10.3.4.2.5

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Question: A lending library has a fixed charge for the first three days and an additional charge for each day thereafter. Saritha paid Rs. 27 for a book kept for seven days, while Susy paid Rs. 21 for the book she kept for five days. Find the fixed charge and the charge for each extra day.

1) **Theoretical Solution:**
Let the fixed charge be Rs. x and the additional charge per day be Rs. y

$$\implies x + 4y = 27 \tag{1.1}$$

$$x + 2y = 21 \tag{1.2}$$

$$\text{From 1.1, } x = (27 - 4y) \tag{1.3}$$

$$\text{Substituting } x \text{ in 1.2, we get } (27 - 4y + 2y) = 21 \tag{1.4}$$

$$\implies y = 3 \tag{1.5}$$

$$x = 15 \tag{1.6}$$

$$\therefore \text{The fixed charge is Rs. 15 and extra charge per day is Rs. 3} \tag{1.7}$$

2) **LU Decomposition using Doolittle's algorithm**
The system of linear equations: $x + 4y = 27$ and $x + 2y = 21$ can be written in matrix form as;

$$A\mathbf{x} = \mathbf{b}, \text{ where}$$

$$A = \begin{bmatrix} 1 & 4 \\ 1 & 2 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} x \\ y \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 27 \\ 21 \end{bmatrix}$$

Doolittle's algorithm decomposes A into a lower triangular matrix L and an upper triangular matrix U such that:

$$A = LU$$

where:

$$L = \begin{bmatrix} 1 & 0 \\ l_{21} & 1 \end{bmatrix}, \quad U = \begin{bmatrix} u_{11} & u_{12} \\ 0 & u_{22} \end{bmatrix}$$

The elements of L and U are computed using:

$$U[i][j] = A[i][j] - \sum_{k=0}^{i-1} L[i][k]U[k][j] \tag{2.1}$$

$$L[i][j] = \frac{A[i][j] - \sum_{k=0}^{j-1} L[i][k]U[k][j]}{U[j][j]} \tag{2.2}$$

with the constraint $L[i][i] = 1$.
Thus, we obtain

$$L = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, \quad U = \begin{bmatrix} 1 & 4 \\ 0 & -2 \end{bmatrix}$$

Solving for x using Forward and Backward Substitution
With $A = LU$, we solve:

$$L\mathbf{y} = \mathbf{b}$$

$$\begin{bmatrix} 1 & 0 \\ l_{21} & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 27 \\ 21 \end{bmatrix}$$

Solving step-by-step:

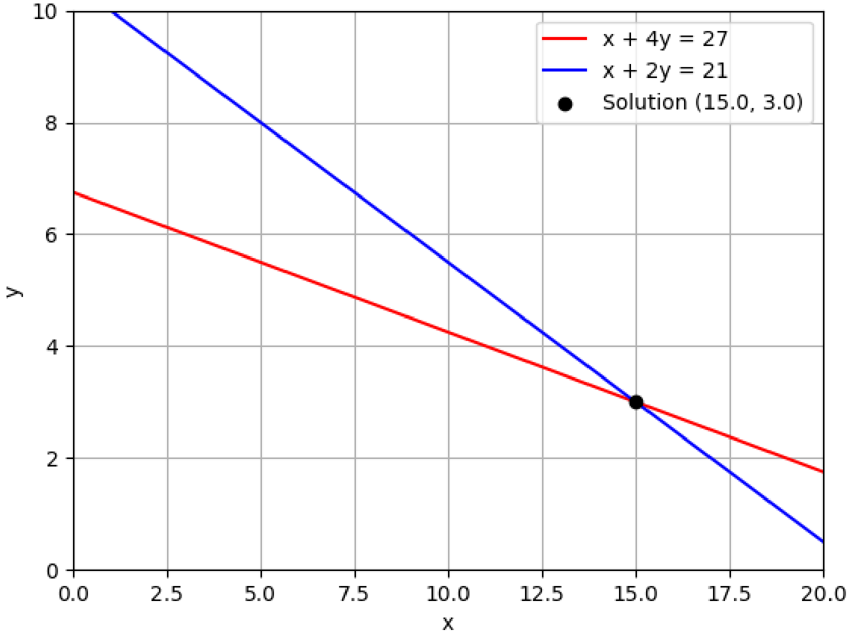
$$\mathbf{y} = \begin{bmatrix} 27 \\ -6 \end{bmatrix}$$

Backward Substitution
Expanding $U\mathbf{x} = \mathbf{y}$:

$$\begin{bmatrix} 1 & 4 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 27 \\ -6 \end{bmatrix}$$

Thus we obtain;

$$\mathbf{x} = \begin{bmatrix} 15 \\ 3 \end{bmatrix}$$


10
8
6
4
2
0
y
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
x
x + 4y = 27
x + 2y = 21
Solution (15.0, 3.0)