EECS 16A Designing Information Devices and Systems I
Summer 2023 Discussion 2A

1. Matrix Multiplication

Consider the following matrices:

$$\mathbf{A} = \begin{bmatrix} 1 & 4 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 3 \\ 2 \end{bmatrix} \qquad \mathbf{C} = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} \qquad \mathbf{D} = \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}$$

$$\mathbf{E} = \begin{bmatrix} 1 & 9 & 5 & 7 \\ 4 & 3 & 2 & 2 \end{bmatrix} \quad \mathbf{F} = \begin{bmatrix} 5 & 5 & 8 \\ 6 & 1 & 2 \\ 4 & 1 & 7 \\ 3 & 2 & 2 \end{bmatrix} \quad \mathbf{G} = \begin{bmatrix} 8 & 1 & 6 \\ 3 & 5 & 7 \\ 4 & 9 & 2 \end{bmatrix} \quad \mathbf{H} = \begin{bmatrix} 5 & 3 & 4 \\ 1 & 8 & 2 \\ 2 & 3 & 5 \end{bmatrix}$$

For each matrix multiplication problem, *if the product exists*, find the product by hand.
Otherwise, explain why the product does not exist.

(a) **A B**

(b) **C D**

(c) **D C**

(d) **C E**

(e) **F E** (only note whether or not the product exists and optionally compute the product if it does)

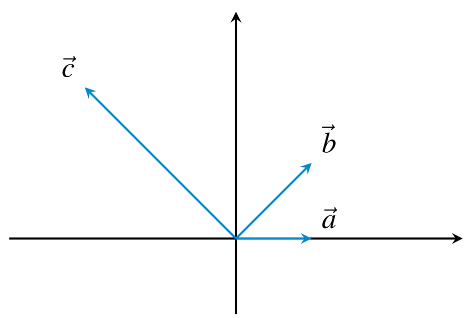
(f) **E F** (only note whether or not the product exists and optionally compute the product if it does)

(g) **G H** (Practice on your own)

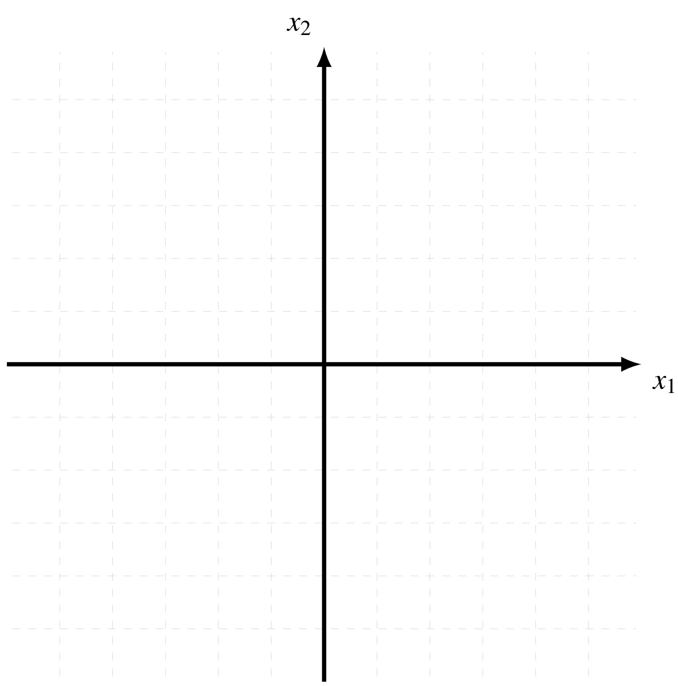
(h) **H G** (Practice on your own)

2. Visualizing Linear Combinations of Vectors

We are given a point $\vec{c}$ that we want to get to, but we can only move in two directions: $\vec{a}$ and $\vec{b}$. We know that to get to $\vec{c}$, we can travel along $\vec{a}$ for some amount α, then change direction and travel along $\vec{b}$ for some amount β. We want to find these two scalars α and β, such that we reach point $\vec{c}$. That is, $\alpha\vec{a} + \beta\vec{b} = \vec{c}$.



(a) First, consider the case where $\vec{x} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $\vec{y} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, and $\vec{z} = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$. Draw these vectors.



(b) We want to find the two scalars α and β, such that by moving α along $\vec{x}$ and β along $\vec{y}$, we can reach $\vec{z}$. Write a system of equations to find α and β in matrix form.

(c) Solve for α, β.

(d) Superimpose the scaled vectors $\alpha\vec{x}$ and $\beta\vec{y}$ on your graph in part (a) and confirm $\alpha\vec{x} + \beta\vec{y} = \vec{z}$.