9.5.17.4

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Question: $y^2dx + \left(x^2 - xy - y^2\right)dy = 0$

1) **Theoretical Solution:**

$$\frac{dy}{dx} = \frac{-y^2}{x^2 - xy - y^2} \tag{1.1}$$

$$\text{taking } y = vx \implies dy = vdx + xdv \tag{1.2}$$

$$\implies v + x\frac{dv}{dx} = \frac{-v^2}{1 - v - v^2} \tag{1.3}$$

$$x\frac{dv}{dx} = \frac{v^3 - v}{1 - v - v^2} \tag{1.4}$$

$$\int \frac{\left(v^2 - 1\right) + v}{v\left(v^2 - 1\right)}dv = \int \frac{-1}{x}dx \tag{1.5}$$

$$\int \left(\frac{1}{v} + \frac{1}{v^2 - 1}\right)dv = \frac{1}{kx} \tag{1.6}$$

$$\ln v + \frac{1}{2}\ln\left(\frac{v-1}{v+1}\right) = \frac{1}{kx} \tag{1.7}$$

$$\implies \frac{1}{kx} = v\sqrt{\frac{v-1}{v+1}} \tag{1.8}$$

$$\text{Substituting } v = \frac{y}{x}; \tag{1.9}$$

$$\frac{1}{k} = y^2\left(\frac{y-x}{y+x}\right) \tag{1.10}$$

$$\text{Taking } x_0 = 0, y_0 = 1 \implies k = 1 \tag{1.11}$$

$$\therefore \text{The final solution is } y^3 - xy^2 - y - x = 0 \tag{1.12}$$

2) **Using Method of Finite Differences:**
We know that;

$$\lim_{x\to 0}\frac{y\left(x+h\right) - y\left(x\right)}{h} = \frac{dy}{dx} \tag{2.1}$$

$$\approx \frac{y_{n+1} - y_n}{h} = \frac{-y_n^2}{x_n^2 - x_ny_n - y_n} \tag{2.2}$$

$$\implies y_{n+1} = y_n + h\left(\frac{-y_n^2}{x_n^2 - x_ny_n - y_n}\right) \tag{2.3}$$

The Method of finite Differences is a numerical technique used to approximate solutions to differential equations.

Now the following steps were used:

a) Initialized $x_0 = 0$ and $y_0 = 1$.
b) h was taken to be 0.001, a small value and number of iterations was taken to be 1000 to ensure accuracy.
c) Now the subsequent points of the curve were generated through iterations by using the below equations;

$$x_{n+1} = x_n + h \tag{2.4}$$

$$y_{n+1} = y_n + h\left(\frac{-y_n^2}{x_n^2 - x_n y_n - y_n}\right) \tag{2.5}$$

The below graph shows the comparison between the curve that is obtained theoretically and the simulation curve(numerically generated points through iterations).