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Manognya Kundarapu - EE24BTECH11037

Question: $e^x dy + (ye^x + 2x)\, dx = 0$

1) **Theoretical Solution:**

$$\frac{dy}{dx} = \frac{-(ye^x + 2x)}{e^x} \tag{1.1}$$
$$\frac{dy}{dx} = -y - \frac{2x}{e^x} \tag{1.2}$$
$$\implies \frac{dy}{dx} + y = \frac{2x}{e^x} \tag{1.3}$$
$$\text{Now, it is in the form of linear differential equation;} \tag{1.4}$$
$$\frac{dy}{dx} + y.p(x) = q(x) \tag{1.5}$$
$$\text{Integrating factor,} I.F = e^{\int 1.dx} \tag{1.6}$$
$$\implies I.F = e^x \tag{1.7}$$
$$y.e^x = \int \frac{-2x}{e^x}.e^x dx \tag{1.8}$$
$$y.e^x = -x^2 + c \tag{1.9}$$
$$\implies y = \frac{-x^2 + c}{e^x} \tag{1.10}$$
$$\text{Taking } x_0 = 0, y_0 = 1 \implies k = 1 \tag{1.11}$$
$$\therefore \text{The final solution is } \frac{-x^2 + 1}{e^x} \tag{1.12}$$

2) **Using Method of Finite Differences:**
The Method of finite Differences is a numerical technique used to approximate solutions to differential equations.
We know that;

$$\lim_{x\to 0} \frac{y(x+h) - y(x)}{h} = \frac{dy}{dx} \tag{2.1}$$
$$\approx \frac{y_{n+1} - y_n}{h} = -y - \frac{2x}{e^x} \tag{2.2}$$
$$\implies y_{n+1} = y_n + h\left(-y_n - \frac{2x_n}{e^{x_n}}\right) \tag{2.3}$$
$$\tag{2.4}$$

Now the following steps were used:

a) Initialized $x_0 = 0$ and $y_0 = 1$.
b) h was taken to be 0.001, a small value and number of iterations was taken to be 1000 to ensure accuracy.
c) Now the subsequent points of the curve were generated through iterations by using the below equations;

$$x_{n+1} = x_n + h \tag{2.5}$$

$$y_{n+1} = y_n + h\left(-y_n - \frac{2x_n}{e^{x_n}}\right) \tag{2.6}$$

The below graph shows the comparison between the curve that is obtained theoretically and the simulation curve(numerically generated points through iterations).