9.2.1

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Question: Check if the differential equation $\frac{d^2y}{dx^2} - \frac{dy}{dx} = 0$ has a solution $y = e^x + 1$ for $y(0) = 2$ and $y'(0) = 1$.

1) **Theoretical Solution:**

$$\text{Taking } \frac{dy}{dx} = t; \tag{1.1}$$

$$\frac{dt}{dx} - t = 0 \tag{1.2}$$

$$\int \frac{1}{t} dt = \int dx \tag{1.3}$$

$$ln(t) = x + k \tag{1.4}$$

$$t = Ce^x \text{ but } y'(0) = 1 \implies \frac{dy}{dx} = e^x \tag{1.5}$$

$$\int dy = \int e^x dx \tag{1.6}$$

$$\implies y = e^x + k \tag{1.7}$$

$$\text{but } y(0) = 2 \implies k = 1 \tag{1.8}$$

$$\therefore y = e^x + 1 \text{ is the solution under given conditions.} \tag{1.9}$$

2) **Using Trapezoidal Rule:**

$$\text{For } \frac{dy}{dx} = f(x, y) \tag{2.1}$$

$$\int_{y_n}^{y_{n+1}} dy = \int_{x_n}^{x_{n+1}} f(x, y)\, dx \tag{2.2}$$

$$y_{n+1} - y_n \approx \frac{h}{2} \left(f(x_n, y_n) + f(x_{n+1}, y_{n+1}) \right) \tag{2.3}$$

$$\implies y_{n+1} = y_n + \frac{h}{2} \left(f(x_n, y_n) + f(x_{n+1}, y_{n+1}) \right) \tag{2.4}$$

$$\text{where } h = x_{n+1} - x_n \tag{2.5}$$

To solve the differential equation $y'' - y' = 0$ numerically using the trapezoidal rule, we first need to rewrite it as a system of first-order differential equations, let $y_1 = y$ and $y_2 = y'$

$$y_1' = y_2 \tag{2.6}$$

$$y_2' = y_2 \tag{2.7}$$

Using the trapezoidal rule, we get the difference equations;

$$y_{1,n+1} = y_{1,n} + \frac{h}{2}\left(y_{2,n} + y_{2,n+1}\right) \tag{2.8}$$

$$y_{2,n+1} = y_{2,n} + \frac{h}{2}\left(y_{2,n} + y_{2,n+1}\right) \tag{2.9}$$

Now the following steps were used:

a) Initialized $x_0 = 0$, $y(0) = 2$ and $y'(0) = 1$.

b) h was taken to be 0.001, a small value and number of iterations was taken to be 1000 to ensure accuracy.

c) Now the subsequent points of the curve were generated through iterations by using the difference equations 2.8 and 2.9 .

The below graph shows the comparison between the curve that is obtained theoretically and the simulation curv(numerically generated points through iterations).