EE 122: Introduction to Control Systems

Lab #3: Proportional Control of Cart Position

Released: Feb 9, 2026 **Due:** Feb 10/11, 2026

Objectives

1. Learn how to derive a transfer function to describe the linear motions of a cart.
2. Gain hands-on experience with experiments and Quanser data acquisition systems to control the position of a cart.
3. Learn how to use Matlab for post-processing experimental data.

1 Background

1.1 Modeling the System

The equations of motion for the cart are given by

$$M\dot{v}_{\text{cart}}(t) + B_{eq}v_{\text{cart}}(t) = A_m v_m(t) \tag{1}$$

where M is the mass of the cart, v_{cart} is the linear velocity of the cart, v_m is the motor input voltage, and B_{eq} is the equivalent damping coefficient. The equation of motion and the variables B_{eq} and A_m are derived from in the Appendix. By taking the Laplace transform, we can obtain the following first-order transfer function for the linear velocity of the IP02 cart with respect to the input motor voltage:

$$\frac{V_{cart}(s)}{V_{motor}(s)} = \frac{K}{\tau s + 1} \tag{2}$$

where V_{cart} is the Laplace transform for the cart velocity $[v_{\text{cart}}(t)]$, V_{motor} is the Laplace transform for the motor input voltage $[v_m(t)]$, K is the steady-state gain, τ is the time constant and s is the Laplace operator.

Pre-Lab Quiz 1.1: Make sure you're ready for the lab!

Derive the transfer function in equation (2) starting from the equations of motion in equation (1).

Pre-Lab Quiz 1.2: Make sure you're ready for the lab!

Plot the pole locations of the transfer function in equation (2) for the case when $M = 0.5$ kg, $B_{eq} = 0.1$ N·s/m, and $A_m = 0.2$ N/V.

Pre-Lab Quiz 1.3: Make sure you're ready for the lab!

Prove that the system is stable for the parameters given in the previous question.

1.2 Position Control Response

For a position control feedback system, we can design the system as shown below, $Y(s)$ being the Laplace transform of our measured output, namely the position of the cart.

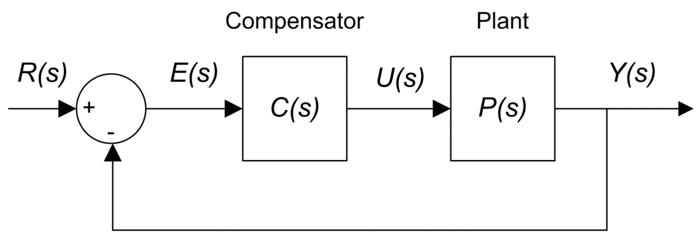

Figure 1: Unity Feedback System

Then $R(s)$, $E(s)$, and $U(s)$ be the Laplace transforms of the reference signal $r(t)$, error $e(t) = r(t) - y(t)$, and the input to the plant $u(t)$.

Pre-Lab Quiz 1.4: Make sure you're ready for the lab!

Repeat the lecture derivation to prove that the closed-loop transfer function is

$$\frac{Y(s)}{R(s)} = \frac{C(s)P(s)}{1 + C(s)P(s)} \tag{3}$$

Hint: Start by writing the output of this system as

$$Y(s) = C(s)P(s)(R(s) - Y(s)) \tag{4}$$

Then, rearrange the equation to get the closed-loop transfer function.

To get the voltage-to-position transfer function $P(s) = Y(s)/U(s)$, we introduced a integrator $\frac{1}{s}$ in series with the voltage-to-speed transfer function from equation (2) (effectively integrating the speed to get position). The result is

$$P(s) = \frac{Y(s)}{U(s)} = \frac{K}{s(\tau s + 1)} \tag{5}$$

In this lab, we will use proportional control law for our control input $u(t)$. This has the form:

$$u(t) = k_p \left[r(t) - y(t) \right] \tag{6}$$

Taking the Laplace transform allows use to represent the input signal $U(s)$ in terms of the reference $R(s)$ and output $Y(s)$:

$$U(s) = k_p \left[R(s) - Y(s) \right] \tag{7}$$

where k_p is the proportional gain, $r = x_d(t)$ is the set-point (desired) cart position, $y = x_a(t)$ is the measured cart position, and $u = v_m(t)$ is the control input or IP02 motor input voltage. Note that the proportional term is based on the error $(e = r - y)$.

We can use equations (5) and (7) to solve for the closed-loop transfer function, which gives:

$$\frac{Y(s)}{R(s)} = \frac{\frac{k_p K}{s(\tau s+1)}}{1 + \frac{k_p K}{s(\tau s+1)}} = \frac{k_p K}{\tau s^2 + s + k_p K}$$

The closed-loop transfer function is a second-order system. From week 3 lectures, we know that the step response of a second-order system is determined by the natural frequency and damping ratio, which are functions of the coefficients in the denominator of the transfer function. By adjusting the proportional gain k_p, we can change the characteristics of the step response.

2 Performance Metrics

Note that, in this case, we originally have a first-order system but the closed loop transfer function is second order. We can compute the step response metrics for this second-order system using the formulas we derived in week 3 lectures. Here we present this theory again.
We start with the standard form of a second-order transfer function:

$$\frac{Y(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \tag{8}$$

where ω_n is the natural frequency and ζ is the damping ratio. We can calculate the values of ω_n and ζ necessary for us to achieve our desired time-domain specifications.

3 Lab Goals: Desired Specifications

The minimum desired time-domain specifications (these are intentionally set losely; you can do better than these but this is the minimum expectation) are:

- peak time, $t_p = 0.15s$
- percent overshoot, $PO = 15\%$
- steady-state error, e_{ss}, as close to zero as possible

Percent overshoot (PO in Figure 2) is defined as:

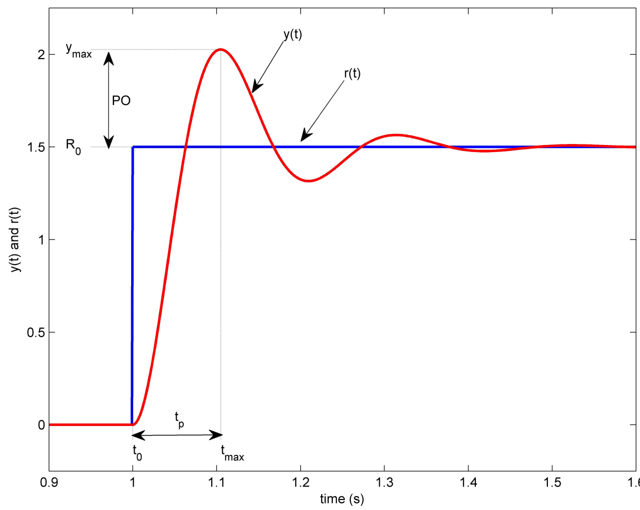

Figure 2: Standard Second-Order Step Response

$$\mathrm{PO} = \frac{100(y_{\max} - R_0)}{R_0} \tag{9}$$

where $y_{\max}$ is maximum output value and R_0 is the magnitude of the reference step input. Percent overshoot is dependent on the damping ratio, ζ, and can be expressed as:

$$\mathrm{PO} = 100 \exp\left\{ \left(\frac{-\pi\zeta}{\sqrt{1-\zeta^2}} \right) \right\} \tag{10}$$

Another quantity we want to control is the peak time, t_p, which is defined as:

$$t_p = t_{\max} - t_0 \tag{11}$$

where $t_{\max}$ is the time at which the maximum output is reached and t_0 is the initial time at which the step input is applied. While the percent overshoot only depends on the damping ratio; the peak time (t_p in Figure 2) needs to include both the damping ratio and the natural frequency:

$$t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} \tag{12}$$

Generally speaking, the damping ratio affects the shape of the response while the natural frequency affects the speed of the response.
The error of the system is given by the difference between the reference (desired cart position) and the output (actual measured cart position):

$$e(t) = r(t) - y(t) \tag{13}$$

Taking the Laplace transform gives:

$$E(s) = R(s) - Y(s) \tag{14}$$

The steady-state error, e_{ss}, is the error as time goes to infinity. Using the Final-Value Theorem, this is given by:

$$e_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s) \tag{15}$$

Pre-Lab Quiz 3.1: Make sure you're ready for the lab!

Compute the theoretical values for the peak time, steady-state error, and percent overshoot for the closed-loop system with proportional control by applying the formula above. Use the values of mass and τ from the previous questions and choose $k_p = 1$, just for this problem.

Pre-Lab Grade and Experimental Steps

Due: Feb 10/11, 2026 **Name:** ______________ **Grade:** _____

1 Experimental Steps

This experiment can only be performed on the cart position control setup. Make sure that MATLAB 2019b (other versions will not connect to the QUARC interface!).

1.1 Simulation

After opening MATLAB, change the workspace directory to the folder on the Desktop labeled "Lab #3". Open `s_ip02_position` on Simulink (see Figure 3 below).

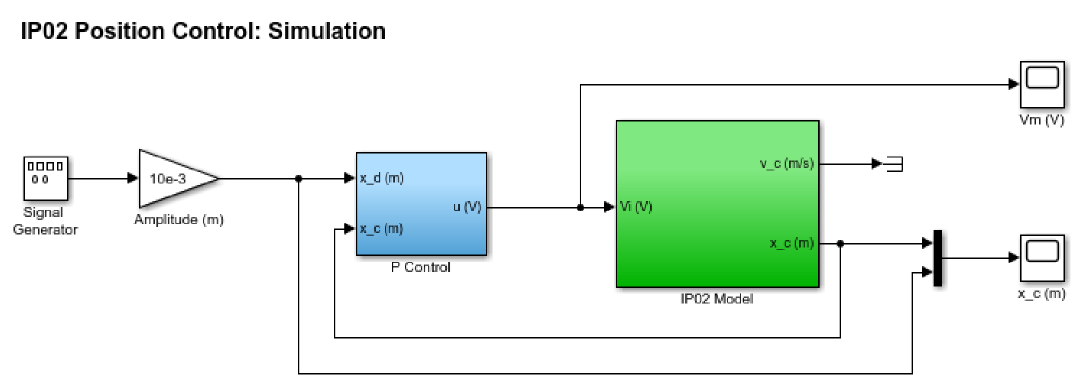

Figure 3: Simulink block diagram for position control simulation

1. Run the `setup_ip02_position` script
2. To generate a step reference, verify the `Signal Generator` is set to the following: Signal Type = `square`, Amplitude = 1, Frequency = 0.66 Hz.
3. Set the `Amplitude` (m) gain block to 0.01 to generate a step of 10 mm.
4. Make sure that the `P-Control` subsystem implements a proportional controller (double click and study the Simulink diagram to make sure you understand).

5. Open the cart position scope, x_cm and the motor input voltage scope, V_mV

6. Start the simulation and observe the response with the parameters assigned; take note of the colored lines in the x_cm graph: Yellow Trace: set-point position Purple Trace: simulated position

7. Change the values for k_p and observe how the changes affect the response. Run the simulation with the k_p values that have the best response within the desired time-domain specifications (refer to the desired specifications in the background section). Note at least 4-5 different values of k_p that you would like to test in the second part.

8. Export the data from the MATLAB workspace variables to your personal computer to compute the steady-state error, percent overshoot, and peak time. Helpful hints on computation:

   – `x_c(m)`: `data_pos`
     * `data_pos(:,1)` = time
     * `data_pos(:,2)` = simulated position
     * `data_pos(:,3)` = set-point position

   – `data_vm` (V): `data_vm`
     * `data_vm(:,1)` = time
     * `data_vm(:,2)` = simulated input voltage

1.2 Position Control Experiment

Open the Simulink file named `q_ip02_position` to view the block diagram as seen in Figure 4.

Experimental steps to get the closed-loop response with the P controller:

1. Run the `setup_ip02_position` script

2. Enter the calculated values for k_p in the MATLAB command line so that the variable k_p exists in the workspace with the desired value set.

3. From the Simulink diagram, adjust the `Signal Generator` to a `square` step reference

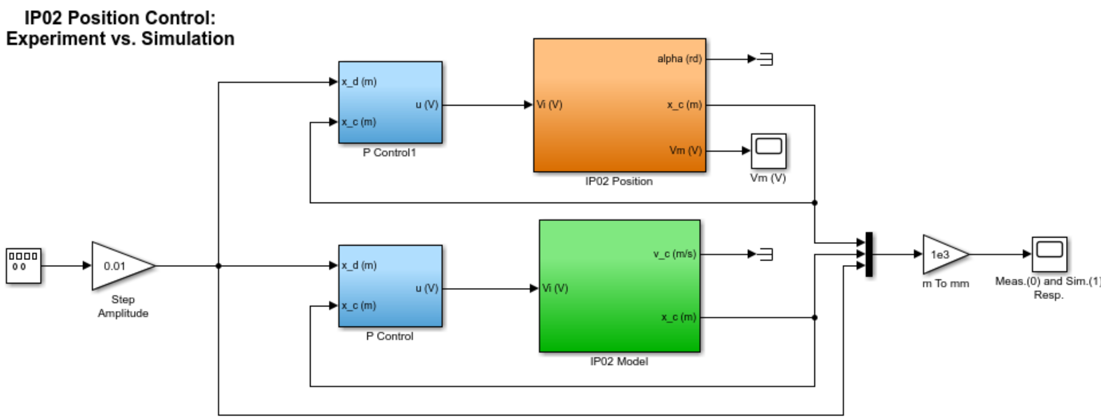

Figure 4: Simulink Block Diagram for Position Control

- Amplitude = 1
- Frequency = 0.66

4. Set the `Amplitude` (m) gain block to 0.01 to generate a step of 10 mm
5. Open the cart position scope (`Meas.(0)` and `Sim.(1) Resp.`) and the motor input voltage scope (`V_m` (V)) to view the graphical data in real-time
6. At the top, open the "`Monitor & Tune`" drop-down menu and click on "`Build for Monitoring`"
   - Verify that the physical amplifier unit is turned on with the gain set to 1x
   - Position the cart so that it is aligned in the center of the railing
7. Once the file has compiled, click on `Connect` then `Start`; click "`Stop`" when you see 3 steps on the graphs
   - Keep clear of the rail the cart travels on until the connection has been stopped
8. Export the data from `data_xc` and `data_vm`

2 Action Items

In your lab report, provide the following:

1. Provide a table with the parameter values used for the position control (k_p)

2. Provide graphs of the collected data.

3. Calculate the steady-state error, the percent overshoot, and the peak time for the Position and Speed data.

4. Briefly discuss any sources of error (between desired specifications and performance that you achieved for the three metrics), and how they affect your final results.

k_p	e_{ss}	% PO	t_p (s)
1. $k_p =$			
2. $k_p =$			
3. $k_p =$			
4. $k_p =$			

Discuss your results in detail here.

Appendix: Relevant Files

File Name	Description
setup_ip02_position.m	Run this file only to set up the experiment's position control parameters.
config_ip02.m	Returns the configuration-based IP02 model specifications Rm, Jm, Kt, Eff_m, Km, Kg, Eff_g, M, r_mp, and Beq
d_ip02_position.m	Determines the response specifications ζ and ω_n, and the control gains k_p and k_v.
d_model_param.m	Determines the model parameters K, and τ
s_ip02_position.mdl	Simulink file that simulates the closed-loop IP02 position control step response
q_ip02_position.mdl	Simulink file that implements the closed-loop IP02 position controller using QUARC

Table 1: Matlab files needed for the Position Control Experiment

File Name	Description
setup_ip02_speed.m	Run this file only to set up the experiment's speed control parameters.
config_ip02.m	Returns the configuration-based IP02 model specifications Rm, Jm, Kt, Eff_m, Km, Kg, Eff_g, M, r_mp, and Beq
d_ip02_speed_lead.m	Determines the control parameters K_c and a, and T.
d_ip02_speed_pi.m	Determines the control parameters K_p and k_i.
d_model_param.m	Determines the PI control parameters K, and τ
s_ip02_speed_lead.mdl	Simulink file that simulates the closed-loop IP02 lead speed control step response
q_ip02_speed_lead.mdl	Simulink file that implements the closed-loop IP02 lead speed controller using QUARC
q_ip02_speed_pi	Simulink file that implements the closed-loop IP02 PI speed controller

Table 2: Matlab files needed for the Speed Control Experiment

Appendix: Equations of Motion Derivation

In this section, we derive the equations of motion for the cart system. By Newton's Second Law, we can represent the force between the cart's DC motor and the motion as follows:

$$M\dot{v}_c(t) = F_c(t) - B_c v_c(t) \tag{16}$$

where M is the mass of the cart, v_c is the linear velocity of the cart, and B_c is the equivalent viscous damping coefficient as seen at the motor pinion. The driving force generated by the motor on the cart can be expressed as:

$$F_c = \frac{\eta_g K_g \tau_m}{r_{mp}} \tag{17}$$

where η_g is the gearbox efficiency, K_g is the gear ratio, τ_m is the torque generated by the motor, and r_{mp} is the motor pinion radius.

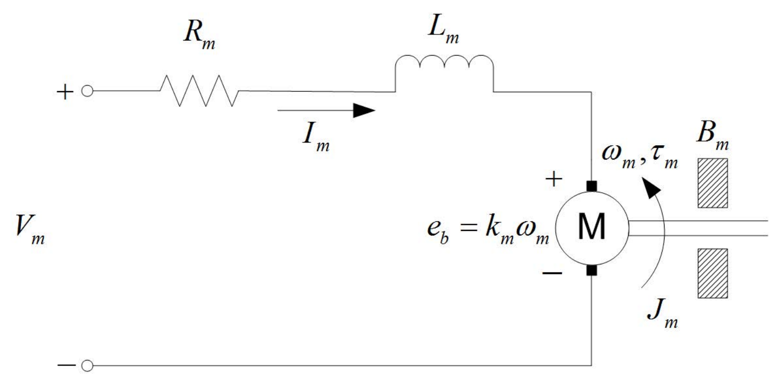

Figure 5: IP02 DC Motor Armature Circuit

Figure 5 displays the circuit for the driving motor where R_m is the motor resistance, L_m is the inductance, B_m is the motor damping, and k_m is the back-emf constant. Using Kirchoff's Voltage Law, we can write the voltage equation to solve for the inductance (given that L_m represents a much smaller value, it can be neglected):

$$V_m(t) - R_m I_m(t) - L_m I_m(t) - k_m \omega_m(t) = 0 \rightarrow I_m(t) = \frac{V_m(t) - k_m \omega_m(t)}{R_m} \tag{18}$$

By introducing the motor torque constant, k_t, we can expand equation (3) by substituting all the components:

$$F_c = \frac{\eta_g K_g \eta_m k_t (V_m(t) - k_m \omega_m(t))}{R_m r_{mp}} \quad where \quad \tau_m(t) = \eta_m k_t I_m(t) \tag{19}$$

The translation between angular to linear velocity from the motor to the cart is as follows

$$F_c = \frac{\eta_g K_g \eta_m k_t (V_m(t) r_{mp} - k_m K_g v_c(t)}{r_{mp}^2 R_m} \quad where \quad \omega_m(t) = \frac{K_g v_c(t)}{r_{mp}} \tag{20}$$

We can now substitute the force function into equation (2):

$$M \dot{v}_c(t) + B_c v_c(t) = \frac{\eta_g K_g \eta_m k_t (V_m(t) r_{mp} - k_m K_g v_c(t))}{r_{mp}^2 R_m} \tag{21}$$

We can rearrange the equation to combine like-terms:

$$M \dot{v}_c(t) + (\frac{k_m \eta_g K_g^2 \eta_m k_t}{r_{mp}^2 R_m} + B_c) v_c(t) = \frac{\eta_g K_g \eta_m k_t V_m(t)}{r_{mp} R_m} \tag{22}$$

This equation can be simplified as follows:

$$M \dot{v}_c(t) + B_{eq} v_c(t) = A_m V_m(t) \tag{23}$$

where the equivalent damping term is:

$$B_{eq} = \frac{\eta_g K_g^2 \eta_m k_t k_m + B_c r_{mp}^2 R_m}{r_{mp}^2 R_m} \tag{24}$$

and the actuator gain is:

$$A_m = \frac{\eta_g K_g \eta_m k_t}{r_{mp} R_m} \tag{25}$$