
7. profit of pulp selling: $g_m^{\text{pulp}} = \sum_{j \in J} \sum_{k \in K} (\omega_j^J)^{m-1} z_{m,j,k}^J (\gamma_{j,k}^J - \delta_{j,k}^J z_{m,j,k}^J)$
8. profit of paper selling: $g_m^{\text{paper}} = \sum_{k \in K} (\omega^{\text{paper}})^{m-1} z_{m,k}^{\text{paper}} (\gamma_k^{\text{paper}} - \delta_k^{\text{paper}} z_{m,k}^{\text{paper}})$
9. cost of capacity expansion:

$$f^{\text{cap}} = \sum_{m=2}^{3} \sigma^{m-2} \left[o^{\text{saw}} (x_m^{\text{saw}} - x_{m-1}^{\text{saw}}) + o^{\text{plywood}} (x_m^{\text{plywood}} - x_{m-1}^{\text{plywood}}) + \sum_{j \in J} o_j^J (x_{m,j}^J - x_{m-1,j}^J) + o^{\text{paper}} (x_m^{\text{paper}} - x_{m-1}^{\text{paper}}) \right]$$

4, Constraints

Besides the constraints that all variables are non-negative, there are ten sets of constraints:

1. limit of timber amount in wood production:

$$h_{m,t} \geq \sum_{i \in I_t^{T1}} a_i^I y_{m,i}^I \quad \forall t \in T^1, m \in M$$

2. limit of timber amount in pulp and paper production:

$$\left(h_{m,t} + \sum_{i \in I_t^{T2}} b_i^I y_{m,i}^I \right) \geq \sum_{j \in J_t^{T2}} a_j^J y_{m,j}^J + a_t^{\text{paper}} y_m^{\text{paper}} \quad \forall t \in T^2, m \in M$$

3. limit of pulp amount in paper production:

$$y_{m,j}^J \geq b_j^{\text{paper}} y_m^{\text{paper}} \quad \forall j \in J, m \in M$$

4. limit of wood amount in selling:

$$y_{m,i}^I \geq \sum_{k \in K} z_{m,i,k}^I \quad \forall i \in I, m \in M$$

5. limit of pulp amount in selling:

$$y_{m,j}^J - b_j^{\text{paper}} y_m^{\text{paper}} \geq \sum_{k \in K} z_{m,j,k}^J \quad \forall j \in J, m \in M$$

6. limit of paper amount in selling:

$$y_m^{\text{paper}} \geq \sum_{k \in K} z_{m,k}^{\text{paper}} \quad \forall m \in M$$

7. limit of capacity in saw mill:

$$\sum_{i \in I^{\text{saw}}} y_{m,i}^I \leq x_m^{\text{saw}} \quad \forall m \in M$$

8. limit of capacity in plywood mill:

$$\sum_{i \in I^{\text{plywood}}} y_{m,i}^I \leq x_m^{\text{plywood}} \quad \forall m \in M$$

9. limit of capacity in pulp production:

$$y_{m,j}^J \leq x_{m,j}^J \quad \forall j \in J, m \in M$$

10. limit of capacity in paper production:

$$y_m^{\text{paper}} \leq x_m^{\text{paper}} \quad \forall m \in M$$

11. relation between capacities

$$\begin{aligned}
&x_1 = r \quad \forall x, r \\
&x_2 \geq x_1 \quad \forall x \\
&x_3 \geq x_2 \quad \forall x \\
&x_m \leq \nu r \quad \forall x, m \in M
\end{aligned}$$

4, Result

obj = $\underline{\underline{1.7241e6}}$

Metsa-Oy Production and Supply Chain 3: Stochastic Dynamic

1, Introduction

2, Definition of Mathematical Expressions

Symbol	Definition	Expression
T^1	type of timber 1	$\{\mathrm{MAT}, \mathrm{KUT}, \mathrm{KOT}\}$
T^2	type of timber 2	$\{\mathrm{MAK}, \mathrm{KUK}, \mathrm{KOK}\}$
T	type of timber	$T^1 \cup T^2$
$I^{T1}_{t=\mathrm{MAT}}$	outputs of wood production corresponding to input MAT	$\{\mathrm{MAS}\}$
$I^{T1}_{t=\mathrm{KUT}}$	outputs of wood production corresponding to input KUT	$\{\mathrm{KUS}, \mathrm{KUV}\}$
$I^{T1}_{t=\mathrm{KOT}}$	outputs of wood production corresponding to input KOT	$\{\mathrm{KOS}, \mathrm{KOV}\}$
$I^{T2}_{t=\mathrm{MAK}}$	outputs of wood production corresponding to output MAK	$\{\mathrm{MAS}\}$
$I^{T2}_{t=\mathrm{KUK}}$	outputs of wood production corresponding to output KUK	$\{\mathrm{KUS}, \mathrm{KUV}\}$
$I^{T2}_{t=\mathrm{KOK}}$	outputs of wood production corresponding to output KOK	$\{\mathrm{KOS}, \mathrm{KOV}\}$
I^{saw}	outputs of wood production in saw mill	$\{\mathrm{MAS}, \mathrm{KUS}, \mathrm{KOS}\}$
I^{plywood}	outputs of wood production in plywood mill	$\{\mathrm{KUV}, \mathrm{KOV}\}$
I	outputs of wood production	$I^{\mathrm{saw}} \cup I^{\mathrm{plywood}}$
J	outputs of pulp production	$\{\mathrm{HSEL}, \mathrm{LSEL}\}$
$J^{T2}_{t=\mathrm{MAK}}$	outputs of pulp production corresponding to input MAK	$\{\mathrm{HSEL}\}$
$J^{T2}_{t=\mathrm{KUK}}$	outputs of pulp production corresponding to input KUK	$\varnothing$
$J^{T2}_{t=\mathrm{KOK}}$	outputs of pulp production corresponding to input KOK	$\{\mathrm{LSEL}\}$
K	regions to sell products	$\{\mathrm{EU}, \mathrm{IE}, \mathrm{PA}, \mathrm{KI}\}$
M	three years in the planning horizon	$\{1, 2, 3\}$
N	four scenarios	$\{1, 2, 3, 4\}$

Table 1, summary of sets

Symbol	Definition	Type	Unit	Set
h_t	purchasing amount of timber t in the first year	integer	$1000m^3$	T
y_i^I	production amount of wood i in the first year	linear	$1000m^3$	I
y_j^J	production amount of pulp j in the first year	linear	$1000m^3$	J
y^{paper}	production amount of paper in the first year	linear	$1000m^3$	$-$
$z_{i,k}^I$	selling amount of wood i in region k in the first year	linear	$1000m^3$	I, K
$z_{j,k}^J$	selling amount of pulp j in region k in the first year	linear	$1000m^3$	J, K
z_k^{paper}	selling amount of paper in region k in the first year	linear	$1000m^3$	K
$h_{m,t}$	purchasing amount of timber t in the year m	integer	$1000m^3$	$M/\{1\}, T$
$y_{m,i}^I$	production amount of wood i in the year m	linear	$1000m^3$	$M/\{1\}, I$
$y_{m,j}^J$	production amount of pulp j in the year m	linear	$1000m^3$	$M/\{1\}, J$
y_m^{paper}	production amount of paper in the year m	linear	$1000m^3$	$M/\{1\}$
$z_{m,i,k}^I$	selling amount of wood in the region k in the year m	linear	$1000m^3$	$M/\{1\}, I, K$
$z_{m,j,k}^J$	selling amount of pulp in the region k in the year m	linear	$1000m^3$	$M/\{1\}, J, K$
$z_{m,k}^{\text{paper}}$	selling amount of paper in the region k in the year m	linear	$1000m^3$	$M/\{1\}, K$
x_m^{saw}	capacity of saw mill	linear	$1000m^3$/year	$M/\{3\}$
x_m^{plywood}	capacity of plywood mill	linear	$1000m^3$/year	$M/\{3\}$
$x_{m,j}^J$	capacity of pulp production line	linear	1000ton/year	$M/\{3\}, J$
x_m^{paper}	capacity of paper production line	linear	1000ton/year	$M/\{3\}$
$x_{n,3}^{\text{saw}}$	capacity of saw mill in the year 3 decide in the year 2	linear	$1000m^3$/year	N
$x_{n,3}^{\text{plywood}}$	capacity of plywood mill in the year 3 decide in the year 2	linear	$1000m^3$/year	N
$x_{n,3,j}^J$	capacity of pulp production line in the year 3 decide in the year 2	linear	1000ton/year	N, J
$x_{n,3}^{\text{paper}}$	capacity of paper production line in the year 3 decide in the year 2	linear	1000ton/year	N

Table 2, summary of decision variables

Symbol	Definition	Unit	Set
$\rho_{n,m}$	price coefficient of the scenario n in the year m	–	N, M
ω_i^I	demand growth coefficient of wood i	–	I
ω_j^J	demand growth coefficient of pulp j	–	J
ω^{paper}	demand growth coefficient of paper	–	–
p^{fuel}	price of fuel wood	euro/(1000m^3)	–
σ	annual discounting factor	–	–
π_n	probability of scenario n	–	N
α_t	fixed cost factor of purchasing timber t	euro/(1000m^3)	T
β_t	unit cost factor of purchasing timber t	euro/(1000m^6)	T
a_i^I	relation of timber input and output in wood production i	–	I
b_i^I	relation of timber output and output in wood production i i	–	I
e_i^I	relation of fuel output and output in wood production i	–	I
c_i^I	cost of wood production i	euro/(1000m^3)	I
r^{saw}	original capacity of saw mill	1000m^3/year	–
r^{plywood}	original capacity of plywood mill	1000m^3/year	–
o^{saw}	capacity expansion cost of saw mill	euro/(1000m^3/year)	–
o^{plywood}	capacity expansion cost of plywood mill	euro/(1000m^3/year)	–
ν^{saw}	max capacity expansion factor of saw mill	–	–
ν^{plywood}	max capacity expansion factor of saw mill	–	–
a_j^J	input and output relation of pulp production j	–	J
c_j^J	cost of pulp production j	euro/(1000ton)	J
r_j^J	original capacity of pulp production j	1000ton/year	J
o_j^J	capacity expansion cost of pulp production j	euro/(1000m^3/year)	–
ν_j^J	max capacity expansion factor of pulp production j	–	J
a_t^{paper}	relation of timber inputs in paper production	–	T
b_j^{paper}	relation of pulp inputs in paper production	–	J
c^{paper}	cost of paper production	euro/(1000ton)	–
r^{paper}	original capacity of paper production	1000ton/year	–
o^{paper}	capacity expansion cost of paper production	euro/(1000m^3/year)	–
ν^{paper}	max capacity expansion factor of paper production	–	–
$\gamma_{m,i,k}^I$	fixed price factor of wood i in the region k in the year m	euro/(1000m^3)	M, I, K
$\delta_{m,i,k}^I$	unit price factor of wood i in the region k in the year m	euro/($10^6 m^6$)	M, I, K
$\gamma_{m,j,k}^J$	fixed price factor of pulp j in the region k in the year m	euro/(1000ton)	M, J, K
$\delta_{m,j,k}^J$	unit price factor of pulp j in the region k in the year m	euro/(10^6ton^2)	M, J, K
$\gamma_{m,k}^{\text{paper}}$	fixed price factor of paper in the region k in the year m	euro/(1000ton)	M, K
$\delta_{m,k}^{\text{paper}}$	unit price factor of paper in the region k in the year m	euro/(10^6ton^2)	M, K

Table 3, summary of constants

3, Objective Function

The Objective function composes of many parts:

$$f_1^{\text{timber}} + f_1^{\text{wood}} + f_1^{\text{pp}} + g_1^{\text{timber}} + g_1^{\text{fuel}} + g_1^{\text{wood}} + g_1^{\text{pulp}} + g_1^{\text{paper}} + f_2^{\text{cap}} +$$
$$\sum_{n\in N} \pi_n \left\{ \sum_{m=2}^{3} \sigma^{m-1} \left[f_{n,m}^{\text{timber}} + f_{n,m}^{\text{wood}} + f_{n,m}^{\text{pp}} + g_{n,m}^{\text{timber}} + g_{n,m}^{\text{fuel}} + g_{n,m}^{\text{wood}} + g_{n,m}^{\text{pulp}} + g_{n,m}^{\text{paper}} \right] + \sigma f_{n,3}^{\text{cap}} \right\}$$

- When m = 1, the variables are here-and-now decisions variables, and this part of the objective function is the same as those in the static model:

1. cost of timber procurement: $f_1^{\text{timber}} = -\sum_{t\in T} h_t(\alpha_t + \beta_t h_t)$
2. cost of wood production: $f_1^{\text{wood}} = -\sum_{i\in I} c_i^I y_i^I$
3. cost of pulp and paper production: $f_1^{\text{pp}} = -\sum_{j\in J} c_j^J y_j^J - c^{\text{paper}} y^{\text{paper}}$
4. profit of left timbers selling:

$$g_1^{\text{timber}} = \sum_{t\in T^1} \alpha_t \left(h_t - \sum_{i\in I_t^{T1}} a_i^I y_i^I \right) + \sum_{t\in T^2} \alpha_t \left(h_t + \sum_{i\in I_t^{T2}} b_i^I y_i^I - \sum_{j\in J_t^{T2}} a_j^J y_j^J - a_t^{\text{paper}} y^{\text{paper}} \right)$$

5. profit of fuel wood selling: $g_1^{\text{fuel}} = \sum_{i\in I} p^{\text{fuel}} e_i^I y_i^I$
6. profit of wood selling: $g_1^{\text{wood}} = \sum_{i\in I} \sum_{k\in K} z_{i,k}^I (\gamma_{i,k}^I - \delta_{i,k}^I z_{i,k}^I)$
7. profit of pulp selling: $g_1^{\text{pulp}} = \sum_{j\in J} \sum_{k\in K} z_{j,k}^J (\gamma_{j,k}^J - \delta_{j,k}^J z_{j,k}^J)$
8. profit of paper selling: $g_1^{\text{paper}} = \sum_{k\in K} z_k^{\text{paper}} (\gamma_k^{\text{paper}} - \delta_k^{\text{paper}} z_k^{\text{paper}})$

- When m = 2 or 3, the variables are wait-and-see decisions variables:

1. cost of timber procurement: $f_{n,m}^{\text{timber}} = -\sum_{t\in T} h_{n,m,t}(\alpha_t + \beta_t h_{n,m,t})$
2. cost of wood production: $f_{n,m}^{\text{wood}} = -\sum_{i\in I} c_i^I y_{n,m,i}^I$
3. cost of pulp and paper production: $f_{n,m}^{\text{pp}} = -\sum_{j\in J} c_j^J y_{n,m,j}^J - \sum_{m\in M} c^{\text{paper}} y_{n,m}^{\text{paper}}$
4. profit of left timbers selling:

$$g_{n,m}^{\text{timber}} = \sum_{t\in T^1} \alpha_t \left(h_{n,m,t} - \sum_{i\in I_t^{T1}} a_i^I y_{n,m,i}^I \right) + \sum_{t\in T^2} \alpha_t \left(h_{n,m,t} + \sum_{i\in I_t^{T2}} b_i^I y_{n,m,i}^I - \sum_{j\in J_t^{T2}} a_j^J y_{n,m,j}^J - a_t^{\text{paper}} y_{n,m}^{\text{paper}} \right)$$

5. profit of fuel wood selling: $g_{n,m}^{\text{fuel}} = \sum_{i\in I} p^{\text{fuel}} e_i^I y_{n,m,i}^I$
6. profit of wood selling: $g_{n,m}^{\text{wood}} = \sum_{i\in I} \sum_{k\in K} \rho_{n,m} (\omega_i^I)^{m-1} z_{n,m,i,k}^I (\gamma_{i,k}^I - \delta_{i,k}^I z_{n,m,i,k}^I)$
7. profit of pulp selling: $g_{n,m}^{\text{pulp}} = \sum_{j\in J} \sum_{k\in K} \rho_{n,m} (\omega_j^J)^{m-1} z_{n,m,j,k}^J (\gamma_{j,k}^J - \delta_{j,k}^J z_{n,m,j,k}^J)$
8. profit of paper selling: $g_{n,m}^{\text{paper}} = \sum_{k\in K} \rho_{n,m} (\omega^{\text{paper}})^{m-1} z_{n,m,k}^{\text{paper}} (\gamma_k^{\text{paper}} - \delta_k^{\text{paper}} z_{n,m,k}^{\text{paper}})$

- Cost of capacity expansion:

$$f_2^{\text{cap}} = o^{\text{saw}} (x_2^{\text{saw}} - x_1^{\text{saw}}) + o^{\text{plywood}} (x_2^{\text{plywood}} - x_1^{\text{plywood}}) + \sum_{j\in J} o_j^J (x_{2,j}^J - x_{1,j}^J) + o^{\text{paper}} (x_2^{\text{paper}} - x_1^{\text{paper}})$$
$$f_{n,3}^{\text{cap}} = o^{\text{saw}} (x_{n,3}^{\text{saw}} - x_2^{\text{saw}}) + o^{\text{plywood}} (x_{n,3}^{\text{plywood}} - x_2^{\text{plywood}}) + \sum_{j\in J} o_j^J (x_{n,3,j}^J - x_{2,j}^J) + o^{\text{paper}} (x_{n,3}^{\text{paper}} - x_2^{\text{paper}})$$

where variables regarding capacities in the first year, like x_1^{saw}, are fixed value variables, those in the second year are here-and-now decision variables, and those in the third year are wait-and-see variables. Those here-and-now decision variables are forced to be equal for different scenarios by constraints 7-10 in the following section.

4, Constraints

When $m = 1$, the constraints are the same as those in dynamic model, which are neglected here. When $m = 2 \text{ or } 3$, besides the constraints that all variables are non-negative, there many ten sets of constraints:

1. limit of timber amount in wood production:

$$h_{n,m,t} \geq \sum_{i\in I_t^{T1}} a_i^I y_{n,m,i}^I \quad \forall t \in T^1, m \in \{2,3\}, n \in N$$

2. limit of timber amount in pulp and paper production:

$$\left(h_{n,m,t} + \sum_{i\in I_t^{T2}} b_i^I y_{n,m,i}^I \right) \geq \sum_{j\in J_t^{T2}} a_j^J y_{n,m,j}^J + a_t^{\text{paper}} y_{n,m}^{\text{paper}} \quad \forall t \in T^2, m \in \{2,3\}, n \in N$$

3. limit of pulp amount in paper production:

$$y^J_{n,m,j} \geq b^{\text{paper}}_j y^{\text{paper}}_{n,m} \quad \forall j \in J, m \in \{2,3\}, n \in N$$

4. limit of wood amount in selling:

$$y^I_{n,m,i} \geq \sum_{k \in K} z^I_{n,m,i,k} \quad \forall i \in I, m \in \{2,3\}, n \in N$$

5. limit of pulp amount in selling:

$$y^J_{n,m,j} - b^{\text{paper}}_j y^{\text{paper}}_{n,m} \geq \sum_{k \in K} z^J_{n,m,j,k} \quad \forall j \in J, m \in \{2,3\}, n \in N$$

6. limit of paper amount in selling:

$$y^{\text{paper}}_{n,m} \geq \sum_{k \in K} z^{\text{paper}}_{n,m,k} \quad \forall m \in \{2,3\}, n \in N$$

7. limit of capacity in saw mill:

$$\begin{aligned} &\sum_{i \in I^{\text{saw}}} y^I_{n,m,i} \leq x^{\text{saw}}_{n,m} \quad \forall m \in \{2,3\}, n \in N \\ &x^{\text{saw}}_{1,2} = x^{\text{saw}}_{n,2} \quad \forall n \in \{2,3,4\} \end{aligned}$$

8. limit of capacity in plywood mill:

$$\begin{aligned} &\sum_{i \in I^{\text{plywood}}} y^I_{m,i} \leq x^{\text{plywood}}_m \quad \forall m \in \{2,3\}, n \in N \\ &x^{\text{plywood}}_{1,2} = x^{\text{plywood}}_{n,2} \quad \forall n \in \{2,3,4\} \end{aligned}$$

9. limit of capacity in pulp production:

$$\begin{aligned} &y^J_{n,m,j} \leq x^J_{n,m,j} \quad \forall j \in J, m \in \{2,3\}, n \in N \\ &x^J_{1,2,j} = x^J_{n,2,j} \quad \forall n \in \{2,3,4\}, j \in J \end{aligned}$$

10. limit of capacity in paper production:

$$\begin{aligned} &y^{\text{paper}}_{n,m} \leq x^{\text{paper}}_{n,m} \quad \forall m \in \{2,3\}, n \in N \\ &x^{\text{paper}}_{1,2} = x^{\text{paper}}_{n,2} \quad \forall n \in \{2,3,4\} \end{aligned}$$

11. relation between capacity expansion factors

$$\begin{aligned} &x_1 = r \quad \forall x, r \\ &x_2 \geq x_1 \quad \forall x \\ &x_3 \geq x_2 \quad \forall x \\ &x_m \leq \nu r \quad \forall x, m \in M \end{aligned}$$

4, Results

obj = $\underline{\underline{2.9696e6}}$