EECS 16A Spring 2023 Designing Information Devices and Systems I Discussion 8A

1. Superposition

For the following circuits, use the superposition theorem to solve for the voltages across the resistor(s).

Solution/Answer:

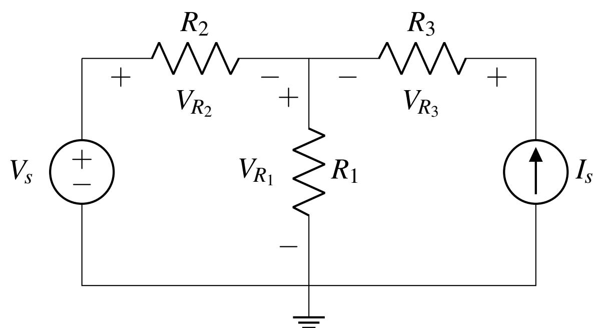
For each circuit;

i. Redraw the circuits with just one source enabled (while disabling/zero-ing all other sources) and solve for circuit voltages and currents.

ii. Repeat for every *independent* source.

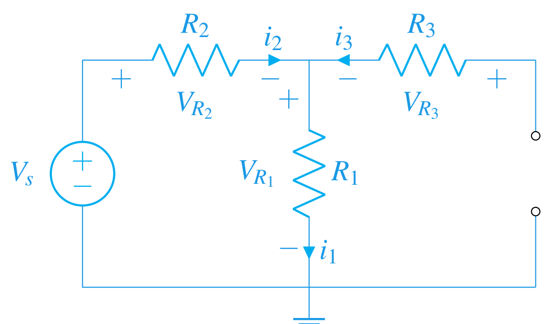
iii. Finally, sum the circuit voltages and currents.

(a)



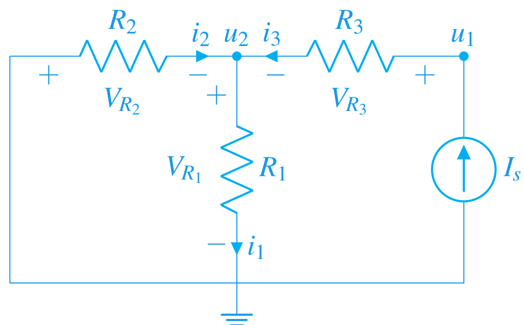
Answer:

First, enable/turn-on only the voltage source, V_s, and disable/zero the current source ($I_s = 0$ A: equivalent to an open circuit). Because of the open circuit, no current flows through resistor R_3 and it can be neglected. What is left is a classic resistive voltage divider across R_2 and R_1. Notice that we have labeled the currents in the below diagram according to passive sign convention. We should use these same labels throughout the superposition process.



$$V_{R_1} = \frac{R_1}{R_1+R_2}V_s$$
$$V_{R_2} = \frac{R_2}{R_1+R_2}V_s$$
$$V_{R_3} = 0$$

Next, enable only the current source, I_s, and disable/zero the voltage source ($V_s = 0\,\text{V}$: equivalent to a short circuit). We will use node voltage analysis to solve for the voltages across the resistors, so we start by labeling the two nodes with unknown potential u_1 and u_2.



- Write KCL equations at u_1 and u_2:

$$u_1 : i_3 = I_s$$
$$u_2 : i_3 + i_2 = i_1$$

- Write element voltages using Ohm's law:

$$V_{R_1} = u_2 - 0 = i_1 R_1 \qquad \rightarrow i_1 = \frac{u_2}{R_1}$$
$$V_{R_2} = 0 - u_1 = i_2 R_2 \qquad \rightarrow i_2 = \frac{-u_2}{R_2}$$
$$V_{R_3} = u_1 - u_2 = i_3 R_3 = I_s R_3 \qquad \rightarrow u_1 = u_2 + I_s R_3$$

- Combine equations to solve for u_1 and u_2:

$$I_s - \frac{u_2}{R_2} = \frac{u_2}{R_1} \text{(plugged into second KCL equation)}$$
$$\left(\frac{1}{R_1} + \frac{1}{R_2}\right)u_2 = I_s$$
$$u_2 = \frac{R_1 R_2}{R_1+R_2} I_s$$
$$u_1 = \left(\frac{R_1 R_2}{R_1+R_2} + R_3\right) I_s$$

Now that we have the node potentials, we can write the voltages across each resistor:

$$V_{R_1} = u_2 = \frac{R_1 R_2}{R_1+R_2} I_s$$
$$V_{R_2} = -u_2 = -\frac{R_1 R_2}{R_1+R_2} I_s$$
$$V_{R_3} = u_1 - u_2 = R_3 I_s$$

Notice that V_{R_3} could've also been calculated directly from Ohm's law.

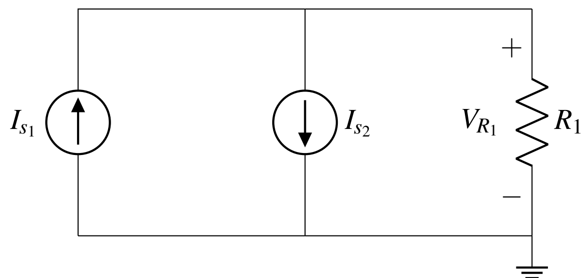
Finally, using superposition we can sum up the contributions from both V_s and I_s to find the resistor voltages of the complete circuit

$$V_{R_1} = V_{R_1}\Big|_{I_s=0} + V_{R_1}\Big|_{V_s=0} = \frac{R_1}{R_1+R_2}V_s + \frac{R_1R_2}{R_1+R_2}I_s$$

$$V_{R_2} = V_{R_2}\Big|_{I_s=0} + V_{R_2}\Big|_{V_s=0} = \frac{R_2}{R_1+R_2}V_s - \frac{R_1R_2}{R_1+R_2}I_s$$

$$V_{R_3} = V_{R_3}\Big|_{I_s=0} + V_{R_3}\Big|_{V_s=0} = I_sR_3$$

(b)



Answer:

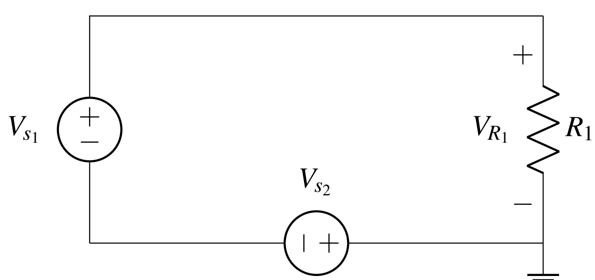
i. *Using superposition:*
Enabling I_{s_1} (and disabling I_{s_2}) gives $V_{R_1} = I_{s_1}R_1$. Enabling I_{s_2} (and disabling I_{s_1}) gives $V_{R_1} = -I_{s_2}R_1$. Finally, the total V_{R_1} is the sum of the individual V_{R_1}'s or

$$V_{R_1} = (I_{s_1} - I_{s_2})R_1$$

ii. *Without superposition:*
Let's approach this holistically. We're looking for the voltage across the resistor, which could be found quickly using Ohm's law if we knew the current. If we were to apply KCL at the node at the top of the circuit, one source is coming in, the other source is leaving, and the current through the resistor is leaving. From KCL, we then know $i_{R_1} = I_{s_1} - I_{s_2}$. Applying Ohm's Law we find:

$$V_{R_1} = (I_{s_1} - I_{s_2})R_1$$

(c) (**PRACTICE**)



Answer:

i. *Using superposition:*
Enabling V_{s_1} (and disabling V_{s_2}) gives $V_{R_1} = V_{s_1}$. Enabling V_{s_2} (and disabling V_{s_1}) gives $V_{R_1} = -V_{s_2}$. Finally, the total V_{R_1} is the sum of the individual V_{R_1}'s or

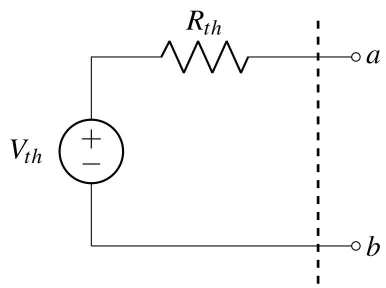
$$V_{R_1} = V_{s_1} - V_{s_2}$$

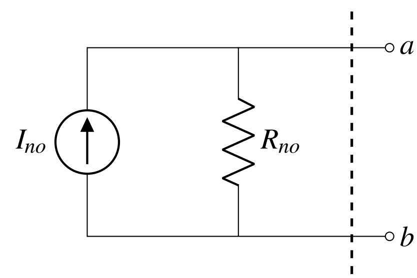
ii. *Without superposition:*
Notice the circuit only has one loop, so use KVL to find the voltage across the resistor.

$$V_{R_1} = V_{s_1} - V_{s_2}$$

2. Thévenin and Norton Equivalence

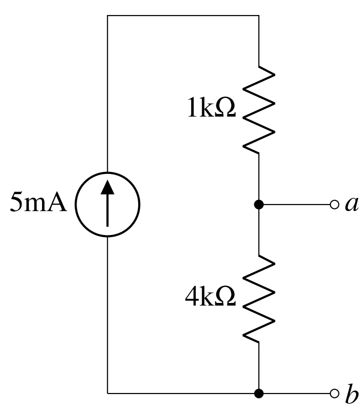
The general Thévenin and Norton equivalent circuits are shown below. Any circuit of any complexity can be represented as an equivalent circuit with either of these simplified forms, from the perspective of a single pair of nodes a and b.





Find the Thévenin and Norton equivalents across terminals a and b for the circuits given below.

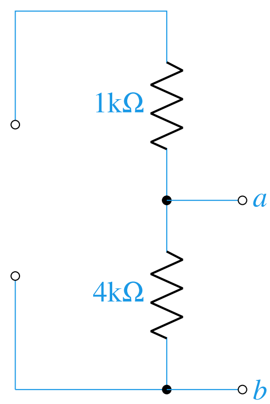
(a)



Answer:
The open circuit voltage across terminals ab is given by Ohm's law:

$$V_{th} = 5\,\text{mA} \cdot 4\,\text{k}\Omega = 20\,\text{V}$$

To find R_{th}, disable/zero out any independent sources (5A current source becomes an open circuit).



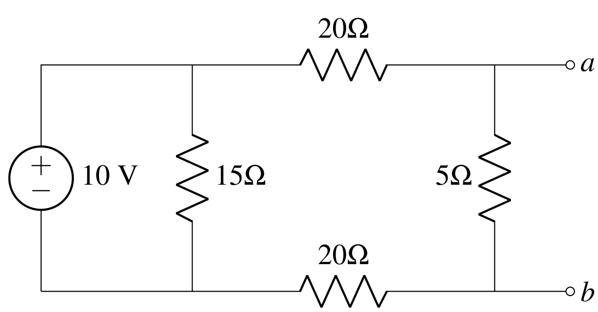
Then, from the open circuit terminals ab, find the equivalent resistance of the rest of the circuit. This is just a single resistor.

$$R_{th} = 4\,\mathrm{k}\Omega = R_{no}$$

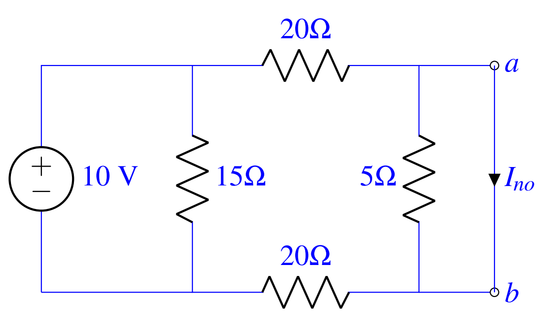
Now to find I_{no},

$$I_{no} = \frac{V_{th}}{R_{th}} = \frac{20\,\mathrm{V}}{4\,\mathrm{k}\Omega} = 5\,\mathrm{mA}$$

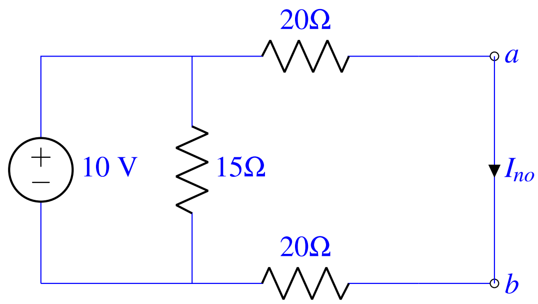
(b)



Answer: Starting with the Norton equivalent current, we connect a short across terminals a and b and find the current through it:



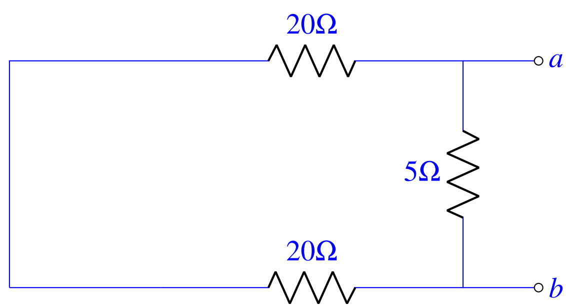
The wire ends up shorting the 5Ω resistor so our equivalent circuit looks like this:



Then, in the branch with current I_{no}, the two 20Ω resistors are in series, so their equivalent resistance is 40Ω, and there is a total voltage of 10V across them. Using Ohm's law,

$$I_{no} = \frac{V}{R} = \frac{10}{40} = 0.25A$$

Next, we want to find the equivalent resistance R_{eq} across terminals a and b by zeroing out all independent sources. Zeroing out the voltage source means replacing it with a short, which shorts the 15Ω resistor. The circuit now looks like this:



From the perspective of terminals a and b, there are two branches: the right branch has the 5Ω resistor, and the left branch has two 20Ω resistors. Therefore, the equivalent resistance can be calculated as 5Ω in parallel with (20Ω in series with 20Ω):

$$R_{eq} = 5||(20+20) = \frac{5 \cdot 40}{5+40} = \frac{40}{9}\Omega$$

Lastly, let's calculate the Thevenin voltage (the equivalent resistance for the Thevenin circuit is the same as for the Norton circuit). We calculate V_{th} by placing an open circuit across terminals a and b and measuring the voltage across it.

From the perspective of the voltage source, there are two branches in the circuit: one with a 15Ω resistor and one with the other 3 resistors. We use the voltage divider equation on the second branch to calculate V_{th}: the total voltage across the branch is 10V, and we are measuring the voltage across the 5Ω resistor:

$$V_{th} = \frac{5}{20+5+20} \cdot 10 = \frac{10}{9}V$$

We can check that $V_{th} = I_{no}R_{eq}$.