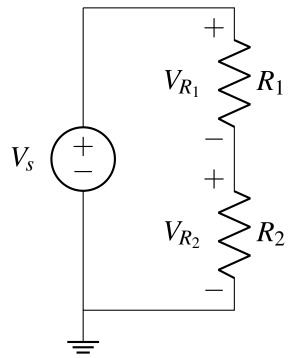
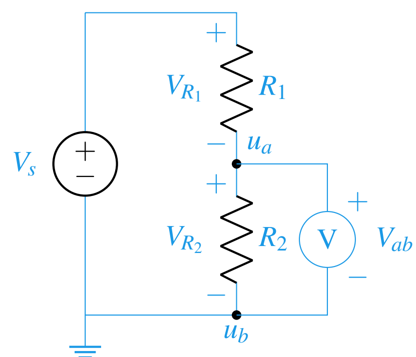
EECS 16A Spring 2023 — Designing Information Devices and Systems I — Discussion 7A

1. Volt and Ammeter

(a) For the voltage divider below, how would we connect a voltmeter to the circuit to measure the voltage V_{R_2}?

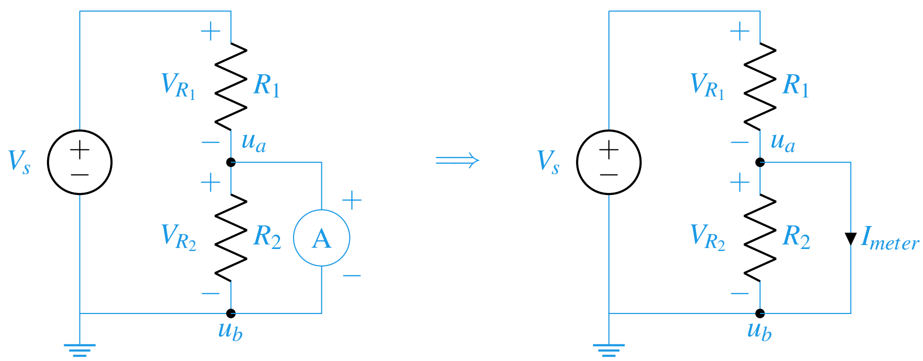


Answer: We connect our voltmeter to the voltage divider such that the voltage across the voltmeter nodes is equal to the voltage we want to measure.
i.e. $V_{ab} = u_a - u_b = V_{R_2}$



(b) What would happen if we accidentally connected an ammeter in the same configuration instead? Assume our ammeter is ideal.

Answer: An ideal ammeter behaves like a wire or a short between two nodes as depicted below:

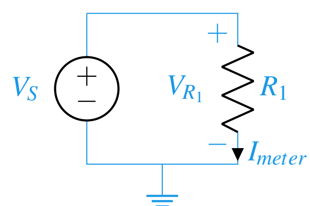


When we have a short, or a new lower resistance path for the current to travel, current will choose to take the path of least resistance. In this case, the ideal wire has no resistance so all of the current leaving R_1 will flow through the wire instead of R_2.

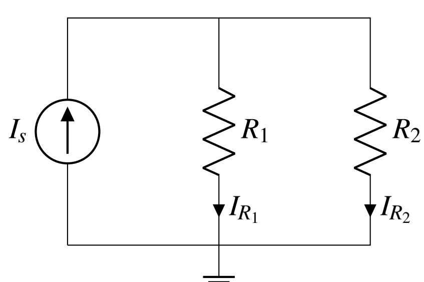
Mathematically, we can see this because this wire has now combined the nodes u_a and u_b into one node. In other words, $u_a = u_b$. We can use this to show that no current is flowing through R_2.

$$I_{R_2} = \frac{V_{R_2}}{R_2} = \frac{u_a - u_b}{R_2} = \frac{0}{R_2} = 0$$

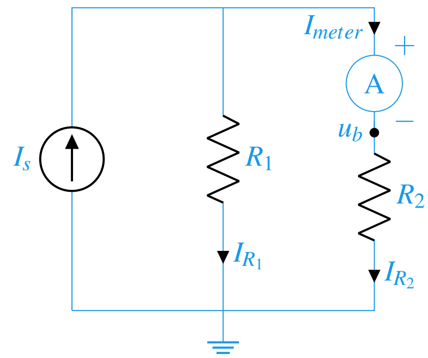
Therefore, an equivalent circuit can be drawn as shown below:



(c) For the current divider below, how would we connect an ammeter to the circuit to measure the current I_{R_2}?

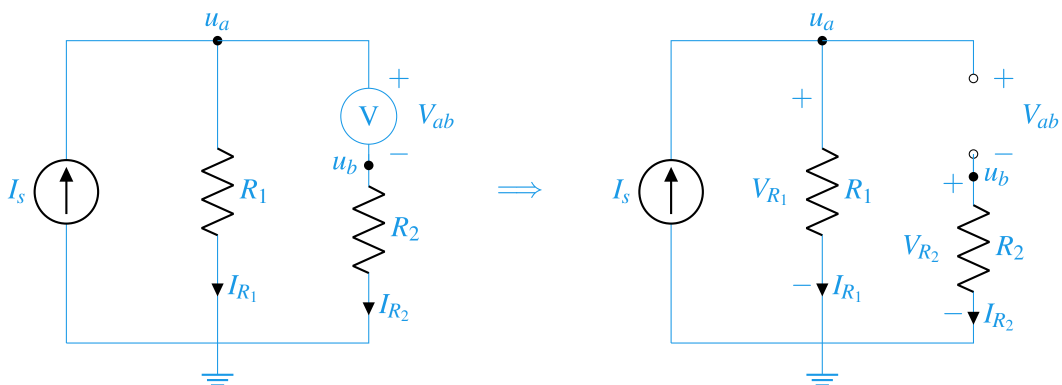


Answer: We connect our ammeter to the current divider such that the current going through the ammeter is equal to the current we want to measure. By doing a KCL at node u_b we can see that $I_{meter} = I_{R_2}$.



(d) What would happen if we accidentally connected a voltmeter in that configuration instead? Assume the voltmeter is ideal.

Answer: An ideal voltmeter behaves like an open circuit as depicted below:



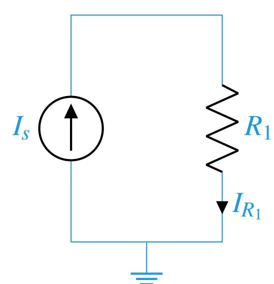
The open circuit creates a dead end and prevents any current from flowing through this circuit branch, therefore $I_{R_2} = 0$ and there is no voltage drop across resistor R_2. As a result, the voltage at node $u_b = V_{R2}$ is now equal to the reference node voltage.

$$u_b - 0 = V_{R2} = I_{R_2} R_2 = 0$$

With this knowledge, we can conclude that the voltmeter will actually read the voltage across the resistor R_1.

$$V_{ab} = u_a - u_b = u_a - 0 = V_{R_1}$$

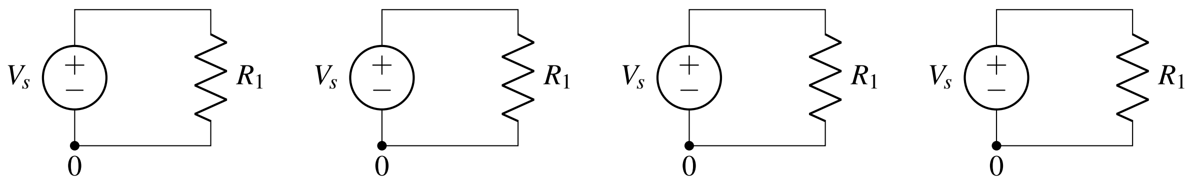
Since $I_{R_2} = 0$, the resistor R_2 has no effect on our circuit and an equivalent circuit can be drawn below:



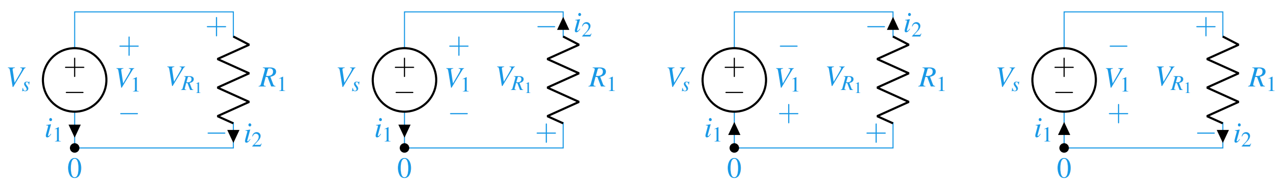
An ideal measurement should be unobtrusive and not change the circuit's behavior.

2. Passive Sign Convention and Power

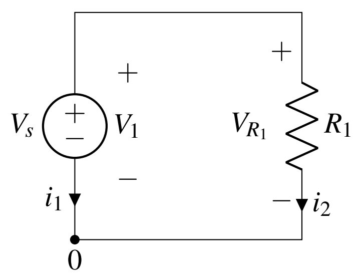
(a) Below are four copies of the same single-resistor circuit. On each copy, provide a distinct choice of labels for each circuit's voltage polarities and current directions (there should be 4 possible choices in total!) while keeping with passive sign convention.



Answer:



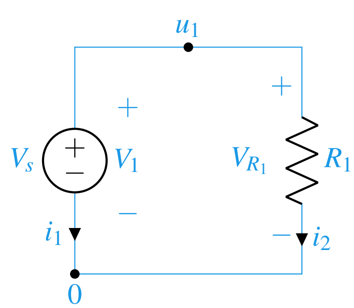
(b) Suppose we consider the following labeling. Calculate the power dissipated or supplied by every element in the circuit. Let $V_s = 5\,\text{V}$ and let $R_1 = 5\,\Omega$. What does a negative value of power represent?



Answer:

Recall that the power dissipated is the rate of electric energy converted into other forms and is given by the equation $P = IV$. A negative value of power dissipated by an element signifies this element is actually supplying/delivering electrical power/energy to the circuit.

We'll start by solving the circuit for the unknown node potentials and currents.



The KCL equation for the one node in this circuit is:

$$i_1 + i_2 = 0$$

The element equations for the two elements in this circuit are:

$$u_1 - 0 = V_1 = V_s$$

$$u_1 - 0 = V_{R_1} = i_2 R_1$$

Solving the above equations with $V_s = 5\,\text{V}$ and $R_1 = 5\,\Omega$:

$$u_1 = 5\,\text{V}$$
$$i_1 = -1\,\text{A}$$
$$i_2 = 1\,\text{A}$$

From above, we can solve for the power dissipated across the resistor:

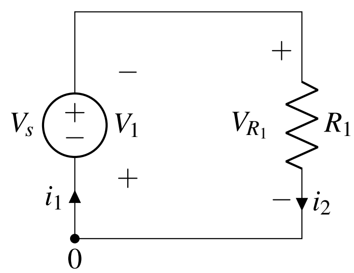
$$P_{R_1} = i_2 V_{R_1} = 1\,\text{A} \cdot 5\,\text{V} = 5\,\text{W}$$

Next we can solve for the power dissipated across the voltage source:

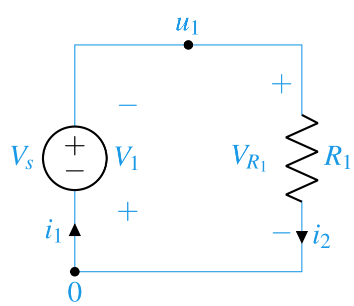
$$P_{V_s} = i_1 V_1 = i_1 V_s = -1\,\text{A} \cdot 5\,\text{V} = -5\,\text{W}$$

Notice we calculate a negative value for the power dissipated by the voltage source, implying the voltage source is adding power to the circuit.

(c) Suppose we choose a second labeling of the circuit as shown below. Calculate the power dissipated or supplied by every element in the circuit. Again, let $V_s = 5\,\text{V}$ and let $R_1 = 5\,\Omega$.



Answer: We'll solve the circuit the same way as last time.



The KCL equation for the one node in this circuit is:

$$-i_1 + i_2 = 0$$

The element equations for the two elements in this circuit are:

$$0 - u_1 = V_1 = -V_s$$

$$u_1 - 0 = V_{R_1} = i_2 R_1$$

Solving the above equations with $V_s = 5\,\mathrm{V}$ and $R_1 = 5\,\Omega$:

$$u_1 = 5\,\mathrm{V}$$
$$i_1 = 1\,\mathrm{A}$$
$$i_2 = 1\,\mathrm{A}$$

From above, we can solve for the power dissipated across the resistor:

$$P_{R_1} = i_2 V_{R_1} = 1\,\mathrm{A} \cdot 5\,\mathrm{V} = 5\,\mathrm{W}$$

Next we can solve for the power dissipated across the voltage source:

$$P_{V_s} = i_1 V_1 = i_1(-V_s) = 1\,\mathrm{A} \cdot -5\,\mathrm{V} = -5\,\mathrm{W}$$

Notice here that the circuit has the same power dissipated by all the elements. This is because with both labeling of currents, we followed the passive sign convention.

(d) Did the values of the element voltages and element currents change with the different labeling? Did the power for each circuit element change? Did the node voltages change? If a quantity didn't change with a difference in labeling, discuss what would have to change for quantity to change.

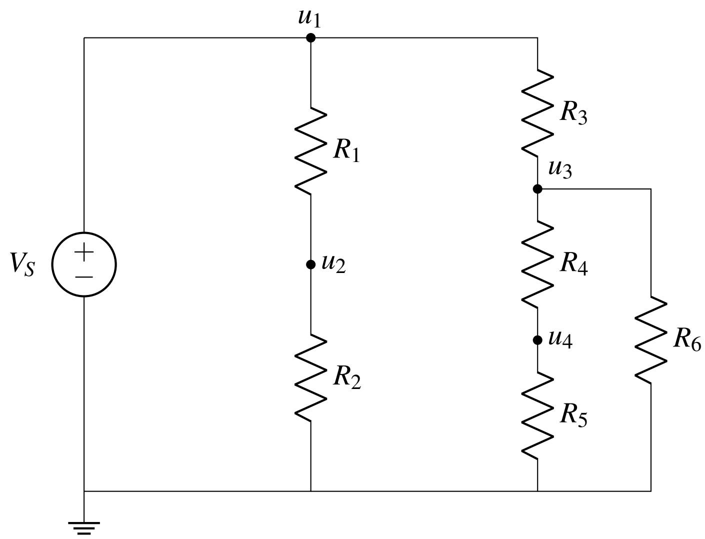
Answer: With a different labeling, element voltages and element currents will change. The quantities were $V_1 = 5\,\mathrm{V}$ and $i_1 = -1\,\mathrm{A}$ in (b) and in (c) $V_1 = -5\,\mathrm{V}$ and $i_1 = 1\,\mathrm{A}$. Flipping the direction of a labeled current or the polarity of a labeled voltage will lead to negation of the value.

The power dissipated by a circuit element will not change because we follow passive sign convention. Passive sign convention requires that if we flip the direction of an element current we also flip the polarity of the corresponding element voltage, so there is a double negation in the computation of power. The only way to get a different value of power would be to change the component values or the circuit diagram itself by removing or adding more circuit elements. A physical system will only have one behavior as governed by the laws of physics - how we compute our answer should not change how it behaves. Our labeled voltage polarities and current directions are more akin to measurement choices which can change what we see.

The node voltages too, did not change. The top node voltage in both labelings were $5\,\mathrm{V}$ and the bottom node voltages were $0\,\mathrm{V}$. What would have to change to alter these values is one of three things: either the location of the reference, the circuit component values, or the circuit diagram itself.

3. NVA Equations

Suppose we have the following circuit. All nodes (including the reference node) have already been labeled for you.

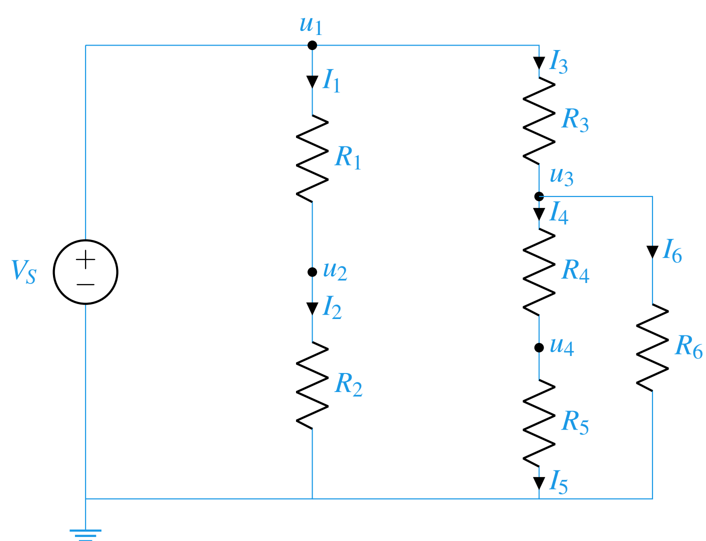


(a) Label all of the branch currents. Do the directions you pick matter?

Answer:

When labeling the currents through branches, the direction you pick does not matter.

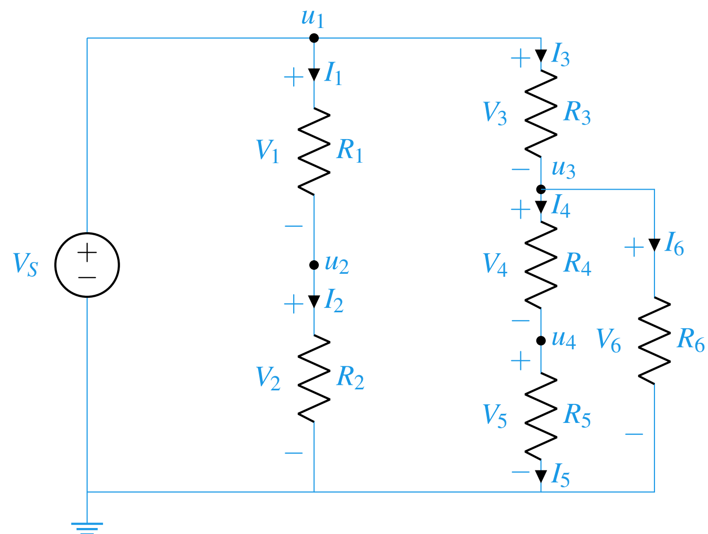
Provided is an example of possible labeling. All future equations will depend on this labeling, and a different labeling scheme would result in different equations (but the same circuit solution).



(b) Draw the $+/-$ voltage labels on every element. What convention must you follow?

Answer:

When drawing the $+/-$ labels, you must follow the passive sign convention. That is, current flows into the $+$ terminal (and out of the $-$ terminal) of every element.



(c) Identify and simplify any redundant currents or known node voltages.

Answer:

The voltage difference across the voltage source is $u_1 - 0 = V_S$, thus the node voltage is $u_1 = V_S$.

The currents I_1 and I_2 are in the same circuit branch (i.e., KCL: $I_1 - I_2 = 0$) and thus can be reduced as $I_1 = I_2$.

The currents I_4 and I_5 are in the same circuit branch (i.e., KCL: $I_4 - I_5 = 0$) and thus can be reduced as $I_4 = I_5$.

(d) Write a KCL equation at every node with an unknown voltage potential.

Answer:

$$\begin{aligned} \text{Node 2}: &\quad I_1 - I_2 = 0 \\ \text{Node 3}: &\quad I_3 - I_4 - I_6 = 0 \\ \text{Node 4}: &\quad I_4 - I_5 = 0 \end{aligned}$$

(e) Use Ohm's law and node voltage differences to find the remaining equations to solve the circuit.

Answer:

Express the element/branch currents as a function of node voltages

$$
\begin{aligned}
I_1R_1 = V_1 = u_1 - u_2 \quad &\implies \quad I_1 = \frac{u_1 - u_2}{R_1} = \frac{V_S - u_2}{R_1} \\
I_2R_2 = V_2 = u_2 - 0 \quad &\implies \quad I_2 = \frac{u_2}{R_2} \\
I_3R_3 = V_3 = u_1 - u_3 \quad &\implies \quad I_3 = \frac{u_1 - u_3}{R_3} = \frac{V_S - u_3}{R_3} \\
I_4R_4 = V_4 = u_3 - u_4 \quad &\implies \quad I_4 = \frac{u_3 - u_4}{R_4} \\
I_5R_5 = V_5 = u_4 - 0 \quad &\implies \quad I_5 = \frac{u_4}{R_5} \\
I_6R_6 = V_6 = u_3 - 0 \quad &\implies \quad I_6 = \frac{u_3}{R_6}
\end{aligned}
$$

(f) Finally, write a system of node voltage equations. How could we solve this system for the unknowns? *In general we can treat the sources and resistances as knowns. The only unknowns should be the node voltages.*

Answer:

Substitute the I-V characteristics from part (e) into the KCL equations from part (f).

$$
\begin{aligned}
\text{Node 2 :} \quad & \frac{V_S - u_2}{R_1} - \frac{u_2}{R_2} = 0 \\
\text{Node 3 :} \quad & \frac{V_S - u_3}{R_3} - \frac{u_3 - u_4}{R_4} - \frac{u_3}{R_6} = 0 \\
\text{Node 4 :} \quad & \frac{u_3 - u_4}{R_4} - \frac{u_4}{R_5} = 0
\end{aligned}
$$

To solve this system we could use substitution, but since there are three equations and three unknowns (u_2, u_3, and u_4), it would be better to set up a matrix vector equation in the form $\mathbf{A}\vec{u} = \vec{b}$

$$
\begin{bmatrix} -\frac{1}{R_1} - \frac{1}{R_2} & 0 & 0 \\ 0 & -\frac{1}{R_3} - \frac{1}{R_4} - \frac{1}{R_6} & -\frac{1}{R_4} \\ 0 & \frac{1}{R_4} & -\frac{1}{R_1} - \frac{1}{R_5} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} -\frac{V_S}{R_1} \\ -\frac{V_S}{R_3} \\ 0 \end{bmatrix} \tag{1}
$$

We can use our linear algebra techniques (e.g., Gaussian Elimination, $\vec{u} = \mathbf{A}^{-1}\vec{b}$) to solve this system for the unknown node voltages.