EECS 16A Designing Information Devices and Systems I
Spring 2023 Homework 8

This homework is due Friday, March 17, 2023 at 23:59. Self-grades are due Friday, March 24, 2023, at 23:59.

Submission Format

Your homework submission should consist of **one** file.

- `hw8.pdf`: A single PDF file that contains all of your answers (any handwritten answers should be scanned).

Submit the file to the appropriate assignment on Gradescope.

Mid Semester Survey

Please fill out the mid semester survey: https://forms.gle/oG9GW9odcW78Nz839.

We highly appreciate your feedback!

1. Reading Assignment and Midterm Survey

For this homework, please read Notes 14 and 15. Note 14 introduces better, but more complex models for the resistive touchscreen. Note 15 covers superposition and equivalence, two techniques to simplify circuit analysis.

(a) Please fill out this survey regarding the first midterm.

(b) For the touch screen model introduced in Note 14, why can't we simultaneously get the horizontal and vertical position of the touch with a single measurement? *Think about how many unknowns there are.*

(c) Explain the connection between the "superposition" you learned about in Note 15 with the "superposition" you learned back in module 1 in the context of linear functions.

Solution:

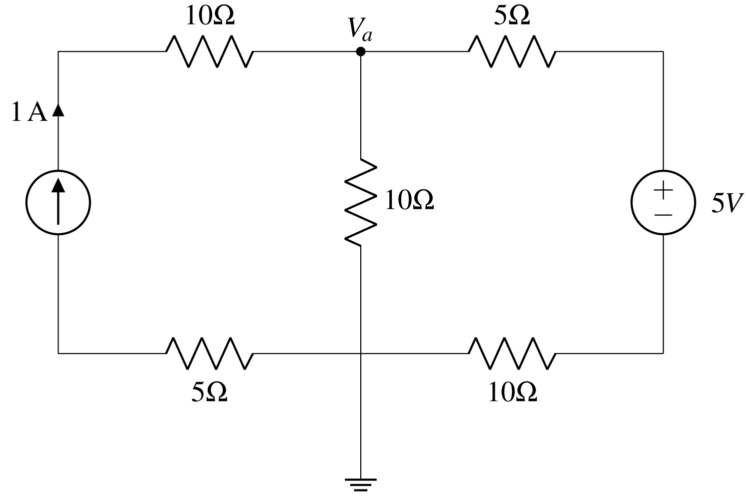
a) Since we have two unknowns—vertical and horizontal positions—we also need two measurements/equations. Thus, for every touch we need to measure the voltage, change the voltage/ground configuration, and measure the voltage again (not super convenient :( ).

b) They are the same idea! We can view the circuits we have seen thus far as linear functions where the inputs are the sources, and the outputs are the unknown circuit quantities. With this view, we know by superposition that we can construct the output for a particular set of input sources by getting the output for each individual source first, then linearly recombining these outputs. That is exactly the superposition introduced in Note 15!

2. Superposition

Learning Goal: *The objective of this problem is to help you practice solving circuits using the principles of superposition.*

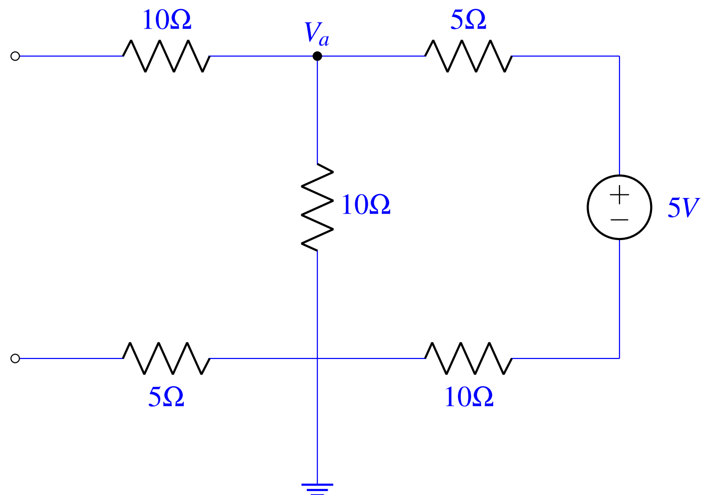
Find the node potential V_a indicated in the diagram using superposition. Be careful when solving to take into account where the reference potential is.



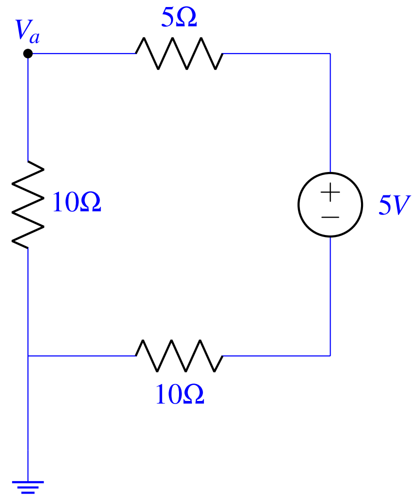
Solution:

Consider the circuits obtained by:

(a) Zeroing out the 1 A current source:

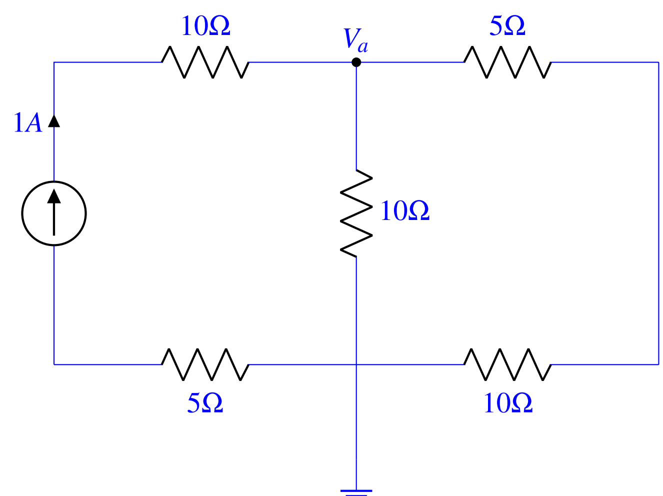


In the above circuit, no current flows through the two resistors in the top left and bottom left, so we can remove them and get the following circuit:

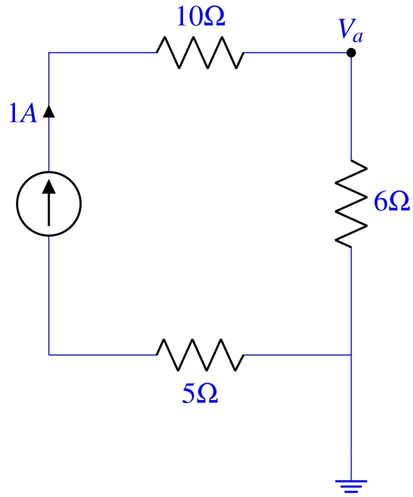


Using NVA, we can find that the voltage drop across the 5 Ω resistor is 1V and the voltage drop across the 10 Ω resistor is 2V. Note that the reference node is not right under the voltage source like we typically see, so V_a is just the voltage drop across the 10 Ω resistor which is 2V. (You can also make this easier by combining resistors and applying the voltage divider, but you still need to take account for where the reference potential is).

(b) Zeroing out the 5 V voltage source:



We can reduce this circuit using resistor equivalences to make it easier to solve. We note that the top right and bottom right resistors are in series, so combined we get 15Ω. Then we can combine this 15Ω resistor with the 10Ω resistor in the middle as they are in parallel to get $\frac{(10\Omega)(15\Omega)}{10\Omega+15\Omega} = 6\Omega$. Our reduced circuit then looks like this:



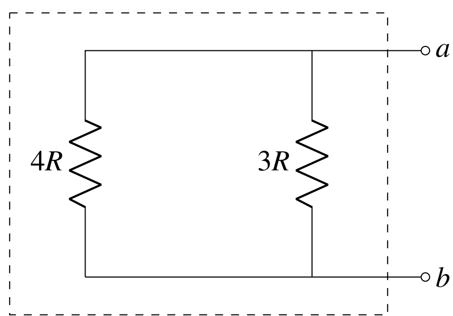
We can then just use Ohm's law to find the voltage drop across the 6Ω to find our node potential at V_a, which is just $(1A)(6\Omega) = 6V$

Now, applying the principle of superposition, we have $V_a = 2V + 6V = 8V$

3. Equivalent Resistance

Learning Goal: *The objective of this problem is to practice finding the equivalent to a series/parallel combination of resistors.*

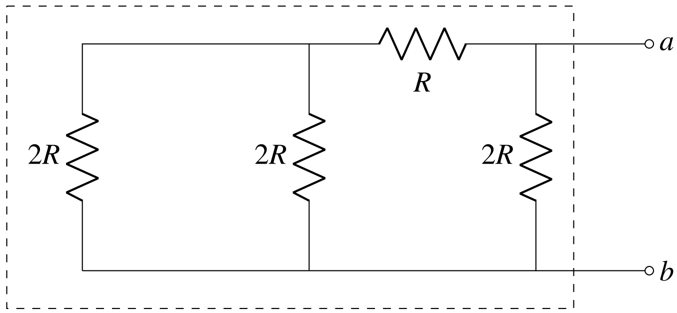
(a) Find the equivalent resistance looking in from points a and b. In other words, express the resistive network in the dashed box as one resistor.



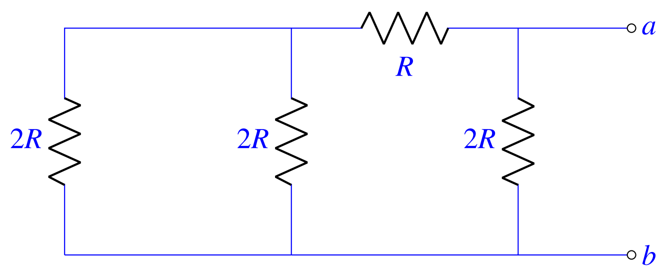
Solution:

$$R_{eq} = 4R \parallel 3R = \frac{4R \cdot 3R}{4R + 3R} = \frac{12}{7}R$$

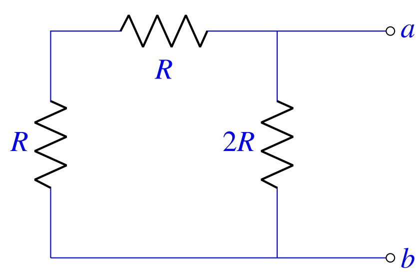
(b) Find the equivalent resistance looking in from points a and b. In other words, express the resistive network in the dashed box as one resistor.



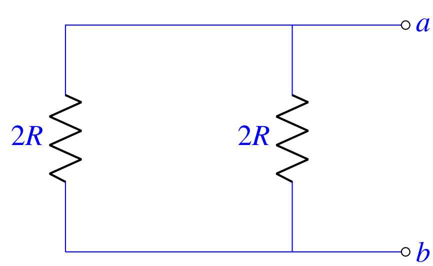
Solution: We find the equivalent resistance for the resistors from left to right. In general, it is easiest to start furthest away from the node voltages we wish to measure.



We can begin by combining the leftmost two resistors, which are in parallel and both have values $2R$. The equivalent resistor would have resistance $R_{eq} = 2R \parallel 2R = \frac{2R \cdot 2R}{2R+2R} = R$, so we replace both resistors with a equivalent resistor of resistance R.

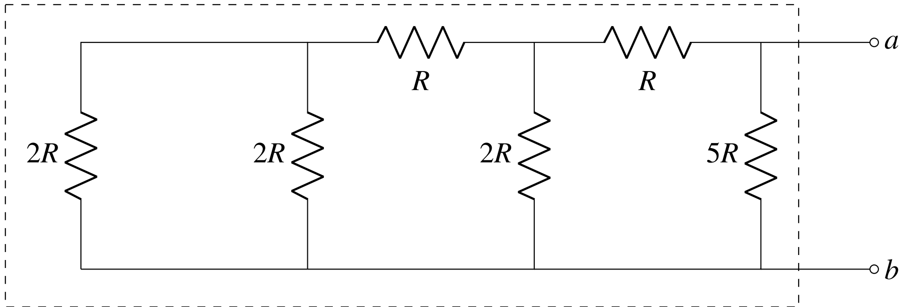


Now, the leftmost resistors are in series, both with values R, so the equivalent resistor would have resistance $R_{eq} = R + R = 2R$.

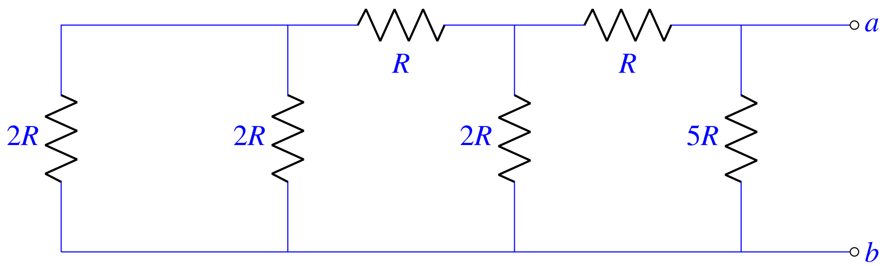


Finally, we are left with a parallel combination of resistors with values $2R$ and $2R$, so the equivalent resistor would have resistance $R_{eq} = 2R \parallel 2R = R$.

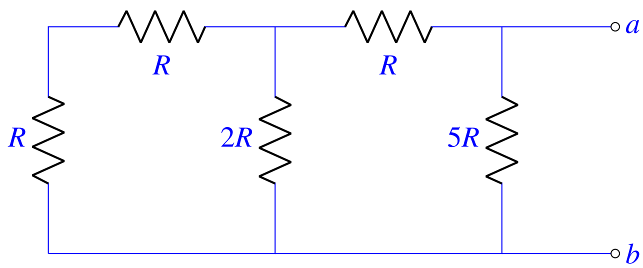
(c) Find the equivalent resistance looking in from points a and b. In other words, express the resistive network in the dashed box as one resistor.



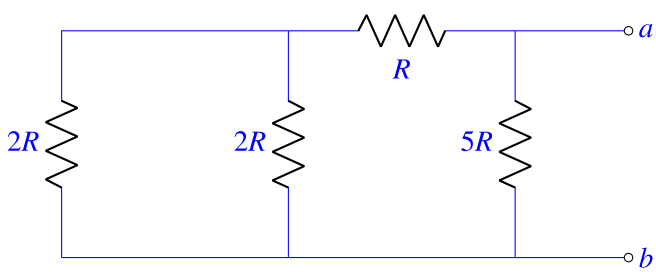
Solution: Again, we find the equivalent resistance for the resistors from left to right.



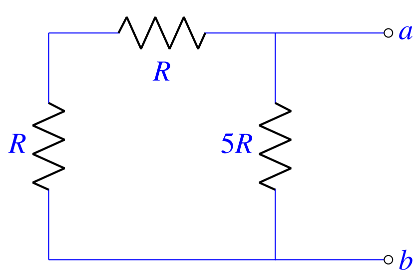
We first combine the two parallel resistors on the left:

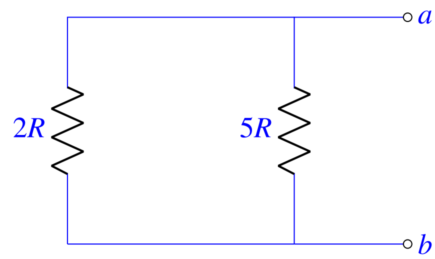


Next, the two series resistors on the left:



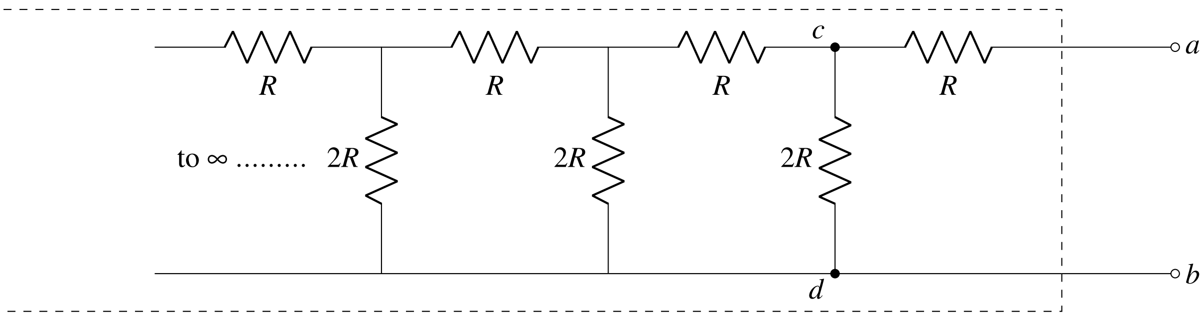
Following the process, we can eventually simplify down to a circuit with a single equivalent resistor.



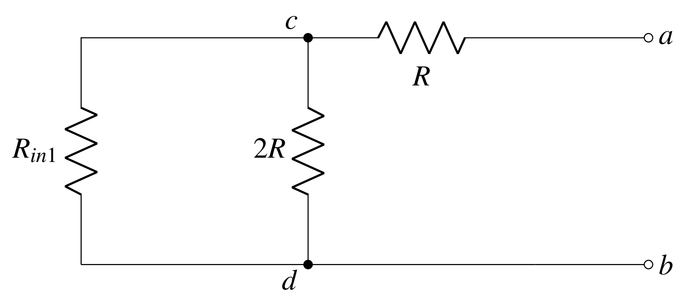


$$R_{eq} = 2R \parallel 5R = \frac{10}{7}R$$

(d) Find the equivalent resistance for the infinite ladder looking in from points a and b. In other words, express the resistive network in the dashed region as one resistor. (*Hint: Let's call the resistance looking in from a and b as R_{in}, and the resistance looking to the left from points c and d as R_{in1}. Replace the entire circuit to the left of points c and d with a resistor whose value is given by R_{in1}. Find the relationship between R_{in} and R_{in1} using this circuit. Find another relationship between R_{in} and R_{in1} using the fact that the the ladder is infinite. For an infinite ladder, adding another branch does not change the equivalent resistance. Think of this as a convergent infinite series.*)



As a first step you can replace the circuit looking to the left from c and d by R_{in1}.



Solution: We wish to compute the equivalent resistance R_{in} looking to the left from nodes a and b. The equivalent resistance looking to the left from nodes c and d is given by R_{in1}. Clearly,

$$R_{in} = (R_{in1}||2R) + R$$

Additionally, since this is an infinite ladder, the equivalent resistance does not change by addition of an extra branch to the right, since having infinity + 1 steps is the same thing as having infinite steps.

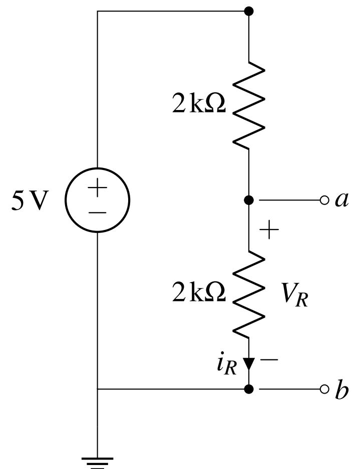
Therefore, $R_{in1} = R_{in}$. Using this result in the previous equation, we have,

$$\begin{aligned}
R_{in} &= (R_{in}||2R) + R \\
R_{in} &= \frac{2R \cdot R_{in}}{2R + R_{in}} + R \\
R_{in}^2 - RR_{in} - 2R^2 &= 0 \\
(R_{in} - 2R)(R_{in} + R) &= 0
\end{aligned}$$

Clearly, $R_{in} = -R$ is not a physically realizable solution. The equivalent resistance looking into this infinite ladder is given by $R_{in} = 2R$.

4. Why Bother With Thévenin Anyway?

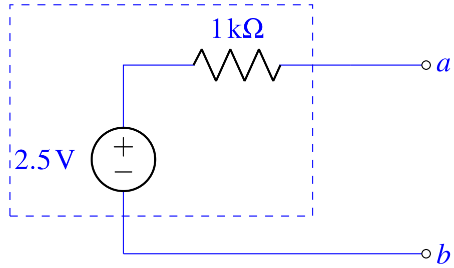
(a) Find a Thévenin eqiuvalent for the circuit shown below looking from the terminals a and b. (*Hint: That is, find the open circuit voltage V_R across the terminals a and b. Also, find the equivalent resistance looking from the terminals a and b when the input voltage source is zeroed.*)



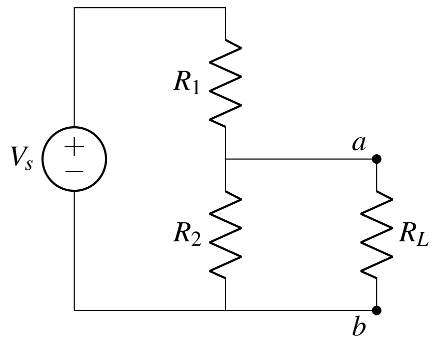
Solution: To find the voltage across terminals a and b, we notice that the circuit is a voltage divider. Therefore, we can use the voltage divider formula to find the voltage across a and b. Then for the equivalent resistance, we zero out the voltage source and notice that the resistors are in parallel with respect to the terminals a and b so we can use the parallel resistor equation to find the equivalent resistance. Be careful! It looks like the the resistors are in series but if we combine them that way, we would be destroying node a!

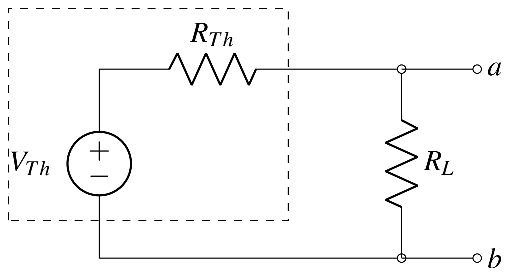
$$V_{Th} = \frac{2\,\text{k}\Omega}{2\,\text{k}\Omega + 2\,\text{k}\Omega} \cdot 5\,\text{V} = 2.5\,\text{V}$$

$$R_{Th} = 2\,\text{k}\Omega \parallel 2\,\text{k}\Omega = 1\,\text{k}\Omega$$

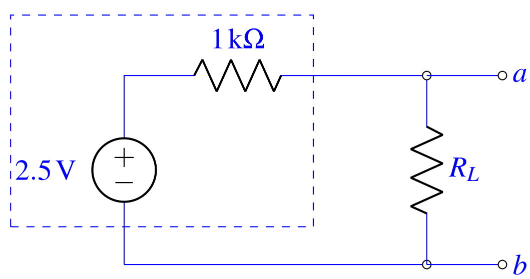


(b) Now consider the circuit shown below where a load resistor of resistance R_L is attached across the terminals a and b. Such a load resistor is often used to a model a device that we want to plug our circuit into, like an audio speaker. Compute the voltage drop V_R across the terminals a and b in this new circuit with the attached load. Express your answer in terms of R_L. *Hint: We have already computed the Thévenin equivalent of the unloaded circuit in part (a). To analyze the new circuit, attach R_L as the load resistance across the Thévenin equivalent computed in part (a), as shown in the figure below. One of the main advantages of using Thévenin (and Norton) equivalents is to avoid re-analyzing different circuits which differ only by the amount of loading (which depends on the device we are connecting!).*





Solution:
We just attach the R_L resistor to our Thévenin equivalent circuit that we found part (a) and calculate the voltage across it.



$$V_R = \frac{R_L}{1\,\mathrm{k\Omega} + R_L} \cdot 2.5\,\mathrm{V}$$

(c) Now compute the voltage drop V_R for three different values of R_L equal to $5/3\,\mathrm{k\Omega}$, $5\,\mathrm{k\Omega}$, and $50\,\mathrm{k\Omega}$? What can you comment on the value of R_L needed to ensure that the loading does not reduce the voltage drop V_R compared to the unloaded voltage V_R computed in part (a)? **Solution:**
$R_L = \frac{5}{3}\mathrm{k\Omega}$:

$$V_R = \frac{\frac{5}{3}\,\mathrm{k}\Omega}{1\,\mathrm{k}\Omega + \frac{5}{3}\,\mathrm{k}\Omega} \cdot 2.5\,\mathrm{V} = 1.56\,\mathrm{V}$$

$R_L = 5\,\mathrm{k}\Omega$:

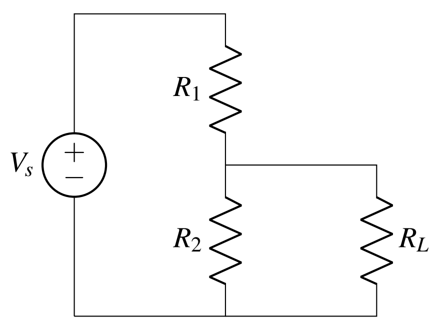
$$V_R = \frac{5\,\mathrm{k}\Omega}{1\,\mathrm{k}\Omega + 5\,\mathrm{k}\Omega} \cdot 2.5\,\mathrm{V} = 2.08\,\mathrm{V}$$

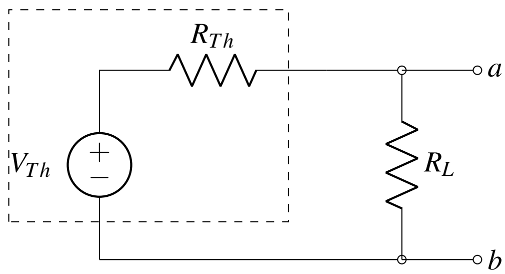
$R_L = 50\,\mathrm{k}\Omega$:

$$V_R = \frac{50\,\mathrm{k}\Omega}{1\,\mathrm{k}\Omega + 50\,\mathrm{k}\Omega} \cdot 2.5\,\mathrm{V} = 2.45\,\mathrm{V}$$

As the value of R_L is increased, the voltage drop V_R approaches the unloaded Thévenin voltage computed in part (a).

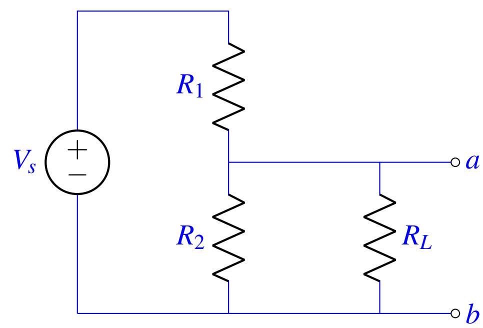
(d) Thus far, we have seen how to use Thévenin equivalents to compute the voltage drop across a load without re-analyzing the entire circuit. We would like to see if we can use the Thévenin equivalent for power computations. Consider the case where the load resistance $R_L = 8\mathrm{k}\Omega$, $V_S = 5V$, $R_1 = R_2 = 2\mathrm{k}\Omega$. Compute the power dissipated across the load resistor R_L both using the original circuit and the Thévenin equivalent. Are they equal? Now, compute the power dissipated by the voltage source V_s in the original circuit. Also, compute the power dissipated by the Thévenin voltage source V_{Th} in the Thévenin equivalent circuit. Is the power dissipated by the two sources equal?



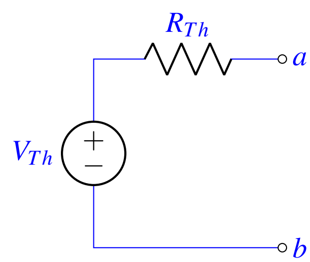


Solution:

We will compare the power dissipation in V_s vs. V_{Th} and R_L in either case. This could be done for the specific example above (with $R_L = 8\mathrm{k}\Omega$), but it's more useful to go through this exercise generally. Thus, we will use the circuit shown below:



Recall that the Thévenin equivalent for the circuit above looks as follows:



where $R_{Th} = \frac{R_1 R_2}{R_1+R_2}$ and $V_{Th} = \frac{R_2}{R_1+R_2} V_s$.
Because we are going to end up writing a few expressions multiple times, we are going to define a new variable:

$$\beta = R_1 R_2 + R_L R_1 + R_L R_2$$

Let's start with our equivalent circuit. In the equivalent circuit, the current through the load resistor and equivalently every other element in the circuit is:

$$I = \frac{V_R}{R_L} = \frac{V_{Th}}{R_L + R_{Th}}$$

With this current, we find the power dissapated across the source and the load resistor.

$$P_{V_{Th}} = -IV = -\frac{V_{Th}^2}{R_L + R_{Th}} = -\frac{V_{Th}^2 (R_1 + R_2)}{\beta} = -\frac{V_s^2 R_2^2}{\beta (R_1 + R_2)} = -0.694\,\mathrm{mW}$$

$$P_{R_L} = I^2 R = \frac{V_{Th}^2}{(R_L + R_{Th})^2} \cdot R_L = \frac{V_{Th}^2 (R_1 + R_2)^2}{\beta^2} \cdot R_L = \frac{V_s^2 R_2^2}{\beta^2} \cdot R_L = 0.617\,\mathrm{mW}$$

Let's try to find the answer from the original circuit. We will begin by calculating the current through the source.

$$I_s = \frac{V_s}{R_{eq}} = \frac{V_s}{R_1 + R_2 \parallel R_L} = \frac{V_s (R_1 + R_2)}{\beta}$$

Now, we can calulate the power through the source.

$$P_{V_s} = -I_s V_s = -\frac{V_s^2 (R_2 + R_L)}{\beta} = -6.94\,\mathrm{mW}$$

The power dissipated by the source in the original circuit is not the same as the power dissipated in the new circuit. What about the load resistor? We will first calculate the voltage across the load resistor.

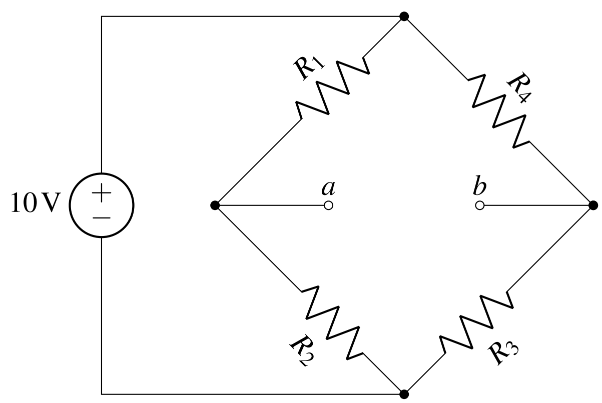
$$V_L = \frac{R_2 \parallel R_L}{R_1 + R_2 \parallel R_L} \cdot V_s = \frac{\frac{R_2 R_L}{R_2+R_L}}{R_1 + \frac{R_2 R_L}{R_2+R_L}} \cdot V_s = \frac{R_2 R_L}{\beta} \cdot V_s$$

$$P_L = \frac{V_L^2}{R_L} = \frac{V_s^2 R_2^2}{\beta^2} R_L = 0.617\,\text{mW}$$

The power through the load is the same! Thévenin equivalents can be used to calculate the power through elements that are not part of the circuit that was transformed.

5. Wheatstone Bridge

Thévenin equivalence is a powerful technique we can use to analyze the Wheatstone bridge circuit shown below. This circuit is used in many sensor applications where resistors R_1, R_2, R_3, R_4 are varying with respect to some external actuation. For example, it can be used to build a strain gauge or a scale. In that case the resistors R_1, R_2, R_3, R_4 would vary with respect to a strain caused by a force, and the Wheatstone Bridge circuit would translate that variation into a voltage difference across the "bridge" terminals a and b. Assume that $R_1 = 2\,\text{k}\Omega$, $R_2 = 2\,\text{k}\Omega$, $R_3 = 1\,\text{k}\Omega$, $R_4 = 3\,\text{k}\Omega$



(a) Calculate the voltage V_{ab} between the two terminals a and b.

Solution:

Notice in the above circuit that there are two voltage dividers, so we can calculate v_a and v_b quickly.

$$v_a = \frac{R_2}{R_1 + R_2} \cdot 10\,\text{V} = 5V$$

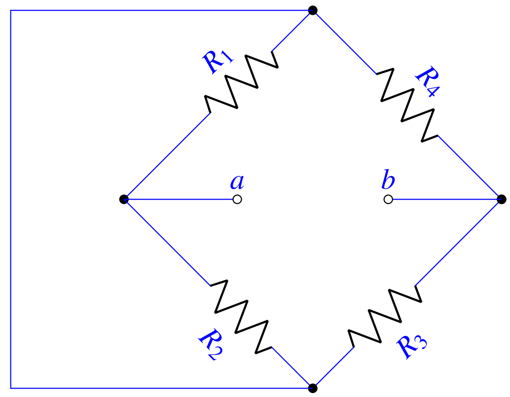
$$v_b = \frac{R_3}{R_3 + R_4} \cdot 10\,\text{V} = 2.5V$$

Thus, the voltage difference between the two terminals a and b is: $V_{ab} = v_a - v_b = 2.5V$.

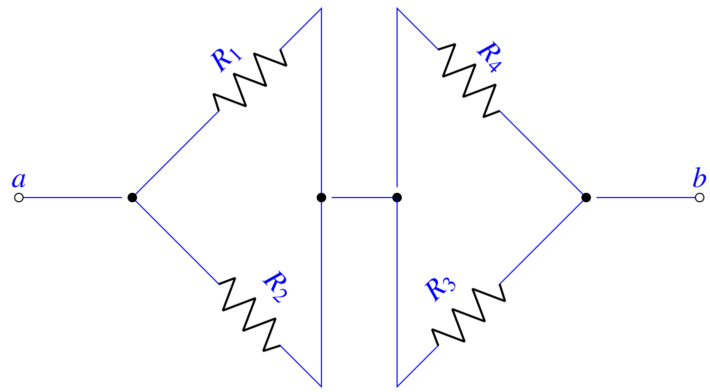
(b) Next, draw the Thévenin equivalent of the Wheatstone bridge circuit.

Solution:

We find the Thévenin resistance by replacing the voltage source with a short and calculating the resistance between the two terminals a and b. The circuit now looks like:



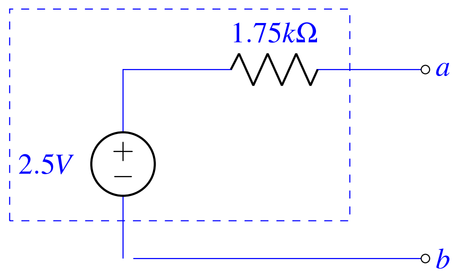
Notice that because the top and bottom node are shorted, we have $R_1 \parallel R_2$ in series with $R_3 \parallel R_4$ between nodes "a" and "b".



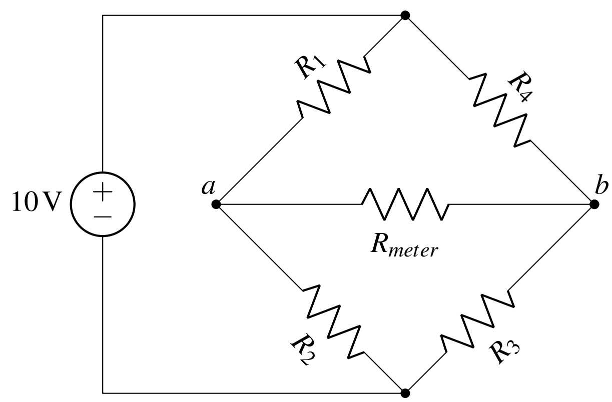
It follows that R_{th} is:

$$\begin{aligned} R_{\text{Th}} &= (R_1 \parallel R_2) + (R_3 \parallel R_4), \text{ where } \parallel \text{ denotes the parallel operator.} \\ &= \frac{R_1 R_2}{R_1 + R_2} + \frac{R_3 R_4}{R_3 + R_4} \\ &= 1.75k\Omega \end{aligned}$$

Using $V_{\text{Th}} = V_{ab} = 2.5V$ from part (a), we can construct the Thévenin equivalent circuit:



(c) Now assume that you are trying to measure the voltage V_{ab} using a voltmeter, whose resistance is R_{meter}, so you end up with the circuit below.

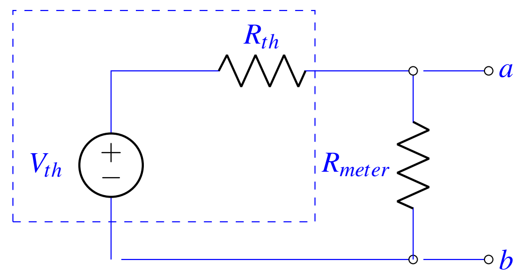


Unfortunately, you voltmeter is far from ideal, so $R_{meter} = 4k\Omega$. Is the voltage V_{ab} you found in part (a) equal to the new voltage $V_{R_{meter}}$ across the voltmeter resistor? Why or why not? Calculate the current $I_{R_{meter}}$ through the voltmeter resistor and the voltage $V_{R_{meter}}$ across the meter resistor.

Solution:

No, the Thévenin voltage we found in part (a) is the open-circuit voltage. If we add R_{meter} back into the original circuit, R_{meter} would load the other resistors (or, equivalently, the Thévenin resistance), so the Thévenin voltage is not equal to the actual voltage across the meter resistor.

Having derived the Thévenin equivalent circuit, we can now draw the following:



Using the facts that, $R_{meter} = 4k\Omega$, $R_{th} = 1.75k\Omega$, $V_{th} = 2.5V$ we can write:

$$I_{R_{meter}} = \frac{2.5V}{1.75k\Omega + 4k\Omega} \approx 0.43\text{mA}$$
$$V_{R_{meter}} = I_{R_{meter}} R_{meter} \approx 1.74V$$

6. Homework Process and Study Group

Who did you work with on this homework? List names and student ID's. (In case you met people at homework party or in office hours, you can also just describe the group.) How did you work on this homework? If you worked in your study group, explain what role each student played for the meetings this week.

Solution:

I first worked by myself for 2 hours, but got stuck on problem 5. Then I met with my study group.

XYZ played the role of facilitator ... etc. We were still stuck on problem 5 so we went to office hours to talk about the problem.

Then I went to homework party for a few hours, where I finished the homework.

