

Exercise Sheet 2 – Probability and Statistics

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Exercise 1

We roll a fair dice three times. What is the probability that two even numbers won't appear in consecutive rolls?

All possible combinations: $6 \cdot 6 \cdot 6 = 216$ Even 2 4 6 Events possible OEO, EOE = 18

$$\text{Probability} = \frac{18}{216} = \frac{1}{12} = \mathbf{0.083}$$

Exercise 2

All possible combinations for 4 babies: $2 \cdot 2 \cdot 2 \cdot 2 = 2^4 = 16$ We can have:

Possibilities

- GGBB
- BBGG
- BGGB
- GBBG
- GBGB
- BGBG

$$\text{Thus: Probability: } \frac{6}{16} = \frac{3}{8} = \mathbf{0.375}$$

Exercise 3

In a family with 3 children, of whom at least one is a girl, what is the probability that the family has at least 2 daughters?

All possible combinations when boy is defined B $2 \cdot 2 = 2^2 = 4$ possibilities

Possibilities:

- BB
- GG
- BG
- GB Thus: Probability = $\frac{3}{4} = \mathbf{0.75}$

Exercise 4

Exercise 4 If A, B are events in sample space S such that $P(A) = P(B|A) = 0.3$ and $P(A|B) = 0.6$, what is the value of P(B)?

$$P(A|B) = P(B|A) \cdot P(A) / P(B)$$

$$0.6 = 0.3 \cdot 0.3 / P(B)$$

$$0.6 = 0.09 / P(B)$$

$$P(B) = 0.09/0.6$$

$$P(B) = \mathbf{0.15}$$

Exercise 5

Almost 10% of all root canal therapies fail. Let X denote the probability variable of number of fails in 4 attempts at root canal.

p = Probability that root canal fails = 0.1

n = numbers of attempts = 4

Attempts are independent so we can use binominal distribution.

1. Write the probability distribution function of the variable X

$$P(X) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

$$P(X) = \frac{4!}{x!(4-x)!} 0.1^x (1-0.1)^{4-x}$$

2. What is the probability that at least one attempt is successful?
This probability that at zero attempt of root canal fails.

$$P(X = 0) = \frac{4!}{0!(4-0)!} 0.1^0 (1-0.1)^{4-0} = 0.9^4 = 0.65$$

$$P(X = 0) = 0.65$$

3. How many successful therapies are expected in 20 attempts?

$$\text{Successful therapies in 20 attempts} = n \cdot (1-p) = 20 \cdot 0.9 = 18$$

Exercise 6

If we have three identical boxes such that the first box contains three white pins and five black pins, the second box contains four white and four black pins, and the third box contains six white and two black pins. If we take two pins of the opposite color from a selected box, what is the probability that the second box is chosen?

We have boxes:

- $B1 = \{3W, 5B\}$
- $B2 = \{4W, 4B\}$
- $B3 = \{6W, 2B\}$

$$P(B1) = P(B2) = P(B3)$$

Let $E1$, $E2$ and $E3$ be the event that a ball is chosen from a box $B1$, $B2$ and $B3$ respectively.

Case white ball EW

$$P(E1) = P(E2) = P(E3) = 1/3$$

$$P(EW|E1) = \frac{3}{8}, P(EW|E2) = \frac{4}{8}, P(EW|E3) = \frac{6}{8}$$

Case black ball EB

$$P(E1) = P(E2) = P(E3) = \frac{1}{3}$$

$$P(EB|E1) = \frac{5}{8}, P(EB|E2) = \frac{4}{8}, P(EB|E3) = \frac{2}{8}$$

Now by Bayes' Theorem for white

$$P(E2|EW) = \frac{P(E2) \cdot P(EW|E2)}{P(E1) \cdot P(EW|E1) + P(E2) \cdot P(EW|E2) + P(E3) \cdot P(EW|E3)}$$

$$P(E2|EW) = \frac{(\frac{1}{3} \cdot \frac{4}{8})}{(\frac{1}{3} \cdot \frac{3}{8}) + (\frac{1}{3} \cdot \frac{4}{8}) + (\frac{1}{3} \cdot \frac{6}{8})} = 0.19$$

Now by Bayes' Theorem for black

$$P(E2|EB) = \frac{P(E2) \cdot P(EB|E2)}{P(E1) \cdot P(EB|E1) + P(E2) \cdot P(EB|E2) + P(E3) \cdot P(EB|E3)}$$

$$P(E2|EB) = \frac{(\frac{1}{3} \cdot \frac{4}{8})}{(\frac{1}{3} \cdot \frac{5}{8}) + (\frac{1}{3} \cdot \frac{4}{8}) + (\frac{1}{3} \cdot \frac{2}{8})} = 0.21$$

To get to know if two pins of the opposite color from a selected box we have to multiply probabilities thus:
 $P(EW|E2 \& EB|E2) = 0.19 * 0.21 = \mathbf{0.039}$

Exercise 7

Our set B can be written as the union of two disjoint sets A and $A^c \cap B$.

So we can have $P(A) + P(A^c \cap B) = P(B) \rightarrow P(A) \leq P(B)$ since $A(A^c \cap B) \geq 0$.

Exercise 8

$$\text{Mean} = \frac{45+70+94+78+61+18+19+34+48+73+56+46+32+95+47+39+96+58+1+23+30+50+21+47+80+23+38+33+5+39+}{30} = 46.633333$$

Sorted sample from probability distribution: [1, 5, 18, 19, 21, 23, 23, 30, 32, 33, 34, 38, 39, 39, 45, 46, 47, 47, 48, 50, 56, 58, 61, 70, 73, 78, 80, 94, 95, 96]

- **Median** = 45.5
- **Mode** = 47
- **75% quantile** = 60.250000
- **Variance** = 655.1367816091955
- **Standard deviation** = 25.595639894505382
- **95% confidence interval** = (37.07576432247458, 56.19090234419208)
- **90% confidence interval** = (38.69313753342548, 54.57352913324119)