EECS 16A Designing Information Devices and Systems I
Fall 2022 Homework 7

This homework is due October 21, 2022, at 23:59.
Self-grades are due October 24, 2022, at 23:59.

Submission Format

Your homework submission should consist of **one** file.

- `hw7.pdf`: A single PDF file that contains all of your answers (any handwritten answers should be scanned).

Submit the file to the appropriate assignment on Gradescope.

Mid Semester Survey

Please fill out the mid semester survey: https://forms.gle/dZVb1uAR9zu8msZr8.

We highly appreciate your feedback!

1. Reading Assignment

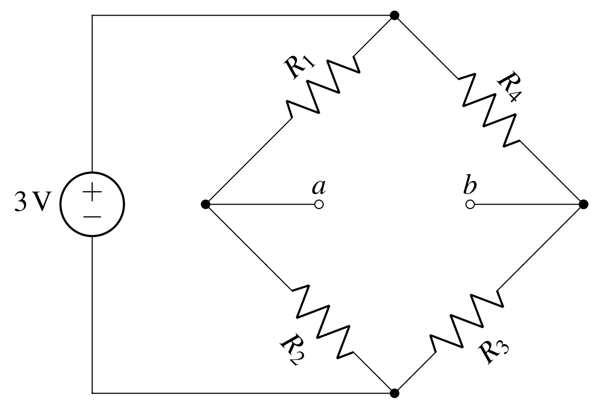
For this homework, please read Note 12, Note 13, and Note 14. Notes 12 and 13 covers voltage divider, how a simple 1-D resistive touchscreen works, physics of circuits, and introduce the notion of power in electric circuits. Note 14 will cover a slightly more complicated 2-D resistive touchscreens and how to analyze them from a circuits perspective.

(a) Describe the key ideas behind how the 1D touchscreen works. In general, why is it useful to be able to convert a “physical" quantity like the position of your finger to an electronic signal (i.e. voltage)?

Solution: You should give yourself full-credit for any reasonable answers.

2. Wheatstone Bridge

A Wheatstone Bridge is a very useful circuit that can be used to help determine unknown resistance values with very high accuracy. This circuit is used in many sensor applications where resistors $R_1 - R_4$ are varying with respect to some external actuation. For example, it can be used to build a strain gauge (`https://en.wikipedia.org/wiki/Strain_gauge`) or a scale. In that case the resistors $R_1 - R_4$ would vary with respect to a strain caused by a force, and the Wheatstone Bridge circuit would translate that variation into a voltage difference across the “bridge” terminals a and b.



(a) Assume that $R_1 = 1\,\text{k}\Omega$, $R_2 = 2\,\text{k}\Omega$, $R_3 = 3\,\text{k}\Omega$, $R_4 = 3\,\text{k}\Omega$. Calculate the voltage V_{ab} between the two terminals a and b.

Hint: It may help to redraw the circuit with each branch containing resistors $R_1 - R_4$ using straight vertical wires, rather than diagonal ones.

Solution:

Notice in the above circuit that there are two voltage dividers, so we can calculate u_a and u_b quickly.

$$u_a = V_{R2} = \frac{R_2}{R_1 + R_2} \cdot 3\,\text{V} = 2V$$

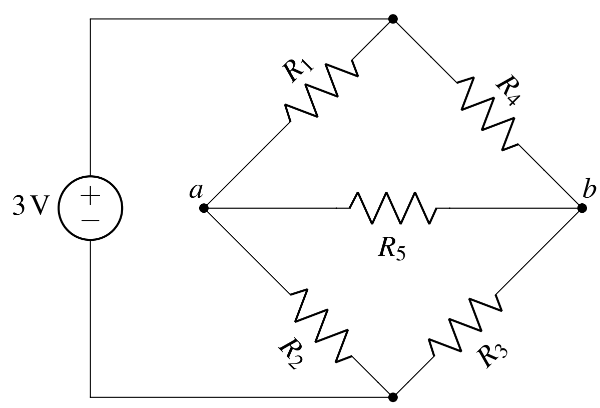
$$u_b = V_{R3} = \frac{R_3}{R_3 + R_4} \cdot 3\,\text{V} = 1.5V$$

Thus, the required voltage difference between the two terminals a and b is: $V_{ab} = u_a - u_b = 0.5V$.

(b) Now assume that you have added an additional resistor, R_5, between terminals a and b as shown below. Assume that $R_1 = 1\,\text{k}\Omega$, $R_2 = 2\,\text{k}\Omega$, $R_3 = 3\,\text{k}\Omega$, $R_4 =?\ \text{k}\Omega$, $R_5 = 5\,\text{k}\Omega$. In the process of constructing this circuit, you notice that you forgot to write down the value of R_4!

However, you notice something very curious about your Wheatstone Bridge circuit: there is no current flowing through resistor R_5.

Based on this observation, what must the value of resistor R_4 be?



Solution:

Because there is no current flowing through resistor R_5, by Ohm's Law we know that the value of V_{ab} must be zero (i.e, $u_a - u_b = 0\,\text{V}$). In other words, the nodes a and b must be at the exact same potential. We can use this fact to backtrack what the value of R_4 must be.

Since $V_{ab} = 0\,\text{V}$, we have actually two separate voltage dividers (R_1 and R_2 form one voltage divider; R_4 and R_3 form the other):

$$u_a = \frac{R_2}{R_1 + R_2} \cdot 3\,\text{V}$$
$$u_b = \frac{R_3}{R_3 + R_4} \cdot 3\,\text{V}$$
$$V_{ab} = u_a - u_b = 0 \implies \frac{R_2}{R_1 + R_2} \cdot 3\,\text{V} = \frac{R_3}{R_3 + R_4} \cdot 3\,\text{V}$$
$$\frac{R_2}{R_1 + R_2} = \frac{R_3}{R_3 + R_4}$$

Plugging in values and solving for R_4:

$$\frac{2\,\text{k}\Omega}{1\,\text{k}\Omega + 2\,\text{k}\Omega} = \frac{3\,\text{k}\Omega}{3\,\text{k}\Omega + R_4}$$
$$\implies R_4 = 1.5\,\text{k}\Omega$$

We note that the ratio of R_1 to R_2 turns out to be the same as the ratio of R_4 to R_3. Maintaining this ratio is what allows the voltage at node a to be equivalent to the voltage at b, and therefore for $V_{ab} = 0\,\text{V}$.

3. Volt and ammeter

Learning Goal: *This problem helps you explore what happens to voltages and currents in a circuit when you connect voltmeters and ammeters in different configurations.*

Use the following numerical values in your calculations: $R_1 = 1\,\text{k}\Omega$, $R_2 = 2\,\text{k}\Omega$, $R_3 = 3\,\text{k}\Omega$, $R_4 = 4\,\text{k}\Omega$, $R_5 = 5\,\text{k}\Omega$, $V_s = 12V$.

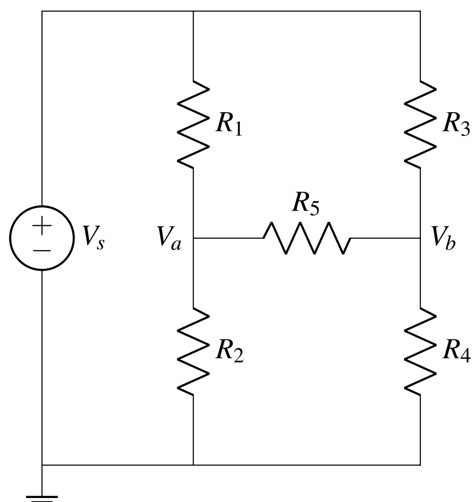
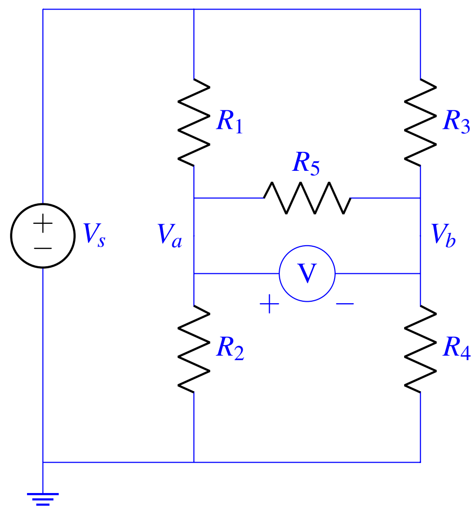


Figure 1: Circuit consisting of a voltage source V_s and five resistors R_1 to R_5

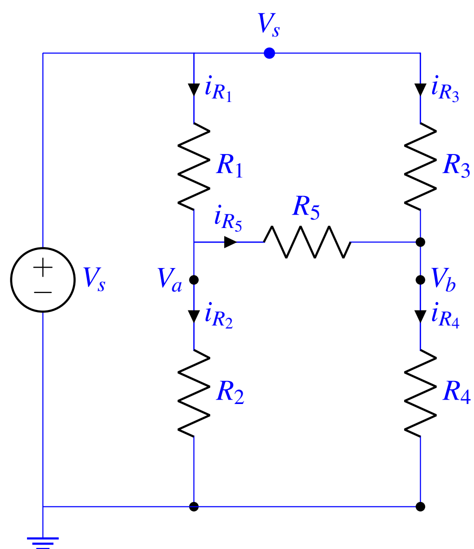
(a) Redraw the circuit diagram shown in Figure 1 by adding a voltmeter (letter V in a circle and plus and minus signs indicating direction) to measure voltage V_{ab} from node V_a (positive) to node V_b (negative).

Calculate the value of V_{ab}. You may use a numerical tool such as IPython to solve the final system of linear equations.

Solution: Below is the redrawn circuit with the voltmeter. Note that it is also correct to have the voltmeter above R_5, as it will still be connected to the same nodes.



Using NVA analysis we need to label our nodes. Instead of labelling u_i at each node, let's directly use the potentials we are given in the problem. V_a and V_b are already labelled. The topmost node is V_s and the bottom most node is our reference. We also label the currents in each element.



Using KCL at node V_a and V_b, we find:

$$i_{R_1} - i_{R_5} - i_{R_2} = 0$$

$$i_{R_5} + i_{R_3} - i_{R_4} = 0$$

Let's substitute IV relationships into the previous equations.

$$\frac{V_s - V_a}{R_1} - \frac{V_a - V_b}{R_5} - \frac{V_a}{R_2} = 0$$

$$\frac{V_a - V_b}{R_5} + \frac{V_s - V_b}{R_3} - \frac{V_b}{R_4} = 0$$

Gathering the V_a and V_b terms:

$$\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_5}\right)V_a - \left(\frac{1}{R_5}\right)V_b = \frac{V_s}{R_1}.$$

$$-\left(\frac{1}{R_5}\right)V_a + \left(\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}\right)V_b = \frac{V_s}{R_3}.$$

Notice that we wrote our unknowns (V_a and V_b) on the left side of the equation. We can then represent this in matrix form as:

$$\begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_5} & -\frac{1}{R_5} \\ -\frac{1}{R_5} & \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5} \end{bmatrix} \begin{bmatrix} V_a \\ V_b \end{bmatrix} = \begin{bmatrix} \frac{V_s}{R_1} \\ \frac{V_s}{R_3} \end{bmatrix}$$

Plugging in the values we were given into the matrix above and using Gaussian elimination we can find the vector of unknowns.

$$\begin{bmatrix} V_a \\ V_b \end{bmatrix} = \begin{bmatrix} 7.896V \\ 7.123V \end{bmatrix}$$

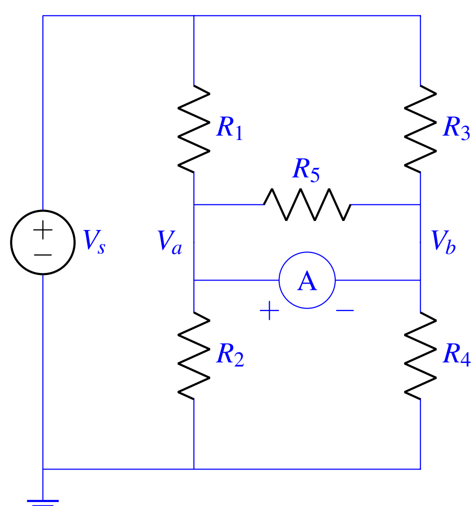
From these node voltages, the voltage V_{ab} can be calculated.

$$V_{ab} = V_a - V_b = 0.773V$$

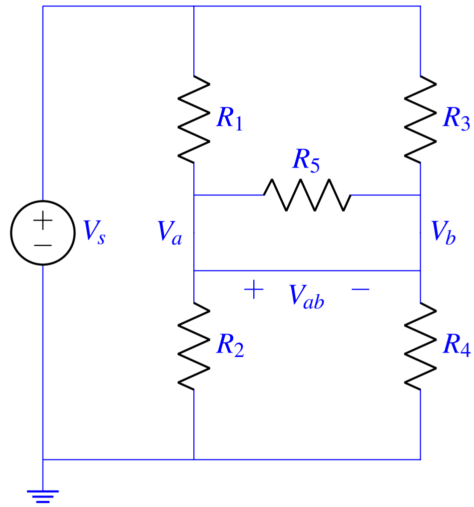
You should give yourself full-credit if your answer is off by a rounding error.

(b) Suppose you accidentally connect an ammeter in part (a) instead of a voltmeter. Calculate the value of V_{ab} with the ammeter connected.

Solution: While you did not have to redraw the circuit, it is depicted below.

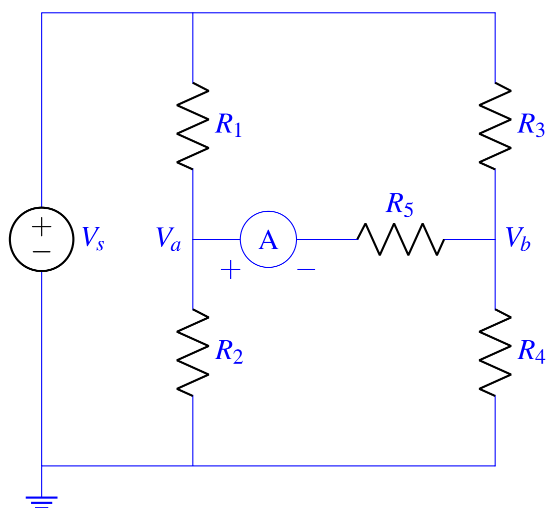


If we assume that the internal resistance of an ammeter is ideally zero, placing it across the nodes V_a and V_b will short them. So $V_a = V_b$. Thus $V_{ab} = 0$. The circuit below shows how the ammeter behaves as a short that unifies the previously separate nodes.



(c) Redraw the circuit diagram shown in Figure 1 by adding an ammeter (letter A in a circle and plus and minus signs indicating direction) in series with resistor R_5. This will measure the current I_{R_5} through R_5. Calculate the value of I_{R_5}.

Solution: The redrawn circuit with the ammeter measuring the current through R_5 is shown in the following circuit. It is also correct to draw the ammeter to the right of R_5 with the orientation of the meter remaining the same: the plus sign should be most proximal to the node labeled V_a, and the minus sign is most proximal to the node labeled V_b.



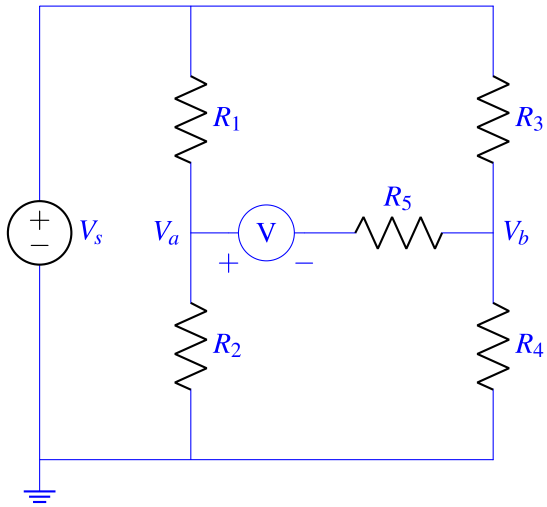
After calculating the node voltages V_a and V_b from part a, we can write:

$$I_{R_5} = \frac{V_a - V_b}{R_5} = 154.6\mu A$$

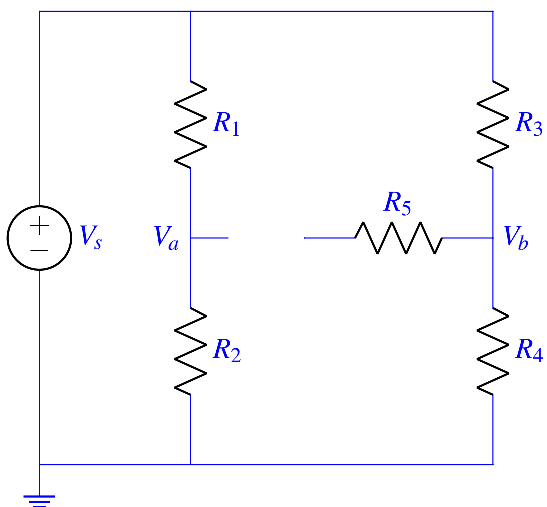
You should give yourself full-credit if your answer is off by a rounding error.

(d) Your friend accidentally connects a voltmeter in part (c) above, rather than an ammeter. Calculate the value of I_{R_5} with the voltmeter connected.

Solution: While you were not required to redraw the new circuit, the circuit is shown below.



The resistance of a voltmeter is infinite and it behaves as an open circuit. There will be no current flowing through R_5. Therefore, $I_{R_5} = 0$. The circuit below depicts how the voltmeter behaves as an open that prevents any current through R_5.

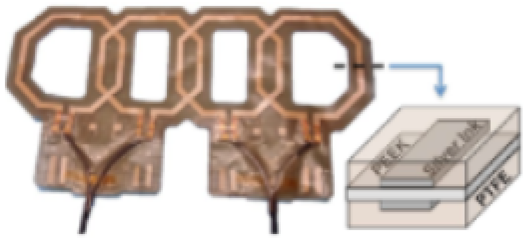


4. Printed electronics

Learning Goal: *This problem will help you practice thinking about electronic materials and their properties.*

All electronic devices require connections to conduct signals. These connections, or traces, are manufactured through different deposition methods such as physical vapor deposition and chemical vapor deposition. Another less traditional technique is printing. Inks can be made from metallic nanoparticles and deposited using inkjet printing, screen printing, and spray coating. A commonly printed metal ink is silver.

Here's an example of a printed MRI antenna coil from research conducted in Prof. Ana Arias's lab.



(a) Say we screenprinted a trace of silver 20 mm in length and 5 µm in width. Given the resistivity should be 0.001 Ω mm, and we measure the resistance of the trace to be 250 Ω, what is the trace thickness?

Solution:

We can rearrange the equation for resistance.

$$R = \rho \frac{L}{Wt}$$
$$t = \rho \frac{L}{WR}$$
$$t = 0.001\,\Omega\,\text{mm} \frac{20\,\text{mm}}{5\,\mu\text{m} \cdot 250\,\Omega}$$
$$t = 16\,\mu\text{m}$$

(b) Nanoparticle inks often require a drying step called *sintering*, during which the nanoparticles coalesce and form condcutive pathways. The manufacturer of our silver paste lists 100 °C and 175 °C as two possible sintering temperatures resulting in resistivities of 0.001 Ω mm and 0.5 Ω µm. Regardless of what you obtained in part (a), assume that we need a trace 20 mm in length, 4 µm in width, and 20 µm in thickness, what is the smallest resistance trace we can obtain and with which sintering temperature?

Solution:

Similarly as in part (a), we can find the resistance resulting from the 100 °C sintering temperature by plugging in values to the equation for resistance

$$R = \rho \frac{L}{Wt} = 0.001\,\Omega\,\text{mm} \frac{20\,\text{mm}}{4\,\mu\text{m} \cdot 20\,\mu\text{m}} = 250\,\Omega$$

Now, from the 175 °C sintering temperature we obtain:

$$R = \rho \frac{L}{Wt} = 0.5\,\Omega\,\mu\text{m} \frac{20\,\text{mm}}{4\,\mu\text{m} \cdot 20\,\mu\text{m}} = 125\,\Omega$$

We would want to use the 175 °C sintering temperature to achieve the lowest resistance trace.

(c) Say the maximum resistance we can tolerate is 125 Ω. What would the cross sectional areas required be from both sintering temperatures to achieve the specified resistance for our 20 mm long trace?

Solution: We can rearrange the resistance equation to find the cross sectional area for the 100 °C sintering temperature:

$$A = \rho \frac{L}{R} = 0.001\,\Omega\,\text{mm} \frac{20\,\text{mm}}{125\,\Omega} = 160\,\mu\text{m}^2$$

For the 175 °C sintering temperature:

$$A = \rho \frac{L}{R} = 0.5\,\Omega\,\mu\text{m} \frac{20\,\text{mm}}{125\,\Omega} = 80\,\mu\text{m}^2$$

(d) Continuing with the design specifications from part (c), if our printing technique has a resolution limit of one micron (meaning the minimum width and minimum length achieveable is one micron) and we want to aim for a trace thickness of at least one hundred micron for good film uniformity, then at which temperature should we sinter our printed silver?

Solution: We should sinter our printed silver at 100 °C. The cross sectional area required by the higher sintering temperature is too small for our printing technique, given that the thickness, or H, already needs to be larger than 100 µm.

(e) One unique advantage of using printing as a deposition technique is that electronic devices can be fabricated on plastic flexible substrates rather than brittle silicon wafers, allowing for applications where lightweight, conformable electronics are needed. However, when heated, plastic substrates can begin to soften and deform. Using your answer from part (c) and part (d) what is one drawback from the lower sintering temperature, and what is one drawback from the higher sintering temperature?

Solution: We see from part (b) that with the same trace dimensions, the lower sintering temperature will result in a higher resistance trace. From part (c) we see that if our circuit design requires a low resistance trace, the higher sintering temperature may require us to print a small feature that is not feasible by our printing technique.

(f) [Challenge] Your manufacuting process wasn't perfect and your resulting trace increases its thickness linearly along the trace, such that the initial trace thickness is 20 µm and the final thickness is 30 µm. Can you compute the resulting resistance of the trace? Assume the trace length is 20 mm, width is 4 µm, and resistivity is 0.001 Ω mm.

Hint: We can write our resistance in a differential form: $dR = \rho(l)\frac{dl}{A(l)}$. *Can we add up all these differential segments of resistance over the trace to get our final resistance value?*

Solution: We have that our thickness is a function of the position $l = 0 \ldots 20\,\text{mm}$:

$$t(l) = 20\,\mu\text{m} + \frac{l}{20\,\text{mm}} \cdot 10\,\mu\text{m}$$

.

Therefore, our area is also a function of the position l:

$$A(l) = W * t(l) = W \cdot (20\,\mu\text{m} + \frac{l}{20\,\text{mm}} \cdot 10\,\mu\text{m})$$

So, we get that our resistance differential is:

$$dR = \frac{\rho dl}{W \cdot (20\,\mu\text{m} + \frac{l}{20\,\text{mm}} \cdot 10\,\mu\text{m})}$$

Integrating we obtain:

$$R = \int_0^{20\,\text{mm}} \frac{\rho dl}{W \cdot (20\,\mu\text{m} + \frac{l}{20\,\text{mm}} \cdot 10\,\mu\text{m})}$$

$$R = \frac{\rho}{W}\int_0^{20\,\text{mm}} \frac{dl}{20\,\mu\text{m} + \frac{l}{20\,\text{mm}} \cdot 10\,\mu\text{m}}$$

$$R = \frac{0.001\,\Omega\,\text{mm}}{4\,\mu\text{m}} \cdot \frac{20\,\text{mm}}{10\,\mu\text{m}} \cdot \ln\left(\frac{20\,\mu\text{m} + 10\,\mu\text{m}}{20\,\mu\text{m}}\right)$$

$$R = 202.73\,\Omega$$

5. Resistive Touchscreen

Learning Goal: *The objective of this problem is to provide insight into modeling of resistive elements. This will also help to apply the concepts from resistive touchscreen.*

In this problem, we will investigate how a resistive touchscreen with a defined thickness, width, and length can actually be modeled as a series combination of resistors. As we know the value of a resistor depends on its length.

Figure 2 shows the top view of a resistive touchscreen consisting of a conductive layer with resistivity ρ_1, thickness t, width W, and length L. At the top and bottom it is connected through perfect conductors ($\rho = 0$) to the rest of the circuit. The touchscreen is wired to voltage source V_s.

Use the following numerical values in your calculations: $W = 50$ mm, $L = 80$ mm, $t = 1$ mm, $\rho_1 = 2\,\Omega\,\text{m}$, $V_s = 10\text{V}$, $x_1 = 20$ mm, $x_2 = 45$ mm, $y_1 = 30$ mm, $y_2 = 60$ mm.

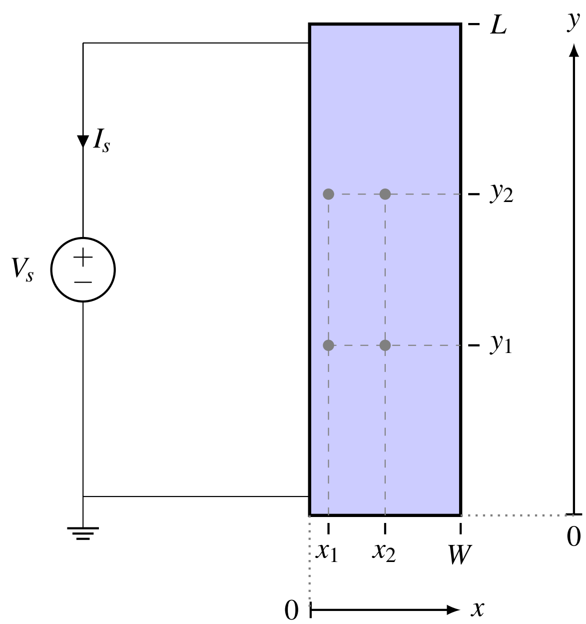
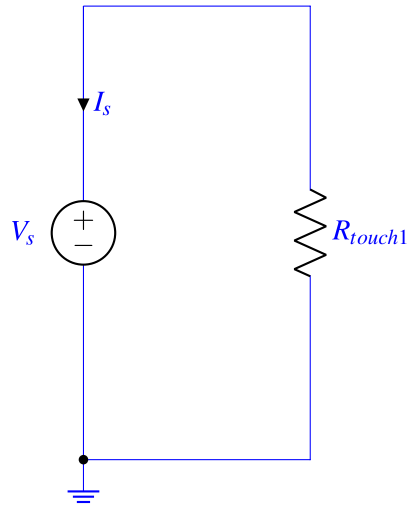


Figure 2: Top view of resistive touchscreen (not to scale). z axis i.e. the thickness not shown (into the page).

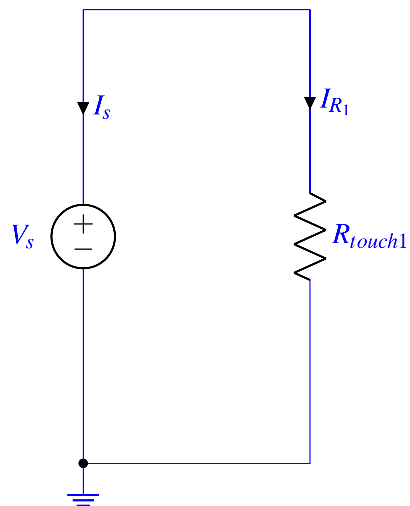
(a) Draw a circuit diagram representing **Figure 2**, where the entire touchscreen is represented as *a single resistor*. **Note that no touch is occurring in this scenario.** Remember that circuit diagrams in general consist of only circuit elements (resistors, sources, etc) represented by symbols, connecting wires, and the reference/ ground symbol. Calculate the value of current I_S based on the circuit diagram you drew. Do not forget to specify the correct unit as always, and double check the direction of I_s!

Solution:



The touchscreen resistance can be found from the following expression:

$$\begin{aligned} R_{touch1} &= \rho_1 \cdot \frac{L}{A} \\ &= \rho_1 \cdot \frac{L}{W \cdot t} \\ &= 2\,\Omega\,\text{m}\left(\frac{80\,\text{mm}}{50\,\text{mm} \cdot 1\,\text{mm}}\right) \\ R_{touch1} &= 3200\,\Omega = 3.2\,\text{k}\Omega \end{aligned}$$



From KCL, we can write:

$$I_s + I_{R_1} = 0 \tag{1}$$

$$I_s = -I_{R_1} \tag{2}$$

Therefore, the current I_{R_1} is equal to:

$$I_{R_1} = \frac{V_s}{R_{touch1}} = \frac{10}{3200}A = 3.125mA$$

And the current I_s is equal to:

$$I_s = -I_{R_1} = -3.125mA$$

(b) Let us assume u_{12} is the node voltage at the node represented by coordinates (x_1, y_2) of the touchscreen, as shown in **Figure 3**. What is the value of u_{12}? You should first draw a circuit diagram representing Figure 3, which includes node u_{12}. Specify all resistance values in the diagram. Does the value of u_{12} change based on the value of the x-coordinate x_1?

Hint: You will need more than one resistor to represent this scenario.

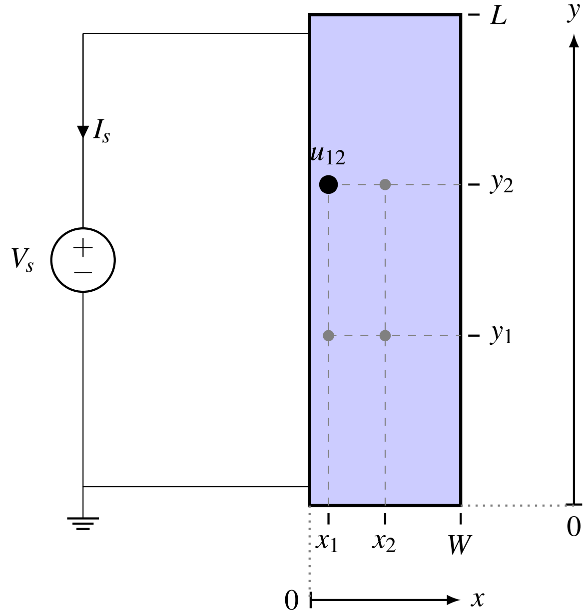
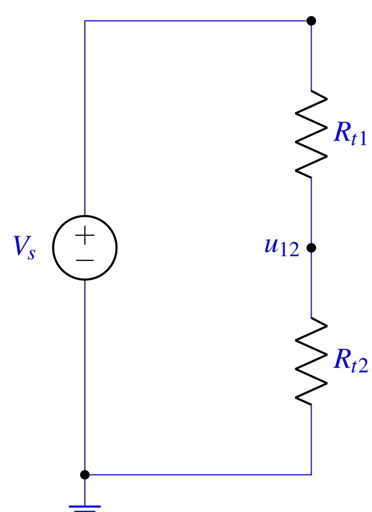


Figure 3: Top view of resistive touchscreen showing node u_{12}.

Solution:

We can represent this setup with the circuit shown below.



Using voltage division, u_{12} can be found from the following expression:

$$u_{12} = V_s \frac{R_{t2}}{R_{t1} + R_{t2}}$$

We know $R_{t1} = \rho_1 \cdot \frac{L-y_2}{W \cdot t}$ and $R_{t2} = \rho_1 \cdot \frac{y_2}{W \cdot t}$. We also know the $\frac{\rho_1}{W \cdot t}$ is common to both R_{t1} and R_{t2}, so those terms will cancel out when we them in.

$$u_{12} = V_s \frac{y_2}{L - y_2 + y_2}$$

$$u_{12} = V_s \frac{y_2}{L}$$

$$u_{12} = 10V \cdot \frac{60\,\text{mm}}{80\,\text{mm}} = 7.5V$$

The value of u_{12} would not change based on the value of the x coordinate, because in our setup the current is flowing from the top to the bottom of the screen. This means that voltage is only dissipated in the y direction, so we can only measure the difference in the y coordinate. We would need another closed circuit where current could flow along the width W to determine where the finger touched in the x direction.

(c) Assume V_{ab} is the voltage measured between the nodes represented by touchscreen coordinates (x_1, y_1) and coordinates (x_1, y_2), as shown in **Figure 4**. Calculate the absolute value of V_{ab}. As with the previous part, you should first draw the circuit diagram representing Figure 4, which includes V_{ab}. Calculate all resistor values in the circuit. *Hint: Try representing the segment of the touchscreen between these two coordinates as a separate resistor itself.*

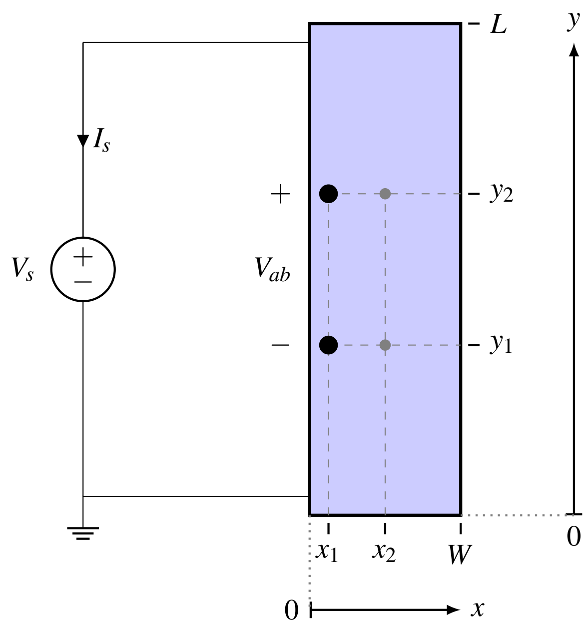
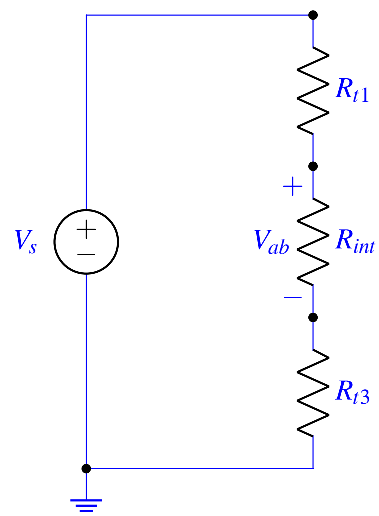
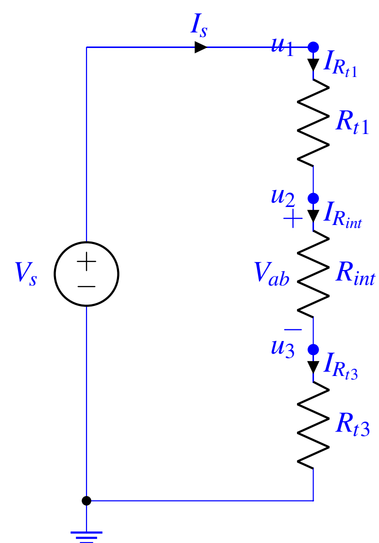


Figure 4: Top view of resistive touchscreen showing voltage V_{ab}.

Solution:



We can use node voltage analysis to find V_{ab}.



Using KCL at the three labelled nodes:

$$I_s = I_{R_{t1}}$$

$$I_{R_{t1}} = I_{R_{t3}}$$

$$I_{R_{t3}} = I_{R_{int}}$$

We see that there is only one current, I_s, going through all resistor elements. Writing the IV relationships for each element:

$$u_1 - u_2 = I_s R_{t1}$$

$$u_2 - u_3 = I_s R_{int}$$

$$u_3 = I_s R_{t3}$$

Knowing that $V_{ab} = u_2 - u_3$, we can write:

$$V_{ab} = u_2 - u_3 = I_s R_{int}$$

Now we just need to find I_s. Looking at the IV relationship equations and using back substitution, we can write:

$$u_1 = V_s = I_{R_{t1}} R_{t1} + I_{R_{int}} R_{int} + I_{R_{t3}} R_{t3}$$

$$I_s = \frac{V_s}{R_{t1} + R_{int} + R_{t3}}$$

Finally, we get:

$$V_{ab} = V_s \frac{R_{int}}{R_{t1} + R_{int} + R_{t3}}$$

Each of the resistances can be calculated as $R_{t1} = \rho_1 \cdot \frac{L-y_2}{W \cdot t}$, $R_{int} = \rho_1 \cdot \frac{y_2 - y_1}{W \cdot t}$ and $R_{t3} = \rho_1 \cdot \frac{y_1}{W \cdot t}$. This gives for V_{ab}:

$$V_{ab} = \frac{y_2 - y_1}{L} V_s = \frac{3}{8} \cdot 10V = 3.75V$$

(d) Calculate (the absolute value of) the voltage between the nodes represented by touchscreen coordinates (x_1, y_1) and coordinates (x_2, y_1) in **Figure 4**.

Solution:

The two points have the same y coordinate, therefore they have the same potential in our touchscreen model. Again, this is because the current is flowing from the top to the bottom of the screen, so the x position does not make a difference. Recall that the touchscreen is effectively being modeled as a single vertical resistor which can be considered as several resistors of varying lengths, all totaling to L. Hence, we do not consider the effect of the x-coordinate on potential – we just need to consider the difference in the y-coordinate between two points. Thus, $\Delta V = 0$.

(e) Calculate (the absolute value of) the voltage between the nodes represented by touchscreen coordinates (x_1, y_1) and coordinates (x_2, y_2) in **Figure 4**.

Solution:

The two points have different x and y coordinates. However, the potential is the same across the x-axis for a fixed y coordinate, as was explained in the solution for part (d). Therefore, the problem is similar to part (c), since the potential is only determined by the y-coordinate of a point. Using the same equivalent circuit in part (c) we have:

$$\Delta V = V_s \frac{R_{int}}{R_{t1} + R_{int} + R_{t3}} = 3.75V$$

(f) **Figure 5** shows a new arrangement with two touchscreens. The two touchscreens are next to each other and are connected to the voltage source in the same way. The second touchscreen (the one on the right) is identical to the one shown in Figure 2, except for different width, W_2, and resistivity, ρ_2.

Use the following numerical values in your calculations: $W_1 = 50$ mm, $L = 80$ mm, $t = 1$ mm, $\rho_1 = 2\,\Omega\,$m, $V_s = 10$V, $x_1 = 20$ mm, $x_2 = 45$ mm, $y_1 = 30$ mm, $y_2 = 60$ mm, which are the same values as before. The new touchscreen has the following numerical values which are different: $W_2 = 85$ mm, $\rho_2 = 1.5\,\Omega\,$m.

Draw a circuit diagram representing **Figure 5**, where the two touchscreens are represented as *two separate resistors*. **Note that no touch is occurring in this scenario.**

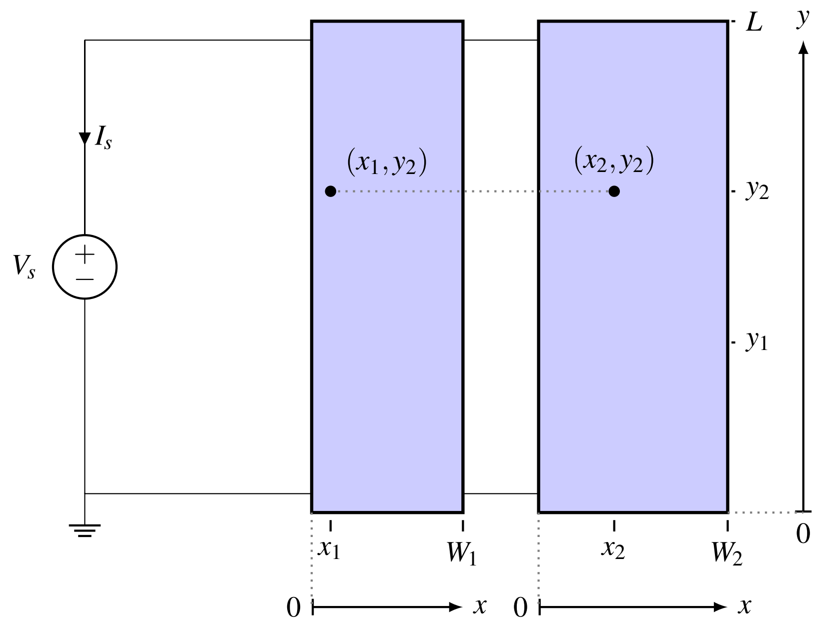
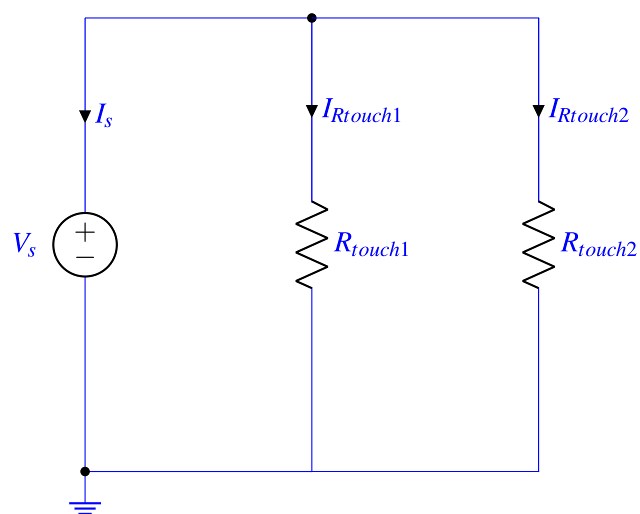


Figure 5: Top view of two touchscreens wired in parallel (not to scale). z axis not shown (into the page).

Solution:



(g) Calculate the value of current I_s for the two touchscreen arrangement based on the circuit diagram you drew in the last part.

Solution:

From KCL, we can write:

$$I_s + I_{Rtouch1} + I_{Rtouch2} = 0 \tag{3}$$

$$I_s = -(I_{Rtouch1} + I_{Rtouch2}) \tag{4}$$

Using Ohm's Law for each element:

$$I_s = -\left(\frac{V_s}{R_{touch1}} + \frac{V_s}{R_{touch2}}\right)$$

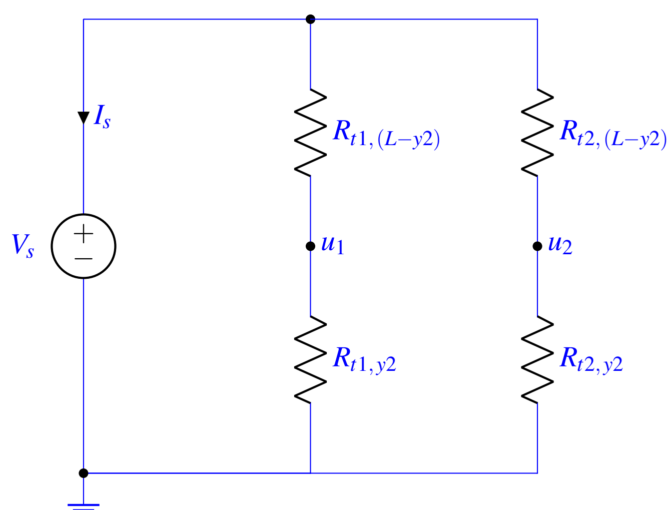
However, the resistance of the second touchscreen can be given by:

$$R_{touch2} = \rho_2 \cdot \frac{L}{W_2 \cdot t} = 1.5\,\Omega\,\text{m}\left(\frac{80\,\text{mm}}{85\,\text{mm} \cdot 1\,\text{mm}}\right)$$
$$R_{touch2} = 1411.8\,\Omega$$

Therefore, using the resistance values for the first and second touchscreen and the applied voltage source, we have:

$$I_s \approx -(3.125\text{mA} + 7.08\text{mA}) = -10.2\text{mA}$$

(h) Consider the two points: (x_1, y_2) in the touchscreen on the left, and (x_2, y_2) in the touchscreen on the right in **Figure 5**. Show that the node voltage at (x_1, y_2) is the same that at (x_2, y_2), i.e. the potential difference between the two points is 0. You can show this without explicitly calculating the node voltages at the two points.
If you were to connect a wire between the two coordinates (x_1, y_2) in the touchscreen on the left, and (x_2, y_2) in the touchscreen on the right, would any current flow through this wire?
Solution:
It will be helpful to first consider the circuit representation of this scenario to understand why the node voltages at the two points on each of the touch screens should be same (and therefore that the potential difference between points (x_1, y_2) and (x_2, y_2) is 0).



Without calculating the node voltages, note that the ratio of the value of $R_{t1,(L-y_2)}$ to $R_{t1,y2}$ is the same as the ratio of the value of $R_{t2,(L-y2)}$ to $R_{t2,y2}$:

$$\frac{R_{t1,y2}}{R_{t1,(L-y2)}} = \frac{\rho_1(y_2)/(W_1 \cdot t)}{\rho_1(L-y_2)/(W_1 \cdot t)} = \frac{y_2}{L-y_2}$$
$$\frac{R_{t2,y2}}{R_{t2,(L-y2)}} = \frac{\rho_2(y_2)/(W_2 \cdot t)}{\rho_2(L-y_2)/(W_2 \cdot t)} = \frac{y_2}{L-y_2}$$

Also note that the ratio of the resistors used in the voltage divider equations can be written as:

$$\frac{R_{t1,y2}}{R_{t1,(L-y2)} + R_{t1,y2}} = \frac{\rho_1(y_2)/(W_1 \cdot t)}{\rho_1(L-y_2)/(W_1 \cdot t) + \rho_1(y_2)/(W_1 \cdot t)} = \frac{y_2}{L}$$
$$\frac{R_{t2,y2}}{R_{t2,(L-y2)} + R_{t2,y2}} = \frac{\rho_2(y_2)/(W_2 \cdot t)}{\rho_2(L-y_2)/(W_2 \cdot t) + \rho_2(y_2)/(W_2 \cdot t)} = \frac{y_2}{L}$$

Beacuse the voltage across the entirety of each of the individual touchscreens is the same: it is $V_s - 0$ or just V_s. The voltage V_s is therefore *divided* between $R_{t1,(L-y2)}$ and $R_{t1,y2}$ exactly the same as it is divided between $R_{t_2,(L-y2)}$ and $R_{t_2,y2}$ because of the ratio argument presented above.

Therefore, the potential difference between u_1 and u_2 will be 0, so long as the y-coordinate value is chosen to be the same.

This also means that there is no current flowing through the wire, since the points in the two touch-screens have the same potential. Therefore,

$$I_{12} = \frac{u_1 - u_2}{R_{wire}} = 0$$

We should note that what causes current to flow is a voltage difference. Here, we have a voltage difference of zero, so the current flowing through the wire is zero.

6. Temperature Sensor

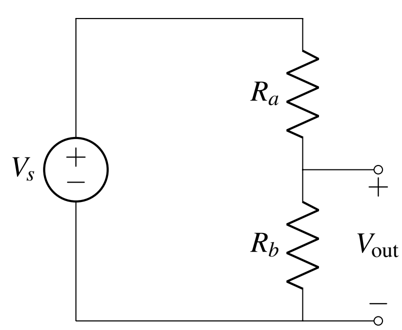
Learning Goal: *This problem will let you apply the tools we have learned so far to a real world circuits application.*

Measuring quantities in the physical world is the job of sensors. This means somehow extracting that information from the world and then converting it into a form that can be observed and processed. Electrical circuits can be very useful for doing this.

For most materials, resistance increases with increasing temperature; that is, a resistor has higher resistance when it is hot than when it is cold. This is often an annoyance to circuit designers who want their circuits to work the same way at different temperatures, but this fact can also be useful. It allows us to convert temperature, a "physical" quantity, into resistance, an "electrical" quantity, to build an electronic thermometer.

In this problem, we are going to explore how effective a particular circuit made out of various types of resistors is at allowing us to measure temperature.

(a) Let's begin by analyzing a common topology, the voltage divider shown below. Find an equation for the voltage V_{out} in terms of R_a, R_b, and V_s.

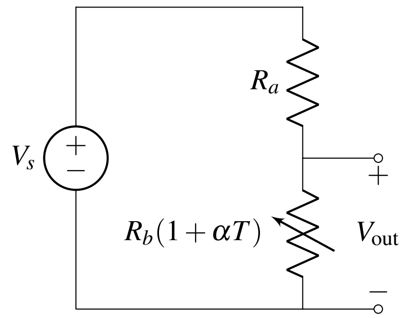


Solution:

We recognize that this circuit is a voltage divider, we can directly write:

$$V_{\text{out}} = \frac{R_b}{R_a + R_b} V_s$$

(b) Now let's suppose that R_a is an ideal resistor that does not depend on temperature, but R_b is a temperature-dependent resistor whose resistance R is set by $R_b' = R_b(1+\alpha T)$, where T is the absolute temperature. Find an equation for the temperature T in terms of the voltage V_{out}, V_s, R_a, R_b, and α.



Solution:

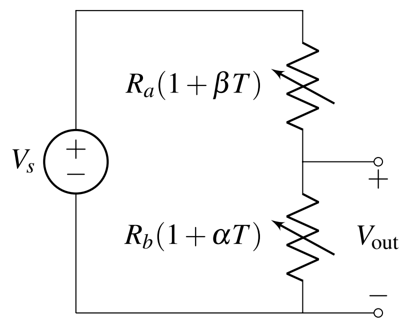
Using the relationship from the earlier part:

$$V_{\text{out}} = \frac{R_b(1+\alpha T)}{R_a + R_b(1+\alpha T)} V_s$$

$$R_a V_{\text{out}} + R_b V_{\text{out}} + R_b \alpha T V_{\text{out}} = R_b V_s + R_b \alpha T V_s$$

$$T = \frac{(R_a + R_b)V_{\text{out}} - R_b V_s}{R_b \alpha (V_s - V_{\text{out}})}$$

(c) It turns out that almost all resistors have some temperature dependence. Consider the same circuit as before, but now, R_a' has a temperature dependence given by $R_a' = R_a(1+\beta T)$. Find an equation for the temperature T in terms of the voltage V_{out}, R_a, R_b, V_s, α, and β.



Solution:

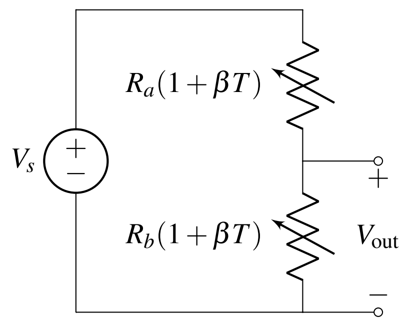
Once again using the equation for the voltage divider:

$$V_{\text{out}} = \frac{R_b(1+\alpha T)}{R_a(1+\beta T) + R_b(1+\alpha T)} V_s$$

$$R_a V_{\text{out}} + R_a \beta T V_{\text{out}} + R_b V_{\text{out}} + R_b \alpha T V_{\text{out}} = R_b V_s + R_b \alpha T V_s$$

$$T = \frac{(R_a + R_b)V_{\text{out}} - R_b V_s}{R_b \alpha (V_s - V_{\text{out}}) - R_a \beta V_{\text{out}}}$$

(d) Your colleague who has not taken EECS16A thinks that they can improve this circuit's ability to measure temperature by making both resistors depend on temperature in the same way. He hence came up with the circuit shown below, where both R_a and R_b have nominally different values, but both vary with temperature as a function of $(1+\beta T)$. Can this circuit be used to measure temperature? Why or why not?



Solution: Using the equation for a voltage divider:

$$V_{\text{out}} = \frac{R_b(1+\beta T)}{R_a(1+\beta T) + R_b(1+\beta T)} V_s = \frac{R_b}{R_a + R_b} V_s$$

Notice this circuit cannot be used to measure temperature because the output voltage is independent of temperature.

7. Prelab Questions

These questions pertain to the Pre-Lab reading for the Touch 2 lab. You can find the reading under the Touch 2 Lab section on the 'Schedule' page of the website.

(a) How many layers are there in the touchscreen and what are they made of?

(b) Provide 2 examples of resistive touchscreens (give one example not listed on the pre-lab reading).

(c) In the circuit given in the reading, what is the current i_3 flowing through resistor R_{h1}?

(d) How do we get touch coordinates in the horizontal direction if you have your circuit that works in the vertical direction?

Solution:

(a) The resistive touchscreen consists of two different layers - a flexible resistive layer on the top and a resistor circuit layer on the bottom.

(b) From the reading: old Nokias, Nintendo DS & Gameboy. Not from the reading: GPS displays, Printers, Airplane Entertainment Screens, Screens that require the use of a stylus.

(c) $0A$. There is no potential difference across the resistor R_{h1} and thus, no current flows through it.

(d) Rotate the orientation of the circuit by 90 degrees.

8. Homework Process and Study Group

Who did you work with on this homework? List names and student ID's. (In case you met people at homework party or in office hours, you can also just describe the group.) How did you work on this homework? If you worked in your study group, explain what role each student played for the meetings this week.

Solution:

I first worked by myself for 2 hours, but got stuck on problem 5. Then I met with my study group.

XYZ played the role of facilitator ... etc. We were still stuck on problem 5 so we went to office hours to talk about the problem.

Then I went to homework party for a few hours, where I finished the homework.