EECS 16A Designing Information Devices and Systems I Homework 12

This homework is due April 14, 2023, at 23:59. Note: Slip days will NOT be allowed on this HW in order to release solutions before Midterm 2.

Self-grades are due April 21, 2023, at 23:59.

# **Submission Format**

Your homework submission should consist of **one** file.

• hw12.pdf: A single PDF file that contains all of your answers (any handwritten answers should be scanned).

Submit the file to the appropriate assignment on Gradescope.

# 1. Reading Assignment

For this homework, please read Notes 19 and 20. They will provide an overview on the "golden rules" of op-amps, various op-amp configurations (non-inverting, inverting, buffers, etc), and designing circuits using op-amps. You are always encouraged to read beyond this as well.

(a) What are the two "golden rules" of ideal op-amps? When do these rules hold true?

**Solution:** The golden rules are the following:

- The currents into the input terminals of the op-amp are zero, i.e.  $I_+ = I_- = 0$ . This rule is always true.
- The error signal going into the op-amp must be zero, i.e.  $u_+ = u_-$ . This rule only holds when there is negative feedback and the op-amp gain is large.
- (b) What is the effect of "loading" and how can op-amps be used to mitigate this effect?

**Solution:** Loading is the effect when a load is connected to the output of a circuit. If we model the circuit output terminal as a Thevenin equivalent, we notice that the load resistance forms a voltage divider with the Thevenin resistance. As a result, the output voltage will decrease due to the loading effect. If we place an op-amp in between the circuit output and the load, we can mitigate the loading effect, since the op-amp will not draw any current on the inputs and thus there will be no voltage drop over the Thevenin resistance. Furthermore, the op-amp output is connected directly to a voltage source and thus will not change no matter what load resistance is attached.

# 2. Op-Amp in Negative Feedback, Round #2

In this problem we will analyze the same op-amp circuit in Problem 3 of last week's Homework 11. However, this time the op-amp is assumed to have infinite gain,  $A \to \infty$ , and we will analyze the circuit directly with KCL, KVL, and the op-amp Golden Rules.

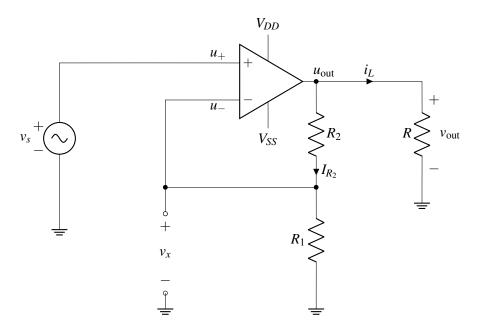
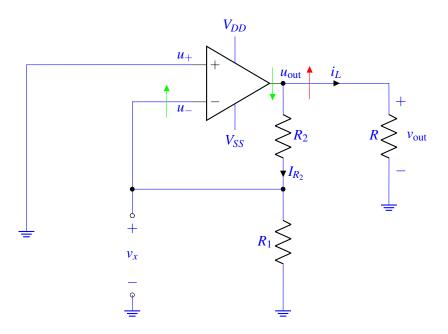


Figure 1: Non-inverting amplifier circuit using an op-amp for feedback

# (a) Is the circuit in Fig. 1 in negative feedback?

Hint: Turn off all independent sources, wiggle the output voltage, and check if the feedback in the loop will counteract the wiggle.

# **Solution:**



First, turn off the independent voltage source  $v_s$  (i.e.,  $v_s = 0$ ) which becomes a short circuit. We notice that no current flows in the feedback loop (since  $i_- = 0$ ), and thus  $u_-$  is the middle node of a voltage divider created by  $R_1, R_2$ . Now, suppose we increase the output voltage  $u_{\text{out}}$  (in red). This would increase the voltage on the terminal  $u_-$  (in green). Then the difference  $u_+ - u_-$  would decrease, causing an *decrease* in the output voltage, since  $v_{\text{out}} = A(u_+ - u_-)$ .

Our initial positive perturbation of  $u_{\text{out}}$  will cause a counteracting decrease in  $u_{\text{out}}$  from the feedback of the op-amp. Thus, the op-amp is in negative feedback.

(b) What is  $u_{+} - u_{-}$ ?

Note: Since in part (a) we confirmed the circuit is in negative feedback, we can apply both op-amp Golden Rules.

**Solution:** For ideal op-amp circuits in negative feedback, the voltage at the two terminals must be equal, so  $u_+ - u_- = 0$ .

- (c) Find  $v_x$  as a function of  $v_{out}$ . Hint: What is the current into the negative terminal  $u_-$  of the op-amp? **Solution:** As we noted in part (a), the current into  $u_-$  is zero and thus the voltage  $v_x$  is the middle node of a voltage divider, so  $v_x = v_{out} \frac{R_1}{R_1 + R_2}$ .
- (d) Now use the results of part (b) and (c) to find  $v_{out}$  as a function of  $v_s$ .

**Solution:** Since  $u_+ = v_s$  and  $u_- = v_x$ , we know that  $v_s = v_x$ . The results of part (c) then becomes

$$v_s = v_{out} \frac{R_1}{R_1 + R_2}$$

which we can rearrange to get

$$v_{out} = v_s \frac{R_1 + R_2}{R_1}.$$

(e) Does the value of the load resistor R affect the output voltage  $v_{out}$ ? Why or why not?

**Solution:** We notice that the expression for  $v_{out}$  that we derived in the previous part does not include the load resistance R. Thus the load resistor does not affect the output voltage. This is because the equivalent circuit model of an op-amp has a voltage-controlled voltage source connected directly to the output. In other words, the output of an op-amp is driven by a voltage source and thus the output voltage will not change based on the resistance at that node.

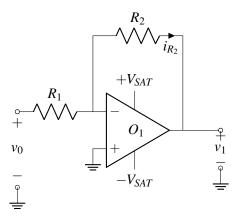
### 3. Integration using Op-amps

Analog circuits can be used to implement many different mathematical functions. In this problem, we will see how we can use an op-amp to create an integrator. An integrator circuit takes a time-varying voltage input  $v_0(t)$  and integrates it over a time period. In other words, we want to build a circuit where the output is of the form

$$v_1(t) = K \int_0^t v_0(\tau) d\tau$$

for some constant K.

(a) Let's analyze the inverting op-amp configuration shown below. For this problem, we will assume that the op-amp is ideal and apply the op-amp golden rules.

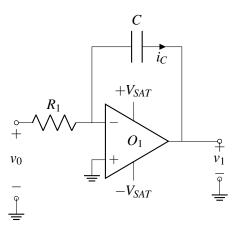


What is the current  $i_{R_2}$  flowing through resistor  $R_2$ ? Write your answer in terms of  $v_0$ ,  $R_1$ , and  $R_2$ .

**Solution:** This is the inverting amplifier we have seen before! We know that this op-amp is in negative feedback (although you may want to check again for youself for practice). Using the op-amp golden rules, we know that  $u^- = u^+ = 0$ . Furthermore since no current flows into the negative input of the op-amp, we must have  $i_{R_1} = i_{R_2}$ . We can then use Ohm's law to find

$$i_{R_2} = i_{R_1} = \frac{v_0 - 0}{R_1} = \frac{v_0}{R_1}.$$

(b) What happens if we replace the resistor in feedback  $R_2$  with a capacitor C instead? Analyze the circuit to find the current through the capacitor  $i_C$  and express your answer in terms of  $v_0$ ,  $R_1$ , and C. How does this current differ from the previous part?



**Solution:** In the previous part, we saw that  $R_2$  does not affect  $i_{R_2}$ . Instead,  $i_{R_1}$  (and thus  $i_{R_2}$ ) is set by the voltage drop  $v_0$  across  $R_1$ . When we replace  $R_2$  with C,  $i_{R_1}$  does not change so we must still have

$$i_C = i_{R_1} = \frac{v_0}{R_1}.$$

The current is the same as in part (a)!

(c) Assume that the capacitor starts uncharged at t = 0 and that  $v_0(t)$  varies with time. Solve for the output voltage  $v_1(t)$  as a function of time t. Express your answer in terms of  $v_0(t)$ ,  $R_1$ , and C. Hint: You may leave your answer as an integral of  $v_0(t)$  as shown in the initial problem statement.

**Solution:** We know that  $u^- = u^+ = 0$  using the second op-amp golden rule. Letting  $V_C$  be the voltage drop across the capacitor, we note that

$$V_C = u^- - v_1 = -v_1$$

since  $i_C$  points to the right and we must follow passive sign convention. In part (b), we found that

$$i_C(t) = \frac{v_0(t)}{R_1}.$$

Recall for a capacitor  $Q = CV_C$ . If we take the derivative with respect to time of both sides, see

$$i_C(t) = \frac{dQ}{dt} = C\frac{dV_C(t)}{dt}.$$

We can now solve this differential equation

$$C\frac{dV_C(t)}{dt} = i_C(t)$$

$$\frac{dV_C(t)}{dt} = \frac{i_C(t)}{C}$$

$$\frac{dV_C(t)}{dt} = \frac{v_0(t)}{R_1C}$$

$$V_C(t) - V_C(0) = \frac{1}{R_1C} \int_0^t v_0(\tau) d\tau$$

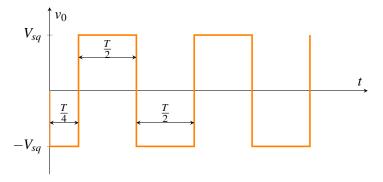
$$V_C(t) = \frac{1}{R_1C} \int_0^t v_0(\tau) d\tau$$

where we have used the fact that  $V_C(0) = 0$  since the capacitor starts uncharged. Finally we can relate

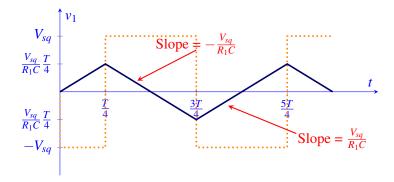
$$v_1(t) = -V_C(t) = -\frac{1}{R_1 C} \int_0^t v_0(\tau) d\tau.$$

We have shown that the output voltage of our op-amp circuit is the integral of the input voltage scaled by the constant  $-\frac{1}{R_1C}$ .

(d) If  $v_0$  varies with time as shown in the following diagram, plot  $v_1$  for t = 0 to t = 1.5T. In your plot indicate an algebraic expression for the slope (as a function of  $R_1$ , C and  $V_{sq}$ ) and add tick marks on the x and y axis indicating the time and voltage values where the ramp slope changes. You may assume again that capacitor C has 0V across it at time t = 0.



**Solution:** When  $v_0(t) = -V_{sq}$  the integrator output will increase linearly with slope  $\frac{V_{sq}}{R_1C}$ . When  $v_0(t) = V_{sq}$  the integrator output will decrease linearly with slope  $-\frac{V_{sq}}{R_1C}$ . Thus the integrator will output a triangle wave!



Previously, we saw that we can construct a triangle wave output voltage using an oscillating current source and capacitor. If we only have access to an oscillating voltage source, we can use the op-amp integrator instead!

#### 4. Cool For The Summer

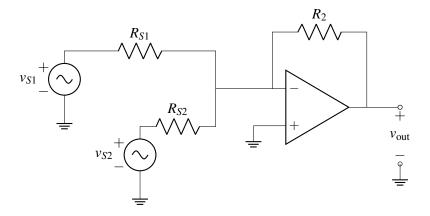
You and a friend want to make a box that helps control an air conditioning unit based on both your inputs. You both have individual dials which you can use to control the voltage. An input of 0 V means that you want to leave the temperature as is. A **negative voltage input** means that you want to **reduce** the temperature. (It's hot out, so we will assume that you never want to increase the temperature – so no, we're not talking about a Berkeley summer...)

Your air conditioning unit, however, responds only to **positive voltages**. The higher the magnitude of the voltage, the stronger it runs. At zero, it is off. You also need a system that **sums up** both you and your friend's control inputs.

Therefore, you need a box that acts as an **an inverting summer** – *it outputs a weighted sum of two voltages where the weights are both negative*. The sum is weighted because one room is bigger, so you need to compensate for this.

(a) You suggest the circuit below, essentially an inverting amplifier with two inputs. Find  $v_{\text{out}}$  in terms of  $v_{S1}$ ,  $v_{S2}$ ,  $R_{S1}$ ,  $R_{S2}$  and  $R_2$ .

Hint: You can solve this problem using either superposition or our tried-and-true KCL analysis.



### **Solution:**

Method 1: Superposition

First, when considering  $v_{S1}$ , we zero out  $v_{S2}$ , and therefore we can disregard  $R_{S2}$ . The reason why we can disregard  $R_{S2}$  is because by the Golden Rules, we know that the voltage at the – terminal of the

op-amp must be equal to the voltage at the + terminal. Therefore, both terminals of  $R_{S2}$  are at 0V, and no current flows through  $R_{S2}$ . With this insight, we recognize that this is just an inverting amplifier! We apply the inverting amplifier gain equation:

$$v_{\text{out}} = -\frac{R_2}{R_{S1}} v_{S1}.$$

Similarly, when  $v_{S2}$  is on and  $v_{S1}$  is zeroed out, we disregard  $R_{S1}$  by the same argument and again have an inverting amplifier. In this case,

$$v_{\text{out}} = -\frac{R_2}{R_{S2}} v_{S2}.$$

Combining the two  $v_{\text{out}}$  equations from superposition, we get

$$v_{\text{out}} = -R_2 \left( \frac{v_{S1}}{R_{S1}} + \frac{v_{S2}}{R_{S2}} \right).$$

# Method 2: KCL without superposition

According to the Golden Rules,  $u_- = u_+ = 0$ V, so we can write a single KCL equation at the  $u_-$  node and solve:

$$\frac{v_{S1}}{R_{S1}} + \frac{v_{S2}}{R_{S2}} + \frac{v_{\text{out}}}{R_2} = 0$$

$$v_{\text{out}} = -v_{S1} \left(\frac{R_2}{R_{S1}}\right) - v_{S2} \left(\frac{R_2}{R_{S2}}\right)$$

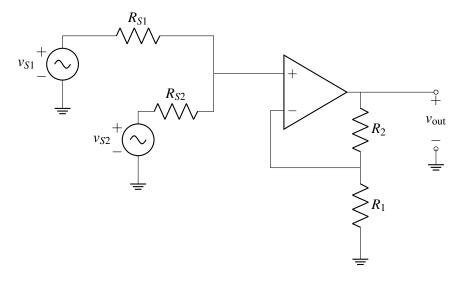
(b) Let's suppose that you want  $v_{\text{out}} = -\left(\frac{1}{4}v_{S1} + 2v_{S2}\right)$  where again  $v_{S1}$  and  $v_{S2}$  represent the input voltages from you and your friend's control knobs. Select resistor values such that the circuit from part (b) implements this desired relationship.

**Solution:** Using the configuration from the previous part, the conditions which need to be satisfied are:

- $\frac{R_2}{R_{S1}} = \frac{1}{4}$   $\frac{R_2}{R_{S2}} = 2$

One possible set of values is  $R_2 = 2k\Omega$ ,  $R_{S1} = 8k\Omega$ , and  $R_{S2} = 1k\Omega$ , but any combination of resistors which satisfies  $R_{S1} = 4R_2 = 8R_{S2}$  are valid solutions.

(c) Your friend has a different circuit idea. He proposes the following circuit below.



Find  $v_{\text{out}}$  in terms of  $v_{S1}$ ,  $v_{S2}$ ,  $R_{S1}$ ,  $R_{S2}$ ,  $R_1$ , and  $R_2$ . Can we also use this circuit to control our AC system? Why or why not?

Hint: How does this circuit relate to the one in question 2?

#### **Solution:**

We notice that the op-amp circuit is just the non-inverting amplifier we studied in question 2! If we can find the voltage at the positive op-amp input  $v_+$  in terms of  $v_{S1}$  and  $v_{S2}$ , then we can use equation we derived:

$$v_{\text{out}} = \frac{R_1 + R_2}{R_1} v_+. \tag{1}$$

Lets again use superposition to find  $v_+$ . Turning off  $v_{S2}$  gives us a voltage divider where  $v_+$  is the middle node. Thus

$$v_+ = v_{S1} \frac{R_{S2}}{R_{S1} + R_{S2}}.$$

If we turn off  $v_{S2}$ , we also get a voltage divider but this time

$$v_+ = v_{S2} \frac{R_{S1}}{R_{S1} + R_{S2}}.$$

We can use superposition which gives

$$v_{+} = v_{S1} \frac{R_{S2}}{R_{S1} + R_{S2}} + v_{S2} \frac{R_{S1}}{R_{S1} + R_{S2}}.$$

Now substituting  $v_+$  into equation 1 gives us

$$v_{\text{out}} = v_{S1} \frac{R_1 + R_2}{R_1} \frac{R_{S2}}{R_{S1} + R_{S2}} + v_{S2} \frac{R_1 + R_2}{R_1} \frac{R_{S1}}{R_{S1} + R_{S2}}.$$

Although this circuit also implements a weighted summer, we see that this one does not invert the output. Namely, the coefficients of  $v_{S1}$ ,  $v_{S2}$  are always positive. In our case we need to convert negative input voltages into a positive output voltage which this circuit cannot do.

### 5. Island Karaoke Machine

**Learning Goal:** The objective of this problem is design a circuit that calculates the difference between two signals and amplifies the result.

You're stuck on a desert island and everyone is bored out of their minds. Fortunately, you have your EECS16A lab kit with op-amps, wires, resistors, and your handy breadboard. You decide to build a karaoke machine. You recover one speaker from the crash remains and use your iPhone as your source. You know that many songs put instruments on either the "left" or the "right" channel, but the vocals are usually present on both channels with equal strength.

In our case, the vocals are present on both left and right channels, but the instruments are only present on the right channel, i.e.

$$v_{\text{left}} = v_{\text{vocals}}$$
  
 $v_{\text{right}} = v_{\text{vocals}} + v_{\text{instrument}},$ 

where the voltage source  $v_{\text{vocals}}$  can have values anywhere in the range of  $\pm 120 \,\text{mV}$  and  $v_{\text{instrument}}$  can have values anywhere in the range of  $\pm 50 \,\text{mV}$ .

What is the goal of a karaoke machine? **The ultimate goal is to** *remove* **the vocals from the audio output.** We're going to do this by first building a circuit that takes the left and right audio outputs of the smartphone and then calculates its **difference**. Let's see what happens.

The equivalent circuit model of the iPhone audio jack and speaker is shown in Figure 2. We model the **audio signals and jack** as  $v_{\text{left}}$  and  $v_{\text{right}}$  with **equivalent source resistance** of the left/right audio channels of  $R_{\text{left}} = R_{\text{right}} = 3\Omega$ . The **speaker** has an equivalent resistance of  $R_{\text{speaker}} = 4\Omega$ .

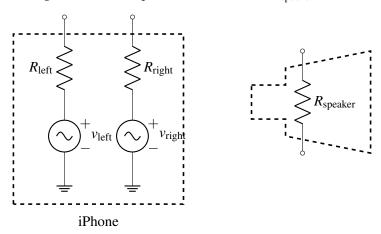


Figure 2: Audio jack and speaker of an iPhone.

(a) One of your island survivors suggests the circuit in Figure 3 to do this. Find the expression for the voltage across the speaker  $R_{\text{speaker}}$  as a function of  $v_{\text{vocals}}$  and  $v_{\text{instruments}}$ .

Does the voltage across the speaker depend on  $v_{\text{vocals}}$ ? In other words, what do you think the islanders will hear – vocals, instruments, or both?

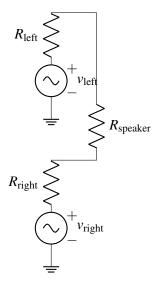
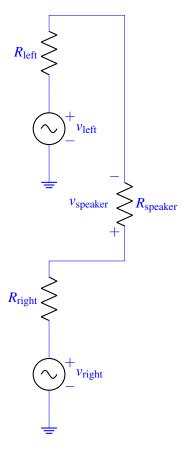


Figure 3: Circuit for part (a).

# **Solution:**

Let's mark the voltage across the speaker,  $v_{\text{speaker}}$ , from bottom to top as in the figure:



We can apply the principle of superposition to solve for  $v_{\text{speaker}}$ . First, we solve for the voltage across the speaker when only  $v_{\text{left}}$  is on. Let's call this  $v_{\text{speaker,left}}$ . Notice that the circuit becomes a voltage divider. Therefore, we get

$$-v_{\text{speaker},\text{left}} = \frac{v_{\text{left}}R_{\text{speaker}}}{R_{\text{speaker}} + R_{\text{left}} + R_{\text{right}}} = \frac{4v_{\text{vocals}}}{10} = 0.4v_{\text{vocals}},$$

giving

$$v_{\text{speaker,left}} = -0.4v_{\text{vocals}}$$
.

Similarly, we solve for the voltage across the speaker when only  $v_{\text{right}}$  is on. Let's call this  $v_{\text{speaker,right}}$ . Again, notice that the circuit becomes a voltage divider. Therefore, we get

$$v_{\text{speaker,right}} = \frac{v_{\text{right}} R_{\text{speaker}}}{R_{\text{speaker}} + R_{\text{left}} + R_{\text{right}}} = \frac{4(v_{\text{vocals}} + v_{\text{instrument}})}{10} = 0.4(v_{\text{vocals}} + v_{\text{instrument}}).$$

Superposition tells us that  $v_{\text{speaker}} = v_{\text{speaker,left}} + v_{\text{speaker,right}} = 0.4v_{\text{instrument}}$ .

What did you notice? The vocals got canceled out! The islanders will only hear the instruments, just as they wanted.

(b) We need to boost the sound level to get the party going. To this end, we want a range of  $\pm 2$  V across the speaker. Design a circuit by completing the Figure 4 below that takes in  $\{v_{\text{left}}, R_{\text{left}}\}$  and  $\{v_{\text{right}}, R_{\text{right}}\}$  combos as inputs and outputs an **amplified version of**  $v_{\text{instrument}}$  **across**  $R_{\text{speaker}}$ . Consider all op-amps to be **ideal** for this problem.

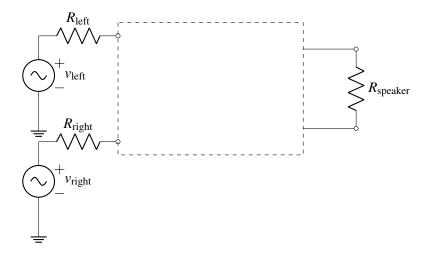


Figure 4: Circuit for part (b).

Hint 1: We need to get an output voltage with the range of  $\pm 2V$ . The input voltage  $v_{instrument}$  can have values anywhere in the range of  $\pm 50\,\text{mV}$ . What gain is needed from the op-amp based amplifier circuits?

Hint 2: Use two op-amps in the non-inverting configuration. The non-ideal voltage source  $\{v_{left}, R_{left}\}$  must be the input to one non-inverting amplifier and the non-ideal voltage source  $\{v_{right}, R_{right}\}$  must be the input to the other non-inverting amplifier. Keep the gain of both amplifiers equal so the vocals still cancel out across the speaker!

*Hint 3:* Connect the speaker  $R_{speaker}$  across the outputs of those two op-amps.

### **Solution:**

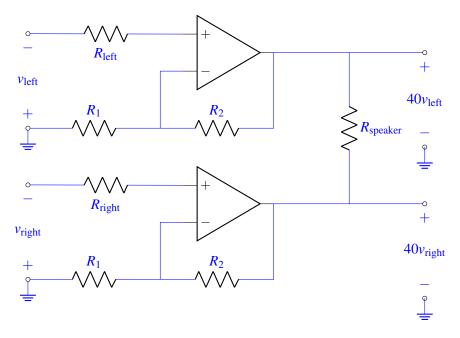
Feed the non-ideal voltage source  $\{v_{\text{left}}, R_{\text{left}}\}$  into a non-inverting amplifier with gain G and the non-ideal voltage  $\{v_{\text{right}}, R_{\text{right}}\}$  into another non inverting amplifier also with gain G. Then connect the two outputs across  $R_{\text{speaker}}$  as shown in the previous part.

In this circuit, the output of the amplifier connected to  $v_{\text{left}}$  is  $Gv_{\text{left}}$  and the output of the amplifier connected to  $v_{\text{right}}$  is  $Gv_{\text{right}}$ . Since  $R_{\text{speaker}}$  is connected between these two nodes, the voltage across the speaker is

$$V_{R_{\text{speaker}}} = Gv_{\text{right}} - Gv_{\text{left}} = G(v_{\text{vocals}} + v_{\text{instrument}} - v_{\text{vocals}}) = Gv_{\text{instrument}}.$$

 $v_{\text{instrument}}$  has voltage range of  $\pm 50 \,\text{mV}$  while we want a voltage range of  $\pm 2 \,\text{V}$  across the speaker. In other words, we need to amplify  $v_{\text{instrument}}$  by  $\frac{2 \,\text{V}}{50 \,\text{mV}} = 40$ .

Therefore, we want to design a non-inverting amplifier with voltage gain of G = 40 using the circuit shown below:



Now, we need to find  $R_1$  and  $R_2$ . For a non-inverting amplifier, we know

$$G = 1 + \frac{R_2}{R_1}$$

Therefore, we can then choose any  $R_1$  and  $R_2$  such that  $\frac{R_2}{R_1} = 39$ . Note that there are multiple ways of choosing them. One such choice is  $R_1 = 1 \text{ k}\Omega$  and  $R_2 = 39 \text{ k}\Omega$ , for instance.

(c) The trouble with the approach in part (b) is that two op-amps are required. Let's say you only have **one op-amp** with you. What would you do? One night in your dreams, you have an inspiration. Why not combine the inverting and non-inverting amplifier into one, as shown in Figure 5!

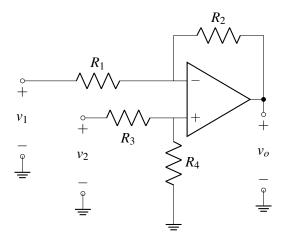


Figure 5: The new amplifier for part (c).

If we set  $v_2 = 0$  V, what is the output  $v_o$  in terms of  $v_1$ ? (This is the inverting path.) Solution:

 $R_3$  and  $R_4$  form a voltage divider with  $v_2$  as the supply and  $u_+$  as the middle node. However if we set  $v_2 = 0 \,\text{V}$ , then we must have  $u_+ = 0 \,\text{V}$  as well. Applying the Golden Rules, we will get  $u_- = u_+ = 0 \,\text{V}$ .

Writing KCL at the - terminal of the op-amp, we get

$$\frac{v_1 - 0}{R_1} = \frac{0 - v_{o,1}}{R_2},$$

which gives

$$v_{o,1} = -\frac{R_2}{R_1}v_1.$$

(d) Consider the circuit in Figure 5 again. If we set  $v_1 = 0$  V, what is the output  $v_o$  in terms of  $v_2$ ? (This is the non-inverting path.)

# **Solution:**

If we set  $v_1 = 0$  V, we can solve for  $u_+ = \frac{v_2 R_4}{R_3 + R_4}$  using the voltage divider equation. By the op-amp golden rule, we know  $u_- = u_+$ . Writing KCL at the – terminal gives

$$\frac{0-u_{-}}{R_{1}}=\frac{u_{-}-v_{o,2}}{R_{2}},$$

which gives

$$v_{o,2} = u_{-} \left( 1 + \frac{R_2}{R_1} \right) = v_2 \left( \frac{R_4}{R_3 + R_4} \right) \left( 1 + \frac{R_2}{R_1} \right).$$

(e) Now, determine  $v_o$  in terms of  $v_1$  and  $v_2$  by superposing your results from the last two parts. Choose values for  $R_1$ ,  $R_2$ ,  $R_3$  and  $R_4$ , such that the output range is  $v_o = \pm 2 \,\mathrm{V}$  for  $v_2 - v_1 = \pm 50 \,\mathrm{mV}$ .

**Solution:** Using the principle of superposition, the resulting output voltage with both sources turned on is just the sum of the two expressions we found in the previous parts.

$$v_o = -\frac{R_2}{R_1}v_1 + \frac{R_4}{R_3 + R_4}\left(1 + \frac{R_2}{R_1}\right)v_2.$$

We noted in part (b) that we need to amplify both  $v_{\text{left}}$  and  $v_{\text{right}}$  by  $40 \times$  in order to get the proper output range. In other words, we want an output of  $v_o = -40v_1 + 40v_2$ . Thus we need to select resistor values that satisfy

$$\frac{R_2}{R_1} = 40$$

and

$$\frac{R_4}{R_3 + R_4} \left( 1 + \frac{R_2}{R_1} \right) = 40$$

$$\frac{R_4}{R_3 + R_4} (41) = 40$$

$$41R_4 = 40(R_3 + R_4)$$

$$\frac{R_4}{R_3} = 40.$$

As long as our resistor ratios satisfy the two equations above, we can select any values. For example, lets choose  $R_1 = 1 \,\mathrm{k}\Omega, R_2 = 40 \,\mathrm{k}\Omega, R_3 = 1 \,\mathrm{k}\Omega, R_4 = 40 \,\mathrm{k}\Omega$ .

(f) We now connect our iPhone and speaker to the inputs and outputs of the circuit we designed. We realize that we forgot to account for the Thevenin resistance of the left and right channel source from the iPhone. How should we modify our resistor value choices from part(e) to account for this?

*Hint:* What is the resistor configuration of  $R_{left}$ ,  $R_1$  and  $R_{right}$ ,  $R_3$ ? Can we combine these pairs into an equivalent resistor for each pair?

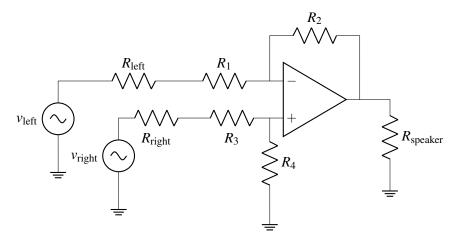


Figure 6: The differential amplifier connected to the source and speaker.

### **Solution:**

 $R_{\text{left}}, R_1$  and  $R_{\text{right}}, R_3$  are in series, so let's replace them with equivalent resistors

$$R_{1,eq} = R_{left} + R_1$$

$$R_{3,eq} = R_{right} + R_3.$$

Now, we have the same circuit we analyzed in part (e) and so again we can choose  $R_{1,eq} = 1 \,\mathrm{k}\Omega$ ,  $R_2 = 40 \,\mathrm{k}\Omega$ ,  $R_{3,eq} = 1 \,\mathrm{k}\Omega$ ,  $R_4 = 40 \,\mathrm{k}\Omega$ . Our new choices for  $R_1$  and  $R_3$  will then be

$$R_1 = R_{1,eq} - R_{\text{left}} = 1 \,\text{k}\Omega - 3 \,\Omega = 997 \,\Omega$$
  
 $R_3 = R_{3,eq} - R_{\text{right}} = 1 \,\text{k}\Omega - 3 \,\Omega = 997 \,\Omega.$ 

We can keep our choices of  $R_2$ ,  $R_4$  but need to subtract  $R_{left}$ ,  $R_{right}$  from the original choices of  $R_1$ ,  $R_3$ .

In part (e), we mentioned that we can select any values of resistors as long as they followed the ratios  $R_2 = 40R_1$  and  $R_4 = 40R_3$ . Here we see one advantage of choosing fairly large values ( $1 \text{ k}\Omega + \text{ as we}$  have done). In this case,  $R_1 \gg R_{\text{left}}$  and thus  $R_{1,eq} \approx R_1$  which means that the Thevenin resistance of the source has little impact on the behavior and gain of the circuit. This means that even if we connect our circuit to a different source with different Thevenin resistance (assuming this resistance is still small), we can still expect our circuit to amplify by around  $40\times$ .

### 6. Homework Process and Study Group

Who did you work with on this homework? List names and student ID's. (In case you met people at homework party or in office hours, you can also just describe the group.) How did you work on this homework? If you worked in your study group, explain what role each student played for the meetings this week.

#### **Solution:**

I first worked by myself for 2 hours, but got stuck on problem 5. Then I met with my study group.

XYZ played the role of facilitator ... etc. We were still stuck on problem 5 so we went to office hours to talk about the problem.

Then I went to homework party for a few hours, where I finished the homework.