

20.1 Design Procedure

Now that we've analyzed many circuits, we are ready to focus on *designing* interesting circuits to perform specific tasks. Circuit design is generally more challenging than circuit analysis. Usually, we are given a prompt about what type of circuit is desired and some specifications, and then we must decide what components to use and how to connect them. Circuit design problems are very similar to proof problems, which we looked at earlier in this course: Both cases are open ended, and we frequently have to integrate many ideas and explore several different possibilities to reach a solution.

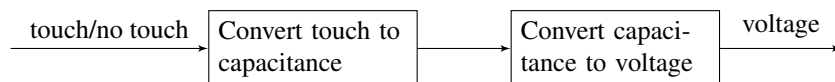
When faced with a design problem, a good place to start is to follow the *design procedure* outlined here:

Step 1 (Specification): Concretely restate the goals for the design.

Frequently, a design prompt will include a lot of text, so we'd like to restate all of the most important features of our design. We'll refer to these specifications later to determine if our design is complete.

Step 2 (Strategy): Describe your strategy (often in the form of a block diagram) to achieve your goal.

To do this, start by thinking about what you can measure vs. what you want to know. For example in our capacitive touchscreen, we want to know if there is a touch and we can measure voltage. Since we know that a touch can change the capacitance, we break this down into the following block diagram:



Step 3 (Implementation): Implement the components described in your strategy.

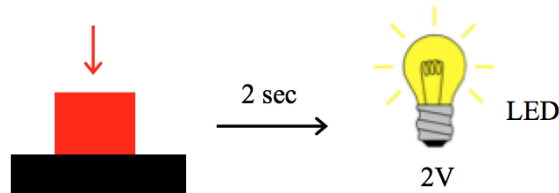
This is where pattern matching is useful: remind yourself of blocks you know, (ex. voltage divider, inverting amplifier) and check if any of these can be used to implement steps of your strategy. If you don't know of a block that does what you want, think about how to modify or extend the blocks you know.

Step 4 (Verification): Check that your design from Step 3 does what you specified in Step 1.

It's tempting to think that you're done after implementation, but verification is critical! In particular, check block-to-block connections, as these are the most common point for problems. Does one block load another block causing it to behave differently than expected? Are there any contradictions (ex. a voltage source with both ends connected by a wire, or a current source directed into an open circuit)? Repeat previous steps if necessary to make sure that your final circuit meets the specifications.

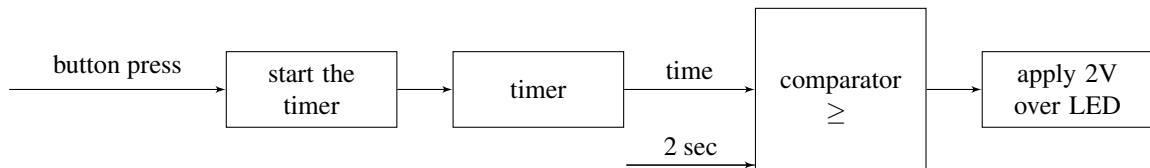
20.2 Design Example: Countdown Timer

Let's practice the design procedure with an example: Your boss comes to you and asks you to build a countdown timer that will turn on an LED 2 sec after a button is pressed. She tells you that the LED turns on when 2V is applied across it.



Step 1 (Specification): Build a circuit that, after a button is pressed, measures 2 seconds and then applies two volts across an LED.

Step 2 (Strategy): Let's determine a strategy to tackle this problem: When the button is pressed we'd like to turn on a timer. Then we'll need to know if the time elapsed has reached 2 sec, which we can determine with a comparator. When our comparator tells us that the time has exceeded 2 sec, we want to apply 2 V across the LED. This is summarized in the following block diagram:



Note that "time" will need to be represented as either a current or voltage, since those are the only circuit quantities we can control. Therefore, we will also have to determine what current/voltage is equivalent to 2 sec in our representation.

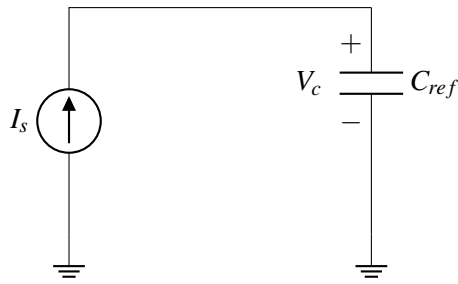
Step 3 (Implementation): Now we'd like to implement each of the blocks above. To do this, we start by considering which component(s) could be used to build each block.

The first block we need to build is to "start the timer." What component can change based on a user input? A switch — we will assume we have a switch that either opens or closes when the button is pressed.

The next block is the timer itself. To build a timer, we need a component that changes either voltage or current as a function of time. A resistor won't work because its IV relationship is static in time. But a capacitor is a good candidate since its IV relationship is time-varying. We know that for a capacitor:

$$I_c = C \frac{dV_c}{dt} \quad (1)$$

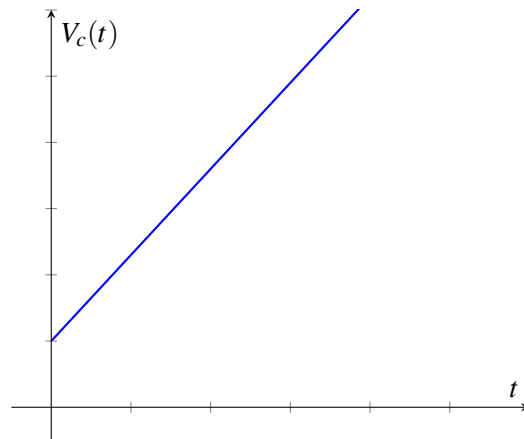
This relationship is easiest to work with when the current is constant, so let's apply a constant current source over the capacitor:



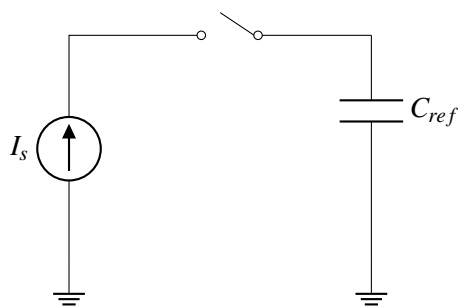
By integrating both sides of Eq. (1) (or referring to the lecture notes on capacitors) we can derive that

$$V_c(t) = \frac{I_s}{C_{ref}}t + V_c(0) \quad (2)$$

where the voltage is linearly increasing with time.

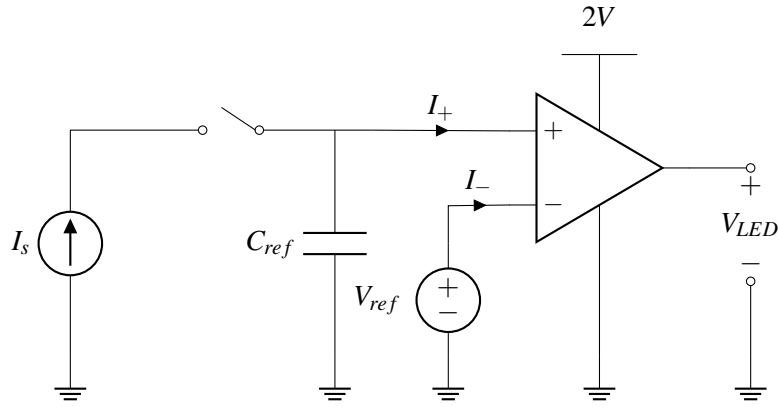


Let's combine our first two blocks: the switch and the timer:



When the switch closes at $t = 0$, we start charging the capacitor. The voltage over the capacitor will indicate how much time has elapsed since the switch closed (i.e. since the button was pressed).

Now let's look at the third block of our block diagram, the comparator. As the name indicates, a good circuit element for this block is an op-amp used as a comparator (meaning, the op-amp is not in negative feedback). Based on our block diagram, we'd like one comparator input to be the timer output, and the other input should be a static voltage representing 2 sec.



We choose V_{ref} to be set equal to the voltage on the capacitor after charging for 2s (assuming the capacitor is initially uncharged):

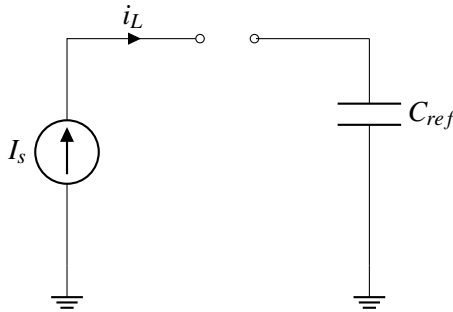
$$V_{ref} = \frac{2I_s}{C_{ref}} \quad (3)$$

If the voltage across C_{ref} becomes higher than V_{ref} , the comparator outputs V_{DD} . From our problem specification, we know that we want to apply 2V over the LED to turn it on, so we set $V_{DD} = 2V$.

Our final block from the diagram is to apply 2V over the LED. We do this by connecting one terminal of the LED to ground and the other to the op-amp output.

Step 4 (Verification): Now, that we have a complete design, we must verify that it works as desired. Since we designed each block separately, there may be mistakes when they are connected.

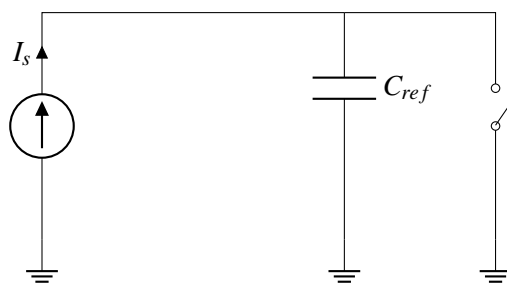
Let's consider what happens before the button is pressed. The left hand side of the circuit looks like this:



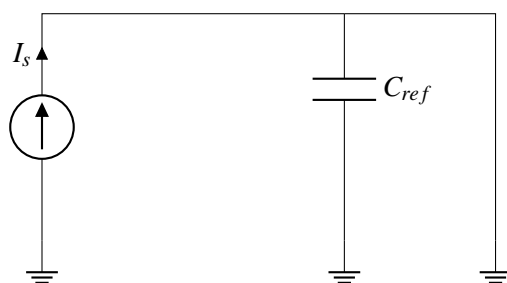
Uh oh – this is a contradiction! To see why, we apply KCL at the upper left node:

$$I_s = I_L \quad (4)$$

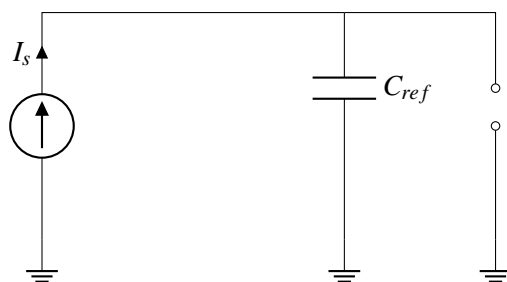
I_s is the constant current supplied by the current source, I_L is the current flow into the switch. Before the button is pressed, I_L must equal 0 since there is an open circuit. However, the current source guarantees that I_s is nonzero. These two equations cannot simultaneously be true, leading to a mathematical contradiction. In real life, at least one of the elements will not behave as expected, possibly causing elements to break. Therefore, we'd like to think of a new circuit that achieves our goals without any mathematical contradictions. To do this we can add another switch in the circuit:



Before the button is pressed, the switch is closed and can be replaced by a wire. In this case, the current from the current source can flow through this wire to ground, eliminating the prior contradiction. In addition, when the switch is closed, it connects the top of the capacitor to ground, so the voltage across the capacitor remains constant at 0 V.

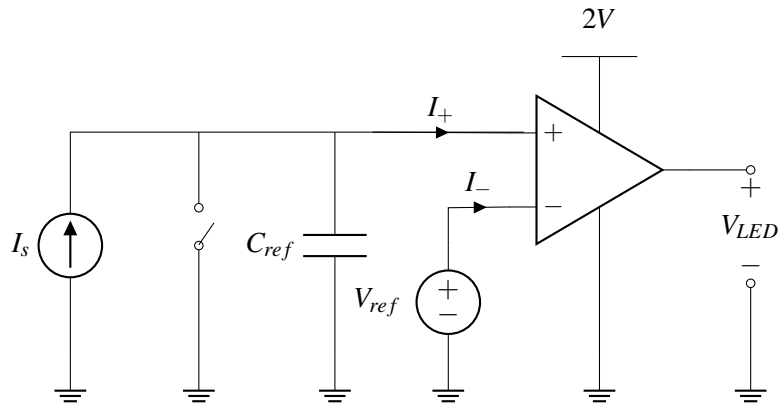


After the button is pressed, the switch is off and can be replaced by an open circuit, and everything functions as it did before.



This change in circuit architecture has another important benefit: it guarantees that the initial voltage on the capacitor is 0 V. We used this assumption when setting V_{ref} in Eq.(3). Without this change, we would not know the initial voltage, and therefore we could not choose V_{ref} to correctly correspond to 2 sec.

We put all of this together into our final design, where the switch is initially closed and it opens when the button is pressed.



Can you build this countdown timer in lab? Almost! The only thing that is missing is the current source. In the next section, we'll explore building an ideal current source out of commonly available components.

20.3 Design Example: “Almost” current source

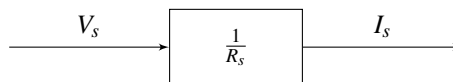
In this section, we will use resistors, voltage sources, and op amps to build a current source. We want a source that can be used as the current source in the countdown timer above. Let's follow our design procedure.

Step 1 (Specification): First we restate the problem clearly: Build a current source that outputs constant current, I_s , regardless of the voltage across it.

Step 2 (Strategy): We can use a voltage source, but we need a way to transform the voltage into a constant current. Ohm's law tell us

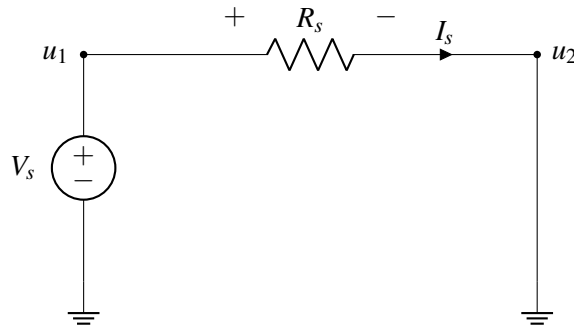
$$I = \frac{V}{R},$$

so maybe we can use a resistor to convert the voltage source output, V_s , into a constant current, I_s . This is summarized in the following block diagram:



Step 3 (Implementation): Based on our strategy from Step 2, we make our first attempt to build the current source. Let's take a voltage source and connect it to a resistor:

Attempt #1

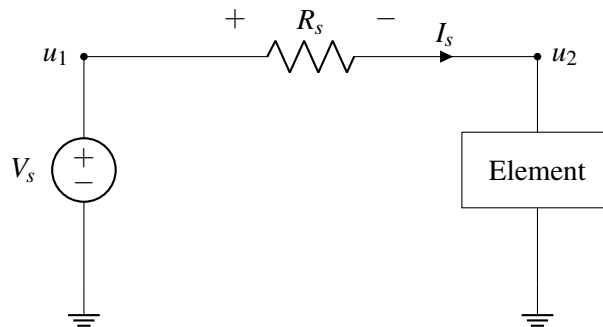


In the above circuit,

$$u_1 = V_s \quad (5)$$

$$I_s = \frac{V_s}{R} \quad (6)$$

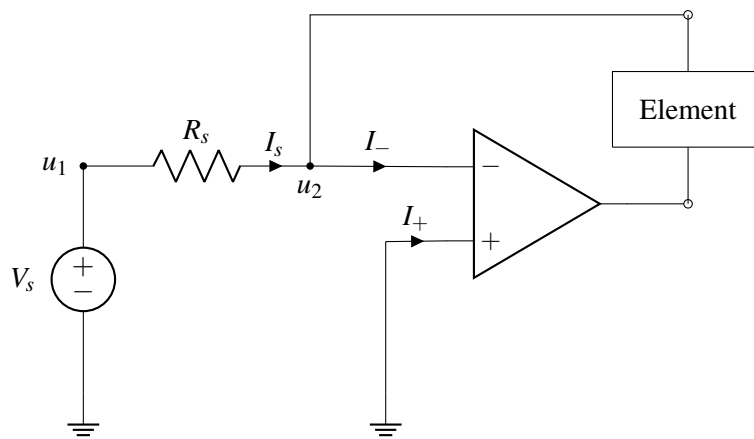
This looks promising, but if we hook up an element between u_2 and ground, we find that the current through R_s is no longer the constant $I_s = \frac{V_s}{R_s}$. For example, if the element is a resistor R_l , then the current drops to $I_s = \frac{V_s}{R_s + R_l}$.



A decent current source needs a constant current output regardless of the element(s) added to the circuit. Thus, we need to modify our design.

Attempt #2 Although our Attempt 1 was unsuccessful, we have learned an important lesson: if we can somehow set u_2 to 0 V without physically connecting it to ground, then the current through R_s will always equal $\frac{V_s}{R_s}$ ($I_s = \frac{u_1 - u_2}{R_s} = \frac{V_s - 0}{R_s}$). Do we know of an element that can set two points to have the same voltage without physically connecting them? Yes! According to golden rule #2, we can set both U_+ and U_- to 0 V if an op amp circuit is in negative feedback.

Indeed, we will now use an op amp to build a current source!



According to golden rules:

$$I_- = 0 \quad (7)$$

$$U_- = U_+ = 0V \quad (8)$$

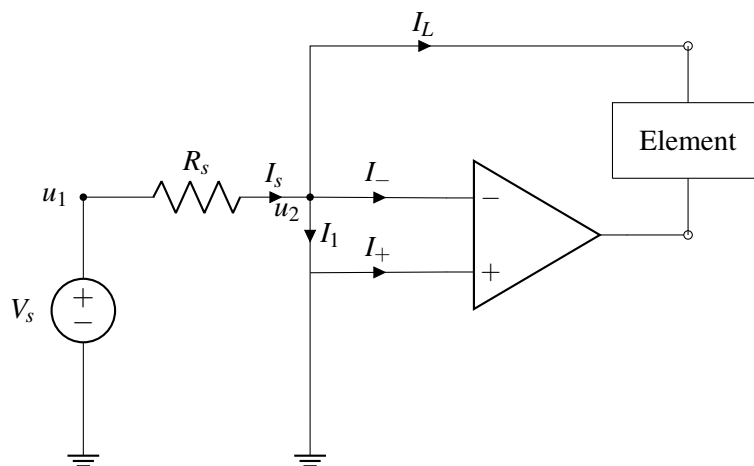
We also know that the current across R_s will always be:

$$I_s = \frac{u_1 - u_2}{R_s} \quad (9)$$

Solving the above equations, we can get the value of I_s :

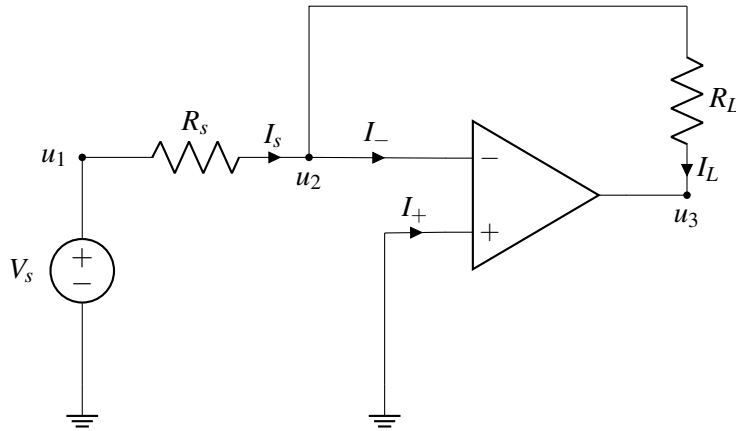
$$I_s = \frac{V_s}{R_s} \quad (10)$$

By setting u_2 (U_-) to 0 V by using a negative feedback circuit, we have successfully built a current source! It is important to keep in mind setting u_2 to 0 V by using a negative feedback circuit is very different from physically connecting the node of u_2 to ground. If we physically connect u_2 to ground by adding a wire between U_- and U_+ our circuit looks something like this:



In this case, I_L will become 0 A because all of the current will flow through the wire between U_- and U_+ (in other words, $I_1 = I_s$). Therefore we cannot physically connect U_- to ground.

Step 4 (Verification): Now let's go back to our correct version (where U_+ and U_- are not connected) and hook up a load resistor R_L to the circuit and prove that current flow through R_L is constant:



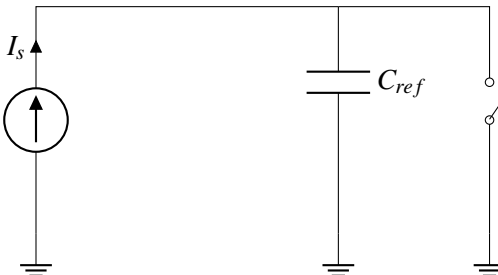
According to KCL:

$$I_L = I_s = \frac{V_s}{R_s} \quad (11)$$

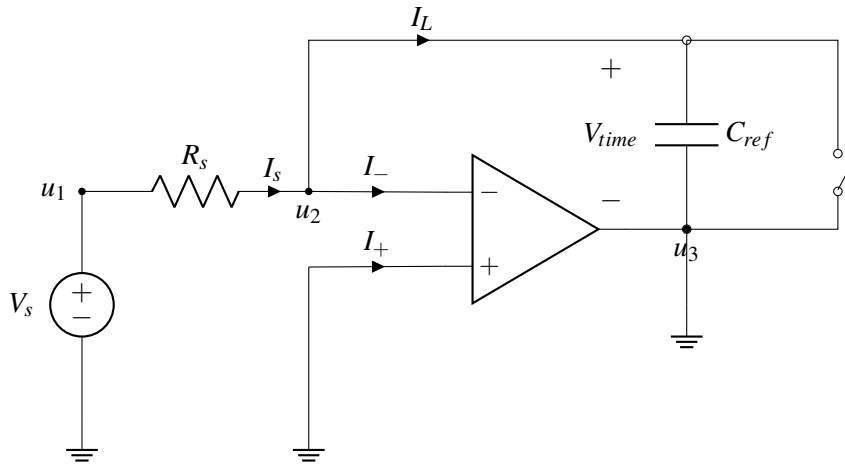
From I_L equation, we can see immediately that I_L is not affected by changes in R_L . How does the circuit maintain the constant current flow through R_L ? The op-amp outputs a negative voltage to maintain the constant current. In other words, the op-amp sets a voltage drop across R_L such that

$$V_{R_L} = \frac{V_s}{R_s} R_L. \quad (12)$$

We've now verified that our current source works for a resistor load. However, we also want this source to work in our countdown timer, which is more complicated. To verify that our complete countdown timer works, we'll connect our current source, then analyze the result. Recall that the first part of the timer looks like this:



When we add our current source, the circuit becomes:

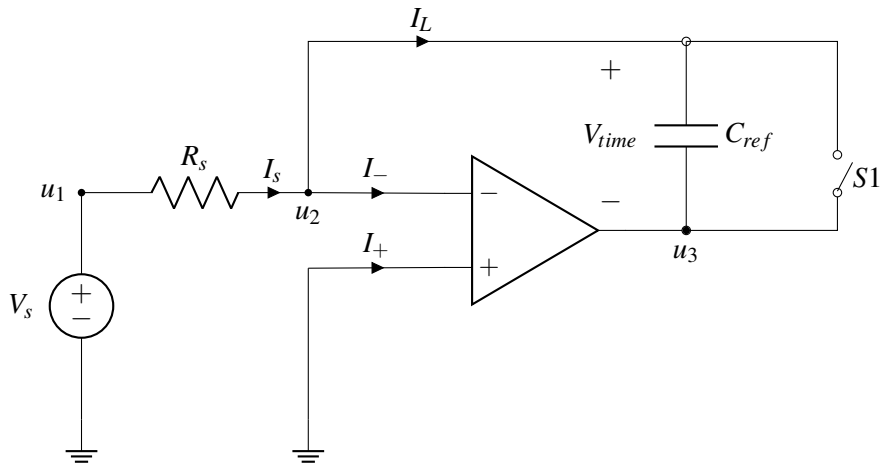


Let's consider the case after the button is pressed and the switch is open. We are supposed to be charging the capacitor to keep track of time. However, u_3 is connected to ground, and $u_2 = 0$ V by the second golden rule. Therefore, the voltage across C_{ref} is

$$V_{time} = u_2 - u_3 = 0 - 0 = 0 \text{ V} \quad (13)$$

which means that the voltage is not increasing with time as desired!

In addition, there is even a worse problem: For the op-amp to be in negative feedback, the controlled voltage source inside the op-amp must make small changes to compensate for changes in the input. However, since the output of the op-amp is always 0 V, there is no feedback and we cannot assume the second golden rule. The fix to solve this problem is to get rid of the ground connection at u_3 .



Now we analyze the circuit above:

$$I_s = C_{ref} \frac{dV_{time}}{dt} \quad (14)$$

$$I_s = C_{ref} \frac{d(u_2 - u_3)}{dt} \quad (15)$$

According to the second golden rule, $u_2 = 0 \text{ V}$.

$$I_s = C_{ref} \frac{d(0 - u_3)}{dt} = C_{ref} \frac{d(-u_3)}{dt} \quad (16)$$

Solving the above equation:

$$u_3(t) = -\frac{I_s}{C_{ref}} \cdot t + u_3(t = 0 \text{ s}) \quad (17)$$

$u_3(t)$ is dependent on the initial value $u_3(t = 0 \text{ s})$. Before touch, the switch $S1$ is closed, which sets $u_3(t = 0 \text{ s})$ to 0 V . Therefore,

$$u_3(t) = -\frac{I_s}{C_{ref}} t = -\frac{V_s}{R_s C_{ref}} t \quad (18)$$

Note there is a term $R_s C_{ref}$ in the denominator. What are the units of $R_s C_{ref}$?

$$\text{Unit of } RC = \frac{\text{volts}}{\text{ampere}} \cdot \frac{\text{coulombs}}{\text{volt}} = \frac{\text{coulombs}}{\text{ampere}} = \frac{\text{coulombs}}{\frac{\text{coulombs}}{\text{second}}} = \text{seconds} \quad (19)$$

The units of $R \cdot C$ are seconds. This comes up frequently in the time-varying analysis of circuits.

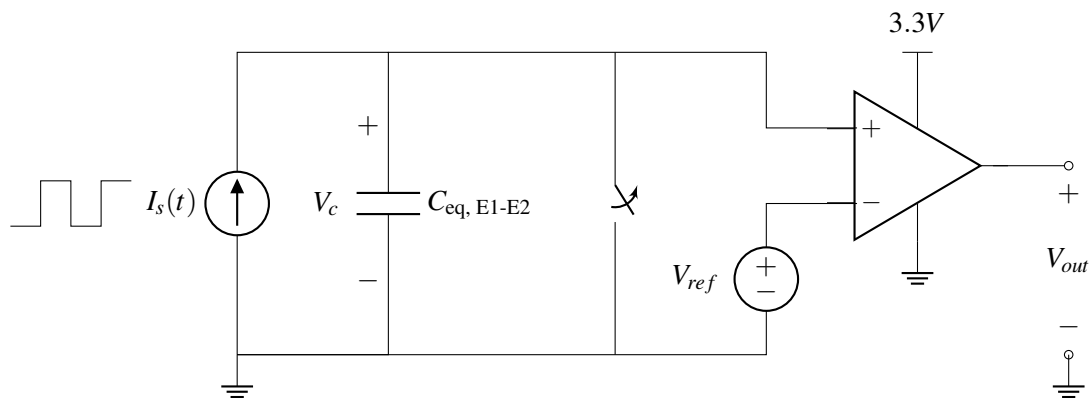
Caveats of the “Almost” Current Source

Although this current source has many things in common with an ideal current source, there are 2 important points where it differs:

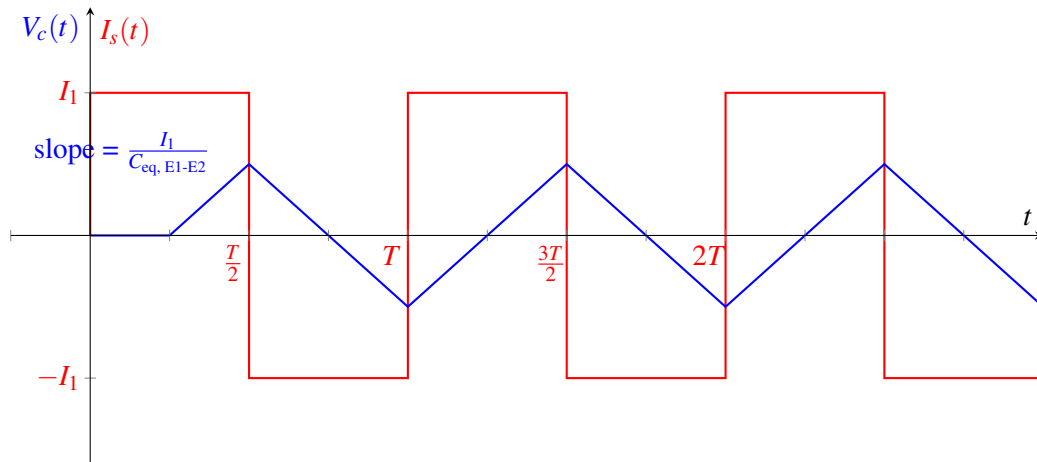
- Do not connect the output of the current source to ground externally. Doing so may force V_{out} of the op-amp to 0 V and lead to non-idealities.
- The circuit element we hook up to the current source must still keep the op-amp circuit in its negative feedback state. Being in negative feedback allows us to set the node u_2 to 0 V without physically connecting it to ground and hence allows a constant current output.

20.4 Capacitive Touchscreen Revisted

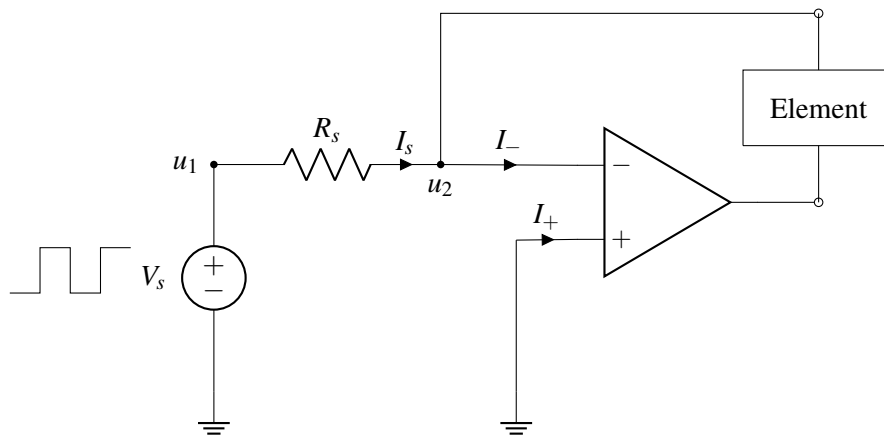
In this section, we'll use our current source design to implement an alternative capacitive touchscreen. Recall that the presence or absence of a finger changes the capacitance between two electrodes, $C_{eq, E1-E2}$. As mentioned in lecture, we can design the following circuit to detect these changes:



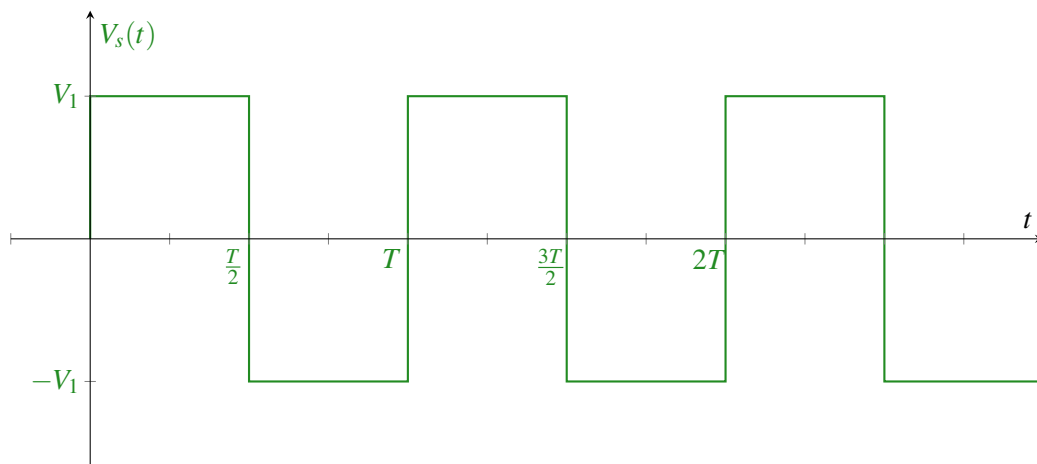
Here the current source, I_s , outputs a periodic square wave alternating between I_1 and $-I_1$ every $T/2$ seconds. The voltage over the capacitor V_c increases linearly, with a slope that depends on $C_{eq, E1-E2}$. The switch is opened at $t = \frac{T}{4}$.



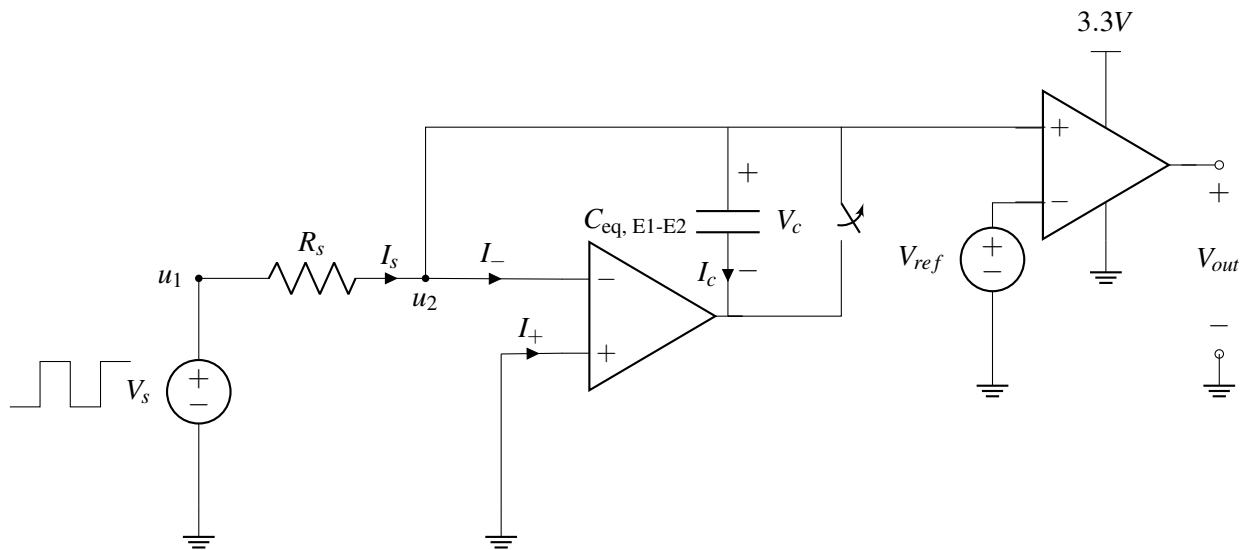
The periodic current source is critical in this design, but not readily available. Can we use our design of the “almost” current source in our capacitive touchscreen? The “almost” current source outputs a constant current, while we need a periodic current (shown in red above). However, we can achieve this by powering the “almost” current source with a periodic voltage instead of a constant voltage:



V_s has the following time-varying response:

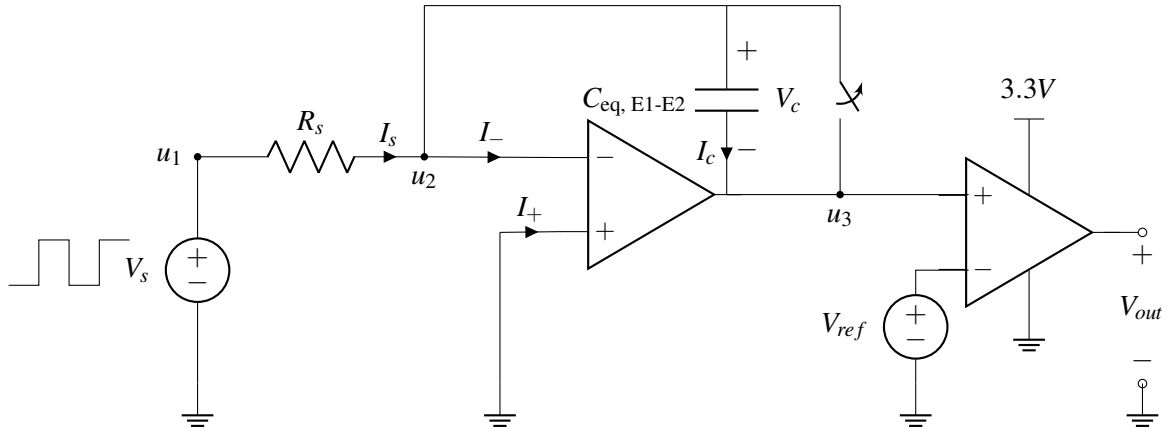


Since $I_s = \frac{V_s}{R_s}$, the current supplied to the load element will be either $\frac{V_1}{R_s}$ or $-\frac{V_1}{R_s}$. Now let's attach this current source to our touchscreen circuit.



Let's look at the comparator/op-amp on the right side. Note that its positive terminal is directly connected to u_2 , and $u_2 = 0$ V. This means that u_+ of the right-most comparator op-amp is always zero and does not depend on the capacitance $C_{eq, E1-E2}$. Therefore, V_{out} cannot detect changes in $C_{eq, E1-E2}$ and the circuit does not work as intended.

How can we fix this? We should not connect u_2 to the op-amp. Instead we can connect the other end of the capacitor as follows:



By the second golden rule (applied to the left op-amp), $u_2 = U_+ = 0$ V. We can use KVL to determine that

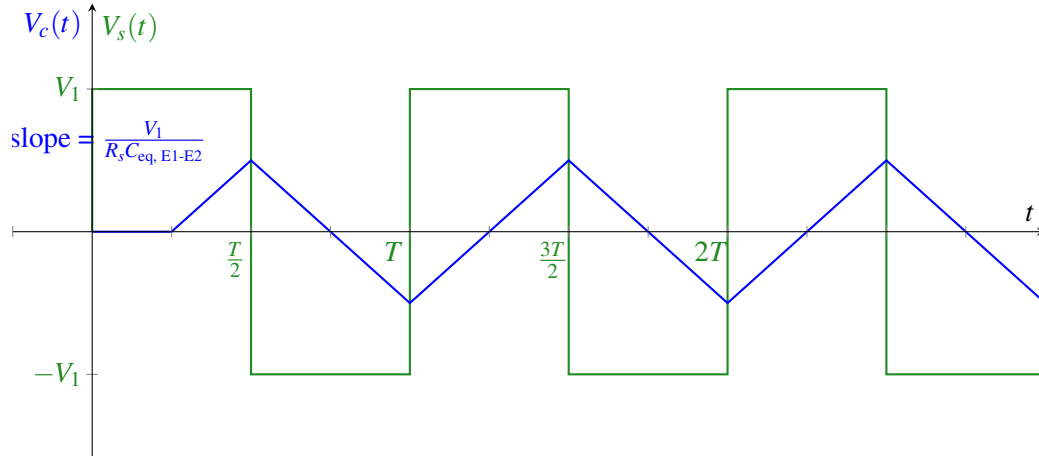
$$\begin{aligned} u_3 &= u_2 - V_c(t) \\ &= -V_c(t) \end{aligned}$$

This is good because u_3 will depend on the capacitance, unlike the previous configuration. Let's analyze what $u_3 = -V_c$ is as a function of the capacitance:

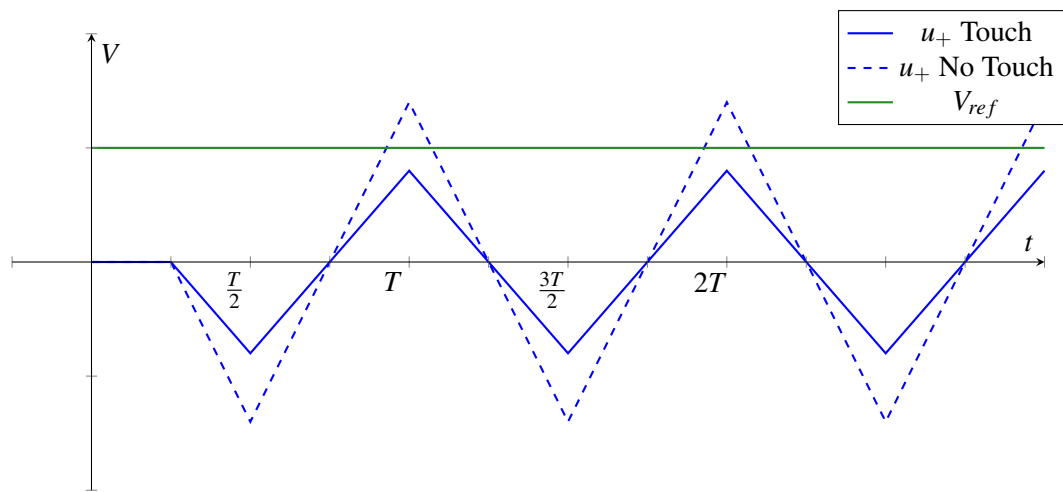
Since no current flows into either op-amp, $I_s = \frac{V_s}{R_s} = I_c$. Using the equation for a capacitor charging under constant current, we find

$$V_c(t) = \frac{I_c}{C_{eq, E1-E2}} t = \frac{V_1}{R_s C_{eq, E1-E2}} t$$

This equation only applies when the current is constant. When V_s becomes negative, I_s will also be negative, and then V_c will decrease at the same rate (see Note 17 for a more in-depth derivation of this effect). Overtime, we get the following waveform for $V_c(t)$ (assuming the switch is opened at $t = \frac{T}{4}$)



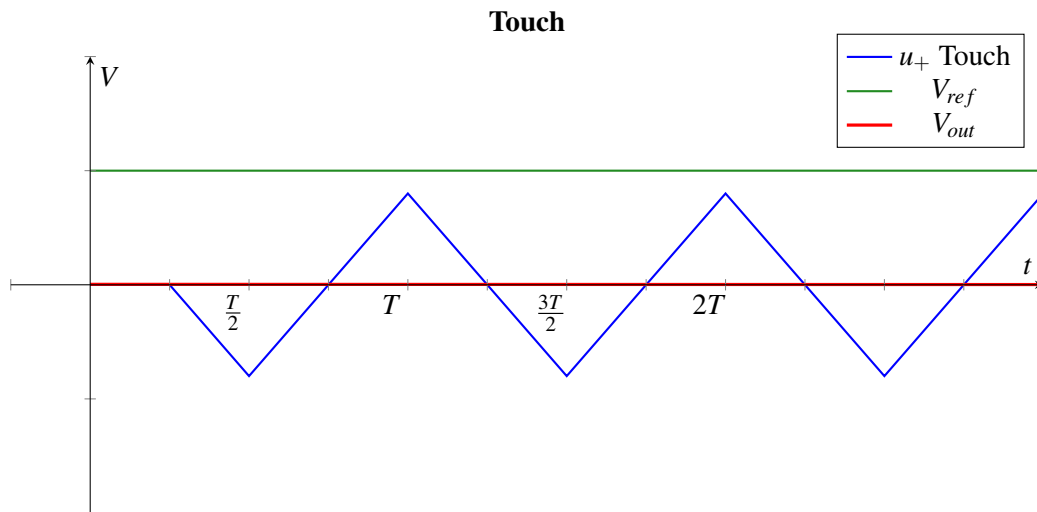
Recalling capacitance $C_{eq, E1-E2}$ increases when there is a touch, we can plot $u_+ = -V_c$ when the finger is either present or absent. We'd like to set V_{ref} halfway between the peak of u_+ with a touch and u_+ without a touch.



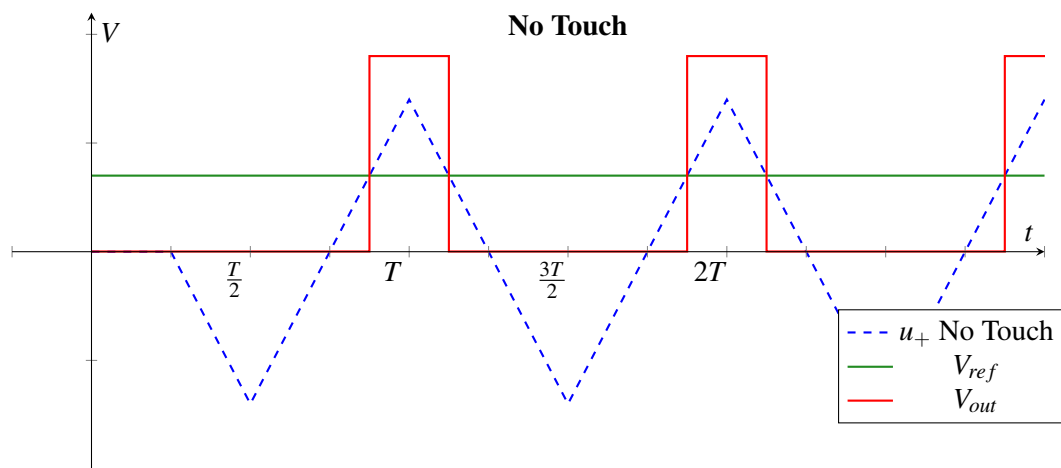
Using the equations for a comparator, we find

$$V_{out} = \begin{cases} 0V & \text{if } u_+ < V_{ref} \\ 3.3V & \text{if } u_+ > V_{ref} \end{cases}$$

We get the following waveforms when there is a touch:



When there is no touch, there is an increase in V_{out} every period, as shown below:



V_{out} has the same structure as when we used an ideal current source! However, since one terminal of our “almost” current source is always 0 V, the analysis was slightly different and we had to rethink the circuit architecture. That’s why Step 4 (Verification) of the design procedure is so important—we can’t assume different circuit blocks will act the same when we connect them.