EECS 16A Designing Information Devices and Systems I
Spring 2023 Homework 7

This homework is due March 10, 2022, at 23:59.
Self-grades are due March 17, 2022, at 23:59.

Submission Format
Your homework submission should consist of **one** file.

- `hw7.pdf`: A single PDF file that contains all of your answers (any handwritten answers should be scanned).

Submit the file to the appropriate assignment on Gradescope.

1. **Reading Assignment**

   For this homework, please read Note 11B, Note 12, and Note 13. Note 11B covers node voltage analysis and goes over an in-depth example of finding voltages and currents in a complex circuit. Notes 12 and 13 cover voltage dividers, how a simple 1-D resistive touchscreen works, the physics of circuits, and introduces the notion of power in electric circuits.

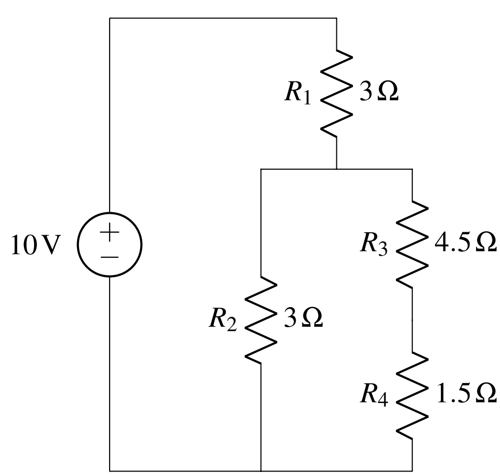
   (a) What are the 7 steps of NVA?

   (b) Describe the key ideas behind how the 1D touchscreen works. In general, why is it useful to be able to convert a “physical" quantity like the position of your finger to an electronic signal (i.e. voltage)?

   Solution: You should give yourself full-credit for any reasonable answers.

2. **Mechanical Circuits**

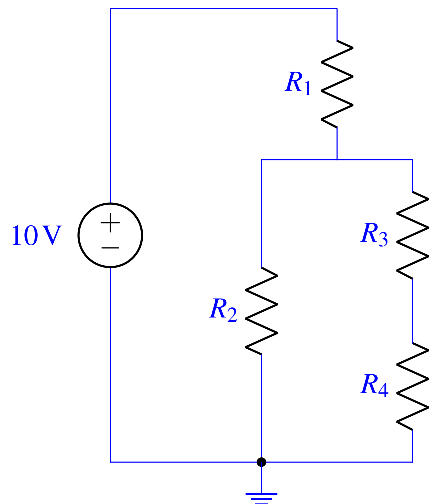
   Find the voltages across and currents flowing through all of the resistors. *Hint: Use the seven steps of node voltage analysis.*



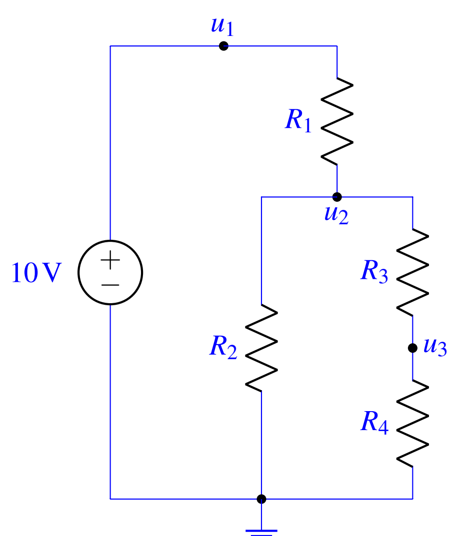
Solution:

Node Voltage Analysis (Seven Step Method):

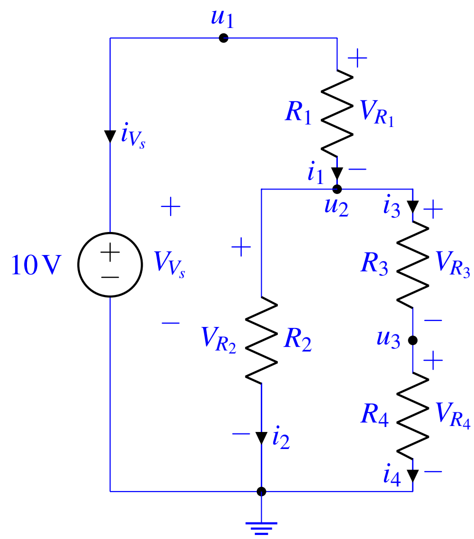
Step 1) Select a ground node. Any choice of ground is valid, but we choose the bottom node:



Step 2) Label all remaining nodes:



Step 3) & 4) Label element voltages and currents. Currents can be labeled in arbitrary directions, but labeling must follow passive sign convention.



Step 5) Identify and reduce unknowns. The voltage source is connected to the top node and ground node. We know that the value of the voltage source $V_s = 10$ V, and that a voltage source forces the difference between its nodes to be V_s. We also know the ground node is 0 V by definition. Therefore, we know that the top node must hold a potential of 10V.

$$u_1 - 0 = V_s = 10\,\text{V}$$

Step 6) Write KCL equations for all nodes with unknowns (u_2 and u_3). The definition of KCL is that the sum of all currents entering and leaving the node must equal 0 A. We will call current entering positive, and current leaving negative. Going in order from node u_2 to node u_3, we set up our KCL expressions:

$$i_1 - i_2 - i_3 = 0 \quad (1)$$
$$i_3 - i_4 = 0 \quad (2)$$

Use element I-V relationships to find equations relating the branch currents to the node voltages. Looking at the differences of node potentials, the top node and u_2 are separated by a resistor, and Ohm's law relates the potential *difference* between each side of the resistor to the current through it, so we have:

$$10V - u_2 = V_{R_1} = i_1 R_1 \implies i_1 = \frac{10V - u_2}{R_1} \quad (3)$$

Similarly for the other resistors

$$u_2 - 0 = V_{R_2} = i_2 R_2 \implies i_2 = \frac{u_2}{R_2} \quad (4)$$
$$u_2 - u_3 = V_{R_3} = i_3 R_3 \implies i_3 = \frac{u_2 - u_3}{R_3} \quad (5)$$
$$u_3 - 0 = V_{R_4} = i_4 R_4 \implies i_4 = \frac{u_3}{R_4} \quad (6)$$

Step 7) Solve the system of equations set up in step 6. Note that we have 6 unknowns $(i_1, i_2, i_3, i_4, u_2, u_3)$ and 6 equations. Substituting equations (3-6) into our KCL equations (1-2), we get

$$\left(\frac{10V-u_2}{R_1}\right)-\frac{u_2}{R_2}-\left(\frac{u_2-u_3}{R_3}\right)=0$$
$$\left(\frac{u_2-u_3}{R_3}\right)-\frac{u_3}{R_4}=0$$

which gives us

$$u_2 = 4\,\mathrm{V}$$
$$u_3 = 1\,\mathrm{V}$$

Using the node voltages, we can derive the voltage across and current through every resistor

$$\begin{aligned}
V_{R_1} = 10V - u_2 = 6\,\mathrm{V}, &\qquad i_1 = \frac{V_{R_1}}{R_1} = 2\,\mathrm{A} \\
V_{R_2} = u_2 = 4\,\mathrm{V}, &\qquad i_2 = \frac{V_{R_2}}{R_2} = 1.33\,\mathrm{A} \\
V_{R_3} = u_2 - u_3 = 3\,\mathrm{V}, &\qquad i_3 = \frac{V_{R_3}}{R_3} = 0.67\,\mathrm{A} \\
V_{R_4} = u_3 = 1\,\mathrm{V}, &\qquad i_4 = \frac{V_{R_4}}{R_1} = 0.67\,\mathrm{A}
\end{aligned}$$

3. Volt and ammeter

Learning Goal: *This problem helps you explore what happens to voltages and currents in a circuit when you connect voltmeters and ammeters in different configurations.*

Use the following numerical values in your calculations: $R_1 = 1\,\mathrm{k}\Omega$, $R_2 = 2\,\mathrm{k}\Omega$, $R_3 = 3\,\mathrm{k}\Omega$, $R_4 = 4\,\mathrm{k}\Omega$, $R_5 = 5\,\mathrm{k}\Omega$, $V_s = 8V$.

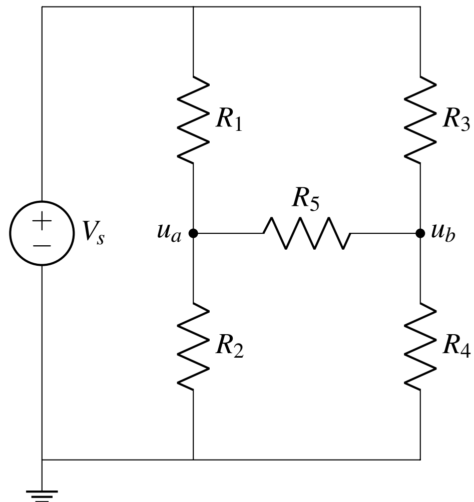
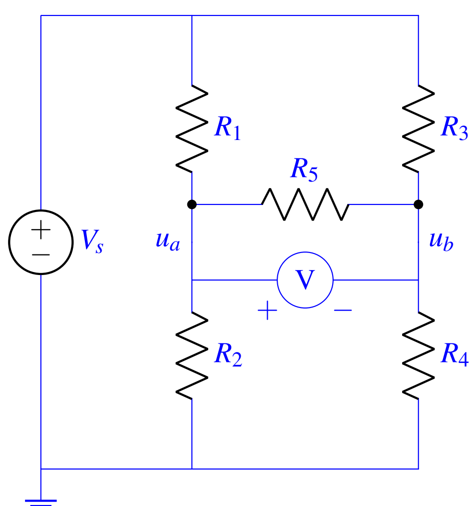


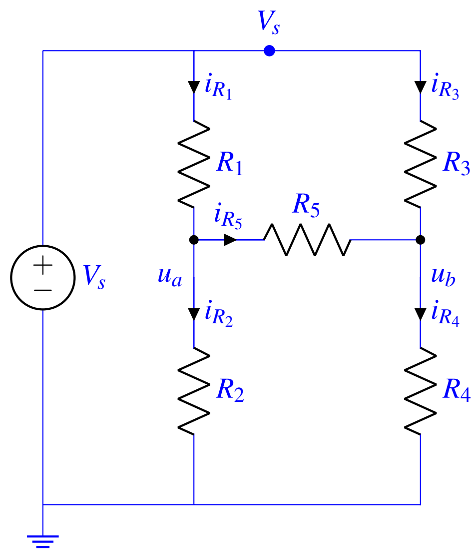
Figure 1: Circuit consisting of a voltage source V_s and five resistors R_1 to R_5

(a) Redraw the circuit diagram shown in Figure 1 by adding a voltmeter (letter V in a circle and plus and minus signs indicating direction) to measure voltage V_{ab} from node u_a (positive) to node u_b (negative). Calculate the value of V_{ab}. You may use a numerical tool such as IPython to solve the final system of linear equations.

Solution: Below is the redrawn circuit with the voltmeter. Note that it is also correct to have the voltmeter above R_5, as it will still be connected to the same nodes.



Using NVA analysis we need to label our nodes. u_a and u_b are already labelled. The topmost node has voltage V_s and the bottom most node is our reference. We also label the currents in each element.



Using KCL at node u_a and u_b, we find:

$$i_{R_1} - i_{R_5} - i_{R_2} = 0$$

$$i_{R_5} + i_{R_3} - i_{R_4} = 0$$

Let's substitute IV relationships into the previous equations.

$$\frac{V_s - u_a}{R_1} - \frac{u_a - u_b}{R_5} - \frac{u_a}{R_2} = 0$$

$$\frac{u_a - u_b}{R_5} + \frac{V_s - u_b}{R_3} - \frac{u_b}{R_4} = 0$$

Gathering the u_a and u_b terms:

$$\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_5}\right)u_a - \left(\frac{1}{R_5}\right)u_b = \frac{V_s}{R_1}.$$

$$-\left(\frac{1}{R_5}\right)u_a + \left(\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}\right)u_b = \frac{V_s}{R_3}.$$

Notice that we wrote our unknowns (u_a and u_b) on the left side of the equation. We can then represent this in matrix form as:

$$\begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_5} & -\frac{1}{R_5} \\ -\frac{1}{R_5} & \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5} \end{bmatrix} \begin{bmatrix} u_a \\ u_b \end{bmatrix} = \begin{bmatrix} \frac{V_s}{R_1} \\ \frac{V_s}{R_3} \end{bmatrix}$$

Plugging in the values we were given into the matrix above and using Gaussian elimination we can find the vector of unknowns.

$$\begin{bmatrix} u_a \\ u_b \end{bmatrix} = \begin{bmatrix} 5.265V \\ 4.748V \end{bmatrix}$$

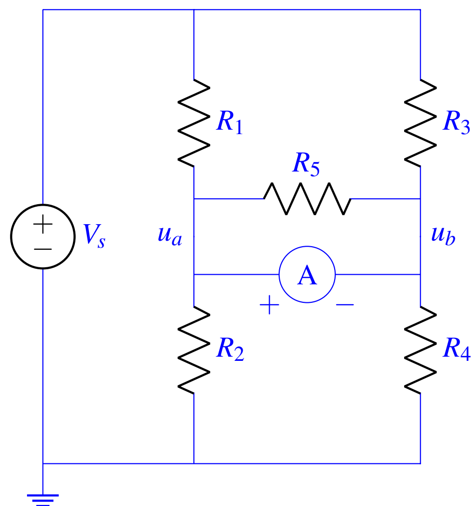
From these node voltages, the voltage V_{ab} can be calculated.

$$V_{ab} = u_a - u_b = 0.516V$$

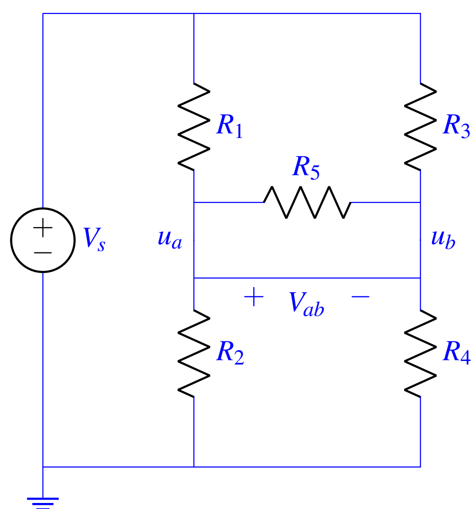
You should give yourself full-credit if your answer is off by a rounding error.

(b) Suppose you accidentally connect an ammeter in part (a) instead of a voltmeter. Calculate the value of V_{ab} with the ammeter connected.

Solution: While you did not have to redraw the circuit, it is depicted below.

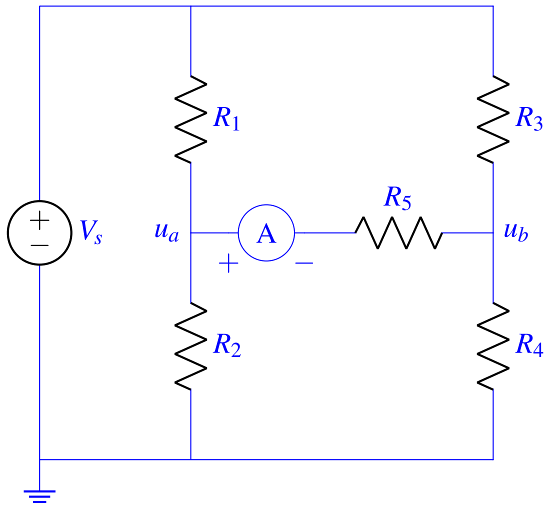


If we assume that the internal resistance of an ammeter is ideally zero, placing it across the nodes u_a and u_b will short them. So $u_a = u_b$. Thus $V_{ab} = 0$. The circuit below shows how the ammeter behaves as a short that unifies the previously separate nodes.



(c) Redraw the circuit diagram shown in Figure 1 by adding an ammeter (letter A in a circle and plus and minus signs indicating direction) in series with resistor R_5. This will measure the current I_{R_5} through R_5. Calculate the value of I_{R_5}.

Solution: The redrawn circuit with the ammeter measuring the current through R_5 is shown in the following circuit. It is also correct to draw the ammeter to the right of R_5 with the orientation of the meter remaining the same: the plus sign should be most proximal to the node labeled u_a, and the minus sign is most proximal to the node labeled u_b.



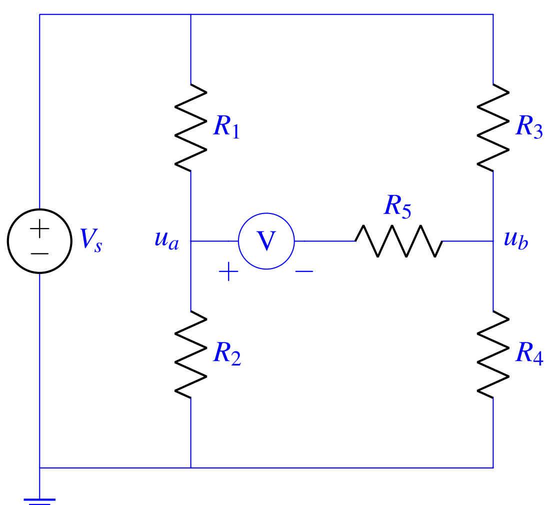
After calculating the node voltages u_a and u_b from part a, we can write:

$$I_{R_5} = \frac{u_a - u_b}{R_5} = 103.2\mu A$$

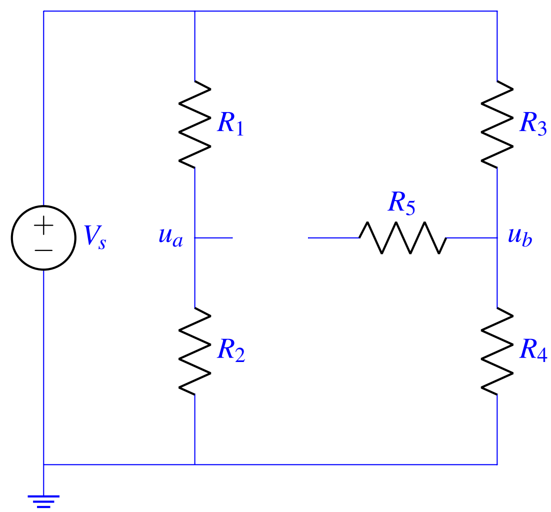
You should give yourself full-credit if your answer is off by a rounding error.

(d) Your friend accidentally connects a voltmeter in part (c) above, rather than an ammeter. Calculate the value of I_{R_5} with the voltmeter connected.

Solution: While you were not required to redraw the new circuit, the circuit is shown below.



The resistance of a voltmeter is infinite and it behaves as an open circuit. There will be no current flowing through R_5. Therefore, $I_{R_5} = 0$. The circuit below depicts how the voltmeter behaves as an open that prevents any current through R_5.


R_1
R_3
R_5
V_s
u_a
u_b
R_2
R_4

4. Cell Phone Battery

As great as smartphones are, one of their drawbacks is that their batteries don't last a long time. For example, a typical smartphone, under average usage conditions (internet, a few cat videos, etc.) uses 0.3W of power. We will model the battery as an ideal voltage source (which maintains a constant voltage across its terminals regardless of current) except that we assume that the voltage drops abruptly to zero when the battery is discharged (in reality, the voltage drops gradually, but let's keep things simple).

Battery capacity is specified in mAh, which indicates how many mA of current the battery can supply for one hour before it needs to be recharged. Suppose the phone's battery has a capacity of 2770mAh at 3.8V. For example, this battery could provide 1000mA (or $P = 1000\,\text{mA} \cdot 3.8\,\text{V} = 3.8\,\text{W}$) for $\frac{2770\text{mAh}}{1000\,\text{mA}} = 2.77$ hours before the voltage abruptly drops from 3.8V to zero.

(a) How long will the phone's full battery last assuming an average power usage of 300 mW?

Solution:

Using out power relation $P = IV$ we see that 300 mW of power at 3.8 V is about 79 mA of current. Our 2770 mA h battery can supply 79 mA for $\frac{2770\,\text{mA h}}{79\,\text{mA}} = 35\,\text{h}$, or about a day and a half.

(b) Suppose the cell phone battery is completely discharged and you want to recharge it completely. How much energy (in J) is this? Recall that a J is equivalent to a W s.

Solution:

The battery capacity is 2770 mA h at 3.8 V. Using $E = Pt = IVt = V(It)$ we see that the battery has a total stored energy of $3.8\,\text{V} \cdot 2770\,\text{mA h} = 10.5\,\text{W h} = 10.5\text{Wh} \cdot \frac{3600\,\text{s}}{1\,\text{h}} = 37.9\text{kJ}$.

(c) The battery has internal circuitry that prevents it from getting overcharged (and possibly exploding!). We will model the battery and its internal circuitry as a resistor R_{bat}. We now wish to charge the battery by plugging it into a wall plug. The wall plug can be modeled as a 5 V voltage source and 200 mΩ resistor, as pictured in Figure 2. What is the power dissipated across R_{bat} for $R_{\text{bat}} = 1\,\Omega$ (i.e. how much power is being supplied to the phone battery as it is charging) and how long will the battery take to charge?

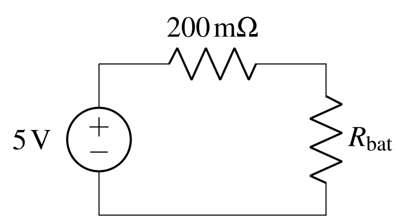


Figure 2: Model of wall plug, wire, and battery.

Solution:

As per the last part, the energy stored in the battery is 2770 mA h at 3.8 V, which is $2.77\,\text{A h} \cdot 3.8\,\text{V} = 10.5\,\text{W h}$. We can find the time to charge by dividing this energy by power dissipated across R_{bat} (in W) to get time in hours. To find the dissipated power, we first need to find the voltage across and current through R_{bat}. We can recognize this circuit as a voltage divider and so we can find the voltage across R_{bat} using our voltage divider equation:

$$V_{bat} = \frac{R_{bat}}{200\,\text{m}\Omega + R_{bat}} * 5\,\text{V} = \frac{1\,\Omega}{1.2\,\Omega} * 5\,\text{V} = 4.167\,\text{V}$$

and the current via Ohm's law

$$I_{bat} = 4.167\,\text{V}/1\,\Omega = 4.167\,\text{A}$$

. With these we can use $P = IV$ to get the dissipated power across R_{bat}.
This gives us

$$P_{bat} = 4.167\,\text{V} \cdot 4.167\,\text{A} = 17.36\,\text{W}$$

and finally the time

$$t = \frac{E}{P_{bat}} = \frac{10.5\,\text{W}\,\text{h}}{17.36\,\text{W}} = 0.6\,\text{h}$$

or about 36 min.

5. Printed electronics

Learning Goal: *This problem will help you practice thinking about electronic materials and their properties.*

All electronic devices require electrical connections to conduct signals. These connections, or traces, are manufactured through different deposition methods such as physical vapor deposition and chemical vapor deposition. Another less traditional technique is printing. Inks can be made from metallic nanoparticles and deposited using inkjet printing, screen printing, and spray coating. A commonly printed metal ink is silver.

(a) Say we screenprinted a trace of silver 20 mm in length and 5 µm in width. Given that the resistivity is 0.001 Ω mm and we measure the resistance of the trace to be 300 Ω, what is the trace thickness?

Solution:

We can rearrange the equation for resistance.

$$R = \rho \frac{L}{Wt}$$

$$t = \rho \frac{L}{WR}$$

$$t = 0.001\,\Omega\,\mathrm{mm} \frac{20\,\mathrm{mm}}{5\,\mu\mathrm{m} \cdot 300\,\Omega}$$

$$t = 13.33\,\mu\mathrm{m}$$

(b) Nanoparticle inks often require a drying step called *sintering*, during which the nanoparticles coalesce and form condcutive pathways. The manufacturer of our silver paste lists 100 °C and 175 °C as two possible sintering temperatures resulting in resistivities of 1 Ω µm and 0.5 Ω µm respectively. Assume that we need a trace 10 mm in length, 2 µm in width, and 10 µm in thickness, what is the smallest resistance trace we can obtain and with which sintering temperature?

Solution:

Similarly as in part (a), we can find the resistance using

$$R = \rho \frac{L}{Wt}$$

Since sintering only affects the resistivity, the sintering temperature with smaller resistivity will result in the smallest trace resistance. Thus we should use the 175 °C sintering temperature which yields a trace resistance of

$$R = 0.5\,\Omega\,\mu\mathrm{m} \frac{10\,\mathrm{mm}}{2\,\mu\mathrm{m} \cdot 10\,\mu\mathrm{m}} = 250\,\Omega$$

(c) Say the maximum resistance we can tolerate is 100 Ω. What would the cross sectional areas required be from both sintering temperatures to achieve the specified resistance for our 20 mm long trace?

Solution: We can rearrange the resistance equation to find the cross sectional area for the 100 °C sintering temperature:

$$A = \rho \frac{L}{R} = 1\,\Omega\,\mu\mathrm{m} \frac{20\,\mathrm{mm}}{100\,\Omega} = 200\,\mu\mathrm{m}^2$$

For the 175 °C sintering temperature:

$$A = \rho \frac{L}{R} = 0.5\,\Omega\,\mu\mathrm{m} \frac{20\,\mathrm{mm}}{100\,\Omega} = 100\,\mu\mathrm{m}^2$$

(d) Continuing with the design specifications from part (c), if our printing technique has a resolution limit of $5\,\mu m$ (meaning the minimum width and minimum length achieveable is $5\,\mu m$) and we want to aim for a trace thickness of at least $25\,\mu m$ for good film uniformity, then at which temperature should we sinter our printed silver?

Solution: We should sinter our printed silver at $100\,^\circ C$. The cross sectional area required by the higher sintering temperature ($100\,\mu m^2$) is too small for our printing technique (minimum $5\,\mu m \cdot 25\,\mu m = 125\,\mu m^2$).

(e) One unique advantage of using printing as a deposition technique is that electronic devices can be fabricated on plastic flexible substrates rather than brittle silicon wafers, allowing for applications where lightweight, conformable electronics are needed. However, when heated, plastic substrates can begin to soften and deform. Using your answers from parts (b-d) what is one drawback from the lower sintering temperature, and what is one drawback from the higher sintering temperature?

Solution: We see from part (b) that with the same trace dimensions, the lower sintering temperature will result in a higher resistance trace. From part (c) and (d) we see that if our circuit design requires a low resistance trace, the higher sintering temperature may require us to print a small feature that is not feasible by our printing technique.

(f) (**OPTIONAL**) Your manufacuting process wasn't perfect and your resulting trace increases its thickness linearly along the trace, such that the initial trace thickness is $20\,\mu m$ and the final thickness is $40\,\mu m$. Can you compute the resulting resistance of the trace? Assume the trace length is $20\,mm$, width is $4\,\mu m$, and resistivity is $1\,\Omega\mu m$.

Hint: We can write our resistance in a differential form: $dR = \rho(l)\frac{dl}{A(l)}$. Can we add up all these differential segments of resistance over the trace to get our final resistance value?

Solution: We have that our thickness is a function of the position $l = 0 \ldots 20\,mm$:

$$t(l) = 20\,\mu m + \frac{l}{20\,mm} \cdot 20\,\mu m = 20\,\mu m + \frac{l}{1000}$$

.

Therefore, our area is also a function of the position l:

$$A(l) = W * t(l) = W \cdot (20\,\mu m + \frac{l}{1000})$$

So, we get that our resistance differential is:

$$dR = \frac{\rho dl}{W \cdot (20\,\mu m + \frac{l}{1000})}$$

Integrating we obtain:

$$R = \int_0^{20\,mm} \frac{\rho dl}{W \cdot (20\,\mu m + \frac{l}{1000})}$$

$$R = \frac{\rho}{W} \int_0^{20\,mm} \frac{dl}{20\,\mu m + \frac{l}{1000}}$$

$$R = \frac{1\,\Omega\mu m}{4\,\mu m} \cdot 1000 \cdot \ln\left(\frac{20\,\mu m + \frac{20\,mm}{1000}}{20\,\mu m}\right)$$

$$R = 173.29\,\Omega$$

6. Resistive Touchscreen

Learning Goal: *The objective of this problem is to provide insight into modeling of resistive elements. This will also help to apply the concepts from resistive touchscreen.*

In this problem, we will investigate how a resistive touchscreen with a defined thickness, width, and length can actually be modeled as a series combination of resistors. As we know the value of a resistor depends on its length.

Figure 3 shows the top view of a resistive touchscreen consisting of a conductive layer with resistivity ρ_1, thickness t, width W, and length L. At the top and bottom it is connected through perfect conductors ($\rho = 0$) to the rest of the circuit. The touchscreen is wired to voltage source V_s.

Use the following numerical values in your calculations: $W = 50$ mm, $L = 80$ mm, $t = 1$ mm, $\rho_1 = 2\,\Omega$m, $V_s = 5$V, $x_1 = 20$ mm, $x_2 = 45$ mm, $y_1 = 30$ mm, $y_2 = 60$ mm.

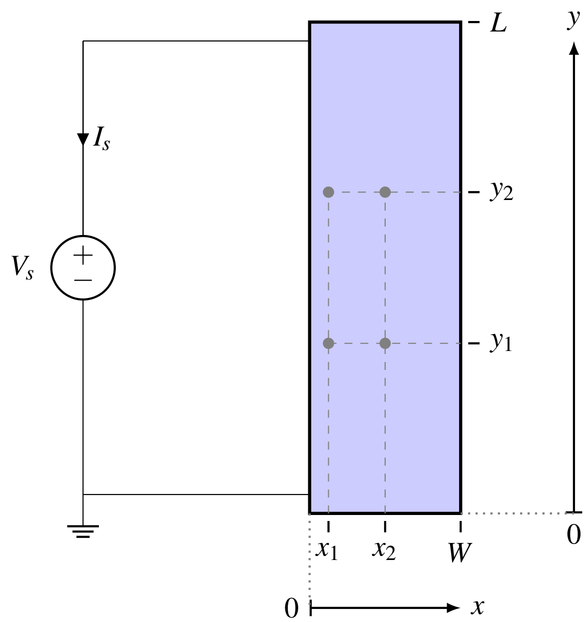
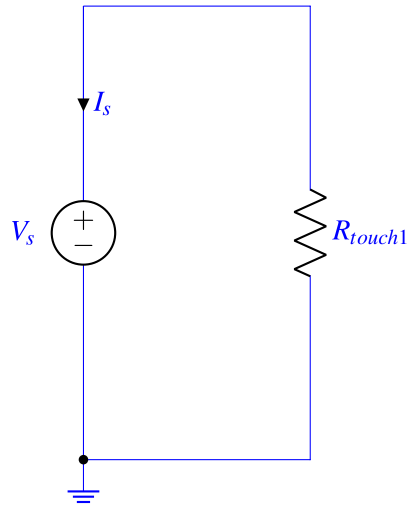


Figure 3: Top view of resistive touchscreen (not to scale). z axis i.e. the thickness not shown (into the page).

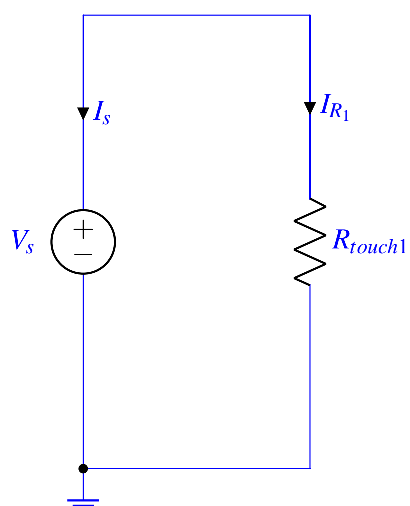
(a) Draw a circuit diagram representing **Figure 3**, where the entire touchscreen is represented as *a single resistor*. **Note that no touch is occurring in this scenario.** Remember that circuit diagrams in general consist of only circuit elements (resistors, sources, etc) represented by symbols, connecting wires, and the reference/ ground symbol. Calculate the value of current I_S based on the circuit diagram you drew. *Do not forget to specify the correct unit as always, and double check the direction of I_s!*

Solution:



The touchscreen resistance can be found from the following expression:

$$\begin{aligned} R_{touch1} &= \rho_1 \cdot \frac{L}{A} \\ &= \rho_1 \cdot \frac{L}{W \cdot t} \\ &= 2\,\Omega\,\mathrm{m}\left(\frac{80\,\mathrm{mm}}{50\,\mathrm{mm} \cdot 1\,\mathrm{mm}}\right) \\ R_{touch1} &= 3200\,\Omega = 3.2\,\mathrm{k}\Omega \end{aligned}$$



From KCL, we can write:

$$I_s + I_{R_1} = 0 \tag{7}$$
$$I_s = -I_{R_1} \tag{8}$$

Therefore, the current I_{R_1} is equal to:

$$I_{R_1} = \frac{V_s}{R_{touch1}} = \frac{5}{3200}A = 1.56mA$$

And the current I_s is equal to:

$$I_s = -I_{R_1} = -1.56mA$$

(b) Let us assume u_{12} is the node voltage at the node represented by coordinates (x_1, y_2) of the touchscreen, as shown in **Figure 4**. What is the value of u_{12}? You should first draw a circuit diagram representing Figure 4, which includes node u_{12}. Specify all resistance values in the diagram. Does the value of u_{12} change based on the value of the x-coordinate x_1?

Hint: You will need more than one resistor to represent this scenario.

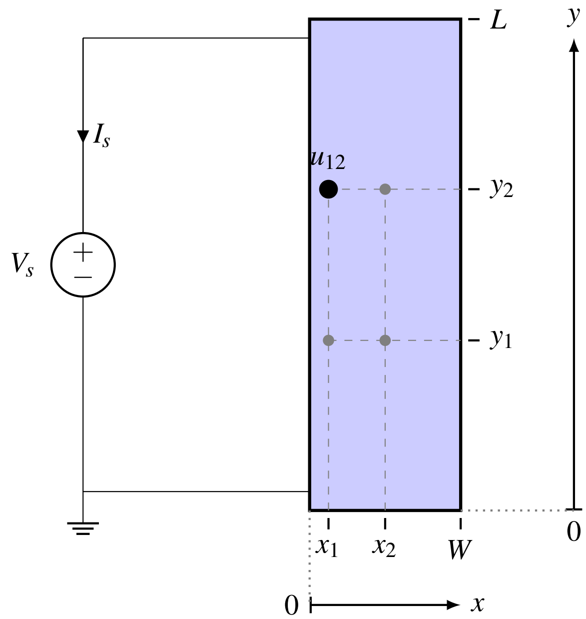
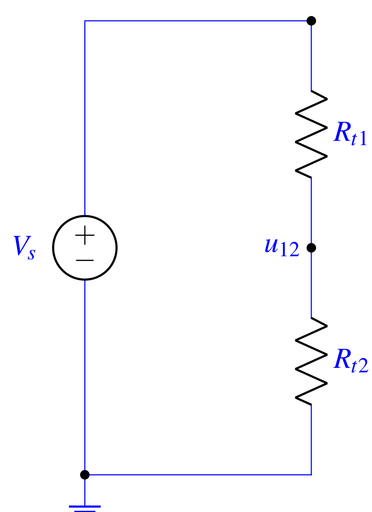


Figure 4: Top view of resistive touchscreen showing node u_{12}.

Solution:

We can represent this setup with the circuit shown below.



Using voltage division, u_{12} can be found from the following expression:

$$u_{12} = V_s \frac{R_{t2}}{R_{t1} + R_{t2}}$$

We know $R_{t1} = \rho_1 \cdot \frac{L-y_2}{W \cdot t}$ and $R_{t2} = \rho_1 \cdot \frac{y_2}{W \cdot t}$. We also know the $\frac{\rho_1}{W \cdot t}$ is common to both R_{t1} and R_{t2}, so those terms will cancel out when we them in.

$$u_{12} = V_s \frac{y_2}{L - y_2 + y_2}$$

$$u_{12} = V_s \frac{y_2}{L}$$

$$u_{12} = 5V \cdot \frac{60\,\text{mm}}{80\,\text{mm}} = 3.75V$$

The value of u_{12} would not change based on the value of the x coordinate, because in our setup the current is flowing from the top to the bottom of the screen. This means that voltage is only dissipated in the y direction, so we can only measure the difference in the y coordinate. We would need another closed circuit where current could flow along the width W to determine where the finger touched in the x direction.

(c) Assume V_{ab} is the voltage measured between the nodes represented by touchscreen coordinates (x_1, y_1) and coordinates (x_1, y_2), as shown in **Figure 5**. Calculate the absolute value of V_{ab}. As with the previous part, you should first draw the circuit diagram representing Figure 5, which includes V_{ab}. Calculate all resistor values in the circuit. *Hint: Try representing the segment of the touchscreen between these two coordinates as a separate resistor itself.*

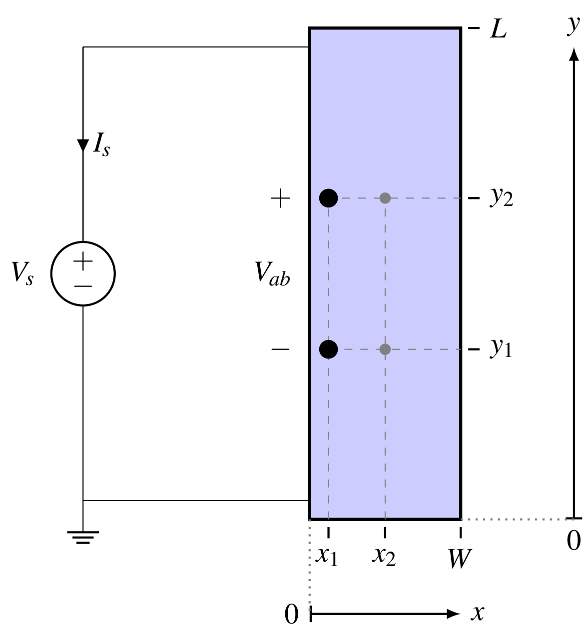
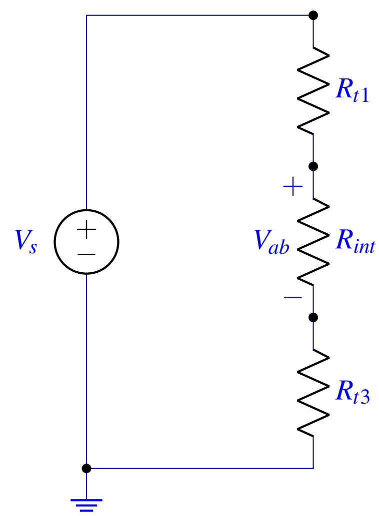
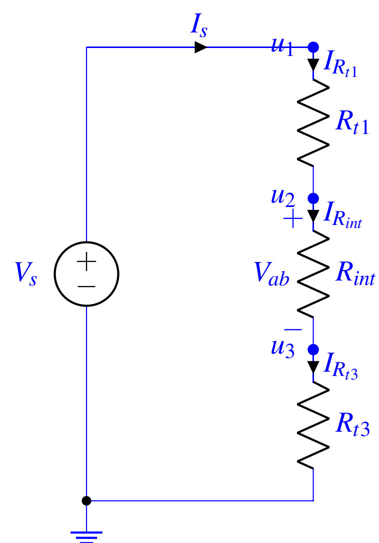


Figure 5: Top view of resistive touchscreen showing voltage V_{ab}.

Solution:



We can use node voltage analysis to find V_{ab}.



Using KCL at the three labelled nodes:

$$I_s = I_{R_{t1}}$$

$$I_{R_{t1}} = I_{R_{t3}}$$

$$I_{R_{t3}} = I_{R_{int}}$$

We see that there is only one current, I_s, going through all resistor elements. Writing the IV relationships for each element:

$$u_1 - u_2 = I_s R_{t1}$$

$$u_2 - u_3 = I_s R_{int}$$

$$u_3 = I_s R_{t3}$$

Knowing that $V_{ab} = u_2 - u_3$, we can write:

$$V_{ab} = u_2 - u_3 = I_s R_{int}$$

Now we just need to find I_s. Looking at the IV relationship equations and using back substitution, we can write:

$$u_1 = V_s = I_{R_{t1}}R_{t1} + I_{R_{int}}R_{int} + I_{R_{t3}}R_{t3}$$
$$I_s = \frac{V_s}{R_{t1} + R_{int} + R_{t3}}$$

Finally, we get:

$$V_{ab} = V_s \frac{R_{int}}{R_{t1} + R_{int} + R_{t3}}$$

Each of the resistances can be calculated as $R_{t1} = \rho_1 \cdot \frac{L-y_2}{W \cdot t}$, $R_{int} = \rho_1 \cdot \frac{y_2-y_1}{W \cdot t}$ and $R_{t3} = \rho_1 \cdot \frac{y_1}{W \cdot t}$. This gives for V_{ab}:

$$V_{ab} = \frac{y_2 - y_1}{L} V_s = \frac{3}{8} \cdot 5V = 1.875V$$

(d) Calculate (the absolute value of) the voltage between the nodes represented by touchscreen coordinates (x_1, y_1) and coordinates (x_2, y_1) in **Figure 5**.

Solution:

The two points have the same y coordinate, therefore they have the same potential in our touchscreen model. Again, this is because the current is flowing from the top to the bottom of the screen, so the x position does not make a difference. Recall that the touchscreen is effectively being modeled as a single vertical resistor which can be considered as several resistors of varying lengths, all totaling to L. Hence, we do not consider the effect of the x-coordinate on potential – we just need to consider the difference in the y-coordinate between two points. Thus, $\Delta V = 0$.

(e) Calculate (the absolute value of) the voltage between the nodes represented by touchscreen coordinates (x_1, y_1) and coordinates (x_2, y_2) in **Figure 5**.

Solution:

The two points have different x and y coordinates. However, the potential is the same across the x-axis for a fixed y coordinate, as was explained in the solution for part (d). Therefore, the problem is similar to part (c), since the potential is only determined by the y-coordinate of a point. Using the same equivalent circuit in part (c) we have:

$$\Delta V = V_s \frac{R_{int}}{R_{t1} + R_{int} + R_{t3}} = 1.875V$$

(f) **Figure 6** shows a new arrangement with two touchscreens. The two touchscreens are next to each other and are connected to the voltage source in the same way. The second touchscreen (the one on the right) is identical to the one shown in Figure 3, except for different width, W_2, and resistivity, ρ_2.

Use the following numerical values in your calculations: $W_1 = 50$ mm, $L = 80$ mm, $t = 1$ mm, $\rho_1 = 2\,\Omega$m, $V_s = 5$V, $x_1 = 20$ mm, $x_2 = 45$ mm, $y_1 = 30$ mm, $y_2 = 60$ mm, which are the same values as before. The new touchscreen has the following numerical values which are different: $W_2 = 85$ mm, $\rho_2 = 1.5\,\Omega$m.

Draw a circuit diagram representing **Figure 6**, where the two touchscreens are represented as *two separate resistors*. **Note that no touch is occurring in this scenario.**

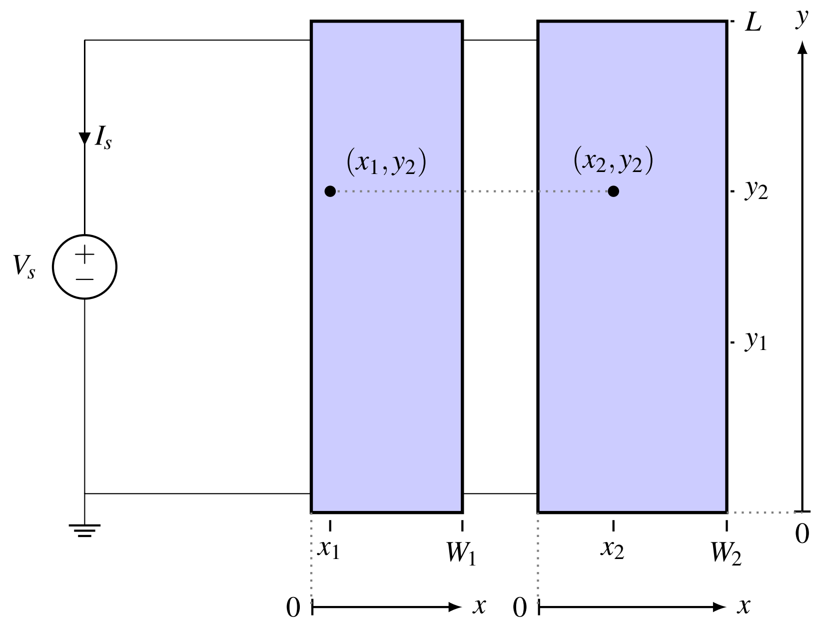
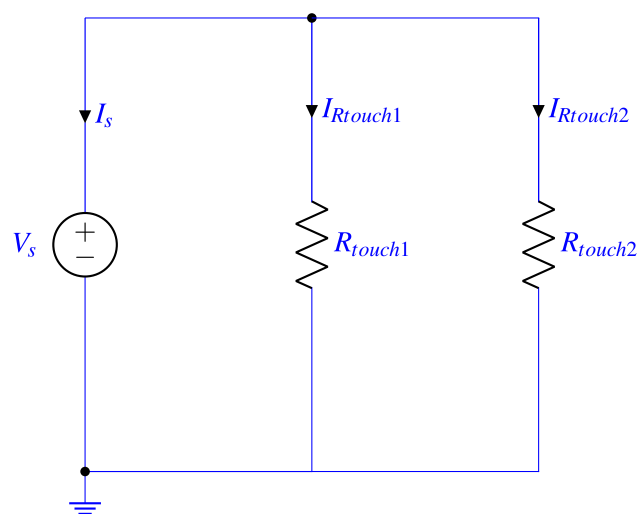


Figure 6: Top view of two touchscreens wired in parallel (not to scale). z axis not shown (into the page).

Solution:



(g) Calculate the value of current I_s for the two touchscreen arrangement based on the circuit diagram you drew in the last part.

Solution:

From KCL, we can write:

$$I_s + I_{Rtouch1} + I_{Rtouch2} = 0 \tag{9}$$

$$I_s = -(I_{Rtouch1} + I_{Rtouch2}) \tag{10}$$

Using Ohm's Law for each element:

$$I_s = -\left(\frac{V_s}{R_{touch1}} + \frac{V_s}{R_{touch2}}\right)$$

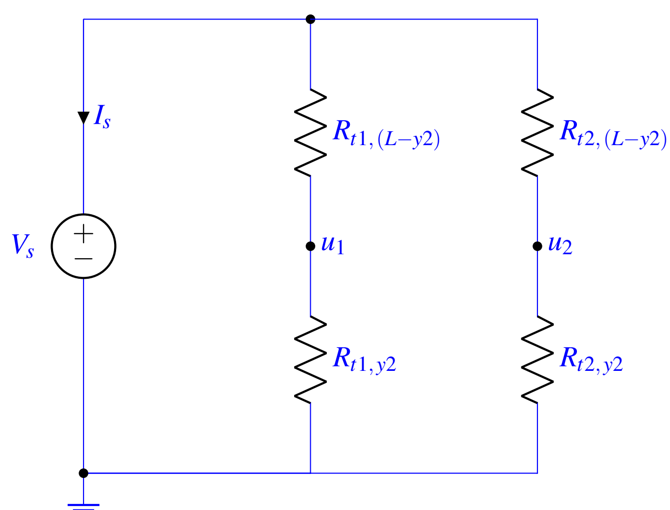
However, the resistance of the second touchscreen can be given by:

$$R_{touch2} = \rho_2 \cdot \frac{L}{W_2 \cdot t} = 1.5\,\Omega\,\mathrm{m}\left(\frac{80\,\mathrm{mm}}{85\,\mathrm{mm} \cdot 1\,\mathrm{mm}}\right)$$
$$R_{touch2} = 1411.8\,\Omega$$

Therefore, using the resistance values for the first and second touchscreen and the applied voltage source, we have:

$$I_s \approx -(1.563\mathrm{mA} + 3.542\mathrm{mA}) = -5.1\mathrm{mA}$$

(h) Consider the two points: (x_1, y_2) in the touchscreen on the left, and (x_2, y_2) in the touchscreen on the right in **Figure 6**. Show that the node voltage at (x_1, y_2) is the same that at (x_2, y_2), i.e. the potential difference between the two points is 0. You can show this without explicitly calculating the node voltages at the two points.
If you were to connect a wire between the two coordinates (x_1, y_2) in the touchscreen on the left, and (x_2, y_2) in the touchscreen on the right, would any current flow through this wire?
Solution:
It will be helpful to first consider the circuit representation of this scenario to understand why the node voltages at the two points on each of the touch screens should be same (and therefore that the potential difference between points (x_1, y_2) and (x_2, y_2) is 0).



Without calculating the node voltages, note that the ratio of the value of $R_{t1,(L-y_2)}$ to $R_{t1,y2}$ is the same as the ratio of the value of $R_{t2,(L-y2)}$ to $R_{t2,y2}$:

$$\frac{R_{t1,y2}}{R_{t1,(L-y2)}} = \frac{\rho_1(y_2)/(W_1 \cdot t)}{\rho_1(L-y_2)/(W_1 \cdot t)} = \frac{y_2}{L-y_2}$$
$$\frac{R_{t2,y2}}{R_{t2,(L-y2)}} = \frac{\rho_2(y_2)/(W_2 \cdot t)}{\rho_2(L-y_2)/(W_2 \cdot t)} = \frac{y_2}{L-y_2}$$

Also note that the ratio of the resistors used in the voltage divider equations can be written as:

$$\frac{R_{t1,y2}}{R_{t1,(L-y2)}+R_{t1,y2}} = \frac{\rho_1(y_2)/(W_1\cdot t)}{\rho_1(L-y_2)/(W_1\cdot t)+\rho_1(y_2)/(W_1\cdot t)} = \frac{y_2}{L}$$
$$\frac{R_{t2,y2}}{R_{t2,(L-y2)}+R_{t2,y2}} = \frac{\rho_2(y_2)/(W_2\cdot t)}{\rho_2(L-y_2)/(W_2\cdot t)+\rho_2(y_2)/(W_2\cdot t)} = \frac{y_2}{L}$$

Beacuse the voltage across the entirety of each of the individual touchscreens is the same: it is $V_s - 0$ or just V_s. The voltage V_s is therefore *divided* between $R_{t1,(L-y2)}$ and $R_{t1,y2}$ exactly the same as it is divided between $R_{t_2,(L-y2)}$ and $R_{t_2,y2}$ because of the ratio argument presented above.

Therefore, the potential difference between u_1 and u_2 will be 0, so long as the y-coordinate value is chosen to be the same.

This also means that there is no current flowing through the wire, since the points in the two touchscreens have the same potential. Therefore,

$$I_{12} = \frac{u_1-u_2}{R_{wire}} = 0$$

We should note that what causes current to flow is a voltage difference. Here, we have a voltage difference of zero, so the current flowing through the wire is zero.

7. Prelab Questions

These questions pertain to the Pre-Lab reading for the Touch 2 lab. You can find the reading under the Touch 2 Lab section on the 'Schedule' page of the website.

(a) How many layers are there in the resistive touchscreen and what are they made of?

(b) Provide 2 examples of resistive touchscreens (give one example not listed on the pre-lab reading).

(c) In the circuit given in the reading, what is the current i_3 flowing through resistor R_{h1}?

(d) How do we get touch coordinates in the horizontal direction if you have your circuit that works in the vertical direction?

Solution:

(a) The resistive touchscreen consists of two different layers - a flexible resistive layer on the top and a resistor circuit layer on the bottom.

(b) From the reading: old Nokias, Nintendo DS & Gameboy. Not from the reading: GPS displays, Printers, Airplane Entertainment Screens, Screens that require the use of a stylus.

(c) $0A$. There is no potential difference across the resistor R_{h1} and thus, no current flows through it.

(d) Rotate the orientation of the circuit by 90 degrees.

8. Homework Process and Study Group

Who did you work with on this homework? List names and student ID's. (In case you met people at homework party or in office hours, you can also just describe the group.) How did you work on this homework? If you worked in your study group, explain what role each student played for the meetings this week.

Solution:

I first worked by myself for 2 hours, but got stuck on problem 5. Then I met with my study group.

XYZ played the role of facilitator ... etc. We were still stuck on problem 5 so we went to office hours to talk about the problem.

Then I went to homework party for a few hours, where I finished the homework.