EECS 16A Designing Information Devices and Systems I
Summer 2023 Homework 5

This homework is due Friday, July 21, 2023 at 23:59.
Self-grades are due Friday, July 28, 2023 at 23:59.

Submission Format
Your homework submission should consist of **one** file.

We strongly recommended that you submit your self-grades PRIOR to taking the midterm on July 24, 2023, since looking at the solutions earlier will help you to study for the midterm.

Mid Semester Survey

Please fill out the mid semester survey: https://forms.gle/XKNPXWDidcsoM7LB9.

We highly appreciate your feedback!

1. **Reading Assignment**

For this homework, please read Note 12, Note 13, Note 14, Note 15, and Note 16. Notes 12 and 13 cover voltage dividers, how a simple 1-D resistive touchscreen works, the physics of circuits, and introduces the notion of power in electric circuits. Note 14 introduces better, but more complex models for the resistive touchscreen. Note 15 covers superposition and equivalence, two techniques to simplify circuit analysis. Note 16 will provide an introduction to capacitors (a circuit element which stores charge), capacitive equivalence, and the underlying physics behind them.

(a) Describe the key ideas behind how the 1D touchscreen works. In general, why is it useful to be able to convert a "physical" quantity like the position of your finger to an electronic signal (i.e. voltage)?

(b) For the touch screen model introduced in Note 14, why can't we simultaneously get the horizontal and vertical position of the touch with a single measurement? *Think about how many unknowns there are.*

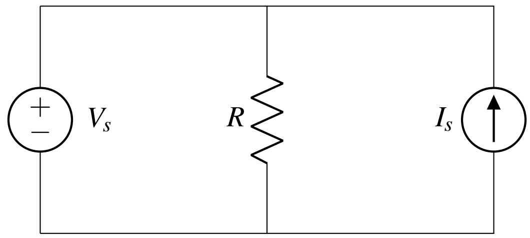
(c) Explain the connection between the "superposition" you learned about in Note 15 with the "superposition" you learned back in module 1 in the context of linear functions.

(d) Describe the short-circuit test to find the Norton equivalent circuit. This test allows us to determine what voltage/current?

(e) How do we calculate the equivalent capacitance of series and parallel capacitors? Compare this with how we calculate resistor equivalences.

2. **Power Analysis**

Learning Goal: *This problem aims to help you practice calculating power dissipation in different circuit elements. It will also give you insights into how power is conserved in a circuit.*



(a) Find expressions for power dissipated by the voltage source (P_{V_s}), the current source (P_{I_s}), and the resistor (P_R) in the circuit above. Remember to label voltage-current pairs using passive sign convention.

(b) Use $R = 5\ \text{k}\Omega$, $V_s = 5$ V, and $I_s = 10$ mA. Calculate the power dissipated by each element. What does a negative value of dissipated power represent? Additionally compute the total power dissipated in all elements.

(c) Once again, let $R = 5\ \text{k}\Omega$, $V_s = 5$ V. What does the value I_s of the current source have to be such that the current source **dissipates** 40 mW? Note that it is possible for a current source to *dissipate* power, i.e. under passive sign convention, $P_{I_s} = +40\text{mW}$. For this value of I_s, compute P_{V_s} and P_R as well.

Hint: If the current source were delivering *power, under passive sign convention the computed power would have been* $P_{I_s} = -40\text{mW}$, *but this is NOT what the question is asking.*

3. Cell Phone Battery

As great as smartphones are, one of their drawbacks is that their batteries don't last a long time. For example, a typical smartphone, under average usage conditions (internet, a few cat videos, etc.) uses 0.3W of power. We will model the battery as an ideal voltage source (which maintains a constant voltage across its terminals regardless of current) except that we assume that the voltage drops abruptly to zero when the battery is discharged (in reality, the voltage drops gradually, but let's keep things simple).

Battery capacity is specified in mAh (this is a unit of charge), which indicates how many mA of current the battery can supply for one hour before it needs to be recharged. Suppose the phone's battery has a capacity of 2770mAh at 3.8V. For example, this battery could provide 1000mA (or $P = 1000\,\text{mA} \cdot 3.8\,\text{V} = 3.8\,\text{W}$) for $\frac{2770\text{mAh}}{1000\,\text{mA}} = 2.77$ hours before the voltage abruptly drops from 3.8V to zero.

(a) How long will the phone's full battery last assuming an average power usage of $300\,\text{mW}$?

(b) Suppose the cell phone battery is completely discharged and you want to recharge it completely. How much energy (in J) is this? Recall that a J is equivalent to a W s.

(c) The battery has internal circuitry that prevents it from getting overcharged (and possibly exploding!). We will model the battery and its internal circuitry as a resistor R_{bat}. We now wish to charge the battery by plugging it into a wall plug. The wall plug can be modeled as a $5\,\text{V}$ voltage source and $200\,\text{m}\Omega$ resistor, as pictured in Figure 1. What is the power dissipated across R_{bat} for $R_{\text{bat}} = 1\,\Omega$ (i.e. how much power is being supplied to the phone battery as it is charging) and how long will the battery take to charge?

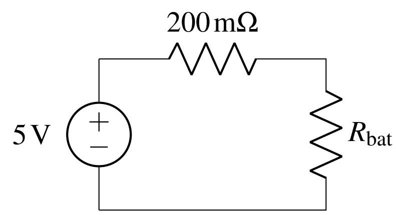


Figure 1: Model of wall plug, wire, and battery.

4. Volt and ammeter

Learning Goal: *This problem helps you explore what happens to voltages and currents in a circuit when you connect voltmeters and ammeters in different configurations.*

Use the following numerical values in your calculations: $R_1 = 1\,\mathrm{k}\Omega$, $R_2 = 2\,\mathrm{k}\Omega$, $R_3 = 3\,\mathrm{k}\Omega$, $R_4 = 4\,\mathrm{k}\Omega$, $R_5 = 5\,\mathrm{k}\Omega$, $V_s = 8V$.

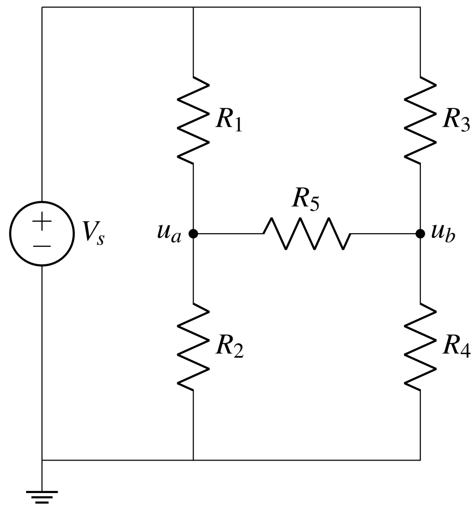


Figure 2: Circuit consisting of a voltage source V_s and five resistors R_1 to R_5

(a) Redraw the circuit diagram shown in Figure 2 by adding a voltmeter (letter V in a circle and plus and minus signs indicating direction) to measure voltage V_{ab} from node u_a (positive) to node u_b (negative). Calculate the value of V_{ab}. You may use a numerical tool such as IPython to solve the final system of linear equations.

(b) Suppose you accidentally connect an ammeter in part (a) instead of a voltmeter. Calculate the value of V_{ab} with the ammeter connected.

(c) Redraw the circuit diagram shown in Figure 2 by adding an ammeter (letter A in a circle and plus and minus signs indicating direction) in series with resistor R_5. This will measure the current I_{R_5} through R_5. Calculate the value of I_{R_5}.

(d) Your friend accidentally connects a voltmeter in part (c) above, rather than an ammeter. Calculate the value of I_{R_5} with the voltmeter connected.

5. Resistive Touchscreen

Learning Goal: *The objective of this problem is to provide insight into modeling of resistive elements. This will also help to apply the concepts from resistive touchscreen.*

In this problem, we will investigate how a resistive touchscreen with a defined thickness, width, and length can actually be modeled as a series combination of resistors. As we know the value of a resistor depends on its length.

Figure 3 shows the top view of a resistive touchscreen consisting of a conductive layer with resistivity ρ_1, thickness t, width W, and length L. At the top and bottom it is connected through perfect conductors ($\rho = 0$) to the rest of the circuit. The touchscreen is wired to voltage source V_s.

Use the following numerical values in your calculations: $W = 50$ mm, $L = 80$ mm, $t = 1$ mm, $\rho_1 = 2\,\Omega\,\mathrm{m}$, $V_s = 5\mathrm{V}$, $x_1 = 20$ mm, $x_2 = 45$ mm, $y_1 = 30$ mm, $y_2 = 60$ mm.

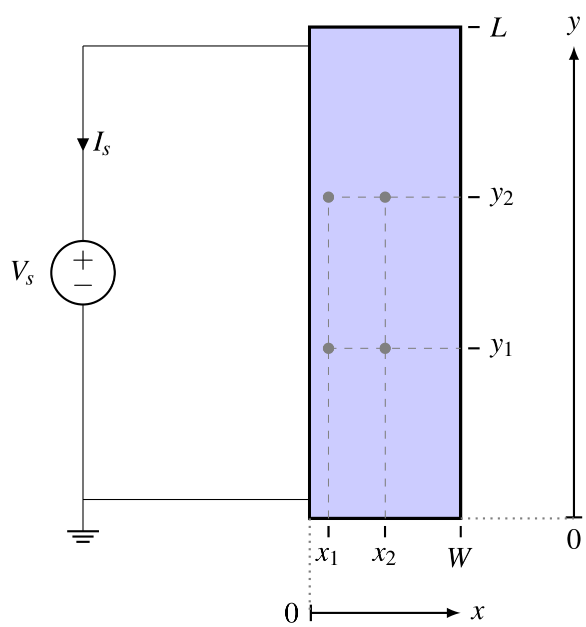


Figure 3: Top view of resistive touchscreen (not to scale). z axis i.e. the thickness not shown (into the page).

(a) Draw a circuit diagram representing **Figure 3**, where the entire touchscreen is represented as *a single resistor*. **Note that no touch is occurring in this scenario.** Remember that circuit diagrams in general consist of only circuit elements (resistors, sources, etc) represented by symbols, connecting wires, and the reference/ ground symbol. Calculate the value of current I_S based on the circuit diagram you drew. *Do not forget to specify the correct unit as always, and double check the direction of I_s!*

(b) Let us assume u_{12} is the node voltage at the node represented by coordinates (x_1, y_2) of the touchscreen, as shown in **Figure 4**. What is the value of u_{12}? You should first draw a circuit diagram representing Figure 4, which includes node u_{12}. Specify all resistance values in the diagram. Does the value of u_{12} change based on the value of the x-coordinate x_1?

Hint: You will need more than one resistor to represent this scenario.

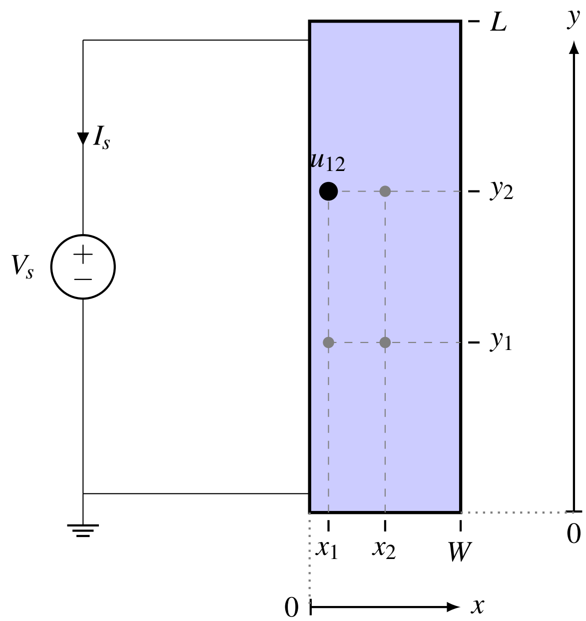


Figure 4: Top view of resistive touchscreen showing node u_{12}.

(c) Assume V_{ab} is the voltage measured between the nodes represented by touchscreen coordinates (x_1, y_1) and coordinates (x_1, y_2), as shown in **Figure 5**. Calculate the absolute value of V_{ab}. As with the previous part, you should first draw the circuit diagram representing Figure 5, which includes V_{ab}. Calculate all resistor values in the circuit. *Hint: Try representing the segment of the touchscreen between these two coordinates as a separate resistor itself.*

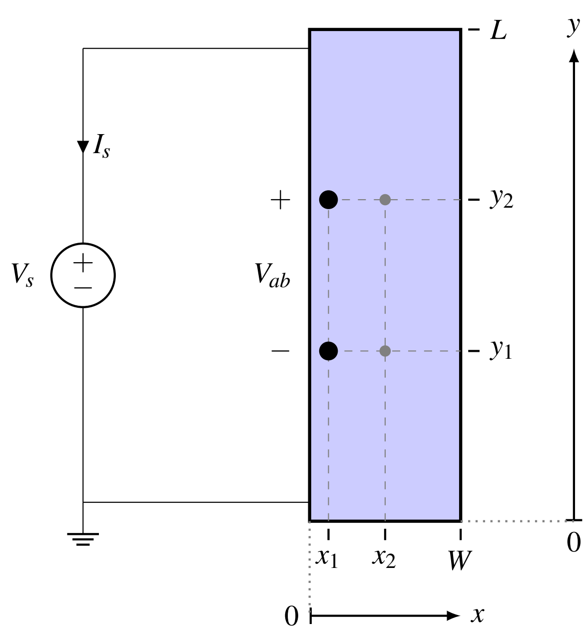


Figure 5: Top view of resistive touchscreen showing voltage V_{ab}.

(d) Calculate (the absolute value of) the voltage between the nodes represented by touchscreen coordinates (x_1, y_1) and coordinates (x_2, y_1) in **Figure 5**.

(e) Calculate (the absolute value of) the voltage between the nodes represented by touchscreen coordinates (x_1, y_1) and coordinates (x_2, y_2) in **Figure 5**.

(f) **Figure 6** shows a new arrangement with two touchscreens. The two touchscreens are next to each other and are connected to the voltage source in the same way. The second touchscreen (the one on the right) is identical to the one shown in Figure 3, except for different width, W_2, and resistivity, ρ_2.

Use the following numerical values in your calculations: $W_1 = 50$ mm, $L = 80$ mm, $t = 1$ mm, $\rho_1 = 2\,\Omega\,$m, $V_s = 5$V, $x_1 = 20$ mm, $x_2 = 45$ mm, $y_1 = 30$ mm, $y_2 = 60$ mm, which are the same values as before. The new touchscreen has the following numerical values which are different: $W_2 = 85$ mm, $\rho_2 = 1.5\,\Omega\,$m.

Draw a circuit diagram representing **Figure 6**, where the two touchscreens are represented as *two separate resistors*. **Note that no touch is occurring in this scenario.**

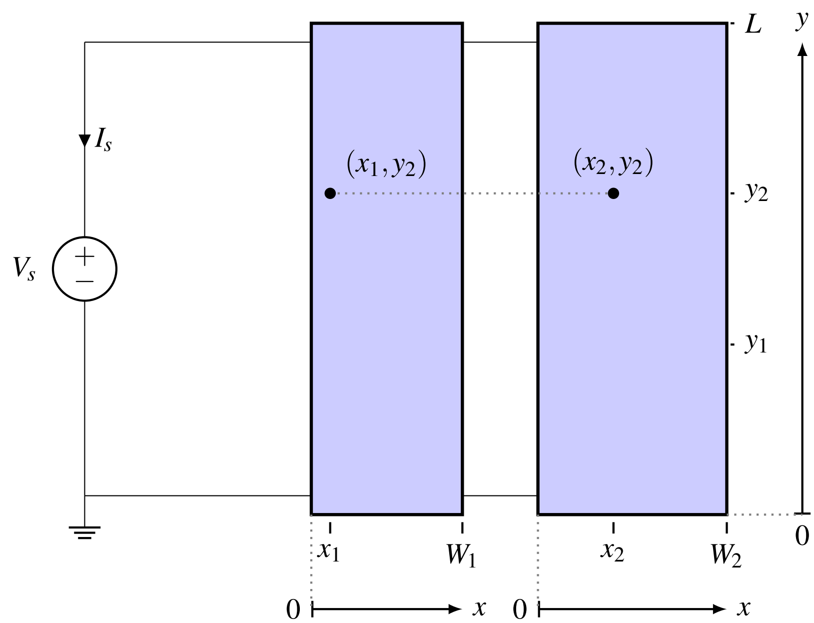


Figure 6: Top view of two touchscreens wired in parallel (not to scale). z axis not shown (into the page).

(g) Calculate the value of current I_s for the two touchscreen arrangement based on the circuit diagram you drew in the last part.

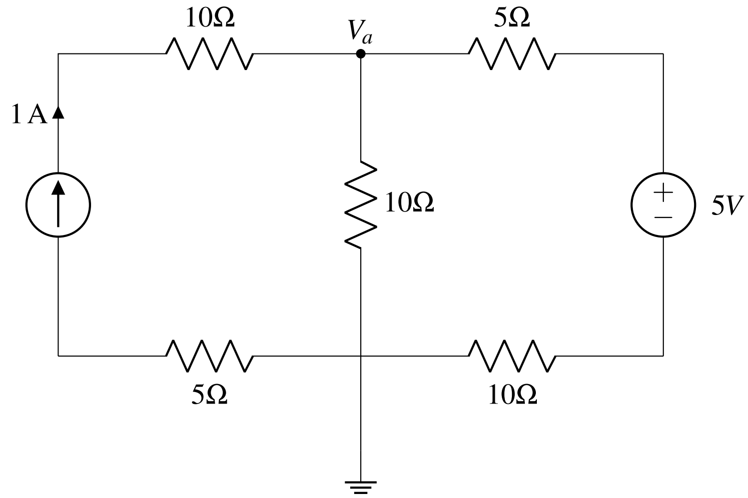
(h) Consider the two points: (x_1, y_2) in the touchscreen on the left, and (x_2, y_2) in the touchscreen on the right in **Figure 6**. Show that the node voltage at (x_1, y_2) is the same that at (x_2, y_2), i.e. the potential difference between the two points is 0. You can show this without explicitly calculating the node voltages at the two points.

If you were to connect a wire between the two coordinates (x_1, y_2) in the touchscreen on the left, and (x_2, y_2) in the touchscreen on the right, would any current flow through this wire?

6. Superposition

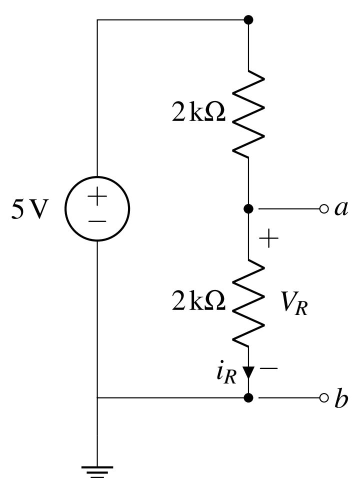
Learning Goal: *The objective of this problem is to help you practice solving circuits using the principles of superposition.*

Find the node potential V_a indicated in the diagram using superposition. Be careful when solving to take into account where the reference potential is.

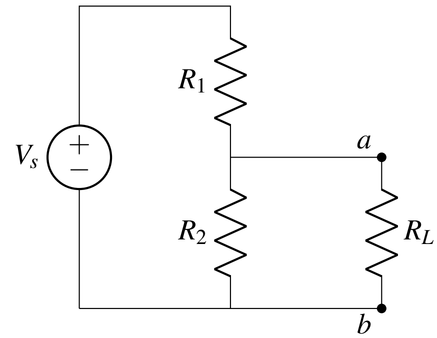


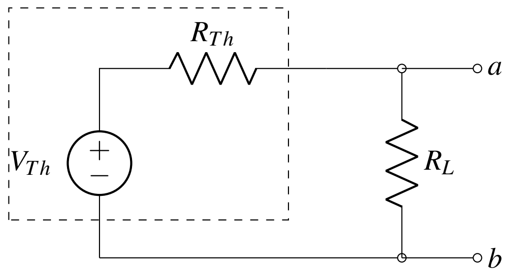
7. Why Bother With Thévenin Anyway?

(a) Find a Thévenin eqiuvalent for the circuit shown below looking from the terminals a and b. (*Hint: That is, find the open circuit voltage V_R across the terminals a and b. Also, find the equivalent resistance looking from the terminals a and b when the input voltage source is zeroed.*)



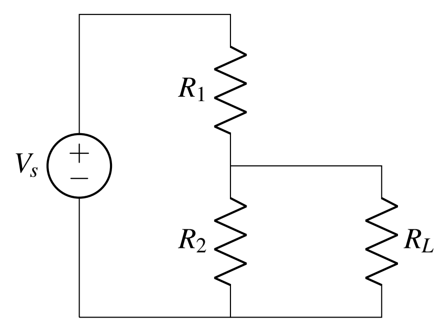
(b) Now consider the circuit shown below where a load resistor of resistance R_L is attached across the terminals a and b. Such a load resistor is often used to a model a device that we want to plug our circuit into, like an audio speaker. Compute the voltage drop V_R across the terminals a and b in this new circuit with the attached load. Express your answer in terms of R_L. *Hint: We have already computed the Thévenin equivalent of the unloaded circuit in part (a). To analyze the new circuit, attach R_L as the load resistance across the Thévenin equivalent computed in part (a), as shown in the figure below. One of the main advantages of using Thévenin (and Norton) equivalents is to avoid re-analyzing different circuits which differ only by the amount of loading (which depends on the device we are connecting!).*

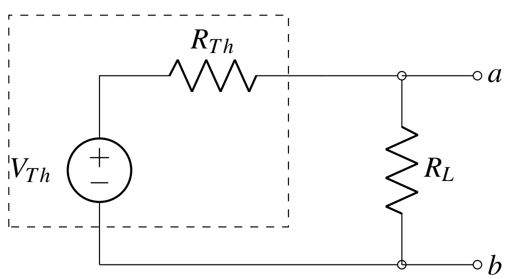




(c) Now compute the voltage drop V_R for three different values of R_L equal to $5/3\,\mathrm{k}\Omega$, $5\,\mathrm{k}\Omega$, and $50\,\mathrm{k}\Omega$? What can you comment on the value of R_L needed to ensure that the loading does not reduce the voltage drop V_R compared to the unloaded voltage V_R computed in part (a)?

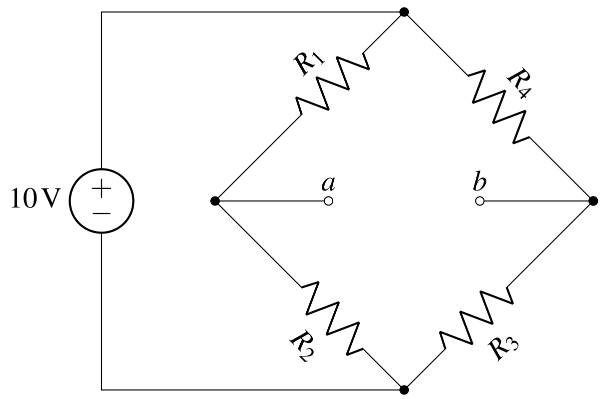
(d) Thus far, we have seen how to use Thévenin equivalents to compute the voltage drop across a load without re-analyzing the entire circuit. We would like to see if we can use the Thévenin equivalent for power computations. Consider the case where the load resistance $R_L = 8\mathrm{k}\Omega$, $V_S = 5V$, $R_1 = R_2 = 2\mathrm{k}\Omega$. Compute the power dissipated across the load resistor R_L both using the original circuit and the Thévenin equivalent. Are they equal? Now, compute the power dissipated by the voltage source V_s in the original circuit. Also, compute the power dissipated by the Thévenin voltage source V_{Th} in the Thévenin equivalent circuit. Is the power dissipated by the two sources equal?





8. Wheatstone Bridge

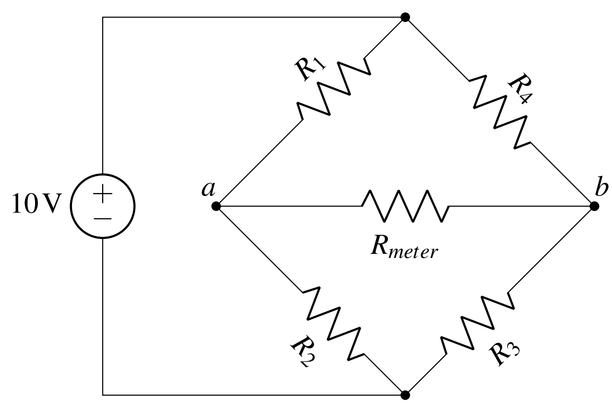
Thévenin equivalence is a powerful technique we can use to analyze the Wheatstone bridge circuit shown below. This circuit is used in many sensor applications where resistors R_1, R_2, R_3, R_4 are varying with respect to some external actuation. For example, it can be used to build a strain gauge or a scale. In that case the resistors R_1, R_2, R_3, R_4 would vary with respect to a strain caused by a force, and the Wheatstone Bridge circuit would translate that variation into a voltage difference across the "bridge" terminals a and b. Assume that $R_1 = 2\,\text{k}\Omega$, $R_2 = 2\,\text{k}\Omega$, $R_3 = 1\,\text{k}\Omega$, $R_4 = 3\,\text{k}\Omega$



(a) Calculate the voltage V_{ab} between the two terminals a and b.

(b) Next, draw the Thévenin equivalent of the Wheatstone bridge circuit.

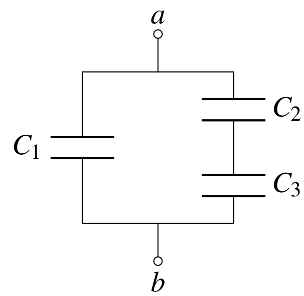
(c) Now assume that you are trying to measure the voltage V_{ab} using a voltmeter, whose resistance is R_{meter}, so you end up with the circuit below.



Unfortunately, you voltmeter is far from ideal, so $R_{meter} = 4k\Omega$. Is the voltage V_{ab} you found in part (a) equal to the new voltage $V_{R_{meter}}$ across the voltmeter resistor? Why or why not? Calculate the current $I_{R_{meter}}$ through the voltmeter resistor and the voltage $V_{R_{meter}}$ across the meter resistor.

9. Equivalent Capacitance

(a) Find the equivalent capacitance between terminals a and b of the following circuit in terms of the given capacitors C_1, C_2, and C_3. Leave your answer in terms of the addition, subtraction, multiplication, and division operators **only**.



(b) Find and draw a capacitive circuit using three capacitors, C_1, C_2, and C_3, that has equivalent capacitance of

$$\frac{C_1(C_2+C_3)}{C_1+C_2+C_3}$$

10. It's finally raining!

A lettuce farmer in the Salinas Valley has grown tired of imprecise online rainfall forecasts. They decide to take matters into their own hands by building a rain sensor. They place a square tank outside and attach two metal plates to two opposite sides in an effort to make a capacitor whose capacitance varies with the amount of water inside.

Note: In practice, water is conductive. However for this problem, assume the metal plates are properly insulated so that no current flows through the water and we can treat it like a dielectric material. In other words, the electric circuit is better modeled as a capacitance and not a resistance.

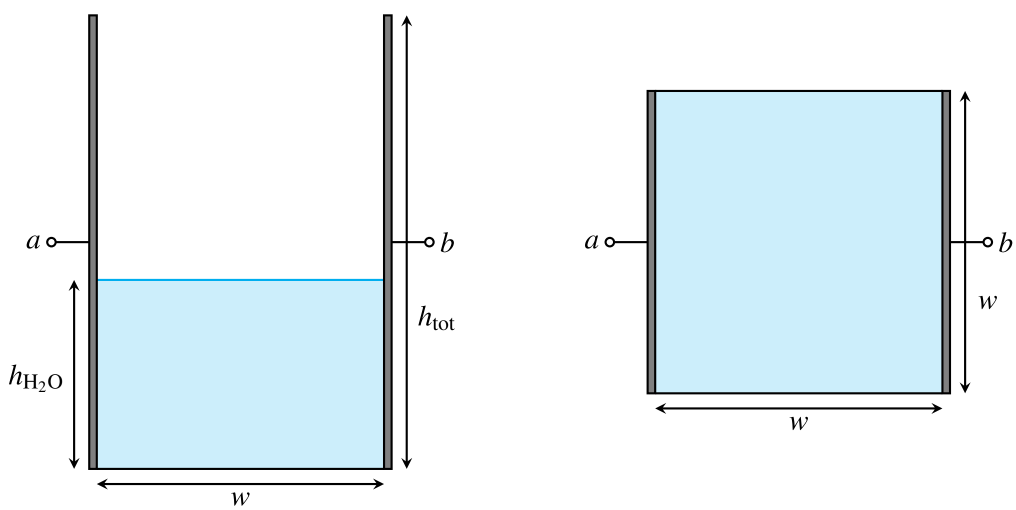


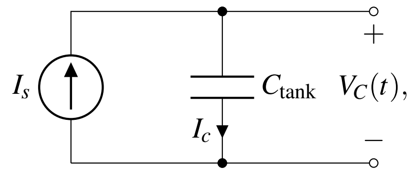
Figure 7: Tank side view (left) and top view (right).

The width and length of the tank are both w (i.e., the base is square) and the height of the tank is h_{tot}.

(a) What is the capacitance between terminals a and b when the tank is full? What about when it is empty? The permittivity of air is $\varepsilon_{\mathrm{air}} = \varepsilon_0$, and the permittivity of rainwater is $\varepsilon_{\mathrm{H_2O}} = 75\varepsilon_0$.

(b) Suppose the height of the water in the tank is $h_{\mathrm{H_2O}}$. Model the tank as a pair of capacitors in parallel, where one capacitor has a dielectric of air, and one capacitor has a dielectric of water. Find the total capacitance C_{tank} between the two metal walls/plates using circuit equivalence.

(c) After building this tank, the farmer consults the internet to assist them with a capacitance-measuring circuit. A fellow internet user recommends building the following circuit:



where C_{tank} is the total tank capacitance between terminals a and b calculated in part (b), and I_s is a known current supplied by a current source.

The user suggests measuring $V_C(t)$ for a brief interval of time, compute the rate of change of V_C, and determine C_{tank}.

Determine $V_C(t)$, where t is the number of seconds elapsed since the start of the measurement. You should assume that before any measurements are taken, the voltage across C_{tank} is initialized to $0\,\text{V}$, i.e. $V_C(0) = 0$.

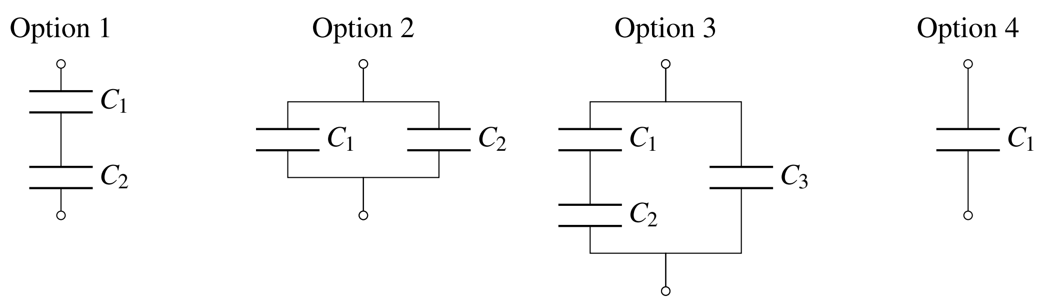
(d) Using the equation you derived for $V_C(t)$, describe how you can use this circuit to determine C_{tank} and $h_{\text{H}_2\text{O}}$.

11. Modeling Weird Capacitors

For each part of this problem,

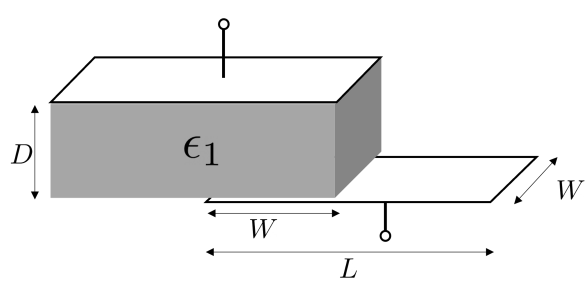
i. Pick the circuit option from below that *best* models the given physical capacitor.

ii. Calculate the total equivalent capacitance of the circuit in terms of the given quantities (e.g. $\varepsilon_1, \varepsilon_2, \varepsilon_3, L, W, D$).

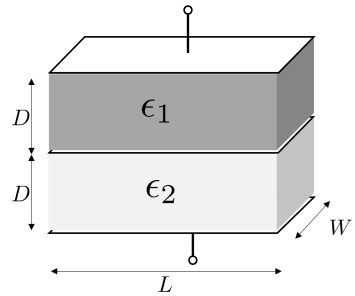


(a) A parallel plate capacitor with plate dimensions L and W, separated by a gap D, is filled with an insulator of permittivity ε_1, with the bottom plate displaced with overlap W as shown below. You can assume $W < L$ and $D << W$.

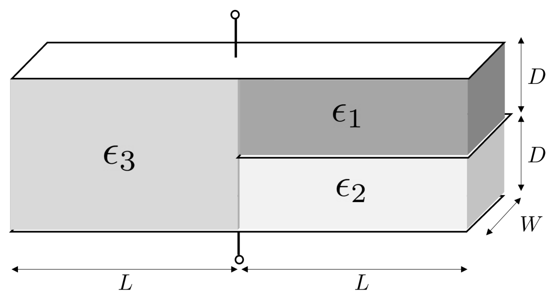
(i) Pick the circuit option from above that best models this physical capacitor, and (ii) calculate the total equivalent capacitance of the circuit in terms of L, W, D, ε_1.



(b) A parallel plate capacitor with plate dimensions L and W, separated by a gap $2 \cdot D$, is filled with two insulators of permittivities ε_1 and ε_2 as shown below. You can assume there's a plate between the two dielectrics.
(i) Pick the circuit option from above that best models this physical capacitor, and (ii) calculate the total equivalent capacitance of the circuit in terms of $L, W, D, \varepsilon_1, \varepsilon_2$.



(c) A parallel plate capacitor with plate dimensions L and W, separated by a gap $2 \cdot D$, is filled with three different materials with permittivities ε_1, ε_2, and ε_3 as shown in the figure below. You can assume there's a plate between the two dielectrics on the right.
(i) Pick the circuit option from above that best models this physical capacitor, and (ii) calculate the total equivalent capacitance of the circuit in terms of $L, W, D, \varepsilon_1, \varepsilon_2, \varepsilon_3$.



12. Prelab Questions

These questions pertain to the prelab reading for the Touch 3A lab. You can find the reading under the Touch 3A Lab section on the 'Schedule' page of the website. We do not expect in-depth answers for the questions. Please limit your answers to a maximum of 2 sentences.

(a) What is the equation for the relationship between the current and voltage of a capacitor? Write out the steps of your derivation. (Hint: Start from the charge equation, and see how you can plug in $I = \frac{dQ}{dt}$)
(b) What is the equation for the C_{eq} of capacitors in series? Write your answer in the form: C_{eq} = .
(c) Does adding another capacitor in series increase or decrease the total capacitance C_{eq}?
(d) Why does touching our touchscreen cause a change in capacitance?
(e) What is the purpose of using a comparator in the capacitive touchscreen that we'll be building?

13. Homework Process and Study Group

Who did you work with on this homework? List names and student ID's. (In case you met people at homework party or in office hours, you can also just describe the group.) How did you work on this homework? If you worked in your study group, explain what role each student played for the meetings this week.