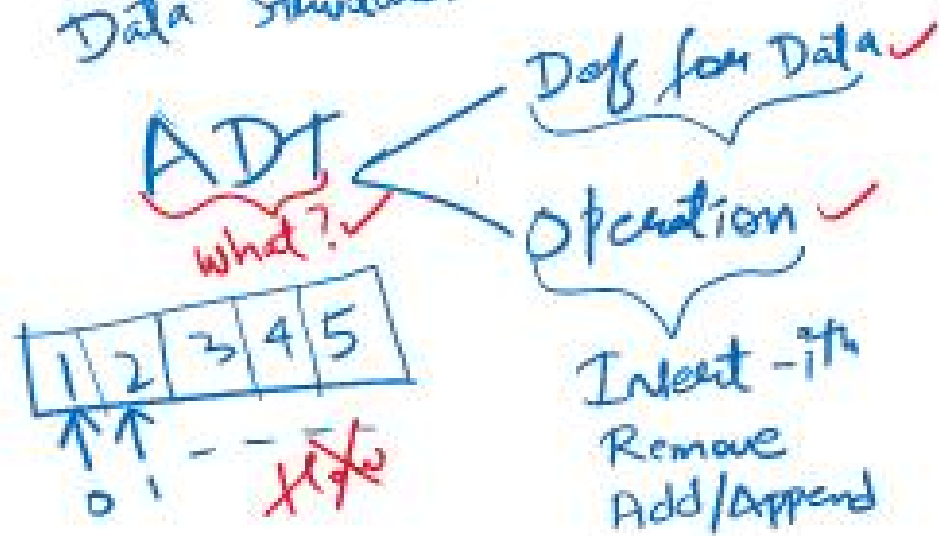


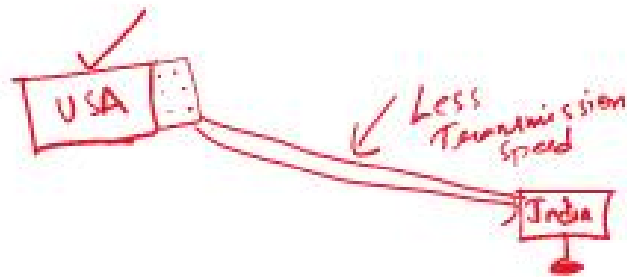
## Data Structures



## Pseudocode? ✓

- Partial Eng + Partial Code
- won't be following Syntax

Intuition ~~Prads~~  
↳ Idea/Feeling



$1 = I \quad 2 = II$   
 $n = n^2 + 2$   
 Big Oh (O)

$$f(n) = O(g(n))$$

$f(n) \leq c \cdot g(n) \quad \forall n \geq n_0$

$f(n) = 2n + 3 \rightarrow 2n + 3 \leq 5n \quad n \geq 1$

$\frac{10n}{7n} \quad f(n) = O(n)$

$2n + 3 \leq 5n^2 \quad f(n) = O(n^2)$

$2n + 3 \leq 55n^3 \quad f(n) = O(n^3)$   
 $n^4 \quad O(n^4)$

$f(n) = 2n^2 + 4$   
 $g(n) = 3n^2 + 4n^2 = 7n^2$   
 $O(g(n)) = O(n^2)$

$n \geq 1$   
 $1 = 6$   
 $2 = 12$   
 $6$   
 $24$   
 $n = 1$

$O(n^2)$   
 $O(n^3)$   
 $O(2^n)$

$f(n) = 2$   
 $O(5) \rightarrow O(1)$   
 $2 \leq 5$   
 $g(n) \uparrow$   
 $O(n)$   
 $O(n^2)$   
 $f(n) = n^3 + 2n^2$   
 $3n^3$   
 $O(n^3)$   
 $n^3 + 2n^3 = 3n^3 \rightarrow O(n^3)$

$f(n) = n + 3n^2$   
 $O(n^2)$   
 $4n^2$   
 $O(n^2)$

$f(n) = n^2 + 2n$   
 $f(n) \approx n^2$   
 $f(n) \approx n^2$   
 $f(n) \approx n^2$   
 $f(n) \approx n$

- # # Time funct Categories
- (i)  $O(1) \rightarrow$  Const  
 (ii)  $O(\log n) \rightarrow$  Logarithmic  $1 < b$   
 (iii)  $O(n) \rightarrow$  Linear  
 (iv)  $O(n^2) \rightarrow$  Quadratic  
 (v)  $O(n^3) \rightarrow$  Cubic  
 (vi)  $O(2^n), O(3^n), \dots, O(a^n), \dots, O(n^n)$   
Exponential

# Order # VV Big N  
 1 <  $\log n$  <  $\sqrt{n}$  <  $n$  <  $n \log n$  <  $n^2$  <  $n^3$  ... <  $2^n$  <  $3^n$  ... <  $n^n$   
 (n)



Poly.

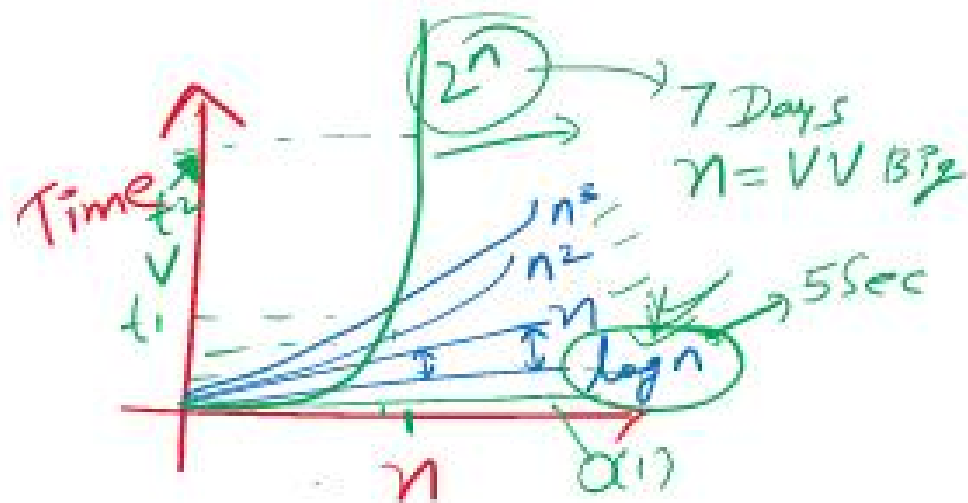
$2^{n+1}$

Exp. ✓

$\log^1$   
 $\log^2$

$\log^2$	$n$	$n^2$	$2^n$
0	< 1	1	2
1	< 2	4	4
2	< 4	16	16
3	< 8	64	256
3.1	< 9	81	512

Big N



Big Oh: Upper Bound #

Upper  ~~$O(1)$~~   ~~$O(n)$~~

$1 < \log n < n < n^2 < n^3 \dots$

~~$O(1)$~~   ~~$O(\log n)$~~   $f(n) = 2n + 3$   $\rightarrow$  Valid ✓

# Omega ( $\Omega$ ) Lower Bound

1)  $\Omega(n), \Omega(n^2) \dots \Omega(1), \dots \Omega(2^n)$

$f(n) = \Omega(g(n))$   
 iff  $f(n) \geq c * g(n)$   $\quad n \geq n_0$

$f(n) = 2n + 3 \geq n$   
 $2n + 3 \geq n$

$\Omega(n)$

#  $1 < \log n < n < n^2 \dots$   
 $\Omega(1), \Omega(\log n), \Omega(n), \Omega(n^2)$

$f(n) \geq c * g(n)$

$f(n) = n^2 + 3$

(i)  $\log n$

(ii) 1

(iii)  $n$

(iv)  $n^2$

$\Omega$

Theorem ( $\Theta$ )

$$f(n) = \Theta(g(n))$$

iff  $\Rightarrow$

$$c_1 \cdot g(n) \leq f(n) \leq c_2 \cdot g(n)$$

$$f(n) = 2n + 3n$$

$$2n \leq 2n + 3 \leq 5n$$

$\Theta(n)$

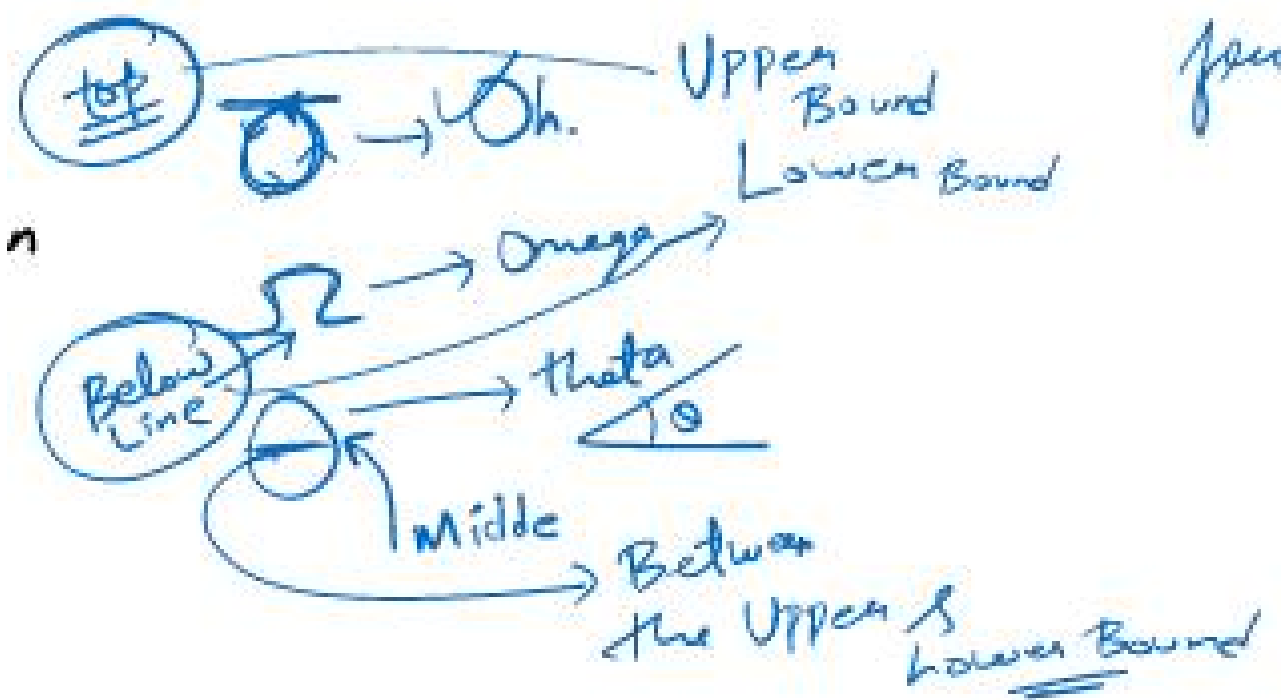
$$2n, 3n, 4n, 5n$$

$$1 < \log n < n < n^2 < \dots < 2^n < n^n$$

$\log n$  is marked with a red X,  $n$  is marked with a blue checkmark,  $n^2$  is marked with a red X, and  $2^n$  is marked with a blue checkmark.



It



Notations are Diff  $O, \Omega, \Theta$

from 

<u>B</u> Care
<u>W</u> Care
<u>A</u> Care


$i++ \Rightarrow \frac{i=i+1}{\text{for}(i=0; i < n; i++)}$   
 $\{$   
 statement  $O(1)$   
 $\}$   
 $O(n)$

$\text{for}(i=n; i > 0; i--)$   
 $\{$   
 $s + O(1)$   
 $\}$   
 $O(n)$

for( $i=0; i < n; i+2$ )  
{  
  st  $O(1)$  ✓  
}

for( $i=0; i < n; i+4$ )  
{ st  $O(1)$      $O(\frac{n}{4})$   
}

1 2 3 4 5 6 7 8



```
for (i=0; i<n; i++)
{
  st o(i)
}
```

$O(n)$

Sum = 0

```
for (i=0; i<n; i++)
{
  for (j=0; j<n; j++)
  {
    Sum = Sum + 1
  }
}
```

$i = 0$   
 $i = 1$   
 $i = 2$   
 $\vdots$   
 $i = n$

$j = 0, 1, 2, \dots, n$   
 $j = 1$   
 $j = 1$   
 $j = 1$   
 $j = 1$   
 $j = 1$

Final Sum = 4

$i = 0$   
 $i = 1$   
 $2 \times 2$   
 $n \times n = n^2$

$j = 0, 1$   
 $j = 0, 1$   
 $4$   
 $n^2$

```

sum = 0
for i in range(0, 2):
    for j in range(0, 2):
        sum = sum + 1
print(sum)

```

$O(1) \times n \times n = O(n^2)$   
 $sum = 4$  (for  $n=2$ )

★

```

sum = 0
for (i = 0; i <= n; i++)
{
    for (j = 0; j <= i; j++)
    {
        print i;
        sum = sum + 1;
    }
}

```

i	j	no. of times
0	X	0
1	0 ✓	1
2	0 ✓ 1 ✓	2
3	0 ✓ 1 ✓ 2 ✓	3
...	...	...
n	...	...

$0 + 1 + 2 + 3 + \dots + n$   
 $\frac{n(n+1)}{2}$   
 $O(\frac{n(n+1)}{2}) \Rightarrow O(n^2)$

$p = 0$ ;  
 $\text{for } (i = 1; i \leq n; i++)$

$\{ p = p + i$

$\text{if } (p > n)$

$i$   
 $1$   
 $2$   
 $3$   
 $4$   
 $\vdots$

$0+1 = 1$   
 $1+2 = 3$   
 $3+3 = 6$

$1+2+3$

$1+2+3+\dots+k \leq n$

$\frac{k(k+1)}{2} \leq n$

$k^2 \leq n$

$k \leq \sqrt{n}$

$O(\sqrt{n})$