EECS 16A Imaging 3

Insert your names here

Last time: Matrix-vector multiplication

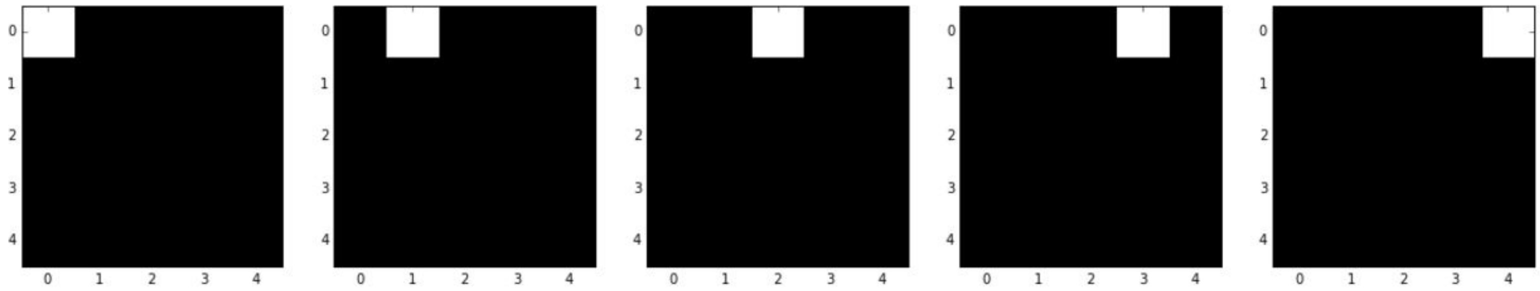
$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \ldots \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & \ldots \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & \ldots \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & \ldots \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & \ldots \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & \ldots \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & \ldots \\ \multicolumn{9}{c}{\ldots} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ \\ \\ \\ i_n \end{bmatrix} = \begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ \\ \\ \\ s_n \end{bmatrix}$$

Masking Matrix H

Unknown, vectorized image, $\vec{i}$

Recorded Sensor readings, $\vec{s}$

Last time: Single-pixel scanning



- Setup a masking matrix where each row is a mask
    - Measured each pixel individually once

$$\vec{s} = H\vec{\imath}$$

- How did we reconstruct our image, once we had s?

Poll Time! (this is review)

What are the requirements of our masking matrix H? (multiple choice)

A. H is invertible
B. H has linearly independent columns
C. H has a trivial nullspace
D. Determinant of H is 0.

$$\vec{s} = H\vec{\iota}$$

Our system

Poll Time! (this is review)

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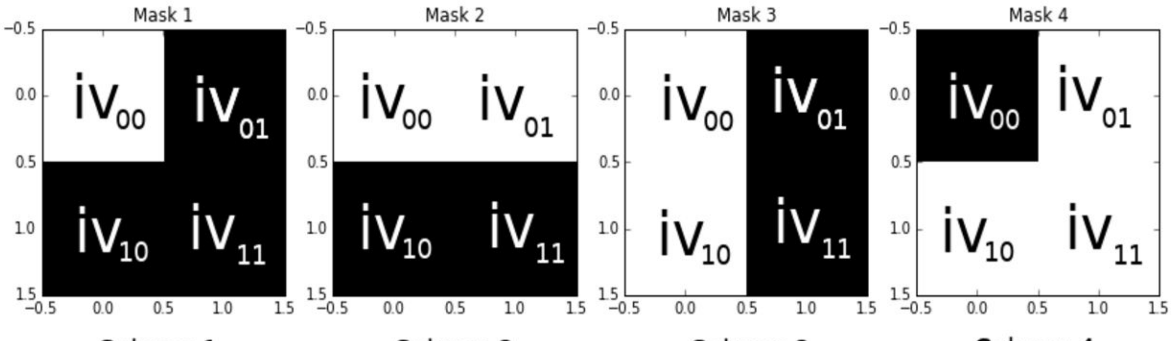
Our system

Questions from Imaging 2

Goal: Understand which measurements are good measurements

- ✓ Can we always reconstruct our image → **need invertible H**
- ? Are all invertible matrices equally good as scanning matrices?
- ? What happens if we mess up a single scan?
- ? What if we use multiple pixel instead of single pixel scan?

Today: Multi-pixel scanning



- **Can we measure multiple pixels at a time?**
  - Measurements are now linear combinations of pixels

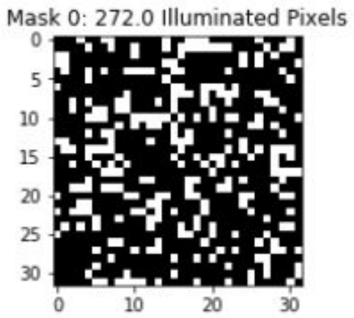
- **How can we reconstruct our scanned image?**
  - Is multi-pixel mask still possible to be linearly independent, aka invertible?

Why do we care?

- Improve image quality by averaging
  - Good measurements → good average

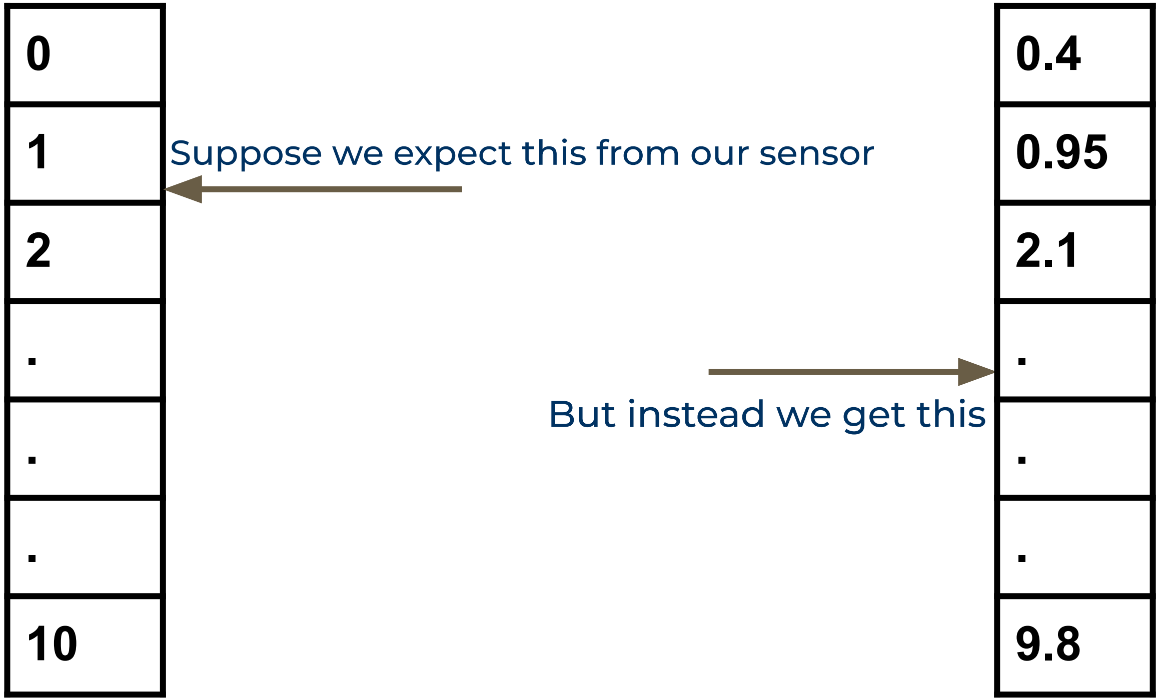
- Redundancy is useful
  - Averaging measurements is better than using bad measurement values
  - Does not “solve” bad measurements, but makes us tolerant of some errors

How do we do it?



- Change masks to illuminate multiple pixels per scan
  - Multiple 1's in each row of masking matrix H
  - Measure linear combinations of pixels instead of single pixels
- BUT multiple pixels → more noise
  - Noise = random variation in our measurement that we don't want (ex: room light getting into box)
  - Signal = data that we do want (light from pixel illumination)
- Too much noise → hard to distinguish signal from noise
  - Want high signal, low noise
  - **Extremely important** → High signal-to-noise ratio (SNR)

What is noise?



What is noise?

Measured values =
ideal vector + noise vector (ω)

$$\underbrace{\begin{bmatrix} 0.4 \\ 0.95 \\ 2.1 \\ . \\ . \\ . \\ 9.8 \end{bmatrix}}_{\vec{S}_{real}} = \underbrace{\begin{bmatrix} 0 \\ 1 \\ 2 \\ . \\ . \\ . \\ 10 \end{bmatrix}}_{\vec{S}_{ideal}} + \underbrace{\begin{bmatrix} 0.4 \\ -0.05 \\ 0.1 \\ . \\ . \\ . \\ -0.2 \end{bmatrix}}_{\vec{w}}$$

How does noise affect our system?

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \ldots \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & \ldots \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & \ldots \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & \ldots \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & \ldots \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & \ldots \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & \ldots \\ & & & & \ldots & & & & \end{bmatrix} \quad \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ \\ \\ \\ i_n \end{bmatrix} + \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \\ \\ \\ \omega_n \end{bmatrix} = \begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ \\ \\ \\ s_n \end{bmatrix}$$

Masking Matrix H

Unknown, vectorized image, $\vec{i}$

Random noise vector, $\vec{w}$

Recorded Sensor readings, $\vec{s}$

A more realistic system

- Sensor readings = image vectors applied to H + noise vector

$$\vec{s} = H\vec{i} + \vec{w}$$

- We can't reconstruct **i**, but we can estimate it

$$\vec{i}_{est} = H^{-1}\vec{s} = \vec{i} + \boxed{H^{-1}\vec{w}}$$

Be careful about the noise term or else it could blow up !!

Eigenvalues for inverse matrices

- H Is an NxN matrix that we know is linearly independent (invertible).
  - No eigenvalue = 0
- Assume H has N linearly independent eigenvectors
- $\mathrm{H}v_i = \lambda_i v_i \text{ for } i = 1 \ldots \mathrm{N}$
- N lin. ind. vectors can span $\mathbb{R}^N$
  - They span the noise vector
- The inverse of H has eigenvalues $\frac{1}{\lambda_1} \cdots \frac{1}{\lambda_N}$ (as proven in homework)

$$\mathrm{H}^{-1} v_i = \frac{1}{\lambda_i} v_i \text{ for } i = 1 \ldots \mathrm{N}$$

How do eigenvalues affect noise?

The noise vector can be written as:

$$\vec{\omega} = \alpha_1 \overrightarrow{v_1} + \alpha_2 \overrightarrow{v_2} + \cdots \alpha_n \overrightarrow{v_n}$$

Including effect of H^{-1}

$$H^{-1}\vec{\omega} = H^{-1}(\alpha_1 \overrightarrow{v_1} + \alpha_2 \overrightarrow{v_2} + \cdots \alpha_n \overrightarrow{v_n})$$

Rewritten with eigenvalues:

$$H^{-1}\vec{\omega} \quad = \frac{1}{\lambda_1} \alpha_1 \overrightarrow{v_1} + \frac{1}{\lambda_2} \alpha_2 \overrightarrow{v_2} + \cdots \frac{1}{\lambda_n} \alpha_n \overrightarrow{v_n}$$

Linking it all together

$$\vec{i}_{est} = H^{-1}\vec{s} + \boxed{H^{-1}\vec{\omega}}$$

$$\boxed{H^{-1}\vec{\omega}} = \frac{1}{\lambda_1}\alpha_1\overrightarrow{v_1} + \frac{1}{\lambda_2}\alpha_2\overrightarrow{v_2} + \cdots \frac{1}{\lambda_n}\alpha_n\overrightarrow{v_n}$$

- Remember: want small noise term for high signal-to-noise ratio
- The noise is directly related to the eigenvalues

Poll Time!

- Do we want small or large eigenvalues for the H matrix in order to get a good image?
  A. Large
  B. The magnitude doesn't matter
  C. Small

- Which of the following equations correctly model our imaging system? (multiple choice)
  A. $s_{ideal} = H.i$
  B. $s_{real} = s_{ideal} + w = H.i + w$
  C. $s_{real} = s_{ideal} + w = H.i + H.w$
  D. $i_{est} = H^{-1}.s_{real} = H^{-1}.s_{ideal} + H^{-1}.w$
  E. $i_{est} = H^{-1}.s_{real} = H^{-1}.s_{ideal} + w$

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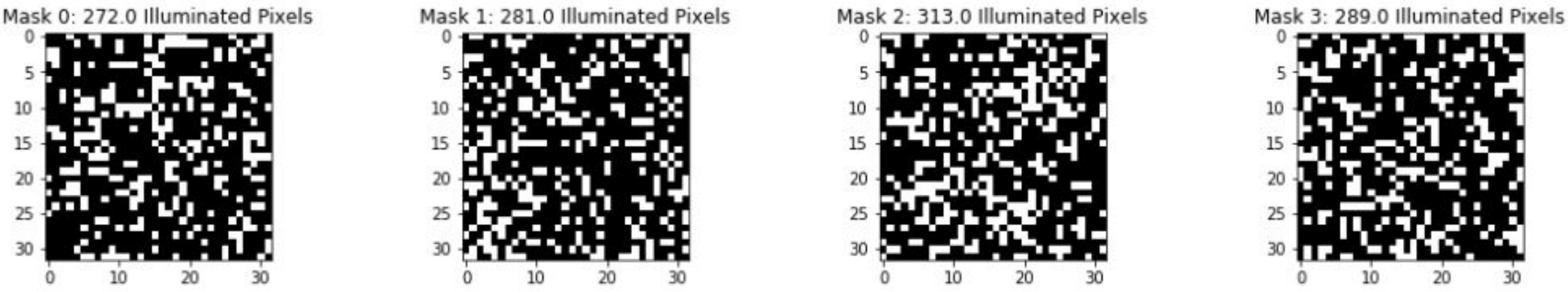
Possible scanning matrix: Random

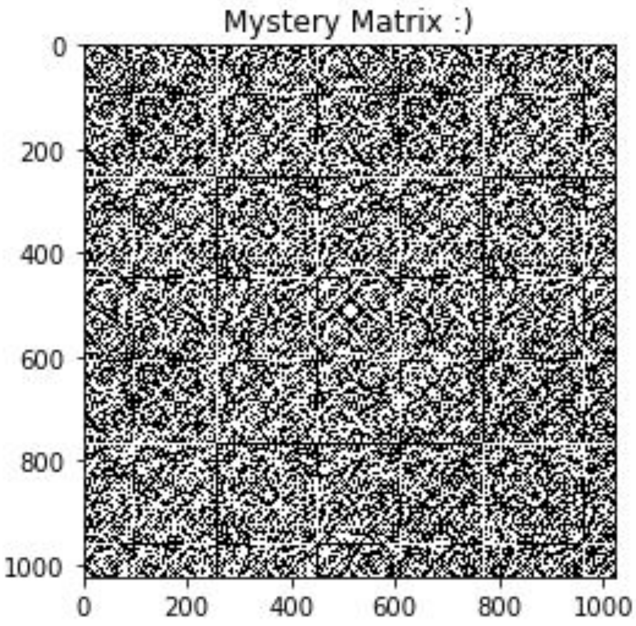


- Illuminate ~300 pixels per scan
  - *Usually* invertible
  - But what are its eigenvalues?

¯_(ツ)_/¯

A more systematic scanning matrix

- Hadamard matrix!
- Constructed to have large eigenvalues
  - Just what we need!



Multipixel Scanning Use Cases

- “Superior technology” is a bit misleading – any practice will have advantages in some cases and disadvantages in others
- Does our setting truly constitute an ideal use-case for multipixel imaging? What are the benefits of multipixel imaging, and can we really reap them?

Multipixel Scanning Use Cases

- “Superior technology” is a bit misleading – any practice will have advantages in some cases and disadvantages in others
- Does our setting truly constitute an ideal use-case for multipixel imaging? What are the benefits of multipixel imaging, and can we really reap them?
  - Multipixel imaging would be useful if we cared about getting *close* to each pixel value, prioritizing getting decent results for each pixel rather than getting really good measurements for some pixels and having others lost entirely.

Multipixel Scanning Use Cases

Do you see the missing pixel?

Multipixel Scanning Expectations

- Given the fact that we’re shining light at multiple portions of an image, it’s surprising that we’re able to reconstruct it at all because our light will easily bleed to the pixels around one region
- Purpose of today’s lab is not to get better results than single-pixel; it’s to show that it is possible to get results using multipixel scanning
- **However, you’ll see an example at the end of lab showing you where multipixel scanning could be useful!**

Projector Setup

- Project masks (rows of H) onto image and “measure” **s** using matrix multiplication
- Multiply with H inverse to find **i** ($=H^{-1}\mathbf{s}$)
- SIMILAR SETUP AS IMG 2 (Different resistor)
  - Don’t forget to adjust projector settings!

Pointers

1. READ CAREFULLY - Long lab with lots of reading; heavily tests understanding of eigen-stuff (important for the exam)
2. Choose an image that focuses on a single object and is not too detailed
3. In case the kernel crashes, simply save your notebook and restart it. You should navigate to the previous import block and run all blocks starting there.

Pointers / Debugging

1. Make sure wires/resistors/light sensor are not loose
2. Light sensor orientation: short leg goes into +
3. Check COM Port
4. Reupload code to launchpad after making any change in circuit
5. Check Baud Rate in Serial Monitor (115200)
6. Projector might randomly restart in the middle of the lab. Make sure brightness 0 contrast 100.
7. Cover box with jacket for dark scanning conditions
8. If you see a very bright corner in the scan, move the light sensor away from the projector