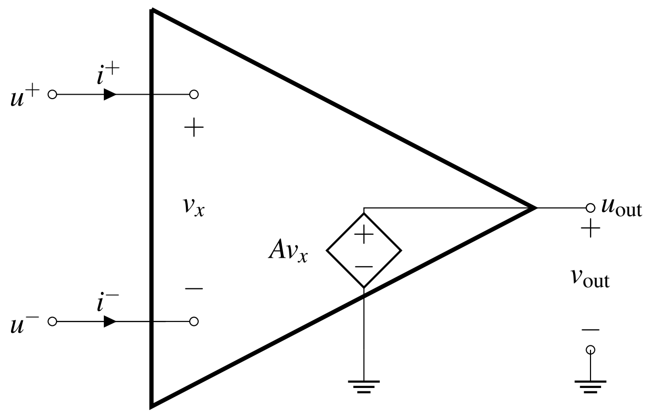
EECS 16A Designing Information Devices and Systems I
Summer 2023 Discussion 6B

1. Op-Amp Rules

Here is an equivalent circuit of an op-amp (where we are assuming that $V_{SS} = -V_{DD}$) for reference:



(a) What are the currents flowing into the positive and negative terminals of the op-amp (i.e., what are i^+ and i^-)? Based on this answer, what are some of the advantages of using an op-amp in your circuit designs?

Answer:

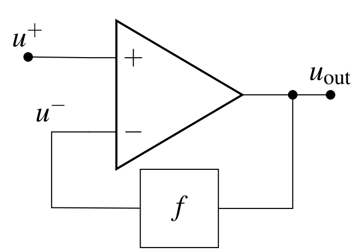
The u^+ and u^- terminals have no closed circuit connection between them, and therefore no current can flow into or out of them. This is very good because we can connect an op-amp to any other circuit, and the op-amp will not disturb that circuit in any way because it does not load the circuit (it is an open circuit).

(b) Suppose we add a resistor of value R_L between u_{out} and ground. What is the value of v_{out}? Does your answer depend on R_L? In other words, how does R_L affect Av_C? What are the implications of this with respect to using op-amps in circuit design?

Answer:

Notice that u_{out} is connected directly to a controlled/dependent voltage source, and therefore v_{out} will always have to be equal to Av_x regardless of what R_L is connected to the op-amp. This is useful because it means that the output voltage of the op-amp is only dependent on the differential input voltage v_x and not dependent on what is connected to the output node u_{out}.

(c) Now suppose our op-amp is connected in negative feedback.



What is the relationship between u^+ and u^-?

Answer: By the 2nd golden rule of op-amps, we know that if an op-amp is in negative feedback, the inputs to the op-amp are the same. In other words

$$u^+ = u^-.$$

Lets prove this mathematically. Assuming that the gain of the op-amp is A, we know that

$$u^- = f u_{\text{out}}$$
$$u_{\text{out}} = A(u^+ - u^-).$$

Substituting u_{out} and combining the two equations we have

$$u^- = fA(u^+ - u^-)$$

which we can rearrange to get

$$\frac{u^+}{u^-} = \frac{1+Af}{Af}.$$

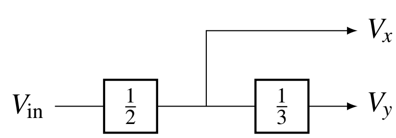
If Af is very large, then we see that

$$\lim_{Af\to\infty} \frac{1+Af}{Af} = 1$$

which means $u^+ = u^-$. In practice, A is very large and as long as we choose a reasonable value of f (i.e. not too small) then our approximation holds.

2. Modular Circuit Buffer

Let's try designing circuits that perform a set of mathematical operations using op-amps. While voltage dividers on their own cannot be combined without altering their behavior, op-amps can preserve their behavior when combined and thus are a perfect tool for modular circuit design. We would like to implement the block diagram shown below:



In other words, create a circuit with two outputs V_x and V_y, where $V_x = \frac{1}{2}V_{\text{in}}$ and $V_y = \frac{1}{3}V_x = \frac{1}{6}V_{\text{in}}$.

(a) Draw two voltage dividers, one for each operation (the $1/2$ and $1/3$ scalings). What relationships hold for the resistor values for the $1/2$ divider, and for the resistor values for the $1/3$ divider?

Answer: Recall our voltage divider consists of V_{in} connected to two resistors (R_1, R_2) in series with R_2 connected to ground and the output voltage between ground and the central node. This yields the formula

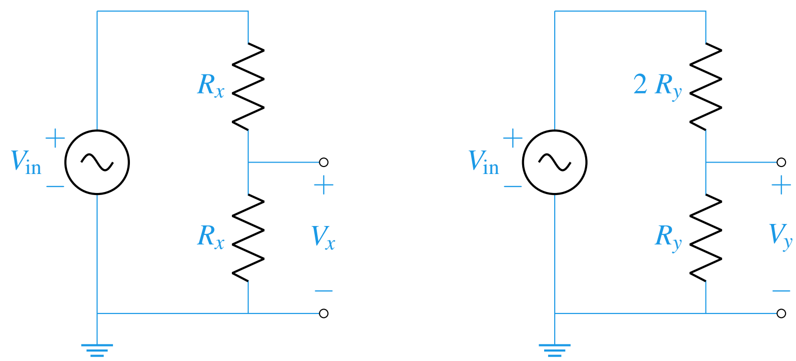
$$V_{\text{out}} = \left(\frac{R_2}{R_1+R_2}\right) V_{\text{in}}.$$

For the $1/2$ operation (V_x output) we recognize

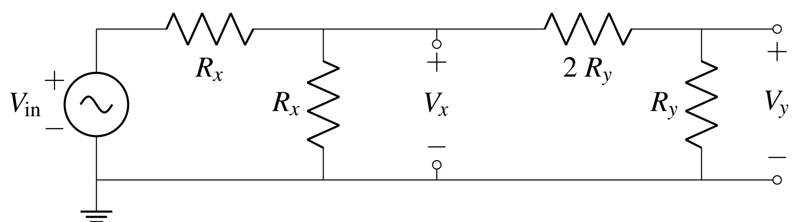
$$\frac{1}{2} = \left(\frac{R_2}{R_1+R_2}\right) \quad \longrightarrow \quad R_1 + R_2 = 2R_2 \quad \longrightarrow \quad R_1 = R_2 \equiv R_x.$$

For the $1/3$ operation (V_y output) we recognize

$$\frac{1}{3} = \left(\frac{R_2}{R_1+R_2}\right) \quad \longrightarrow \quad R_1 + R_2 = 3R_2 \quad \longrightarrow \quad \frac{R_1}{2} = R_2 \equiv R_y.$$



(b) If you combine the voltage dividers, made in part (a), as shown by the block diagram (output of the $1/2$ voltage divider becomes the source for the $1/3$ voltage divider circuit), do they behave as we hope (meaning $V_{\text{in}} = 2V_x = 6V_y$)?



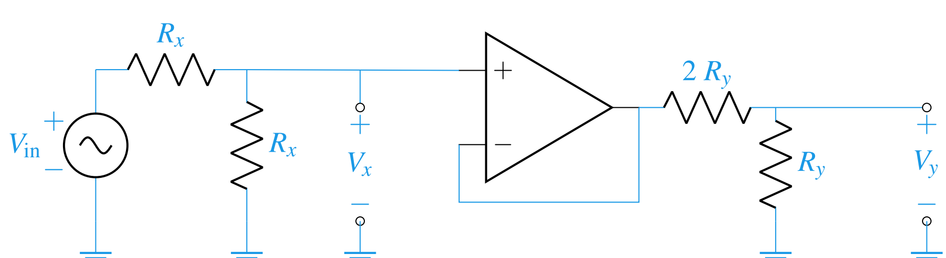
Answer: To quickly access this combined system, we may identify V_x as the result of a new equivalent voltage divider (recognizing the R_y resistors in series and that series is in parallel with R_x). The load resistor becomes $R_{eq} = \frac{3R_xR_y}{R_x+3R_y}$. This yields

$$V_x = \left(\frac{R_{eq}}{R_x+R_{eq}}\right)V_{\text{in}} = \left(\frac{1}{2+\frac{R_x}{3R_y}}\right)V_{\text{in}} \qquad V_y = \frac{1}{3}V_x = \left(\frac{1}{6+\frac{R_x}{R_y}}\right)V_{\text{in}}$$

From this stage it is evident that combining our dividers changes their behavior (although they preserve behavior in the limit $R_y >> R_x$). The second divider draws current from middle node of the first divider and so we can longer apply the voltage divider equation. The new values for V_x, V_y are dependent on values from both dividers, which means they can't be treated independently!

(c) Perhaps we could use an op-amp (in negative-feedback) to achieve our desired behavior.
Modify the implementation you tried in part (b) using a negative feedback op-amp in order to achieve the desired V_x, V_y relations $V_x = \frac{V_{\text{in}}}{2}$ and $V_y = \frac{V_x}{3} = \frac{V_{\text{in}}}{6}$.
HINT: Place the op-amp in between the dividers such that the V_x node is an input into the op-amp, while the source of the 2nd divider is the output of the op-amp!

Answer: Use the op-amp as a voltage buffer.



Since no current flows into the positive op-amp input, we've successfully isolated the dividers so they can be used in a modular fashion! $\square$

NOTE: The V_x, V_y outputs from this configuration would change with the addition of a load on either terminal. As a follow-up, think about ways to make each output agnostic to the loads attached!

