EE 122: Introduction to Control Systems

Problem Set #4: Your first controller

Name: ______________ **Submission Date:** ______________

Problem 1 The Chapter 4 of the FBS textbook (2nd Edition) is on various system examples and the kinds of control problems that arise in those systems. The chapter discusses the following examples:

1. Cruise Control
2. Bicycle Dynamics
3. Operational Amplifier Circuits
4. Computing Systems and Networks
5. Atomic Force Microscopy
6. Drug Administration
7. Population Dynamics

Your task in this problem is to thoroughly read ONE of the examples above and apply the techniques you have learned in this course so far to describe the system mathematically, analyze the system properties, and discuss at least one control problem (mathematically) for that system.

(a) Choose one of the examples above and do the following:

- [**10 points**] Describe the system mathematically by writing down the governing equations (ODEs, PDEs, TF, FSM etc.) that describe the system dynamics. You can use the equations in the textbook as a starting point. Once you have the model, describe whether the model is linear or nonlinear, time-invariant or time-varying, and SISO or MIMO (single-input single-output or multi-input multi-output).
- [**20 points**] Find at least two properties / metrics that are relevant for the system you chose. For example, if you choose the cruise control example, you can find the rise time and the overshoot of the step response.

- **[20 points]** Discuss at least one control problem for the system you chose. You may need to read external material to identify control problem for the system of your choice. No need to solve the problem fully — a description with at least 1-2 accompanying equations about the control problem is sufficient.

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Problem 2 The goal of this problem is to design a "bang-bang" controller for a thermostat system. A bang-bang controller is a simple on-off controller that applies maximum control effort when the system is below a certain threshold and turns off the control effort when the system is above that threshold. Consider a simple model of a thermostat system where the temperature $T(t)$ of a room is governed by the following first-order ODE:

$$\frac{dT(t)}{dt} = -\alpha(T(t) - T_{\mathrm{a}}) + \beta u(t)$$

where $\alpha > 0$ is the cooling coefficient, T_{a} is the ambient temperature, $\beta > 0$ is the heating coefficient, and $u(t)$ is the control input. Your task is to design a bang-bang controller that maintains the room temperature at a desired setpoint T_{set}. The controller should turn the heater on when the temperature is below T_{set} and turn it off when the temperature is above T_{set}. When you turn the heater on, the control input $u(t)$ should be set to a maximum value u_{max}, and when you turn it off, $u(t)$ should be set to zero.

(a) [**5 points**] Draw a block diagram of the closed-loop system with the controller. Label the input, output, and the controller clearly.

(b) [**5 points**] Write down the transfer function of the system without the controller (i.e., the open-loop transfer function from $u(t)$ to $T(t)$)..

(c) [**15 points**] Derive the expression for the control input $u(t)$ as a function of the temperature $T(t)$ and the setpoint T_{set}. Is this controller linear or nonlinear?

(d) [**20 points**] Simulate the closed-loop system with your bang-bang controller for a given set of parameters $\alpha, \beta, T_{\mathrm{a}}, T_{\mathrm{set}}$ and initial temperature $T(0)$. Plot the temperature response $T(t)$ over time and discuss the behavior of the system under your controller. You may use MATLAB or Python for the simulation. You must include your code and the resulting plot with your submission.

(e) [**5 points**] Discuss the advantages and disadvantages of using your controller. Propose another controller that could potentially improve the performance of the system and briefly describe how it would work (no need to simulate the new controller, just describe it).