
EECS 16A Designing Information Devices and Systems I Homework 7

This homework is due October 21, 2022, at 23:59.

Self-grades are due October 24, 2022, at 23:59.

Submission Format

Your homework submission should consist of **one** file.

- `hw7.pdf`: A single PDF file that contains all of your answers (any handwritten answers should be scanned).

Submit the file to the appropriate assignment on Gradescope.

Mid Semester Survey

Please fill out the mid semester survey: <https://forms.gle/dZVb1uAR9zu8msZr8>.

It is optional, but we we highly appreciate your feedback!

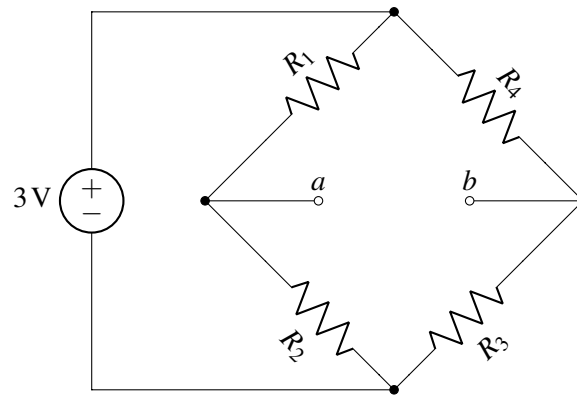
1. Reading Assignment

For this homework, please read [Note 12](#), [Note 13](#), and [Note 14](#). Notes 12 and 13 covers voltage divider, how a simple 1-D resistive touchscreen works, physics of circuits, and introduce the notion of power in electric circuits. Note 14 will cover a slightly more complicated 2-D resistive touchscreens and how to analyze them from a circuits perspective.

- (a) Describe the key ideas behind how the 1D touchscreen works. In general, why is it useful to be able to convert a “physical” quantity like the position of your finger to an electronic signal (i.e. voltage)?

2. Wheatstone Bridge

A Wheatstone Bridge is a very useful circuit that can be used to help determine unknown resistance values with very high accuracy. This circuit is used in many sensor applications where resistors $R_1 - R_4$ are varying with respect to some external actuation. For example, it can be used to build a strain gauge (https://en.wikipedia.org/wiki/Strain_gauge) or a scale. In that case the resistors $R_1 - R_4$ would vary with respect to a strain caused by a force, and the Wheatstone Bridge circuit would translate that variation into a voltage difference across the “bridge” terminals a and b .



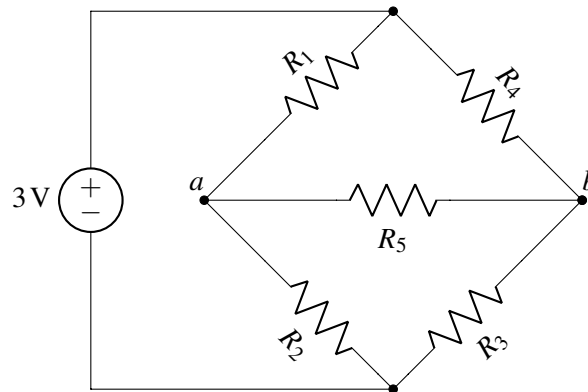
- (a) Assume that $R_1 = 1\text{ k}\Omega$, $R_2 = 2\text{ k}\Omega$, $R_3 = 3\text{ k}\Omega$, $R_4 = 3\text{ k}\Omega$. Calculate the voltage V_{ab} between the two terminals a and b .

Hint: It may help to redraw the circuit with each branch containing resistors $R_1 - R_4$ using straight vertical wires, rather than diagonal ones.

- (b) Now assume that you have added an additional resistor, R_5 , between terminals a and b as shown below. Assume that $R_1 = 1\text{ k}\Omega$, $R_2 = 2\text{ k}\Omega$, $R_3 = 3\text{ k}\Omega$, $R_4 = ?\text{ k}\Omega$, $R_5 = 5\text{ k}\Omega$. In the process of constructing this circuit, you notice that you forgot to write down the value of R_4 !

However, you notice something very curious about your Wheatstone Bridge circuit: there is no current flowing through resistor R_5 .

Based on this observation, what must the value of resistor R_4 be?



3. Volt and ammeter

Learning Goal: This problem helps you explore what happens to voltages and currents in a circuit when you connect voltmeters and ammeters in different configurations.

Use the following numerical values in your calculations: $R_1 = 1\text{ k}\Omega$, $R_2 = 2\text{ k}\Omega$, $R_3 = 3\text{ k}\Omega$, $R_4 = 4\text{ k}\Omega$, $R_5 = 5\text{ k}\Omega$, $V_s = 12\text{ V}$.

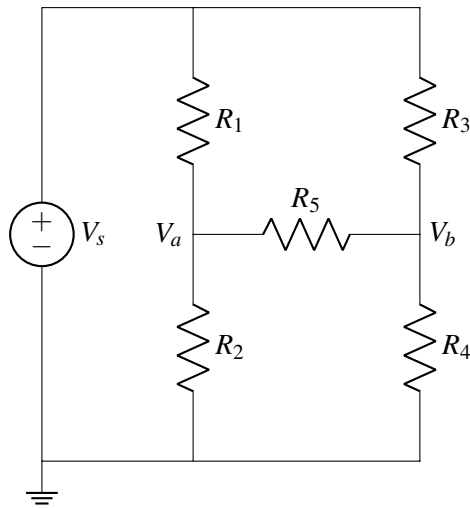


Figure 1: Circuit consisting of a voltage source V_s and five resistors R_1 to R_5

- Redraw the circuit diagram shown in Figure 1 by adding a voltmeter (letter V in a circle and plus and minus signs indicating direction) to measure voltage V_{ab} from node V_a (positive) to node V_b (negative). Calculate the value of V_{ab} . You may use a numerical tool such as IPython to solve the final system of linear equations.
- Suppose you accidentally connect an ammeter in part (a) instead of a voltmeter. Calculate the value of V_{ab} with the ammeter connected.
- Redraw the circuit diagram shown in Figure 1 by adding an ammeter (letter A in a circle and plus and minus signs indicating direction) in series with resistor R_5 . This will measure the current I_{R_5} through R_5 . Calculate the value of I_{R_5} .
- Your friend accidentally connects a voltmeter in part (c) above, rather than an ammeter. Calculate the value of I_{R_5} with the voltmeter connected.

4. Printed electronics

Learning Goal: This problem will help you practice thinking about electronic materials and their properties.

All electronic devices require connections to conduct signals. These connections, or traces, are manufactured through different deposition methods such as physical vapor deposition and chemical vapor deposition. Another less traditional technique is printing. Inks can be made from metallic nanoparticles and deposited using inkjet printing, screen printing, and spray coating. A commonly printed metal ink is silver.

Here's an example of a printed MRI antenna coil from research conducted in Prof. Ana Arias's lab.



- (a) Say we screenprinted a trace of silver 20 mm in length and 5 μm in width. Given the resistivity should be $0.001 \Omega\text{mm}$, and we measure the resistance of the trace to be 250Ω , what is the trace thickness?
- (b) Nanoparticle inks often require a drying step called *sintering*, during which the nanoparticles coalesce and form conductive pathways. The manufacturer of our silver paste lists 100°C and 175°C as two possible sintering temperatures resulting in resistivities of $0.001 \Omega\text{mm}$ and $0.5 \Omega\mu\text{m}$. Regardless of what you obtained in part (a), assume that we need a trace 20 mm in length, 4 μm in width, and 20 μm in thickness, what is the smallest resistance trace we can obtain and with which sintering temperature?
- (c) Say the maximum resistance we can tolerate is 125Ω . What would the cross sectional areas required be from both sintering temperatures to achieve the specified resistance for our 20 mm long trace?
- (d) Continuing with the design specifications from part (c), if our printing technique has a resolution limit of one micron (meaning the minimum width and minimum length achievable is one micron) and we want to aim for a trace thickness of at least one hundred micron for good film uniformity, then at which temperature should we sinter our printed silver?
- (e) One unique advantage of using printing as a deposition technique is that electronic devices can be fabricated on plastic flexible substrates rather than brittle silicon wafers, allowing for applications where lightweight, conformable electronics are needed. However, when heated, plastic substrates can begin to soften and deform. Using your answer from part (c) and part (d) what is one drawback from the lower sintering temperature, and what is one drawback from the higher sintering temperature?
- (f) [Challenge] Your manufacturing process wasn't perfect and your resulting trace increases its thickness linearly along the trace, such that the initial trace thickness is 20 μm and the final thickness is 30 μm . Can you compute the resulting resistance of the trace? Assume the trace length is 20 mm, width is 4 μm , and resistivity is $0.001 \Omega\text{mm}$.

Hint: We can write our resistance in a differential form: $dR = \rho(l) \frac{dl}{A(l)}$. Can we add up all these differential segments of resistance over the trace to get our final resistance value?

5. Resistive Touchscreen

Learning Goal: The objective of this problem is to provide insight into modeling of resistive elements. This will also help to apply the concepts from resistive touchscreen.

In this problem, we will investigate how a resistive touchscreen with a defined thickness, width, and length can actually be modeled as a series combination of resistors. As we know the value of a resistor depends on its length.

Figure 2 shows the top view of a resistive touchscreen consisting of a conductive layer with resistivity ρ_1 , thickness t , width W , and length L . At the top and bottom it is connected through perfect conductors ($\rho = 0$) to the rest of the circuit. The touchscreen is wired to voltage source V_s .

Use the following numerical values in your calculations: $W = 50$ mm, $L = 80$ mm, $t = 1$ mm, $\rho_1 = 2\Omega\text{m}$, $V_s = 10\text{V}$, $x_1 = 20$ mm, $x_2 = 45$ mm, $y_1 = 30$ mm, $y_2 = 60$ mm.

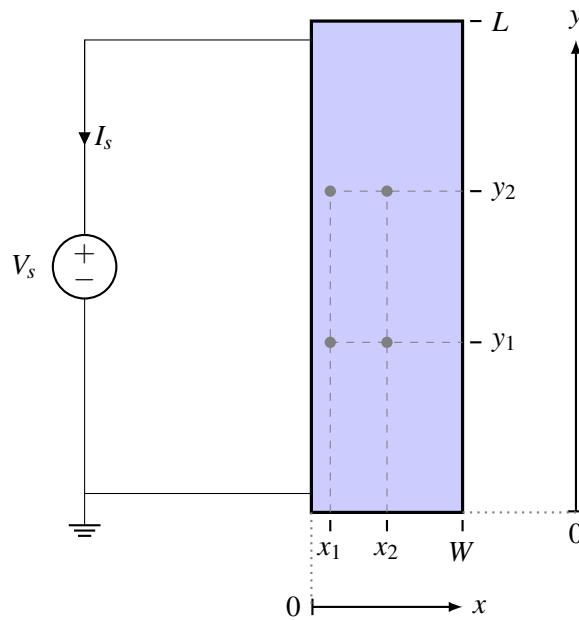
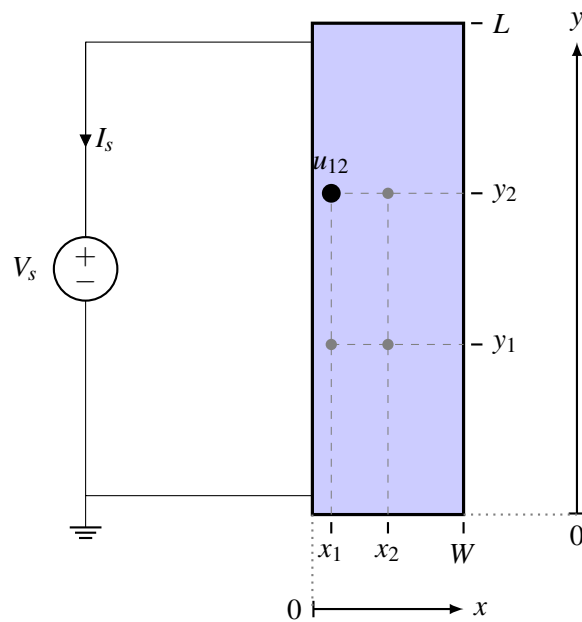


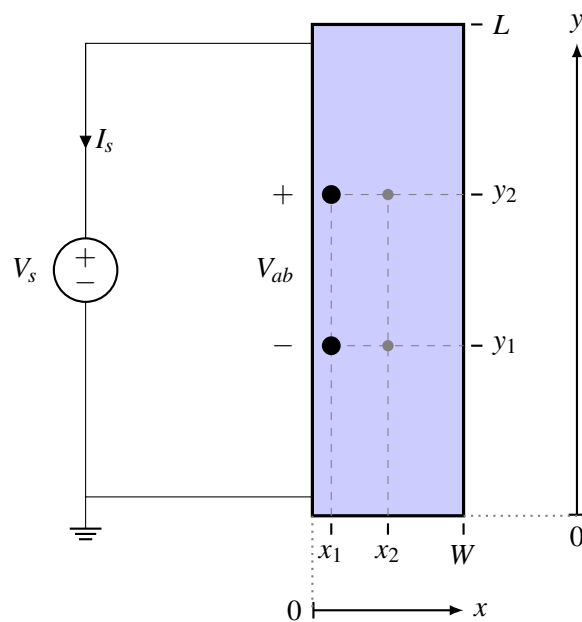
Figure 2: Top view of resistive touchscreen (not to scale). z axis i.e. the thickness not shown (into the page).

- Draw a circuit diagram representing **Figure 2**, where the entire touchscreen is represented as a *single resistor*. **Note that no touch is occurring in this scenario.** Remember that circuit diagrams in general consist of only circuit elements (resistors, sources, etc) represented by symbols, connecting wires, and the reference/ ground symbol. Calculate the value of current I_s based on the circuit diagram you drew. Do not forget to specify the correct unit as always, and double check the direction of I_s !
- Let us assume u_{12} is the node voltage at the node represented by coordinates (x_1, y_2) of the touchscreen, as shown in **Figure 3**. What is the value of u_{12} ? You should first draw a circuit diagram representing Figure 3, which includes node u_{12} . Specify all resistance values in the diagram. Does the value of u_{12} change based on the value of the x -coordinate x_1 ?

Hint: You will need more than one resistor to represent this scenario.

Figure 3: Top view of resistive touchscreen showing node u_{12} .

- (c) Assume V_{ab} is the voltage measured between the nodes represented by touchscreen coordinates (x_1, y_1) and coordinates (x_1, y_2) , as shown in **Figure 4**. Calculate the absolute value of V_{ab} . As with the previous part, you should first draw the circuit diagram representing Figure 4, which includes V_{ab} . Calculate all resistor values in the circuit. *Hint: Try representing the segment of the touchscreen between these two coordinates as a separate resistor itself.*

Figure 4: Top view of resistive touchscreen showing voltage V_{ab} .

- (d) Calculate (the absolute value of) the voltage between the nodes represented by touchscreen coordinates (x_1, y_1) and coordinates (x_2, y_1) in **Figure 4**.
- (e) Calculate (the absolute value of) the voltage between the nodes represented by touchscreen coordinates (x_1, y_1) and coordinates (x_2, y_2) in **Figure 4**.
- (f) **Figure 5** shows a new arrangement with two touchscreens. The two touchscreens are next to each other and are connected to the voltage source in the same way. The second touchscreen (the one on the right) is identical to the one shown in Figure 2, except for different width, W_2 , and resistivity, ρ_2 . Use the following numerical values in your calculations: $W_1 = 50$ mm, $L = 80$ mm, $t = 1$ mm, $\rho_1 = 2 \Omega\text{m}$, $V_s = 10\text{V}$, $x_1 = 20$ mm, $x_2 = 45$ mm, $y_1 = 30$ mm, $y_2 = 60$ mm, which are the same values as before. The new touchscreen has the following numerical values which are different: $W_2 = 85$ mm, $\rho_2 = 1.5 \Omega\text{m}$. Draw a circuit diagram representing **Figure 5**, where the two touchscreens are represented as *two separate resistors*. **Note that no touch is occurring in this scenario.**

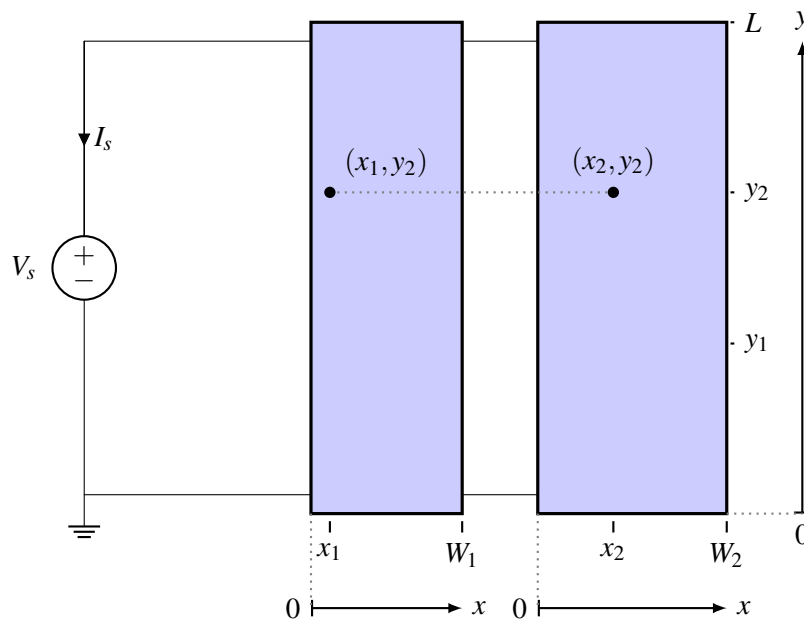


Figure 5: Top view of two touchscreens wired in parallel (not to scale). z axis not shown (into the page).

- (g) Calculate the value of current I_s for the two touchscreen arrangement based on the circuit diagram you drew in the last part.
- (h) Consider the two points: (x_1, y_2) in the touchscreen on the left, and (x_2, y_2) in the touchscreen on the right in **Figure 5**. Show that the node voltage at (x_1, y_2) is the same that at (x_2, y_2) , i.e. the potential difference between the two points is 0. You can show this without explicitly calculating the node voltages at the two points.
- If you were to connect a wire between the two coordinates (x_1, y_2) in the touchscreen on the left, and (x_2, y_2) in the touchscreen on the right, would any current flow through this wire?

6. Temperature Sensor

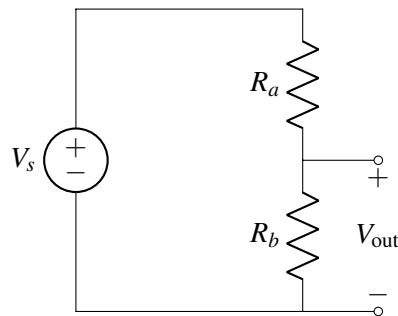
Learning Goal: This problem will let you apply the tools we have learned so far to a real world circuits application.

Measuring quantities in the physical world is the job of sensors. This means somehow extracting that information from the world and then converting it into a form that can be observed and processed. Electrical circuits can be very useful for doing this.

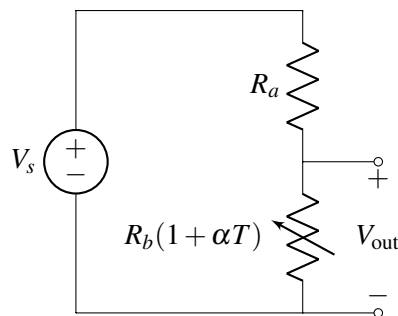
For most materials, resistance increases with increasing temperature; that is, a resistor has higher resistance when it is hot than when it is cold. This is often an annoyance to circuit designers who want their circuits to work the same way at different temperatures, but this fact can also be useful. It allows us to convert temperature, a “physical” quantity, into resistance, an “electrical” quantity, to build an electronic thermometer.

In this problem, we are going to explore how effective a particular circuit made out of various types of resistors is at allowing us to measure temperature.

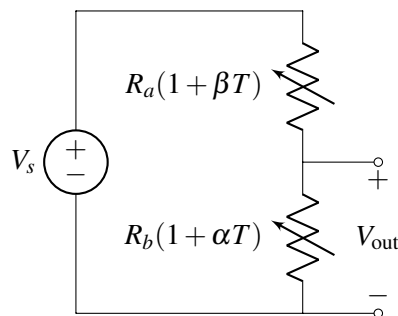
- (a) Let’s begin by analyzing a common topology, the voltage divider shown below. Find an equation for the voltage V_{out} in terms of R_a , R_b , and V_s .



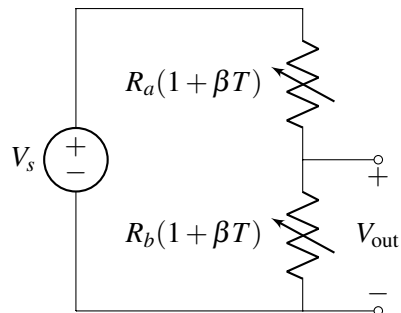
- (b) Now let’s suppose that R_a is an ideal resistor that does not depend on temperature, but R_b is a temperature-dependent resistor whose resistance R is set by $R'_b = R_b(1 + \alpha T)$, where T is the absolute temperature. Find an equation for the temperature T in terms of the voltage V_{out} , V_s , R_a , R_b , and α .



- (c) It turns out that almost all resistors have some temperature dependence. Consider the same circuit as before, but now, R'_a has a temperature dependence given by $R'_a = R_a(1 + \beta T)$. Find an equation for the temperature T in terms of the voltage V_{out} , R_a , R_b , V_s , α , and β .



- (d) Your colleague who has not taken EECS16A thinks that they can improve this circuit's ability to measure temperature by making both resistors depend on temperature in the same way. He hence came up with the circuit shown below, where both R_a and R_b have nominally different values, but both vary with temperature as a function of $(1 + \beta T)$. Can this circuit be used to measure temperature? Why or why not?



7. Prelab Questions

These questions pertain to the Pre-Lab reading for the Touch 2 lab. You can find the reading under the Touch 2 Lab section on the 'Schedule' page of the website.

- How many layers are there in the touchscreen and what are they made of?
- Provide 2 examples of resistive touchscreens (give one example not listed on the pre-lab reading).
- In the circuit given in the reading, what is the current i_3 flowing through resistor R_{h1} ?
- How do we get touch coordinates in the horizontal direction if you have your circuit that works in the vertical direction?

8. Homework Process and Study Group

Who did you work with on this homework? List names and student ID's. (In case you met people at homework party or in office hours, you can also just describe the group.) How did you work on this homework? If you worked in your study group, explain what role each student played for the meetings this week.