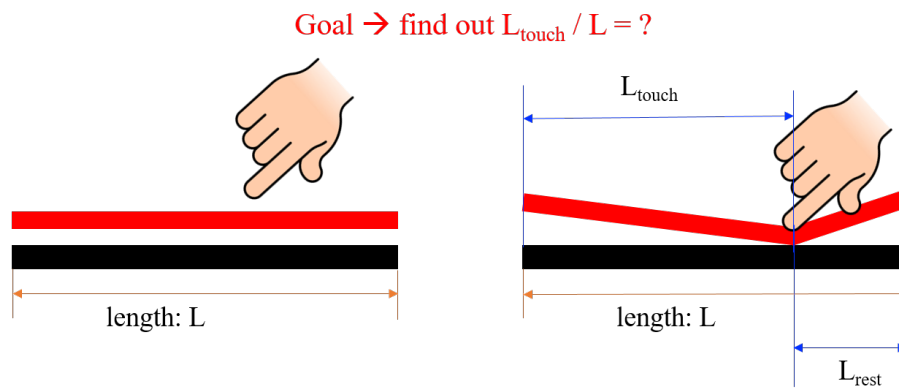## 13.1   Resistive Touchscreen - Expanding the Model

Recall the physical structure of the simple resistive touchscreen given in the prior lecture:

Previously, we ignored the black "bottom plate". But why do we want it at all? Let's return to the model of our touchscreen and build from that. The simple version is shown below, where u_{mid} represents the point where the finger is touching. Note that a voltage source has been connected around it, with the positive terminal connected to one edge of the top plate and the negative terminal connected to the opposite edge.
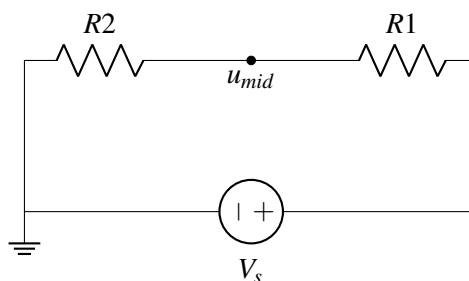
To understand what the bottom plate is doing, we will first assume the bottom plate is a perfect conductor ($\rho = 0$), so the circuit will look as follows. Notice again that a voltage source is connected to the terminating edges of the top plate:
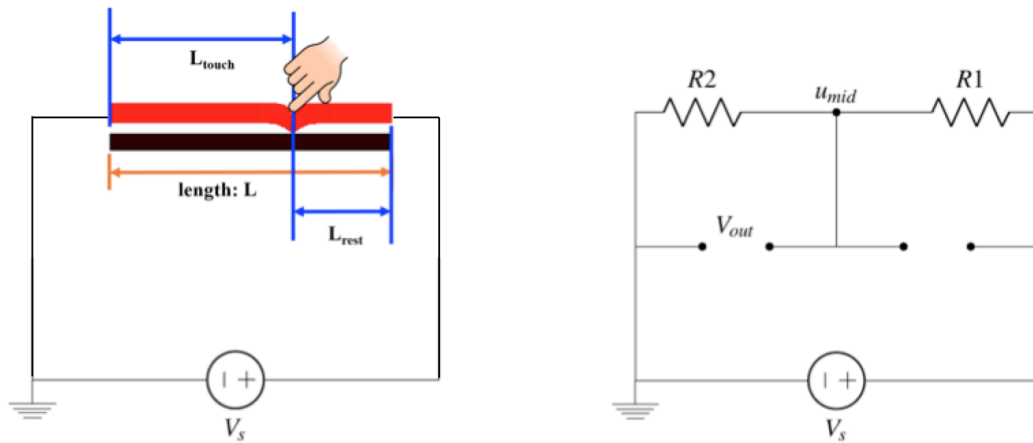
Does the voltage at u_{mid} change in this case? Thinking about it in terms of nodal analysis, we have not actually added any new nodes to our circuit - ideal wires (with zero voltage drop) are all that we have actually added. Therefore, we see that with a perfect conductor (analogous to our ideal wire), $V_{out} = u_{mid}$. Note here that V_{out} is the voltage drop across an open circuit that is placed in parallel with resistor R_2 to capture $V_{R_2} = u_{mid}$.

So why have the bottom plate at all? It lets us take the measurement for V_{out} using connection points on the edge of the plate instead of having to put a probe or wire at the actual touch point every time! As we have shown, in the case of an ideal bottom plate this does not affect the measurement result itself.

What if the bottom plate is not a perfect conductor? This means that the bottom plate would also have an associated resistance, calculated just like the resistance of the top electrode. Our new circuit schematic would look like this:
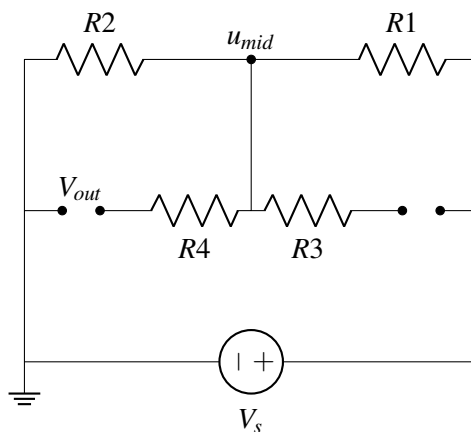
There are now more nodes in the circuit. We now have two options; we can either go back and perform nodal analysis with this new expanded circuit, or we can try and build off previous understanding to see if we can discern how the addition of resistors R_3 and R_4 affect circuit behavior.

Consider this zoomed-in schematic:
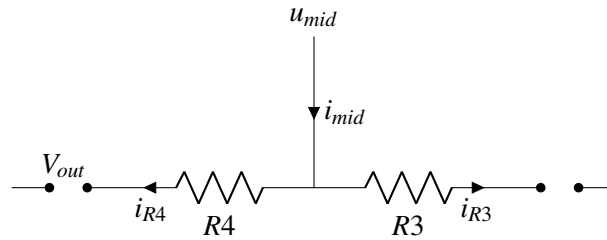
Note that there is a current shown through the two new resistors, which may lead one to think that we need to conduct nodal analysis again. However, both of these resistors are followed by an open circuit. From the definition of an open circuit, we know that zero current will flow through it. If $i_{mid} = i_{R4} + i_{R3}$ from KCL, and $i_{R4} = i_{R3} = 0$ from the definition of an open circuit (Ohm's Law says that $0V = R * 0A$), the voltage across these new resistors will be 0. This means that, even with an imperfectly conductive bottom plate, the voltage V_{out} will still be equal to u_{mid}, even with the addition of these new resistors.

To measure an output voltage, we need to put some device at the open circuit labeled V_{out}. Therefore, from the analysis above, we would like our device to function like an "open circuit". In the prior case, it allowed us to neglect the non-idealities of our bottom plate. This extends to an important question: **How can I guarantee that whatever I connect to my circuit to measure V_{out} or i_{out} does not influence/change the circuit itself?** To seek a general answer, we will introduce a new quantity: power.

13.2 Power

Note the two fundamental quantities that we may want to measure, voltage and current. To measure these quantities we use tools called a *voltmeter* and an *ammeter*, respectively. We can draw an abstracted diagram that looks like these for each case:
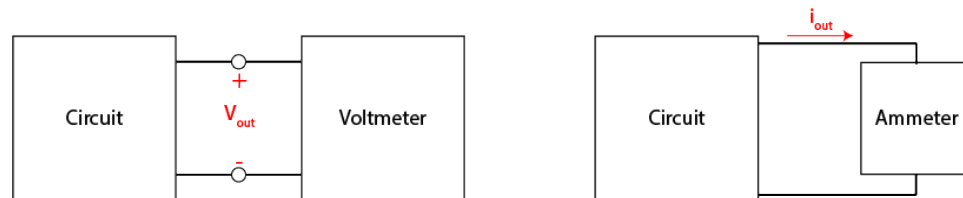
Note that the voltmeter measures voltage *across* (i.e. in parallel with) the circuit, while the ammeter needs to be put in-line (i.e. in series) with the circuit so that the current flows *through* the ammeter. It turns out that the most complete and concise way of guaranteeing these measurement tools do not influence the circuit is to state that **they do not allow any power dissipated through the measurement device**.

So what exactly *is* power? Power P is the rate of change of energy, i.e. $P = \frac{dE}{dt}$. Recall that voltage V is the amount of energy needed to move a unit charge between two points (see Note 12). Thus, $dE = V \, dQ$ where dE is a differential unit of energy and dQ is a differential unit of charge. Taking the time derivative of both sides yields:
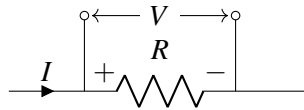
$$\frac{dE}{dt} = V\frac{dQ}{dt} \tag{1}$$

$$P = V\frac{dQ}{dt} \tag{2}$$

Recall that current I is defined as the rate of change of charge, $\frac{dQ}{dt}$. Therefore,

$$P = V\frac{dQ}{dt} \tag{3}$$

$$P = VI \tag{4}$$

The power dissipated (positive) or supplied (negative) by a component with voltage V and current I is thus equal to their product IV. Note that according to passive sign convention, positive current goes into the positive terminal of the component.

When $P = IV$ is positive, power is being dissipated; when $P = IV$ is negative, power is being generated/delivered. Being able to tell when power is dissipated versus delivered using the sign is the reason why passive sign convention was introduced in the first place! Note that, once we have completed nodal analysis, we should therefore be able to solve for power delivered/dissipated in every circuit element using this relationship.

$P = IV$ is a fundamental relationship that will be used repeatedly and is worth memorizing. It can also take other useful forms through combination with Ohm's Law, given here for reference:

$$P = IV \tag{5}$$

$$P = \frac{V^2}{R} \tag{6}$$

$$P = I^2 R \tag{7}$$

Returning to the question of how to build an ideal measurement device, remember that we want the device to have zero effect on the power dissipation/generation of the circuit.
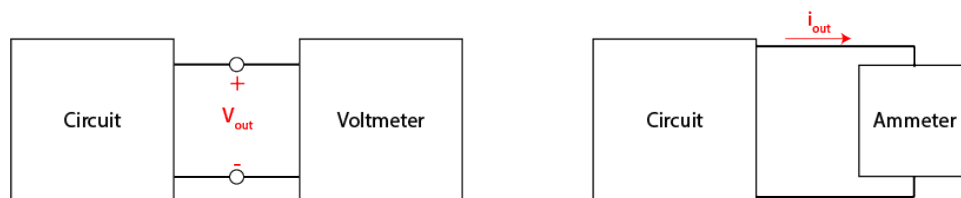
Because our voltmeter is made to measure voltage, we can naturally assume that the voltage will or can be non-zero; this means that a voltmeter *must* have 0 current going into it to ensure $P = IV = 0$. In the ideal

case, the voltmeter will resemble an open-circuit, which has exactly zero current through. If we instead assume that the voltmeter does have some associated resistance, what do we want it to be? Remember Ohm's Law, $V = IR$. For a given voltage, the higher the associated resistance, the lower the current and therefore the lower the dissipated power. Therefore, in the ideal case, we would want our voltmeter to have infinite input resistance.

On the other hand, the ammeter has the circuit's current flowing *through* it. Therefore, in order to ensure $P = IV = 0$, the ammeter needs V across it to equal zero. In the ideal case, the ammeter will therefore resemble a short-circuit (ideal wire), which has exactly zero voltage drop across it. If we instead assume that the ammeter does have some associated resistance, what do we want it to be? Remember Ohm's Law, $V = IR$. For a given current, the lower the associated resistance, the lower the associated voltage drop and therefore the lower the dissipated power. Therefore, in the ideal case, we would want our ammeter to have negligible input resistance.

13.3 Practice Questions

These practice problems are also available in an interactive form on the course website.

1. What does it mean for the power of a particular component to be negative, for the following components?

   (a) Resistor
   (b) Voltage source
   (c) Current source

2. How much power is dissipated in R_2 in the following circuit?
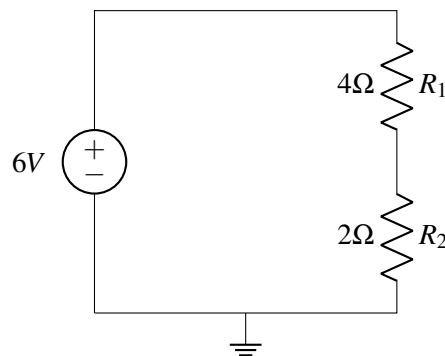
3. In the same circuit as above, how much power is dissipated in the voltage source?