

EECS 16A Designing Information Devices and Systems I Discussion 14A
Spring 2023

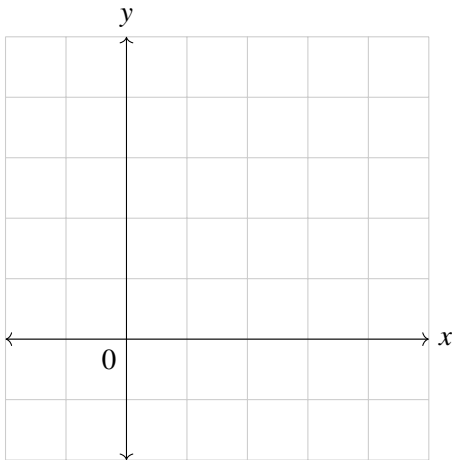
1. Mechanical Projection

In \mathbb{R}^n , the vector valued projection of vector \vec{b} onto vector \vec{a} is defined as:

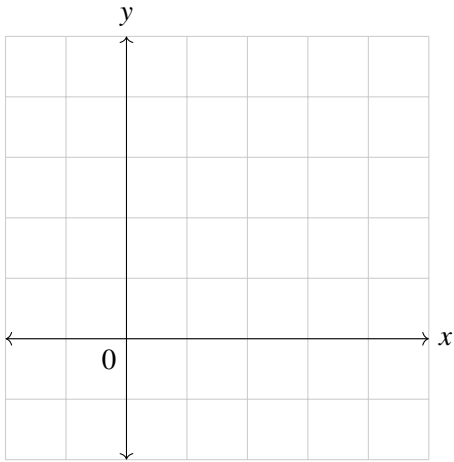
$$\text{proj}_{\vec{a}}(\vec{b}) = \frac{\langle \vec{a}, \vec{b} \rangle}{\|\vec{a}\|^2} \vec{a}.$$

Recall $\|\vec{a}\|^2 = \langle \vec{a}, \vec{a} \rangle$.

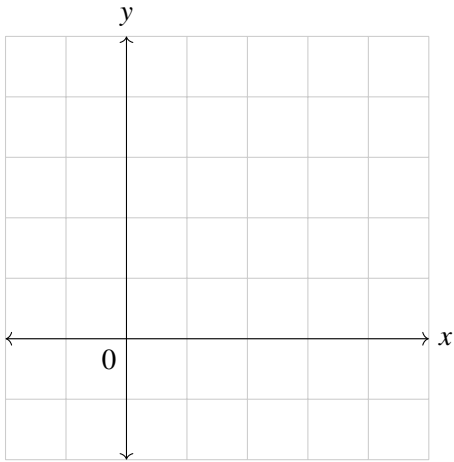
(a) Project $\begin{bmatrix} 5 \\ 2 \end{bmatrix}$ onto $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ — that is, onto the y-axis. Graph these two vectors and the projection.



(b) Project $\begin{bmatrix} 4 \\ -2 \end{bmatrix}$ onto $\begin{bmatrix} -2 \\ 1 \end{bmatrix}$. Graph these two vectors and the projection.



(c) Project $\begin{bmatrix} 4 \\ -2 \end{bmatrix}$ onto $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$. Graph these two vectors and the projection.



2. Least Squares with Orthogonal Columns

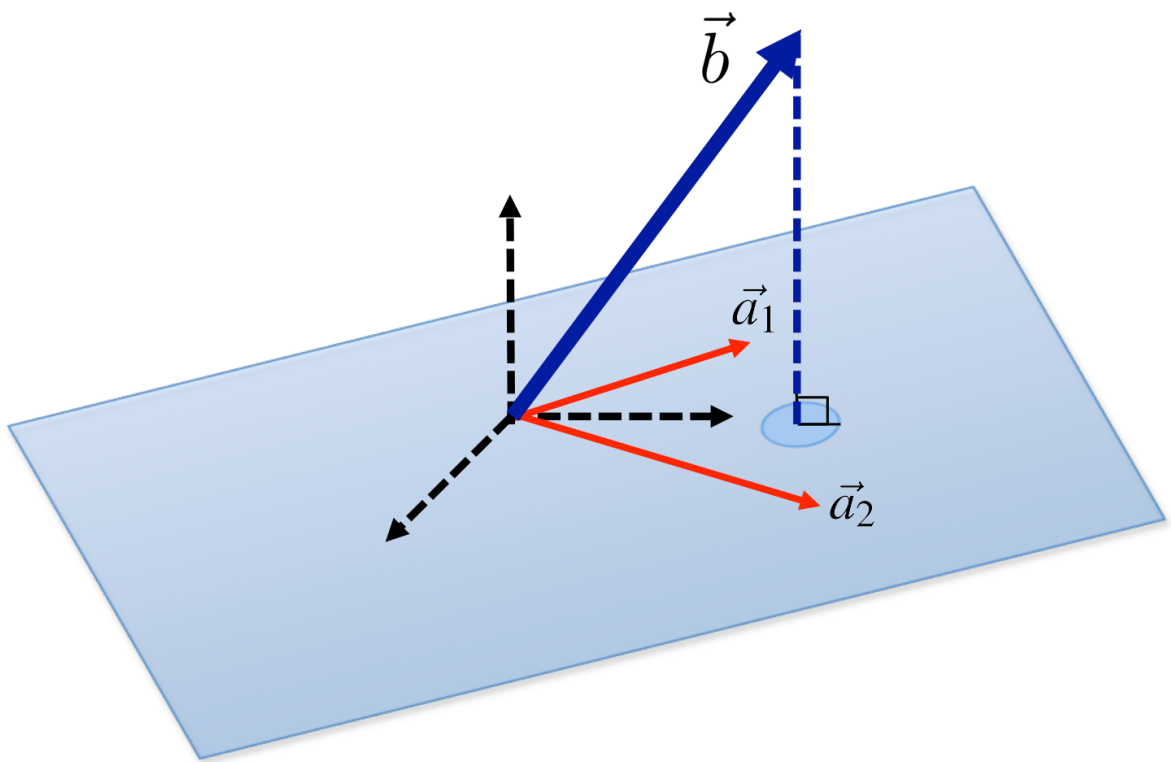
(a) Consider a least squares problem of the form

$$\min_{\vec{x}} \left\| \vec{b} - \mathbf{A}\vec{x} \right\|^2 = \min_{\vec{x}} \left\| \mathbf{A}\vec{x} - \vec{b} \right\|^2 = \min_{\vec{x}} \left\| \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} - \begin{bmatrix} | & | \\ \vec{a}_1 & \vec{a}_2 \\ | & | \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \right\|^2$$

Let the solution be $\vec{\hat{x}} = \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix}$.

Label the following elements in the diagram below.

$\text{span}\{\vec{a}_1, \vec{a}_2\}$, $\vec{e} = \vec{b} - \mathbf{A}\vec{\hat{x}}$, $\mathbf{A}\vec{\hat{x}}$, $\vec{a}_1\hat{x}_1$, $\vec{a}_2\hat{x}_2$, $\text{colspace}(\mathbf{A})$



- (b) We now consider the special case of least squares where the columns of \mathbf{A} are orthogonal. Given that $\vec{x} = (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \vec{b}$ and $A\vec{x} = \text{proj}_{\mathbf{A}}(\vec{b}) = \hat{x}_1 \vec{a}_1 + \hat{x}_2 \vec{a}_2$, show that

$$\text{proj}_{\vec{a}_1}(\vec{b}) = \hat{x}_1 \vec{a}_1$$

$$\text{proj}_{\vec{a}_2}(\vec{b}) = \hat{x}_2 \vec{a}_2$$

(c) Compute the least squares solution to

$$\min_{\vec{x}} \left\| \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \right\|^2.$$