Supplementary Material for "Approximating Input-Output Curve of Pumped Storage Hydro Plant: A Disjunctive Convex Hull Method"

Implementation of ZZI Formulation

The detailed formulation of the integer zig-zag (ZZI) piecewise linear approximation method is implemented in (S-1). The subscripts h, t for all variables are eliminated for convenience. To enable a fair comparison to the proposed method in this paper, we modified the model in [20] in the following aspects:

1) Volume at the beginning and end of time period t (i.e., $v_{r,t-1}$) is used to model generating and pumping input-output curves, which is consistent with that in (4) in the main text.

2) Realistic head-dependent bounds for flow rate are defined in (S-1l).

3) The volume variable modeling for the idle mode, as shown in (S-1a)-(S-1b), is consistent with that in (15) in the main text.

$$v = \sum_{i=1}^{m^{\mathrm{g}}}\sum_{j=1}^{n} \hat{v}_j^{\mathrm{g}} \cdot \phi_{i,j}^{\mathrm{g}} + \sum_{i=1}^{m^{\mathrm{p}}}\sum_{j=1}^{n} \hat{v}_j^{\mathrm{p}} \cdot \phi_{i,j}^{\mathrm{p}} + v^{\mathrm{i}} \tag{S-1a}$$

$$(1-u^{\mathrm{g}}-u^{\mathrm{p}})\cdot \underline{V} \le v^{\mathrm{i}} \le (1-u^{\mathrm{g}}-u^{\mathrm{p}})\cdot \overline{V} \tag{S-1b}$$

$$q^{\mathrm{g}} = \sum_{i=1}^{m^{\mathrm{g}}}\sum_{j=1}^{n} \hat{q}_i^{\mathrm{g}} \cdot \phi_{i,j}^{\mathrm{g}}, \quad p^{\mathrm{g}} = \sum_{i=1}^{m^{\mathrm{g}}}\sum_{j=1}^{n} \hat{p}_{i,j}^{\mathrm{g}} \cdot \phi_{i,j}^{\mathrm{g}} \tag{S-1c}$$

$$p^{\mathrm{p}} = \sum_{i=1}^{m^{\mathrm{p}}}\sum_{j=1}^{n} \hat{p}_i^{\mathrm{p}} \cdot \phi_{i,j}^{\mathrm{p}}, \quad q^{\mathrm{p}} = \sum_{i=1}^{m^{\mathrm{p}}}\sum_{j=1}^{n} \hat{q}_{i,j}^{\mathrm{p}} \cdot \phi_{i,j}^{\mathrm{p}} \tag{S-1d}$$

$$u^{\mathrm{g}} = \sum_{i=1}^{m^{\mathrm{g}}}\sum_{j=1}^{n} \phi_{i,j}^{\mathrm{g}}, \quad \phi_{i,j}^{\mathrm{g}} \ge 0 \tag{S-1e}$$

$$u^{\mathrm{p}} = \sum_{i=1}^{m^{\mathrm{p}}}\sum_{j=1}^{n} \phi_{i,j}^{\mathrm{p}}, \quad \phi_{i,j}^{\mathrm{p}} \ge 0 \tag{S-1f}$$

$$\sum_{i=1}^{m^{\mathrm{g}}}\left(C_{i-1,k}^{r^{\mathrm{g}}} \cdot \sum_{j=1}^{n} \phi_{i,j}^{\mathrm{g}}\right) \le \varsigma_k^{\mathrm{g}} \tag{S-1g}$$

$$\le \sum_{i=1}^{m^{\mathrm{g}}}\left(C_{i,k}^{r^{\mathrm{g}}} \cdot \sum_{j=1}^{n} \phi_{i,j}^{\mathrm{g}}\right) \tag{S-1h}$$

$$\sum_{i=1}^{m^{\mathrm{p}}}\left(C_{i-1,k'}^{r^{\mathrm{p}}} \cdot \sum_{j=1}^{n} \phi_{i,j}^{\mathrm{p}}\right) \le \varsigma_{k'}^{\mathrm{p}} \tag{S-1i}$$

$$\le \sum_{i=1}^{m^{\mathrm{p}}}\left(C_{i,k'}^{r^{\mathrm{p}}} \cdot \sum_{j=1}^{n} \phi_{i,j}^{\mathrm{p}}\right) \tag{S-1j}$$

$$\sum_{j=1}^{n}\left(C_{j-1,l}^{s} \cdot \left(\sum_{i=1}^{m^{\mathrm{g}}} \phi_{i,j}^{\mathrm{g}} + \sum_{i=1}^{m^{\mathrm{p}}} \phi_{i,j}^{\mathrm{p}}\right)\right) \le \zeta_l$$

$$\le \sum_{j=1}^{n}\left(C_{j,l}^{s} \cdot \left(\sum_{i=1}^{m^{\mathrm{g}}} \phi_{i,j}^{\mathrm{g}} + \sum_{i=1}^{m^{\mathrm{p}}} \phi_{i,j}^{\mathrm{p}}\right)\right) \tag{S-1k}$$

$$\sum_{j=1}^{n}\left(\underline{Q}_j^{\mathrm{g}} \cdot \sum_{i=1}^{m^{\mathrm{g}}} \phi_{i,j}^{\mathrm{g}}\right) \le q^{\mathrm{g}} \le \sum_{j=1}^{n}\left(\overline{Q}_j^{\mathrm{g}} \cdot \sum_{i=1}^{m^{\mathrm{g}}} \phi_{i,j}^{\mathrm{g}}\right) \tag{S-1l}$$

$$\varsigma_k^{\mathrm{g}}, \varsigma_{k'}^{\mathrm{p}}, \zeta_l \in \mathbb{Z} \qquad \forall k=1,\dots,r^{\mathrm{g}},\ \forall k'=1,\dots,r^{\mathrm{p}},\ \forall l=1\dots,s \tag{S-1m}$$

$$\sum_{(i,j)\in S_1} \phi_{i,j}^{\mathrm{g(p)}} \le z_1^{\mathrm{g(p)}}, \qquad \sum_{(i,j)\in S_2} \phi_{i,j}^{\mathrm{g(p)}} \le 1 - z_1^{\mathrm{g(p)}} \tag{S-1n}$$

$$\sum_{(i,j)\in S_3} \phi_{i,j}^{\mathrm{g(p)}} \le z_2^{\mathrm{g(p)}}, \qquad \sum_{(i,j)\in S_4} \phi_{i,j}^{\mathrm{g(p)}} \le 1 - z_2^{\mathrm{g(p)}} \tag{S-1o}$$

$$z_1^{\mathrm{g(p)}}, z_2^{\mathrm{g(p)}} \in \{0,1\} \tag{S-1p}$$

where $r^{\mathrm{g}} = \lceil \log_2(m^{\mathrm{g}}-1)\rceil$, $r^{\mathrm{p}} = \lceil \log_2(m^{\mathrm{p}}-1)\rceil$, and $s = \lceil \log_2(n-1)\rceil$. The sets S_1 to S_4 are defined in (S-2). $a \equiv b \pmod c$ means a and b are congruent modulo c, i.e., $a - c\cdot\lfloor a/c\rfloor = b - c\cdot\lfloor a/c\rfloor$.

$$S_1 = \{(i,j): i \equiv j \pmod 2 \text{ and } i+j \equiv 2 \pmod 4\} \tag{S-2a}$$

$$S_2 = \{(i,j): i \equiv j \pmod 2 \text{ and } i+j \equiv 0 \pmod 4\} \tag{S-2b}$$

$$S_3 = \{(i,j): i \not\equiv j \pmod 2 \text{ and } i+j \equiv 3 \pmod 4\} \tag{S-2c}$$

$$S_4 = \{(i,j): i \not\equiv j \pmod 2 \text{ and } i+j \equiv 1 \pmod 4\} \tag{S-2d}$$

Variable $\phi_{i,j}^{\mathrm{g(p)}}$ is convex combination coefficient variable for zig-zag formulations; $\tilde{\varsigma}_k^{\mathrm{g}}, \tilde{\varsigma}_{k'}^{\mathrm{p}}, \tilde{\zeta}_l$ are integer decision variables for integer zig-zag formulation; $z_1^{\mathrm{g(p)}}, z_2^{\mathrm{g(p)}}$ are binary decision variables for triangular selections. Other variables are defined similar to those in the main text.

Coefficients $m^{\mathrm{g}}, m^{\mathrm{p}}, n$ are numbers of discretization points on the generating flow rate, pumping power, and volume axis (the numbers of pieces are $m^{\mathrm{g}}-1, m^{\mathrm{p}}-1, n-1$, respectively); $r^{\mathrm{g}}, r^{\mathrm{p}}, s$ are sizes of variables ς_k^{g}, $\varsigma_{k'}^{\mathrm{p}}$, and ζ_l; $(\hat{q}_i^{\mathrm{g}}, \hat{v}_j, \hat{p}_{i,j}^{\mathrm{g}})$ are points obtained from $m^{\mathrm{g}}\times n$ discretization of the generating input-output curve; $(\hat{p}_i^{\mathrm{p}}, \hat{v}_j, \hat{q}_{i,j}^{\mathrm{p}})$ are points obtained from $m^{\mathrm{p}}\times n$ discretization of the pumping input-output curve; C^r is coefficient matrix for zig-zag formulations; $C_{i,k}^r$ is entry in row i and column k of matrix C^r. The values of these coefficients are defined in [20].

Testing Data for LMP

Eight LMP profiles used in the case study are given in Table S-I, which is shown on the next page. These historical LMP profiles are from MISO day-ahead market [35]. We assume 50% initial available volume percentage in the base test cases. The inflow and outflow are set as zero in this case.

TABLE S-I
LMP PROFILE DATA IN CASE STUDY

hour	LMP 1	LMP 2	LMP 3	LMP 4	LMP 5	LMP 6	LMP 7	LMP 8
1	20.48	23.03	16.49	18.43	24.81	27.57	45.79	38.08
2	20.07	21.79	16.01	17.40	23.22	24.64	45.10	36.76
3	20.26	21.58	17.42	16.41	22.68	23.63	42.19	37.94
4	20.31	21.62	16.37	16.85	22.71	23.52	43.92	38.58
5	20.58	21.68	18.64	19.05	22.48	23.42	44.88	40.40
6	21.19	23.55	22.37	22.94	21.70	25.97	47.43	43.20
7	25.86	29.17	25.59	25.29	21.87	29.55	48.86	47.21
8	32.75	34.68	24.15	25.18	24.83	34.34	49.02	46.59
9	30.55	33.80	25.22	25.56	29.46	41.04	51.16	50.07
10	29.06	32.93	26.15	26.44	33.93	42.12	57.41	53.44
11	27.88	32.11	26.01	27.27	38.74	49.05	59.61	52.18
12	26.48	29.69	27.13	27.36	41.18	61.71	60.19	46.68
13	23.76	27.57	28.09	28.44	46.08	68.33	62.86	45.41
14	22.61	25.95	31.25	28.24	56.43	72.34	65.92	45.13
15	21.91	23.78	30.28	29.10	59.79	76.63	73.85	44.80
16	21.42	23.60	34.78	26.84	64.52	81.68	78.52	46.64
17	22.14	24.09	37.91	27.63	69.06	83.14	81.35	49.98
18	27.20	30.70	34.76	28.79	66.94	78.18	76.94	61.87
19	28.27	33.11	27.94	26.55	59.65	69.90	71.28	73.88
20	29.00	31.88	29.44	28.44	51.74	55.93	62.31	54.53
21	27.07	29.80	24.77	26.25	42.74	47.08	54.45	47.83
22	25.61	27.56	22.32	23.95	39.03	40.11	49.65	42.64
23	23.43	24.98	21.46	23.70	34.17	33.79	46.40	41.98
24	22.87	22.79	19.32	22.32	29.98	29.46	44.40	38.47