6.6.23

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Question: The normal to the curve $x^2 = 4y$ passing through $(1, 2)$ is

1) **Theoretical Solution:**
   Let the slope of the normal be 'm' at a point (x_0, y_0) on the curve. We know;

$$2x_0 = 4\frac{dy}{dx} \tag{1.1}$$

$$\implies m = \frac{-2}{x_0} \tag{1.2}$$

$$\text{Equation of normal is given by } y - y_0 = \frac{-2}{x_0}(x - x_0) \tag{1.3}$$

$$\text{Substituting (1,2) in above } 2 - y_0 = \frac{-2}{x_0}(1 - x_0) \tag{1.4}$$

$$\text{also } (x_0)^2 = 4y \tag{1.5}$$

$$\text{On solving we get } x_0 = 2, y_0 = 1 \tag{1.6}$$

$$\implies \text{ required normal is } x + y = 3 \tag{1.7}$$

2) **Using Gradient descent method:**
   We want to find the normal line to the curve $x^2 = 4y$ at a specific point (x_0, y_0), such that this normal passes through the point (1,2).

   Thus, if we know the coordinates (x_0, y_0), we can form the equation of the normal using point-slope form: $(y - y_0) = \frac{-2}{x_0}(x - x_0)$
   $\implies y = mx + b$

   Objective function (Minimization goal):
   We want to minimize the distance between the point (1,2) and the normal line. In other words, we want to find the point (x_0, y_0) on the curve where the normal passes through the point (1,2).

   $\implies d = \frac{|y_0 - mx_0 - b|}{\sqrt{1+m^2}}$

   Gradient descent:
   Gradient descent is an optimization algorithm used to minimize a function iteratively. Here's how it works:
   - We start with an initial guess for (x_0, y_0)
   - We compute the gradient (partial derivatives) of the objective function with respect to x_0 and y_0
   - We update x_0 and y_0 in the direction that reduces the distance (minimizes the objective function).

- This is done iteratively until the values converge to a point where the gradient is close to zero (indicating a minimum).

Steps in the Gradient Descent Algorithm:

- Start with initial guesses for x_0 and y_0 (we chose $x_0 = 2, y_0 = 1$ as a starting point).
- The gradient of the objective function tells us how to change x_0 and y_0 to minimize the distance. In the gradient descent process, we calculate the partial derivatives of the distance with respect to x_0 and y_0
- Adjust x_0 and y_0 by a small amount in the direction of the negative gradient (i.e., move in the direction that reduces the distance).

$$x_0 \leftarrow x_0 - \text{learning rate} \times \frac{\partial \text{Distance}}{\partial x_0}$$

$$y_0 \leftarrow y_0 - \text{learning rate} \times \frac{\partial \text{Distance}}{\partial y_0}$$

- Continue this process for a set number of iterations or until the changes become very small (i.e., convergence)

The below graph shows the comparison between the curve that is obtained theoretically and the simulation curve(numerically generated points through iterations).