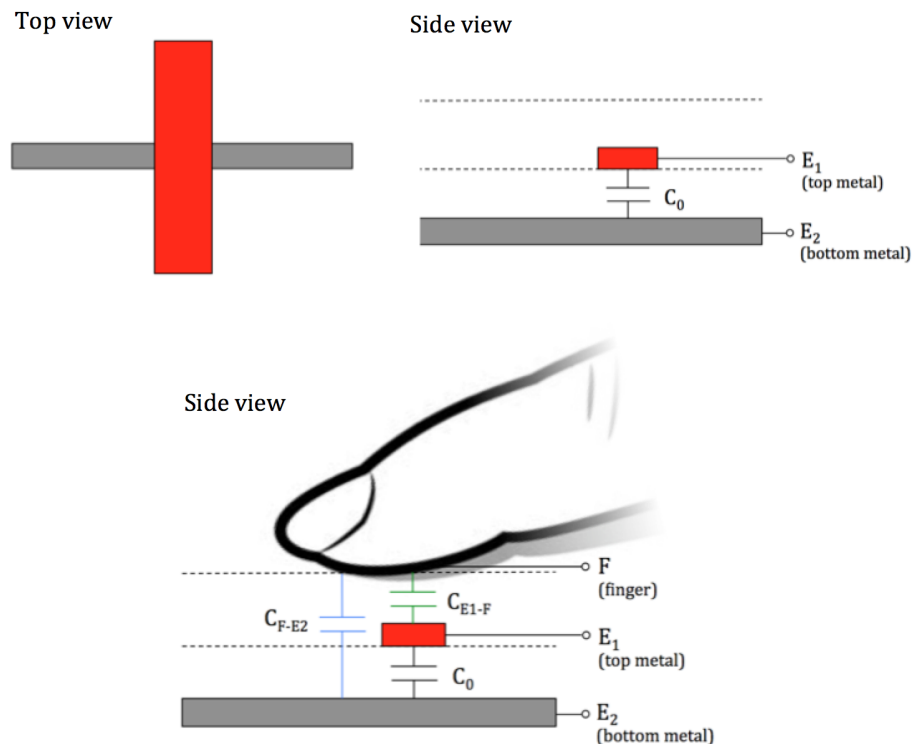


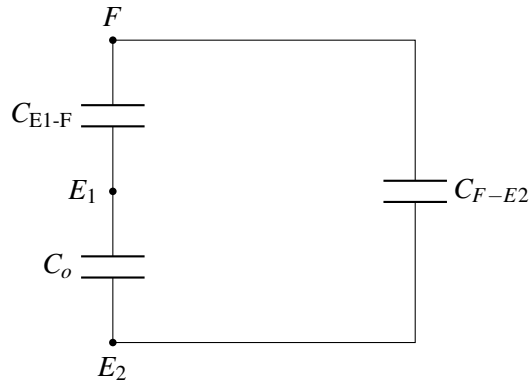
17.1 Capacitive Touchscreen

In the last note, we saw that a capacitor consists of two pieces of conductive material separated by a non-conductive material. In a typical off-the-shelf capacitor, the conductive material is a metal. However, our skin is mostly water and therefore also acts as a conductor. This means we can form a capacitor by holding a finger over a sheet of metal! Let's look at how we can use this concept to design a touchscreen based on capacitance.

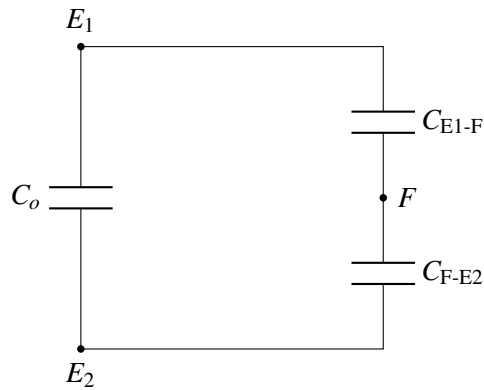
Each pixel of our touchscreen consists of two pieces of metal with an insulator (non-conductive material) between them. This forms a capacitor with capacitance C_0 . Another insulator, such as glass, goes on top of the upper (red) piece of metal. We want to be able to detect if there is a finger touch on top of this insulator.



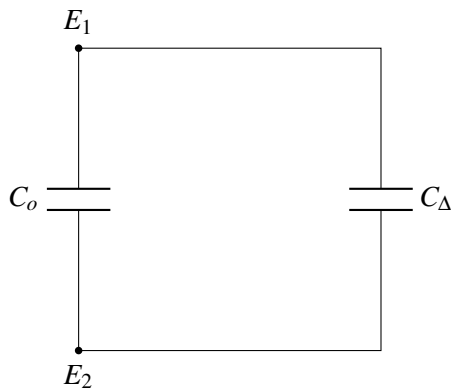
When the finger touches the top insulator, two new capacitors are formed between the finger and each of the two electrodes. C_{F-E2} is the capacitance between our finger (F) and electrode $E2$, and C_{E1-F} is the capacitance between our finger (F) and electrode $E1$. Now, we can draw the equivalent circuit model corresponding to the pixel we're looking at. Note that the nodes are labeled with the electrode or finger that the node represents.



Our touchscreen can only measure things that are between electrodes E_1 and E_2 as it is impractical to physically connect a measuring device to our finger. So, we redraw our circuit as follows:

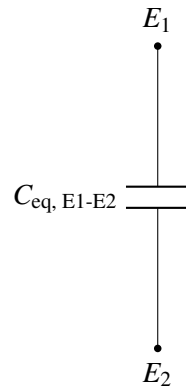


We can further simplify the circuit by replacing the series combination of C_{E1-F} and C_{F-E2} by some capacitance C_Δ . When the finger is present, C_{E1-F} and C_{F-E2} have positive capacitance, so we know C_Δ has some capacitance greater than zero.



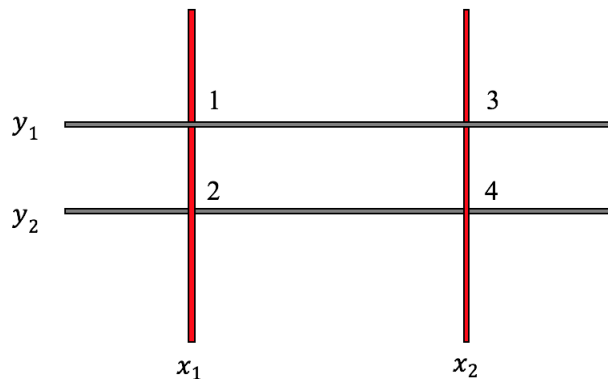
Finally we consider the parallel combination of C_o and C_Δ as some equivalent capacitance $C_{eq, E1-E2} = C_o + C_\Delta$. When the finger is present, $C_\Delta > 0$, and the equivalent capacitance $C_{eq, E1-E2}$ will be greater than

C_o . When there is no finger present, the capacitors forming C_Δ do not exist, and therefore the equivalent capacitance $C_{eq, E1-E2}$ is equal to C_o . If we can measure this change in capacitance, we can detect the presence or absence of a finger.



17.1.1 Touchscreen with multiple pixels

Our analysis from the previous section only tells us if a finger is present or absent. How can we determine *where* the finger is? We can create a grid of the individual pixels described above. Here's an example with four pixels:



The four pixels are labeled 1, 2, 3, 4. If we want to know if a finger is present at pixel 1, we can measure the capacitance between electrodes x_1 and y_1 – if the capacitance is increased, we know there is a finger pressing at pixel 1. Similarly, to know if there is a finger pressing on top of pixels 2, 3, 4, we can measure the capacitances between (x_1, y_2) , (x_2, y_1) , (x_2, y_2) . Since each pixel acts separately, we can detect multiple presses simultaneously – this enables many of the touchscreen gestures we're familiar with, such as pinching to zoom.

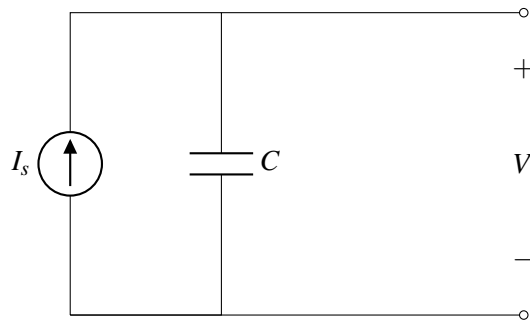
17.2 Capacitance Measurement

How can we detect changes in capacitance? We can't measure capacitance directly, but if we can transform capacitance into a voltage, we can measure that. There are multiple ways of doing this, but in this note we

will cover only one specific method of measuring capacitance. First, let's review some basic physics for a capacitor. From our last note,

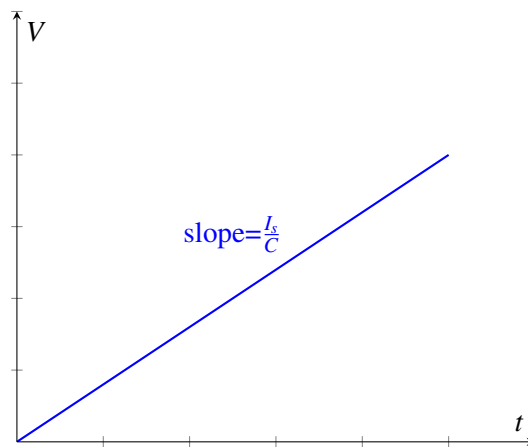
$$I = C \frac{dV}{dt} \quad (1)$$

Based on this equation, we hypothesize that if we connect a known current source I_s to the capacitor and measure the voltage, we might be able to solve for the capacitance C . For our first attempt, we build the following circuit to measure C :



If the capacitor starts off at $t = 0$ with no voltage across it, we find that V is:

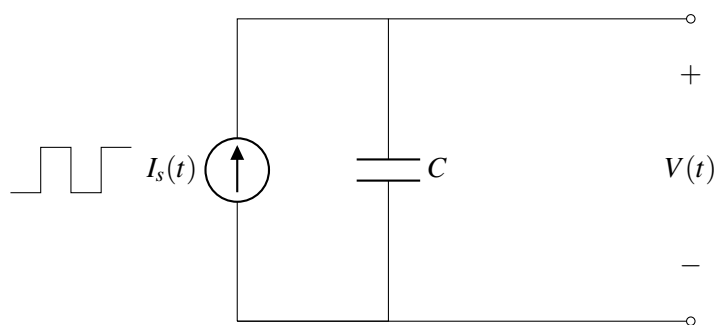
$$V = \frac{I_s t}{C} \quad (2)$$



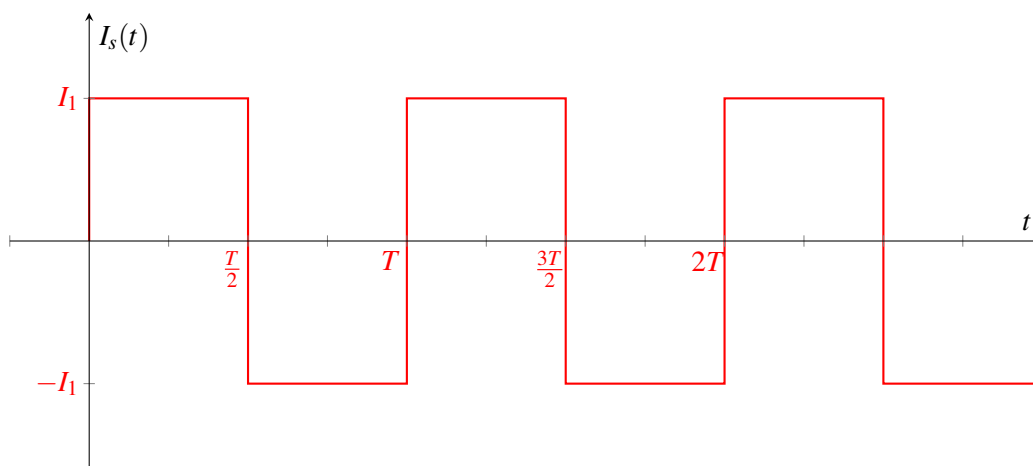
where t is the time passed since we started charging the capacitor. If we measure the voltage at a known time t , we can solve for the capacitance. However, as time continues to pass, the voltage across the capacitor (and also the charge stored on the capacitor) will grow to infinity. It is very challenging to build an ideal current source that works over this large range of voltages, so our model quickly becomes unrealistic.

17.2.1 Measuring capacitance with a periodic current source

To remedy this, instead of applying a constant current source, we apply a *periodic* current source



where the current I_s is a function of time as follows:



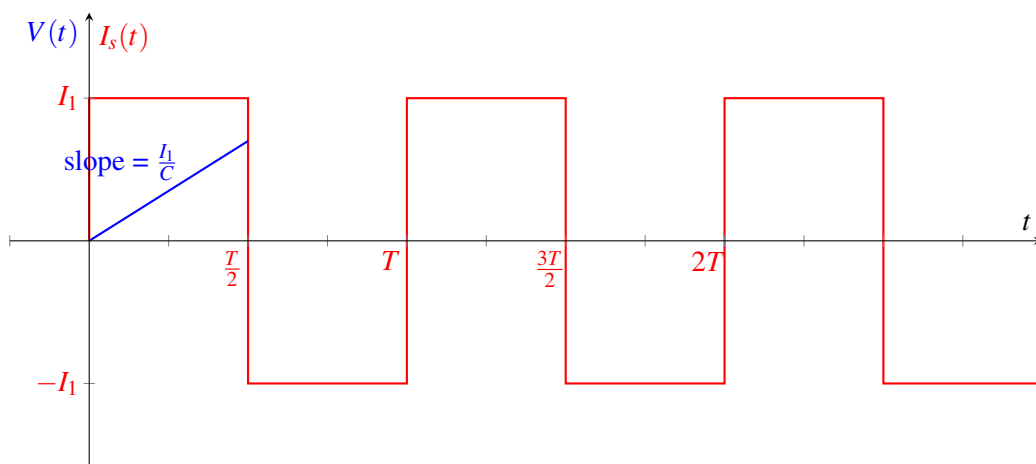
What does the voltage V look like with this current source? Let's assume that the capacitor is initially uncharged (ie. $Q = 0$). Since $Q = CV$, this means that at time $t = 0$ the voltage $V = 0$. (We will revisit this assumption later.)

When a constant current source is applied to a capacitor, we know that the voltage obeys the following equation

$$V_C(t) = \frac{I}{C}t + V_C(0). \quad (3)$$

Our periodic current source I_s is constant from $t = 0$ to $t = \frac{T}{2}$, so we can apply equation 3 over this time period. We know the initial voltage is zero, so:

$$V(t) = \frac{I_1}{C}t \quad \text{when } 0 \leq t \leq \frac{T}{2}$$



In order to figure out what happens next, let's consider a more generic version of equation 3:

$$V_C(t) = \frac{I}{C}(t - t_0) + V_C(t_0). \quad (4)$$

With this equation, we can consider an arbitrary starting time t_0 instead of always starting at $t = 0$. Plugging in $t_0 = 0$ yields equation 3. Like equation 3, the above equation is only true when the current is constant.

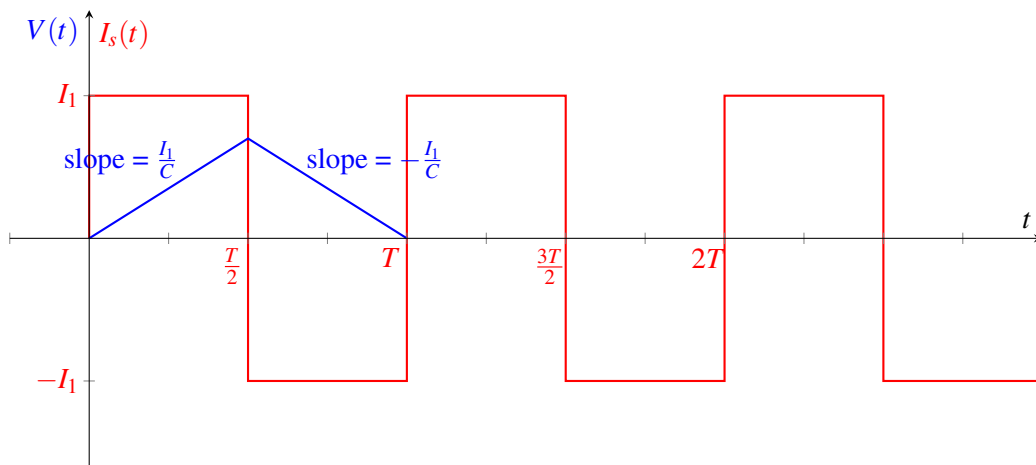
The next time period with constant current is from $t = \frac{T}{2}$ to $t = T$. Over this time, the current through the capacitor is $-I_1$. Since we are starting at time $\frac{T}{2}$, we set $t_0 = \frac{T}{2}$ and plug into equation 4.

$$V(t) = \frac{-I_1}{C} \left(t - \frac{T}{2} \right) + V \left(\frac{T}{2} \right)$$

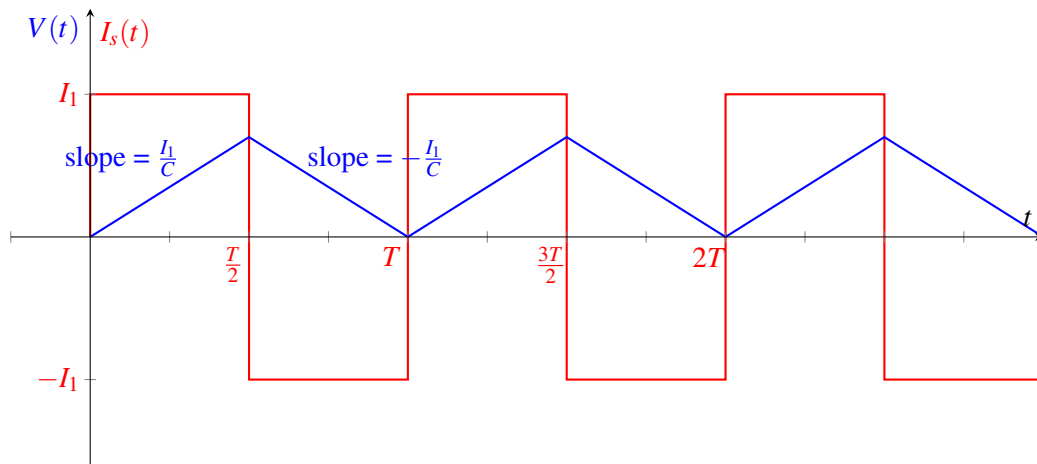
$$V(t) = \frac{-I_1}{C} \left(t - \frac{T}{2} \right) + \frac{I_1 T}{2C}$$

Combining with our previous relationship yields:

$$V_C(t) = \begin{cases} \frac{I_1}{C}t & \text{when } 0 \leq t \leq \frac{T}{2} \\ \frac{-I_1}{C} \left(t - \frac{T}{2} \right) + \frac{I_1 T}{2C} & \text{when } \frac{T}{2} < t \leq T \end{cases}$$



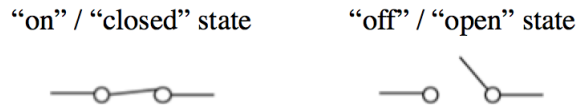
To determine the full behavior of $V(t)$, we could continue to apply equation 4 for each period of constant current. However, we notice that at $t = T$, the voltage and current are the same as they were at $t = 0$. Since the current source is periodic (repeats every T), the voltage pattern will also repeat.



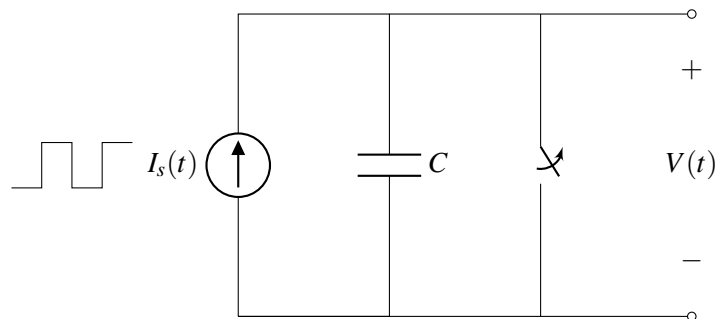
17.2.2 Switches

In the model above, we made the assumption that the capacitor was initially uncharged. This assumption tells us that $V(0) = 0$, and our subsequent analysis relies on this fact. How can we ensure that this assumption is true? Let's introduce a new element, a switch, to help us.

A switch has two states, at its *on* state (or “closed state”), the switch becomes a perfectly conducting wire with zero resistance, and when the switch is turned *off* (or the switch is “opened”), it becomes an open circuit.



We can modify our design from above to include a switch in parallel with the capacitor:



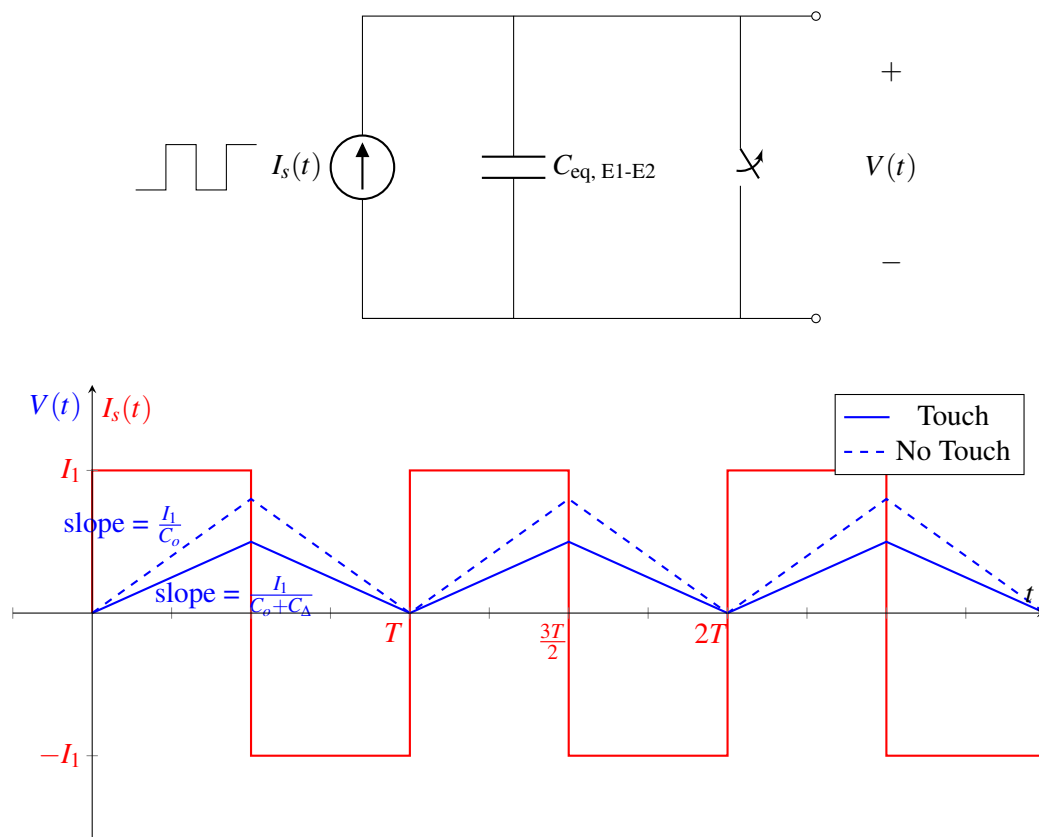
When the switch is closed, it acts as an ideal wire, so the voltage across the capacitor is 0. Since $Q = CV$, this means that the charge is zero as well, so sometimes we refer to this switch as “discharging” the capacitor.

To ensure that $V(0) = 0$, we can keep the switch closed for $t < 0$. At $t = 0$, the switch is opened, turning it into an open circuit. Now the circuit behaves the same way as it did before adding the switch, but we've ensured that the voltage over the capacitor is 0 at $t = 0$.

17.2.3 Effect of finger touch on the measurement

Remember that we'd like to use our circuit to detect if there is a finger pressing on a given pixel. Recall that when the finger is absent, the equivalent capacitance between the electrodes is C_o . When the finger is present, the equivalent capacitance is $C_o + C_\Delta$ where $C_\Delta > 0$. This difference in capacitance will change the slope of $V(t)$ in our measurement.

We use the same circuit design as before and measure the capacitance between the two touchscreen electrodes.



When there is no finger present, the slope of the rising edge of $V(t)$ is $\frac{I_1}{C_o}$. When there is a finger pressing, the slope is smaller: $\frac{I_1}{C_o + C_\Delta}$. This results in a smaller peak voltage when there is a finger press.

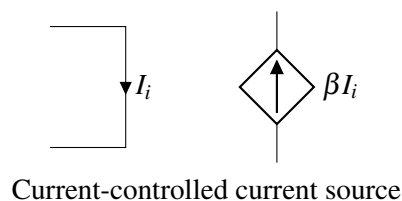
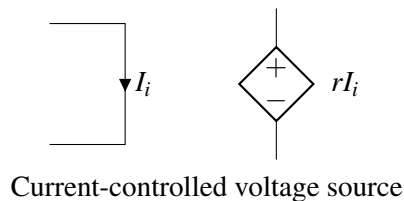
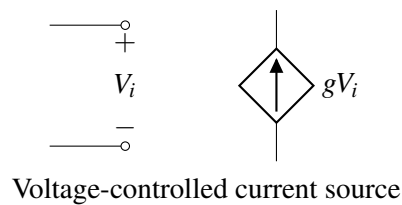
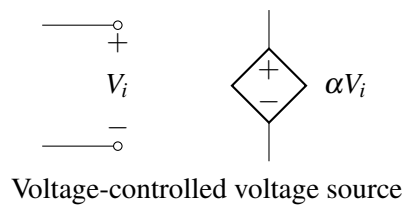
This change in voltage can be quite small since C_Δ can be very small. Next we will introduce a *comparator* that will allow us to measure very small voltage changes.

17.3 Op-amp and Comparator Fundamentals

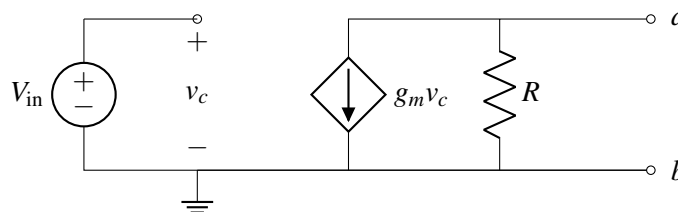
In this section, we will introduce a new circuit component called an *op-amp*. As we'll see shortly, an op-amp can be used as a *comparator* – something that compares two voltages and indicates which is larger. However, op-amps have many other uses too, and we'll explore these more in the next note.

17.3.1 Dependent Sources

At this point, we will introduce a new circuit element: dependent sources. Dependent sources act like the independent sources we've studied so far, but instead of producing a single voltage/current, the voltage/current is *controlled* by something else in the circuit. There are four different types of dependent sources, shown below:



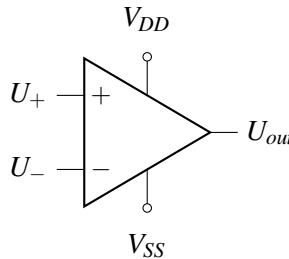
Here, the diamond symbol represents the source, which produces voltage or current proportional to a different voltage or current elsewhere in the circuit. Let's make this more concrete by looking at an example:



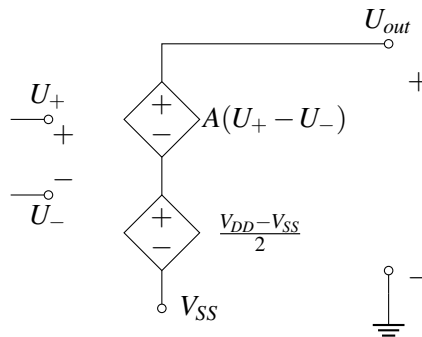
In this case, the current in the dependent source is equal a constant (g_m) times the voltage v_c , which is defined elsewhere in the circuit. Just like an independent current source, this dependent source will produce the same current, regardless of what is connected to it (unless v_c changes – then the current will change appropriately). As we'll see in future notes, dependent sources are useful tools for modeling many advanced circuit elements.

17.3.2 Op-amp Model

By definition, an amplifier is something that can transform something small into something much bigger. For example, a speaker is an audio amplifier — if you connect your smart phone to a speaker, it can generate sounds much louder than your phone can. An **op-amp (operational amplifier)** is a device that transforms a small voltage difference into a very large voltage difference. The circuit symbol for an op-amp is shown below:



An op-amp has two input terminals marked (+) and (−) with potentials U_+ and U_- , two power supply terminals called V_{DD} and V_{SS} , and one output terminal with potential U_{out} . The internal circuit design of an op-amp is beyond the scope of this course (if you are curious, you can search for schematics online), but we can model the op-amp with the following circuit:



This circuit includes two voltage-controlled voltage sources. (Note that the dependent voltage sources in the schematic above are referenced to V_{SS} , not ground.) The bottom voltage source adds the voltage $\frac{V_{DD}-V_{SS}}{2}$ so that when $U_+ = U_-$, the output voltage is the midpoint between V_{DD} and V_{SS} . The top source creates a voltage $A(U_+ - U_-)$, which amplifies the difference between the two input terminals.

In a good op-amp, the constant A is very large — approaching infinity. If A is very large, does the op-amp produce infinite voltage? It might seem like this is the case from the equivalent circuit schematic, but the op-amp's power must come from somewhere – it comes from the supply voltages V_{SS} and V_{DD} . This means that U_{out} cannot be greater than V_{DD} or less than V_{SS} .

Following the schematic given above, the behavior of an op-amp can be summarized with the equation

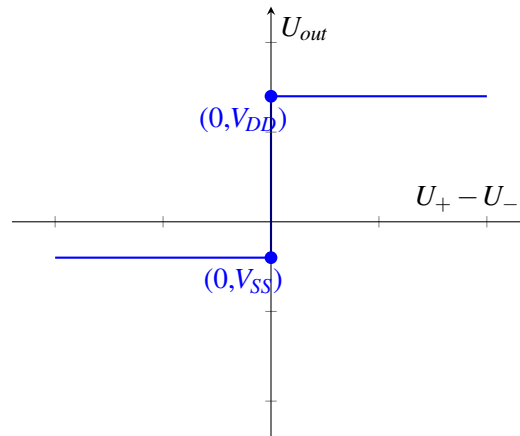
$$U_{out} = A(U_+ - U_-) + \frac{V_{DD} + V_{SS}}{2},$$

except U_{out} cannot be above V_{DD} or below V_{SS} . So U_{out} is actually given by the following equation, where the first and third cases are the cutoffs:

$$U_{out} = \begin{cases} V_{SS} & A(U_+ - U_-) + \frac{V_{DD}+V_{SS}}{2} < V_{SS} \\ A(U_+ - U_-) + \frac{V_{DD}+V_{SS}}{2} & V_{SS} \leq A(U_+ - U_-) + \frac{V_{DD}+V_{SS}}{2} \leq V_{DD} \\ V_{DD} & V_{DD} < A(U_+ - U_-) + \frac{V_{DD}+V_{SS}}{2} \end{cases} \quad (5)$$

17.3.3 Op-amps as Comparators

For very large A (close to infinity), the non-clipped output $A(U_+ - U_-) + \frac{V_{DD}+V_{SS}}{2}$ is infinity if $U_+ - U_- > 0$, and negative infinity if $U_+ - U_- < 0$. Then, we are always in either the first or third case, which gives us the following plot:

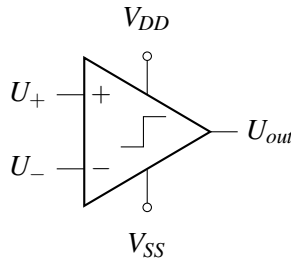


Since A is very large, there is a very sharp transition in the center of the plot. Therefore, when $U_+ < U_-$ (or equivalently, when $U_+ - U_- < 0$), then U_{out} equals V_{SS} . Similarly, when $U_+ > U_-$, then U_{out} equals V_{DD} . In this case, the op-amp is acting as a comparator since it indicates which voltage (U_+ or U_-) is larger, even if the difference between them is very small.

17.3.4 Comparators in Practice

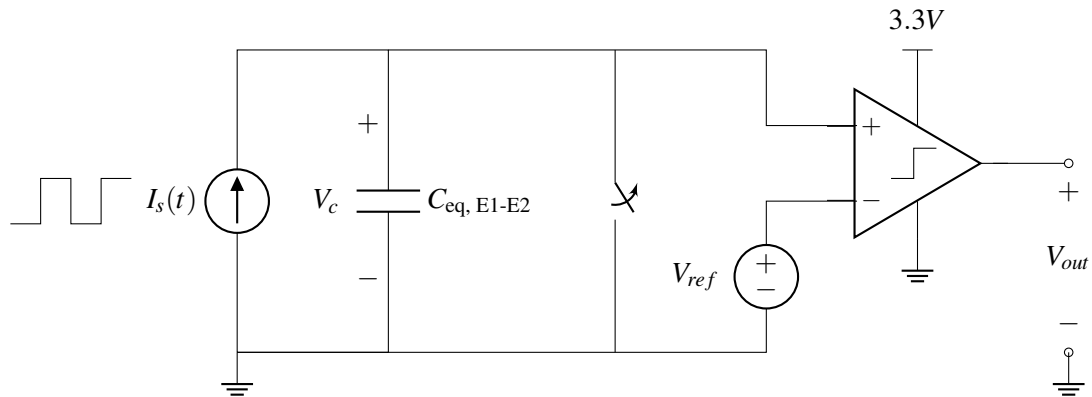
In practice, circuit designers rarely use op-amps to compare inputs but instead use special components called *comparators*. As demonstrated above, op-amps can function as comparators; however, they are rather slow and imprecise since they're optimized for signal amplification with output voltages always staying between the rails. Dedicated comparators, by contrast, are designed to output only the supply voltages and are preferred to op-amps for their faster operation. The plot of input and output voltages for a comparator is identical to that of an ideal op-amp in the comparator configuration, shown in the previous subsection.

The circuit symbol for a comparator is shown below:



17.4 Capacitive Touchscreen with Comparator

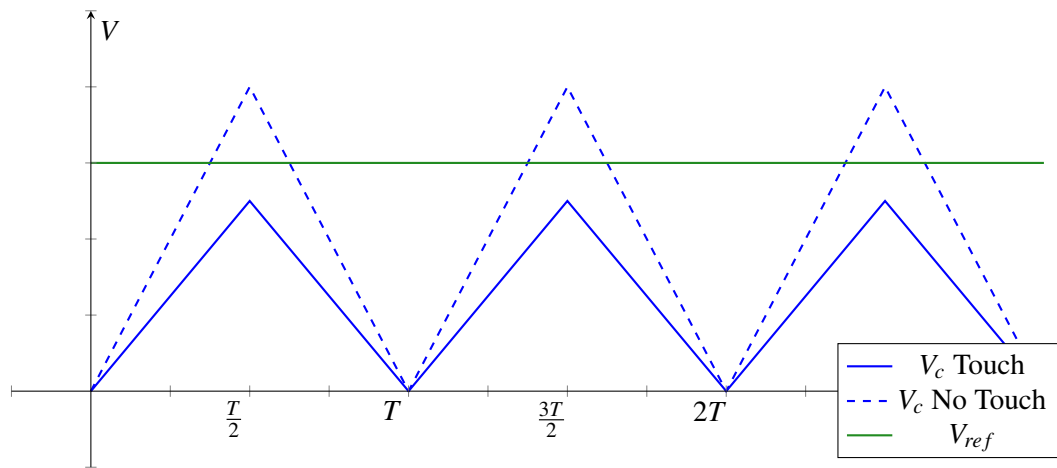
Now we can complete our touchscreen by adding a comparator to detect if a finger is pressing at a given pixel.



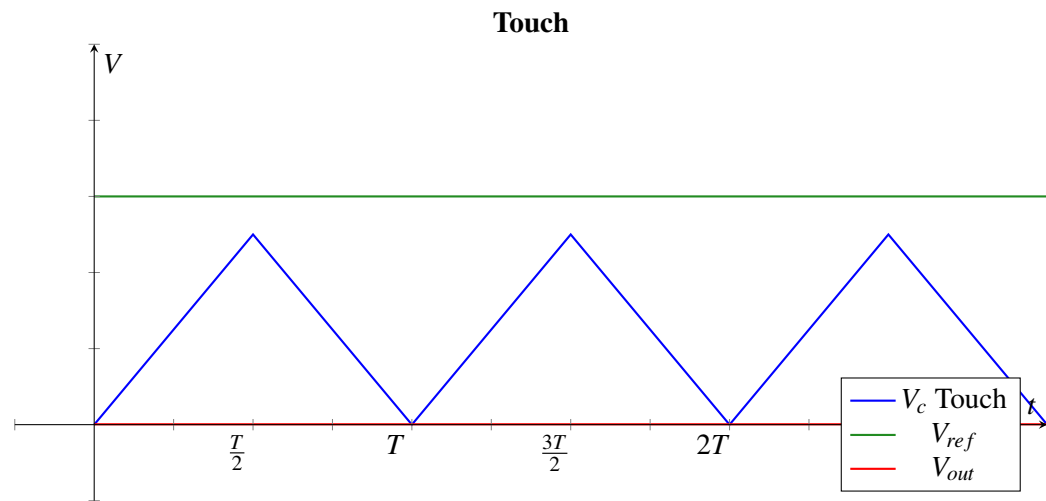
First, note that the voltage over the capacitor V_c is equal to the voltage at the positive terminal of the comparator ($U_+ = V_c$). The negative terminal of the comparator is connected to V_{ref} so $U_- = V_{ref}$. Using the equations for a comparator, we can write

$$V_{out} = \begin{cases} 3.3V & \text{if } V_c > V_{ref} \\ 0V & \text{if } V_c < V_{ref} \end{cases}$$

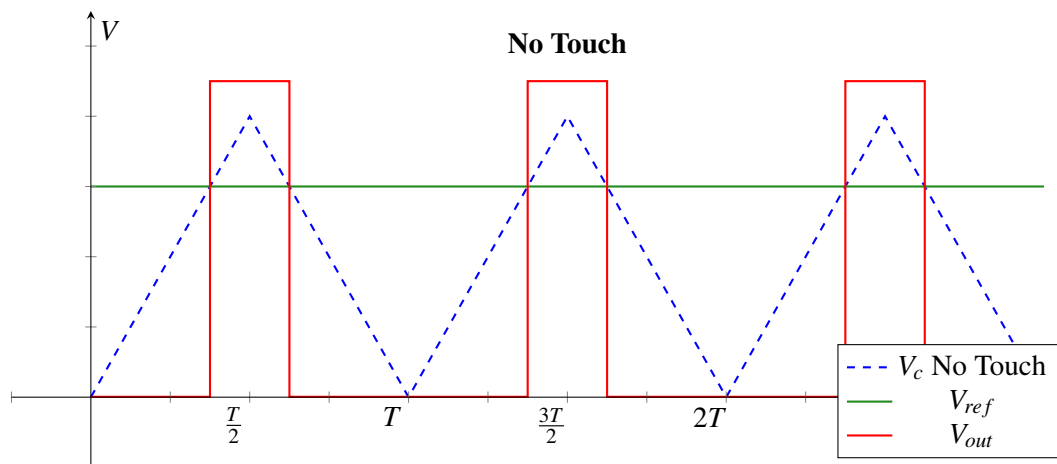
How should we choose V_{ref} ? We would like the behavior of V_{out} to depend on the presence or absence of a finger touch, so we should set V_{ref} to be somewhere in between the peak of V_c with a finger and the peak of V_c without a finger. In fact, to be most robust to noise, V_{ref} should be exactly between the two peaks.



When there is a finger present, V_c is always less than V_{ref} so the output voltage is 0:



When there is no finger present, then there will be an increase in V_{out} when V_c is higher than V_{ref} :



Every period of the current source cycle, we can measure if there is a finger touching this pixel. Later in this course, we will learn how to build a periodic current source like the one needed in this design — after that you will be ready to completely build your own capacitive touchscreen!