Exercise 3.2-7:

$\phi = \frac{1+\sqrt{5}}{2}$

$\hat{\phi} = \frac{1-\sqrt{5}}{2}$

$\therefore F_1 = \frac{\phi-\hat{\phi}}{\sqrt{5}} = 1$

$\therefore F_2 = \frac{\phi^2-\hat{\phi}^2}{\sqrt{5}} = 2$

$\therefore Let's\ suppose\ F_n = \frac{\phi^n-\hat{\phi}^n}{\sqrt{5}}$

$\therefore F_{n-1} = \frac{\phi^{n-1}-\hat{\phi}^{n-1}}{\sqrt{5}}$

$$\begin{aligned}\therefore F_{n+1} &= F_{n-1} + F_n \\ &= \frac{\phi^{n-1} \cdot \frac{3+\sqrt{5}}{2} - \hat{\phi}^{n-1} \cdot \frac{3-\sqrt{5}}{2}}{\sqrt{5}} \\ &= \frac{\phi^{n+1} - \hat{\phi}^{n+1}}{\sqrt{5}}\end{aligned}$$

$\therefore F_{n+1} = \frac{\phi^{n+1}-\hat{\phi}^{n+1}}{\sqrt{5}}$

$\therefore The\ induction\ holds\ for\ n = N+1$

$\therefore F_i = \frac{\phi^i-\hat{\phi}^i}{\sqrt{5}}$