

**Exercise 2.3-3:**

$$\therefore T(n) = \begin{cases} 2 & \text{if } n = 2, \\ 2T(n/2) + n & \text{if } n = 2^k, \text{ for } k > 1 \end{cases}$$

$\therefore$  When  $n = 2$ ,  $T(n) = 2$

$\therefore$  When  $n = 4$ ,  $T(n) = 2T(2) + 4$

$\therefore T(n) = n \lg n$ , for  $n = 2, 4$

$\therefore$  Suppose for  $k \in \{1, 2, \dots, K\}$ ,  $K \in N^*$ , when  $n = 2^k$ ,  $T(n) = n \lg n = k2^k$

$\therefore$  When  $k = K + 1$ ,  $n = 2^{K+1}$

$$T(n) = T(2^{K+1})$$

$$= 2T(2^K) + 2^{K+1}$$

$$= 2K2^K + 2^{K+1}$$

$$= (K + 1)2^{K+1}$$

$$= n \lg n$$

$\therefore$  It holds for  $k = K + 1$

$\therefore T(n) = n \lg n$