EECS 16A Designing Information Devices and Systems I
Spring 2023 Homework 6

This homework is due Friday, March 3rd, 2023, at 23:59.
Self-grades are due Friday, March 10th, 2023, at 23:59.

Submission Format
Your homework submission should consist of **one** file.

- `hw6.pdf`: A single PDF file that contains all of your answers (any handwritten answers should be scanned).

Submit the file to the appropriate assignment on Gradescope.

1. **Reading Assignment**

   For this homework, please review and read Note 11A/B, which introduces the basics of circuit analysis and node voltage analysis. You are always welcome and encouraged to read beyond this as well.

2. **It's a Triforce!**

   Learning Goal: *This problem explores passive sign convention and nodal analysis in a slightly more complicated circuit.*

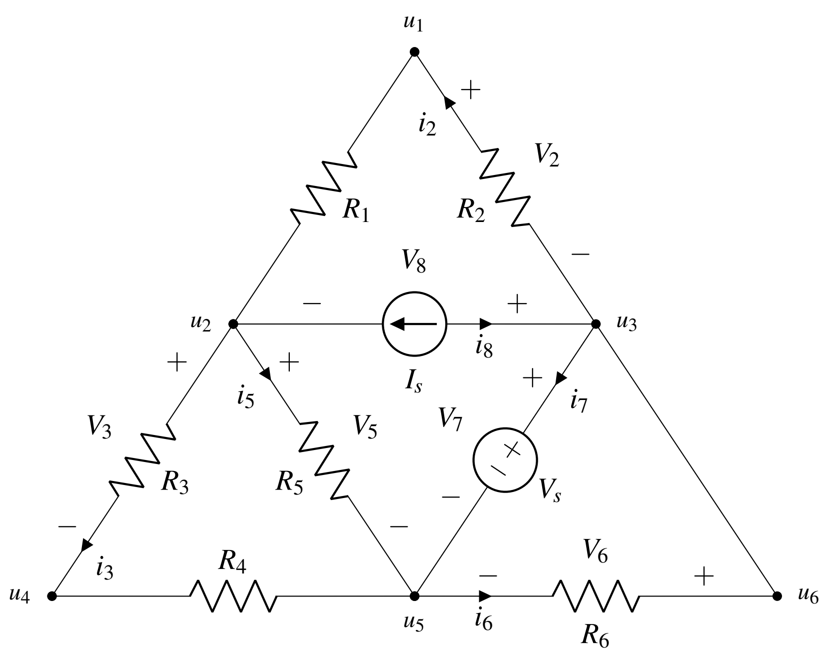


Figure 1: A triangular circuit consisting of a voltage source V_s, current source I_s, and resistors R_1 to R_6.

   (a) Which elements I_s, V_s, R_2, R_3, R_5, or R_6 in Figure 1 have current-voltage labeling that violates *passive sign convention*? Explain your reasoning.

Solution:

Recall *passive sign convention* dictates that positive current should *enter* the positive voltage terminal and *exit* the negative voltage terminal.

The elements associated with I_s, R_2, and R_6 have (external) voltage/current labelings which violate passive sign convention as the current is depicted leaving the positive terminal (or entering the negative terminal). This could be corrected either by swapping the voltage polarity or the current direction.

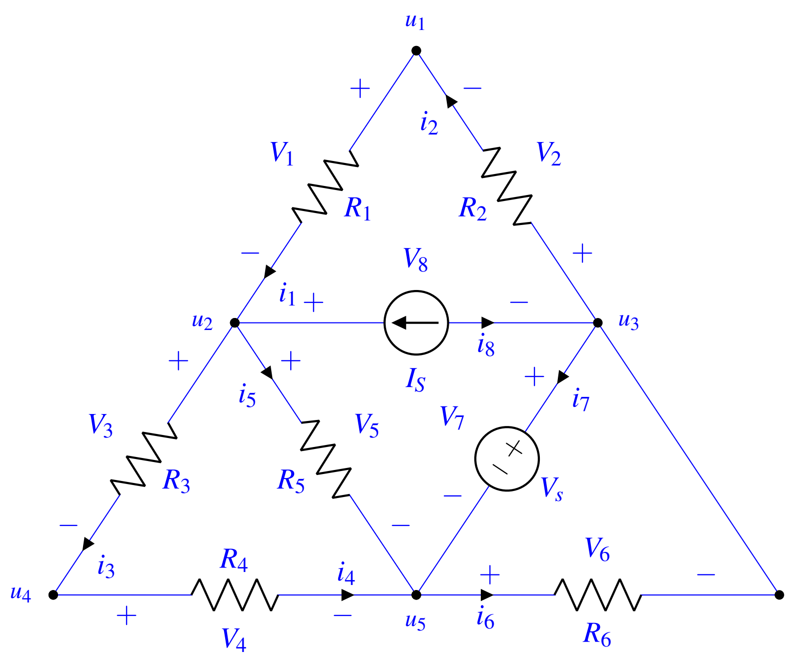
(b) In Figure 1, the nodes are labeled with u_1, u_2, ... etc. There is a subset of u_i's in the given circuit that are redundant, i.e. there might be more than one label for the same node. Which node(s) do not have a unique label? Justify your answer.

Solution:

The nodes u_3 and u_6 are redundant and can be represented with the same node since there is a short connecting them.

(c) Redraw the circuit diagram by correctly labeling *all* the element voltages and element currents according to passive sign convention. The component labels that were violating passive sign convention in part (a) should be corrected by *swapping the element voltage polarity*. Additionally, label the elements that have not been labeled yet.

Solution:



This is one of the possible correct labelings. Since the problem statement disallows swapping the current direction, the only other valid answers would include variations of the completely unlabeled elements R_1 and R_4. The flipping of *both* current *and* voltage labelings for R_1 and/or R_4 is also a correct answer.

(d) Write an equation to describe the current-voltage relationship for element R_4 in terms of the relevant i's, R's, and node voltages in this circuit. Your final expression should include u_4, u_5, and i_4.

Solution:

The resulting equation should look like:

$$R_4 = \frac{u_4 - u_5}{i_4}$$

Alternatively, if you labelled the current i_4 going in the opposite direction (going from u_5 to u_4), the resulting equation should look like:

$$R_4 = \frac{u_5 - u_4}{i_4}$$

(e) Write the KCL equation for node u_2 in terms of the node voltages and other circuit elements.

Solution:

KCL at u_2 gives us:

$$i_1 - i_8 - i_3 - i_5 = 0$$

The equations for the current through each of the branches are:

$$i_1 = \frac{u_1 - u_2}{R_1}$$
$$i_3 = \frac{u_2 - u_4}{R_3}$$
$$i_8 = -I_s$$
$$i_5 = \frac{u_2 - u_5}{R_5}$$

The final expression is then:

$$\frac{u_1 - u_2}{R_1} + I_s - \frac{u_2 - u_5}{R_5} - \frac{u_2 - u_4}{R_3} = 0$$

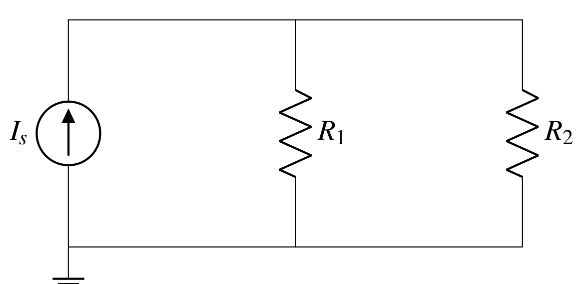
Or any equivalent equation.

3. Circuit Analysis

Learning Goal: *This problem will help you practice circuit analysis using the node voltage analysis (NVA) method.*

Using the steps outlined in lecture or in Note 11, analyze the following circuits to calculate the currents through each element and the voltages at each node. Use the ground node labelled for you. You may use a numerical tool such as IPython to solve the final system of linear equations.

(a) $I_s = 3\,\text{mA}, R_1 = 2\,\text{k}\Omega, R_2 = 4\,\text{k}\Omega$



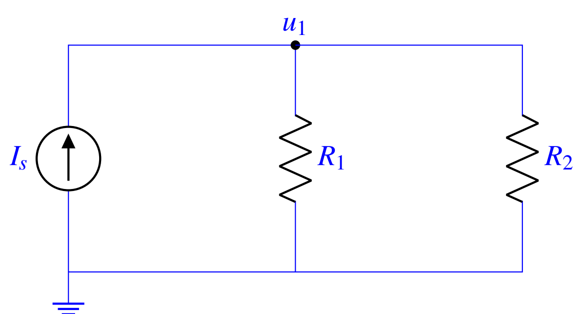
Solution:

Step 1)Pick a junction and label it as u = 0 ("ground"), meaning that we will measure all of the voltages in the rest of the circuit relative to this point.

Step 2) Label all remaining junctions as some "u_i", representing the voltage at each junction relative to the zero junction/ground.

Step 3) Label Remaining Nodes

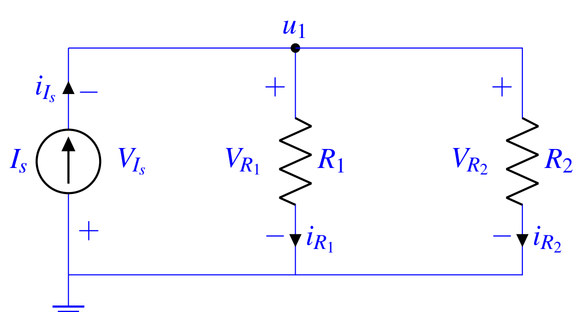
Label the current through every element in the circuit "in". Every element in the circuit that was listed above should have a current label, including ideal wires. The direction of the arrow which direction of current flow you are considering to be positive. At this stage of the algorithm, you can pick the direction of all of the current arrows arbitrarily - as long as you are consistent with this choice and follow the rules described in the rest of this algorithm, the math will work out correctly



Step 4) Label Element Voltages and Currents

Next we mark all element voltages and currents.

Start with the current. The direction is arbitrary (top to bottom, bottom to top, it won't matter, but stick with your choice in subsequent steps). Then mark the element voltages following the passive sign convention, i.e. the voltage and current point in the "same" direction.



Step 5) KCL Equations

Write KCL equations for all nodes with unknown voltage, which is only u_1.

$$(-)i_{I_s} + i_1 + i_2 = 0$$

Step 6) Element Currents

Find expressions for all element currents in terms of voltage and element characteristics (e.g. Ohm's law) for all circuit elements except voltage sources. In this circuit there are three, R_1, R_2, and I_s.

$$i_{I_s} = I_s$$
$$i_{R_1} = \frac{V_{R_1}}{R_1}$$
$$i_{R_2} = \frac{V_{R_2}}{R_2}$$

Step 7) Element Voltages
Rewrite the element voltages using the node differences.

$$i_{I_s} = I_s$$
$$i_{R_1} = \frac{u_1}{R_1}$$
$$i_{R_2} = \frac{u_1}{R_2}$$

Step 8) Substitute Element Currents in KCL Equations
Now we substitute the expressions derived in Step 7 into the KCL equations from Step 5.

$$-I_s + \frac{u_1}{R_1} + \frac{u_1}{R_2} = 0$$

We can isolate the unknown terms (u_1) on the left and the known on the right

$$\frac{u_1}{R_1} + \frac{u_1}{R_2} = I_s$$

We only have one equation to solve. Setting up the matrix equation would just be the same as solving this equation. Solving for u_1, we get

$$u_1 = I_s \frac{R_1 R_2}{R_1 + R_2}$$

Plugging in the values we were given, we get

$$\begin{aligned} u_1 &= 3\,\mathrm{mA} \frac{2\,\mathrm{k}\Omega \cdot 4\,\mathrm{k}\Omega}{2\,\mathrm{k}\Omega + 4\,\mathrm{k}\Omega} \\ &= 4\,\mathrm{V} \end{aligned}$$

Node u_1 is $4\,\mathrm{V}$ relative to the ground node we defined. If we had defined the top node as ground, then the bottom node would have measured as $-4\,\mathrm{V}$. Similarly, as drawn, we have $V_{R_1} = V_{R_2} = 4\,\mathrm{V}$; if we flipped the polarities, i.e. swapped + and -, we would have $V_{R_1} = V_{R_2} = -4\,\mathrm{V}$.
The branch currents can be obtained from the node voltages and element equations. Therefore, we can write:

$$i_{R_1} = \frac{u_1}{R_1} = 2\,\mathrm{mA}$$
$$i_{R_2} = \frac{u_1}{R_2} = 1\,\mathrm{mA}$$

Note that

$$i_{R_1} = I_s \frac{R_2}{R_1 + R_2} \qquad i_{R_2} = I_s \frac{R_1}{R_1 + R_2}$$

These are very similar equations to the voltage divider circuit. We call this circuit a current divider.

4. Pre-lab Questions

These questions pertain to the pre-lab reading for the *Touch 1* lab. You can find the reading under the *Touch 1* Lab section on the 'Schedule' page of the website. We do not expect in-depth answers for the questions. Please limit your answers to a maximum of 2 sentences.

(a) What are the three terminals of a potentiometer?

(b) How can you make a voltage divider using a potentiometer?

(c) What is the threshold voltage of the red LED we use in the lab?

Solution:

(a) A potentiometer (a type of variable resistor) has three terminals: two end terminals and one movable middle terminal that acts as the output.

(b) Connect the voltage source to one end terminal and ground to another end of the potentiometer then measure voltage on the output terminal.

(c) $V_{\text{LED}} = +1.6\text{ V}$

5. Homework Process and Study Group

Who did you work with on this homework? List names and student ID's. (In case you met people at homework party or in office hours, you can also just describe the group.) How did you work on this homework? If you worked in your study group, explain what role each student played for the meetings this week.

Solution:

I first worked by myself for 2 hours, but got stuck on problem 5. Then I met with my study group.

XYZ played the role of facilitator ... etc. We were still stuck on problem 5 so we went to office hours to talk about the problem.

Then I went to homework party for a few hours, where I finished the homework.