School of Engineering
University of Calirfornia, Merced

ME140 Vibration and Control

Experiment No. One

Estimation of m-c-k Coefficients of SODF Oscillators

Objectives

1. Learn how to determine the spring constant and the viscous damping coefficient of SODF spring-mass-damper oscillators.
2. Gain hands-on experience of experiments and data acquisition system.
3. Learn how to use Matlab for post-processing experimental data.

Background

An understanding of simple harmonic motion is fundamental to an engineer performing vibration analysis. Simple harmonic oscillator has three elements: an inertial element, a damping element and a stiffness element.
The experimental setup consists of a mass attached to a spring and a damper that is oscillating horizontally on a tabletop. The restoring force of the spring on the mass is proportional to the displacement from the equilibrium point where the mass is at rest. The damper force is proportional to the relative velocity across the damper. This damper can be made of a real dashpot, or can be viewed as a model of all the damping effect due to friction, air and the energy loss in the spring.

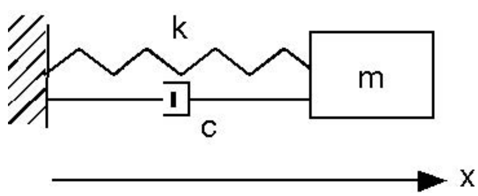

Figure 1: The physical model of the experiment.

We can define the *undamped natural frequency* ω_n of the system as follows:

$$\omega_n = \sqrt{\frac{k}{m}}$$

Applying Newton's secon law to the system shown in Figure 1, we yield,

$$m\ddot{x} + c\dot{x} + kx = 0 \quad (1)$$

The general solution of the equation is:

$$x(t) = C_1 e^{(-\frac{c}{2m}+\sqrt{(\frac{c}{2m})^2-\frac{k}{m}})}t + C_2 e^{-\frac{c}{2m}-\sqrt{(\frac{c}{2m})^2-\frac{k}{m}}}t \quad (2)$$

We define the *critical damping constant* c_c such that $(\frac{c_c}{2m})^2 - \frac{k}{m} = 0$. From this condition, we have $c_c = 2m\omega_n$. Conceptually, c_c is the lowest value of c for which the system's free response to initial conditions will *not* be oscillatory. The *damping ratio* ζ is defined as $\zeta = \frac{c}{c_c}$. Equation (2) can be rewritten as:

$$x(t) = C_1 e^{(-\zeta+\sqrt{\zeta^2-a})}\omega_n t + C_2 e^{(-\zeta-\sqrt{\zeta^2-1})}\omega_n t \quad (3)$$

System Classification

Vibrational systems are often classified according to the value of ζ in the following manner:
$\zeta < 1$: = Underdamped System (oscillatory response)
$\zeta = 1$: = Critically Damped System (non-oscillatory response)
$\zeta > 1$: = Overdamped System (non-oscillatory response)
The spring-mass-damper system we use in this lab falls in the underdamped category, which means that $\zeta^2 - 1 < 0$, and hence $x(t)$ will be oscillatory. A real valued solution of $x(t)$ in the underdamped case is:

$$x(t) = e^{-\zeta\omega_n t}(C'_1 cos\sqrt{1-\zeta^2}\omega_n t + C'_2 sin\sqrt{1-\zeta^2}\omega_n t) \quad (4)$$

or

$$x(t) = X_0 e^{-\zeta\omega_n t} cos(\omega_d t - \phi_o) \quad (5)$$

where $(C_1^{‘}, C_2^{‘})$ and (X_o, ϕ_0) are arbitrary constants to be determined from the initial conditions. The *damped natural frequency* ω_d of the system is defineed as:

$$\omega_d = \omega_n\sqrt{1-\zeta^2} \quad (6)$$

Logarithmic Decrement

The *logarithmic decrement* δ represents the rate at which the amplitude of a free damped vibration decreases. If we choose two times such that $t_2 = t_1 = \tau_d$ where τ_d is the period of the damped oscillator, then

$$\frac{x_1}{x_2} = \frac{X_0 e^{-\zeta\omega_n t_1} cos(\omega_d t_1 - \phi_0)}{X_0 e^{-\zeta\omega_n t_2} cos(\omega_d t_2 - \phi_0)} = \frac{e^{-\zeta\omega_n t_1}}{e^{-\zeta\omega_n(t_1+\tau_d)}} = e^{\zeta\omega_n\tau_d} \quad (7)$$

We are now ready to define δ:

$$\delta = ln(\frac{x_1}{x_2}) = \zeta\omega_n\tau_d = \frac{c}{2m}\tau_d \quad (8)$$

At this point we have defined all relevant system variables, and it is important to note that each of these variables $(\zeta, \omega_n, \omega_d, \delta, \tau_d)$ can be readily calculated if we know m, c, and k. In this lab, however, m, c, and k are all unknown. The objective of the experiment is to measure $(\zeta, \omega_n, \omega_d, \delta, \tau_d)$ experimentally so as to calculate m, c, and k.

Estimate of Mass

The experiment yields a set $(\zeta, \omega_n, \omega_d, \delta, \tau_d)$ values, which ultimately give us two ratios: c/m and k/m. Our goal is to determine m, c, and k. Obviously we need another equation. It should be pointed out that the overall system mass includes the unknown masses of the cart, armature and other motor components contributing to inertia. We can obtain the mass, m_c of the cart, amrature, and other motor components by comparing the k/m ratios for experiments with different added masses as follows:

$$\frac{k/(m_c + m_w^A)}{k/(m_c + m_w^B)} = \frac{C_A}{C_B} \tag{9}$$

where the added weights m_w^A and m_w^B are known, and C_A and C_B are experimentally determined k/m ratios for trials A and B, respectively. From Equation (9), one can readily calculate m_c.

Procedure

In the experiment, we will displace an underdamped spring-loaded cart with an added and known mass by a certain distance, and then record the oscillatory response position of the cart after it is released. There will be six trials altogether, according to the following chart:

Trial No.	1	2	3	4	5	6
Added Mass (kg)	0.1	0.1	0.1	0.5	0.5	0.5
Initial Displacement	1 cm	2 cm	3 cm	1 cm	2 cm	3 cm

Table 1: The weight of the added mass may be differnt, but will be known.

Each trial will proceed as follows:

1. Open ECP Executive software from the desktop icon. Go to <Command>\Trajectory>\and click <step>. Click <setup>. Choose open loop, and set step size = 0, dwell time = 3,000 ms, and Number of Reps = 1. Click <OK>and close window.

2. Go to <Command>\<Execute>and choose normal data sampling. On the physical setup, one group member displaces cart 1 at the appropriate distance. Click <run>and release the cart approximetly 1 second later.

3. Go to <Plotting>\<Setup Plot>. Choose 'encoder 1 position' only in the left axis box, then click <plot data. The plot will show the damped oscillations =of the dcart. The plot is intended to give you a qualitative feel for how the system reponds, the actual data processing will be done with Matlab.

4. Go to <Data>\<Export raw data>. Pick an appropriate file name, and export the data to a folder on your own storage. Close the ECP software after all trials are done.

5. Find a computer with Matlab installed. Save the sample file "damping.m" in a folder with your data on the computer. Open Matlab sample file and follow instructions as indicated in the program.

Action Items

1. Calculate an estimate of the mass of the empty cart and armature (m_c) using the method listed in the background section. You should obtain three m_c values by using the following ratios: $\frac{Trial_4}{Trial_1}, \frac{Trial_5}{Trial_2}, \frac{Trial_6}{Trial_3}$. Report the average of these three values as your experimental m_c value.

2. Create a data table which includes (ζ, ω_n, ω_d, δ, τ_d, k, c, m) for all 6 trials. The mass here is the overall system mass for that particular trial: $m = m_c + m_w$.

3. Briefly discuss any sources of error, and how they affect your final results.

4. Write a paragraph of constructive criticism for the lab manual and the performance of the TA. All suggestions are welcomed and appreciated.