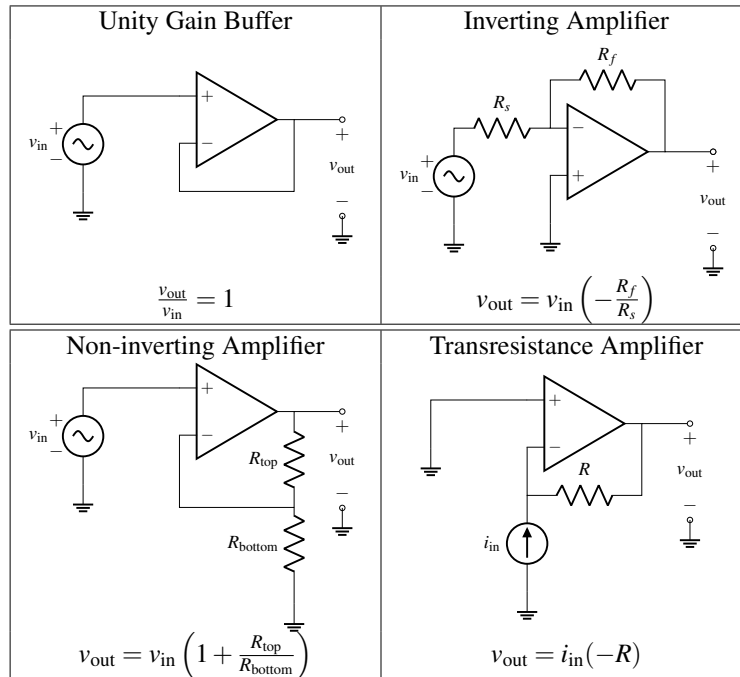## EECS 16A   Designing Information Devices and Systems I
Spring 2023
Discussion 12A

1. Reference Circuits

Below are some common circuit topologies and their corresponding input-output relationships. *Note: it is possible to reference the amplifying circuits to a nonzero constant voltage V_{REF} in order to add an offset to the gain.*

Unity Gain Buffer

$$\frac{v_{\text{out}}}{v_{\text{in}}} = 1$$

Inverting Amplifier

$$v_{\text{out}} = v_{\text{in}}\left(-\frac{R_f}{R_s}\right)$$

Non-inverting Amplifier

$$v_{\text{out}} = v_{\text{in}}\left(1 + \frac{R_{\text{top}}}{R_{\text{bottom}}}\right)$$

Transresistance Amplifier

$$v_{\text{out}} = i_{\text{in}}(-R)$$

2. Review: Design Procedure

Now that we've analyzed many circuits, we are ready to focus on *designing* interesting circuits to perform specific tasks. Circuit design is generally more challenging than circuit analysis. Usually, we are given a prompt about what type of circuit is desired and some specifications, and then we must decide what components to use and how to connect them. Circuit design problems are very similar to proof problems, which we looked at earlier in this course: Both cases are open ended, and we frequently have to integrate many ideas and explore several different possibilities to reach a solution.

When faced with a design problem, a good place to start is to follow the *design procedure* outlined here:

Step 1 (Specification): Concretely restate the goals for the design.
   Frequently, a design prompt will include a lot of text, so we'd like to restate all of the most important features of our design. We'll refer to these specifications later to determine if our design is complete.

Step 2 (Strategy): Describe your strategy (often in the form of a block diagram) to achieve your goal.
   To do this, start by thinking about what you can measure vs. what you want to know. For example in our capacitive touchscreen, we want to know if there is a touch and we can measure voltage. Since we know that a touch can change the capacitance, we break this down into the following block diagram:
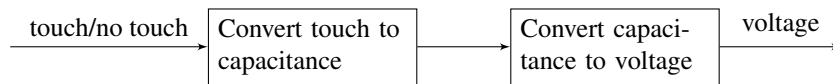
Step 3 (Implementation): Implement the components described in your strategy.

This is where pattern matching is useful: remind yourself of blocks you know, (ex. voltage divider, inverting amplifier) and check if any of these can be used to implement steps of your strategy. If you don't know of a block that does what you want, think about how to modify or extend the blocks you know.

Step 4 (Verification): Check that your design from Step 3 does what you specified in Step 1.

It's tempting to think that you're done after implementation, but verification is critical! In particular, check block-to-block connections, as these are the most common point for problems. Does one block load another block causing it to behave differently than expected? Are there any contradictions (ex. a voltage source with both ends connected by a wire, or a current source directed into an open circuit)? Repeat previous steps if necessary to make sure that your final circuit meets the specifications.

3. Noise Cancelling Headphones

The basic goal of a pair of noise cancelling headphones is for the user to hear only the desired audio signal and not any other sounds from external sources. In order to achieve this goal, noise cancelling headphones include at least one microphone that listens to what you might have otherwise heard from external sources, and then feeds a signal in to your speakers that cancels (subtracts out) that externally-generated sound.

You may find the following voltage summer circuit useful:
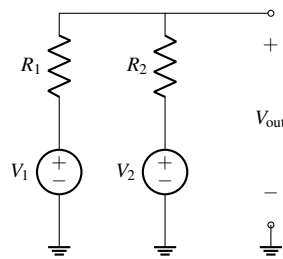
$$\text{Where } V_{\text{out}} = V_1 \left(\frac{R_2}{R_1+R_2} \right) + V_2 \left(\frac{R_1}{R_1+R_2} \right)$$

Answer: For design problems, there are often multiple solutions that work. In general, if you design a circuit that meets the given constraints and provides the desired behavior, your answer will be considered correct, even if it doesn't match the solutions.

(a) Let's start by looking at the most basic part of the headphones: driving the speaker itself with the audio stream we would like to hear. In our system, the source of the audio comes from a digital-to-analog converter (DAC) that translates digital bits to analog voltages (you will learn more about DACs in EECS16B!). It can be modeled as a voltage source with min/max values of $0\,\text{V}$ and $1\,\text{V}$ and a $50\,\Omega$ source resistance. The speaker can be modeled as an $8\,\Omega$ resistor, but in order to produce loud enough sounds and not damage the speaker (driving the speaker with non-zero average voltage can damage the transducer within the speaker), it needs to be driven with a range of $-1.5\,\text{V}$ to $1.5\,\text{V}$ (relative to the ground connected to the DAC, which is the same ground used throughout the system).
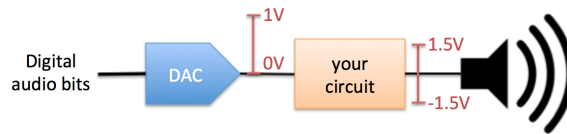
You are provided two voltage sources with values $-1.5\,\text{V}$ and $1.5\,\text{V}$, an op-amp, and any resistors you would like. Design a circuit that could drive the speaker while meeting the specifications above. *Hint: shifting a signal is the same as adding a fixed value to it.*

Suggested procedure:

i. Specification: restate the goals for this circuit.

ii. Strategy: Draw a block a diagram to achieve your goals. Think about what you can measure and what circuits you have seen, e.g. adders and multipliers. Try to break down the goals into multiple steps.

iii. Implementation: Build each of the blocks you described above. Use the reference circuits and pattern match.

iv. Verification: Put your circuit together and make sure it works. Especially look at when you put blocks together, and watch out for loading effects.

Let's go through the design method.

i. **Step 1: Specification**:

The goal of this circuit is to transform an input that ranges from $0\,\text{V}$ to $1\,\text{V}$ into an output that ranges from $-1.5\,\text{V}$ to $1.5\,\text{V}$.

We are given a DAC, modelled by a voltage source and a resistor, and a speaker, modelled by a resistor.


ii. **Step 2: Strategy**:

To achieve the goal, we need two things:

i. Shift the signal to center at $0\,\text{V}$.

ii. Provide gain to the signal to achieve a $3\,\text{V}$ range.

The order of shifting and amplifying is determined by the supply voltage of the op-amps. Notice that if we amplify our signal before we shift, our op-amp needs to output up to $3\,\text{V}$, while we only have voltage sources of $+1.5\,\text{V}$ or $-1.5\,\text{V}$ to power the op-amp with. When we shift the input signal, the shifted signal does not have to have the same amplitude, so we can adjust the gain of the amplifier accordingly. If we make the amplitude of the shifted input signal N, the block diagram is shown below.
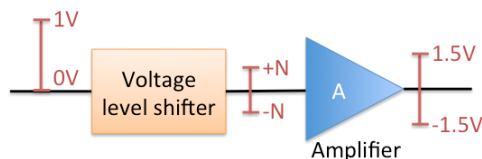
iii. **Step 3: Implementation**:

We can use a non-inverting amplifier circuit to provide a gain of more than 1, solving the second point. Thus, we are left with the voltage shifter.

It turns out we can use a voltage summer – after all, shifting a voltage is nothing more than subtracting a fixed value.
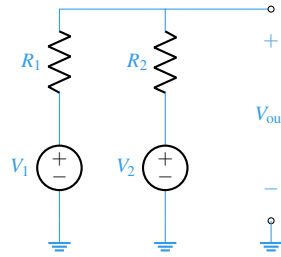
We already derived that the output of a voltage summer is $kV_1 + (1-k)V_2$, where $k = \frac{R_2}{R_1+R_2}$, so now this is a matter of picking appropriate values for V_2 and k and using V_1 as the input. We also realize that there is some gain $k < 1$ applied to the input. Thus, we have to adjust the amplifier gain to $\frac{A}{k} = \frac{3}{k}$ ($A = 3$ since our output range is $3\,\text{V}$ while our input range is $1\,\text{V}$).

Let's build the voltage shifter. We can only use the $-1.5\,\text{V}$ voltage source for V_2 if we want to shift the signal level down, i.e. subtract voltage. We also know that we want the shift to be half of the actual signal, since we want to center the signal around 0. Since V_1, the DAC voltage, ranges between $0\,\text{V}$ and $1\,\text{V}$, the median voltage is $\frac{1}{2}\text{V}$:

$$kV_{1,med} + (1-k)V_2 = 0$$

$$(1-k)V_2 = -k\cdot(V_{1,med})$$

$$(1-k)\cdot\left(-\frac{3}{2}\right) = -k\left(\frac{1}{2}\right)$$

$$3 - 3k = k$$

$$k = \frac{3}{4}$$

Knowing our equation $k = \frac{R_2}{R_1+R_2}$, we can pick values for the resistors such that the ratio is $\frac{3}{4}$. However, remember that our DAC has a source resistance of $50\,\Omega$. We can use this resistance as R_1 since it is in series with the voltage source. It follows that $R_2 = 150\,\Omega$. Our voltage shifter circuit is shown below. We label the output as V_{in} to the next stage: the amplifier.
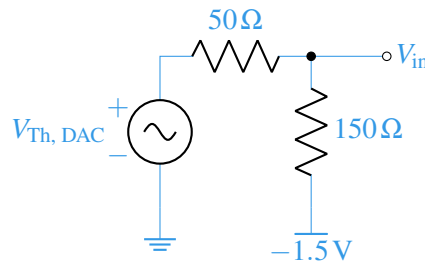

Remember that this voltage shifter actually has a gain. Specifically, $V_{in} = kV_{Th,\,DAC} + (1-k)(-1.5) = 0.75\,V_{Th,DAC} - 0.375$. Thus, this shifter has a gain of 0.75. Following our discussion before, we need to adjust our amplifier to have a gain of $\frac{3}{0.75} = 4$ and build the appropriate non-inverting amplifier. Since the gain of a non-inverting amplifier is $\frac{R_2+R_1}{R_1}$, we can choose $R_1 = 1\,\text{k}\Omega$ and $R_2 = 3\,\text{k}\Omega$. The final circuit is shown below.
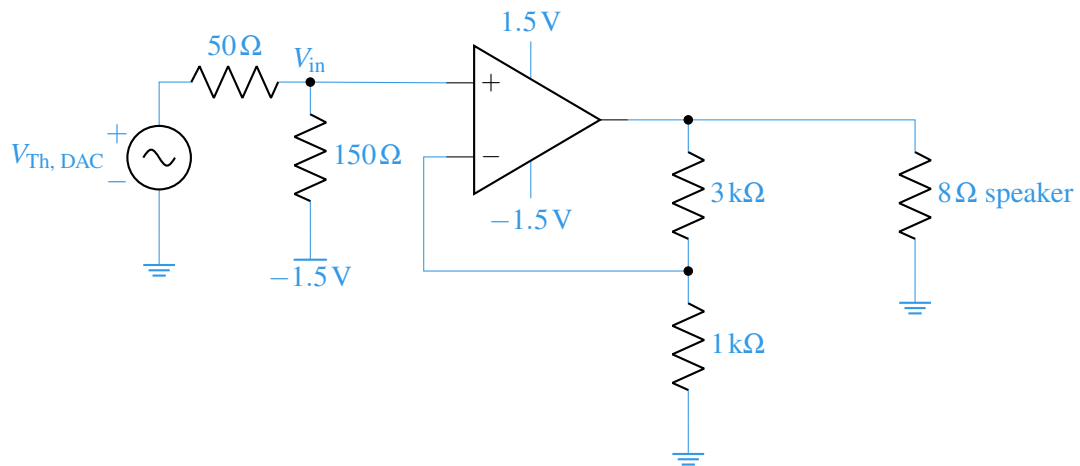
iv. **Step 4: Verification** The first place to check when verifying your circuit is the connection between blocks, where loading effects may unintentionally change our circuit's behavior. At the output of the level shifter / voltage divider is the input to an op-amp. Since no current flows into the op-amp, there is no loading, and so the level shifter is unaffected by cascading them. At the output of the non-inverting amplifier, we have our load resistor (a.k.a. the speaker). Since the output of the amplifier is a voltage source (from the op-amp), the load resistor does not affect the function of the amplifier. So this circuit should work!

(b) Now, let's look at implementing the noise cancellation. In this problem, we will assume that we do not have access to software and therefore cannot digitally remove the noise (as do most noise cancelling headphones). We will therefore focus on implementing the cancellation physically, which is to directly take the (analog) voltage produced by the microphone and subtract it out from the voltage we feed to the speaker.
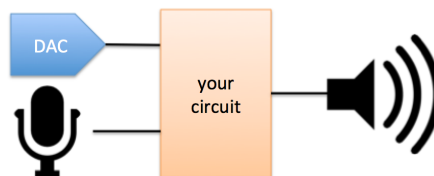
Let's assume that the microphone can be modeled as a voltage source with min/max values of $0\,\text{V}$ and $1\,\text{V}$ (relative to the DAC's ground) and a $10\,\text{k}\Omega$ source resistance. However, because the materials in the headphones attenuate some of the external sound, the loudest signals picked up by the microphone should correspond to a voltage range of only $-125\,\text{mV}$ to $+125\,\text{mV}$ driven onto the speaker.

Design a circuit that takes in the signal from the microphone and outputs the appropriate signal driven to the speaker. We will assume the following circuit will sum this signal and the one from the previous part to accomplish noise cancellation, so we want this circuit to be invert. You can use op-amps and

resistors to do this, but no new voltage sources (you still have the 1.5 and -1.5V sources from the previous part).

i. **Step 1: Specification**
The new circuit takes in a signal from the microphone from $0\,\text{V}$ to $1\,\text{V}$ and outputs a signal from $-125\,\text{mV}$ to $125\,\text{mV}$. The microphone has a $10\,\text{k}\Omega$ resistance, compared to the $50\,\Omega$ resistance in the DAC.

ii. **Step 2: Strategy**
Like in part (a), we first need to shift the range of the microphone voltage such that it centers around $0\,\text{V}$, then apply gain or some multiplication to reduce the range. We will have a similar block diagram as the previous part, but the output range is now $-125\,\text{mV}$ to $125\,\text{mV}$ instead of -1.5V to 1.5V

iii. **Step 3: Implementation**
The most intuitive solution is to use a similar level-shifter and amplifier combination from the circuit in part (a). Since we need to subtract the microphone signal, we can use an inverting amp instead of a non-inverting amp.
We use a similar level shifting circuit, and its resistor values can be found in a similar process to (a). We are given a source resistance, so we use that again. Centering the mic voltage to 0 again gives us a full range of 0.75 V. Now however, we want our output range to be only $-125\,\text{mV}$ to $+125\,\text{mV}$, i.e. a full range of 0.25 V, so we need a smaller final gain. Also, for this gain, instead of a non-inverting amp, we use an inverting amp. This takes care of the "subtraction" aspect.
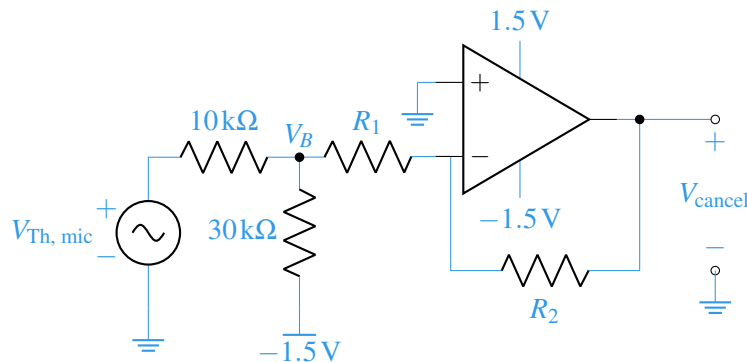
We know the gain of the inverting amp is $-\frac{R_2}{R_1}$. We want this to be $-\frac{0.25\,\text{V}}{0.75\,\text{V}} = -\frac{1}{3}$. So we set $R_1 = 3\,\text{k}\Omega, R_2 = 1\,\text{k}\Omega$.

However, there is an issue with the circuit we just built. There will be a current running through R_1 from the V_B node so it is loading the level shifting circuit. That means the voltage at V_B is affected. It is important then to add a voltage buffer at this node to prevent loading. Note that this is not a problem when are using a non-inverting amplifier, since the input to that is an op-amp terminal, not a resistor.
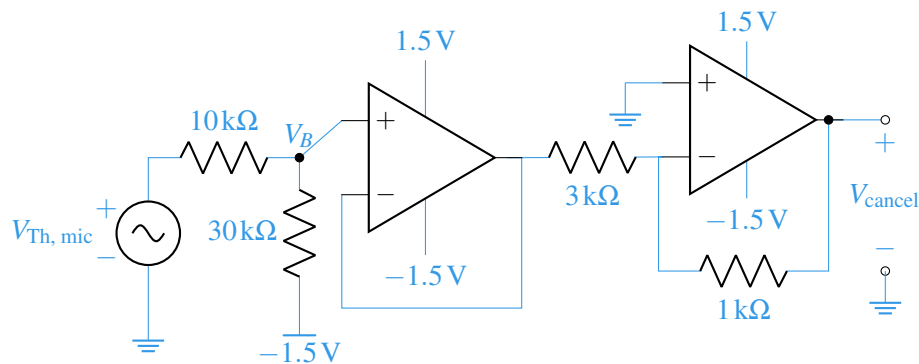
iv. **Step 4: Verification** We have effectively completed this step already, but here are useful things to check at this point:

   i. Check between the circuit blocks, e.g. summer circuit, level shifter circuit, etc., to make sure we have no loading effects, i.e. one circuit doesn't affect another. For example, we saw that putting the inverting amplifier right after the level shifter affects the level shifter's voltage, resulting in a voltage buffer.

   ii. Write the equations for each of the blocks and make sure the circuit completes the computation you expect.

   iii. Make sure you don't have any circuit no-nos, i.e. no short circuits and no mistaken open circuits. Make sure your op-amps inputs are labelled correctly to give you the circuit you think you have.

(c) (Optional)

We now have a circuit that takes in the DAC voltage and produces the correct voltage for the speaker, and a second circuit that takes in the microphone voltage and produces the correct voltage for the speaker. Now we simply have to add these together.

Design a circuit that combines these two circuits by adding their outputs to drive a speaker. We don't need any gain from this circuit, but since we are driving the $8\,\Omega$ speaker, we may want to have some isolation between the speaker and these first two circuits.

You may use op-amps and resistors to complete this circuit. Note that since our speaker driver now needs to handle both the cancellation and the desired audio signal, you can assume that the supply voltages fed to the op-amp have sufficiently large magnitude to ensure that they never clip (reach the power rails). In other words, you should continue to assume that you have $\pm 1.5\,\text{V}$ voltage sources available to use in the rest of your circuit, but the op-amps are now supplied by a separate set of larger, sufficient voltage sources.

i. **Step 1: Specification**
We previously built the individual circuits needed to drive the speaker. In this final step, we want to sum those two circuits. We also want to do something to prevent the speaker loading.

ii. **Step 2: Strategy**

We will use the exact circuits from before, then sum those outputs. We could use a voltage summer with just two resistors, but that fails to give us isolation. So we should try to include some op-amp circuit.

The complete block diagram of the full noise cancelling circuit will look as follows:
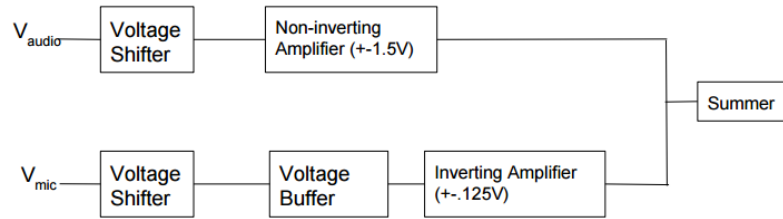
iii. **Step 3: Implementation** Since we took care of the gains and level shift already, we can just directly add the outputs of the previous two circuits without any further scaling, i.e. take the exact sum of them. We can use the following "summing amplifier" circuit, which combines a voltage summer and a non-inverting amplifier. We will need the non-inverting gain because a voltage summer always applies a linear combination with a gain on each term of less than 1.
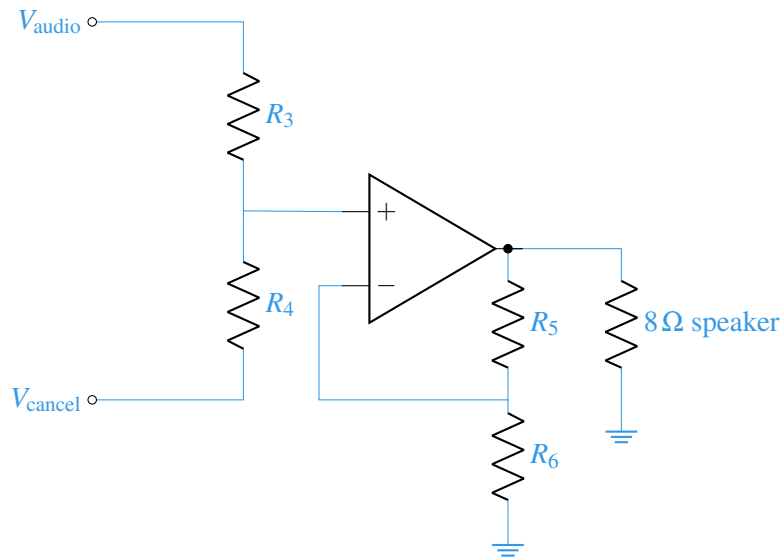
Note that since both of the inputs to this circuit will be driven with op amps, we do not have to worry about loading the previous parts, i.e. affecting their output voltages.

Setting up the equality of what we want and the equation that this circuit implements, we get:

$$V_{out} = \left(\frac{R_4}{R_3 + R_4} V_{audio} + \frac{R_3}{R_3 + R_4} V_{cancel} \right) \left(\frac{R_5 + R_6}{R_6} \right)$$

$$= \left(\frac{R_4}{R_3 + R_4} \right) \left(\frac{R_5 + R_6}{R_6} \right) V_{audio} + \left(\frac{R_3}{R_3 + R_4} \right) \left(\frac{R_5 + R_6}{R_6} \right) V_{cancel}$$

$$= V_{audio} + V_{cancel}$$

The easiest solution to this is $R_3 + R_4 = R_5 + R_6$ and $R_6 = R_3 = R_4 = R_5$. We pick all of the resistors to be $1\,\text{k}\Omega$.

We can also take this time to select the rails for this op-amp. We specified that we can use larger rails here to handle the bigger range. The maximum output voltage we can achieve is 1.5 V from the audio + 0.125 V from the mic = 1.625 V, so we need at least that large of a supply on both ends.

Putting this all together we get the following circuit:
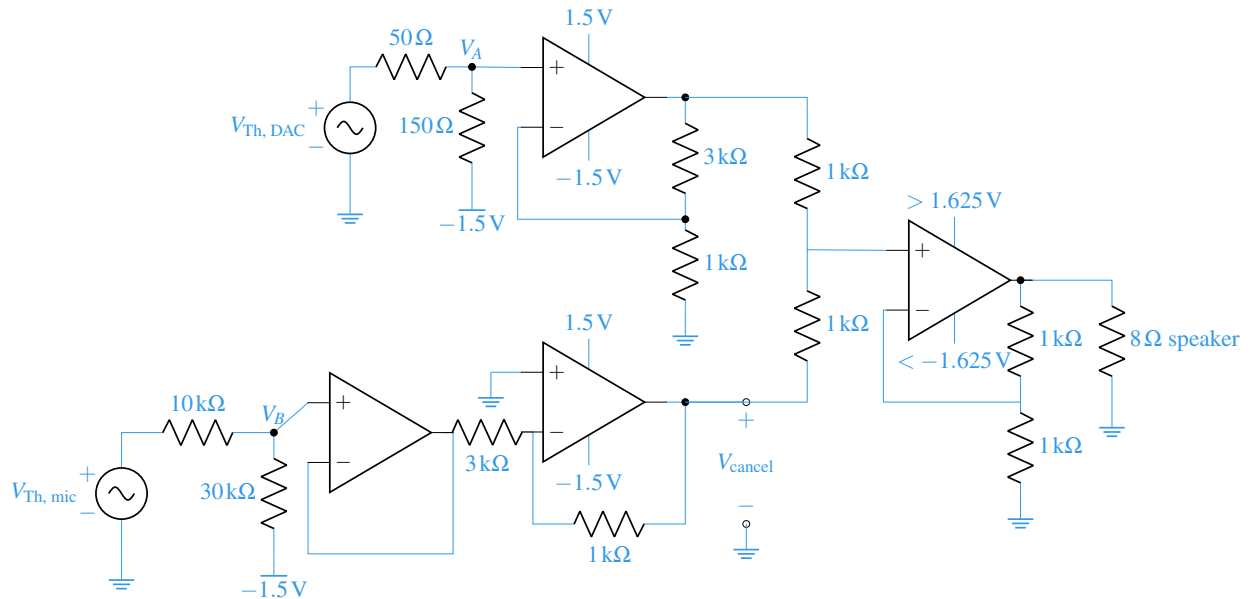
iv. **Step 4: Verification** Since we combined multiple blocks together, it is useful to check to make sure the input to one block does not affect the output to another. In the first two parts, we picked blocks that gave us op-amps at the output, so even though we have resistors at the input to the next stage, we will not be affected by loading.

There are more advanced solutions to this that use fewer op-amps by using more advanced op-amp topologies. If you are curious about those, please ask your TAs. The benefit of this version of the circuit, and its worth looking at, is because it has great modularity and is easy to understanding the building blocks and how they come together.