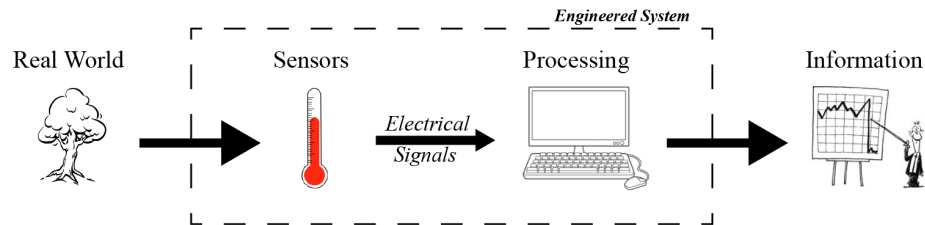


11.1 Introduction to Electrical Circuit Analysis



Our ultimate goal is to design systems that solve people’s problems. To do so, it’s critical to understand how we go from real-world events all the way to useful information that the system might then act upon. The most common way an engineered system interfaces with the real world is by using sensors and/or actuators that are often composed of electronic circuits; these communicate via electrical signals to processing units, which are *also* composed entirely of electronic circuits. In order to fully understand and *design* a useful system, we will need to first understand **Electrical Circuit Analysis**.

In this note, we will be intentionally ignoring the underlying physics of electrical circuits, and will instead focus on a standard procedure and set of rules that will allow us to systemically “solve” such circuits, (i.e., given a circuit diagram, solve for all of the relevant electrical quantities in that circuit). The reason for this approach is that one does not need to understand any real physics in order to be able to analyze electrical circuits, and in fact, impartial or incorrect physical intuition often leads to errors and then confusion. By abstracting the physics away during the analysis step, we hope to emphasize that the analysis in and of itself is simply a matter of taking a visual diagram (representing an electrical circuit) and applying a set of rules to it that will convert the diagram in to a set of (linear) equations that can then be solved using the techniques we have developed in the first module.

11.2 Basic Circuit Quantities

Let’s start with some definitions of basic quantities present in an electrical circuit. **Current** is the flow of charges (e.g. electrons) in the circuit, and **voltage** is the potential energy (per charge) between two points in the circuit. This potential energy is what causes charge to flow (ie. causes current). **Resistance** is the material’s tendency to resist the flow of current.

Quantity	Symbol	Units
Voltage	V	Volts (V)
Current	I	Amperes (A)
Resistance	R	Ohms (Ω)

We use these quantities in a **Circuit Diagram**, a visual representation of how a collection of circuit elements are connected. Each circuit element has some voltage *across* it and some current *through* it.

Why is voltage “across” a circuit element? Voltage, or electric potential, is only defined *relative* to another point. A simple analogy is elevation: A mountain’s summit could be 9,000 ft above sea level, but 21,000 ft above the bottom of the ocean. In both cases, the elevation is only meaningful relative to another point. For convenience, we frequently define sea level as a reference point with “0 ft of elevation” – then we can state elevation as a single number which is implicitly referenced to sea level (ex. the mountain is 9,000 ft tall). Similarly, in circuits, we will frequently define a reference point, called **ground**, against which other voltages can be measured.

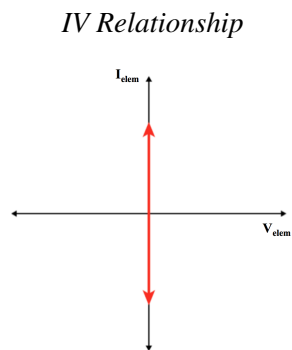
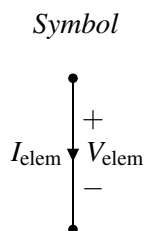
11.3 Basic Circuit Elements

How do our basic circuit quantities interact? It depends on the circuit element! For each element there is a relationship between the voltage across the element and the current through it, call an “IV Relationship.” Let’s look at some of the most common circuit elements and their IV relationships.

Wire: The most common element in a schematic is the wire, drawn as a solid line. The IV relationship for a wire is:

$V_{elem} = 0$ A wire is an ideal connection with zero voltage across it.

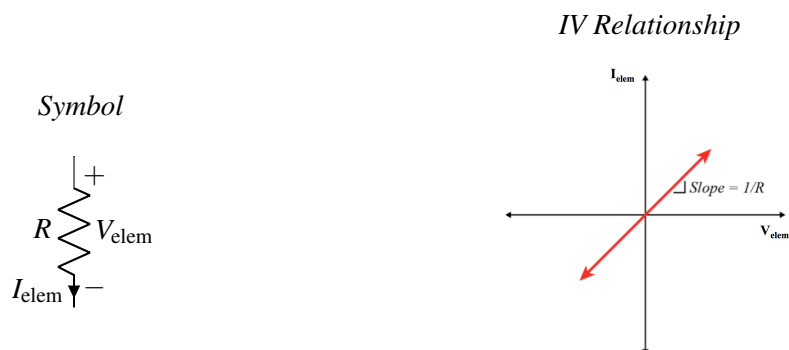
$I_{elem} = ?$ The current through a wire can take any value, and is determined by the rest of the circuit.



Resistor: The IV relationship of a resistor is called “Ohm’s Law.”

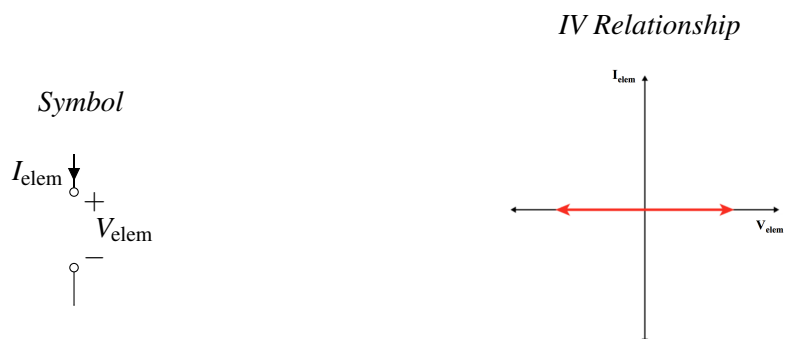
$V_{elem} = I_{elem}R$ The voltage across a resistor is determined by Ohm’s Law.

$I_{elem} = \frac{V_{elem}}{R}$ The current through a resistor is determined by Ohm’s Law.



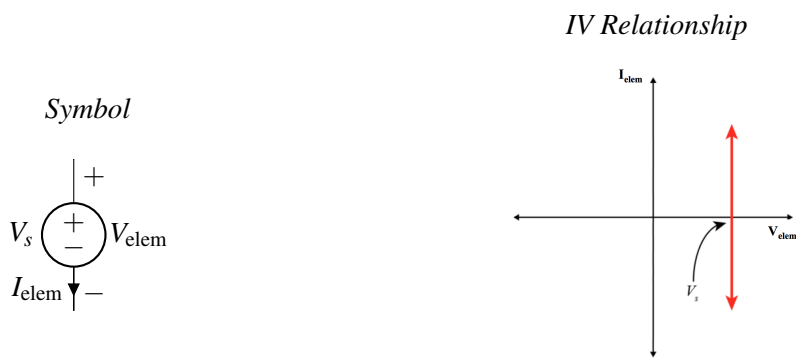
Open Circuit: This element is the dual of the wire.

$V_{elem} = ?$ The voltage across an open circuit can take any value, and is determined by the rest of the circuit.
 $I_{elem} = 0$ No current is allowed to flow through an open circuit.



Voltage Source: A voltage source is a component that forces a specific voltage across its terminals. The + and - sign indicates which direction the voltage is pointing. The voltage difference between the “+” terminal and the “-” terminal is always equal to V_s , no matter what else is happening in the circuit.

$V_{elem} = V_s$ The voltage across the voltage source is always equal to the source value.
 $I_{elem} = ?$ The current through a voltage source is determined by the rest of the circuit.



Current Source: A current source forces current in the direction specified by the arrow indicated on the schematic symbol. The current flowing through a current source is always equal to I_s , no matter what else is happening in the circuit. Note the duality between this element and the voltage source.

$$\begin{aligned} V_{elem} &= ? && \text{The voltage across a current source is determined by the rest of the circuit.} \\ I_{elem} &= I_s && \text{The current through a current source is always equal to the source value.} \end{aligned}$$

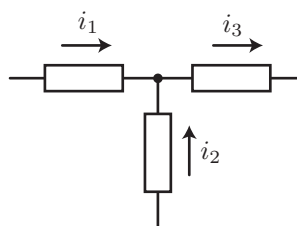


11.4 Rules for Circuit Analysis

In addition to the IV relationships for a single elements, there are also rules govern the current and voltage relationships between multiple elements.

11.4.1 Kirchhoff's Current Law (KCL)

A place in a circuit where two or more of the above circuit elements meet is called a **node**. Kirchhoff's Current Law (KCL) states that the net current flowing out of (or equivalently, into) any node of a circuit is zero. To put this more simply, the current flowing into a node must equal the current flowing out of that node.



For example, consider the circuit shown above. We define current flowing out of the node to be positive, and therefore, current flowing into the node is negative. For example, from the left branch, there is i_1 current flowing into the node, so there is $-i_1$ current flowing out of the node. Similarly, there is $-i_2$ current flowing out of the node and i_3 current flowing out of the node. Therefore, the currents must satisfy

$$(-i_1) + (-i_2) + i_3 = 0 \text{ or } i_1 + i_2 = i_3 \quad (1)$$

11.4.2 Kirchhoff's Voltage Law (KVL)

Kirchhoff's Voltage Law (KVL) states that the sum of voltages across the elements connected in a loop must be equal to zero. In our elevation analogy for voltage, this is equivalent to saying “what goes up must come down”. We actually will not use KVL in the analysis procedure we outline next, but it is still an important tool and can be used as an additional rule to double check your final answers.

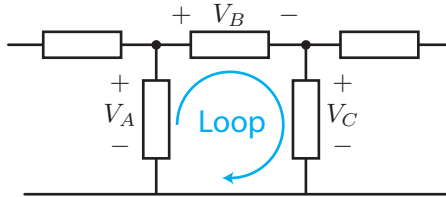


Figure 1: KVL illustration

Mathematically, KVL states that:

$$\sum_{\text{Loop}} V_k = 0. \quad (2)$$

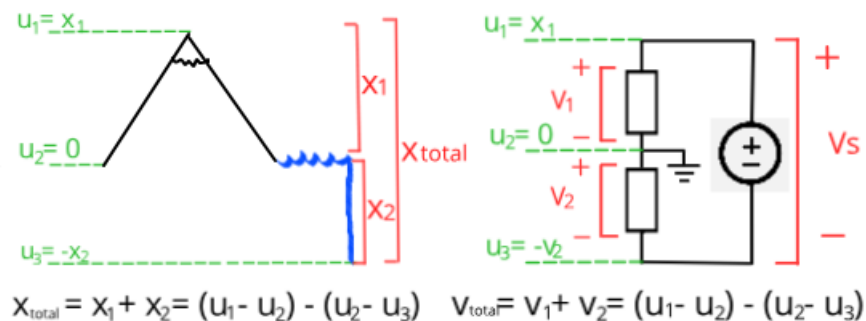
When adding the voltage “drops” around the loop we must follow a convention. If the arrow corresponding to the loop goes into the “+” of an element we subtract the voltage across that element. (In our elevation analogy, we went “downhill” from higher voltage to lower voltage so we lost “elevation.”) Conversely, if the arrow goes into the “-” of an element, we add the voltage across that element (this is like going “uphill”). Following this convention for the example in Figure 1 we find:

$$V_A - V_B - V_C = 0. \quad (3)$$

Note that if we had defined the loop in the opposite direction,

$$-V_A + V_C + V_B = 0. \quad (4)$$

Thinking about an elevation analogy to voltage can help give some intuition to KVL: If you walk in a circle (a loop) so that you end up back where you started, then your total change in elevation must be zero, no matter how much you go up or down. If you walk in a line, ending up somewhere different, then your total change in elevation is equal to the sum of all of the elevation changes along the way.



11.4.3 Ohm's Law and Resistors

As already described when we introduced resistors as an element, for these elements, the voltage across them is directly proportional to the current that flows through them, where the proportionality constant is the "resistance" (R) of the device. This relationship is known as **Ohm's Law**.

$$V_{\text{elem}} = I_{\text{elem}}R. \quad (5)$$

The unit of R is Volts/Amperes, or more commonly "Ohms" (Ω).

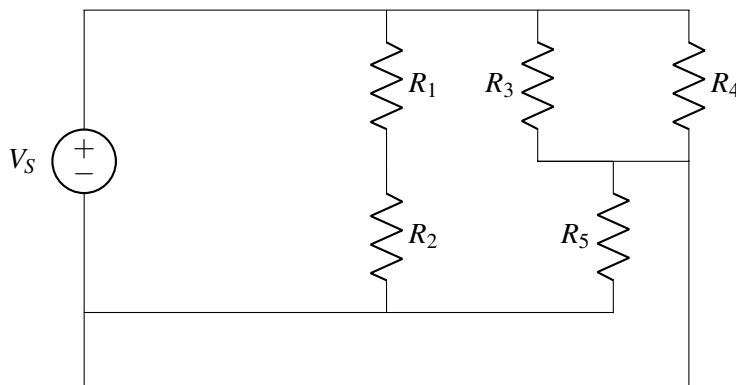
11.5 Circuit Analysis Algorithm

In this course, we will learn how to take a real world system and build a circuit diagram that models the behavior of that system, and we will design our own circuits for specific real world tasks. For now, we will assume that we already have an accurate circuit diagram, and will learn how to analyze the circuit.

For a given circuit, we would like to find all of the voltages and currents sometimes we call this "solving" the circuit. The procedure for doing this is described in a separate note: eecs16a.org/lecture/mod2/nva.html

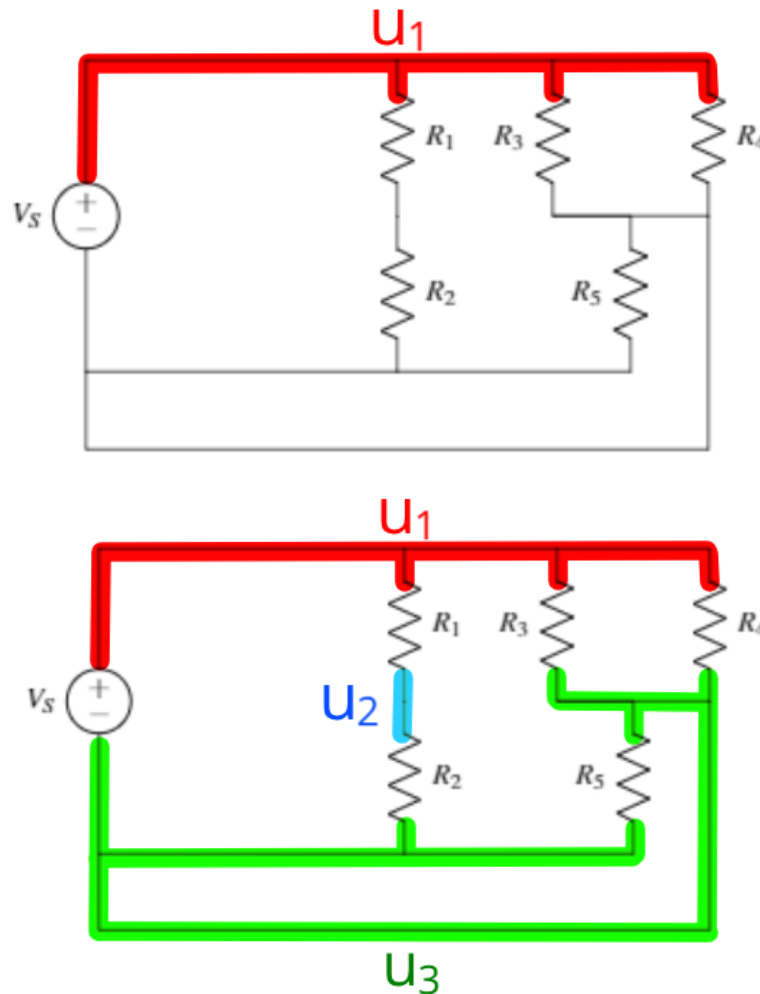
11.6 Guide to Finding Nodes

Here, we will go over a method you can use to correctly identify all of the nodes in a circuit. We'll go through this method while applying it to an example circuit, shown below.



We'll find nodes one at a time with this method (the order we find the nodes is arbitrary). We start by choosing a color to represent the first node (red, in this example). Then we choose a starting point on the circuit, such as the upper left corner. From this point, we trace (in red) along all of the connected wires until we hit a non-wire component. Everything traced in red is part of a single node.

Then we choose a new color and a new point on the circuit *that is not already colored*. We repeat the process: tracing over all wires that are connected to this new point and stop when a non-wire element is reached. Repeat this process for new colors until all of the wires have been identified.



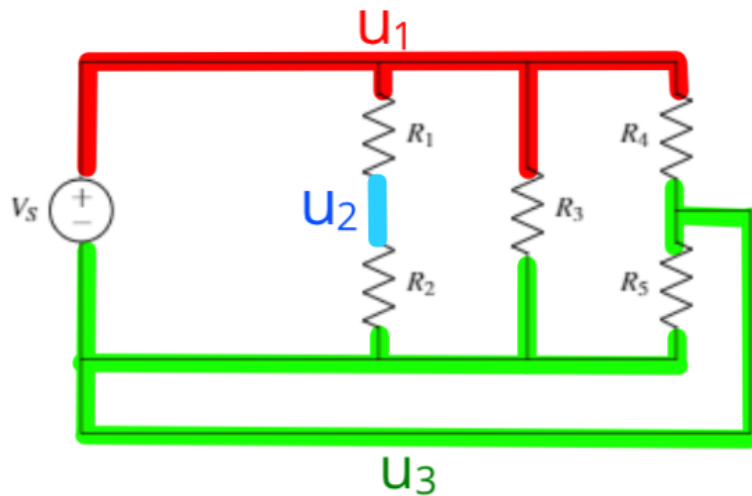
We've now identified all of the nodes! There are three in this example circuit. Note that one of these would be labeled "ground" in the seven step circuit analysis procedure, so we would only have to solve for two unknown node potentials.

Once we've identified the nodes, we can use this knowledge to help us redraw the circuit in a way that the currents and voltages in each element don't change. In other words, we want to use a different diagram to represent the same circuit behavior. This is useful because sometimes it is easier to see patterns in a circuit diagram if it's drawn differently. However, we must be careful to not change the circuit when we redraw it.

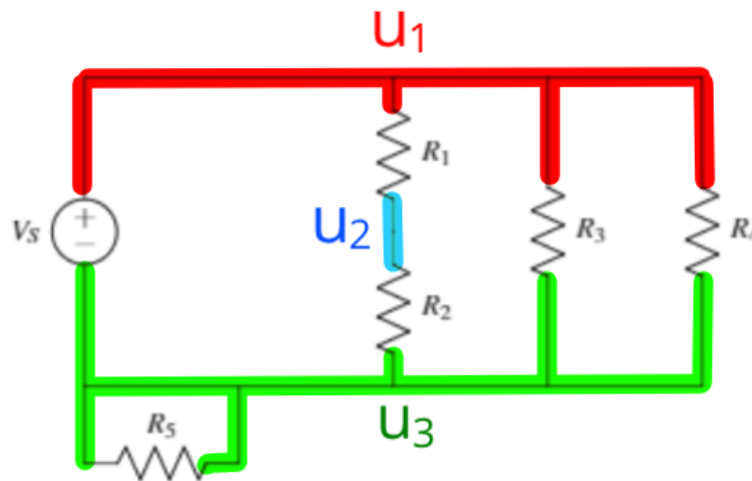
If we don't want the circuit to change when we redraw it, **each non-wire component must be connected to the same nodes on either end**. This is because the voltage drop over each element is dependent only on the nodes it is connected to, and the current through each element is determined by the voltage drop and the IV relationship of that element.

For example, we can redraw our example circuit with R_3 in a different location, as long as one end of R_3 is still connected to the red node (u_1) and the other end of R_3 is still connected to the green node (u_3):

We can similarly rearrange other components if needed. In this case, we move R_5 but maintain that it is connected to the green node, u_3 , on both sides. Note that all of the elements have the same node connections



as they did in the original circuit. Therefore this circuit will have the exact same behavior as the original:

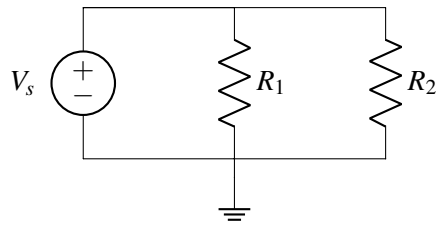


Sometimes redrawing a circuit can make it easier to analyze. However, it is important to stay consistent with node labeling and make sure the redrawn circuit is still the same as the original.

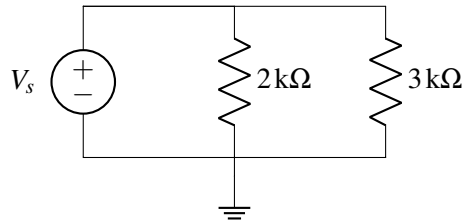
11.7 Practice Problems

These practice problems are also available in an interactive form on the course website (<http://ee16a.com/hw-practice#/>).

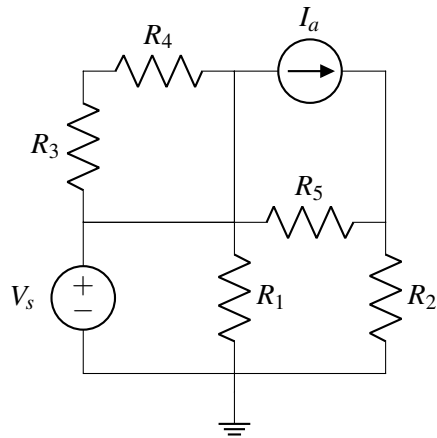
1. True or False: A voltage source can have any current through it.
2. True or False: A current source can have any voltage across it.
3. True or False: The voltage across R_1 and across R_2 is the same.



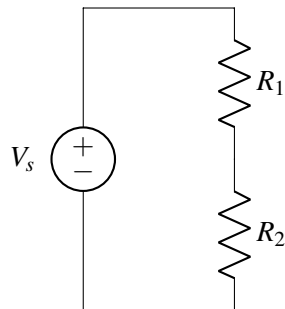
4. True or False: The current through the resistors is the same.



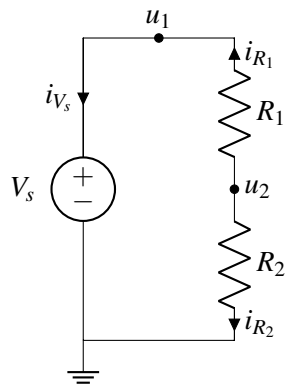
5. If you have n nodes in a circuit with k non-wire elements connecting the nodes, how many equations do you need to solve for all node potentials and element currents? Remember that one node needs to be grounded.
6. How many nodes would you need to label to perform nodal analysis? Include nodes for ground and for V_s .



7. How many nodes are in the following circuit?



8. Assume that you have picked the ground node and labeled the node potentials and branch currents as follows.



What are the $+/-$ labels for R_1 and R_2 according to passive sign convention?

9. For the same circuit as above, formulate a system of equations to solve for all node potentials and branch currents.

