

9.7.18

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Question: $e^x dy + (ye^x + 2x) dx = 0$

1) Theoretical Solution:

$$\frac{dy}{dx} = \frac{-(ye^x + 2x)}{e^x} \quad (1.1)$$

$$\frac{dy}{dx} = -y - \frac{2x}{e^x} \quad (1.2)$$

$$\Rightarrow \frac{dy}{dx} + y = \frac{2x}{e^x} \quad (1.3)$$

Now, it is in the form of linear differential equation; (1.4)

$$\frac{dy}{dx} + y.p(x) = q(x) \quad (1.5)$$

$$\text{Integrating factor, } I.F = e^{\int 1 \cdot dx} \quad (1.6)$$

$$\Rightarrow I.F = e^x \quad (1.7)$$

$$y \cdot e^x = \int \frac{-2x}{e^x} \cdot e^x dx \quad (1.8)$$

$$y \cdot e^x = -x^2 + c \quad (1.9)$$

$$\Rightarrow y = \frac{-x^2 + c}{e^x} \quad (1.10)$$

$$\text{Taking } x_0 = 0, y_0 = 1 \Rightarrow k = 1 \quad (1.11)$$

$$\therefore \text{The final solution is } \frac{-x^2 + 1}{e^x} \quad (1.12)$$

2) Using Method of Finite Differences:

The Method of finite Differences is a numerical technique used to approximate solutions to differential equations.

We know that;

$$\lim_{x \rightarrow 0} \frac{y(x+h) - y(x)}{h} = \frac{dy}{dx} \quad (2.1)$$

$$\approx \frac{y_{n+1} - y_n}{h} = -y - \frac{2x}{e^x} \quad (2.2)$$

$$\Rightarrow y_{n+1} = y_n + h \left(-y - \frac{2x}{e^x} \right) \quad (2.3)$$

$$(2.4)$$

Now the following steps were used:

- Initialized $x_0 = 0$ and $y_0 = 1$.
- h was taken to be 0.001, a small value and number of iterations was taken to be 1000 to ensure accuracy.
- Now the subsequent points of the curve were generated through iterations by using the below equations;

$$x_{n+1} = x_n + h \quad (2.5)$$

$$y_{n+1} = y_n + h \left(-y - \frac{2x}{e^x} \right) \quad (2.6)$$

The below graph shows the comparison between the curve that is obtained theoretically and the simulation curve (numerically generated points through iterations).

