EECS 16A Designing Information Devices and Systems I
Fall 2022 Homework 6

This homework is due Friday, October 14, 2022, at 23:59.
Self-grades are due Monday, October 17, 2021, at 23:59.

Submission Format

Your homework submission should consist of **one** file.

- `hw6.pdf`: A single PDF file that contains all of your answers (any handwritten answers should be scanned) as well as your IPython notebook saved as a PDF.
  If you do not attach a PDF "printout" of your IPython notebook, you will not receive credit for problems that involve coding. Make sure that your results and your plots are visible. Assign the IPython printout to the correct problem(s) on Gradescope.

Submit the file to the appropriate assignment on Gradescope.

1. **Reading Assignment**

   For this homework, please review and read Note 11A/B, which introduces the basics of circuit analysis and node voltage analysis. You are always welcome and encouraged to read beyond this as well.

2. **It's a Triforce!**

   Learning Goal: *This problem explores passive sign convention and nodal analysis in a slightly more complicated circuit.*

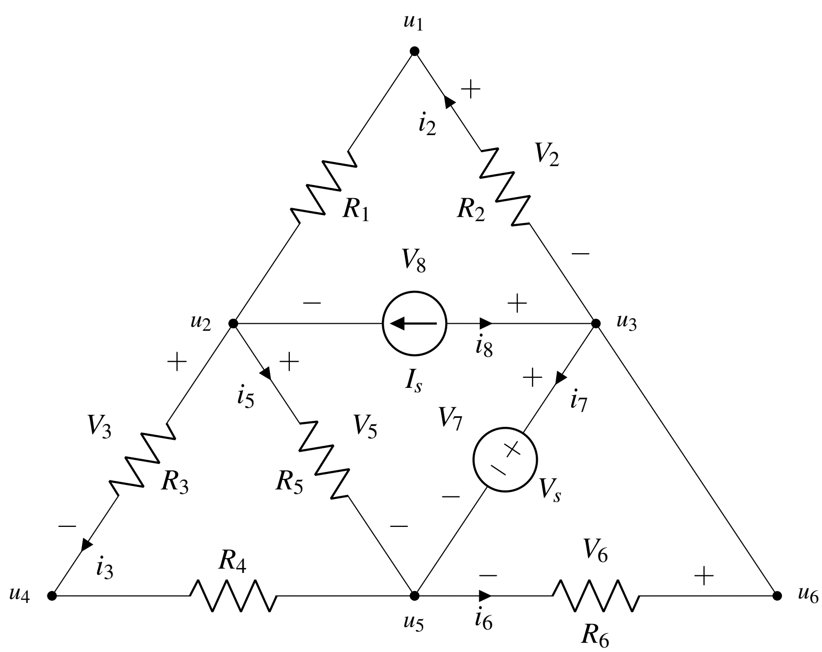


Figure 1: A triangular circuit consisting of a voltage source V_s, current source I_s, and resistors R_1 to R_6.

(a) Which elements I_s, V_s, R_2, R_3, R_5, or R_6 in Figure 1 have current-voltage labeling that violates *passive sign convention*? Explain your reasoning.

Solution:

Recall *passive sign convention* dictates that positive current should *enter* the positive voltage terminal and *exit* the negative voltage terminal.

The elements associated with I_s, R_2, and R_6 have (external) voltage/current labelings which violate passive sign convention as the current is depicted leaving the positive terminal (or entering the negative terminal). This could be corrected either by swapping the voltage polarity or the current direction.

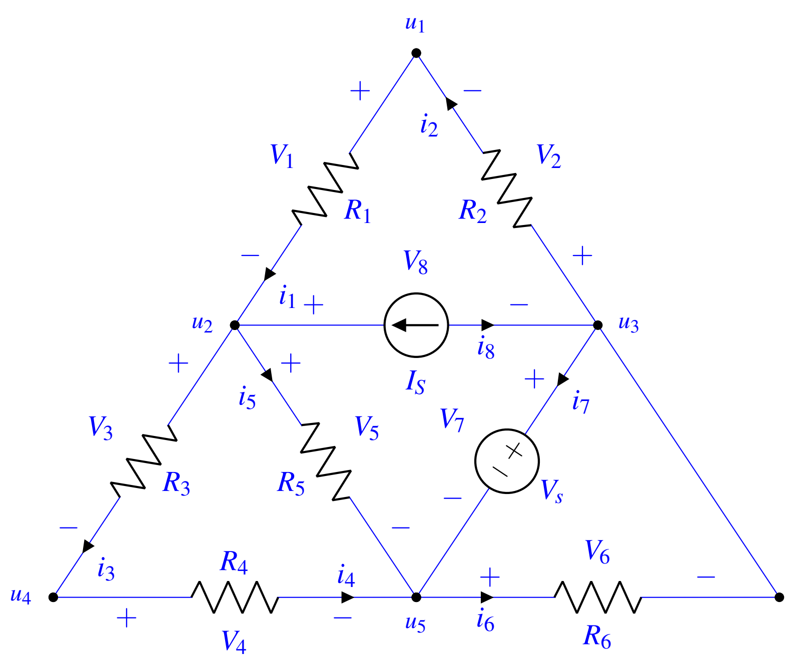
(b) In Figure 1, the nodes are labeled with u_1, u_2, ... etc. There is a subset of u_i's in the given circuit that are redundant, i.e. there might be more than one label for the same node. Which node(s)? Justify your answer.

Solution:

The nodes u_3 and u_6 are redundant and can be represented with the same node since there is a short connecting them.

(c) Redraw the circuit diagram by correctly labeling *all* the element voltages and element currents according to passive sign convention. The component labels that were violating passive sign convention in part (a) should be corrected by *swapping the element voltage polarity*. Also you should now label the elements that have not been labeled yet.

Solution:



This is one of the possible correct labelings. Since the problem statement disallows swapping the current direction, the only other valid answers would include variations of the completely unlabeled elements R_1 and R_4. The flipping of *both* current *and* voltage labelings for R_1 and/or R_4 is also a correct answer.

(d) Write an equation to describe the current-voltage relationship for element R_4 in terms of the relevant i's, R's, and node voltages in this circuit.

Solution:

The resulting equation should look like:

$$R_4 = \frac{u_4 - u_5}{i_4}$$

Alternatively, if you labelled the current i_4 going in the opposite direction (going from u_5 to u_4), the resulting equation should look like:

$$R_4 = \frac{u_5 - u_4}{i_4}$$

(e) Write the KCL equation for node u_2 in terms of the node voltages and other circuit elements.

Solution:

KCL at u_2 gives us:

$$i_1 - i_8 - i_3 - i_5 = 0$$

The equations for the current through each of the branches are:

$$i_1 = \frac{u_1 - u_2}{R_1}$$
$$i_3 = \frac{u_2 - u_4}{R_3}$$
$$i_8 = -I_s$$
$$i_5 = \frac{u_2 - u_5}{R_5}$$

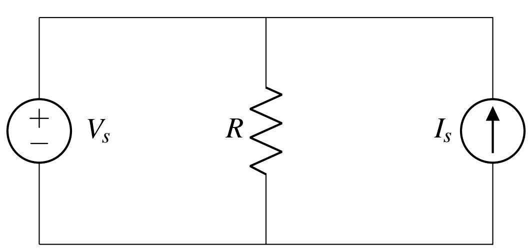
The final expression is then:

$$\frac{u_1 - u_2}{R_1} + I_s - \frac{u_2 - u_5}{R_5} - \frac{u_2 - u_4}{R_3} = 0$$

Or any equivalent equation.

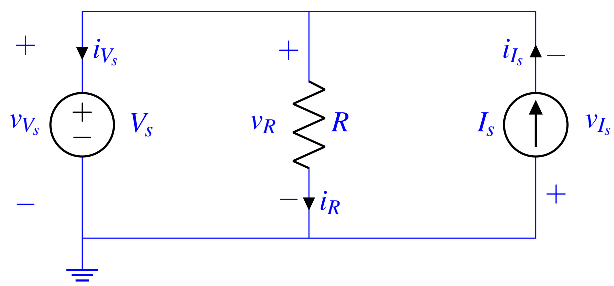
3. Power Analysis

Learning Goal: *This problem aims to help you practice calculating power dissipation in different circuit elements. It will also give you insights into how power is conserved in a circuit.*



(a) Find expressions for power dissipated by the voltage source (P_{V_s}), the current source (P_{I_s}), and the resistor (P_R) in the circuit above. Remember to label voltage-current pairs using passive sign convention.

Solution: We label a reference node, and then solve for the currents i_V, i_R and the voltages V_R, V_I.



Solving the above circuit using nodal analysis, we get

$$i_R = \frac{V_s}{R}$$

$$i_{V_s} = I_s - \frac{V_s}{R}$$

$$v_{I_s} = -V_s$$

$$v_R = V_s$$

Using this we can calculate

$$P_{V_s} = V_{V_s} \cdot i_V = I_s \cdot V_s - \frac{{V_s}^2}{R}$$

$$P_{I_s} = i_{I_s} \cdot v_{I_s} = -I_s \cdot V_s$$

$$P_R = i_R \cdot v_R = \frac{{V_s}^2}{R}$$

Note that $P_{V_s} + P_I + P_R = 0$, i.e. energy provided is energy dissipated, which verifies our intuition about conservation of energy.

(b) Use $R = 5\ \text{k}\Omega$, $V_s = 5$ V, and $I_s = 10$ mA. Calculate the power dissipated by each element. What does a negative value of dissipated power represent? Additionally compute the total power dissipated in all elements.

Solution:

$P_{V_S} = (0.01\text{A})(5\text{V}) - \frac{(5\text{V})^2}{5000\Omega} = 0.045W$
$P_{I_s} = -(0.01\text{A})(5\text{V}) = -0.05\text{W}$
$P_R = \frac{(5\text{V})^2}{5000\Omega} = 0.005\text{W}$

A negative value of dissipated power means the element is *delivering* power.
The total power dissipated in all elements is $P_{V_S} + P_I + P_R = 0$.

(c) Once again, let $R = 5\ \text{k}\Omega$, $V_s = 5$ V. What does the value I_s of the current source have to be such that the current source **dissipates** 40 mW? Note that it is possible for a current source to *dissipate* power, i.e. under passive sign convention, $P_{I_s} = +40\text{mW}$. For this value of I_s, compute P_{V_s} and P_R as well.

Hint: If the current source were delivering *power, under passive sign convention the computed power would have been* $P_{I_s} = -40\text{mW}$, *but this is NOT what the question is asking.*

Solution:

Remember that using passive sign convention, an element whose power is negative is supplying power, and an element whose power is positive is dissipating power. Therefore, we want $P_{I_s} = 40\text{mW}$. We know that $P_{I_s} = -I_s \cdot V_s$. Therefore, $I_s = -\frac{0.04\text{W}}{5\text{V}} = -0.008\text{A}$.

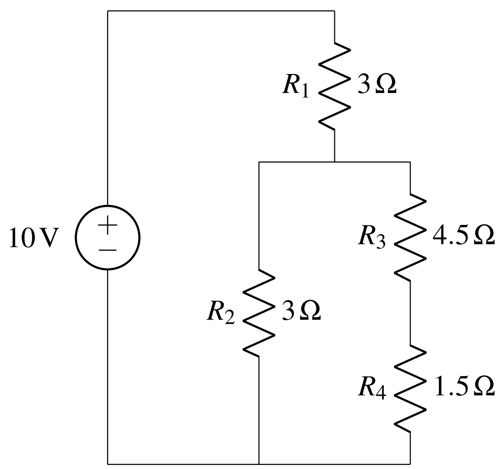
$P_{V_s} = (-0.008\text{A})(5\text{V}) - \frac{(5\text{V})^2}{5000\Omega} = -0.045W$

$P_{I_s} = -(-0.008\text{A})(5\text{V}) = 0.04\text{W}$
$P_R = \frac{(5\text{V})^2}{5000\Omega} = 0.005\text{W}$

Note that still $P_{V_S} + P_I + P_R = 0$.

4. Mechanical Circuits

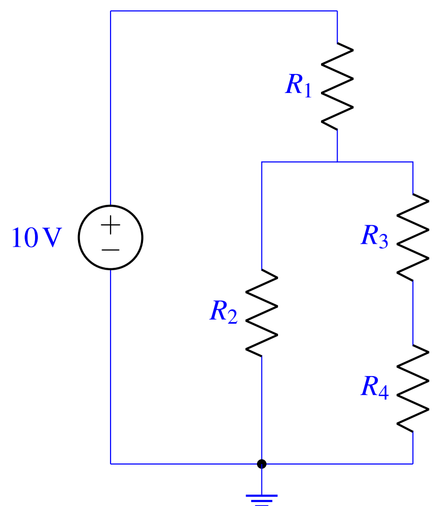
Find the voltages across and currents flowing through all of the resistors.



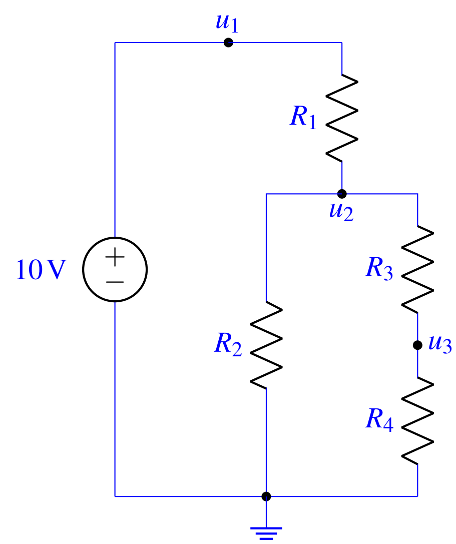
Solution:

Node Voltage Analysis (Seven Step Method):

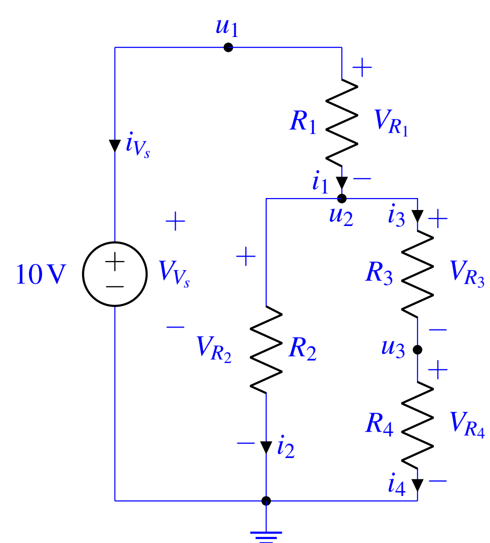
Step 1) Select a reference node. Any choice is valid, but we choose the bottom node:



Step 2) Label all remaining nodes:



Step 3) & 4) Label element voltages and currents



Step 5) Write KCL equations for all nodes with unknowns (u_1, u_2, u_3). The definition of KCL is that the sum of all currents entering and leaving the node must equal 0 A. We will call current entering positive, and current leaving negative. Going node by node from u_1 to u_3 in order, we set up our KCL expressions:

$$i_1 - i_2 - i_3 = 0$$
$$i_3 - i_4 = 0$$

Step 6) Use element I-V relationships to find equations relating the branch currents to the node voltages. Remember that only potential (voltage) *differences* make sense physically, thus we will look at the differences in node potentials and use the appropriate I-V relationship to try and get useful equations.

The voltage source is connected to u_1 and reference. We know that the value of the voltage source $V_s = 10$ V, and that a voltage source forces the difference between its nodes to be V_s. Since we know the reference node is 0 V by definition, we get

$$u_1 - 0 = V_{V_s} = V_s = 10$$

Continuing in a similar manner looking at the differences of node potentials, u_1 and u_2 are separated by a resistor, and Ohm's law relates the potential *difference* between each side of the resistor to the current through it, so we have:

$$u_1 - u_2 = V_{R_1} = i_1 R_1 \implies i_1 = \frac{u_1}{R_1}$$

Similarly for the other resistors

$$\begin{aligned} u_2 - 0 = V_{R_2} = i_2 R_2 \quad &\implies \quad i_2 = \frac{u_2}{R_2} \\ u_2 - u_3 = V_{R_3} = i_3 R_3 \quad &\implies \quad i_3 = \frac{u_2 - u_3}{R_3} \\ u_3 - 0 = V_{R_4} = i_4 R_4 \quad &\implies \quad i_4 = \frac{u_3}{R_4} \end{aligned}$$

We can thus substitute these node voltages into the KCL equations from Step 5.

$$\begin{aligned} \left(\frac{u_1 - u_2}{R_1}\right) - \frac{u_2}{R_2} - \left(\frac{u_2 - u_3}{R_3}\right) &= 0 \\ \left(\frac{u_2 - u_3}{R_3}\right) - \frac{u_3}{R_4} &= 0 \end{aligned}$$

We know that $u_1 = 10$ V, therefore we can solve for the two unknowns $(u_2,\ u_3)$ with the system of two equations. We find that

$$\begin{aligned} u_2 &= 4 \text{ V} \\ u_3 &= 1 \text{ V} \end{aligned}$$

5. Pre-lab Questions

These questions pertain to the pre-lab reading for the *Touch 1* lab. You can find the reading under the *Touch 1* Lab section on the 'Schedule' page of the website. We do not expect in-depth answers for the questions. Please limit your answers to a maximum of 2 sentences.

(a) What are the three terminals of a potentiometer?

(b) What is the common threshold voltage for an LED?

Solution:

(a) A potentiometer (a type of variable resistor) has three terminals: two end terminals and one movable middle terminal that acts as the output.

(b) $V_{\text{LED}} = +0.7$ V

6. Homework Process and Study Group

Who did you work with on this homework? List names and student ID's. (In case you met people at homework party or in office hours, you can also just describe the group.) How did you work on this homework? If you worked in your study group, explain what role each student played for the meetings this week.

Solution:

I first worked by myself for 2 hours, but got stuck on problem 5. Then I met with my study group.

XYZ played the role of facilitator ... etc. We were still stuck on problem 5 so we went to office hours to talk about the problem.

Then I went to homework party for a few hours, where I finished the homework.

