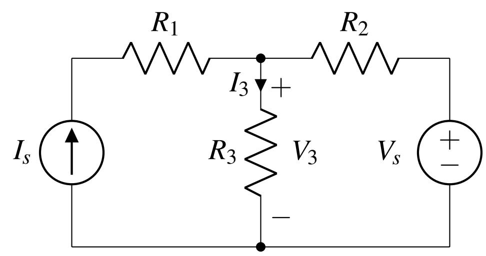
EECS 16A Designing Information Devices and Systems I
Spring 2023 Discussion 13A

1. Superposition

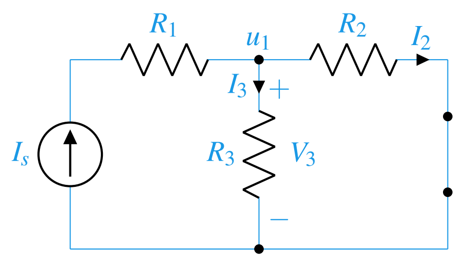
Consider the following circuit:



(a) With the current source turned on and the voltage source turned off, find the current I_3.

Answer:

The voltage source turned off ($V_s = 0$) is equivalent to a short circuit.



Write a KCL equation at node u_1 and apply Ohm's Law to R_2 and R_3

$$I_s - I_3 - I_2 = 0$$
$$I_s - \frac{u_1}{R_3} - \frac{u_1}{R_2} = 0$$

which simplifies to

$$u_1 = \left(\frac{1}{R_2} + \frac{1}{R_3}\right)^{-1} \cdot I_s$$
$$= (R_2 || R_3) \cdot I_s$$

Finally, the current I_3 through resistor R_3 is found using Ohm's Law as

$$I_3\Big|_{V_s=0} = \frac{u_1}{R_3} = \frac{R_2||R_3}{R_3} \cdot I_s = \frac{R_2}{R_2 + R_3} I_s$$

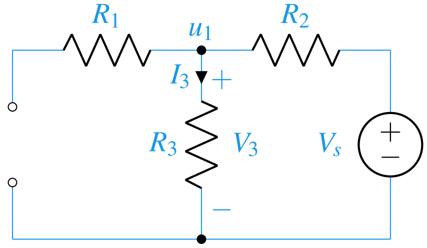
Alternatively, we note that I_s is split between R_2 and R_3, and therefore we can use the current divider relationship:

$$I_3\Big|_{V_s=0} = \frac{R_2}{R_2 + R_3} I_s$$

(b) With the voltage source turned on and the current source turned off, find the voltage drop V_3 across R_3.

Answer:

We note that when the current source is turned off, it becomes an open circuit.



There is no current flowing through resistor R_1, thus we are left with a voltage divider.

$$V_3\Big|_{I_s=0} = \frac{R_3}{R_2+R_3}V_s$$

(c) Find the power dissipated by R_3.

Answer:

To compute the power dissipated/generated by the resistor R_3 (or any circuit element), we need to find the voltage across and current through it. However, we have only partially determined these quantities. First find the missing quantities by completing the process of superposition. The voltage drop across R_3 when only the current source on is

$$V_3\Big|_{V_s=0} = I_3\Big|_{V_s=0} \cdot R_3 = \frac{I_sR_2R_3}{R_2+R_3}$$

and the current through R_3 when only the voltage source on is given by:

$$I_3\Big|_{I_s=0} = \frac{V_3\Big|_{I_s=0}}{R_3} = \frac{V_s}{R_2+R_3}$$

Then we can compute the full voltage and current using superposition

$$V_3 = V_3\Big|_{I_s=0} + V_3\Big|_{V_s=0} = \frac{V_sR_3+I_sR_2R_3}{R_2+R_3}$$

$$I_3 = I_3\Big|_{I_s=0} + I_3\Big|_{V_s=0} = \frac{I_sR_2+V_s}{R_2+R_3}$$

The dissipated power is finally calculated as

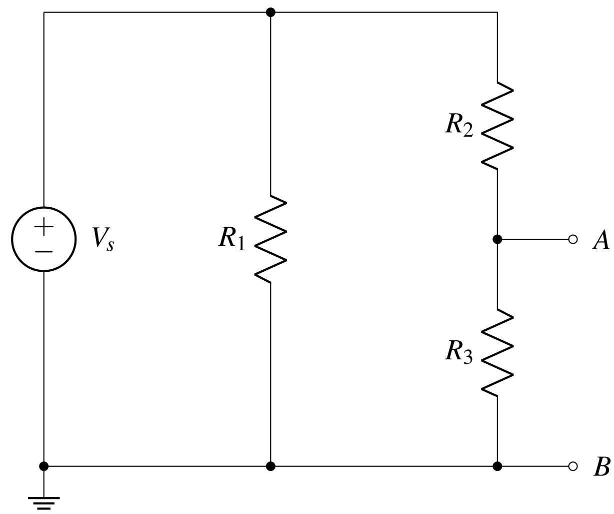
$$P_3 = I_3V_3 = \left(\frac{I_sR_2+V_s}{R_2+R_3}\right)\cdot\left(\frac{V_sR_3+I_sR_2R_3}{R_2+R_3}\right) = \frac{R_3(I_sR_2+V_s)^2}{(R_2+R_3)^2}$$

Note that the total dissipated power cannot be found by summing the dissipated power in each superposition sub-circuit. This is because the formula for power $P = I\cdot V$ does not satisfy properties of linearity with respect to both voltage and current.

$$\begin{aligned} P_3 &\neq P_3\Big|_{I_s=0} + P_3\Big|_{V_s=0} \\ &\neq I_3\Big|_{I_s=0}\cdot V_3\Big|_{I_s=0} + I_3\Big|_{V_s=0}\cdot V_3\Big|_{V_s=0} \end{aligned}$$

2. Thévenin/Norton Equivalence

(a) Find the Thévenin resistance R_{th} of the circuit shown below, with respect to its terminals A and B.

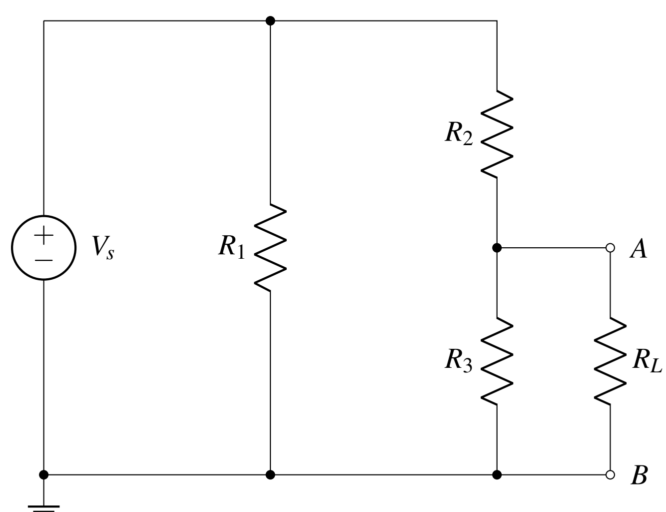


Answer: To find the Thévenin resistance, we null out the voltage source (which shorts out R_1) and find the equivalent resistance, which is:

$$R_{th} = R_2 \parallel R_3$$

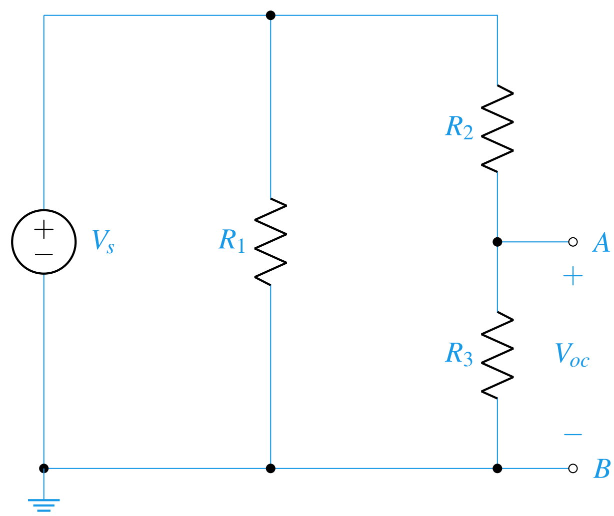
since resistors R_2 and R_3 are in parallel.

(b) Now a load resistor, R_L, is connected across terminals A and B, as shown in the circuit below. Using Thévenin equivalence, find the power dissipated in the load resistor in terms of the given variables.



Answer:

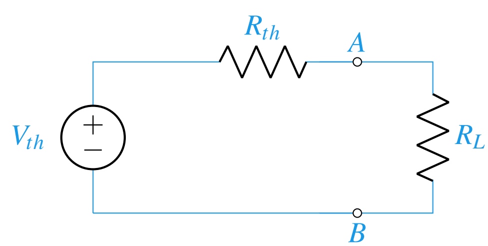
To help simplify the analysis, we replace the circuit with its Thévenin equivalent circuit. In order to do so, we first need to find the Thévenin voltage, V_{th}. One way to determine V_{th} is to find the open circuit voltage, $V_{AB} = V_{oc}$, in the original circuit when an open circuit is connected externally to terminals A and B.



The open circuit can be derived from a voltage divider:

$$V_{th} = V_{oc} = \frac{R_3}{R_2 + R_3} V_s$$

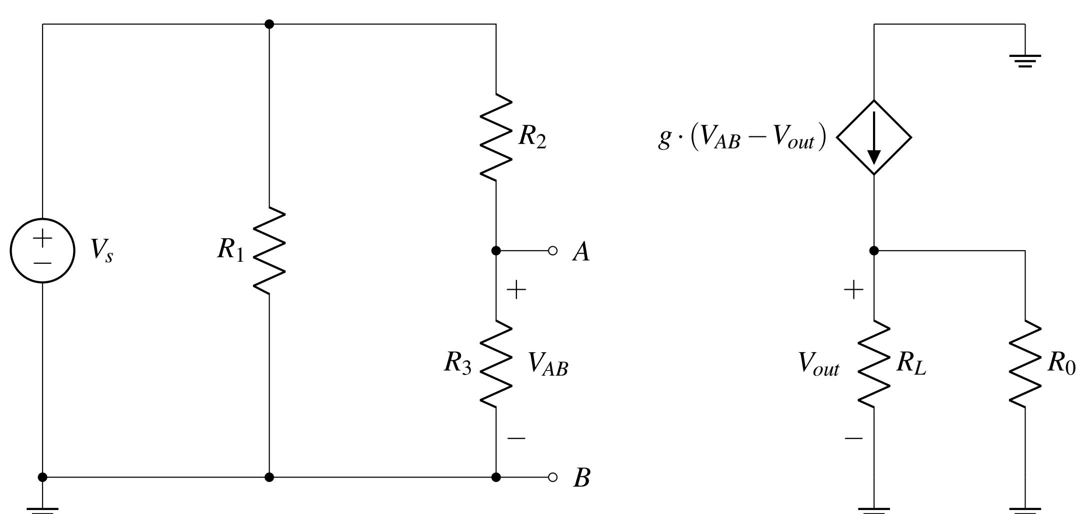
We also already know the Thévenin resistance $R_{th} = R_2 \parallel R_3$ from part (a).
Thus, the circuit can be simplified to:



The power through the load resistor is then given by:

$$P_{R_L} = V_{R_L} \cdot I_{R_L} = I_{R_L}^2 \cdot R_L = \left(\frac{V_{th}}{R_L + R_{th}}\right)^2 R_L = \left(\frac{R_3}{R_2 + R_3} V_s \cdot \frac{1}{R_L + R_2 \parallel R_3}\right)^2 R_L$$

(c) We modify the circuit as shown below, where g is a known constant:



Find a symbolic expression for V_{out} as a function of V_s.

Hint: Redraw the left part of the circuit using its Thévenin equivalent.

Answer:

We note that $V_{AB} = V_{th} = \frac{R_3}{R_2+R_3}V_s$ from part (b). Then, since R_0 and R_L are in parallel, we have that $V_{out} = I \cdot (R_0 \parallel R_L) = g \cdot (V_{th} - V_{out})(R_0 \parallel R_L)$. Solving for V_{out}, we get:

$$V_{out} = \frac{R_2}{R_2+R_3} V_s \frac{g \cdot R_L \parallel R_0}{1 + g \cdot R_L \parallel R_0}$$

What happens to the voltage V_{out} as the gain of the dependent current source, g, approaches infinity? The output voltage V_{out} simplifies to

$$V_{out} = \frac{R_2}{R_2+R_3} V_s$$

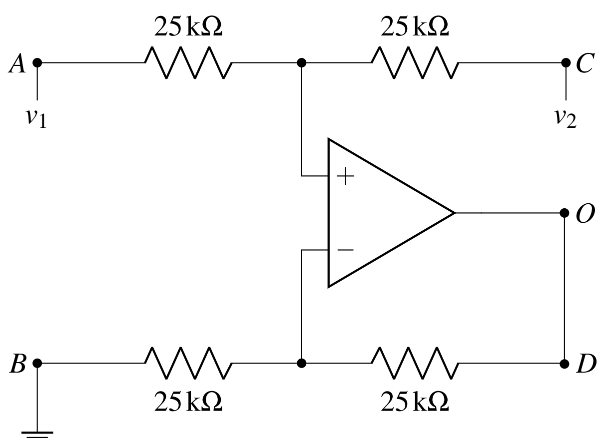
This circuit is in negative feedback! And it effectively allows V_{out} to be independent on the load resistor R_L.

Although it might not look like it, the op-amp is configured as a unity gain buffer.

3. A Versatile Opamp Circuit

For each circuit configuration, determine the voltage at O, given that v_1 and v_2 are voltage sources. All circuit configurations are in negative feedback.

(a) Configuration 1:



Answer:

By recognizing a voltage summer (or by using superposition), we note that the voltage at v^+ is given by:

$$v^+ = \frac{25\,\text{k}\Omega}{25\,\text{k}\Omega + 25\,\text{k}\Omega} v_1 + \frac{25\,\text{k}\Omega}{25\,\text{k}\Omega + 25\,\text{k}\Omega} v_2 = \frac{v_1 + v_2}{2}$$

Method 1: Voltage Divider

The circuit connected to the negative op-amp terminal v^- also forms a voltage divider since the input current $I^- = 0$, thus

$$v^- = \frac{25\,\text{k}\Omega}{25\,\text{k}\Omega + 25\,\text{k}\Omega} v_O = \frac{v_O}{2}$$

Since the circuit is in negative feedback, we can apply the second Golden Rule: $v^+ = v^-$. Thus,

$$v^- = v^+$$
$$\frac{v_O}{2} = \frac{v_1 + v_2}{2}$$
$$v_O = v_1 + v_2$$

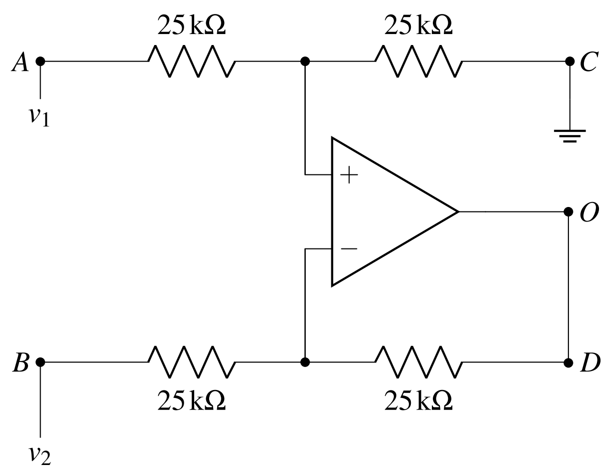
Method 2: Non-Inverting Amplifier

Alternatively, the rest of the circuit looks like a non-inverting amplifier with a gain of $1 + \frac{25\,\mathrm{k}\Omega}{25\,\mathrm{k}\Omega} = 2$. Therefore, the output voltage is:

$$v_O = 2v^+ = v_1 + v_2$$

This circuit configuration takes the sum of v_1 and v_2.

(b) Configuration 2:



Answer:

By recognizing a voltage divider and applying the formula, we note that the voltage at the positive input terminal v^+ of the op-amp is given by

$$v^+ = \frac{25\,\mathrm{k}\Omega}{25\,\mathrm{k}\Omega + 25\,\mathrm{k}\Omega} v_1 = \frac{v_1}{2}$$

By applying the Golden Rule: $I^- = 0$, we can write a KCL equation at the negative input terminal v^-

$$\frac{v^- - v_2}{25\,\mathrm{k}\Omega} + \frac{v^- - v_O}{25\,\mathrm{k}\Omega} + 0 = 0$$

Solving for v_O yields

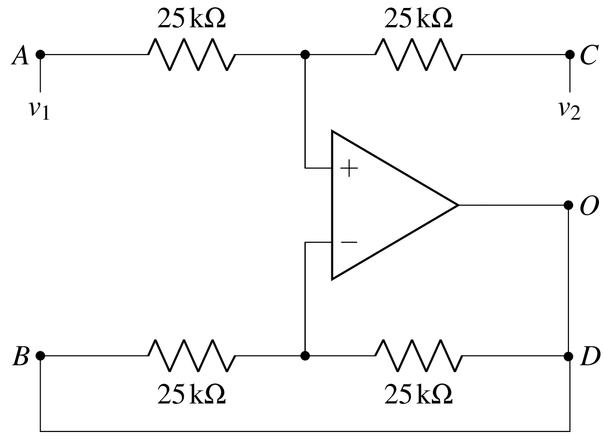
$$v_O = 2v^- - v_2$$

Since the op-amp is in negative feedback, we can apply the second Golden Rule $v^+ = v^-$. Thus

$$v_O = 2v^- - v_2 = 2v^+ - v_2 = 2\left(\frac{v_1}{2}\right) - v_2$$
$$v_O = v_1 - v_2$$

This circuit configuration takes the difference of v_1 and v_2.

(c) Configuration 3:



Answer:

Like Configuration 1 in part (a), we note that the circuit connected to node v^+ is a voltage summer, thus

$$v^+ = \frac{v_1 + v_2}{2}$$

We note that the resistors connected to B and D do not affect the circuit as no current is flowing through them. Therefore,

$$v_O = v^- = v^+ = \frac{v_1 + v_2}{2}$$