

# Supplementary Material for “A Resilience-Oriented Multi-Stage Adaptive Distribution System Planning Considering Multiple Extreme Weather Events”

## MIXED-INTEGER LINEAR PROGRAM IMPLEMENTATION FOR OPTIMIZATION PROBLEM IN (24)

The optimization problem in (24) is equivalent to (S-1).

$$\begin{aligned} & \hat{\kappa}_s^{(J,M)} + \max_{\eta, \mathbf{u}_s} \eta \\ \text{s.t. } & \eta \leq \left[ \mathbf{b}_s^{\text{xtm}} + \mathbf{C}_s^{\text{u}} \mathbf{u}_s - \mathbf{C}_s^{\text{inv}} \hat{\mathbf{x}}^J - \mathbf{C}_s^{\text{adp}} \hat{\mathbf{x}}^{\text{adp}(J,M)} \right. \\ & \quad \left. - \mathbf{C}_s^{\text{d}} \hat{\mathbf{y}}_s^{\text{d}(n)} \right]^\top \boldsymbol{\lambda}_s^{(n)}, \quad \forall n = 1, \dots, N-1 \quad (\text{S-1a}) \end{aligned}$$

$$\mathbf{C}_s^{\text{c}\top} \boldsymbol{\lambda}_s^{(n)} = \mathbf{0}, \quad \forall n = 1, \dots, N-1 \quad (\text{S-1b})$$

$$-\mathbf{C}_s^{\sigma\top} \boldsymbol{\lambda}_s^{(n)} \leq \mathbf{M}_\sigma \cdot \mathbf{1}, \quad \forall n = 1, \dots, N-1 \quad (\text{S-1c})$$

$$\boldsymbol{\lambda}_s^{(n)} \leq \mathbf{0}, \quad \forall n = 1, \dots, N-1 \quad (\text{S-1d})$$

$$\mathbf{u}_s \in \mathcal{U}_s^{\text{xtm}} \quad (\text{S-1e})$$

There is a nonlinear term  $(\mathbf{C}_s^{\text{u}} \mathbf{u}_s)^\top \boldsymbol{\lambda}_s^{(n)}$  in (S-1a). Let  $\mathbf{C}_s^{\text{u}}(p, q)$ ,  $\boldsymbol{\lambda}_s^{(n)}(p)$ , and  $\mathbf{u}_s(q)$  denote the  $(p, q)$ -th entry (row  $p$ , column  $q$ ) of matrix  $\mathbf{C}_s^{\text{u}}$ , the  $p$ -th entry of vector  $\boldsymbol{\lambda}_s^{(n)}(p)$ , and the  $q$ -th entry of vector  $\mathbf{u}_s(q)$ , respectively. Define  $\alpha_{p,q} = \boldsymbol{\lambda}_s^{(n)}(p) \cdot \mathbf{u}_s(q)$ . We have  $(\mathbf{C}_s^{\text{u}} \mathbf{u}_s)^\top \boldsymbol{\lambda}_s^{(n)} = \boldsymbol{\lambda}_s^{(n)\top} \mathbf{C}_s^{\text{u}} \mathbf{u}_s = \sum_{(p,q) \in \mathcal{NZ}} \mathbf{C}_s^{\text{u}}(p, q) \cdot \boldsymbol{\lambda}_s^{(n)}(p) \cdot \mathbf{u}_s(q) = \sum_{(p,q) \in \mathcal{NZ}} \mathbf{C}_s^{\text{u}}(p, q) \cdot \alpha_{p,q}$ , where  $\mathcal{NZ}$  is the set of indices for non-zero entries of matrix  $\mathbf{C}_s^{\text{u}}$ .

The production term of continuous variable  $\boldsymbol{\lambda}_s^{(n)}(p)$  and binary variable  $\mathbf{u}_s(q)$  in the definition of  $\alpha_{p,q}$  can be linearized with the technique in (15), as shown in (S-2).

$$\begin{aligned} \underline{\boldsymbol{\lambda}}_s^{(n)}(p) \cdot \mathbf{u}_s(q) & \leq \alpha_{p,q} \leq \overline{\boldsymbol{\lambda}}_s^{(n)}(p) \cdot \mathbf{u}_s(q), \\ & \forall (p, q) \in \mathcal{NZ}, \forall n = 1, \dots, N-1 \quad (\text{S-2a}) \end{aligned}$$

$$\begin{aligned} \boldsymbol{\lambda}_s^{(n)}(p) - \overline{\boldsymbol{\lambda}}_s^{(n)}(p) \cdot (1 - \mathbf{u}_s(q)) & \leq \alpha_{p,q} \\ & \leq \underline{\boldsymbol{\lambda}}_s^{(n)}(p) - \underline{\boldsymbol{\lambda}}_s^{(n)}(p) \cdot (1 - \mathbf{u}_s(q)), \\ & \forall (p, q) \in \mathcal{NZ}, \forall n = 1, \dots, N-1 \quad (\text{S-2b}) \end{aligned}$$

where,  $\underline{\boldsymbol{\lambda}}_s^{(n)}(p)$  and  $\overline{\boldsymbol{\lambda}}_s^{(n)}(p)$  are lower and upper bounds for  $\boldsymbol{\lambda}_s^{(n)}(p)$ , respectively.

Finally, the optimization problem in (S-1) can be reformulated to a mixed-integer linear program by: 1) replacing  $(\mathbf{C}_s^{\text{u}} \mathbf{u}_s)^\top \boldsymbol{\lambda}_s^{(n)}$  in (S-1a) to  $\sum_{(p,q) \in \mathcal{NZ}} \mathbf{C}_s^{\text{u}}(p, q) \cdot \alpha_{p,q}$ , and 2) adding constraints in (S-2) to (S-1).

## MODIFIED 141-BUS SYSTEM DIAGRAM

The 141-bus system data is originally from [30]. Numerous typo corrections can be found in MATPOWER 141-bus test case [S1]. To test our distribution system planning approach, candidate lines, tie-lines, existing and candidate renewable energy sources are added. The diagram for the modified system is shown in Fig. S-1 on the next page.

## REFERENCES

- [S1] R. D. Zimmerman, C. E. Murillo-Sánchez, and R. J. Thomas, “Matpower: Steady-state operations, planning, and analysis tools for power systems research and education,” *IEEE Trans. Power Syst.*, vol. 26, no. 1, pp. 12–19, 2011.

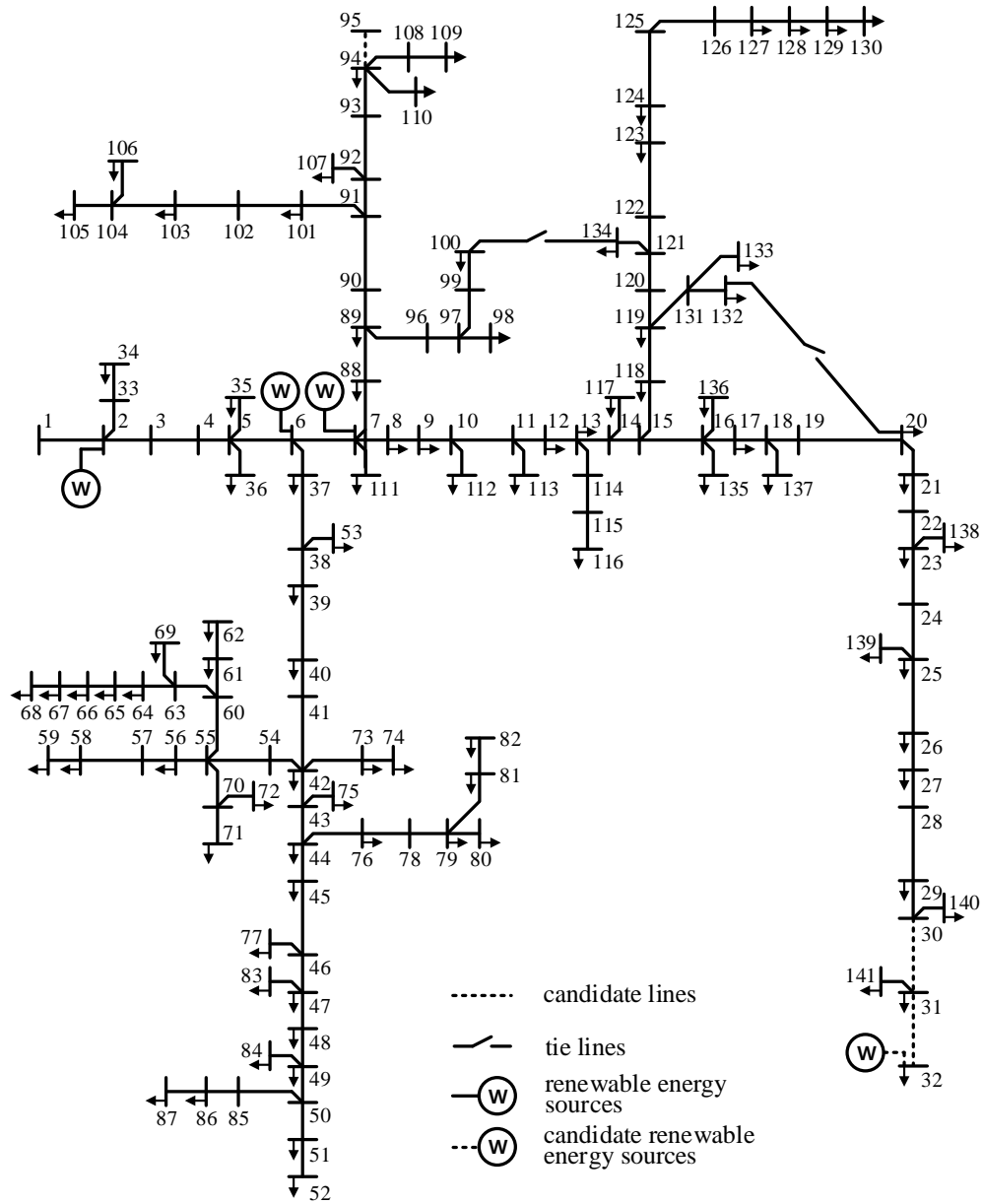


Fig. S-1. Diagram for modified 141-bus System.