

**Exercise 2-1:**

a. Answer:

For an array of size  $k$ , sorting it using insertion sort will cost  $\Theta(k^2)$  time.

Thus for  $n/k$  such sublists, the total running time is  $\Theta(n/k \cdot k^2) = \Theta(nk)$

b. Answer:

During the merging process, we pick the smallest element and put it in the merged list.

That's the " $n$ " in  $\Theta(n \lg(n/k))$

By using a minimal heap, we can find the smallest element in  $O(1)$  time.

But adjusting the heap requires  $O(\lg(n/k))$  time. Thus the total running time will be  $\Theta(n \lg(n/k))$  in the worst case.

c. Answer:

$$\text{Let } n \lg n = nk + n \lg(n/k)$$

$$\therefore n \lg n = nk + n \lg n - n \lg k$$

$$\therefore n \lg k = nk$$

$$\therefore k = \lg k, \text{ no solution.}$$

$$\therefore \text{But there is actually a constant } c \neq 1, \text{ s.t. } cn \lg n = nk + n \lg(n/k)$$

$$\therefore \text{There could be a } k \text{ that make them equal, but it will be fairly small.}$$

d. Answer:

When  $k=1$ ,  $T(n)=\Theta(n \lg n)$ , it's degenerated to heap sort, which is good enough.

When  $k=n$ ,  $T(n)=\Theta(n^2)$ , it's degenerated to insertion sort, which is good enough for small data sets.

So, make it small. How about 5?