13. Properties of Triangle

Manognya Kundarapu
(EE24BTECH11037)

Section-A JEE Advanced/IIT-JEE

E. Subjective Problems

1) A triangle ABC has sides $AB = AC = 5cm$ and $BC = 6cm$. Triangle $A'B'C'$ is the reflection of the triangle ABC in a line parallel to AB is placed at a distance $2cm$ from AB, outside the triangle ABC. Triangle $A''B''C^{'}$ is the reflection of the triangle $A'B'C'$ in a line parallel to $B'C'$ placed at a distance of $2cm$ from outside $B'C'$ the triangle $A'B'C'$ Find the distance between A and A''. (1978)
2) (a) If a circle is inscribed in aright angled triangle ABC with the right angle at B, show that the diameter of the circle is equal to $AB + BC - AC$.
(b) If a triangle is inscribed in a circle, then the product of any two sides of the triangle is equal to the product of the diameter and the perpendicular distance of the third side from the opposite vertex. Prove the above statement. (1979)
3) (a) A ballon is observed simultaneously from three points A, B and C on a straight road directly beneath it. The angular elevation at B is twice that at A and the angular elevation at C is thrice that at A. If the distance between A and B is a and the distance between B and C is b, find the height of the balloon in terms of a and b.
(b) Find the area of the smaller part of a disc of radius $10cm$, cut off by a chord AB which subtends an angle of $22\frac{1}{2}°$at the circumference. (1979)
4) ABC is a triangle. D is the middle point of BC. If AD is perpendicular to AC,then prove that $\cos(A)\cos(C) = \frac{2(c^2-a^2)}{3ac}$ (1980)
5) If in a triangle ABC, $\cos(A)\cos(B) + \sin(A)\sin(B)\sin(C) = 1$, Show that $a : b : c = 1 : 1 : \sqrt{2}$ (1986-5 Marks)
6) A sign-post in the form of an isosceles triangle ABC is mounted on a pole of height h fixed to the ground. The base BC of the triangle is parallel to the ground. A man standing on the ground at a distance d from the sign-post finds that the top vertex A of the triangle subtends an angle β and either of the other two vertices subtends the same angle α at his feet. Find the area of the triangle. (1988-5 Marks)
7) ABC is a triangular park with $AB = aAC = 100m$. A television tower stands at the midpoint of BC. The angles of elevation of the top of the tower at A, B, C are 45°, 60°, 60°, respectively. Find the height of the tower. (1989-5 Marks)
8) Let the angles A, B, C of a triangle ABC be in A.P.and let $b : c = \sqrt{3} : \sqrt{2}$.Find the angle A. (1981-2 Marks)
9) A vertical pole stands at a point Q on a horizontal ground. A and B are points on the ground, d metres apart. The pole subtends angles α and β at A and B respectively. AB subtends an angle γ at Q. Find the height of the pole. (1982-3 Marks)
10) The ex-radii r_1, r_2, r_3 of $\triangle ABC$ are in H.P. Show that its sides a, b, c are in A.P. (1983-3 Marks)
11) For a triangle ABC it is given that $\cos(A) + \cos(B) + \cos(C) = \frac{3}{2}$. Prove that the triangle is equilateral. (1984-4 Marks)
12) With usual notation, if in a triangle ABC; $\frac{b+c}{11} = \frac{c+a}{12} = \frac{a+b}{13}$ then prove that $\frac{\cos(A)}{7} = \frac{\cos(B)}{19} = \frac{\cos(C)}{25}$. (1984-4 Marks)
13) A ladder rests against a wall at an angle α to the horizontal. Its foot is pulled away from the wall through a distance a, so that it slides a distance b down the wall making an angle β with the horizontal,

Show that $a = \tan\frac{1}{2}(\alpha + \beta)$ (1985-5 Marks)

14) ABC is a triangle with $AB = AC$. D is any point on the side BC.E and F are points on the side AB and AC, respectively, such that DE is parallel to AC, and DF is parallel to AB. Prove that
$DF + FA + AE + ED = AB + AC$
(1980)

15) In a triangle ABC, the median to the side BC is of length $\frac{1}{\sqrt{11-6\sqrt{3}}}$ and it divides the angle A into angles 30°and 45°.Find the length of the side BC. (1985-5 Marks)