

EECS 16A Designing Information Devices and Systems I
Summer 2023 Discussion 06A

1. Dynamical Systems (Spring 2020 Midterm 1 Question 7)

Define matrices $Q, R \in \mathbb{R}^{2 \times 2}$ according to

$$Q = \begin{bmatrix} 0 & 3/4 \\ 1 & 1/4 \end{bmatrix}, \quad R = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}.$$

- (a) Find the eigenvalues for the matrix Q .

- (b) Consider a system with state vector $\vec{x}[n] \in \mathbb{R}^2$ at time $n \geq 1$ given by

$$\vec{x}[n] = Q\vec{x}[n-1].$$

Is there a non-zero vector \vec{x} satisfying $\vec{x} = Q\vec{x}$? If yes, give one such vector.

(c) Draw the state-transition diagram for the system in part (b). Label your nodes "A" and "B".

(d) Now, consider a system with state vector $\vec{w}[n] \in \mathbb{R}^2$ at time $n \geq 1$ given by:

$$\vec{w}[n] = \begin{cases} Q\vec{w}[n-1] & \text{if } n \text{ is odd} \\ R\vec{w}[n-1] & \text{if } n \text{ is even.} \end{cases}$$

Write expressions for $\vec{w}[1]$, $\vec{w}[2]$, $\vec{w}[3]$ and $\vec{w}[4]$ in terms of $\vec{w}[0]$ and Q and R . Write each answer in the form of a matrix-vector product.

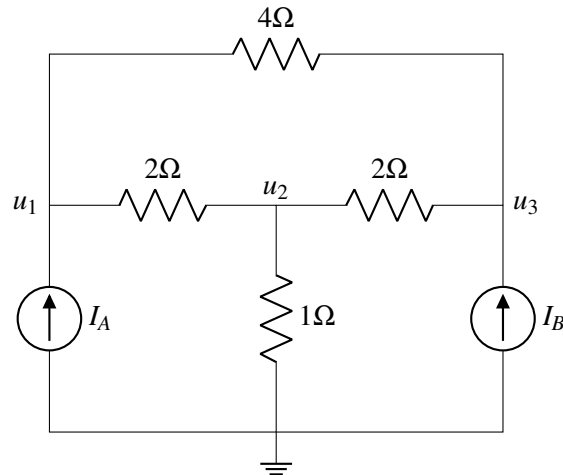
- (e) Suppose we start the system of part (d) with state $\vec{w}[0] = [11/14 \quad 3/14]^\top$. Find expressions for \vec{w}_{even} and \vec{w}_{odd} , which are defined according to

$$\vec{w}_{\text{even}} = \lim_{k \rightarrow \infty} \vec{w}[2k], \quad \vec{w}_{\text{odd}} = \lim_{k \rightarrow \infty} \vec{w}[2k+1].$$

In words, \vec{w}_{even} and \vec{w}_{odd} describe the long-term behavior of the system at even and odd time-instants, respectively. (Hint: you can avoid computation by thinking about the system at even time-instants in terms of a state-transition diagram.)

2. Superposition (Fall 2020 Midterm 2 Question 7)

For this question, we will analyze the circuit shown below with the two current sources of strength I_A and I_B as inputs. It may be observed that the network of resistors shown in the circuit is symmetric. We will first solve this circuit for symmetric inputs $I_A = I_B$, and then for anti-symmetric inputs $I_A = -I_B$. Using these two results, we we will solve the circuit for arbitrary inputs I_A, I_B .

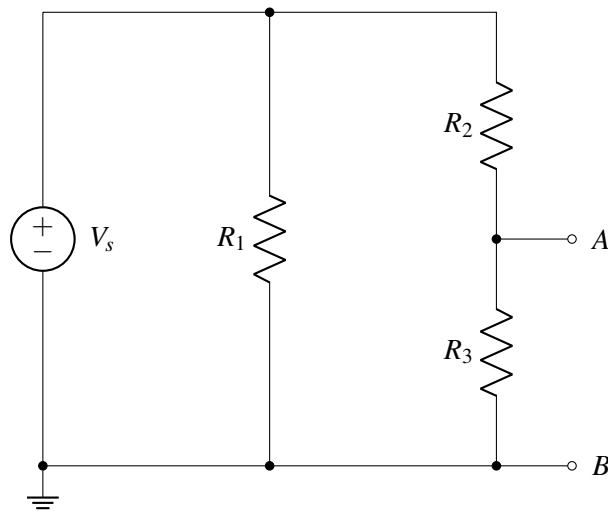


- (a) Consider the circuit above with symmetric inputs, $I_A = I_B = 1\text{A}$. Using superposition, solve for the node voltages at the nodes marked u_1 , u_2 and u_3 .

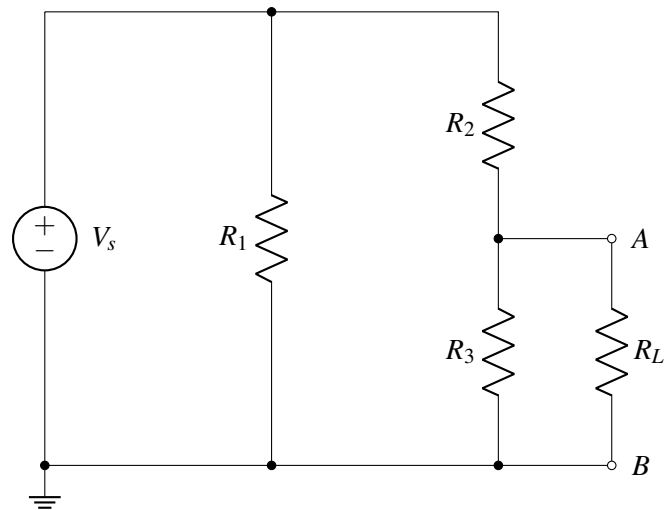
- (b) Consider the same circuit as before with anti-symmetric inputs, $I_A = 1\text{ A}$ and $I_B = -1\text{ A}$. Using superposition solve for the node voltages at the nodes marked u_1 , u_2 and u_3 .

3. Thévenin/Norton Equivalence

- (a) Find the Thévenin resistance R_{th} of the circuit shown below, with respect to its terminals A and B .



- (b) Now a load resistor, R_L , is connected across terminals A and B , as shown in the circuit below. Using Thévenin equivalence, find the power dissipated in the load resistor in terms of the given variables.



4. OPTIONAL: Power to Resist (from Spring 2018 midterm 2)

Find the power dissipated by the voltage source in the circuit below. Be sure to use passive sign convention.

