
EECS 16A
Spring 2023

Designing Information Devices and Systems I

Discussion 1B

1. Gaussian Elimination

Use Gaussian elimination to solve the following systems. Does a solution exist? Is it unique?

(a)

$$\left[\begin{array}{ccc|c} 2 & 0 & 4 & 6 \\ 0 & 1 & 2 & -3 \\ 1 & 2 & 0 & 3 \end{array} \right]$$

(b)

$$\left[\begin{array}{ccc|c} 1 & 4 & 2 & 2 \\ 1 & 2 & 8 & 0 \\ 1 & 3 & 5 & 3 \end{array} \right]$$

(c)

$$\left[\begin{array}{ccc|c} 2 & 2 & 3 & 7 \\ 0 & 1 & 1 & 3 \\ 2 & 0 & 1 & 1 \end{array} \right]$$

(d) True or False: A system of equations with more equations than unknowns will always have either infinite solutions or no solutions.

(e) **(Practice)**

$$\left[\begin{array}{ccc|c} 3 & -1 & 2 & 1 \\ 0 & 0 & 2 & 1 \end{array} \right]$$

(f) **(Practice)**

$$\left[\begin{array}{rrcr} 2x & + & 4y & + & 2z & = & 8 \\ x & + & y & + & z & = & 6 \\ x & - & y & - & z & = & 4 \end{array} \right]$$

2. Finding The Bright Cave

Nara the one-handed druid and Kody the one-handed ranger find themselves in dire straits. Before them is a cliff with four cave entrances arranged in a square: two upper caves and two lower caves. Each entrance emits a certain amount of light, and the two wish to find exactly the amount of light coming from each cave. Here's the catch: after contracting a particularly potent strain of ghoulish fever, our intrepid heroes are only able to see the total intensity of light before them (so their eyes operate like a single-pixel camera). Kody and Nara are capable adventurers, but they don't know any linear algebra – and they need your help.

Kody proposes an imaging strategy where he uses his hand to completely block the light from two caves at a time. He is able to take measurements using the following four masks (black means the light is blocked from that cave):

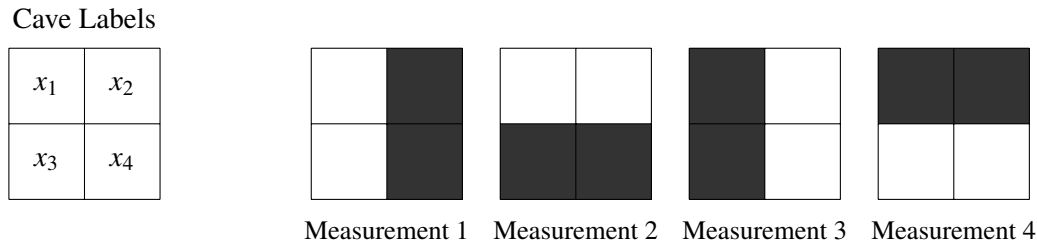


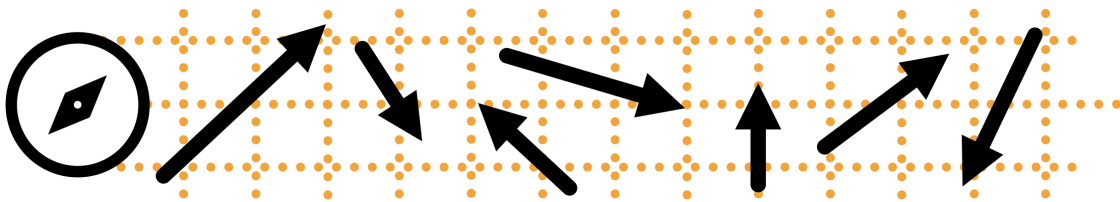
Figure 1: Four image masks.

- Let x_1 , x_2 , x_3 , and x_4 represent the magnitude of light emanating from the four cave entrances shown in the image above. Write an equation for each masking process in Figure 1 which results in the four measurements of total light: m_1 , m_2 , m_3 , and m_4 . Then, create an augmented matrix that represents this system.
- Does Kody's set of masks give us a unique solution for all four caves' light intensities? Why or why not?
- Nara, in her infinite wisdom, places her one hand diagonally across the entrances, covering two of the cave entrances. However, her hand is not wide enough, letting in 50% of the light from the caves covered and 100% of the light from the caves not covered. The following diagram shows the percentage of light let through from each cave:

50%	100%
100%	50%

Does this additional measurement give them enough information to solve the problem? Why or why not?

3. Vectors



A vector is an ordered list of numbers. For instance, a point on a plane (x, y) is a vector! We label vectors using an arrow overhead \vec{v} , and since vectors can live in ANY dimension of space we'll need to leave our notation general $\vec{v} = (v_1, v_2, \dots)$. Below are a few more examples (the left-most form is the general definition):

$$\vec{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \in \mathbb{R}^n \qquad \vec{a} = \begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix} \in \mathbb{R}^3 \qquad \vec{b} = \begin{bmatrix} 2.4 \\ 5.3 \end{bmatrix} \in \mathbb{R}^2$$

Just to unpack this a bit more, $\vec{b} \in \mathbb{R}^3$ in English means "vector \vec{b} lives in 3-dimensional space."

- The \in symbol literally means "in"
- The \mathbb{R} stands for "real numbers" (FUN FACT: \mathbb{Z} means "integers" like $-2, 4, 0, \dots$)
- The exponent $\mathbb{R}^n \leftarrow$ indicates the dimension of space, or the number of elements in the vector.

One last thing: it is standard to write vectors in column-form, like seen with $\vec{a}, \vec{b}, \vec{x}$ above. We call these *column vectors*, in contrast to horizontally written vectors, which we call *row vectors*. Row vectors are denoted with a transpose symbol; for instance, \vec{x}^T denotes a row vector, which is simply \vec{x} but expressed horizontally. This will become important later on when we discuss the importance of dimension matching.

Okay, let's dig into a few examples:

(a) Which of the following vectors lives in \mathbb{R}^2 space?

$$i. \begin{bmatrix} 3 \\ 6 \end{bmatrix} \quad ii. \begin{bmatrix} 5 \\ 0 \\ 3 \\ 5 \end{bmatrix} \quad iii. \begin{bmatrix} -4.76 \\ 1.32 \\ 0.01 \end{bmatrix} \quad iv. \begin{bmatrix} -20 \\ 100 \end{bmatrix}$$

(b) Graphically show the vectors (either in a sketch with axes, or a plot on a computer):

$$i. \begin{bmatrix} 2 \\ 5 \end{bmatrix} \quad ii. \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

(c) Compute the sum $\vec{a} + \vec{b} = \vec{c}$ from the vectors below, and then graphically sketch or plot these vectors. (show them in a way that forms a triangle; also, is there only one possible triangle?)

$$\vec{a} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad \vec{b} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$