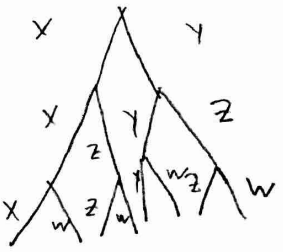
Eli Zolle
Homework 3

1) P_1: $Y >_1 Z >_1 X >_1 W$

P_2: $X >_2 Y >_2 W >_2 Z$

P_3: $W >_3 Z >_3 X >_3 Y$



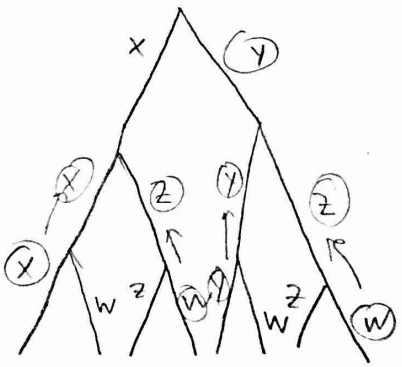
a) $A = (X, Y, Z, W)$

Assuming all people vote sincerely,

X beats Y, then Z beats X, then W beats Z so W wins

b) If people vote sophisticatedly,

$W > Z, Y > Z, Z > X, X > W, X > Y, Y > W$



In this case, X would be the Sophisticated winner because X beats W and Y but not Z. So if X could beat Z in round 2 X would win the whole thing

$A' = (X\ Y\ W\ W)$

c) In the second round vote between X and Z, if player 1 votes for X over Z, then X would win However, P1 would do better than if they voted sincerely because $X >_1 W$.

2) P1: $y >_1 z >_1 x >_1 w$

P2: $x >_2 y >_2 w >_2 z$

P3: $w >_3 z >_3 x >_3 y$

And $y > z, y > w, x > y, x > w, z > x, w > z$

a) the tournament described above is

$T = \{(x,y), (y,z), (y,w), (x,w), (z,x), (w,z)\}$

b) The copland winners can be computed as follows:

$|T^+(x)| = 2, |T^+(y)| = 2, |T^+(z)| = 1, |T^+(w)| = 1$

So the winners are x and y

c) x can beat y and w directly, y beats z so $x \in$ top-cycle set

y can beat w and z directly, z beats x so $y \in TCS$

z can beat x directly, x beats y and w so $z \in TCS$

w can beat z directly, z beats x so $w \notin TCS$

Then $TCS = \{x, y, z\}$

3) $q = .6$, $w_1 = .1$, $w_2 = .2$, $w_3 = .3$, $w_4 = .4$

a) Decisive coalitions of voters:

$\{(w_1, w_2, w_3, w_4), (w_1, w_2, w_3), (w_2, w_3, w_4), (w_2, w_4), (w_3, w_4), (w_1, w_3, w_4)$
$(w_1, w_3, w_4)\}$

b) Compute the Banzhaf indicies for these voters

w_1 has 1 Point
w_2 has 4 Points
w_3 has 4 points
w_4 has 5 Points
So the Banzhaf indicies are; $w_1: \frac{1}{14}$, $w_2: \frac{4}{14}$, $w_3: \frac{4}{14}$, $w_4: \frac{5}{14}$

c) Now $q = .65$

Decisive coalitions are:

$\{(w_1, w_2, w_3, w_4), (w_2, w_3, w_4), (w_3, w_4), (w_1, w_2, w_4), (w_1, w_3, w_4)\}$

w_1 has 1 Point
w_2 has 1 Point
w_3 has 3 Points
w_4 has 5 Points
So the indicies are; $w_1: \frac{1}{10}$, $w_2: \frac{1}{10}$, $w_3: \frac{3}{10}$, $w_4: \frac{5}{10}$

d) Neither of these examples have dummy voters because all voters are critical for at least 1 coalition in both cases.