1 Objectives

This lab has four learning objectives:

    a. become familiar with disturbance-response experiments and repeatable data collection in SE1 172,

    b. quantify an external disturbance applied to a pendulum at rest by estimating an *impulse magnitude*,

    c. connect time-domain decay measurements to frequency-domain pole locations (and a theoretical decay rate), and

    d. convert a second-order model to state space and generate a step response using `ct.step_response` (Python) or `step` (MATLAB).

Reminder from Lab #1: You already identified the *characteristic equation* of your pendulum setup by estimating ζ and ω_n from free-decay data. In this lab, you will **fix** ζ and ω_n to your Lab #1 values for your assigned setup and use them to quantify a disturbance event.

2 Pre-lab theory (must complete before lab)

2.1 Model (fixed from Lab #1)

We model the pendulum as a rigid body rotating in a vertical plane about a pivot. Let $\alpha(t)$ be the angular displacement from the stable downward equilibrium. In the small-angle regime, the linearized equation of motion is

$$J\,\ddot{\alpha}(t) + b\,\dot{\alpha}(t) + mg\ell\,\alpha(t) = r(t), \tag{1}$$

where $r(t)$ is an externally applied torque (input).

It is common to rewrite the transfer function in standard second-order form,

$$G(s) \equiv \frac{A(s)}{R(s)} = \frac{K\,\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}, \tag{2}$$

where the poles (and hence the natural response) are completely determined by ζ and ω_n.

Important: In Lab #1, you did *not* apply a measured input $r(t)$, so you could not determine the gain K from data. In Lab #2, we will focus on:

- poles $\Rightarrow$ decay rate and oscillation frequency (these depend only on ζ, ω_n),

- a disturbance impulse magnitude $\Rightarrow$ a quantity you can estimate from the response amplitude *up to an unknown gain.*

2.2 Disturbance as an impulse (what a "hit" means)

In this lab, you will start with the pendulum at rest near the downward equilibrium and apply a brief disturbance by tapping the pendulum (a "hit"). We model this as a short torque pulse that is well-approximated by an impulse:

$$r(t) \approx I\,\delta(t),$$

where I is the *impulse magnitude* (units: N·m·s) and $\delta(t)$ is the Dirac delta. Taking Laplace transforms, $\mathcal{L}\{\delta(t)\} = 1$, so the input is

$$R(s) = I.$$

Therefore, the output in the Laplace domain is

$$A(s) = G(s)\,R(s) = I\,G(s). \tag{3}$$

This is the frequency-domain statement that "a unit impulse excites the system's modes".

2.3 Poles and theoretical decay rate (frequency-domain view)

The poles of $G(s)$ are the roots of

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0.$$

For the underdamped case $0 < \zeta < 1$, the poles are

$$s_{1,2} = -\zeta\omega_n \pm j\,\omega_d, \qquad \omega_d = \omega_n\sqrt{1 - \zeta^2}. \tag{4}$$

The real part

$$\sigma \equiv \zeta\omega_n$$

is the **decay rate** of the exponential envelope. Specifically, the oscillation envelope decays like $e^{-\sigma t}$.

Pre-Lab Quiz 2.1: Make sure you're ready for the lab!

Using equation (4), show that the impulse response (and any underdamped natural response) has an exponentially decaying envelope of the form $e^{-\sigma t}$, where $\sigma = \zeta\omega_n$.

2.4 Measured decay rate from peak ratios (time-domain view)

Let α_k and α_{k+1} be the magnitudes of two successive same-sign peaks separated by one period T. For underdamped motion, the envelope is approximately exponential, so

$$|\alpha_{k+1}| \approx |\alpha_k|\,e^{-\sigma T}.$$

Taking logs,

$$\ln\left(\frac{|\alpha_k|}{|\alpha_{k+1}|}\right) \approx \sigma T.$$

Define the log decrement for one period as

$$\delta_k \equiv \ln\left(\frac{|\alpha_k|}{|\alpha_{k+1}|}\right).$$

Averaging over several peak pairs,

$$\bar{\delta} \approx \frac{1}{N-1}\sum_{k=1}^{N-1}\ln\left(\frac{|\alpha_k|}{|\alpha_{k+1}|}\right), \qquad T \approx \frac{1}{N-1}\sum_{k=1}^{N-1}(t_{k+1} - t_k).$$

This gives a measured decay rate

$$\sigma_{\text{meas}} \approx \frac{\bar{\delta}}{T}. \tag{5}$$

You will compare this to the **theoretical** decay rate

$$\sigma_{\text{theory}} = \zeta\omega_n$$

using your fixed Lab #1 values of ζ and ω_n.

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2.5 Settling time for a decaying oscillation

In a disturbance response experiment, the output returns to zero. We define settling time as the first time after which $\alpha(t)$ stays within a small band around zero.

Let $\alpha_{\max}$ be the magnitude of the first (or largest) peak after the hit. For a given tolerance ε (e.g., $\varepsilon = 0.02$ for a 2% band), define the settling time t_s as the smallest time such that

$$|\alpha(t)| \leq \varepsilon\,\alpha_{\max} \quad \text{for all } t \geq t_s.$$

Since the envelope decays like $e^{-\sigma t}$, a *theoretical* settling time estimate is obtained from

$$e^{-\sigma t_s} \approx \varepsilon \quad \Rightarrow \quad t_s \approx \frac{1}{\sigma} \ln\left(\frac{1}{\varepsilon}\right).$$

For $\varepsilon = 0.02$, $\ln(1/0.02) \approx 3.912$, so

$$t_{s,2\%} \approx \frac{3.912}{\sigma} = \frac{3.912}{\zeta \omega_n}. \tag{6}$$

2.6 Estimating an impulse magnitude from the response amplitude

From equation (3), the impulse response scales linearly with the impulse magnitude I. However, the absolute mapping from torque impulse to angle depends on the gain K, which you did not identify in Lab #1.

Therefore, in this lab you will estimate an **effective impulse magnitude**

$$I_{\text{eff}} \equiv I\,K, \tag{7}$$

which is identifiable from output data using fixed ζ, ω_n. This is still a meaningful quantity: it measures "how hard you hit" *in the units of your identified model.*

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One practical approach: fit the underdamped impulse response form. For $0 < \zeta < 1$, the impulse response of (2) has the form

$$\alpha(t) = I_{\text{eff}}\, \omega_n^2 \cdot \frac{1}{\omega_d}\, e^{-\sigma t} \sin(\omega_d t)\, u(t), \tag{8}$$

where $u(t)$ is the unit step (causality), $\sigma = \zeta \omega_n$, and $\omega_d = \omega_n \sqrt{1 - \zeta^2}$. With ζ, ω_n fixed, the only unknown scaling in (8) is I_{eff}. You can estimate I_{eff} by matching the measured amplitude of $\alpha(t)$ to this template (for example, using least squares over a time window after the hit).

Pre-Lab Quiz 2.4: Make sure you're ready for the lab!

Assume ζ and ω_n are fixed. Using equation (8), explain why I_{eff} is the only unknown scale factor in the impulse response. What information would you need (in principle) to separate I and K?

2.7 State-space model and step response

A convenient normalized second-order model is

$$\ddot{\alpha}(t) + 2\zeta \omega_n \dot{\alpha}(t) + \omega_n^2 \alpha(t) = \omega_n^2\, u(t), \tag{9}$$

which corresponds to the transfer function

$$\frac{A(s)}{U(s)} = \frac{\omega_n^2}{s^2 + 2\zeta \omega_n s + \omega_n^2}.$$

(Here, the input $u(t)$ is a *normalized* input; this avoids needing the unknown gain K.)

Define the state vector $x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} \alpha(t) \\ \dot{\alpha}(t) \end{bmatrix}$. Then

$$\dot{x}(t) = Ax(t) + Bu(t), \qquad y(t) = Cx(t) + Du(t), \tag{10}$$

with $y(t) = \alpha(t)$.

Pre-Lab Quiz 2.5: Make sure you're ready for the lab!

Starting from equation (9), write down A, B, C, D for the state-space model (10) with $y = \alpha$.

2.8   Before proceeding any further...

Show your pre-lab work, including your solutions to all pre-lab quizzes, to your TA. Your TA will assign points to you out of 30 for the pre-lab work. But if you have 0/30 points, you will not be allowed to proceed with the lab.

3   Experimental setup and data collection

The SE1 172 lab has three pendulum setups: the rotary pendulum with servo (Quanser), the cart pendulum (Quanser), or the ECP pendulum apparatus. In this lab, you will record the pendulum angle $\alpha(t)$ as a function of time after an externally applied disturbance (a brief hit/tap) while the system is otherwise in open loop. Before powering up, verify that the pendulum can swing freely, that cables are not in the path of motion, and that you are familiar with the emergency procedures.

Step 0: Open MATLAB / ECP and set path to EE 122 lab files. These steps are dependent on your setup. Follow the specific instructions for your assigned setup for your test run and then follow the directions in Steps 1 to 7 to collect data for your analysis.

Rotary pendulum setup: Go to C:\Desktop\EE 122 \Rotary Inverted Pendulum\Software\. Open `setup_rotpen.m` file to see the setup parameters and **run** the file. Make sure that the voltage amplifier is powered ON (green light on in the front). Open the Simulink model `q_rotpen_mdl_student_lab2.slx` and **click** on "Monitor & Tune", then on "Build for Monitoring". After building, **click** on "Connect" and then "Start". Open the Scope window showing the pendulum angle (α). Hold the servo angle still (someone else should hold it) for consistent measurements.

Cart pendulum setup: Open `setup_ip02_sip.m` file and **run** it. Then, follow the same steps as the rotary pendulum setup in the appropriate "quick start" Simulink model.

ECP pendulum setup: Open ECP software. Set the pendulum at rest and go to Utility and **click** on "Zero Position". This setup requires you to apply a calibration formula to get angles in degrees from the counts that you observe: `1 degree is 44.4 counts`. After zero positioning, set up data acquisition for the encoder angle and export raw data after your run.

Step 1: Confirm the sign convention and the units (you must use angles in radians — so if degrees are reported convert them!). With a few trial runs, understand how the data stream is being stored in MATLAB / ECP software.

Divide clear roles in your group: who will control the software, who will apply the disturbance, who will record the time of the hit, who will do the calculations, and who will fill out the worksheet.

Step 2: Bring the pendulum to the downward equilibrium and let it come to rest. Your goal is to apply a brief disturbance *starting from rest*. Wait until the angle trace is approximately constant (near zero) before applying the hit.

Step 3: Start recording $\alpha(t)$. Record at a high enough sampling rate so that you can capture multiple oscillations clearly. Note the sampling rate f_s (Hz).

Step 4: Apply a brief disturbance (a quick tap/hit) to the pendulum. Do **not** hit hard. The disturbance should be short compared to the oscillation period, and the resulting motion should remain in the small-angle regime (target peak magnitude less than $15°$). If your response is too large (nonlinear) or too small (buried in noise), repeat with a better disturbance.

Step 5: Continue recording until the oscillations visibly decay and the pendulum settles back near zero. Record long enough so that you can capture at least 6–8 same-sign peaks (for better averaging).

Step 6: Save your data! Email/send the data to yourself and move on from the experimental setup so that another team can perform the data collection.

A note on data export: When you hit "Stop", the data collection stops. Try to finish your run within 50 seconds from when you click "Start". After hitting "Stop", a `.mat` file will be saved in the appropriate EE122 lab folder. Sort the files in the folder by "Date modified" to find your file. YOU MUST RENAME THE FILE to include your group number and then email this file to yourself so that you can leave the setup for the next group. You should perform the analysis on your own computer.

TA note: If data export does not work, make sure to go to "Control Panel" on the "Hardware" tab and set the data archival settings to save the data to a `.mat` file and set the signal trigger to record data for at least 50 seconds.

Step 7: From your recorded data, identify same-sign peaks (t_k, α_k) for $k = 1, 2, \ldots$ after the hit. Use at least 5 peak pairs so you can average the decay and the period. Then:

- compute σ_{meas} from your data,
- compute $\sigma_{\text{theory}} = \zeta \omega_n$ from your Lab #1 values,

- compute and compare settling times (measured vs theory),

- estimate I_{eff} as the disturbance impulse scale.

Setup information

Your setup name (cart-pole, rotary, ECP): _____

Sampling rate f_s (Hz): _____

Your Lab #1 values for this setup:

$$\zeta = \text{_____} \qquad \omega_n \text{ (rad/s)} = \text{_____}$$

Peak table (after the hit)

Record at least 6 same-sign peaks after the disturbance. Use magnitudes $|\alpha_k|$ so the decay ratio is positive:

$$\delta_k = \ln\left(\frac{|\alpha_k|}{|\alpha_{k+1}|}\right).$$

Peak k	time t_k (s)	peak $|\alpha_k|$ (rad)	δ_k
1			
2			
3			
4			
5			
6			–

Measured decay rate vs theoretical decay rate

Compute the average period T using same-sign peaks:

$$T \approx \frac{1}{5}\sum_{k=1}^{5}(t_{k+1} - t_k), \qquad \omega_d = \frac{2\pi}{T}.$$

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Compute the average log decay $\bar{\delta}$:

$$\bar{\delta} \approx \frac{1}{5} \sum_{k=1}^{5} \ln\left(\frac{|\alpha_k|}{|\alpha_{k+1}|}\right).$$

Then compute the measured decay rate:

$$\sigma_{\text{meas}} \approx \frac{\bar{\delta}}{T}.$$

Compute the theoretical decay rate using Lab #1 values:

$$\sigma_{\text{theory}} = \zeta\omega_n.$$

Your computed values:
T (s): _____, $\bar{\delta}$: _____, σ_{meas} (1/s): _____

σ_{theory} (1/s): _____, ω_d (rad/s): _____

Compare σ_{meas} and σ_{theory}. Report a percent difference. Give one reason your measured decay rate may differ from theory (noise, nonlinear motion, imperfect "impulse", friction effects, etc.).

Settling time (measured vs theory)

Define α_{max} as the magnitude of the first (or largest) peak after the hit. Choose a tolerance band $\varepsilon = 0.02$ (2%).

$$\text{Threshold} = \varepsilon\,\alpha_{\text{max}}.$$

Measured settling time: from your time series, find the first time t_s after which $|\alpha(t)| \leq \varepsilon\alpha_{\text{max}}$ for all later times.

Theoretical settling time: use

$$t_{s,2\%} \approx \frac{3.912}{\sigma_{\text{theory}}} = \frac{3.912}{\zeta\omega_n}.$$

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Your values:

$\alpha_{\max}$ (rad): _____, Threshold $0.02\alpha_{\max}$ (rad): _____

$t_{s,\text{meas}}$ (s): _____, $t_{s,2\%,\text{theory}}$ (s): _____

Describe how you measured settling time from your data (what threshold you
used and how you decided it stayed within the band). Compare measured vs
theory and comment on agreement.

Estimating the disturbance impulse magnitude (effective scale)

In this lab you will estimate an *effective* impulse magnitude $I_{\text{eff}} = IK$, which scales
the impulse response.
Using the fixed ζ, ω_n, compute:

$$\sigma_{\text{theory}} = \zeta\omega_n, \qquad \omega_d = \omega_n\sqrt{1-\zeta^2}.$$

The impulse response template (up to scale) is:

$$\alpha(t) \approx I_{\text{eff}}\,\omega_n^2 \cdot \frac{1}{\omega_d}\,e^{-\sigma_{\text{theory}}t}\,\sin(\omega_d t).$$

Estimate I_{eff} by matching this template to your measured $\alpha(t)$ after the hit (for
example, by least squares fitting over a time window that includes multiple oscilla-
tions).

Your estimated value:

$$I_{\text{eff}} = \underline{\hspace{4cm}}$$

Explain (briefly) how you estimated I_{eff} from your data and report the time window used. If your estimate is sensitive to the chosen window, explain why.

State-space model and step response plot

Use the normalized second-order model:

$$\ddot{\alpha}(t) + 2\zeta\omega_n\dot{\alpha}(t) + \omega_n^2\alpha(t) = \omega_n^2\,u(t),$$

with states $x_1 = \alpha$, $x_2 = \dot{\alpha}$, output $y = \alpha$.

(A) Write A, B, C, D**:**

$$A = \begin{bmatrix} \rule{2cm}{0.4pt} & \rule{2cm}{0.4pt} \\ \rule{2cm}{0.4pt} & \rule{2cm}{0.4pt} \end{bmatrix}, \quad B = \begin{bmatrix} \rule{2cm}{0.4pt} \\ \rule{2cm}{0.4pt} \end{bmatrix}, \quad C = \begin{bmatrix} \rule{2cm}{0.4pt} & \rule{2cm}{0.4pt} \end{bmatrix}$$

(B) Generate a step response plot using A, B, C, D**:**

- In Python, use `control` (`ct`) and `ct.step_response`.

- In MATLAB, use `ss(A,B,C,D)` then `step(sys)`.

Use your Lab #1 values of ζ and ω_n. Include your plot in your submission.

Attach/insert your step response plot here (or include in your report). From the plot, estimate rise time, percent overshoot, and settling time of the normalized step response. Comment on how these time-domain properties relate to ζ and ω_n.