Supplementary Material for "A Resilience-Oriented Multi-Stage Adaptive Distribution System Planning Considering Multiple Extreme Weather Events"

Mixed-Integer Linear Program Implementation for Optimization Problem in (24)

The optimization problem in (24) is equivalent to (S-1).

$$\hat{\kappa}_s^{(J,M)} + \max_{\eta, \boldsymbol{u}_s} \eta$$
$$\text{s.t.} \quad \eta \leq \Big[\boldsymbol{b}_s^{\mathrm{xtm}} + \boldsymbol{C}_s^{\mathrm{u}}\boldsymbol{u}_s - \boldsymbol{C}_s^{\mathrm{inv}}\hat{\boldsymbol{x}}^J - \boldsymbol{C}_s^{\mathrm{adp}}\hat{\boldsymbol{x}}_s^{\mathrm{adp}(J,M)} - \boldsymbol{C}_s^{\mathrm{d}}\hat{\boldsymbol{y}}_s^{\mathrm{d}(n)}\Big]^\top \boldsymbol{\lambda}_s^{(n)},\ \forall n = 1,\ldots,N-1 \tag{S-1a}$$
$$\boldsymbol{C}_s^{\mathrm{c}\top}\boldsymbol{\lambda}_s^{(n)} = \boldsymbol{0},\ \forall n = 1,\ldots,N-1 \tag{S-1b}$$
$$-\boldsymbol{C}_s^{\sigma\top}\boldsymbol{\lambda}_s^{(n)} \leq \mathrm{M}_\sigma \cdot \boldsymbol{1},\ \forall n = 1,\ldots,N-1 \tag{S-1c}$$
$$\boldsymbol{\lambda}_s^{(n)} \leq \boldsymbol{0},\ \forall n = 1,\ldots,N-1 \tag{S-1d}$$
$$\boldsymbol{u}_s \in \mathcal{U}_s^{\mathrm{xtm}} \tag{S-1e}$$

There is a nonlinear term $(\boldsymbol{C}_s^{\mathrm{u}}\boldsymbol{u}_s)^\top \boldsymbol{\lambda}_s^{(n)}$ in (S-1a). Let $\boldsymbol{C}_s^{\mathrm{u}}(p,q)$, $\boldsymbol{\lambda}_s^{(n)}(p)$, and $\boldsymbol{u}_s(q)$ denote the (p,q)-th entry (row p, column q) of matrix $\boldsymbol{C}_s^{\mathrm{u}}$, the p-th entry of vector $\boldsymbol{\lambda}_s^{(n)}(p)$, and the q-th entry of vector $\boldsymbol{u}_s(q)$, respectively. Define $\alpha_{p,q} = \boldsymbol{\lambda}_s^{(n)}(p) \cdot \boldsymbol{u}_s(q)$. We have $(\boldsymbol{C}_s^{\mathrm{u}}\boldsymbol{u}_s)^\top \boldsymbol{\lambda}_s^{(n)} = \boldsymbol{\lambda}_s^{(n)\top}\boldsymbol{C}_s^{\mathrm{u}}\boldsymbol{u}_s = \sum_{(p,q)\in\mathcal{NZ}} \boldsymbol{C}_s^{\mathrm{u}}(p,q) \cdot \boldsymbol{\lambda}_s^{(n)}(p) \cdot \boldsymbol{u}_s(q) = \sum_{(p,q)\in\mathcal{NZ}} \boldsymbol{C}_s^{\mathrm{u}}(p,q) \cdot \alpha_{p,q}$, where $\mathcal{NZ}$ is the set of indices for non-zero entries of matrix $\boldsymbol{C}_s^{\mathrm{u}}$.

The production term of continuous variable $\boldsymbol{\lambda}_s^{(n)}(p)$ and binary variable $\boldsymbol{u}_s(q)$ in the definition of $\alpha_{p,q}$ can be linearized with the technique in (15), as shown in (S-2).

$$\underline{\boldsymbol{\lambda}}_s^{(n)}(p) \cdot \boldsymbol{u}_s(q) \leq \alpha_{p,q} \leq \overline{\boldsymbol{\lambda}}_s^{(n)}(p) \cdot \boldsymbol{u}_s(q),\quad \forall (p,q) \in \mathcal{NZ}, \forall n = 1,\ldots,N-1 \tag{S-2a}$$
$$\boldsymbol{\lambda}_s^{(n)}(p) - \overline{\boldsymbol{\lambda}}_s^{(n)}(p) \cdot (1 - \boldsymbol{u}_s(q)) \leq \alpha_{p,q} \leq \boldsymbol{\lambda}_s^{(n)}(p) - \underline{\boldsymbol{\lambda}}_s^{(n)}(p) \cdot (1 - \boldsymbol{u}_s(q)),\quad \forall (p,q) \in \mathcal{NZ}, \forall n = 1,\ldots,N-1 \tag{S-2b}$$

where, $\underline{\boldsymbol{\lambda}}_s^{(n)}(p)$ and $\overline{\boldsymbol{\lambda}}_s^{(n)}(p)$ are lower and upper bounds for $\boldsymbol{\lambda}_s^{(n)}(p)$, respectively.

Finally, the optimization problem in (S-1) can be reformulated to a mixed-integer linear program by: 1) replacing $(\boldsymbol{C}_s^{\mathrm{u}}\boldsymbol{u}_s)^\top \boldsymbol{\lambda}_s^{(n)}$ in (S-1a) to $\sum_{(p,q)\in\mathcal{NZ}} \boldsymbol{C}_s^{\mathrm{u}}(p,q) \cdot \alpha_{p,q}$, and 2) adding constraints in (S-2) to (S-1).

Modified 141-bus System Diagram

The 141-bus system data is originally from [30]. Typo corrections can be found in MATPOWER 141-bus test case [31]. To test our distribution system planning approach, candidate lines, tie-lines, existing and candidate renewable energy sources are added. The diagram for the modified system is shown in Fig. S-1 on the next page.

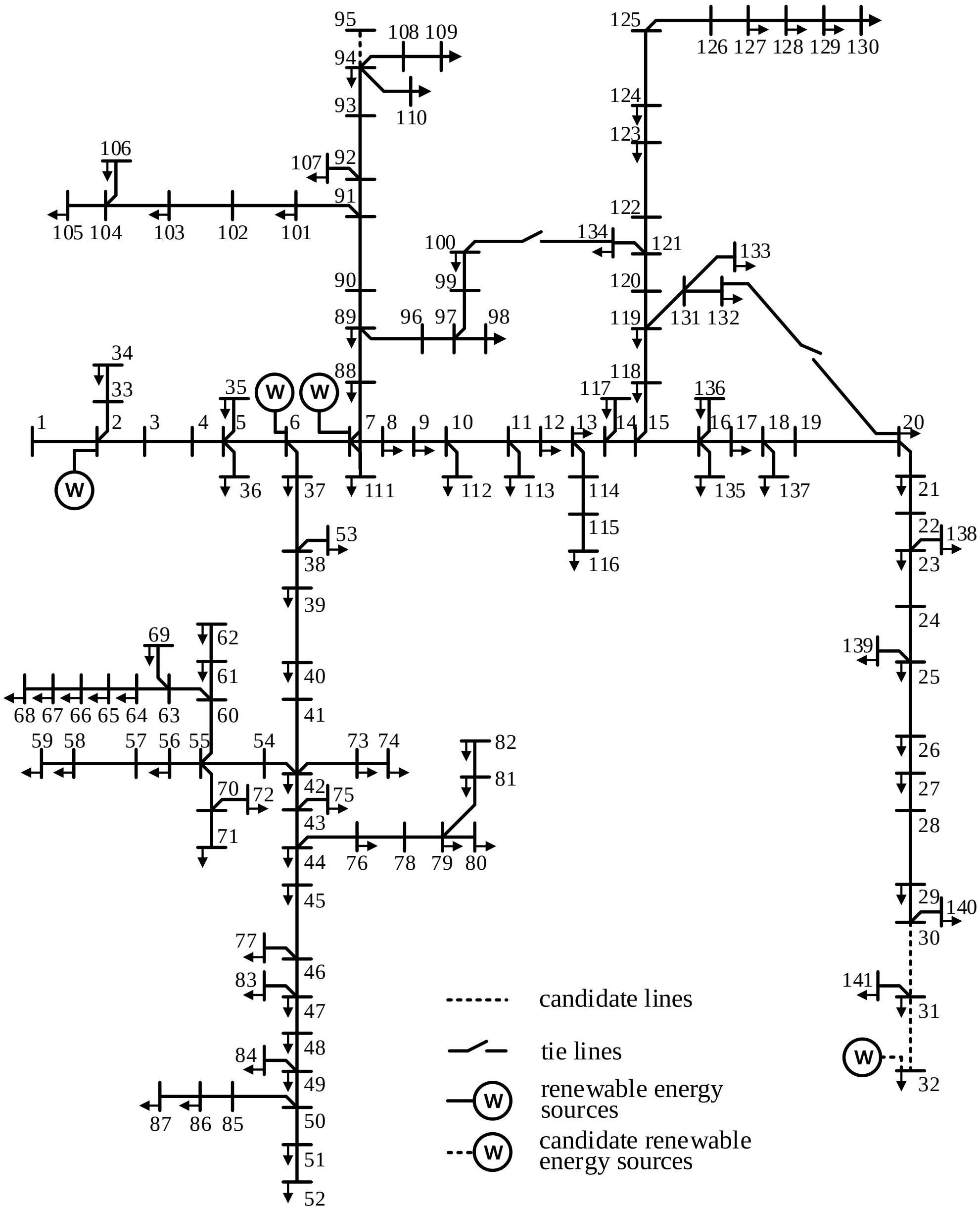

Fig. S-1. Diagram for modified 141-bus System.