EECS 16A Designing Information Devices and Systems I
Spring 2023 Discussion 11A

1. Unity Gain Feedback

Below is the general block diagram for negative feedback where A represents the gain of our system and f is the feedback factor.

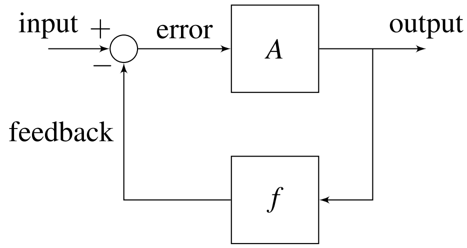


Figure 1: Block diagram for negative feedback.

In this problem, we will look at a specific and important case where $f = 1$, also known as unity gain feedback.

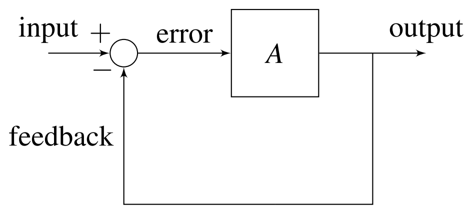


Figure 2: Block diagram for unity gain feedback.

(a) What is the transfer function (ratio of output to input) of the unity gain feedback system? What happens when the system gain is very large, i.e. $A \to \infty$.

Answer: Let's label our input and output to the system S_{in} and S_{out} respectively. In lecture we found that the transfer function of the general case negative feedback system is given by Black's formula:

$$\frac{S_{out}}{S_{in}} = \frac{A}{1+Af}.$$

In the case of unity gain feedback, $f = 1$ and thus our transfer function becomes

$$\frac{S_{out}}{S_{in}} = \frac{A}{1+A}.$$

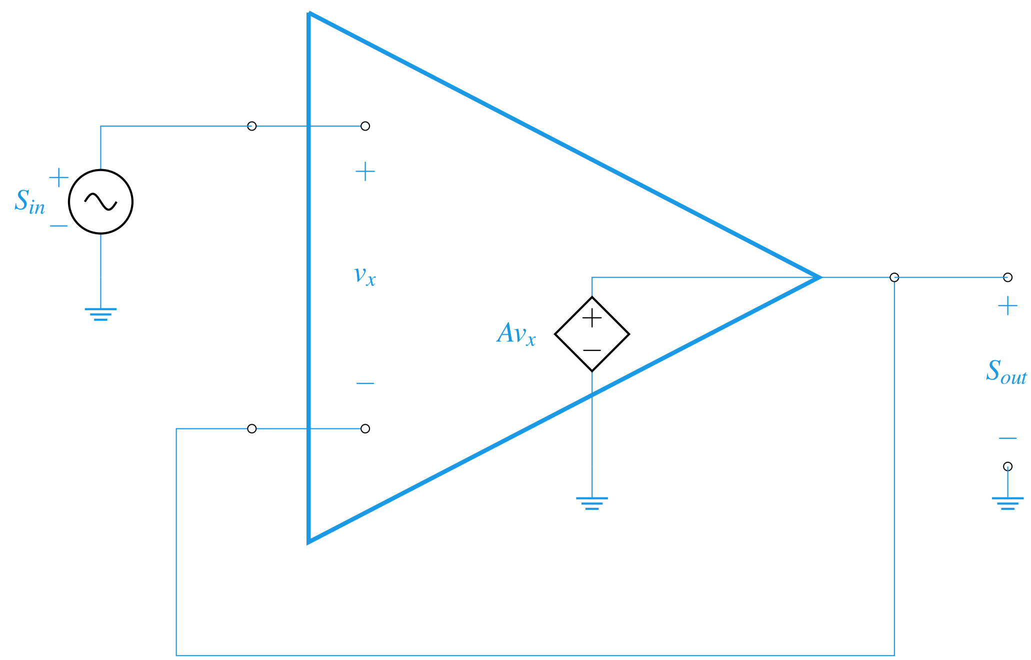
As A grows very large, we have

$$\lim_{A\to\infty} \frac{A}{1+A} = 1$$

which means $S_{out} = S_{in}$ and explains the name-sake "unity gain". This system is often referred to as a "unity gain buffer", "voltage follower", or just "buffer".

(b) How can we implement the unity-gain feedback system using a single op-amp?

Answer: The first thing we notice is the subtraction between the input and negative feedback in our system. In an op-amp, we note that the two inputs are subtracted then amplified at the output. Thus we should connect the input of the system S_{in} to the positive input of the op-amp and the output of the op-amp to the negative input of the op-amp to create a negative feedback loop. The output of the system S_{out} can be connected to the output of the op-amp.



(c) Although the gain of an op-amp is very large (usually in the tens or hundreds of thousands), it is very difficult to precisely control the gain during manufacturing. Assume we have three op-amps which have different gains of $A = 50\,000, 75\,000, 100\,000$. For each value of A, what is the resulting transfer function of the op-amp in unity-gain configuration? What is an advantage of placing an op-amp in negative feedback?

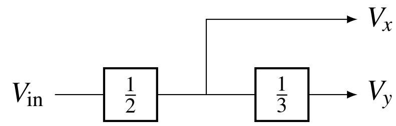
Answer: Plugging in our values of A into the transfer function calculated in part (a), we see

$$\frac{S_{out}}{S_{in}} = \begin{cases} 0.999\,980 & A = 50\,000 \\ 0.999\,987 & A = 75\,000 \\ 0.999\,990 & A = 100\,000 \end{cases}.$$

Since A is fairly large in each case, we see that the change in $\frac{A}{1+A}$ is very small. Even though the actual op-amp gain values vary by up to $2\times$, negative feedback ensures that the transfer function of the system remains stable with different values of A as long as A is large.

2. Modular Circuit Buffer

Let's try designing circuits that perform a set of mathematical operations using op-amps. While voltage dividers on their own cannot be combined without altering their behavior, op-amps can preserve their behavior when combined and thus are a perfect tool for modular circuit design. We would like to implement the block diagram shown below:



In other words, create a circuit with two outputs V_x and V_y, where $V_x = \frac{1}{2}V_{in}$ and $V_y = \frac{1}{3}V_x = \frac{1}{6}V_{in}$.

(a) Draw two voltage dividers, one for each operation (the $1/2$ and $1/3$ scalings). What relationships hold for the resistor values for the $1/2$ divider, and for the resistor values for the $1/3$ divider?

Answer: Recall our voltage divider consists of V_{in} connected to two resistors (R_1, R_2) in series with R_2 connected to ground and the output voltage between ground and the central node. This yields the formula

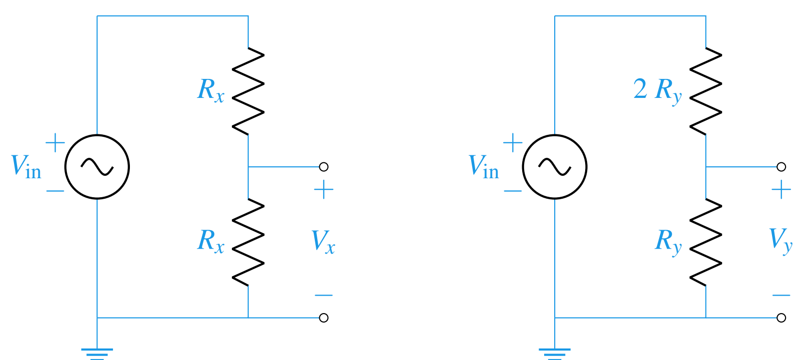
$$V_{out} = \left(\frac{R_2}{R_1 + R_2}\right) V_{in}.$$

For the $1/2$ operation (V_x output) we recognize

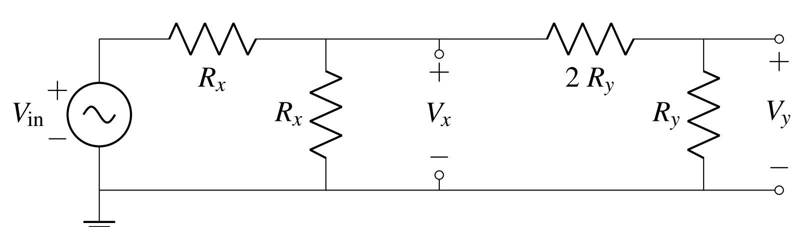
$$\frac{1}{2} = \left(\frac{R_2}{R_1 + R_2}\right) \longrightarrow R_1 + R_2 = 2R_2 \longrightarrow R_1 = R_2 \equiv R_x.$$

For the $1/3$ operation (V_y output) we recognize

$$\frac{1}{3} = \left(\frac{R_2}{R_1 + R_2}\right) \longrightarrow R_1 + R_2 = 3R_2 \longrightarrow \frac{R_1}{2} = R_2 \equiv R_y.$$



(b) If you combine the voltage dividers, made in part (a), as shown by the block diagram (output of the $1/2$ voltage divider becomes the source for the $1/3$ voltage divider circuit), do they behave as we hope (meaning $V_{in} = 2V_x = 6V_y$)?



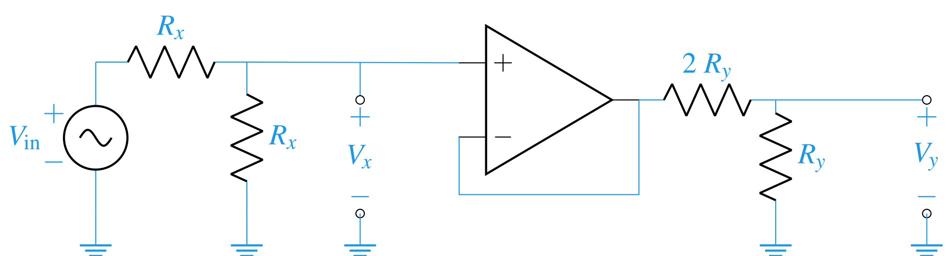
Answer: To quickly access this combined system, we may identify V_x as the result of a new equivalent voltage divider (recognizing the R_y resistors in series and that series is in parallel with R_x). The load resistor becomes $R_{eq} = \frac{3R_xR_y}{R_x+3R_y}$. This yields

$$V_x = \left(\frac{R_{eq}}{R_x + R_{eq}}\right) V_{\text{in}} = \left(\frac{1}{2 + \frac{R_x}{3R_y}}\right) V_{\text{in}} \qquad V_y = \frac{1}{3} V_x = \left(\frac{1}{6 + \frac{R_x}{R_y}}\right) V_{\text{in}}$$

From this stage it is evident that combining our dividers changes their behavior (although they preserve behavior in the limit $R_y >> R_x$). The second divider draws current from middle node of the first divider and so we can longer apply the voltage divider equation. The new values for V_x, V_y are dependent on values from both dividers, which means they can't be treated independently!

(c) Perhaps we could use an op-amp (in negative-feedback) to achieve our desired behavior.
Modify the implementation you tried in part (b) using a negative feedback op-amp in order to achieve the desired V_x, V_y relations $V_x = \frac{V_{\text{in}}}{2}$ and $V_y = \frac{V_x}{3} = \frac{V_{\text{in}}}{6}$.
HINT: Place the op-amp in between the dividers such that the V_x node is an input into the op-amp, while the source of the 2nd divider is the output of the op-amp!

Answer: Use the op-amp as a voltage buffer.



Since no current flows into the positive op-amp input, we've successfully isolated the dividers so they can be used in a modular fashion! □

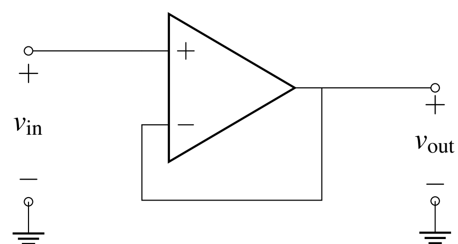
NOTE: The V_x, V_y outputs from this configuration would change with the addition of a load on either terminal. As a follow-up, think about ways to make each output agnostic to the loads attached!

3. Testing for Negative Feedback

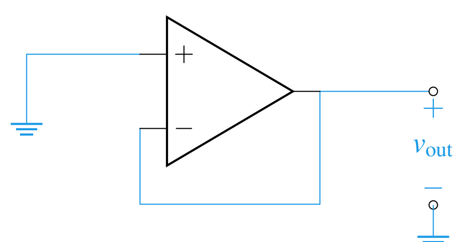
While it is tempting to say "if the feedback voltage is connected to the negative op-amp terminal, then we have negative feedback", this is not always true. Here is a two-step procedure for determining if a circuit is in negative feedback:

- **Step 1: Zero out all independent sources**, replacing voltage sources with wires and current sources with opens as we did in superposition. You do not need to zero out the voltage sources that serve as the power supplies to the op-amp, since they are not treated as signals and are not considered part of the op-amp.
- **Step 2: Wiggle the output and check the loop**. Assume that the output increases slightly. Check the direction of change of the feedback signal and the error signal from the circuit. Any change in the error signal will cause a new change in the output. This change is the feedback loop's response to the initial change.
  - If the error signal decreases, then the output must also decrease. This is the *opposite direction* we initially assumed, i.e. the loop is trying to correct for the change. So the circuit is in negative feedback.
  - If the error signal instead increased, then the output would also increase. This is the *same direction* we initially assume, i.e. the initial increase lead to further increase. We call this positive feedback.

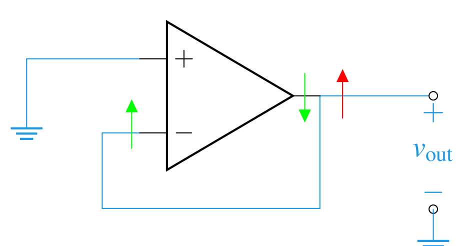
(a) Show that the voltage buffer circuit is in negative feedback. Note that here v_{in} is acting as a voltage source.



Answer: First, zero out all independent sources. For this problem, we just need to tie the input to ground.

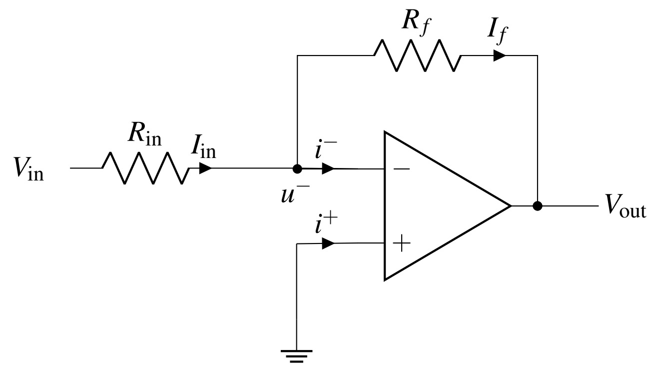


Next, wiggle the output and check the loop. Below, we label the initial change in the output in red and label subsequent changes in green:

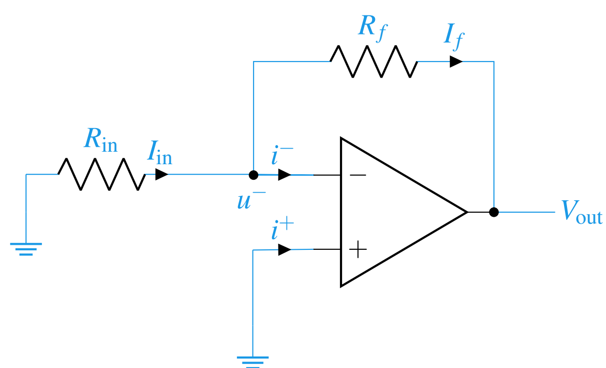


We suppose that the v_{out} increases. Output voltage v_{out} is connected to the negative terminal input u^- so u^- also increases. Our op-amp equation is $v_{out} = A \cdot (u^+ - u^-)$, so increasing u^- will cause v_{out} to decrease. This is the opposite of what initially happened, so we are in negative feedback.

(b) Show that the inverting amplifier circuit is in negative feedback.

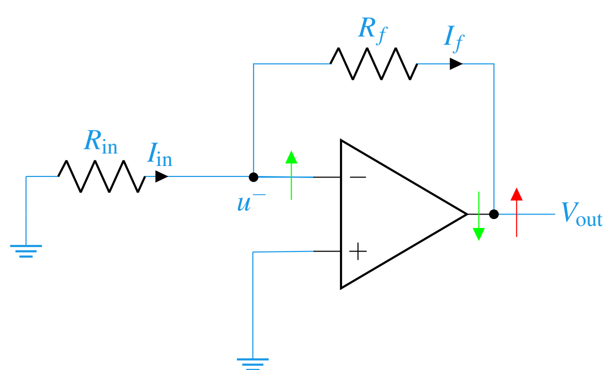


Answer: First, zero out all independent sources. For this problem, we just need to tie the input to ground.



Note that R_f and R_{in} now form a voltage divider.

Next, wiggle the output and check the loop. Below, we label the initial change in the output in red and label subsequent changes in green:



We suppose that the v_{out} increases. Output voltage v_{out} is connected to the negative terminal input u^- so u^- also increases. Our op-amp equation is $v_{out} = A \cdot (u^+ - u^-)$, so increasing u^- will cause v_{out} to decrease. This is the opposite of what initially happened, so we are in negative feedback.