EECS 16A Designing Information Devices and Systems I
Spring 2023 Discussion 5B

1. Steady and Unsteady States

You're given the matrix $\mathbf{M}$:

$$\mathbf{M} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ 0 & 1 & -2 \\ 0 & 0 & 2 \end{bmatrix}$$

which generates the next state of a physical system from its previous state: $\vec{x}[k+1] = \mathbf{M}\vec{x}[k]$.

(a) The eigenvalues of $\mathbf{M}$ are $\lambda_1 = 1, \lambda_2 = 2, \lambda_3 = \frac{1}{2}$. Define $\vec{x} = \alpha\vec{v}_1 + \beta\vec{v}_2 + \gamma\vec{v}_3$, a linear combination of the eigenvectors corresponding to the eigenvalues. For each of the cases in the table, determine if

$$\lim_{n\to\infty} \mathbf{M}^n\vec{x}$$

converges. If it does, what does it converge to?

α	β	γ	Converges?	$\lim_{n\to\infty} \mathbf{M}^n\vec{x}$
0	0	$\neq 0$		
0	$\neq 0$	0		
0	$\neq 0$	$\neq 0$		
$\neq 0$	0	0		
$\neq 0$	0	$\neq 0$		
$\neq 0$	$\neq 0$	0		
$\neq 0$	$\neq 0$	$\neq 0$		

(b) (**Practice**) Find the eigenspaces associated with the eigenvalues:

i. span($\vec{v}_1$), associated with $\lambda_1 = 1$
ii. span($\vec{v}_2$), associated with $\lambda_2 = 2$
iii. span($\vec{v}_3$), associated with $\lambda_3 = \frac{1}{2}$

2. Steady State Reservoir Levels

We have 3 reservoirs: A, B, and C. The pumps system between the reservoirs is depicted in Figure 1.

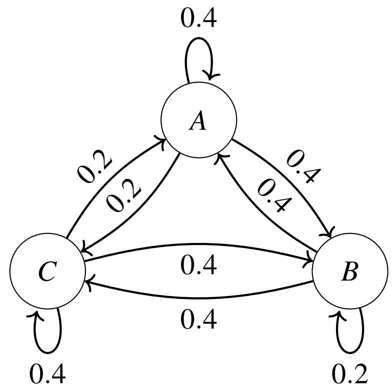

Figure 1: Reservoir pumps system.

(a) Assuming

$$\vec{x} = \begin{bmatrix} x_A \\ x_B \\ x_C \end{bmatrix}$$

is a vector representing the amount of water in tank A, B, and C respectively, write out the corresponding transition matrix $\mathbf{T}$ representing the pumps system.

(b) You are told that $\lambda_1 = 1$, $\lambda_2 = \frac{1}{5}$, $\lambda_3 = -\frac{1}{5}$ are the eigenvalues of $\mathbf{T}$. Find a steady state vector $\vec{x}$, i.e. a vector such that $T\vec{x} = \vec{x}$.

(c) What does the magnitude of the other two eigenvalues λ_2 and λ_3 say about the steady state behavior of their associated eigenvectors?

(d) Assuming that you start the pumps with the water levels of the reservoirs at $A_0 = 150, B_0 = 250, C_0 = 200$ (in kiloliters), what would be the steady state water levels (in kiloliters) according to the pumps system described above?

