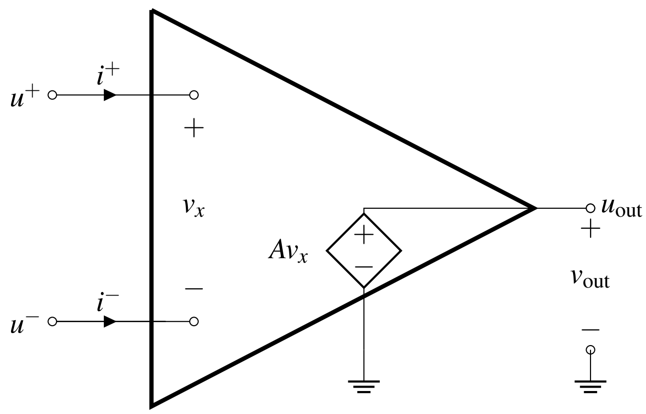
EECS 16A Designing Information Devices and Systems I
Spring 2023 Discussion 11B

1. Op-Amp Rules

Here is an equivalent circuit of an op-amp (where we are assuming that $V_{SS} = -V_{DD}$) for reference:



(a) What are the currents flowing into the positive and negative terminals of the op-amp (i.e., what are i^+ and i^-)? Based on this answer, what are some of the advantages of using an op-amp in your circuit designs?

Answer:

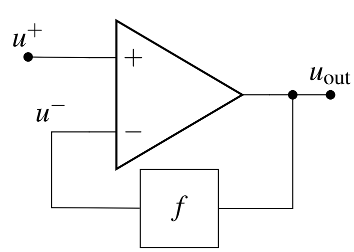
The u^+ and u^- terminals have no closed circuit connection between them, and therefore no current can flow into or out of them. This is very good because we can connect an op-amp to any other circuit, and the op-amp will not disturb that circuit in any way because it does not load the circuit (it is an open circuit).

(b) Suppose we add a resistor of value R_L between u_{out} and ground. What is the value of v_{out}? Does your answer depend on R_L? In other words, how does R_L affect Av_C? What are the implications of this with respect to using op-amps in circuit design?

Answer:

Notice that u_{out} is connected directly to a controlled/dependent voltage source, and therefore v_{out} will always have to be equal to Av_x regardless of what R_L is connected to the op-amp. This is useful because it means that the output voltage of the op-amp is only dependent on the differential input voltage v_x and not dependent on what is connected to the output node u_{out}.

(c) Now suppose our op-amp is connected in negative feedback.



What is the relationship between u^+ and u^-?

Answer: By the 2nd golden rule of op-amps, we know that if an op-amp is in negative feedback, the inputs to the op-amp are the same. In other words

$$u^+ = u^-.$$

Lets prove this mathematically. Assuming that the gain of the op-amp is A, we know that

$$u^- = f u_{\text{out}}$$
$$u_{\text{out}} = A(u^+ - u^-).$$

Substituting u_{out} and combining the two equations we have

$$u^- = fA(u^+ - u^-)$$

which we can rearrange to get

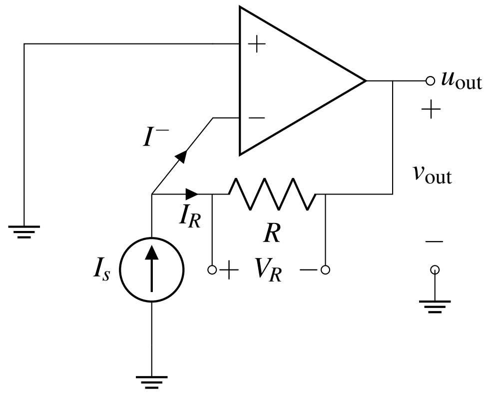
$$\frac{u^+}{u^-} = \frac{1+Af}{Af}.$$

If Af is very large, then we see that

$$\lim_{Af\to\infty} \frac{1+Af}{Af} = 1$$

which means $u^+ = u^-$. In practice, A is very large and as long as we choose a reasonable value of f (i.e. not too small) then our approximation holds.

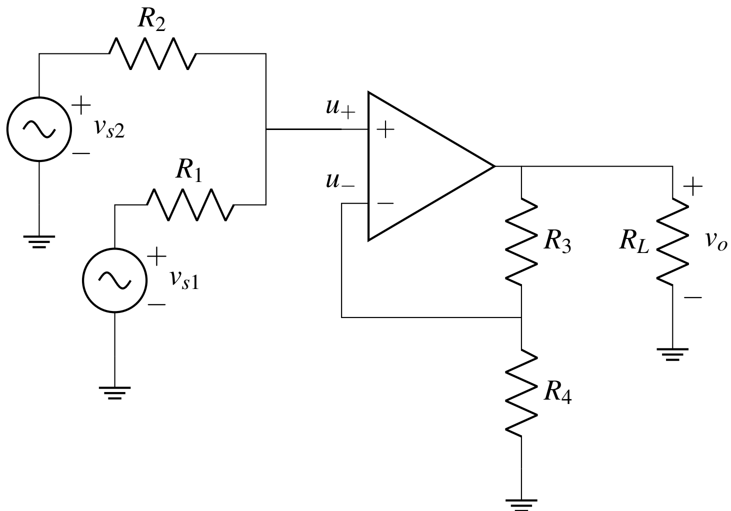
2. A Trans-Resistance Amplifier



Calculate v_{out} as a function of I_s and R.

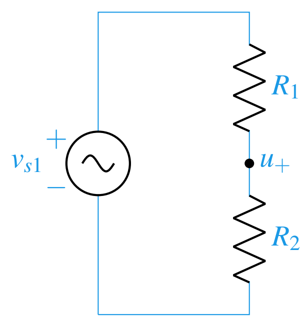
Hint: First show that the op-amp is in negative feedback and then apply the golden rules.

3. Multiple Inputs To One Op-Amp



(a) First, let's focus on the left part of the circuit containing the voltage sources v_{s1} and v_{s2}, and resistances R_1 and R_2. Solve for u_+ in the circuit above. *(Hint: Use superposition.)*

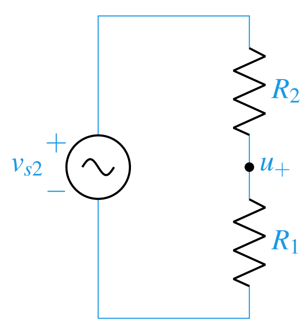
Answer: Let's call the potential at the positive input of the op-amp u_+. Using superposition, we first turn off v_{s2} and find u_+. The circuit then looks like:



We recognize the above circuit as a voltage divider. Thus,

$$u_{+,vs1} = \frac{R_2}{R_1 + R_2} v_{s1}$$

By symmetry, we expect v_{s2} to have a similar circuit and expression. The circuit for v_{s2} looks like:



The expression for u_+ with v_{s2} is then:

$$u_{+,vs2} = \frac{R_1}{R_1+R_2} v_{s2}$$

From superposition, we know the output must be the sum of these.

$$u_+ = \frac{R_2}{R_1+R_2} v_{s1} + \frac{R_1}{R_1+R_2} v_{s2}$$

(b) How would you choose R_1 and R_2 that produce a voltage $u_+ = \frac{1}{2}V_{s1} + \frac{1}{2}V_{s2}$? Could you also achieve $u_+ = \frac{1}{3}V_{s1} + \frac{2}{3}V_{s2}$?

Answer:

We found that the output voltage for any two resistors R_1 and R_2 is:

$$v_+ = \frac{R_2}{R_1+R_2} V_1 + \frac{R_1}{R_1+R_2} V_2$$

Thus, to create the $\frac{1}{2}$ - $\frac{1}{2}$ ratio, we can use any nonzero resistances with value R such that:

$$R_1 = R \qquad R_2 = R$$

Similarly, to create the $\frac{1}{3}$ - $\frac{2}{3}$ ratio, we can use:

$$R_1 = 2R \qquad R_2 = R$$

In general, you can achieve anything of the form $u_+ = kV_1 + (1-k)V_1$ with $k \in (0,1)$! This is a very useful topology to combine two voltages.

(c) Now, for the whole circuit, find an expression for v_o.

Answer:

With u_+ determined, we can find the output voltage directly from the formula for a non-inverting amplifier. We can also derive it using the process below.

From the negative feedback rule, $u_+ = u_-$. Using voltage dividers, we can express u_- in terms of v_o:

$$u_- = \frac{R_4}{R_3+R_4} v_o$$

$$v_o = \left(1 + \frac{R_3}{R_4}\right) u_- = \left(1 + \frac{R_3}{R_4}\right) u_+$$

Now, to find the final output, we can set u_+ to our earlier expression.

$$v_o = \left(1 + \frac{R_3}{R_4}\right) \left(\frac{R_2}{R_1+R_2} v_{s1} + \frac{R_1}{R_1+R_2} v_{s2}\right)$$

(d) How should we select our values R_1, R_2, R_3, R_4 to find the sum of different signals, i.e. $V_{s1} + V_{s2}$? What about taking the sum and multiplying by 2, i.e. $2(V_{s1} + V_{s2})$?

Answer:

The circuit already finds the weighted sum of two inputs. By setting $R_1 = R_2$ and $R_3 = R_4$, we can take the exact sum of two inputs.

$$v_o = \left(1 + \frac{R_3}{R_4}\right) \left(\frac{R_2}{R_1+R_2} v_{s1} + \frac{R_1}{R_1+R_2} v_{s2}\right) = (1+1) \left(\frac{1}{2} v_{s1} + \frac{1}{2} v_{s2}\right) = v_{s1} + v_{s2}$$

Notice that the first half of this circuit (R_1 and R_2) form a voltage summer with coefficients less than one; the second half is just a non-inverting amplifier. Thus we can always use R_1 and R_2 to take an equally weighted sum of the inputs and then multiply greater than 1 using the non-inverting amplifier. If we set $R_1 = R_2$, we get $\left(\frac{1}{2}v_{s1} + \frac{1}{2}v_{s2}\right)$ into the op-amp. To get a total gain of 2, then the non-inverting op-amp needs a gain of 4, so we can pick $R_3 = 3R_4$.

