Structure for indexing texts

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Plan

1. Introduction
2. Suffix Tree
3. Suffix Automaton
4. Compact Suffix Automaton

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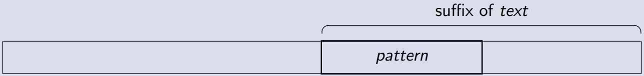
Indexes

- **Pattern matching** in static texts
- **Basic operations**
  - existence of patterns in the text
  - number of occurrences of patterns
  - list of positions of occurrences
- **Other applications**
  - finding repetitions in texts
  - finding regularities in texts
  - approximate matchings
  - ...

Implementation of indexes



Implementation

with efficient data structures

- **Suffix Trees**
  digital trees, PATRICIA tree (compact trees)
- **Suffix Automata or DAWG's**
  minimal automata, compact automata

with efficient algorithm

- **Suffix Arrays**
  binary search in the ordered list of suffixes

Efficient constructions

- **Position tree, suffix tree**
  [Weiner 1973], [McCreight, 1976], [Ukkonen, 1992] [Farach, 1997]
- **Suffix DAWG, suffix automaton, factor automaton**
  [Blumer *et al.*, 1983], [Crochemore, 1984]
- **Suffix array, PAT array**
  [Manber, Myers, 1993], [Gonnet, 1987]
  [Kärkkäinen, Sanders, 2003]
  [Kim et al., 2003], [Ko, Aluru, 2003]
- **Some other implementations of suffix trees**
  [Andersson, Nilsson, 1993], [Irving, 1995]
  [Kärkkäinen, 1995], [Munro et al., 1999]
- **For external memory** (*SB-trees*)
  [Ferragina, Grossi, 1995]
- **Compact suffix automaton**
  [Crochemore, Vérin, 1997], [Inenaga et al., 2001]

Suffixes

Text $y \in \Sigma^*$

- $\mathit{Suff}(y)$ = set of suffixes of y,
- $\mathrm{card}\,\mathit{Suff}(y) = |y| + 1$

Example

$\mathit{Suff}(\mathtt{ababbb})$

i	0	1	2	3	4	5		
$y[i]$	a	b	a	b	b	b		
							position	
	a	b	a	b	b	b	0	
		b	a	b	b	b	1	
			a	b	b	b	2	
				b	b	b	3	
					b	b	4	
						b	5	
						ε	6	(empty string)

Plan

Trie of suffixes

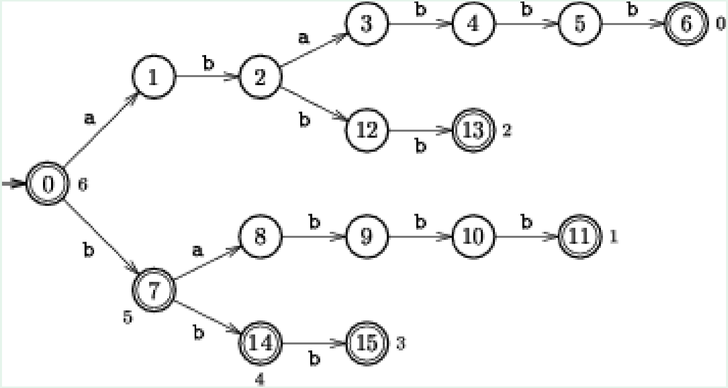
Text $y \in \Sigma^*$, *Suff*(y) set of suffixes of y

$\mathcal{T}(y)$ = digital tree which branches are labeled by suffixes of y
= tree-like deterministic automaton accepting *Suff*(y)

- **Nodes**
  identified with subwords of y
- **Terminal nodes**
  identified with suffixes of y, output = position of the suffix

Trie of suffixes

Suffix trie of ababbb



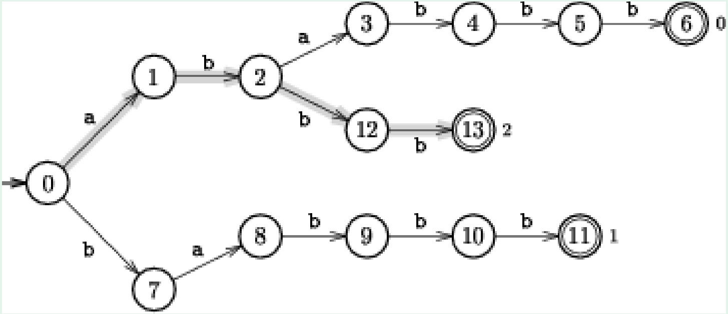
Forks

Insertion of $u = y[i \mathinner{.\,.} n-1]$ in the structure accepting longer suffixes

- **Head** of u: longest prefix $y[i \mathinner{.\,.} k-1]$ of u occurring before i
- **Tail** of u: rest $y[k \mathinner{.\,.} n-1]$ of suffix u
- $y =$ ababbb; head of abbb is ab; tail of abbb is bb
- **Fork**
  any node that has outdegree 2 at least,
  or that both has outdegre 1 and is terminal
- **Note**: the node associated with the head of u is a fork
  initial node is a fork iff y non empty

Forks

Example

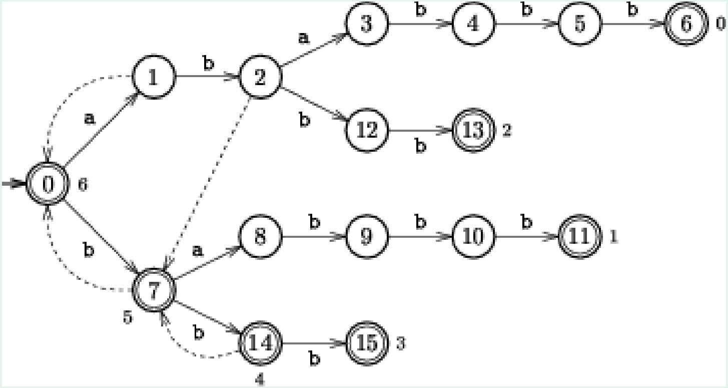


Suffix link

- **Function** s_y, *suffix link*
  if node p identified with subword av, $a \in \Sigma, v \in \Sigma^*$
  $s_y(p) = q$, node identified with v
- **Use**
  creates shortcuts used to accelerate heads computations
- **Useful for forks only**
  undefined on initial node
- **Note**: if p is a fork, so is $s_y(p)$

Suffix link

Example



Suffix Tree

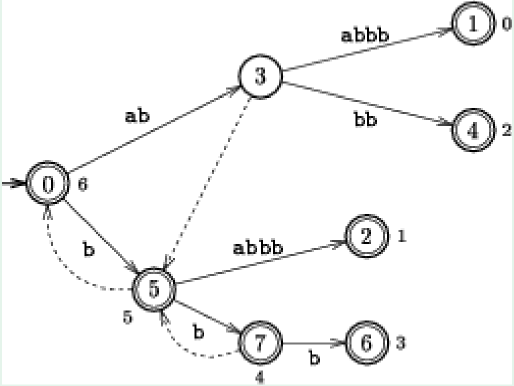
Text $y \in \Sigma^*$ of length n

$\mathcal{S}(y)$ suffix tree of y: compact trie accepting *Suff* (y)

- **Definition**
  tree obtained from the suffix trie of y by deleting all nodes having outdegree 1 that are not terminal
- **Edges labeled by subwords** of y instead of letters
- **Number of nodes**: no more than $2n$ (if $n > 0$)
  because all internal nodes have two children at least and there are at most n external nodes

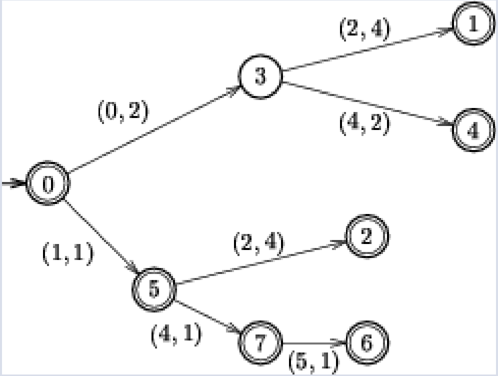
Suffix Tree

Example



Labels of edges

- **Labels represented by pairs** $(pos, Length)$



i	0	1	2	3	4	5
$y[i]$	a	b	a	b	b	b

- Requires to have y in main memory
- **Size of** $\mathcal{S}(y)$: $O(n)$

Scheme of suffix tree construction

```
SUFFIX-TREE(y)
T ← NEW-TREE()
for i ← 0 to n − 1 do
        find fork of head of y[i . . n − 1] using
            FAST-FIND from node s[r], and then SLOW-FIND
        k ← position of tail of y[i . . n − 1]
        if k < n then
                q ← NEW-STATE()
                Adj[fork] ← Adj[fork] ∪ {((k, n − k), q)}
                output[q] ← i
        else output[fork] ← i
return T
```

Adjacency-list representation of labeled arcs

Straight insertion

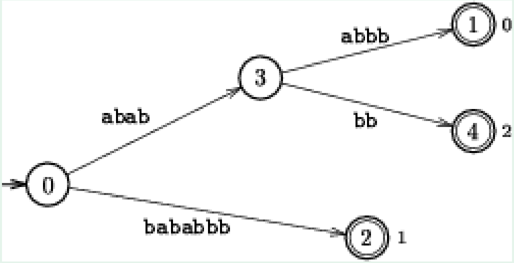
- Insertion of suffix ababbb is done by letter comparisons from the initial node (current node)

- It leads to create node 3 which suffix link is still undefined,
- and node 4 associated with suffix ababbb at position 2
- Head is abab, tail is bb

Straight insertion

Results of the insertion of ababbb

i	0	1	2	3	4	5	6	7
$y[i]$	a	b	a	b	a	b	b	b



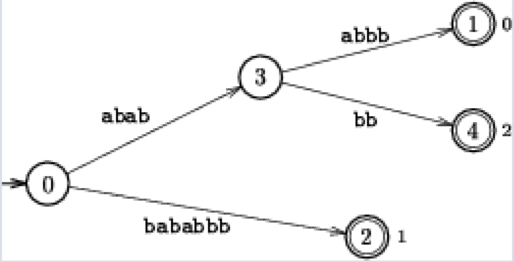
Slow find

$\textsc{Slow-Find}(p, k)$

```
while k < n and TARGET(p, y[k]) ≠ NIL do
    q ← TARGET(p, y[k])
    (j, ℓ) ← label(p, q)
    i ← j
    do  i ← i + 1
        k ← k + 1
    while i < j + ℓ and k < n and y[i] = y[k]
    if i < j + ℓ then
        Adj[p] ← Adj[p] \ {((j, ℓ), q)}
        r ← NEW-STATE()
        Adj[p] ← Adj[p] ∪ {((j, i − j), r)}
        Adj[r] ← Adj[r] ∪ {((j + i − j, ℓ − i + j), q)}
        return (r, k)
    p ← q
return (p, k)
```

New suffix link

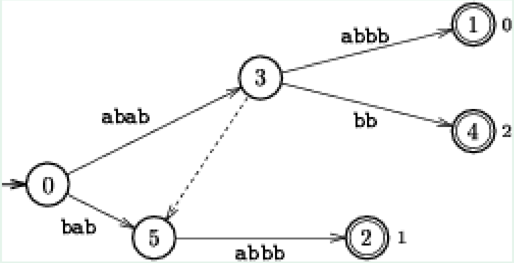
- Computing $s[3] = s_y(3)$ remains to find the node associated with bab



- Arc $(0,(1,7),2)$ is split into $(0,(1,3),5)$ and $(5,(4,4),2)$
- Execution in constant time (here)
- In general, iteration in time proportional to the number of nodes along the path (and not proportional to the length of the string)

New suffix link

i	0	1	2	3	4	5	6	7
$y[i]$	a	b	a	b	a	b	b	b



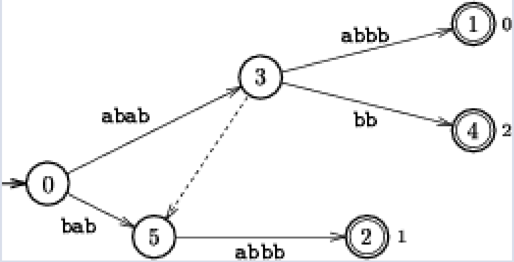
Fast find

$\text{Fast-Find}(r, j, k)$

$\triangleright$ computes $\text{Target}(r, y[j \mathinner{.\,.} k-1])$
if $j \geqslant k$ **then**
  return r
else $q \leftarrow \text{Target}(r, y[j])$
  $(j', \ell) \leftarrow \mathit{label}(r, q)$
  if $j + \ell \leqslant k$ **then**
    return $\text{Fast-Find}(q, j+\ell, k)$
  else $\mathit{Adj}[r] \leftarrow \mathit{Adj}[r] \setminus \{((j', \ell), q)\}$
    $p \leftarrow \text{New-state}()$
    $\mathit{Adj}[r] \leftarrow \mathit{Adj}[r] \cup \{((j, k-j), p)\}$
    $\mathit{Adj}[p] \leftarrow \mathit{Adj}[p] \cup \{((j'+k-j, \ell-k+j), q)\}$
    return p

Next insertion

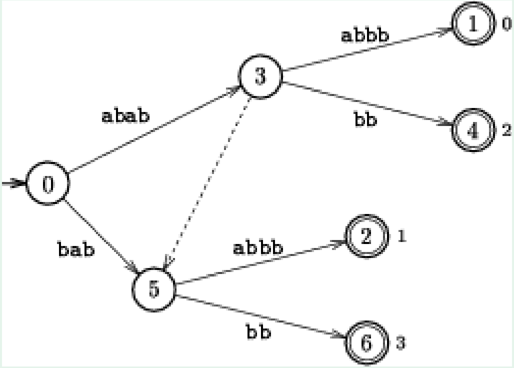
- End of insertion of suffix babbb



- Execution in constant time
- Head is bab, tail is bb

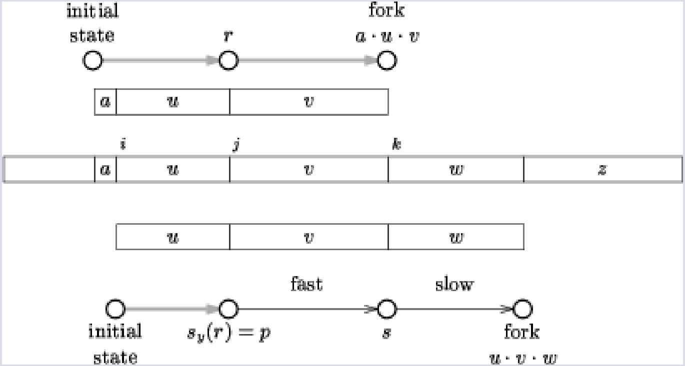
Next insertion

i	0	1	2	3	4	5	6	7
$y[i]$	a	b	a	b	a	b	b	b



Scheme for insertion

- Scheme for the insertion of suffix $y[i \,.\,.\, n-1] = u \cdot v \cdot w \cdot z$



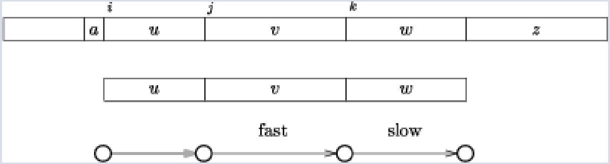
- It first computes $p = \textsc{Target}(s[r], v)$ with $\textsc{Fast-Find}$ (if necessary)
- then the fork of the current suffix with $\textsc{Slow-Find}$

Complete algorithm

```
SUFFIX-TREE(y)
T ← NEW-TREE()
s[initial[T]] ← initial[T]
(fork, k) ← (initial[T], 0)
for i ← 0 to n − 1 do
    k ← max{k, i}
    if s[fork] = NIL then
        r ← parent of fork
        (j, ℓ) ← label(r, fork)
        if r = initial[T] then
            ℓ ← ℓ − 1
        s[fork] ← FAST-FIND(s[r], k − ℓ, k)
    (fork, k) ← SLOW-FIND(s[fork], k)
    if k < n then
        q ← NEW-STATE()
        Adj[fork] ← Adj[fork] ∪ {((k, n − k), q)}
        output[q] ← i
    else output[fork] ← i
return T
```

Running time

- Scheme for insertion



- Main iteration increments i, which never decreases
- Iteration in $\textsc{Fast-Find}$ increments j, which never decreases
- Iteration in $\textsc{Slow-Find}$ increments k, which never decreases
- Basic operations run in constant time or in time $O(\log \operatorname{card} \Sigma)$

Theorem

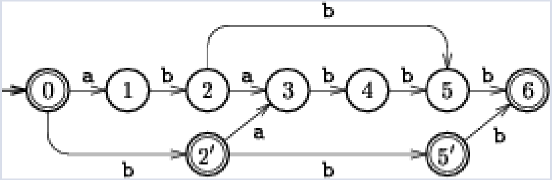
Execution of $\textsc{Suffix-tree}(y) = \mathcal{S}(y)$ *takes* $O(|y| \times \log \operatorname{card} \Sigma)$ *time in the comparison model.*

Plan

Suffix Automaton

Text $y \in \Sigma^*$ of length n
$\mathcal{A}(y)$ = minimal deterministic automaton accepting *Suff* (y)

- **Minimization** of the trie of suffixes



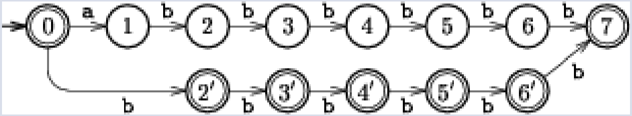
- States are classes of factors (subwords) of y
- **Size**:

$$n + 1 \leqslant \#states \leqslant 2n - 1$$

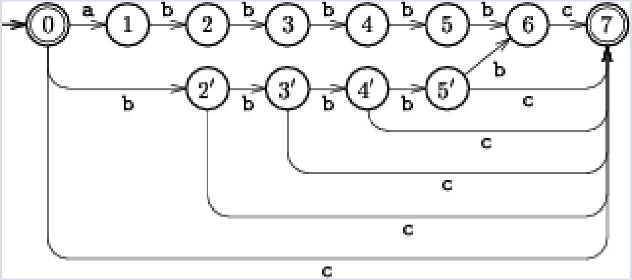
$$n \leqslant \#arcs \leqslant 3n - 4$$

Maximal size

- Maximal number of states

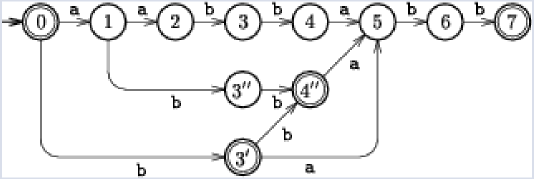


- Maximal number of arcs



Suffix link

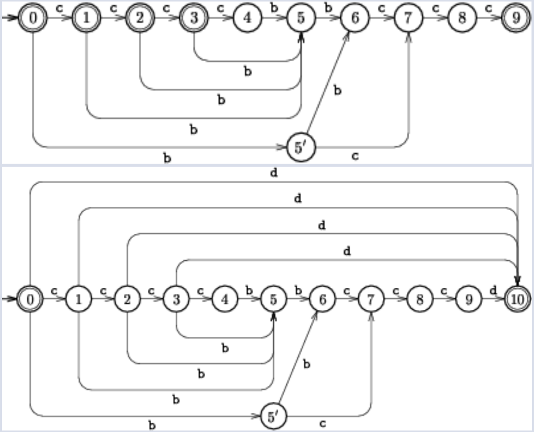
- **Function** f_y, *suffix link*
  let $p = \textsc{Target}(\textit{initial}[\mathcal{A}], v,)$, $v \in \Sigma^+$
  $f_y(p) = \textsc{Target}(\textit{initial}[\mathcal{A}], u,)$, where u is the longest suffix of v occurring in a different right context



- $s[1] = 0$, $s[2] = 1$, $s[3] = 3''$, $s[3''] = 3'$, $s[3'] = 0$, $s[4] = 4''$, $s[4''] = 3'$, $s[5] = 1$, $s[6] = 3''$, $s[7] = 4''$.
- **Suffix path**
  example for state 7: $\langle 7, 4'', 3', 0 \rangle$, sequence of terminal states
- **Use**
  same but more efficient than suffix link in suffix trees

One step (1)

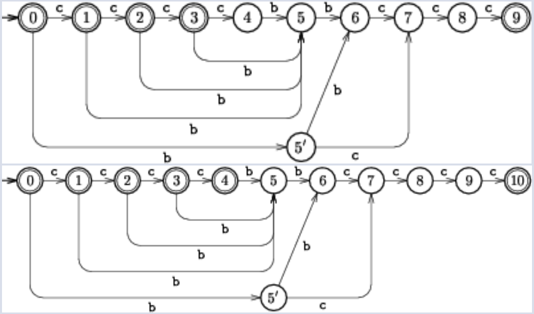
- **From** $\mathcal{A}(\text{ccccbbccc})$ **to** $\mathcal{A}(\text{ccccbbcccd})$



- New arcs from states of the suffix path $\langle 9, 3, 2, 1, 0 \rangle$.

One step (2)

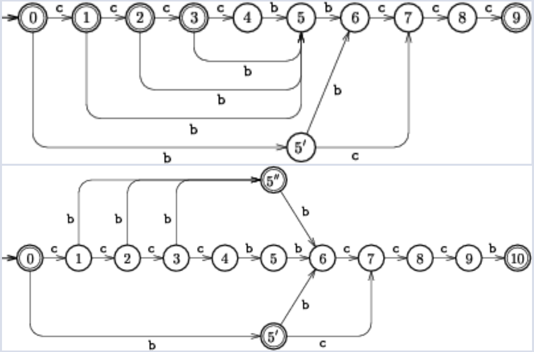
- **From** $\mathcal{A}(\texttt{ccccbbccc})$ **to** $\mathcal{A}(\texttt{ccccbbcccc})$



- Link $3 = s[9]$ and solid arc $(3, \mathtt{c}, 4)$ (not a shortcut) then, $s[10] = \textsc{Target}(3, \mathtt{c}) = 4$

One step (3)

- **From** $\mathcal{A}(\text{ccccbbccc})$ **to** $\mathcal{A}(\text{ccccbbcccb})$



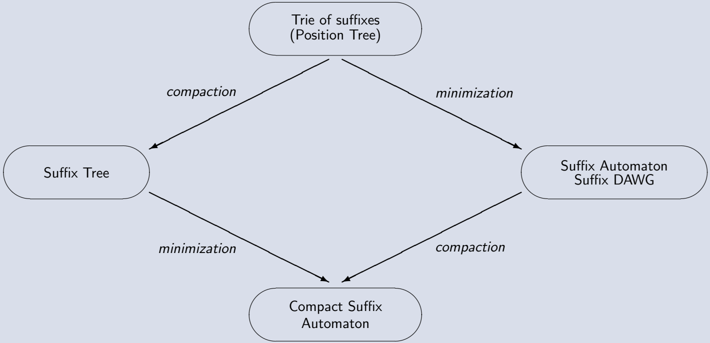
- Link $3 = s[9]$, non-solid arc $(3, \text{b}, 5)$, cccb suffix but ccccb not state 5 is cloned into $5'' = s[10] = s[5]$, $s[5''] = 5'$ arcs $(3, \text{b}, 5)$, $(2, \text{b}, 5)$ et $(1, \text{b}, 5)$ are redirected onto $5''$

Operations on indexes

Text y of length n

- **Index implemented by suffix tree or suffix automaton of y**
  memory space $O(n)$, construction time $O(n \times \log \operatorname{card} \Sigma)$
- **String matching**
  searching y for x of length m: time $O(m \times \log \operatorname{card} \Sigma)$
  number of occurrences of x in y: same complexity after $O(n)$ preprocessing
- **All occurrences**
  finding all occurrences of x in y: time $O(m \times \log \operatorname{card} \Sigma) + |output|)$
- **Repetitions**
  computing a longest subword of y occurring at least k times: time $O(n)$
- **Marker**
  computing a shortest subword of y occurring exactly once: time $O(n)$

Saving space



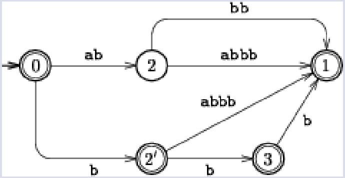
Plan

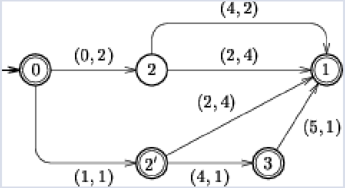
Compact Suffix Automaton

Text $y \in \Sigma^*$ of length n

$\mathcal{A}^{\mathsf{C}}(y)$ = compact minimal automaton accepting *Suff* (y)

- **Compaction** of $\mathcal{A}(y)$, or **minimization** of $\mathcal{S}(y)$





i	0	1	2	3	4	5
$y[i]$	a	b	a	b	b	b

- **Linear size**: $O(n)$

Direct construction of CSA

- Similar to both
  - Suffix Tree construction
  - Suffix Automaton construction
- Sequential addition of suffixes in the structure from the longest to the shortest
- Used features:
  - "slow-find" and "fast-find" procedures
  - suffix links
  - solid and non-solid arcs
  - state splitting
  - re-directions of arcs
- **Complexity**: $O(n \log \operatorname{card} \Sigma)$ time, $O(n)$ space
  50% **saved** on space of Suffix Automaton

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