

THIS IS YOUR MACHINE LEARNING SYSTEM?

YUP! YOU POUR THE DATA INTO THIS BIG  
PILE OF LINEAR ALGEBRA, THEN COLLECT  
THE ANSWERS ON THE OTHER SIDE.

WHAT IF THE ANSWERS ARE WRONG?

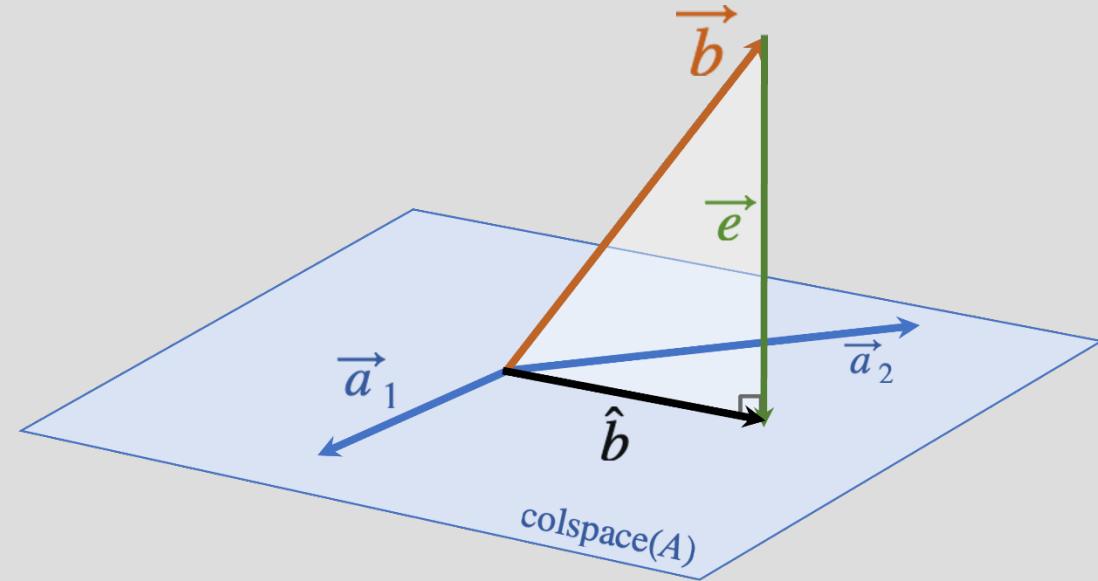
JUST STIR THE PILE UNTIL  
THEY START LOOKING RIGHT.



**EECS 16A**  
**fun stuff!**

# Overdetermined system: use least squares

$$A \quad x = b$$

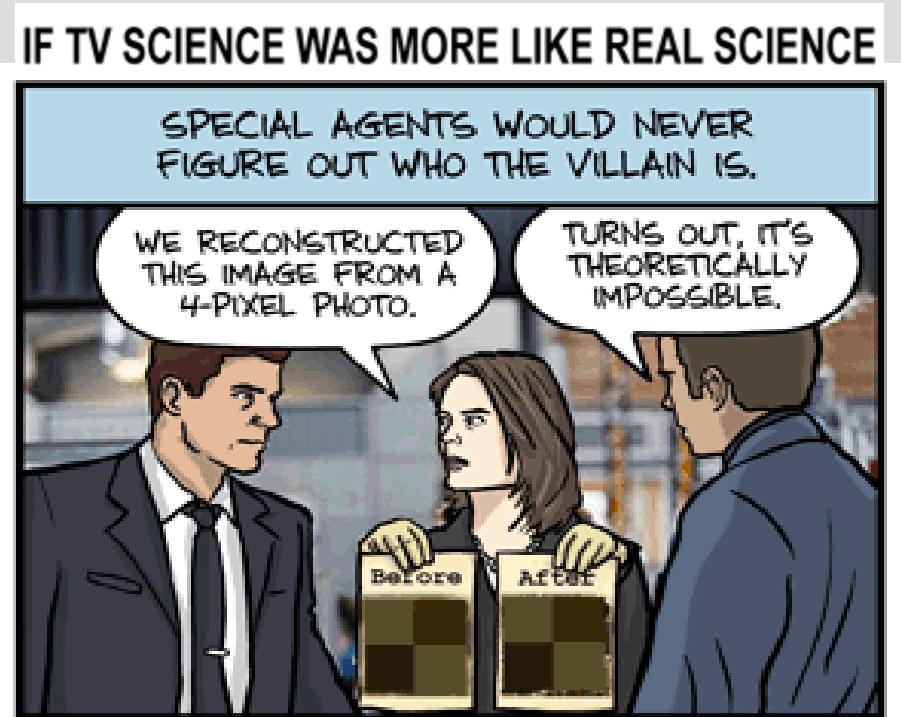


- the least-squares solution “minimally perturbs”  $b$

$$\hat{x} = (A^T A)^{-1} A^T \vec{b}$$

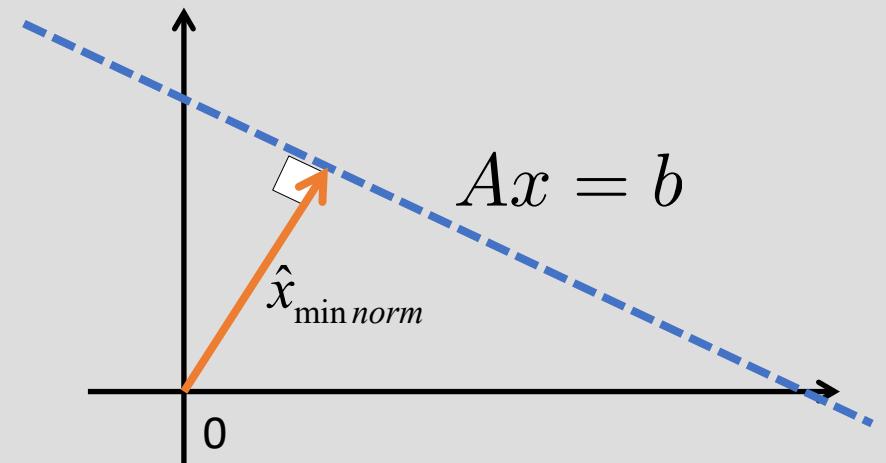
# Underdetermined system: ???

$$A \quad x = b$$



- Can be infinite valid solutions!
- Ideas: pick the ‘smallest’ one? The ‘sparsest’?
  - e.g. min norm:

$$\hat{x}_{\min norm} = \mathbf{A}^T (\mathbf{A}\mathbf{A}^T)^{-1} \vec{b}$$



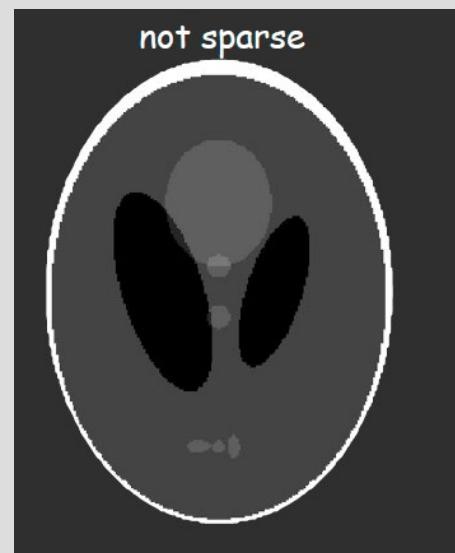
# 'Sparsity' tells us how 'dense' the solution is

Dense Matrix											
1	2	31	2	9	7	34	22	11	5		
11	92	4	3	2	2	3	3	2	1		
3	9	13	8	21	17	4	2	1	4		
8	32	1	2	34	18	7	78	10	7		
9	22	3	9	8	71	12	22	17	3		
13	21	21	9	2	47	1	81	21	9		
21	12	53	12	91	24	81	8	91	2		
61	8	33	82	19	87	16	3	1	55		
54	4	78	24	18	11	4	2	99	5		
13	22	32	42	9	15	9	22	1	21		

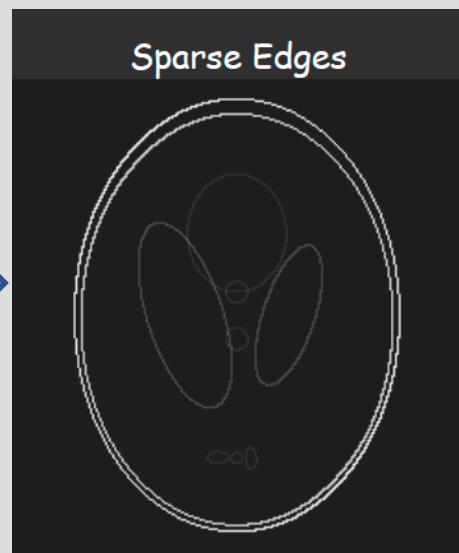
Sparse Matrix											
1	.	3	.	9	.	3	.	.	.		
11	.	4	.	.	.	.	.	2	1		
.	.	1	.	.	.	4	.	1	.		
8	.	.	.	3	1	.	.	.	.		
.	.	.	9	.	.	1	.	17	.		
13	21	.	9	2	47	1	81	21	9		
.	.	.	.	.	.	.	.	.	.		
.	.	.	.	19	8	16	.	.	55		
54	4	.	.	.	11	.	.	.	.		
.	.	2	.	.	.	22	.	.	21		

The fraction of non-zero elements in a matrix is called the **sparsity**



Take |derivative|

Sometimes things are sparse in a different way



# Example: image compression

Reduce memory by smartly choosing which information to throw away



No compression



23:1 compression



144:1 compression



bus.rar

# Sparse $x$ means only a few columns of $A$ ‘matter’

$$\begin{matrix} \text{A sparse matrix } A \\ \text{with many zero entries} \end{matrix} = \begin{matrix} \text{A tall, narrow column vector } x \\ \text{with many zero entries} \end{matrix}$$

If we knew which elements were non-zero, we could solve a small least squares problem:

$$\begin{matrix} \text{Matrix A} & \times & \text{Vector b} & = & \text{Vector y} \\ \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & -2 & 1 \\ 3 & -2 & 1 & 2 \\ 4 & 1 & 2 & -1 \end{bmatrix} & \times & \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} & = & \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \end{matrix}$$

# Can we compress data at the capture stage?

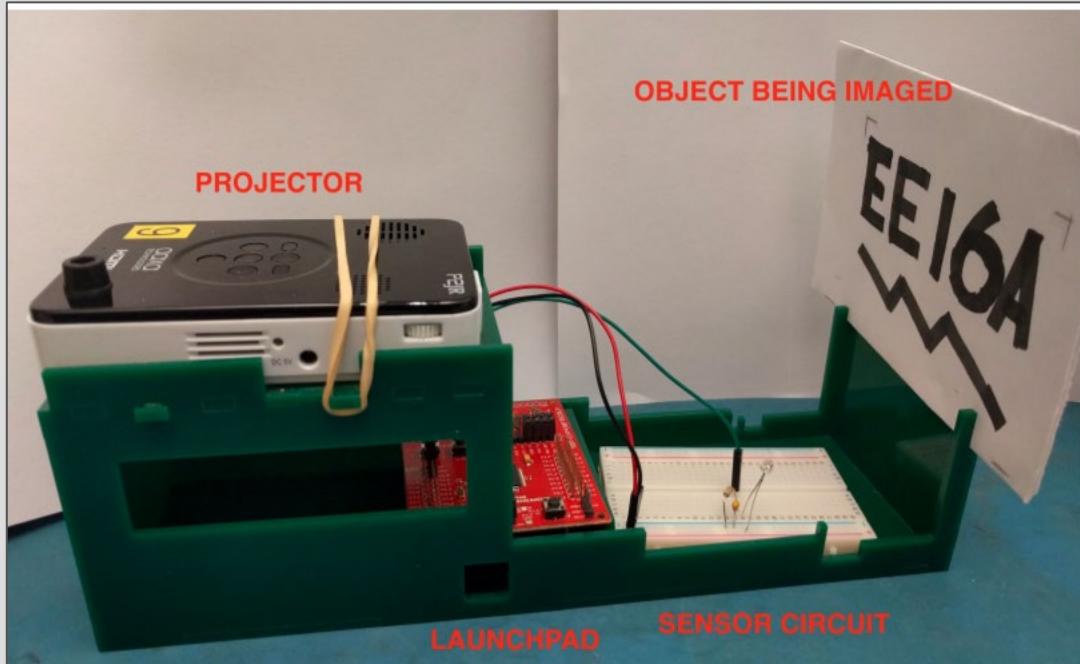


compress

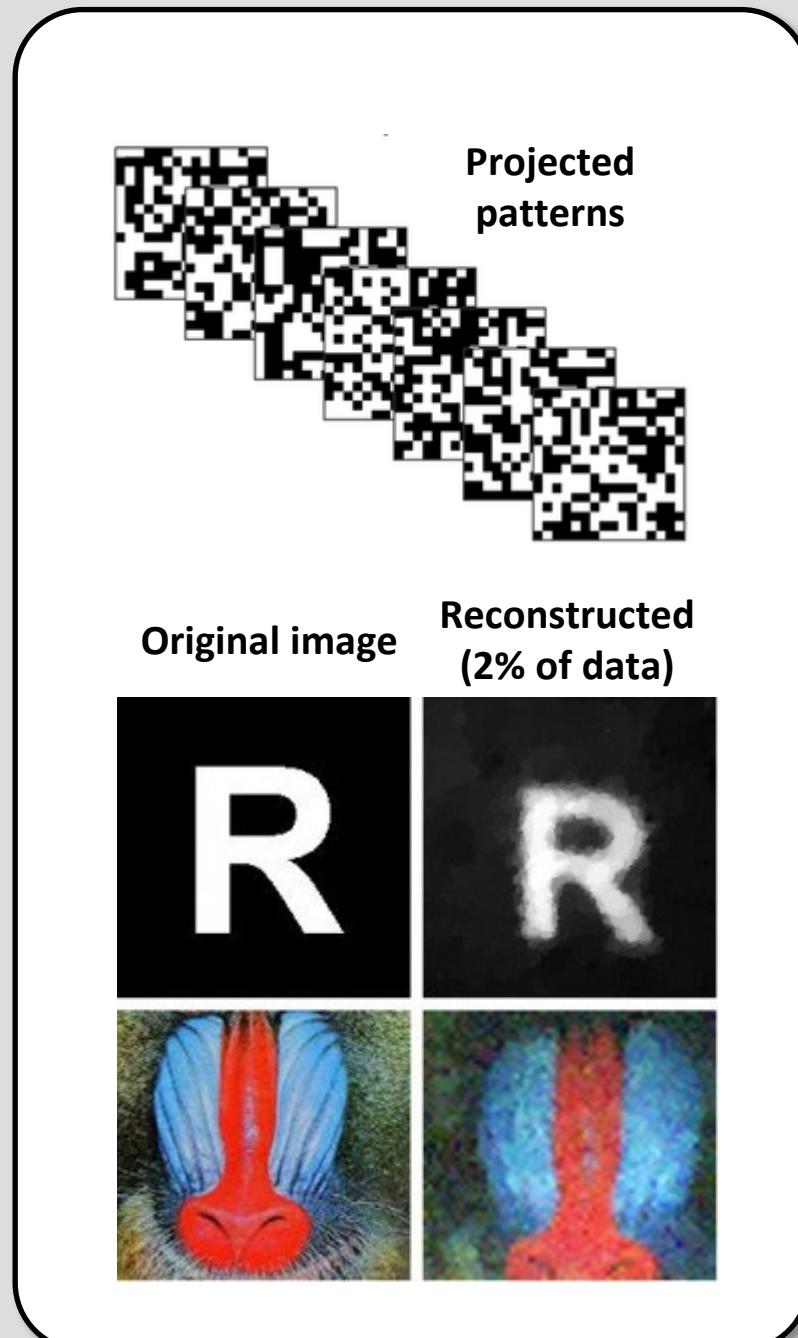


# Yes! With compressed sensing!

## Example: single-pixel camera



If you design the patterns on your imaging lab well, and images are compressible, you could solve with very little data!



# We usually take direct measurements

$$\begin{matrix} b \\ = \\ \text{A} \\ x \end{matrix}$$

# Multiplexed measurements

$$\begin{matrix} b \\ \hline \end{matrix} = \begin{matrix} A \\ \hline \end{matrix} \begin{matrix} x \\ \hline \end{matrix}$$

What makes a good A matrix?

A is “orthogonal”

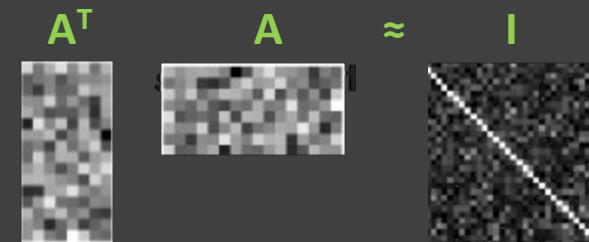
$$A^T \quad A \quad = \quad I$$

# Compressed sensing solves underdetermined problems

$$\begin{matrix} b \\ \hline \end{matrix} = \begin{matrix} A \\ \hline \end{matrix} \begin{matrix} x \\ \hline \end{matrix}$$

What makes a good  $A$  matrix?

$A$  is (almost) orthogonal

$$A^T \quad A \quad \approx \quad I$$


# Computational Imaging

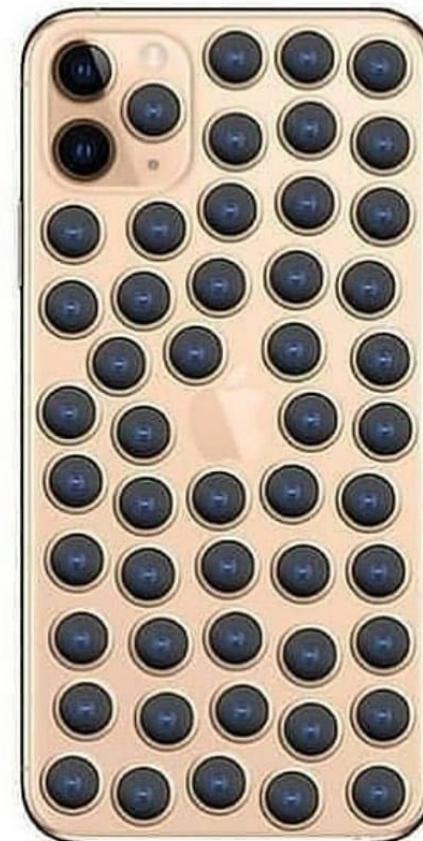
**2019**

iPhone 11 Pro



**2029**

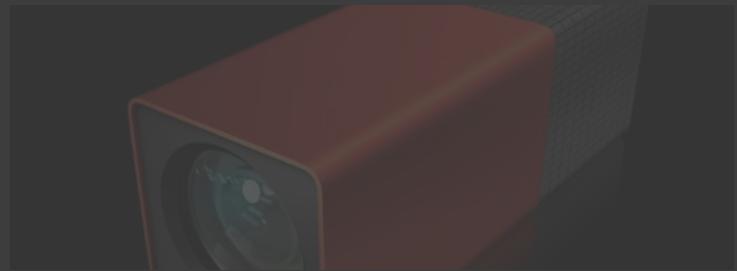
iPhone 21 Pro



# Computational Imaging @ Berkeley

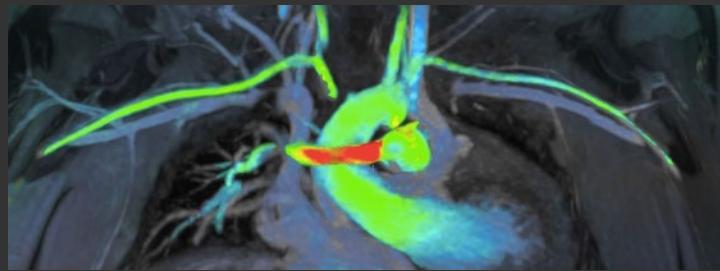
Light Field Cameras

Ren Ng



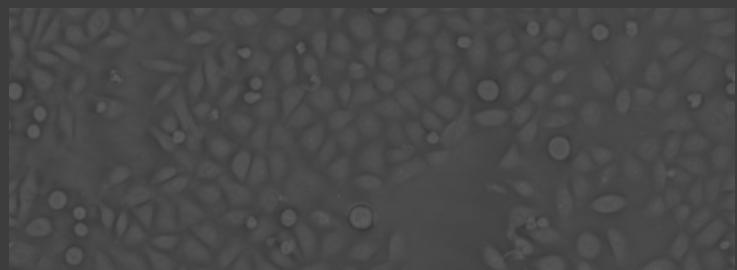
Compressed Sensing MRI

Michael Lustig



Comp. Illumination Microscopy

Laura Waller



Full-stack Optimization

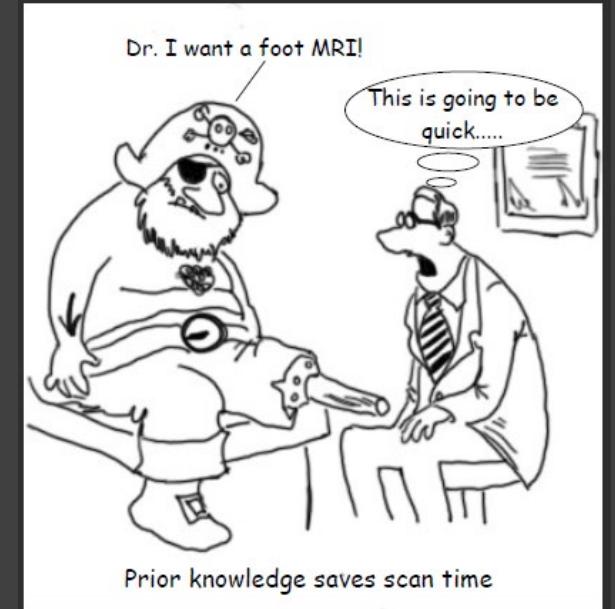
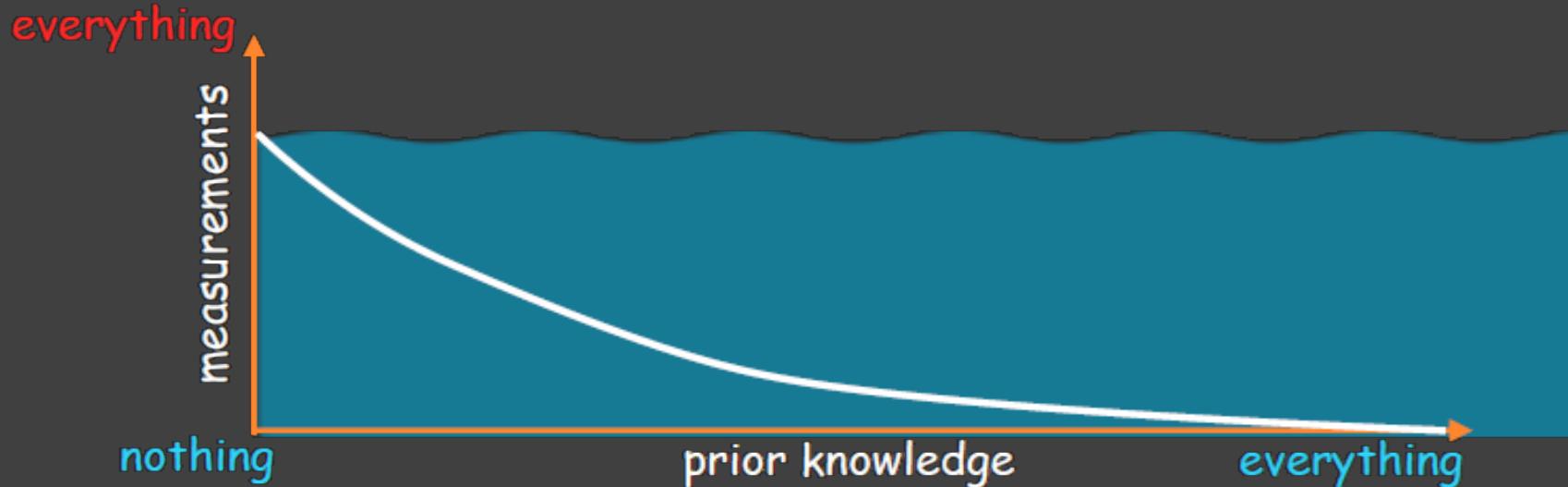
Ben Recht



$$x(s) = \text{PSF} \circledast \left\{ \sum_{j=1}^k c_j \delta(s - s_j) \right\}$$

# Compressed sensing is all about using prior knowledge

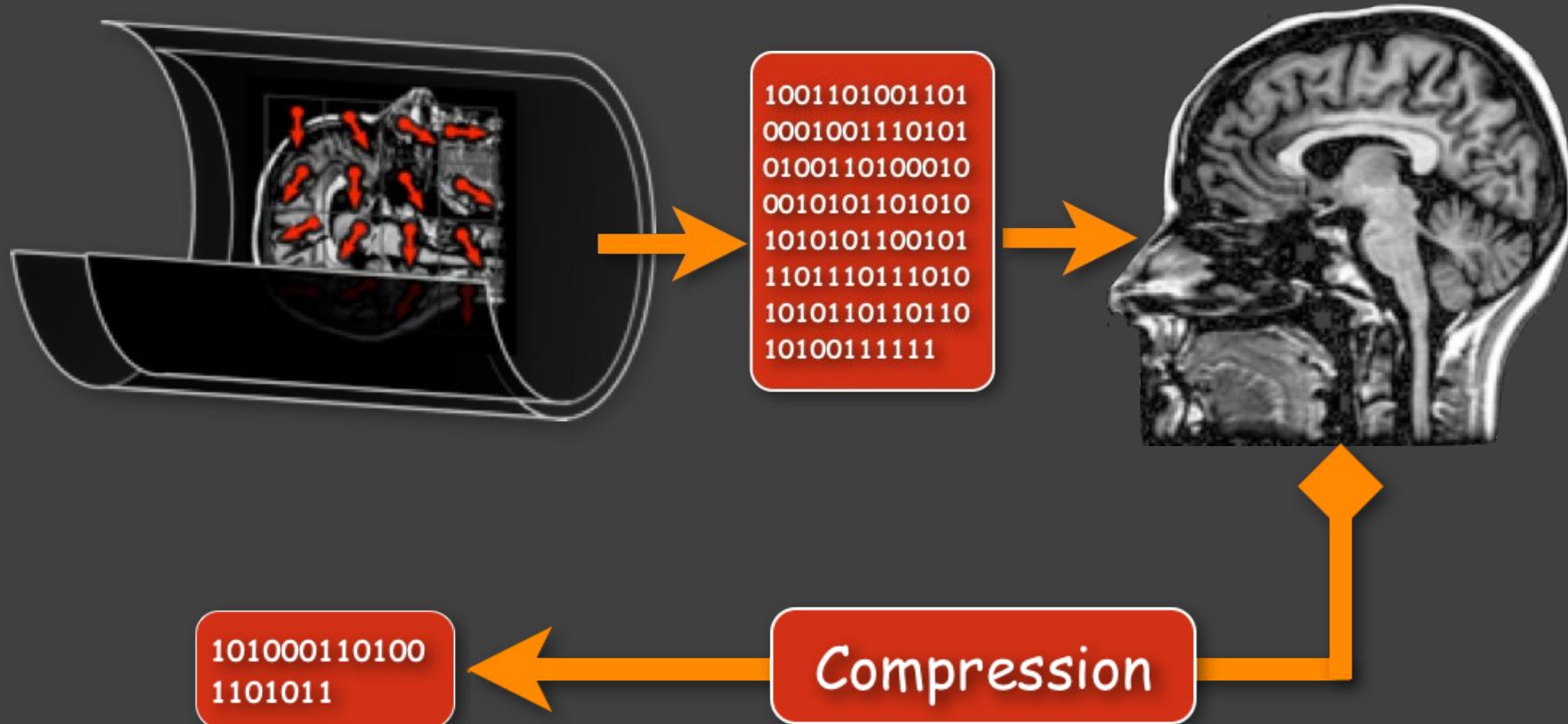
- Redundancy reduces sampling requirements  
(The more you know, the less you need)



# Compressed Sensing MRI

Medical images are compressible

Standard approach: First collect, then compress

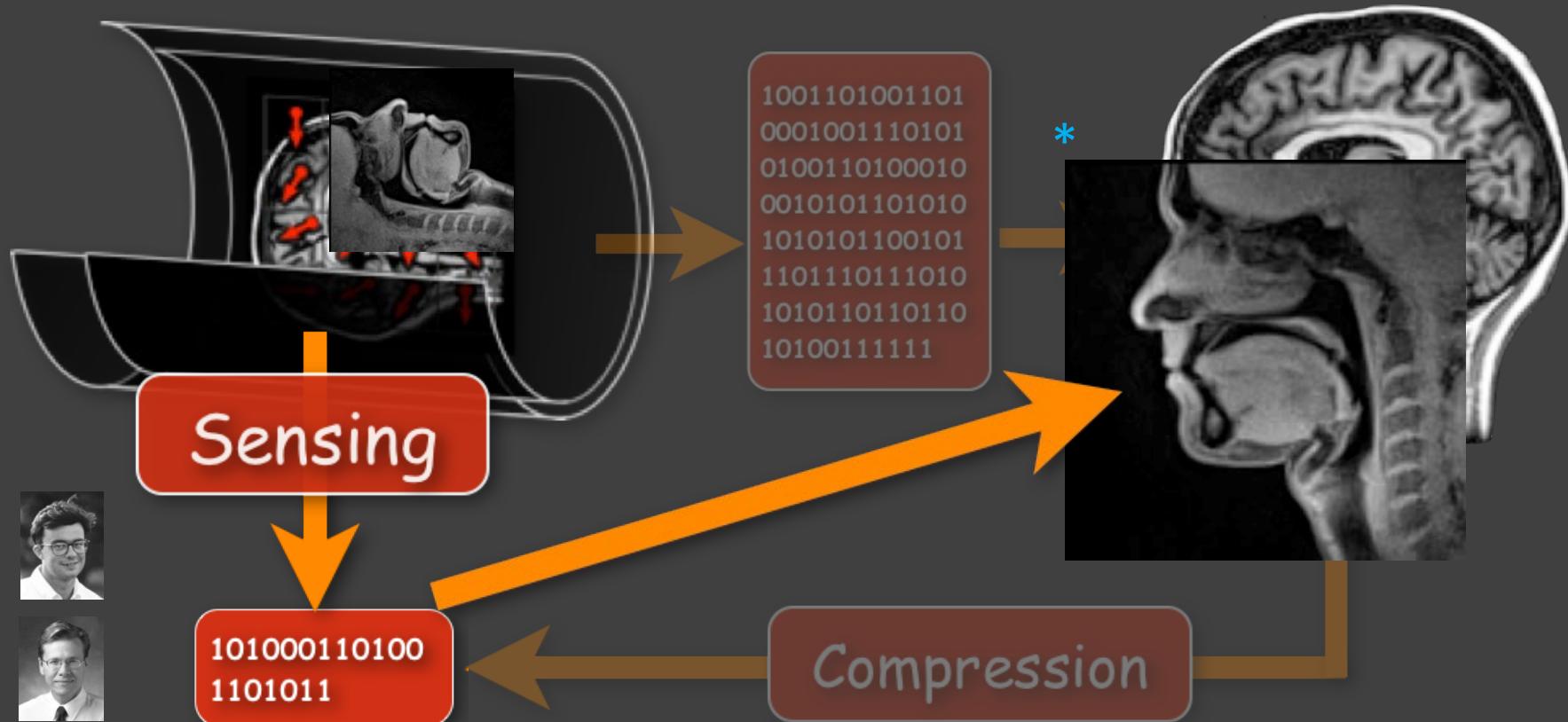


Michael Lustig's Lab

# Compressed Sensing MRI

Medical images are compressible

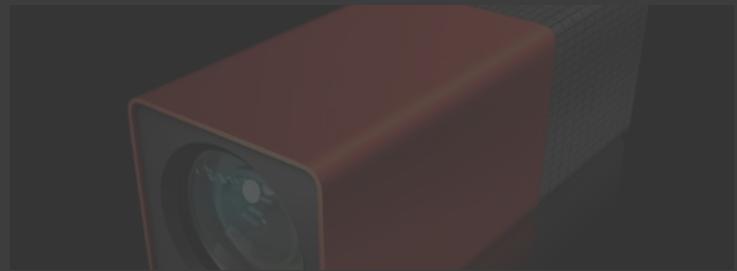
New approach: Acquire “compressed” data directly!



# Computational Imaging @ Berkeley

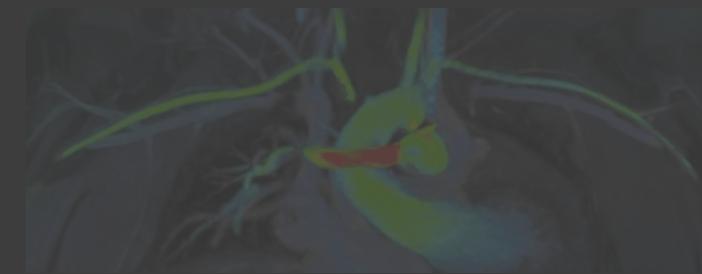
Light Field Cameras

Ren Ng



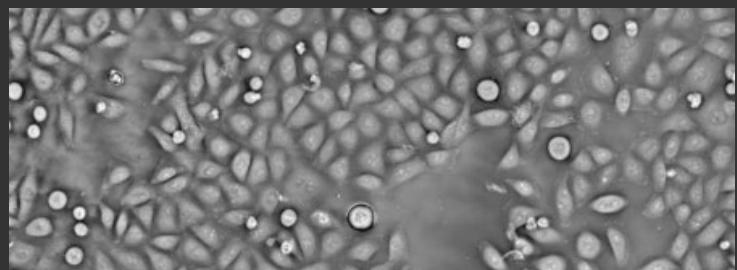
Compressed Sensing MRI

Michael Lustig



Comp. Illumination Microscopy

Laura Waller



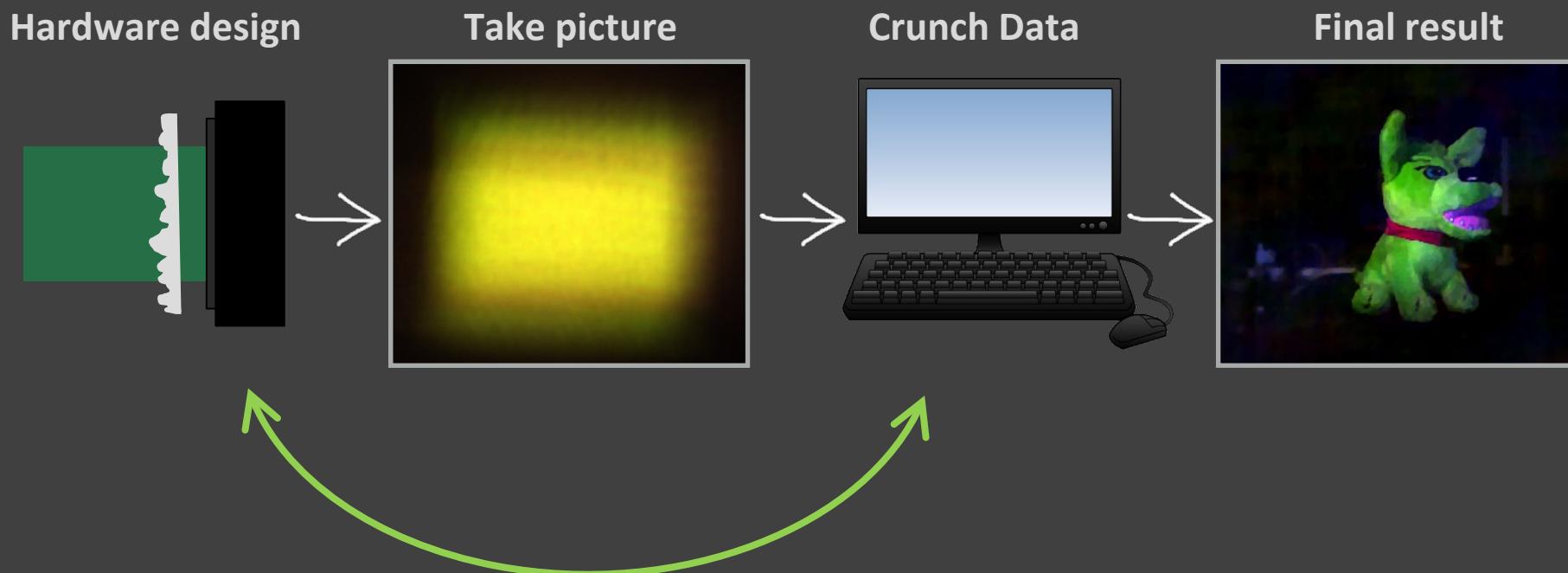
Full-stack Optimization

Ben Recht

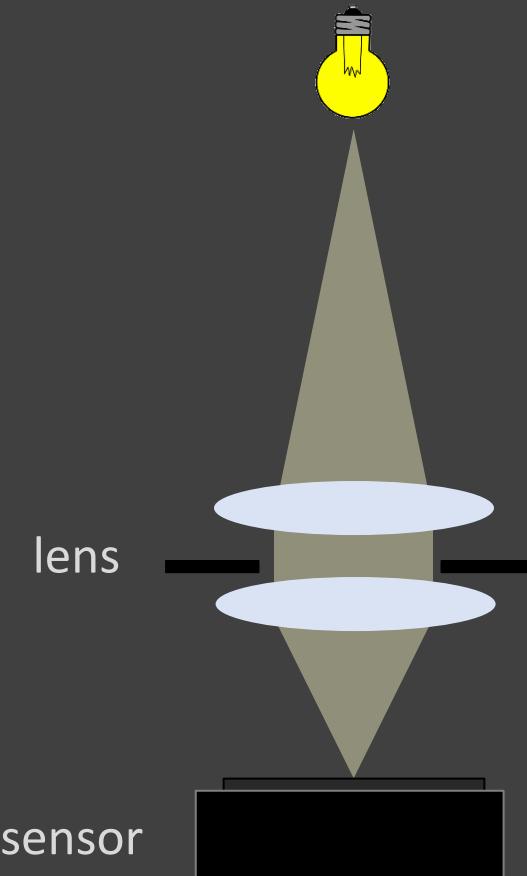


$$x(s) = \text{PSF} \circledast \left\{ \sum_{j=1}^k c_j \delta(s - s_j) \right\}$$

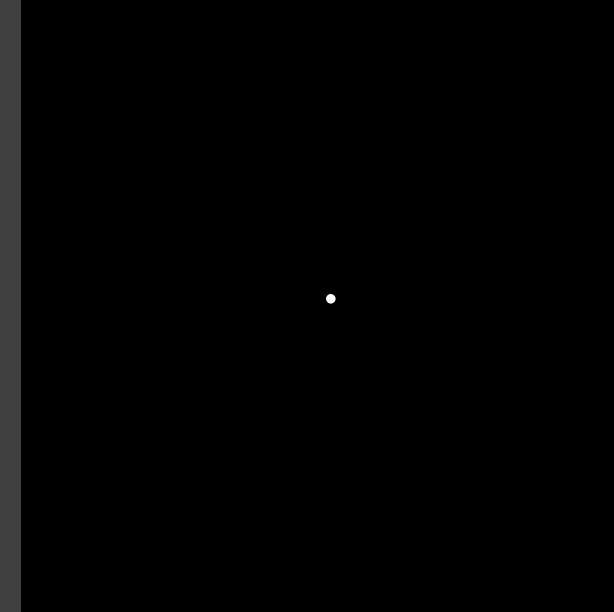
# Computational imaging pipeline



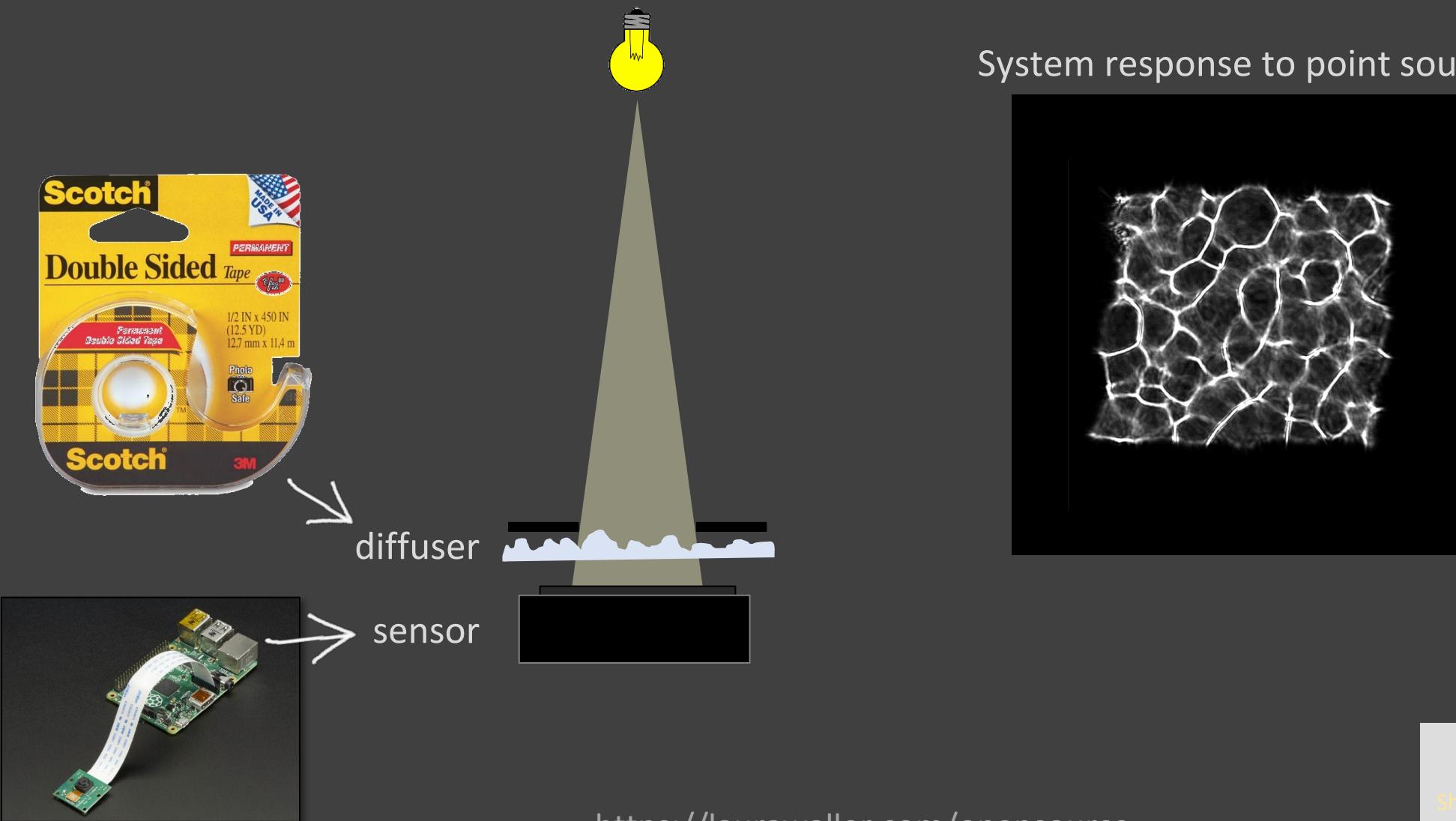
# Lenses map points to points



System response to point source



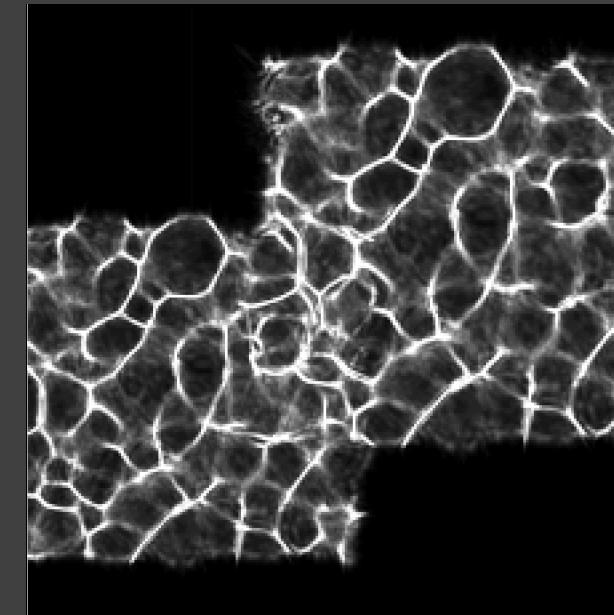
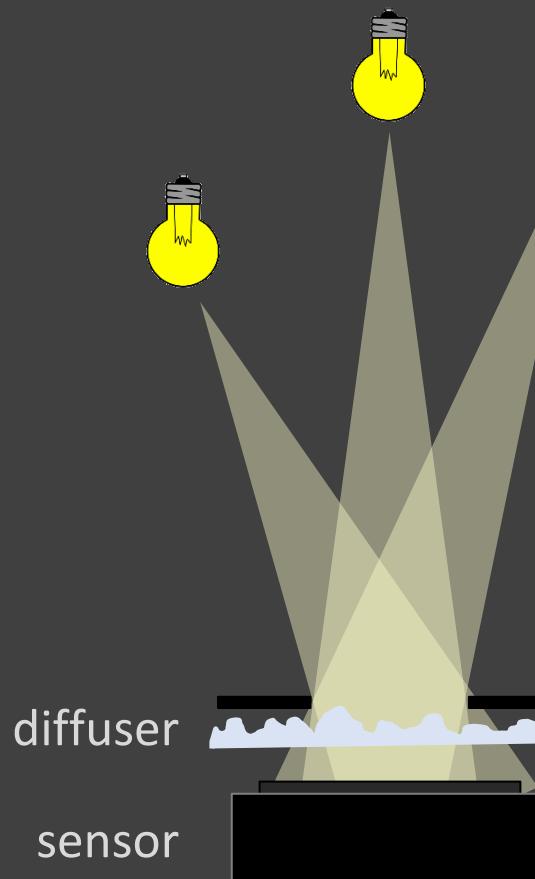
# DiffuserCam: stick a scatterer on a sensor



Camille Biscarrat  
Shreyas Parthasarathy



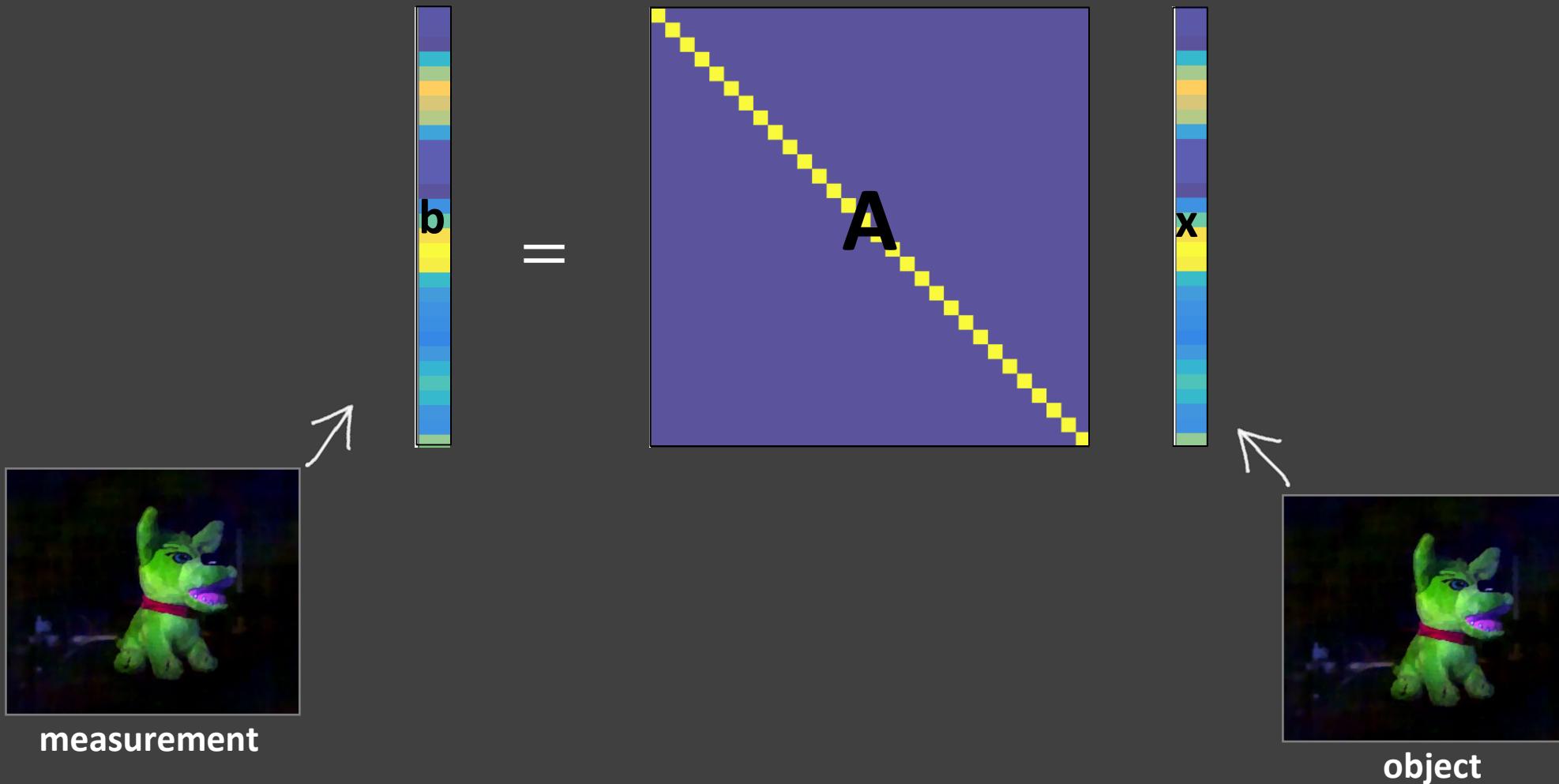
# DiffuserCam: stick a scatterer on a sensor



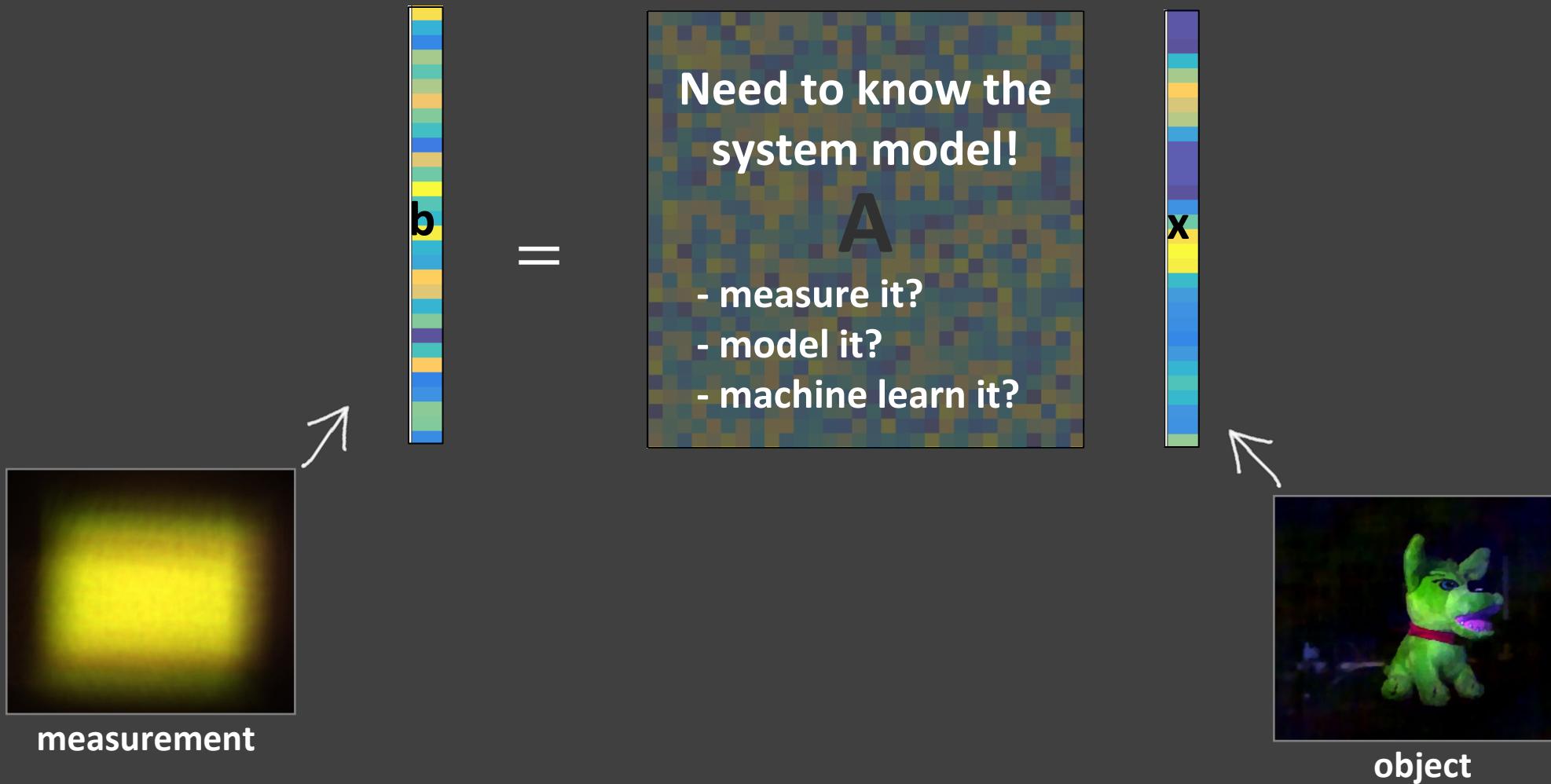
Grace Kuo  
Nick Antipa



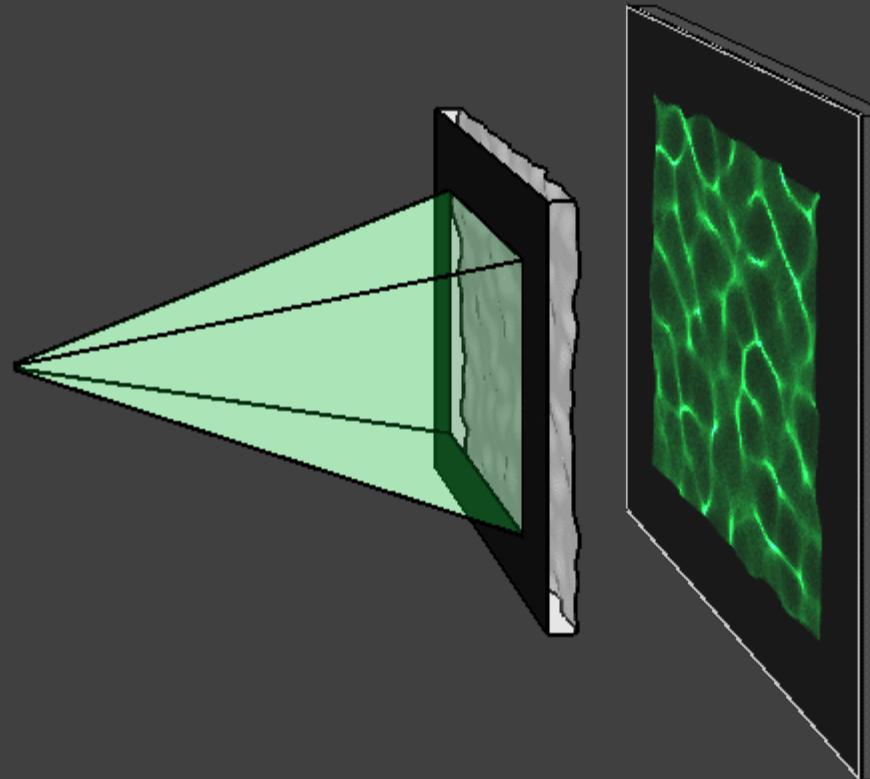
# Traditional cameras take direct measurements



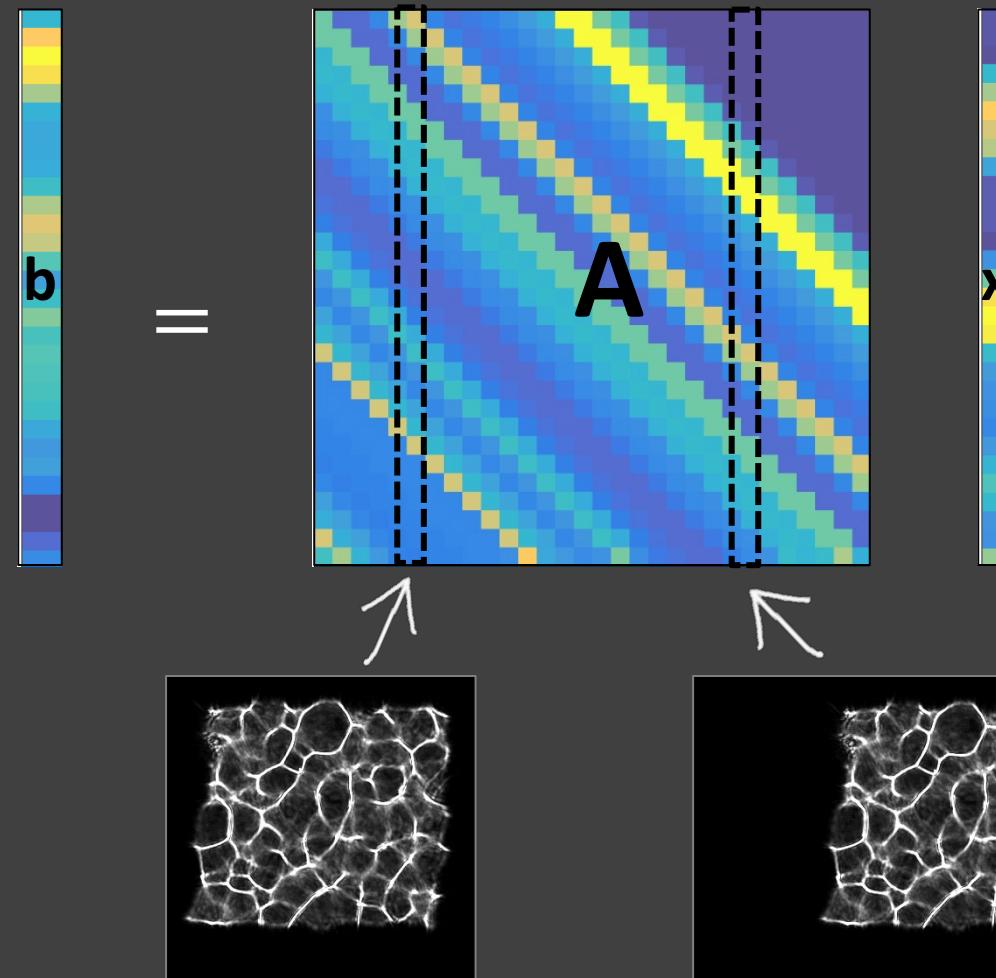
# Computational cameras can multiplex



# System response shifts with position

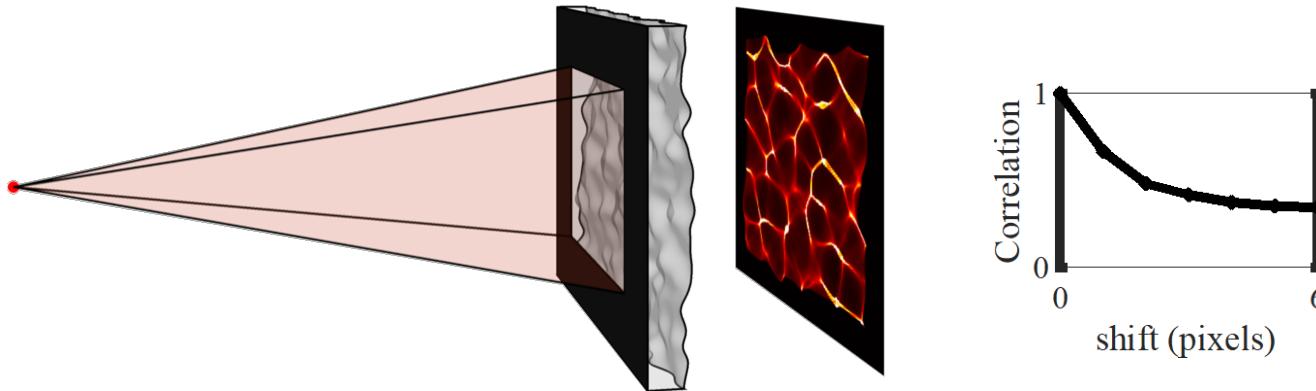


# DiffuserCam system model is a ‘shift-invariant’



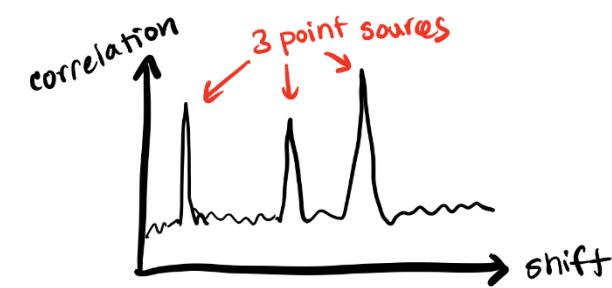
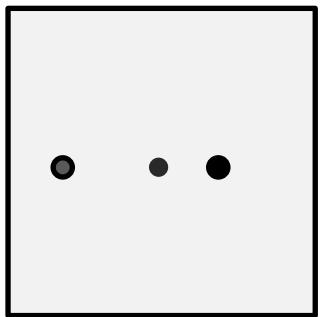
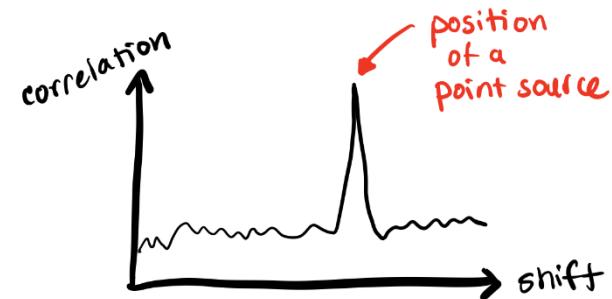
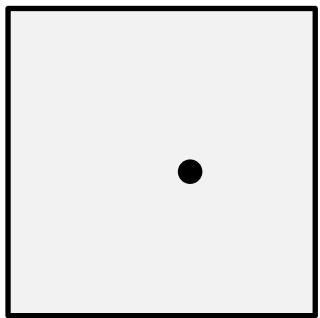
System response is same but shifted  
for different image pixels

We could find location of a point by correlating image captured with shifts in system response!



# Reconstruction finds strength of each ‘point source’:

Looks a lot like our GPS problem! (especially if image is sparse)





raw sensor data



recovered scene

\*solver is ADMM with TV reg in Halide

Grace Kuo  
Nick Antipa





raw sensor data



recovered scene

\*solver is ADMM with TV reg in Halide

Grace Kuo  
Nick Antipa



# Image reconstruction is nonlinear optimization

$$\arg \min_{\geq 0} \left\| \begin{matrix} \text{b} \\ - \end{matrix} - \begin{matrix} \text{A} \\ * \end{matrix} \begin{matrix} \text{x} \\ \Phi \end{matrix} \right\|_2^2 + \lambda \left\| \begin{matrix} \Phi \\ \downarrow \\ \text{Sparsity basis} \end{matrix} \right\|_1$$

\*solved with ADMM in Halide

S. Boyd, et al. *Foundations and Trends in Machine Learning* (2011)

J. Ragan-Kelley, et al. *AMC SIGPLAN* (2013)



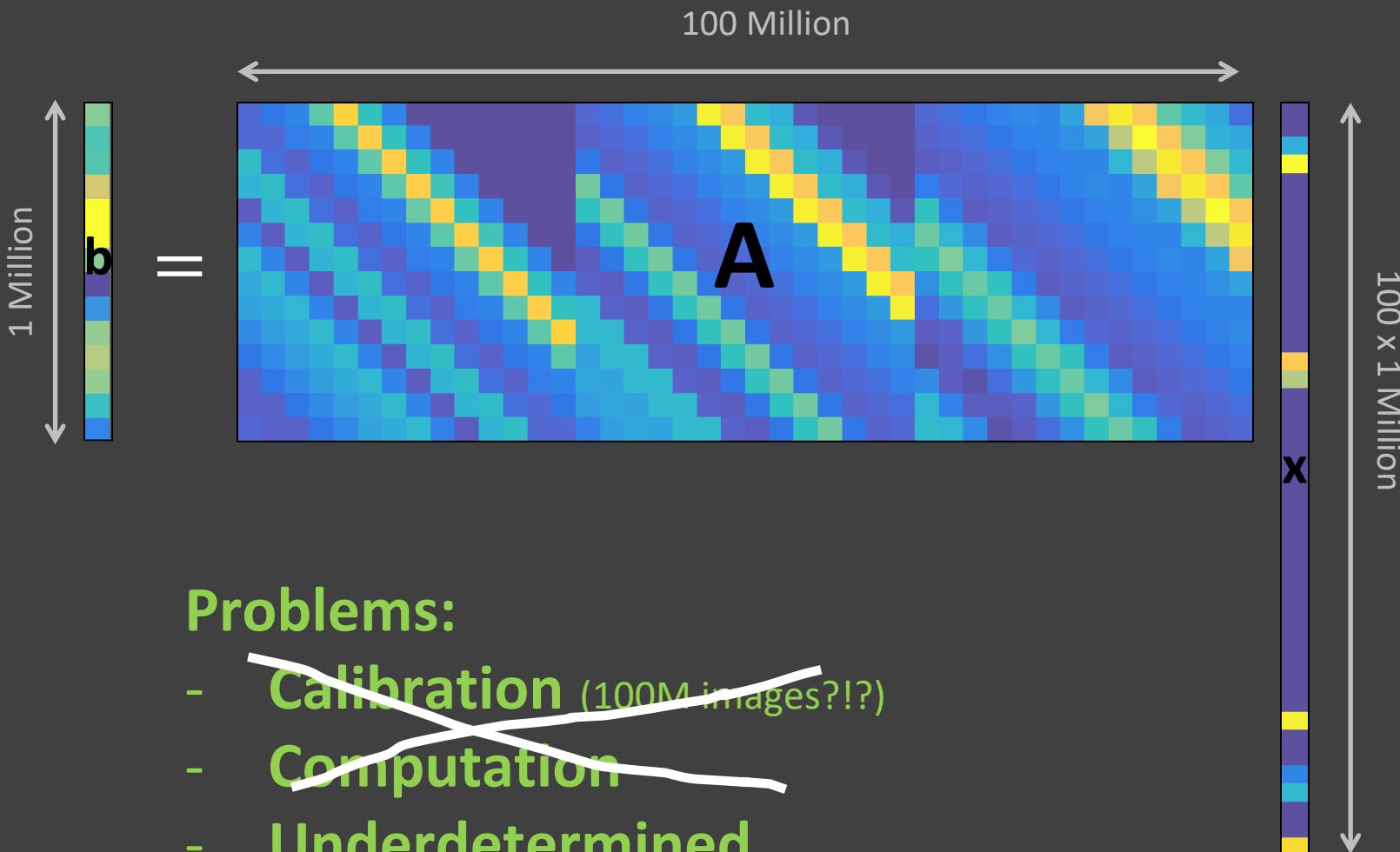
Cute! But what's it good for?

**2D**

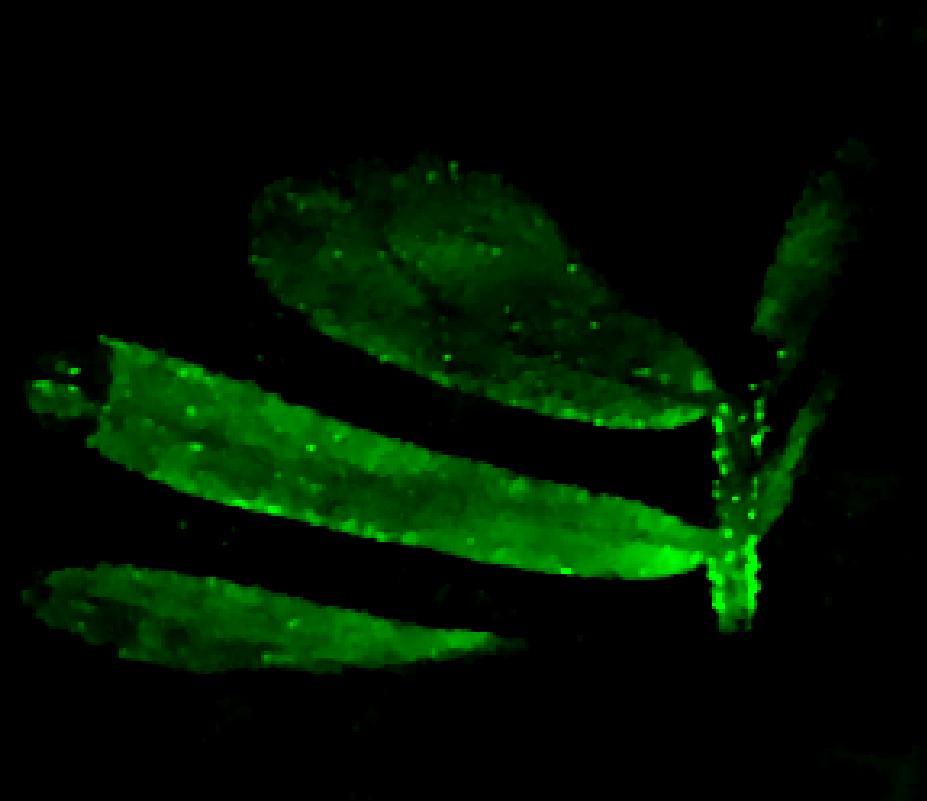
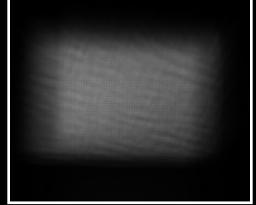


**3D**

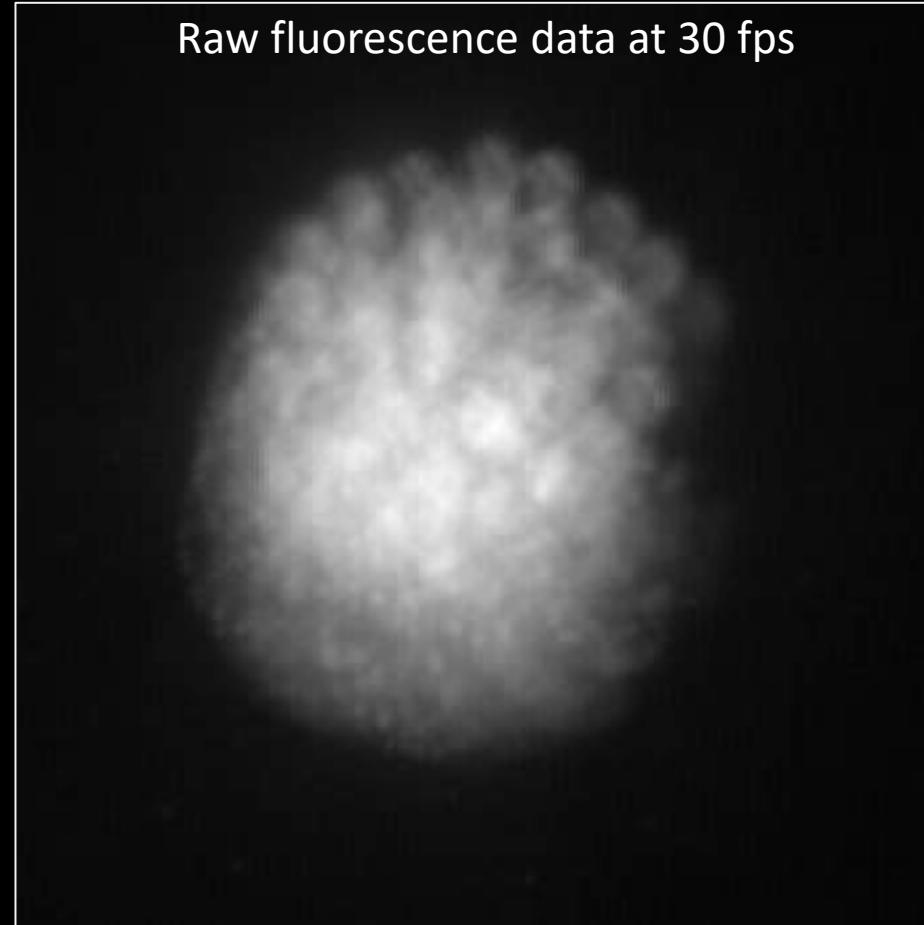
# Single-shot 3D is underdetermined



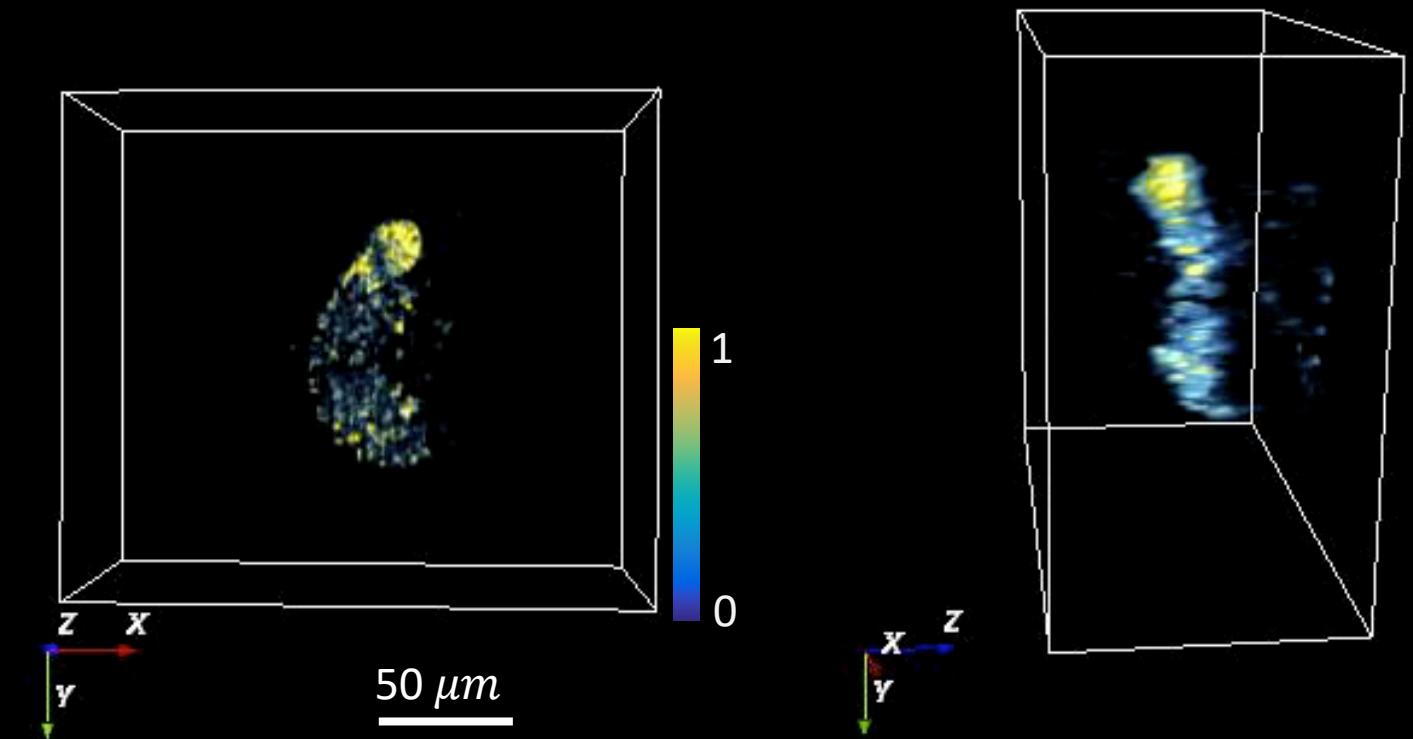
**128X** more voxels for **FREE!**



Raw fluorescence data at 30 fps



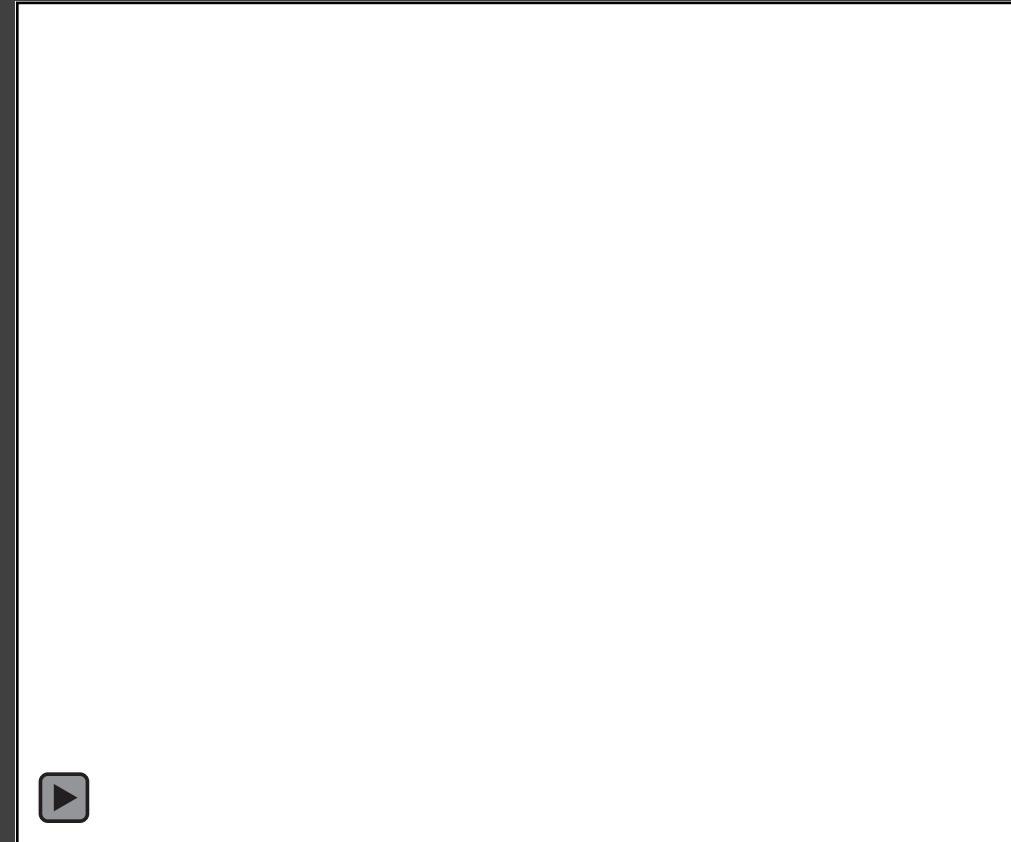
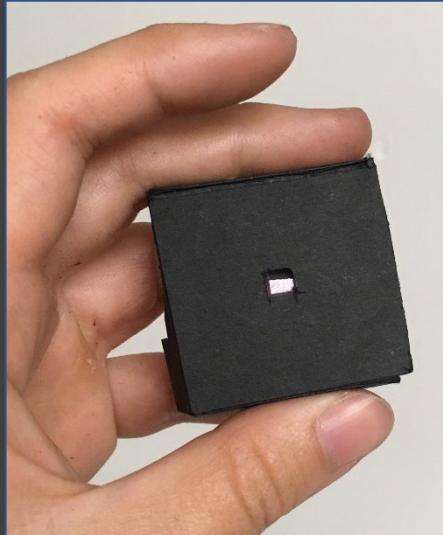
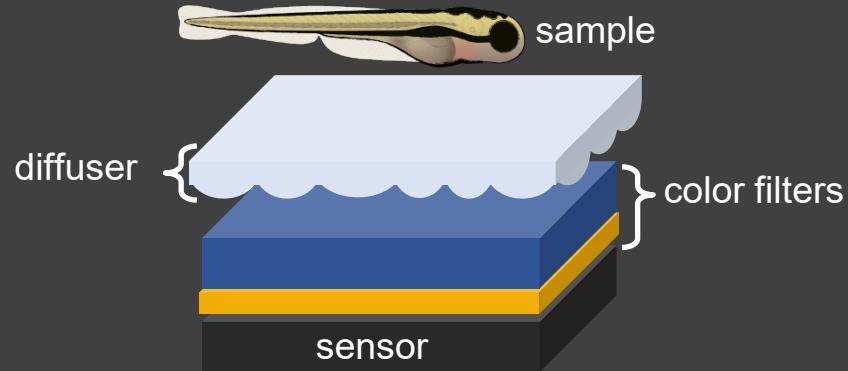
3D video reconstruction



Kyrollos Yanny  
Nick Antipa



# Neural activity tracking with flat DiffuserScope

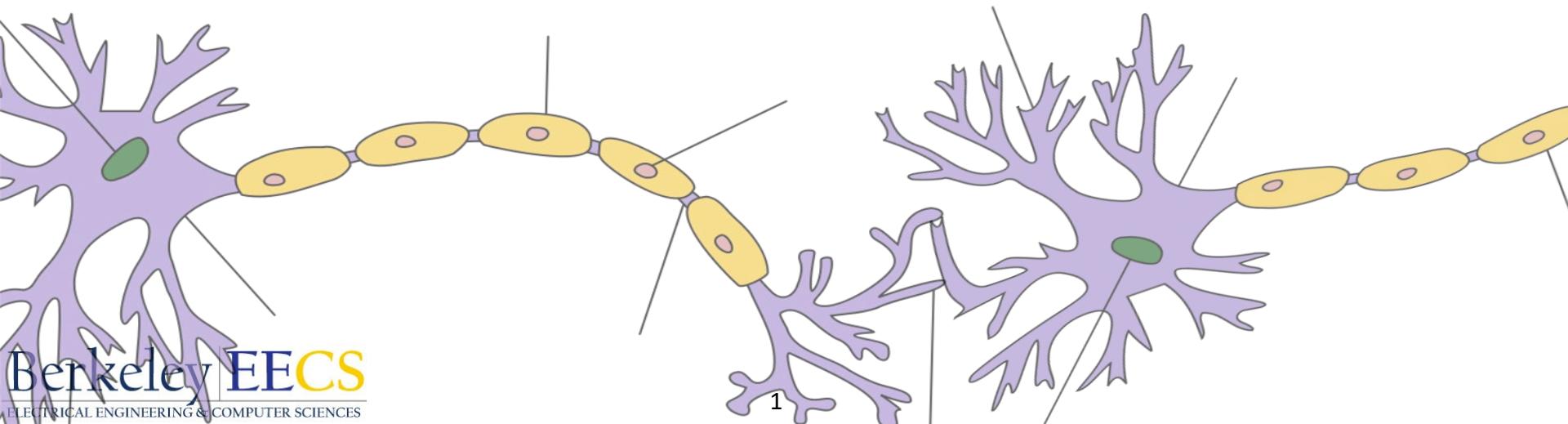


Grace Kuo

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# EECS 16A

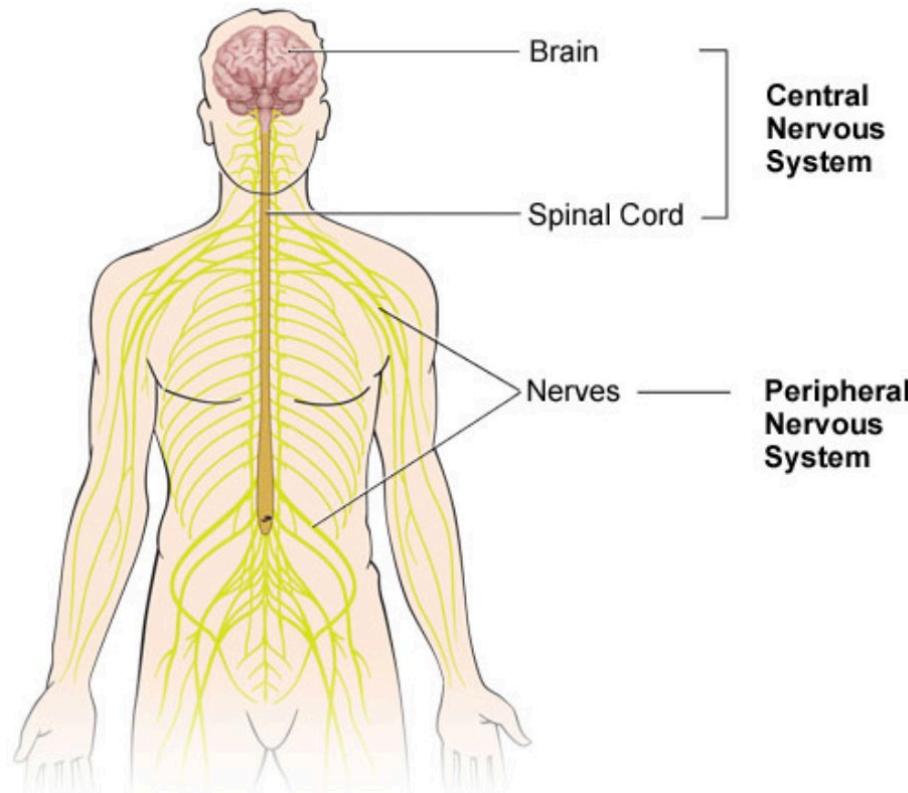
## Neurons are Circuits!



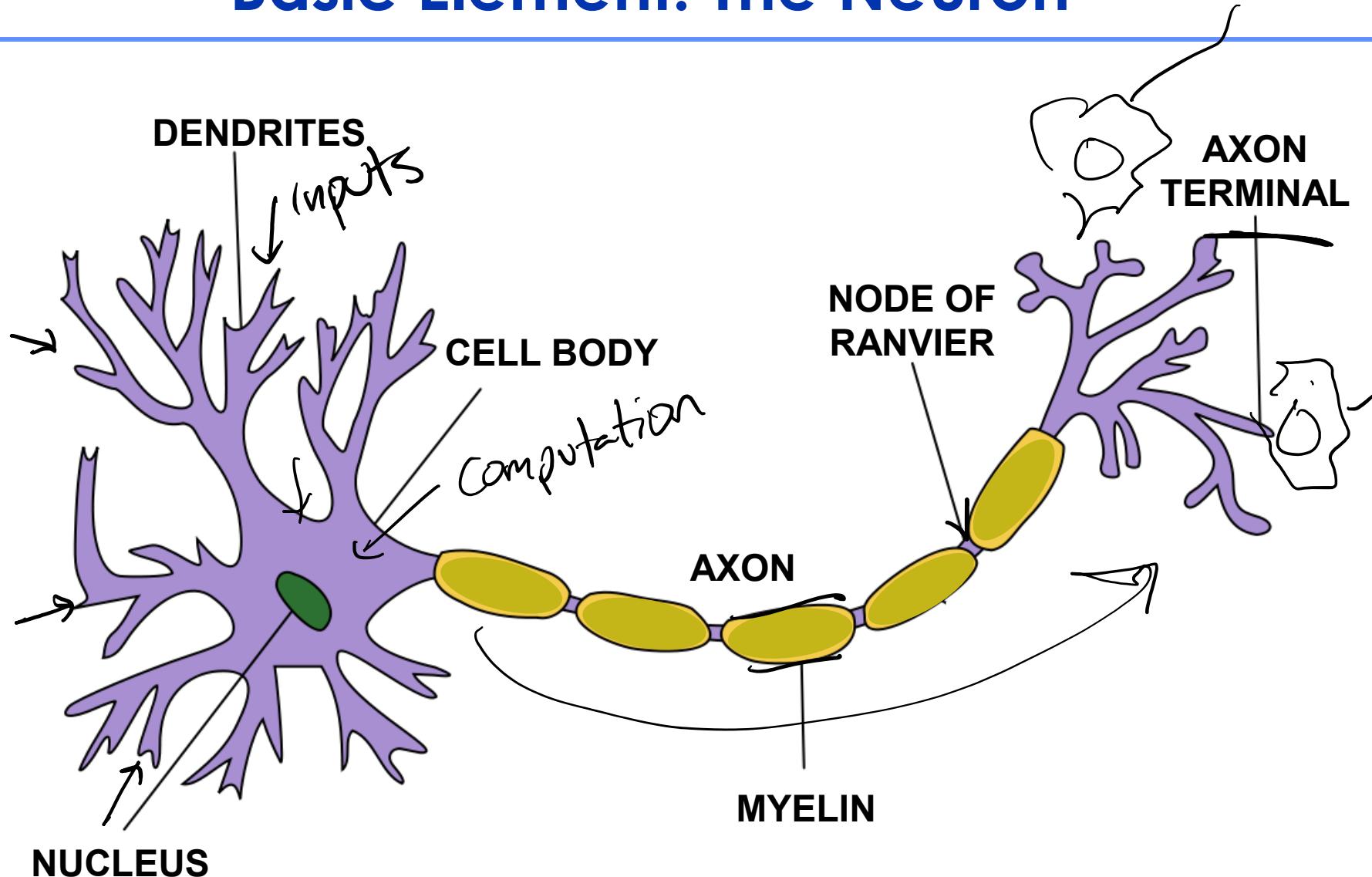
# The Body Electric - Nervous System

- **There are two distinct parts of the nervous system**

- Central Nervous system: Brain, Spinal Cord
- Peripheral Nervous system: All other neural elements, including the peripheral nerves (motor and sensory) and the autonomic nerves (regulate internal organs)

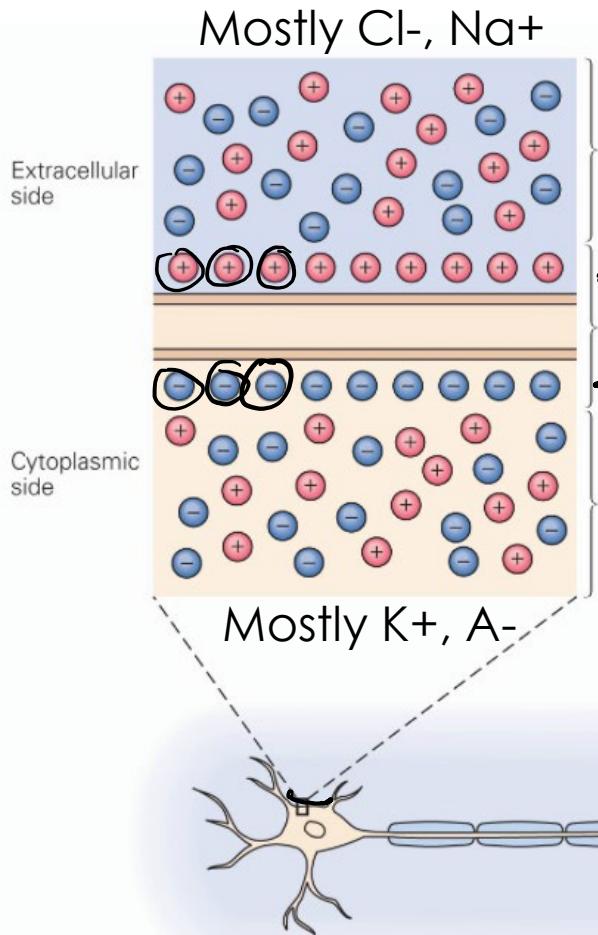


# Basic Element: The Neuron



You have 100 billion of these

# Resting Membrane Potential



- Resting channels are permeable to  $\text{K}^+$  diffusing out of the cell causing (+) charges to accumulate at the cell surface and (-) charges inside
- This self limits when the electrical force negates the chemical force

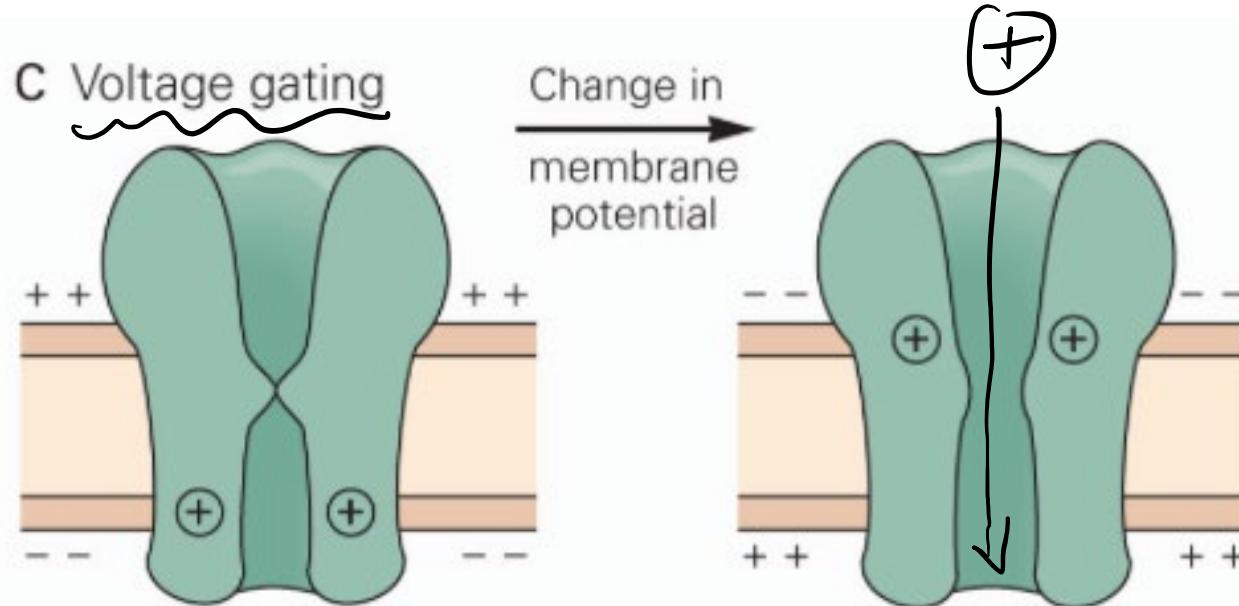
- Nernst Equation ( $E \rightarrow$  potential)

$$E_x = \frac{RT}{zF} \ln \frac{[X]_o}{[X]_i}$$

← Concentration

- R – gas constant
- T – Temperature (K)
- F – Faraday constant
- z – Valence of ion
- $RT/zF \sim 25\text{mV}$  at  $25\text{C}$ ,  $\text{K}^+$   
– Equivalent to  $kT/q!$
- $E_k$  is typically  $\sim -70$  to  $\underline{\underline{-80\text{mV}}}$

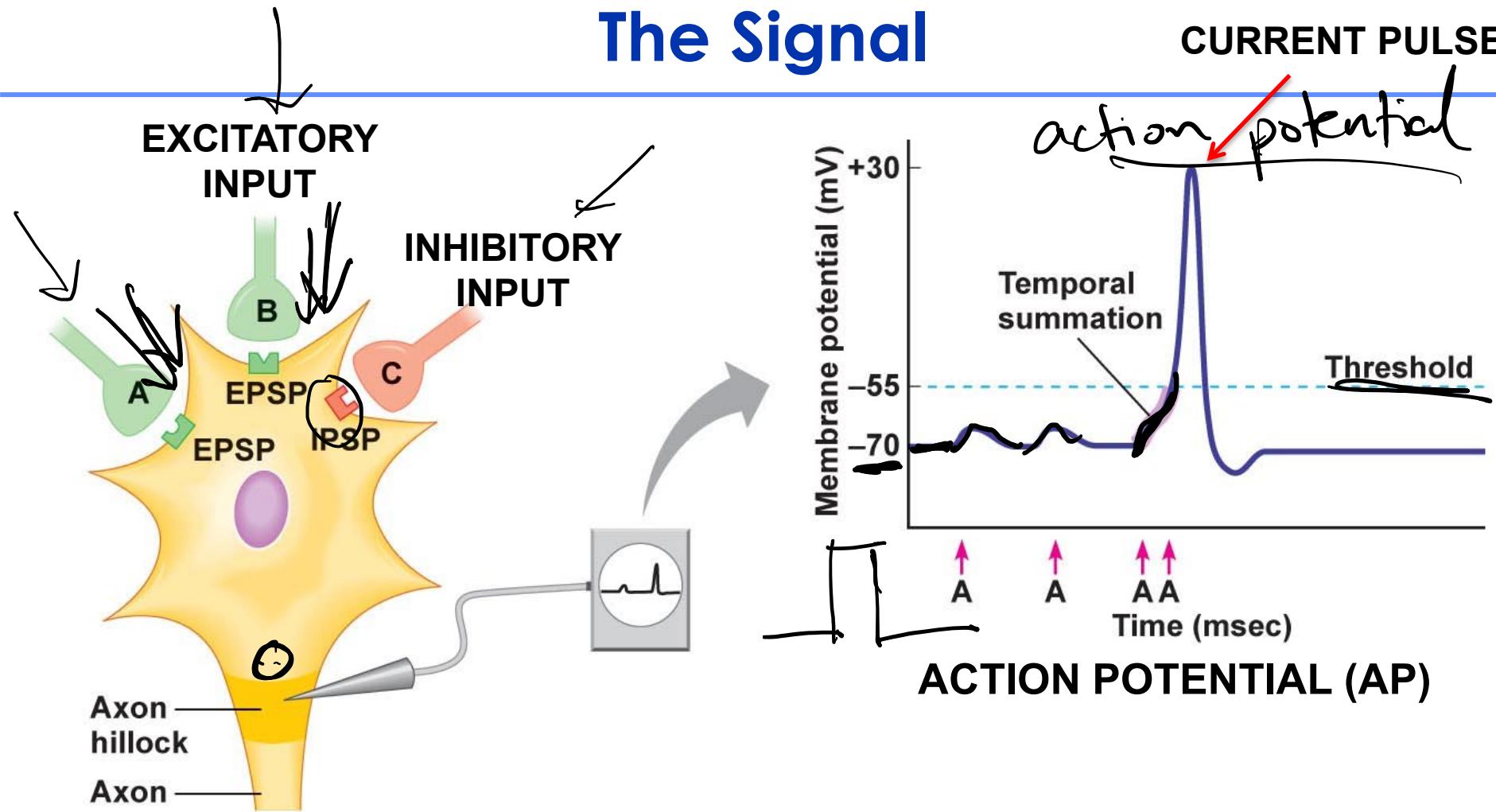
# Ion Channels (switches)



- There are several types of stimuli controlling ion channels opening and closing
  - These can be chemical, electrical or mechanical

# The Signal

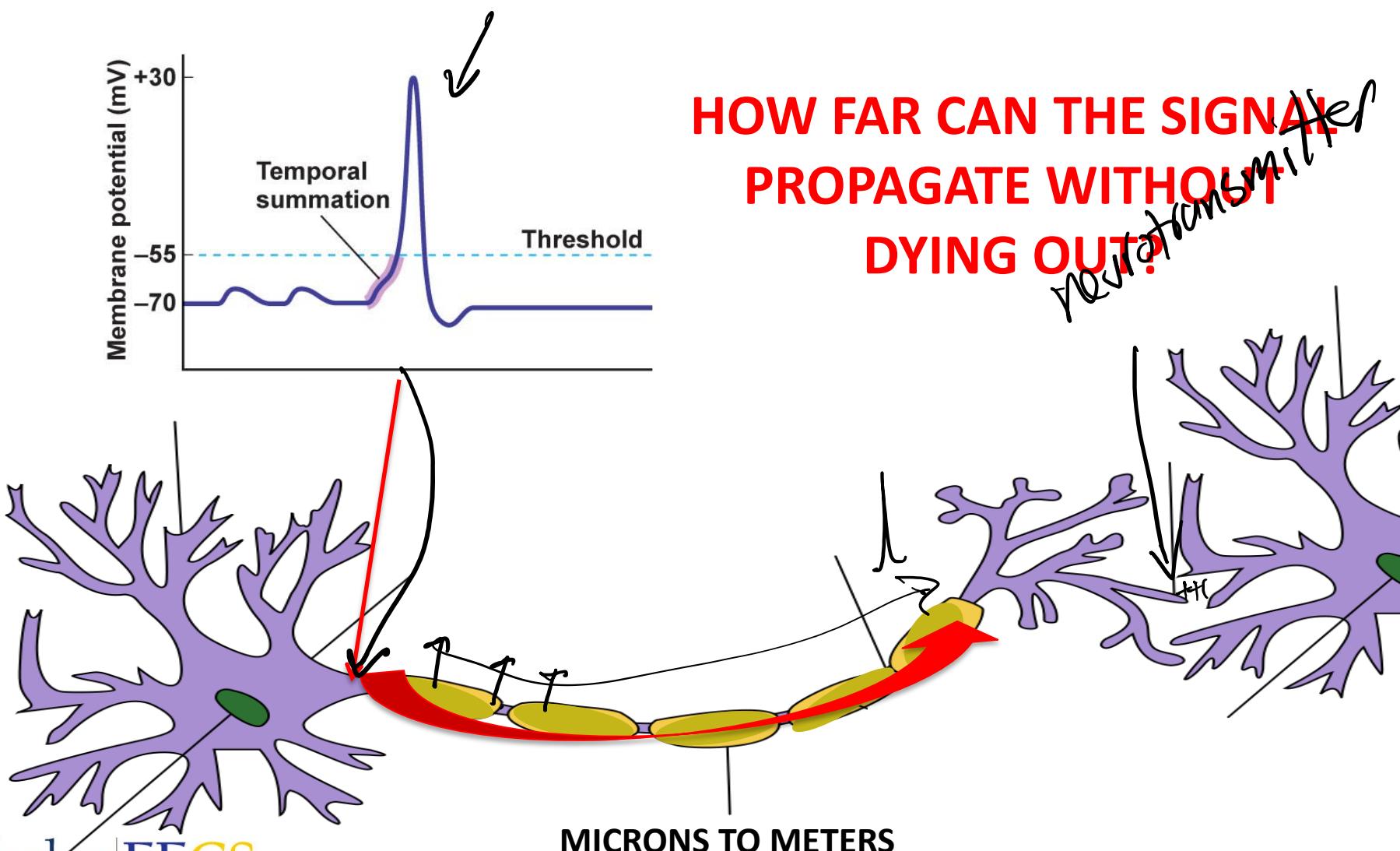
CURRENT PULSE



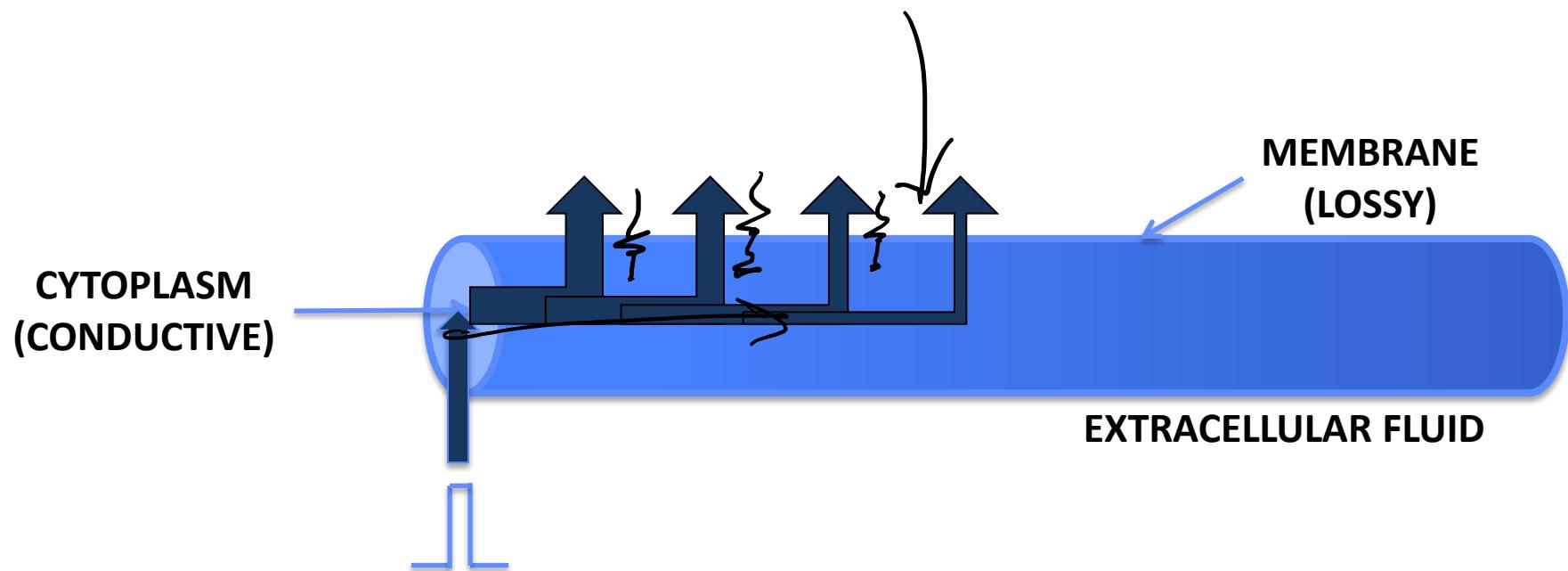
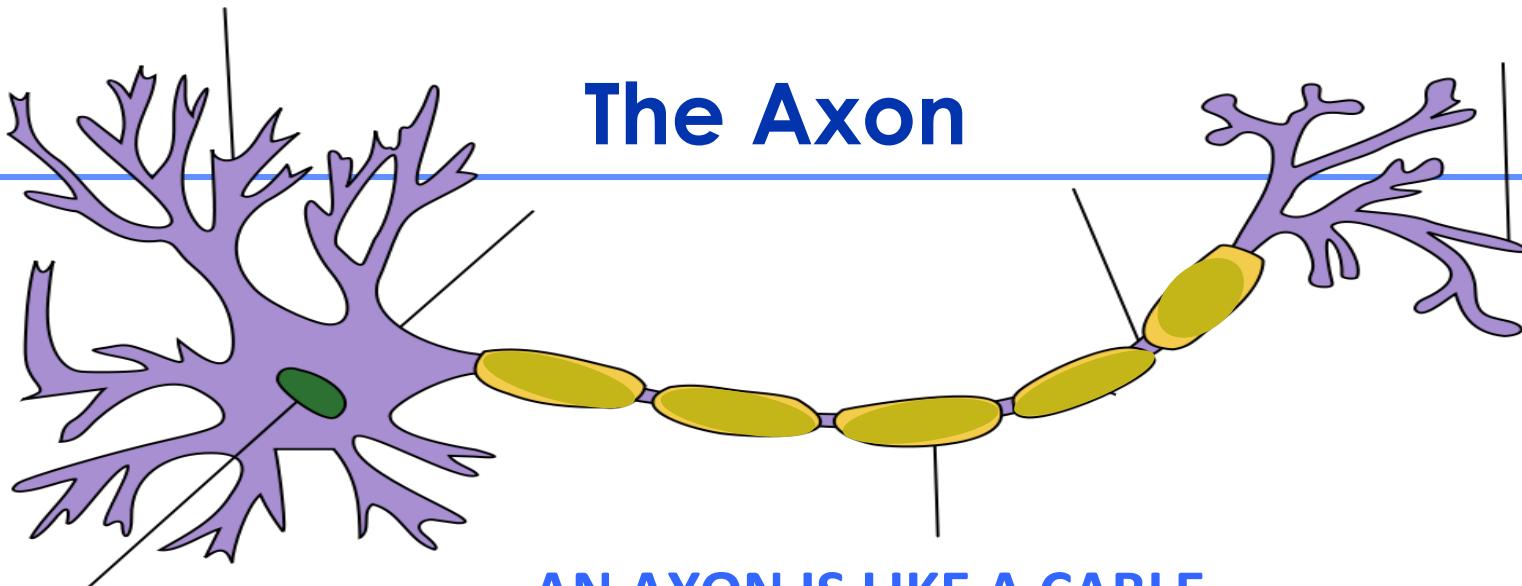
© 2011 Pearson Education, Inc.

- Ion channels open in response to stimuli causing the cell to depolarize
- There is temporal and spatial summation
- Once the membrane potential goes above threshold, it starts an AP

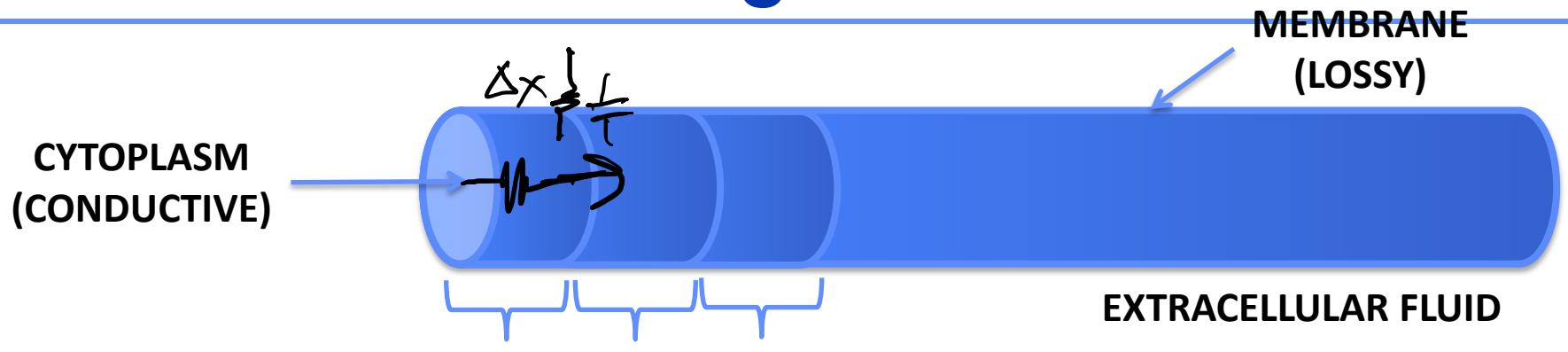
# Action Potential Propagation



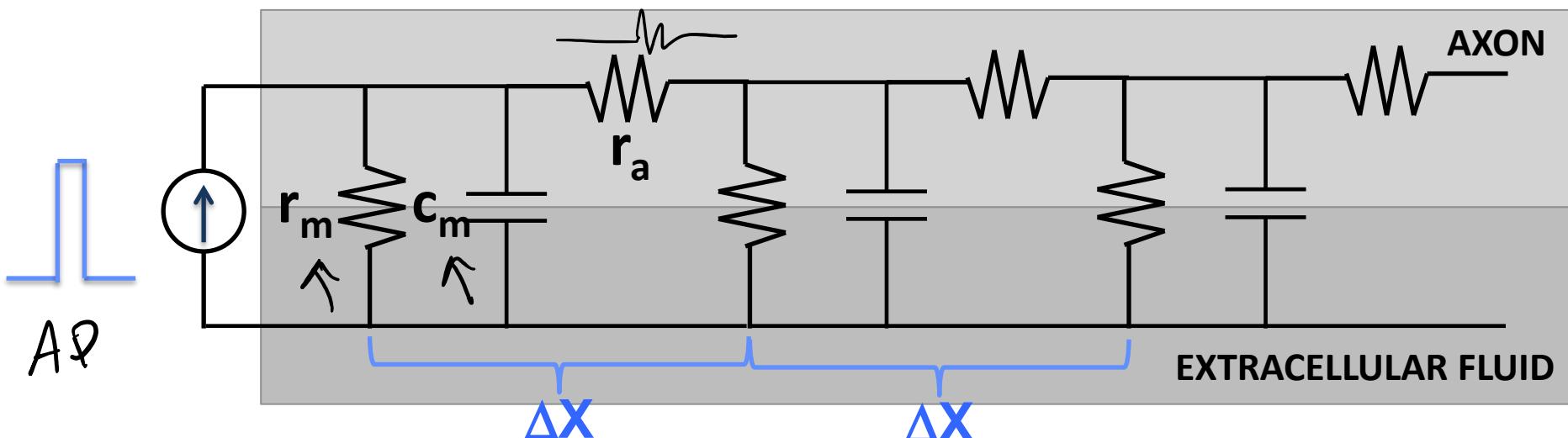
# The Axon



# Modeling the Axon



SEGMENT  $\Delta x$

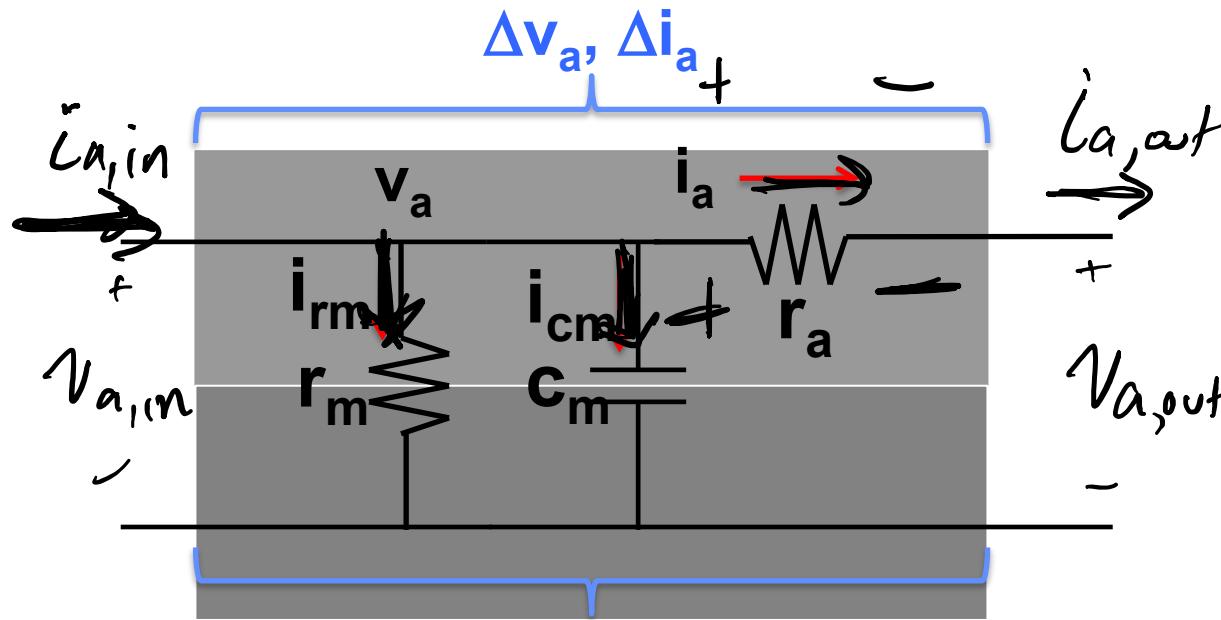


$r_m$  = membrane resistance [ $\Omega \cdot m$ ];

$r_a$  = axon resistance [ $\Omega/m$ ]

$c_m$  = membrane capacitance [ $F/m$ ]

# A Single Axon Segment



KIRCHHOFF'S VOLTAGE LAW

$\Delta x$

KIRCHHOFF'S CURRENT LAW

$$\underline{\Delta v_a} = -\underline{i_a} \underline{r_a} \underline{\Delta x}$$

$$\left( \frac{\Delta v_a}{\Delta x} = -i_a r_a \right)$$

$$\underline{\Delta i_a} = \left( \underline{i_{rm}} + \underline{i_{cm}} \right) \underline{\Delta x}$$

$$\frac{\Delta i_a}{\Delta x} = -\frac{v_a}{r_m} - C_m \frac{\partial v_a}{\partial t}$$

# The Rest is Math

EECS 16B  
120

$$\frac{\Delta v_a}{\Delta x} = -i_a r_a$$

$$\underline{\Delta x \rightarrow 0}$$

$$-\frac{1}{r_a} \frac{\partial v_a}{\partial x} = i_a$$

$$-\frac{1}{r_a} \frac{\partial^2 v_a}{\partial x^2} = \frac{\partial i_a}{\partial x}$$

$$\frac{\Delta i_a}{\Delta x} = -\frac{v_a}{r_m} - c_m \frac{\partial v_a}{\partial t}$$

$$\frac{\partial i_a}{\partial x} = -\frac{v_a}{r_m} - c_m \frac{\partial v_a}{\partial t}$$



\*

$$-\frac{1}{r_a} \frac{\partial^2 v_a}{\partial x^2} = -\frac{v_a}{r_m} - c_m \frac{\partial v_a}{\partial t}$$



# The Equation

$$-\frac{1}{r_a} \frac{\partial^2 v_a}{\partial x^2} = -\frac{v_a}{r_m} - c_m \frac{\partial v_a}{\partial t}$$

REARRANGE  
TERMS

$$\left. \begin{aligned} & -\frac{r_m}{r_a} \frac{\partial^2 v_a}{\partial x^2} + c_m r_m \frac{\partial v_a}{\partial t} + v_a = 0 \\ & \quad \underbrace{\qquad}_{\text{SPACE DEPENDENT}} \quad \underbrace{\qquad}_{\text{TIME DEPENDENT}} \end{aligned} \right\} \text{final equation}$$

LET'S ANALYZE ONE AT A TIME

# Time Dependence

$$-\frac{r_m}{r_a} \frac{\partial^2 v_a}{\partial x^2} + c_m r_m \frac{\partial v_a}{\partial t} + v_a = 0$$

HOLD SPACE  
CONSTANT

$$\cancel{c_m r_m} \frac{\partial v_a}{\partial t} + v_a = 0$$

ODE WITH STANDARD SOLUTION

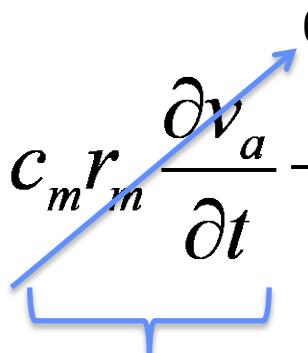
$$\rightarrow \boxed{v_a(t) = v_o e^{-t/\tau}}$$

$$\tau = \underbrace{r_m c_m}$$

TIME CONSTANT

# Space Dependence

$$-\frac{r_m}{r_a} \frac{\partial^2 v_a}{\partial x^2} + c_m r_m \frac{\partial v_a}{\partial t} + v_a = 0$$



HOLD TIME  
CONSTANT

$$-\frac{r_m}{r_a} \frac{\partial^2 v_a}{\partial x^2} + v_a = 0$$

ODE WITH STANDARD SOLUTION

$$v_a(x) = v_o e^{-x/\lambda}$$

$$\lambda = \sqrt{\frac{r_m}{r_a}}$$

SPACE CONSTANT



# What does it mean?

$$\lambda = \sqrt{\frac{r_m}{r_a}}$$

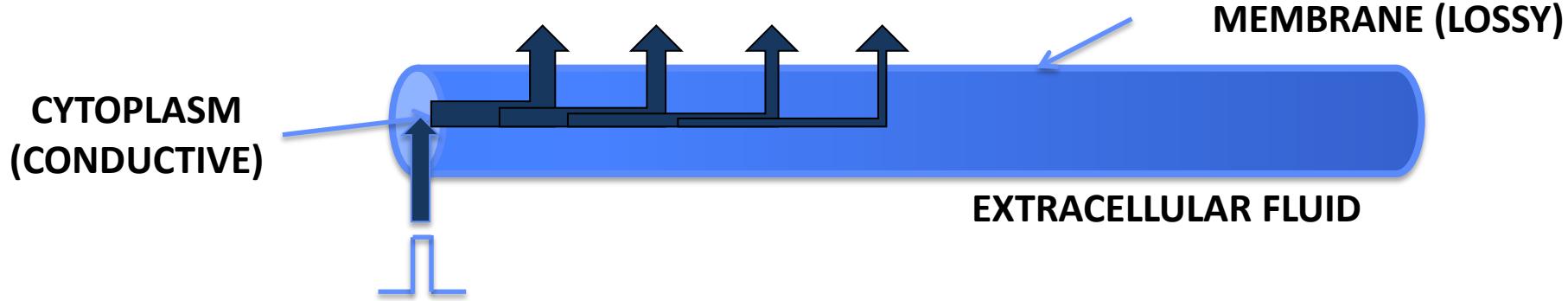
\***SPACE CONSTANT**

The distance it takes for a signal to decay away

$$\tau = r_m C_m$$

\***TIME CONSTANT**

The amount of time it takes for a signal to decay away

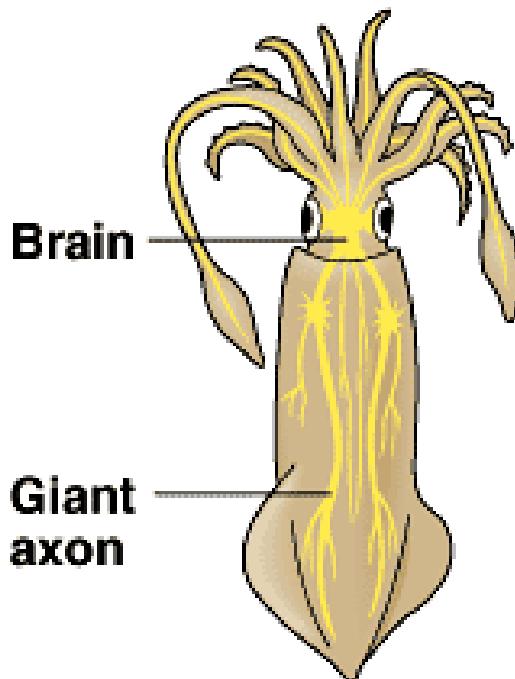


**HOW CAN IT BE IMPROVED?**

# Evolution

$$\uparrow \lambda = \sqrt{\frac{r_m \uparrow}{r_a \downarrow}}$$

SPACE CONSTANT

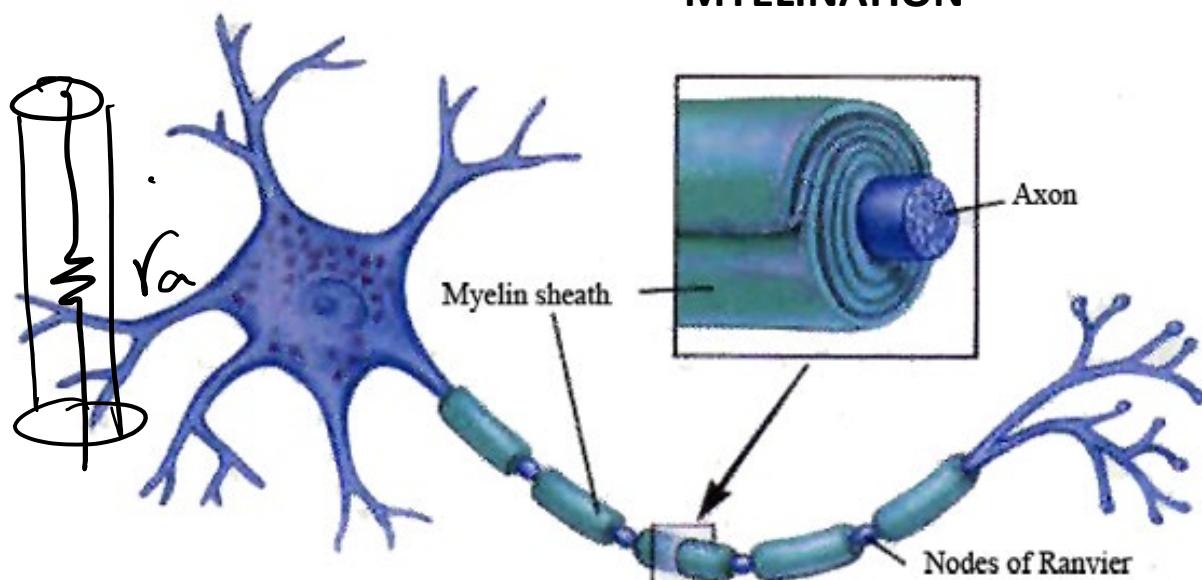


$$r_a \downarrow, \lambda \uparrow$$

$$\uparrow \tau = r_m c_m$$

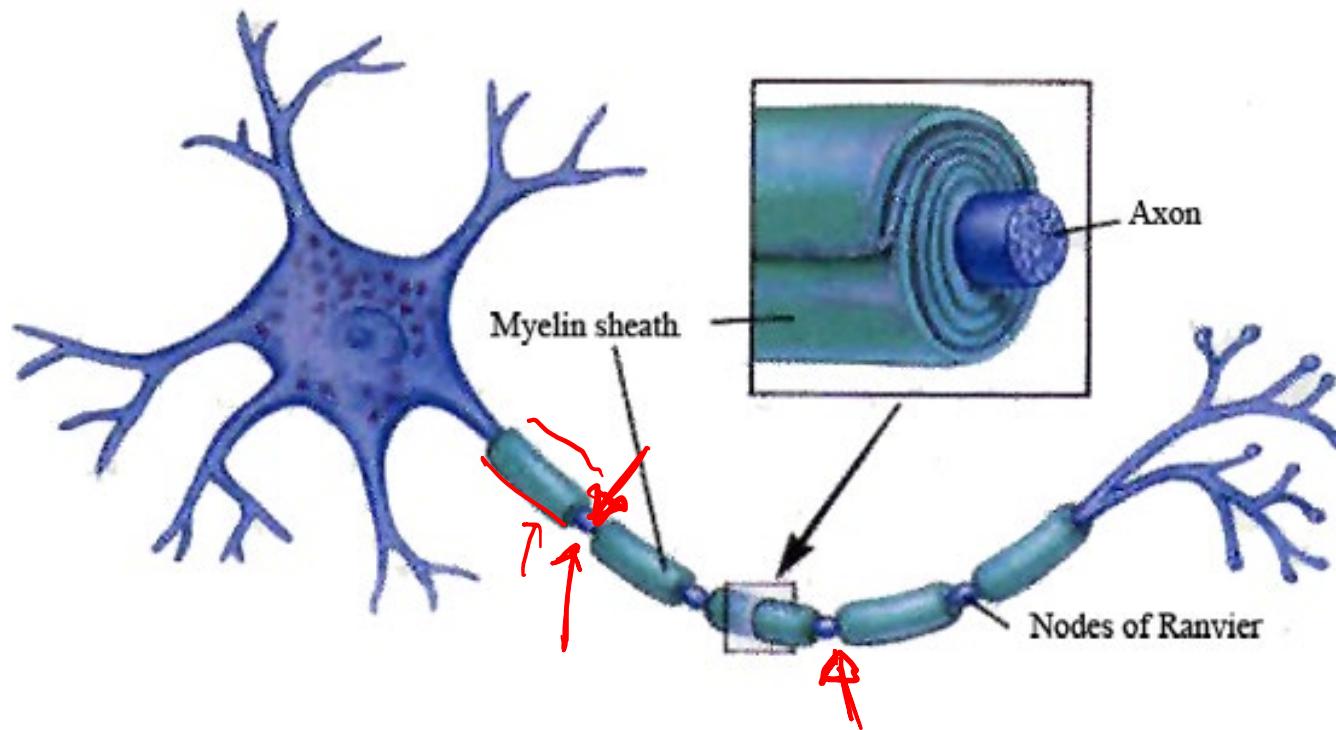
TIME CONSTANT

MYELINATION



$$r_m \uparrow, c_m \downarrow, \lambda \uparrow$$

# Nodes of Ranvier

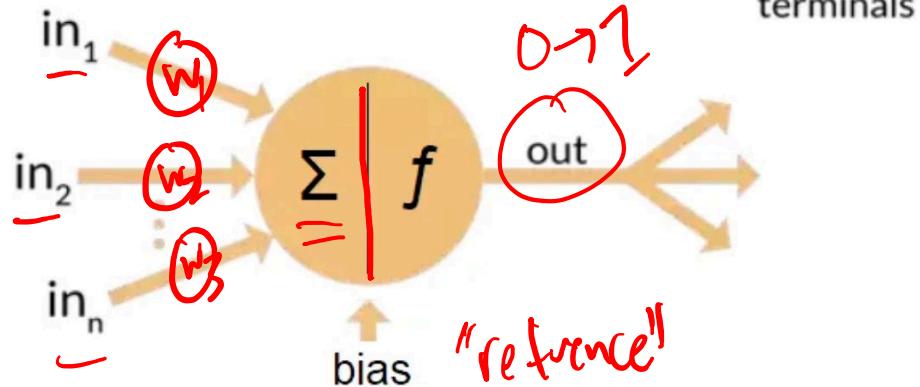
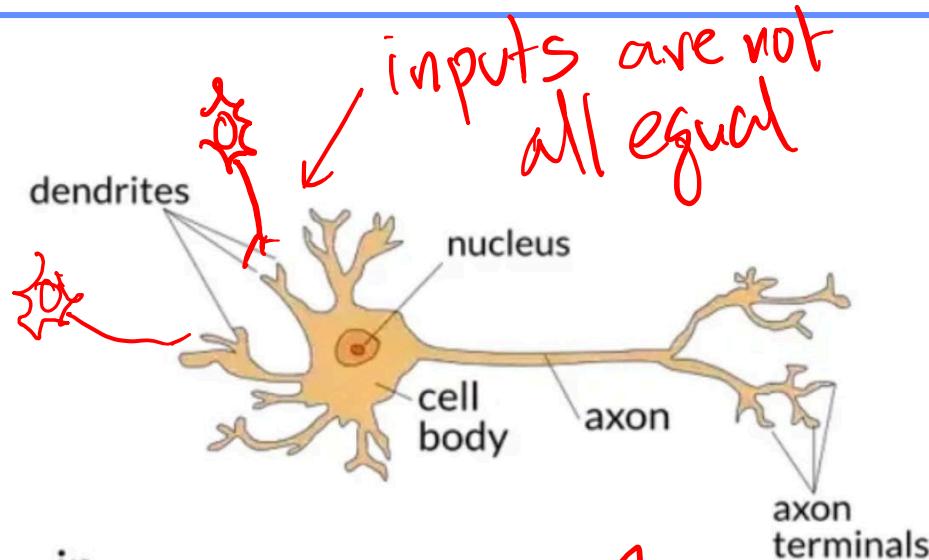


What do you think Nodes of Ranvier are for?

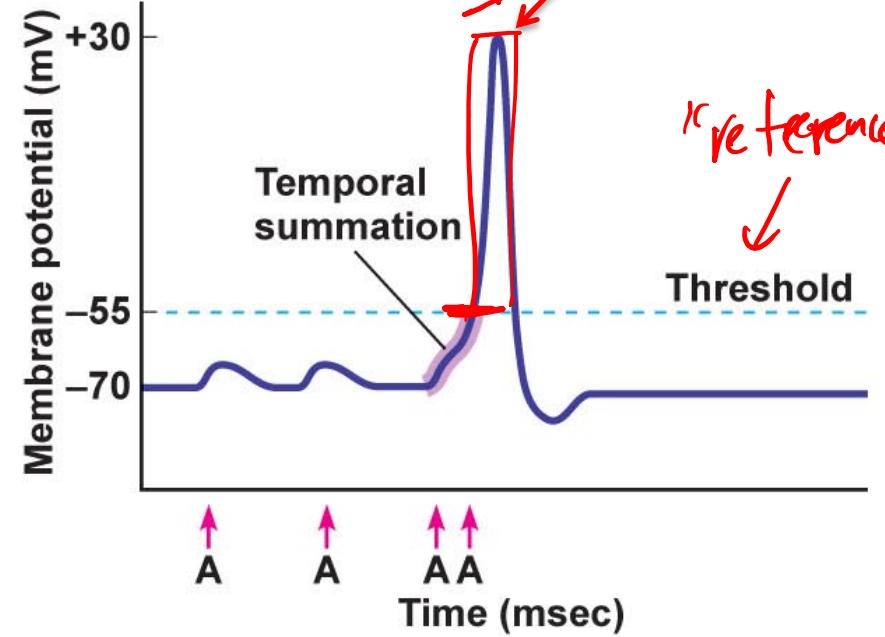
*Hint: think about a digital signal propagating across a very long wire..*

digital repeater  
EECS 151

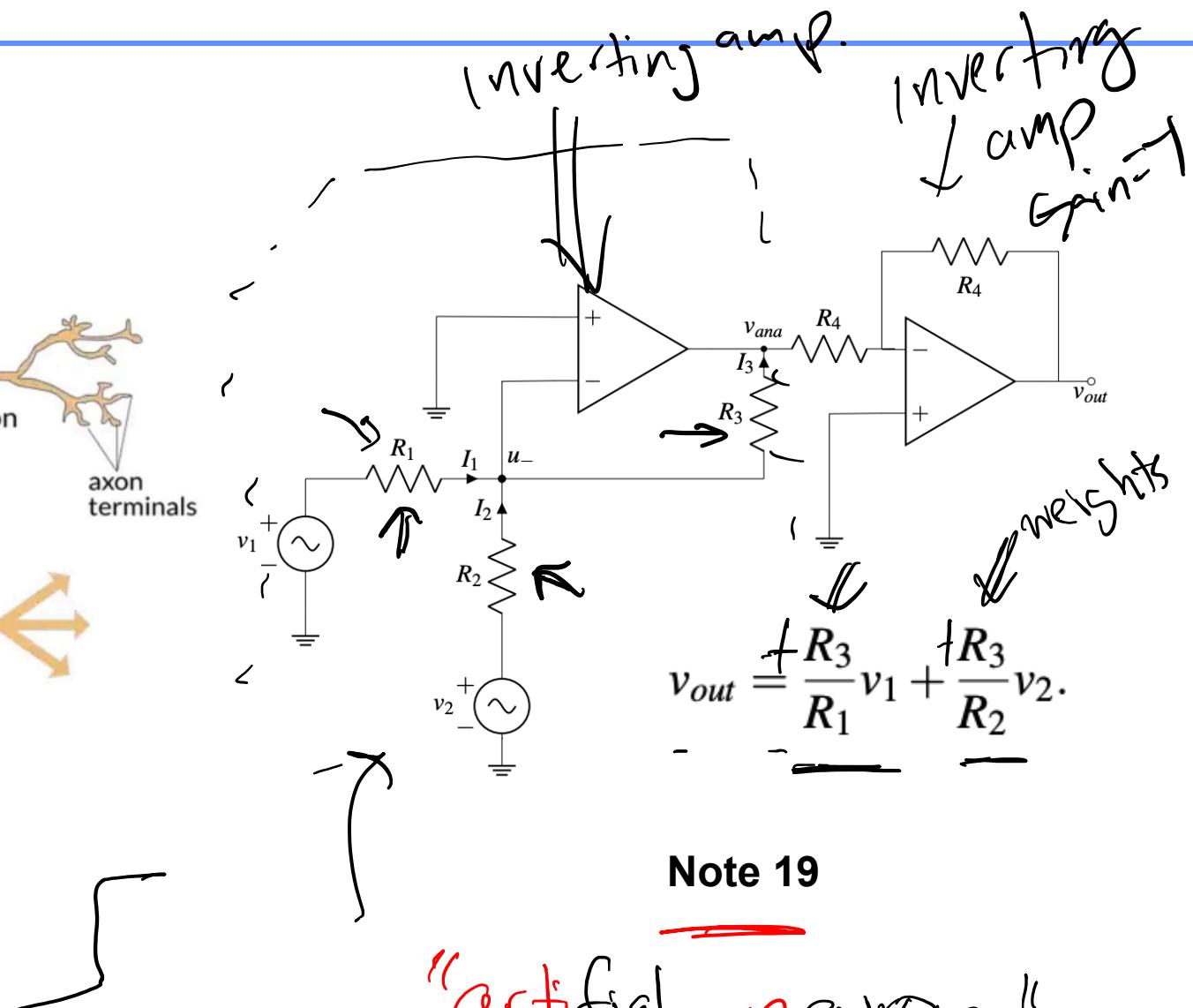
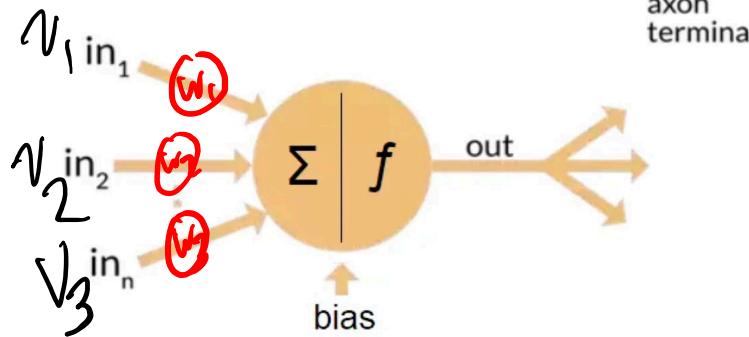
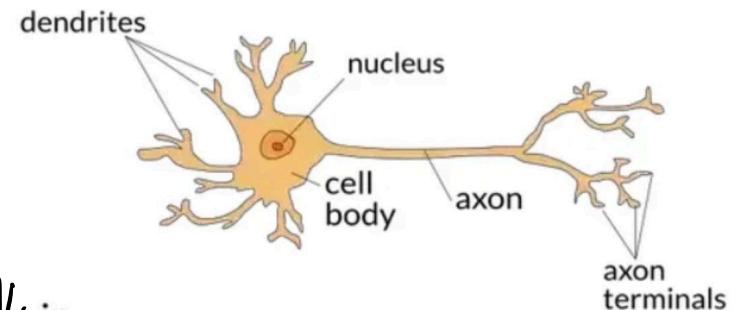
# Neural Networks



circuits?

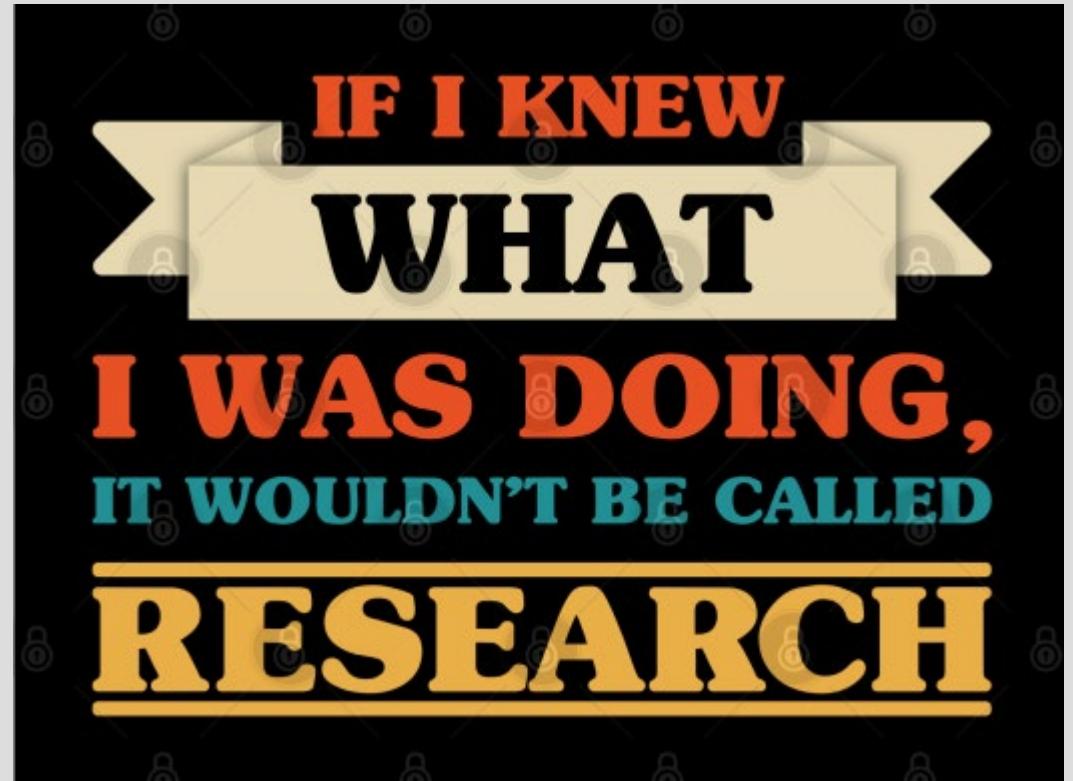


# Artificial Neuron

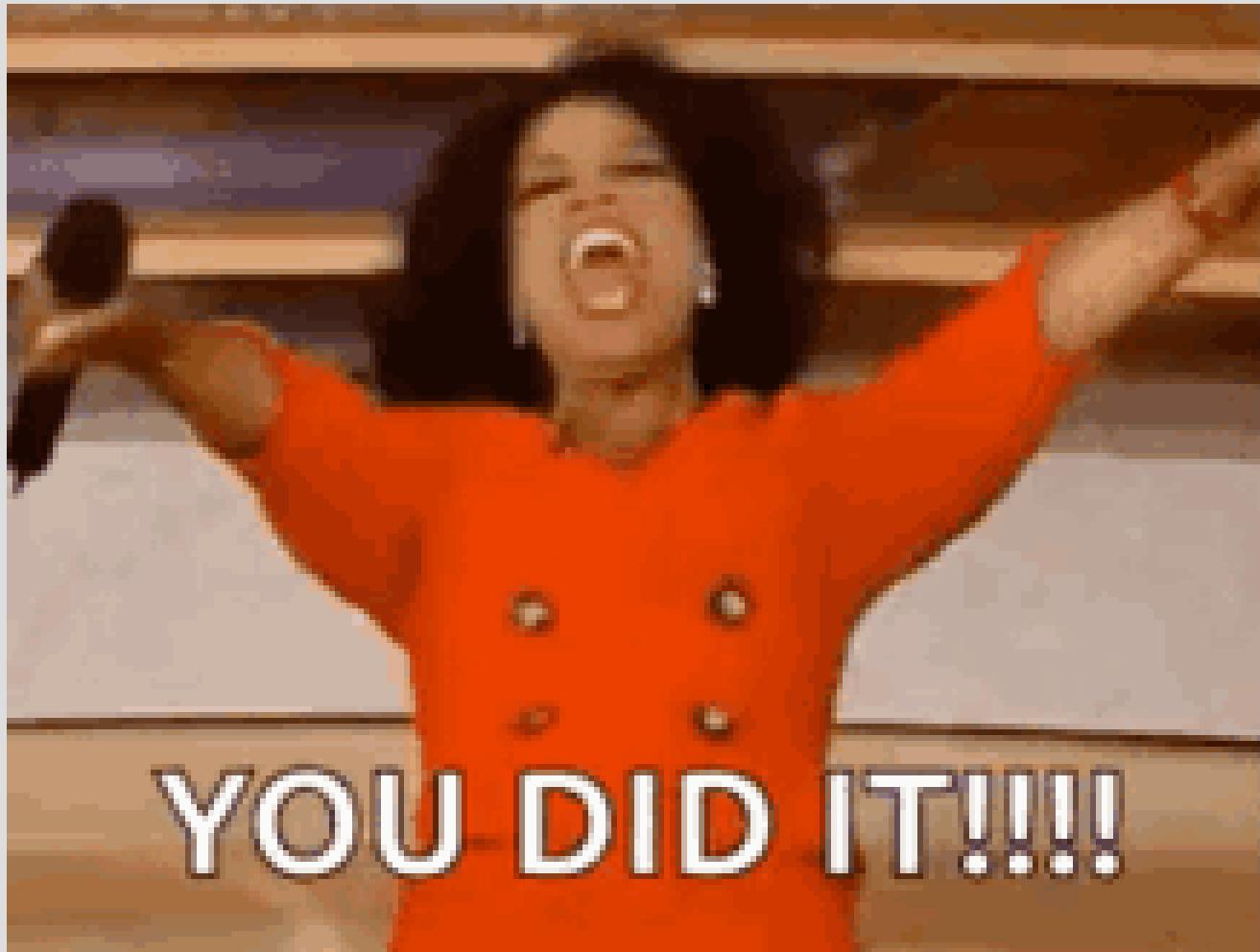


# How to get involved in research

- If you're interested in research:
  - Talk to your TAs
  - Talk to professors
  - Look for openings on Beehive/Dare/URAP websites



**Enough about me...**



- Congrats!
- What you have accomplished this semester:
  - Built a camera
  - Built two types of touchscreens
  - Built your own GPS system
- If you liked the class, please:
  - Thank your TAs!
  - apply to become one!

# Learning Goals

Stuff We did:

## EECS 16A

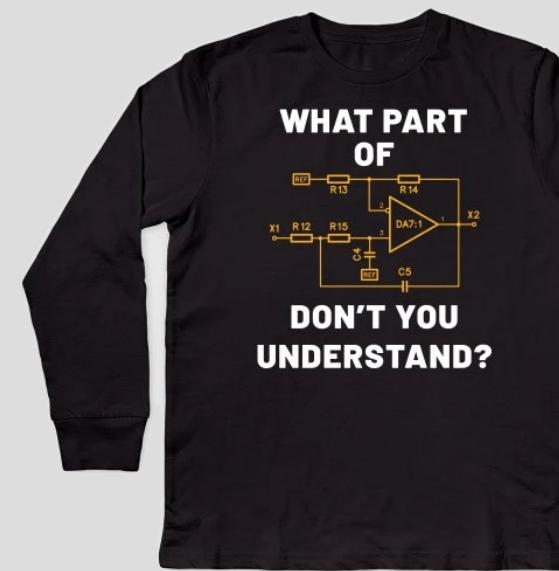
- Module 1: Introduction to systems
  - How do we collect data? build a model?
- Module 2: Introduction to circuits and design
  - How do we use a model to solve a problem
- Module 3: Introduction Signal Processing and Machine Learning
  - How do we “learn” models from data, and make predictions?



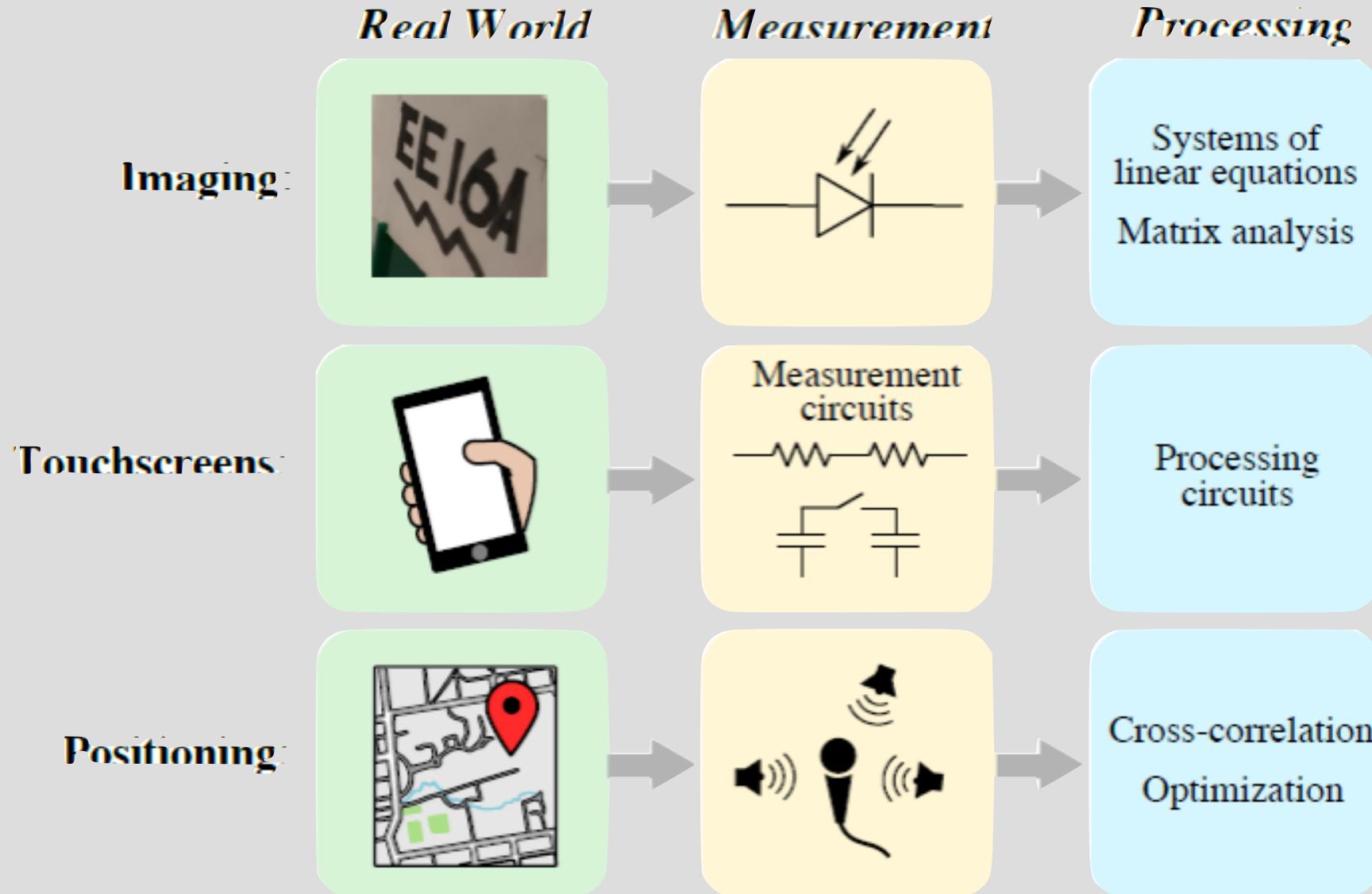
Stuff you will do next

## EECS 16B

- Module 4: Advanced circuit design / analysis
- Module 5: Introduction to control and robotics
- Module 6: Introduction to data analysis and signal processing

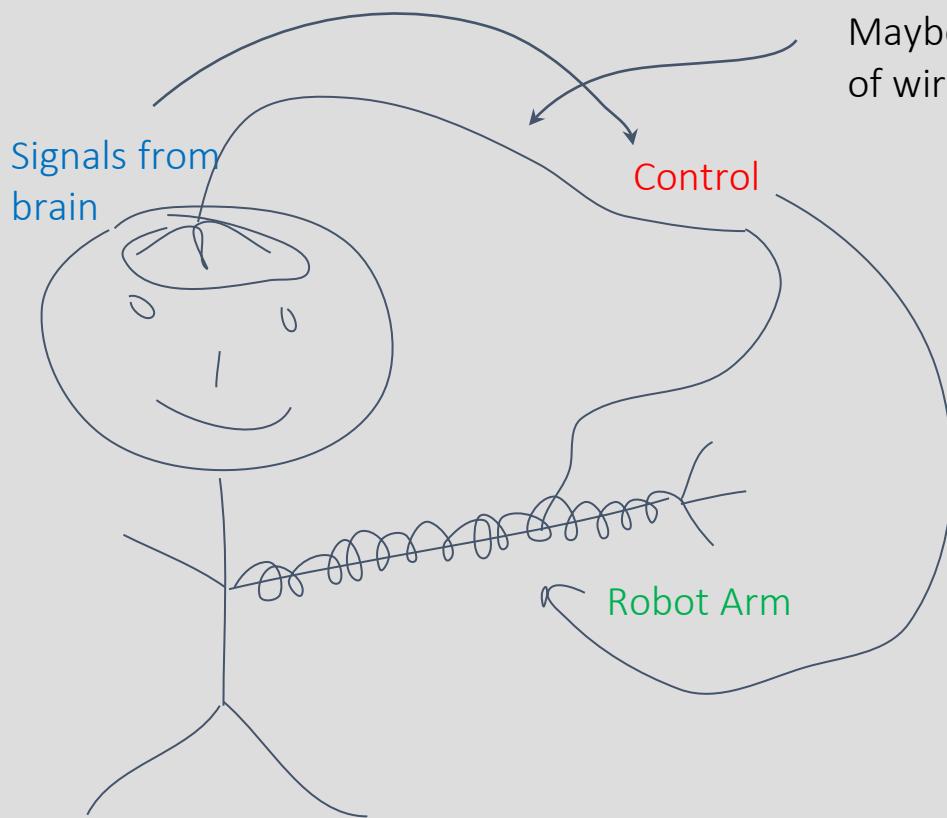


# What you built:



# EECS16B: Designing Information Devices and Systems II

**Big goal:** Get signals from brain and interpret them



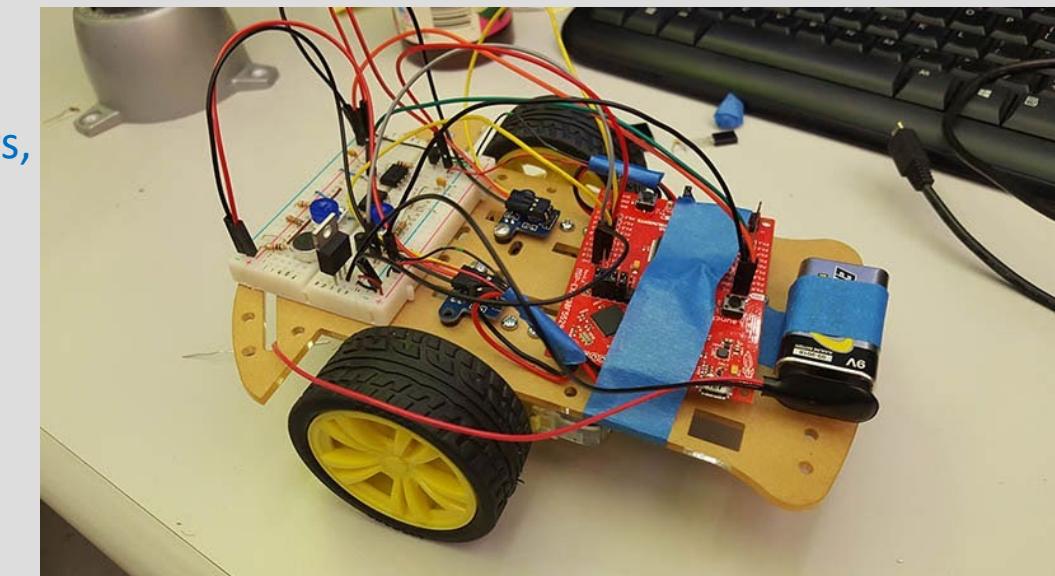
OpAmp Filters,  
ADCs/DACs,  
uController,  
SysID,  
Feedback,  
SVD, PCA

**Module 1 – Circuits: Interfaces (brain, voice)**

**Module 2 – Control: Controls (feedback, stability)**

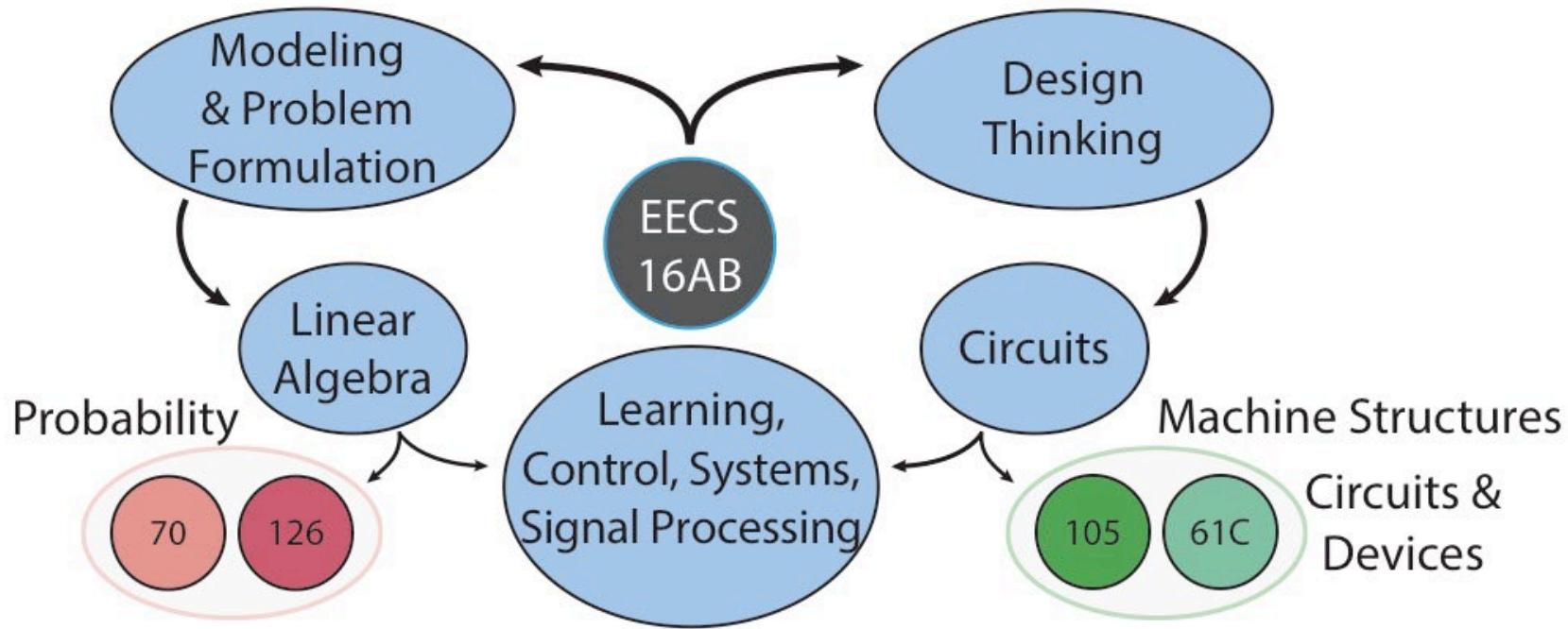
**Module 3 - Classification: Figuring out the intention**

**Voice controlled robo car lab project – from scratch!**



[Demo video](#)

**Design Contest**  
**(make our SIXT33N better!)**

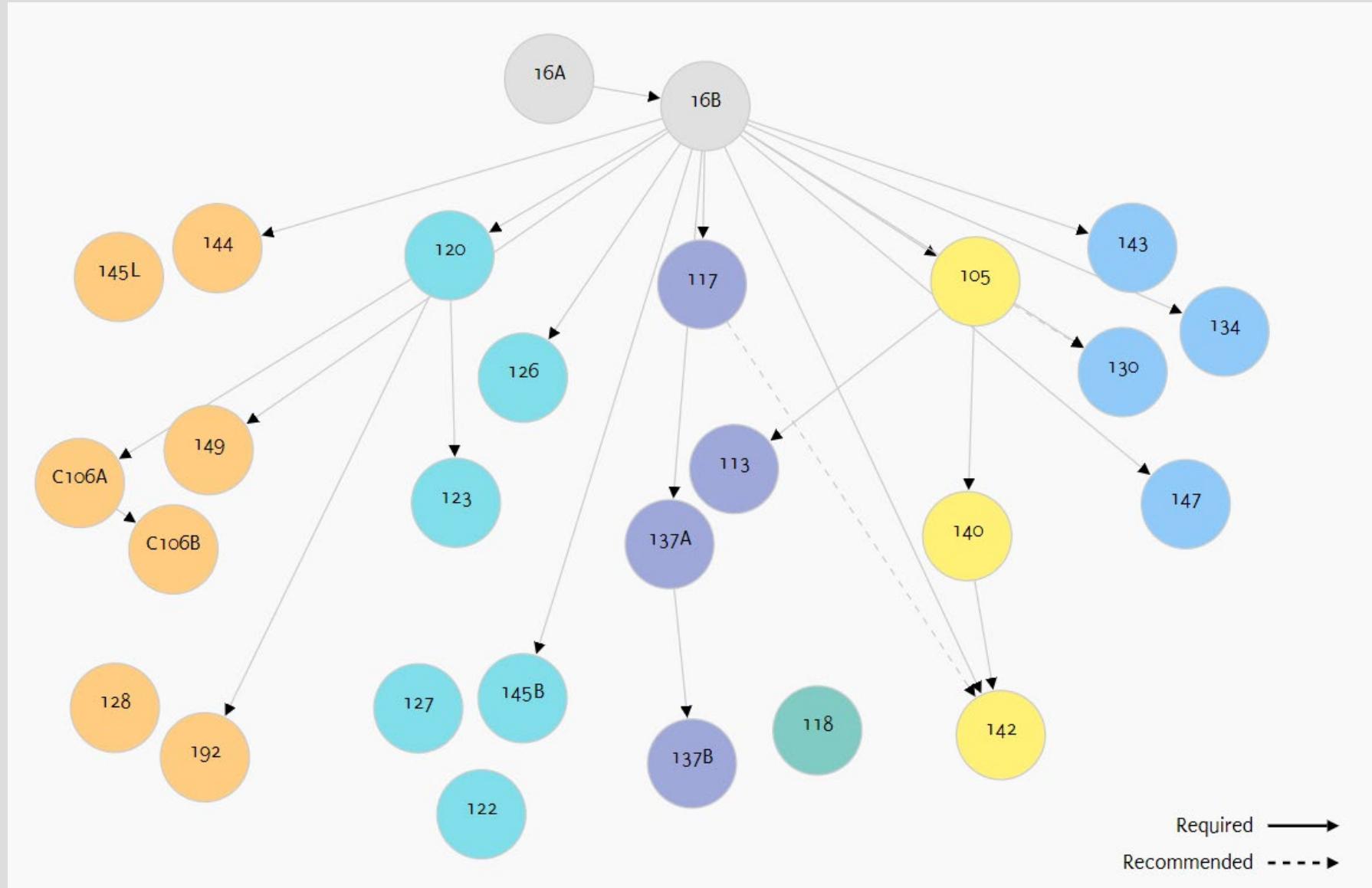


How to approach something unfamiliar  
and systematically build understanding

Linear Algebra: conceptual tools to model  
Circuits: How to go from model to design, grounded in physical world

Intro to foundational concepts in Machine Learning

# EECS course map



## CS COURSE MAP



core



hardware



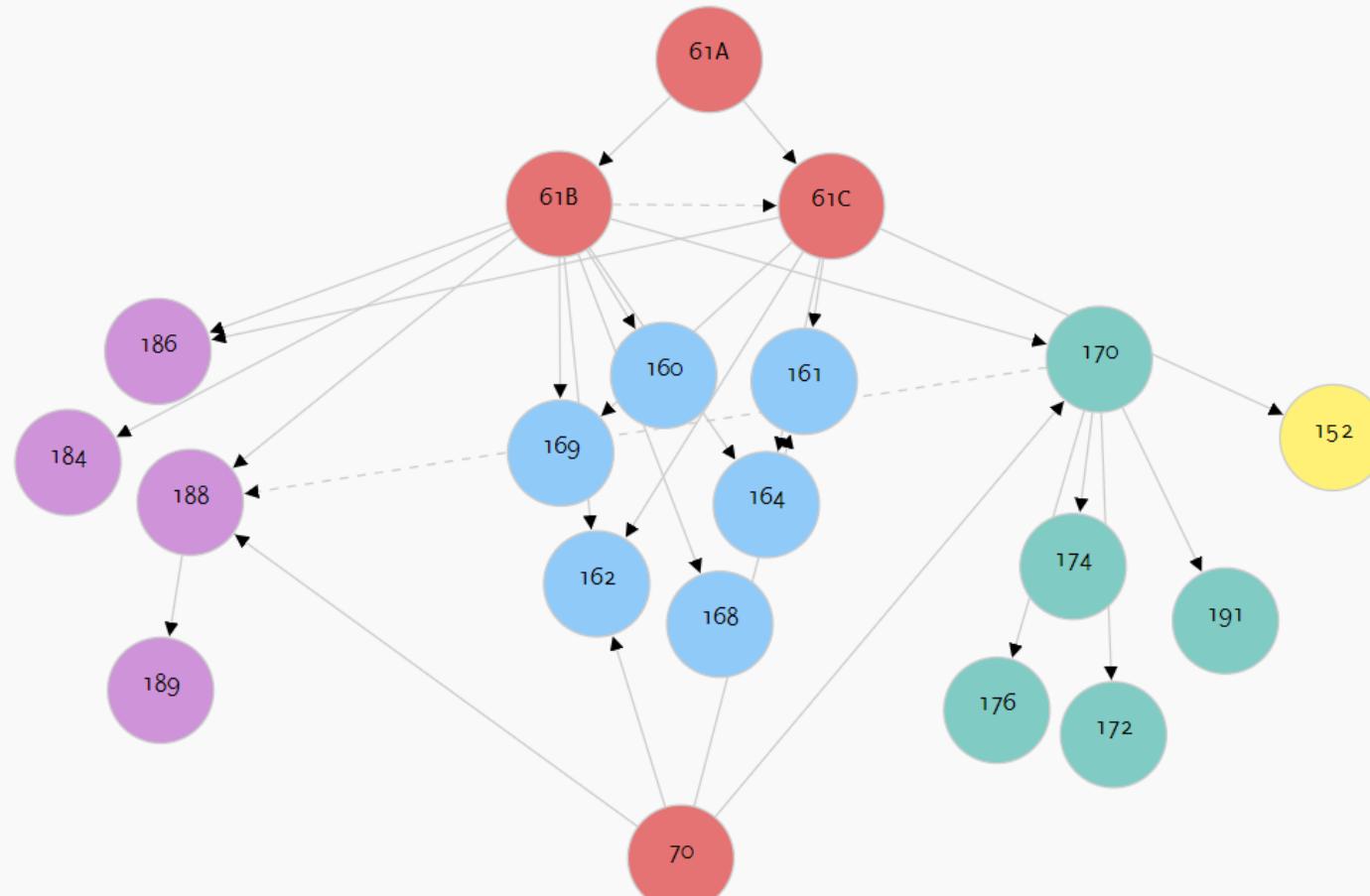
software



theory



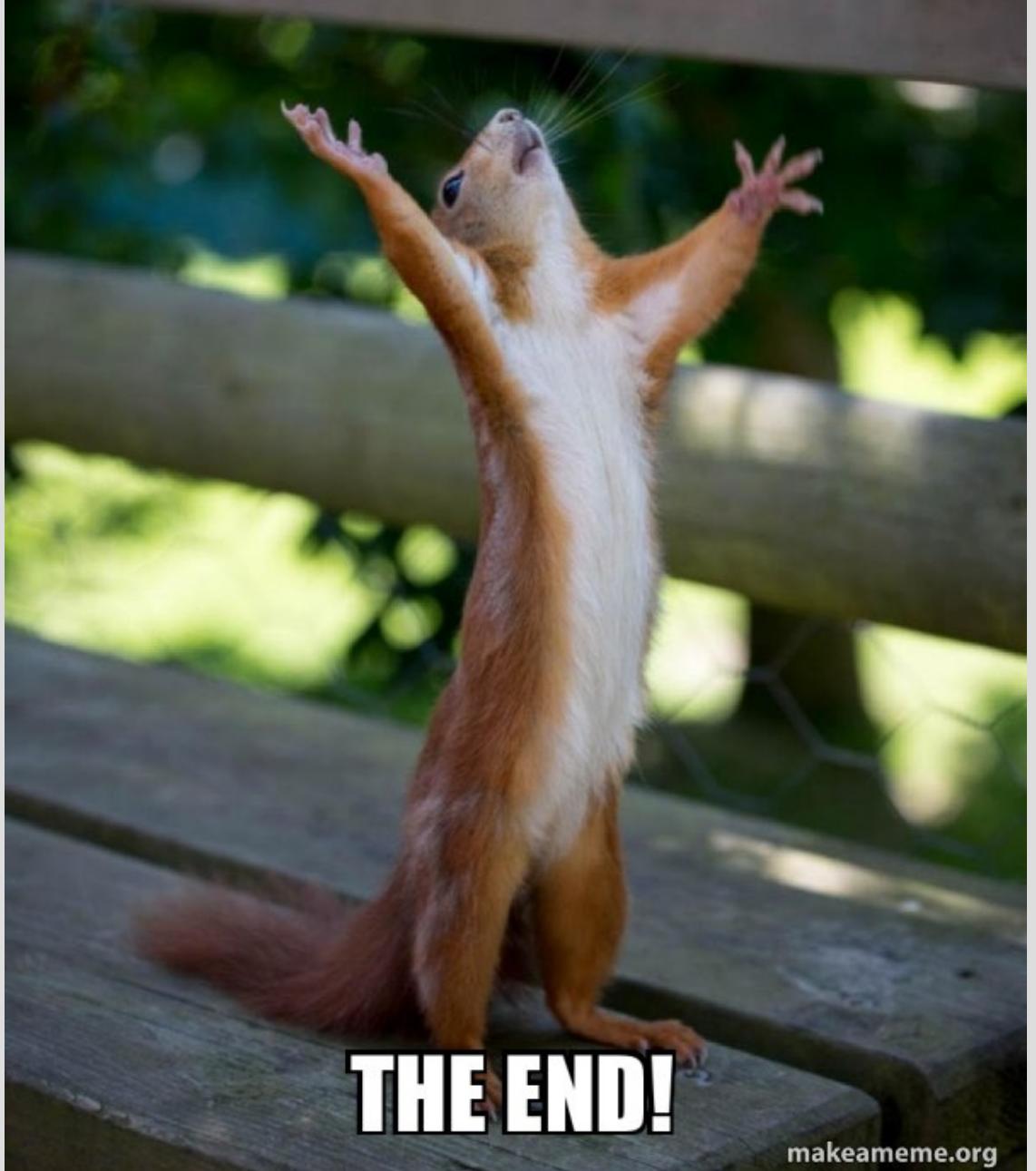
applications



Required →

Recommended - - - →

**FINALLY!**



**THE END!**

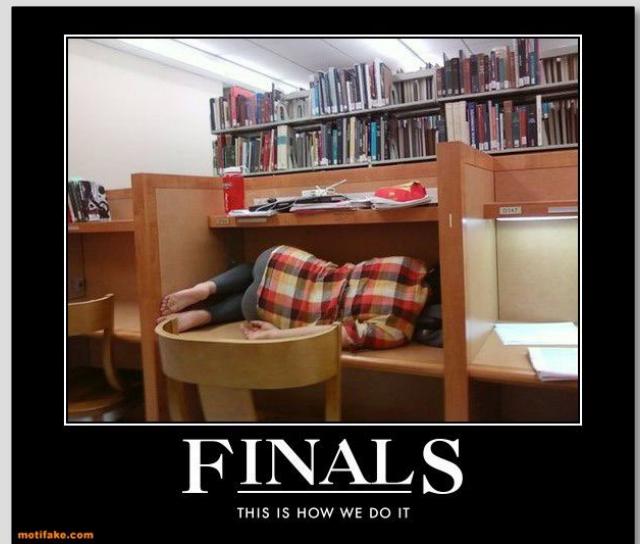
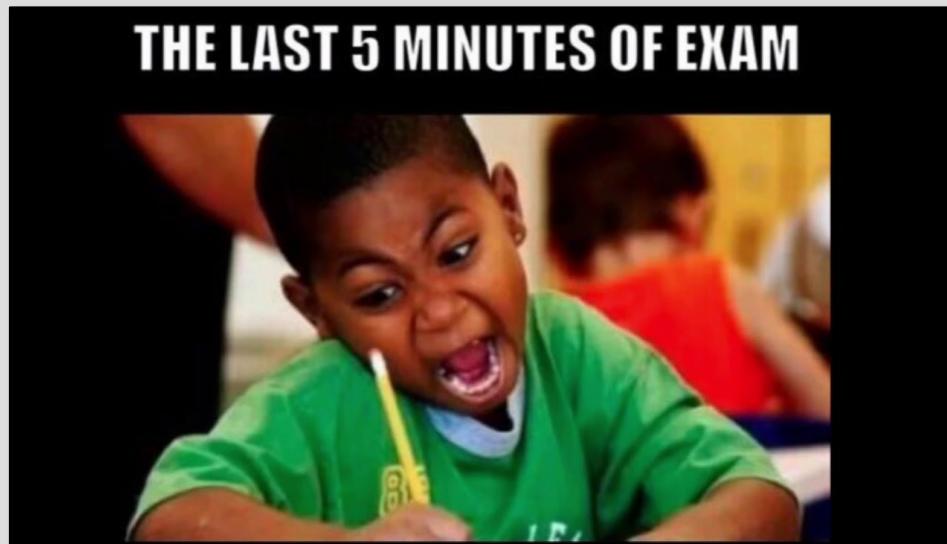
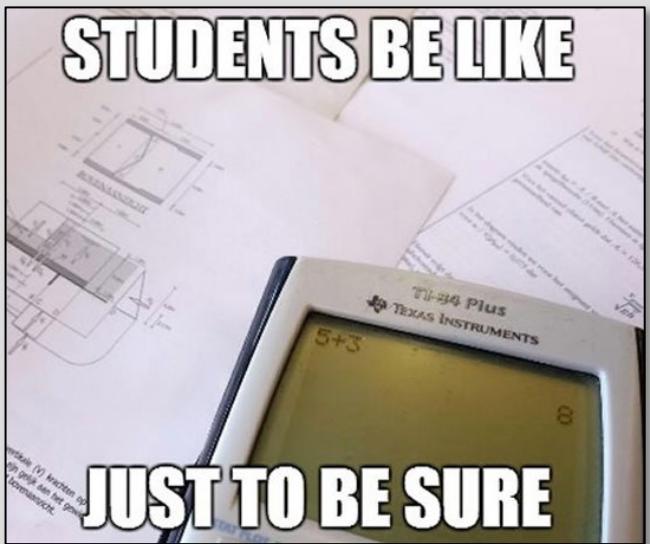
**The End**

# Oh, except for the final exam...

**IN FINAL EXAM..  
WHEN YOU DON'T KNOW THE  
ANSWER OF QUESTION... BUT  
YOU CAN'T LEAVE IT BLANK**



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