

EE 102: Signal Processing and Linear Systems

Homework #11: Sampling

Name: _____

Submission Date: _____

Problem 1 You know the following properties of a continuous-time signal $x(t)$:

- It is dominated by five main frequency components: 100 Hz, 300 Hz, 600 Hz, 900 Hz, and 1200 Hz, with other frequencies between 100 Hz and 1500 Hz having negligible amplitudes.
- The maximum amplitude of the signal in time-domain is 2.
- It is known that the signal can be uniquely reconstructed from its samples with a sampling frequency of 10,000 Hz.

(a) [5 points] Using the information provided, sketch the frequency domain representation $X(\omega)$. You will not be able to draw an exact sketch.

(b) [10 points] For what values of the sampling frequency f_s (in Hz) is $X(\omega)$ guaranteed to be zero?

(c) [5 points] What is the Nyquist sampling rate for this signal? Justify your answer.

(d) [5 points] Sketch a block diagram that shows the process of sampling and perfect reconstruction of the signal $x(t)$. Label all important components in your diagram and label the signal properties (e.g., sampling frequency, bandwidth, etc.) at each stage of the diagram.

(e) [5 points] By exploring the musical notes or human voice frequency ranges, suggest a real-world signal that could have similar frequency characteristics as the signal described above.

Problem 2 [Adapted from Problem 7.2 in Oppenheim & Willsky]

A signal $x(t)$ with Fourier transform $X(j\omega)$ undergoes impulse-train sampling (like what we discussed during the lectures) to generate

$$x_p(t) = \sum_{n=-\infty}^{\infty} x(nT) \delta(t - nT)$$

where $T = 10^{-4}$. For each of the following sets of constraints on $x(t)$ and/or $X(j\omega)$, does the sampling theorem guarantee that $x(t)$ can be recovered exactly from $x_p(t)$? Clearly justify your answers.

[5 points each]

- (a) $X(j\omega) = 0$ for $|\omega| > 5000\pi$
- (b) $X(j\omega) = 0$ for $|\omega| > 15000\pi$
- (c) $\operatorname{Re}\{X(j\omega)\} = 0$ for $|\omega| > 5000\pi$
- (d) $x(t)$ real and $X(j\omega) = 0$ for $\omega < -5000\pi$
- (e) $x(t)$ real and $X(j\omega) = 0$ for $\omega < -15000\pi$
- (f) $X(j\omega) * X(j\omega) = 0$ for $|\omega| > 15000\pi$
- (g) $|X(j\omega)| = 0$ for $\omega > 5000\pi$

Problem 3 [Adapted from Problem 7.22 in Oppenheim & Willsky]

The signal $y(t)$ is generated by convolving a band-limited signal $x_1(t)$ with another band-limited signal $x_2(t)$, that is,

$$y(t) = x_1(t) * x_2(t)$$

where

$$X_1(j\omega) = 0 \quad \text{for } |\omega| > 1000\pi, \quad X_2(j\omega) = 0 \quad \text{for } |\omega| > 2000\pi.$$

Impulse-train sampling is performed on $y(t)$ to obtain

$$y_p(t) = \sum_{n=-\infty}^{\infty} y(nT) \delta(t - nT).$$

(a) [15 points] Specify the range of values for the sampling period T which ensures that $y(t)$ is recoverable from $y_p(t)$.

(b) [20 points] Now, for a different pair of signals, $x_1(t) = \cos(2\pi t) + \cos(20\pi t)$ and $x_2(t) = \text{sinc}(\pi t)$ (where $\text{sinc}(t) = \frac{\sin(t)}{t}$, with $\text{sinc}(0) = 1$), write a computer program (in Python or MATLAB) to verify the theoretically derived range of values for T that ensure perfect reconstruction of $y(t)$ from $y_p(t)$. Then, visually show how the reconstruction of $y(t)$ from $y_p(t)$ fails when you have higher values of T (i.e., lower sampling frequency) and succeeds when you have lower values of T (i.e., higher sampling frequency). Include your code and plots in your submission.