

Welcome to EECS 16A!

Designing Information Devices and Systems I



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Fall 2021

Module 2
Lecture 2
Introduction to Circuit Analysis
(Note 12)



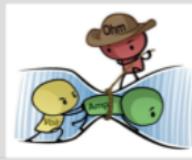
Last Class (0203)...

Quantities	Analytical Symbol	Units
Current	I	Amperes (A)
Voltage	V	Volts (V)
Resistance	R	Ohms (Ω)

$I \Rightarrow$ flows through an element

$V \Rightarrow$ applied across an element

$R \Rightarrow$ opposition to current flow



Electronic Materials

- Conductors
- Semiconductors
- Insulators

* **Charge** \Rightarrow Can be either positive or negative; basic element of electric flow. Unit: Coulomb [C]

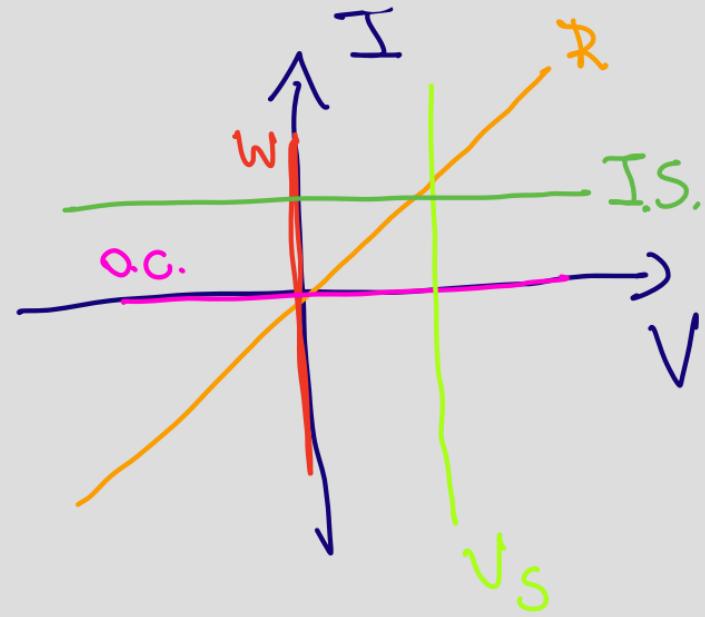
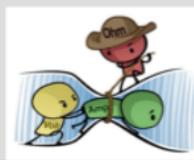
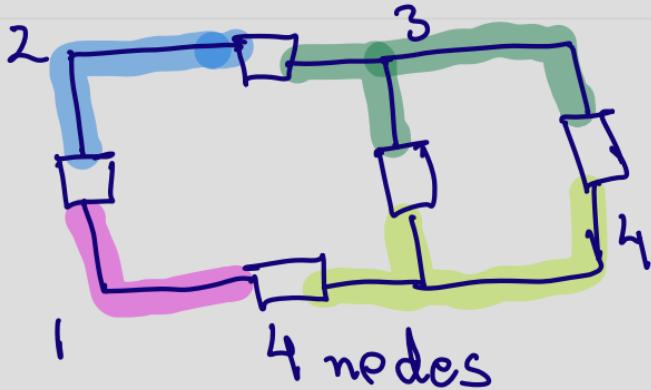
* **Current** \Rightarrow Net amount of charge that passes through some cross-section area over a period of time.

$$[A] \rightarrow I = \frac{dQ}{dt} \frac{[C]}{[s]}$$

Last Class (0203)...

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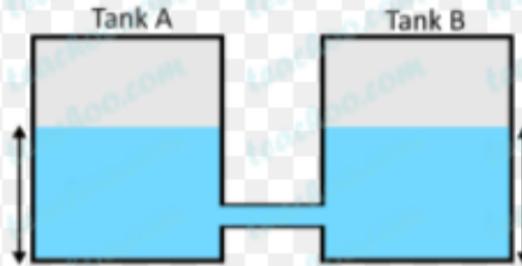


- Today
- KCL ; KVL \Rightarrow RULES
 - Circuit analysis
 - First operator !

Potential Energy

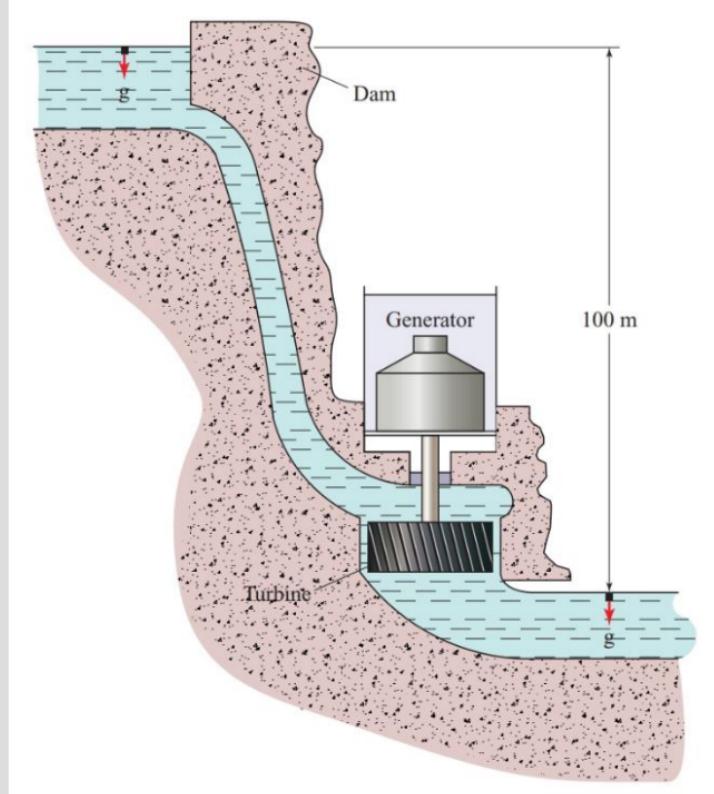
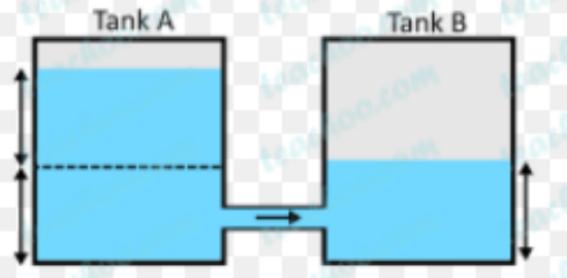
Water is exact the same level in both Tanks

So, water will **not** flow

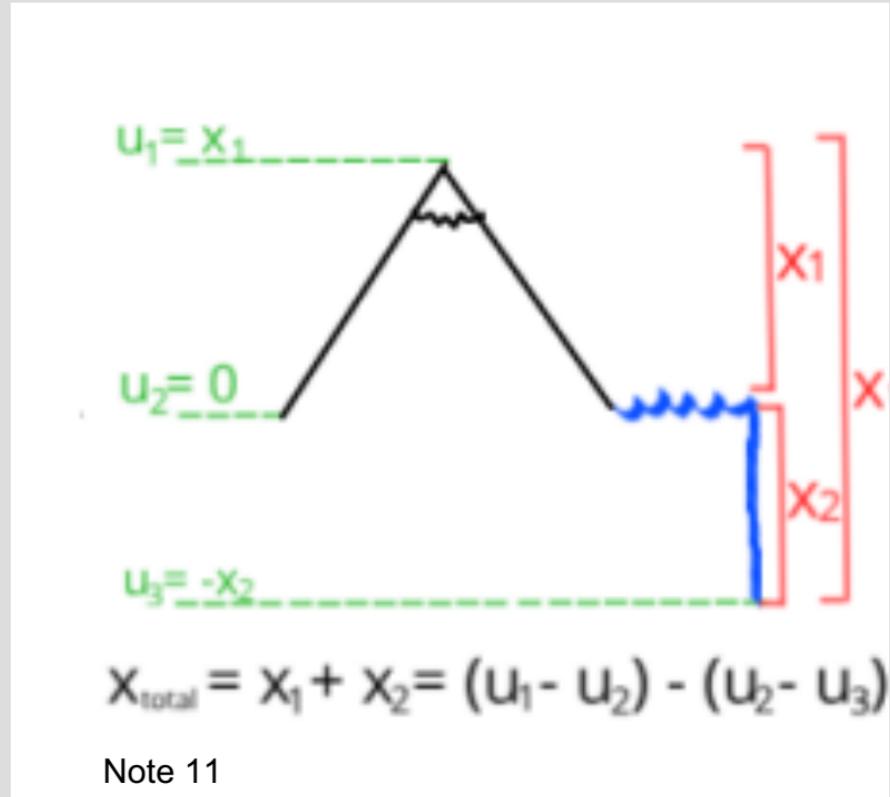
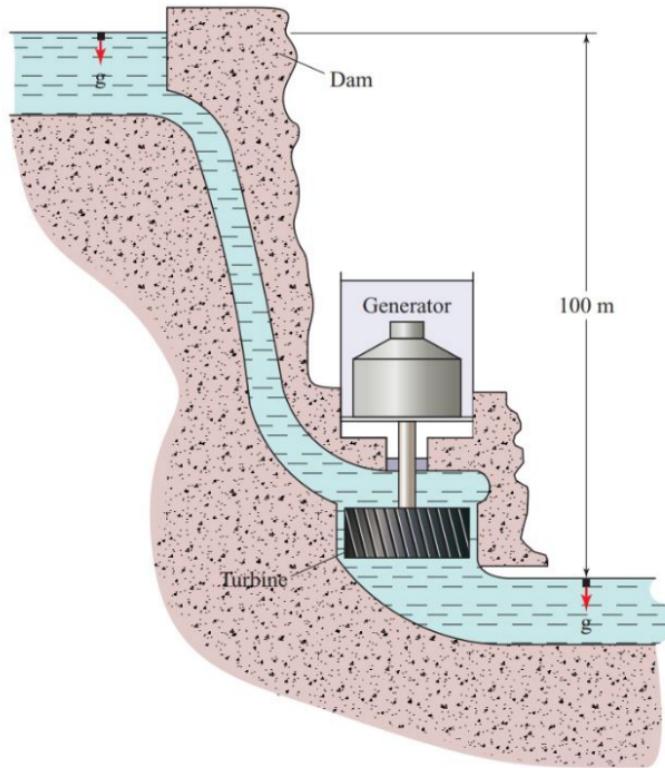


Since Tank A has higher water level,

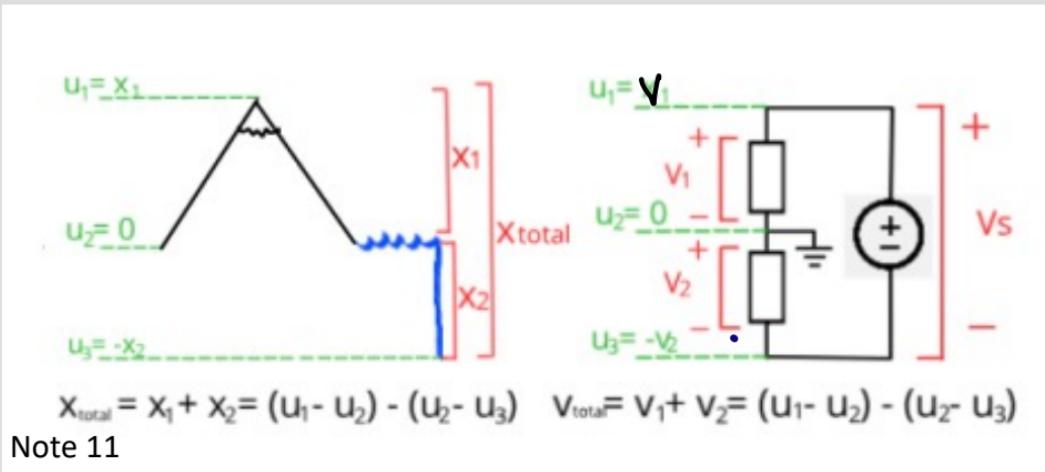
So, water will **flow** from Tank A to Tank B



Potential and Voltage



Electronic Devices depend on movement of charges



We always need to define a reference for potentials.

Ground = 0

U_1, U_2, U_3
potentials

$$V_1 = U_1 - U_2$$

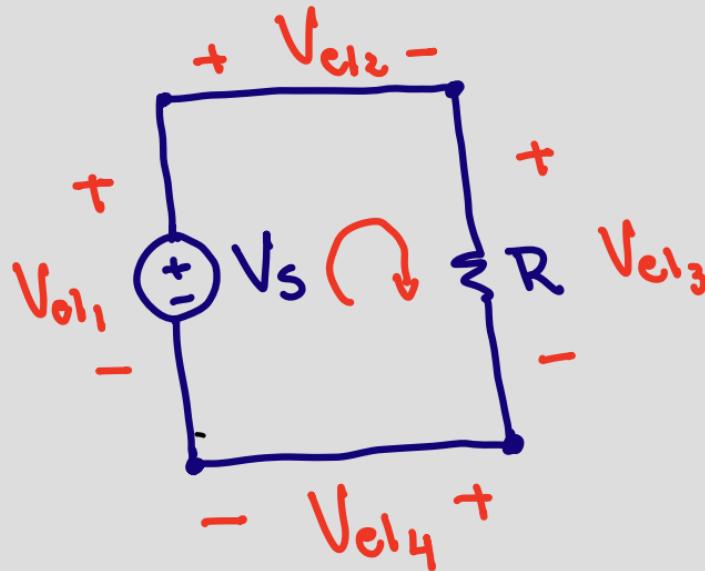
$$V_2 = U_2 - U_3$$

$$V_{\text{total}} = V_1 + V_2$$

$$V_{\text{total}} = V_s$$

Rules for circuit analysis: Kirchoff's Voltage Law (KVL)

Sum of Voltages across the elements in a loop equal zero



$$V_{el1} - V_{el2} - V_{el3} - V_{el4} = 0$$

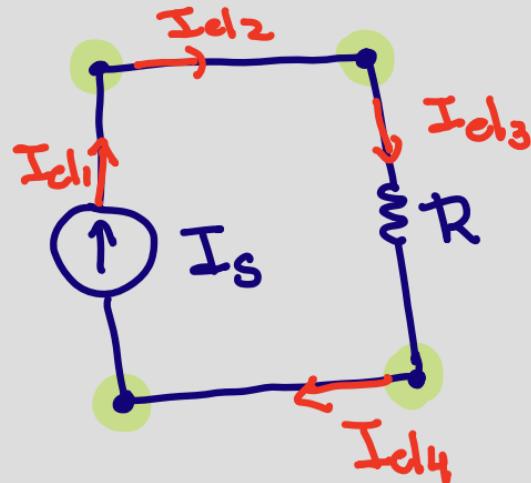
$$\begin{array}{rcl} + & \stackrel{+}{\textcircled{z}} & V_{el2} \\ V_{el1} & \stackrel{-}{\textcircled{z}} & + V_{el3} \\ - & \stackrel{-}{\textcircled{z}} & - V_{el4} \end{array}$$

$$V_{el1} = V_s$$

$$V_{el3} = ?$$

Rules for circuit analysis: Kirchoff's Current Law (KCL)

The current flowing into any junction must equal the current flowing out



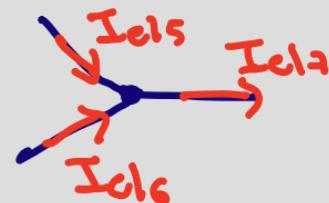
$$I_{cl1} = I_{cl2}$$

$$I_{cl2} = I_{cl3}$$

$$I_{cl3} = I_{cl4}$$

$$I_{cl4} = I_{cl1}$$

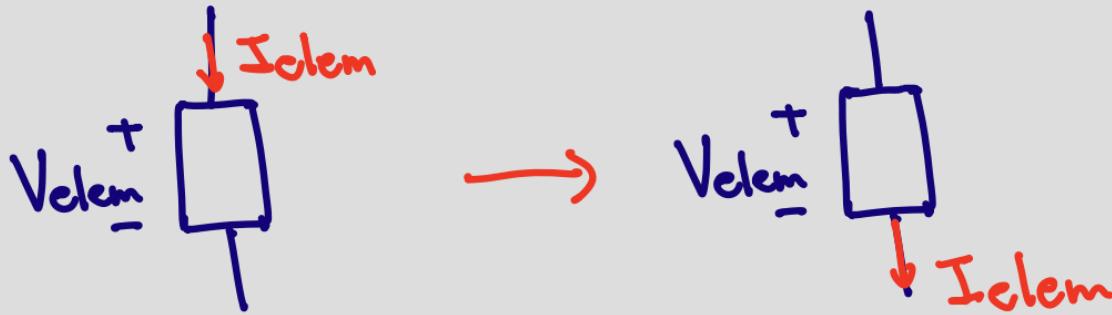
Example 2:



$$I_{cl5} + I_{cl6} = I_{cl7}$$

Rules for circuit analysis: KCL within the element

The current flowing into any junction must equal the current flowing out



Same current!

Both are allowed.

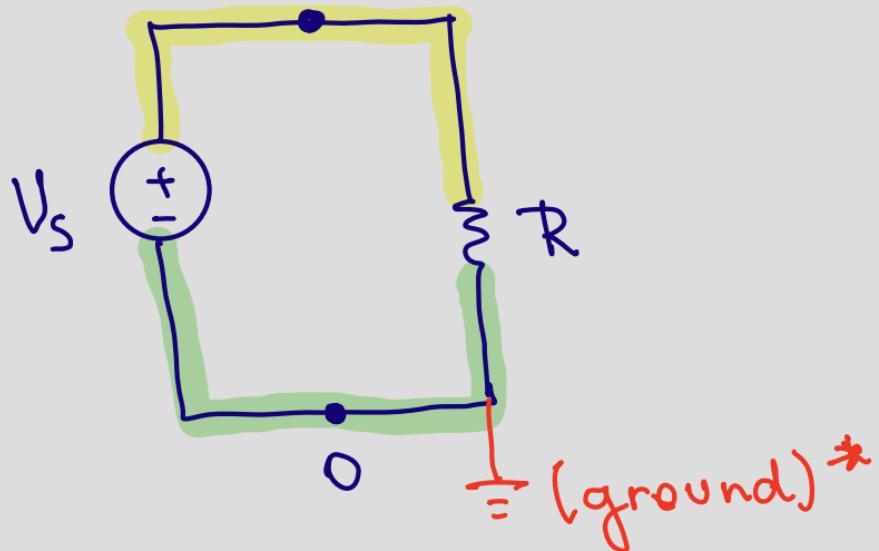
I_{elem} goes
into a +
or out of
a - terminal



* Passive sign
convention

Circuit Analysis Algorithm : step 1

Pick a reference node and label it as 0 potential. All voltages measured relative to this node.

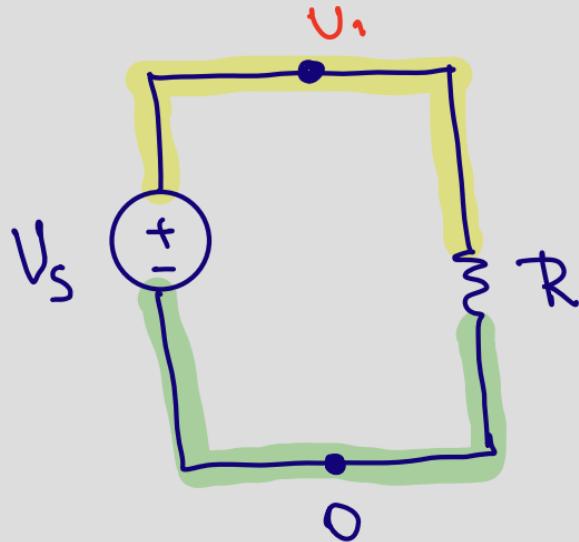


* Tells you where
the reference
is.

Circuit Analysis Algorithm : step 2

Label all remaining nodes as potentials U_i
 $[U_1 \dots U_{N-1}]$

$$U_1 - 0 = U_1 = V$$



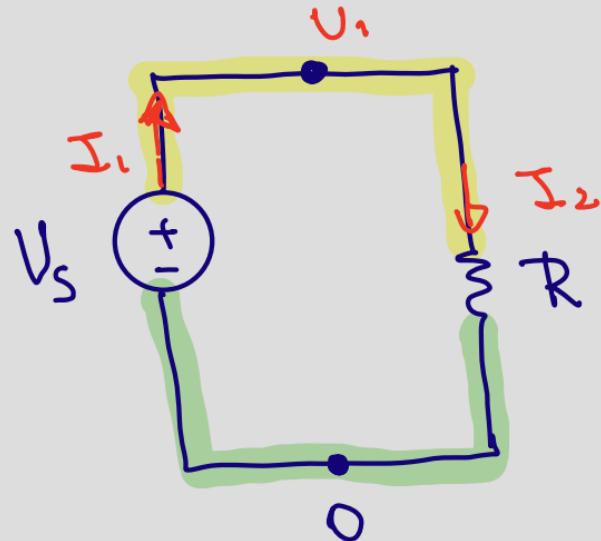
Voltage between
node \downarrow and node "0".

Circuit Analysis Algorithm : step 3

Label all branch currents with I_m

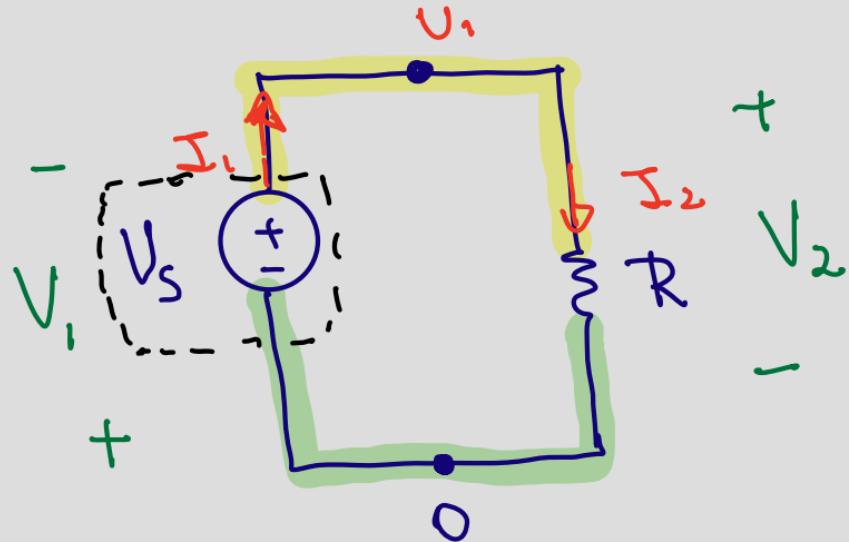
Arbitrarily pick directions of I_m

$[I_1 \dots I_k]$



Circuit Analysis Algorithm : step 4

Add signs + and – element voltages to each element following the passive sign convention



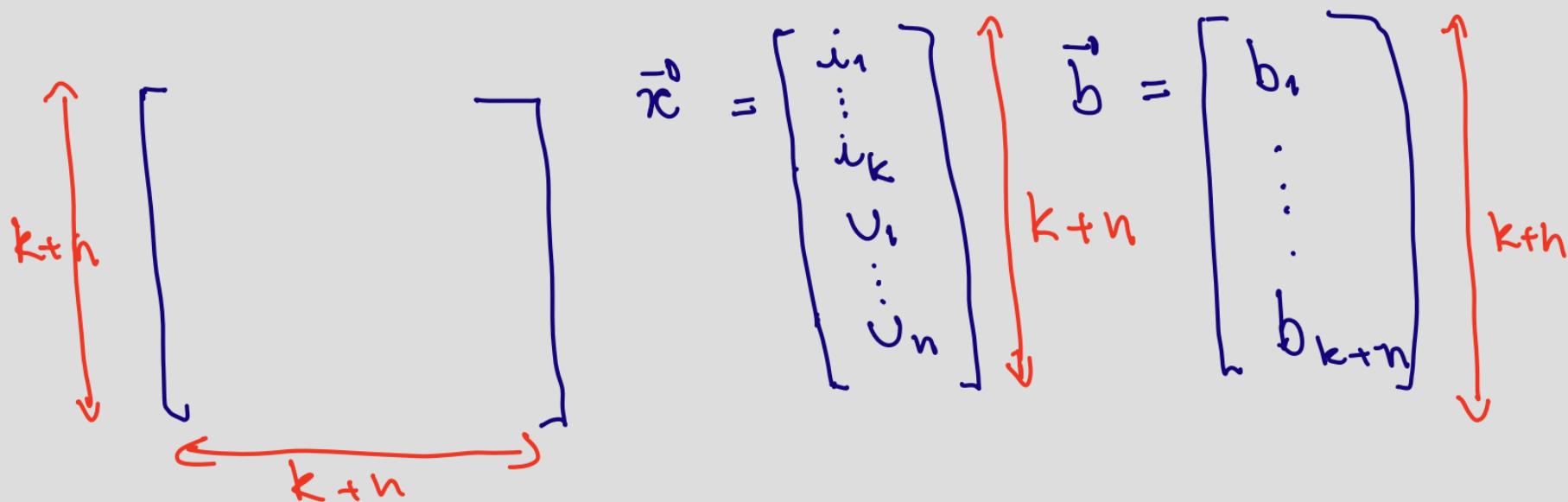
Circuit Analysis Algorithm : step 5

$$A \vec{x} = \vec{b}$$

\vec{x} :: unknowns

\vec{b} :: knowns/ constants

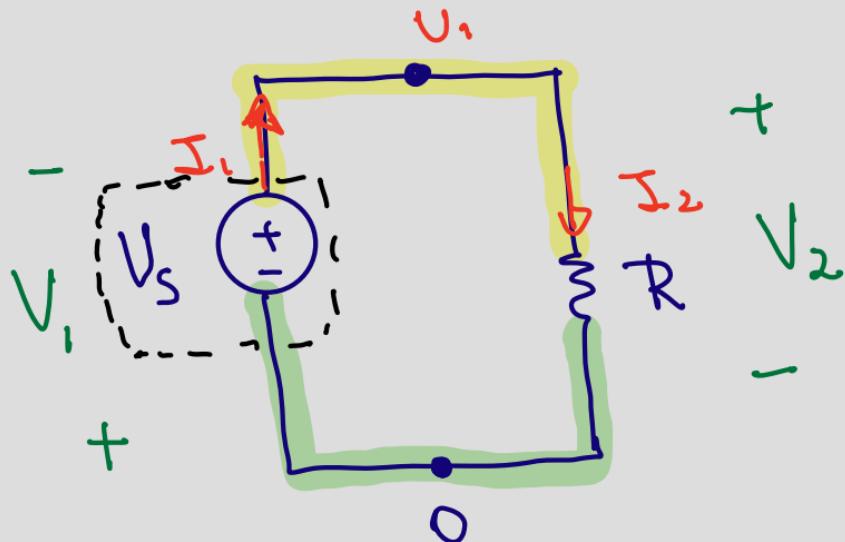
A :: knowns/ constants



Circuit Analysis Algorithm : step 6

Use KCL to fill as many rows of A as possible (linear independence) # Nodes

$$-1 = N-1$$



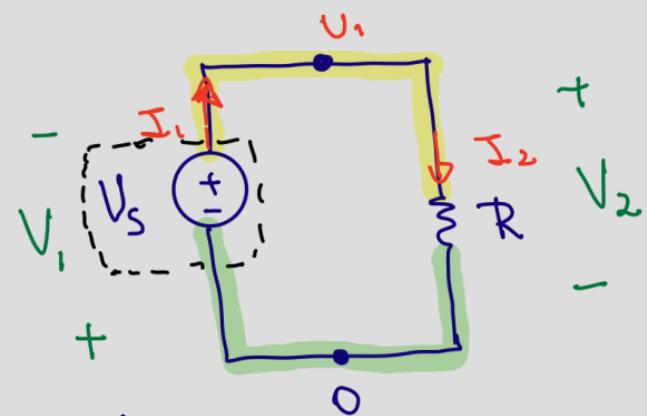
$$I_1 = I_2$$

$$I_1 - I_2 = 0$$

$$\begin{bmatrix} 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ U_1 \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix}$$

Circuit Analysis Algorithm : step 7

Use current-voltage relationships for each element to fill the rest of the A matrix



$$\text{Voltage Element} = -V_s$$

$$V_{el1} = 0 - V_1$$

$$V_1 = V_1 = V_s$$

$$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & R & -1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ V_1 \end{bmatrix} = \begin{bmatrix} 0 \\ V_s \\ 0 \end{bmatrix}$$

$$V_1 = V_s$$

$$I_1 = V_s / R$$

$$I_2 = V_s / R$$

Resistor

$$V_{el2} = I_2 \cdot R$$

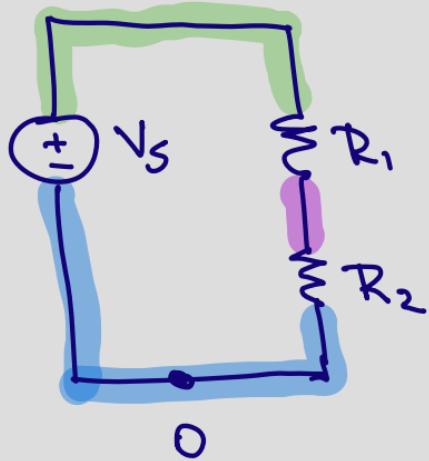
$$V_{el2} = V_1 - 0 = V_1$$

$$V_1 = I_2 \cdot R$$

$$I_2 \cdot R - V_1 = 0$$

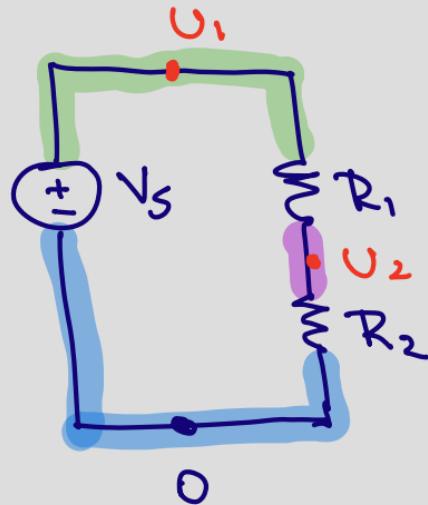
Node Voltage Analysis – Voltage Divider (Operator) \rightarrow make circuit analysis faster

✓ Step 1 – Pick a node and label it as ground



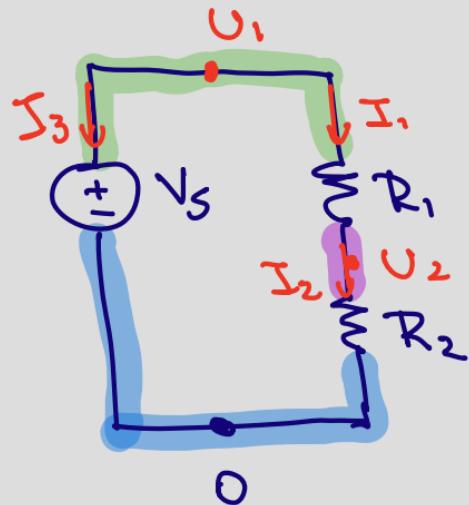
Node Voltage Analysis – Voltage Divider

Step 2 – Label all remaining nodes as some potential U_i .



Node Voltage Analysis – Voltage Divider

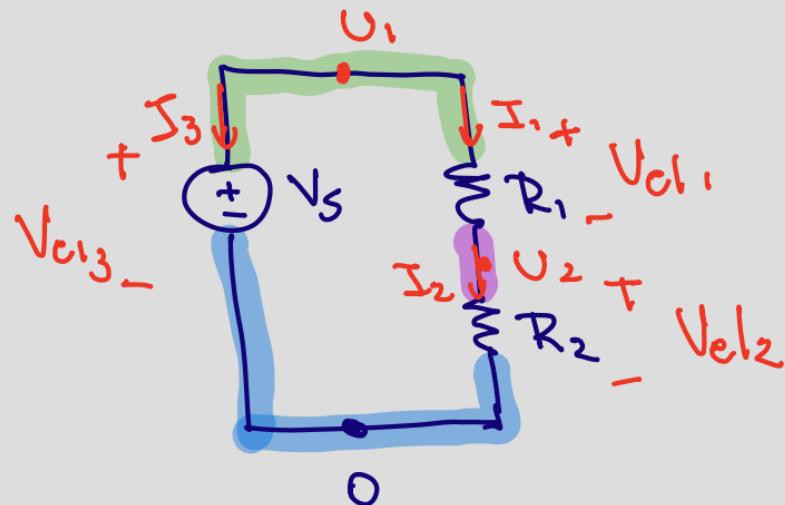
Step 3 – Label the current through every non-wire element in the circuit with I_n . ✓



Node Voltage Analysis – Voltage Divider

Requires some thinking

Step 4 – Add +/- labels on each non-wire element, following the passive sign convention.



* Important!

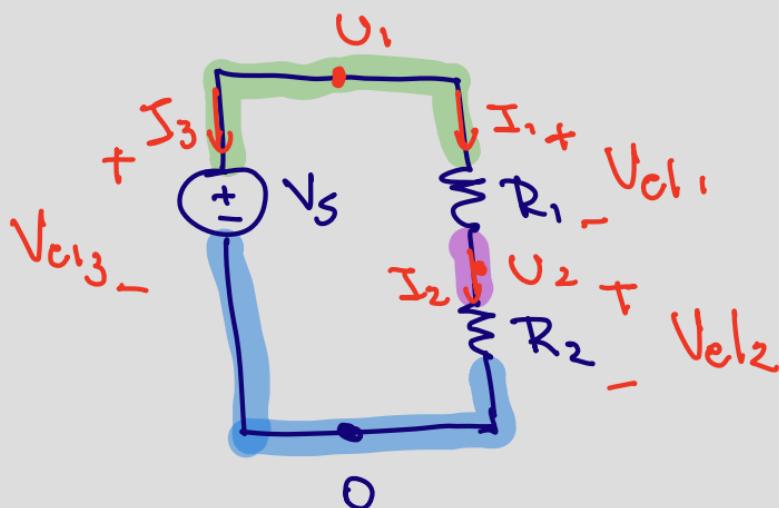
Passive sign convention: the current enters at the positive terminal and exits at the negative terminal.

Node Voltage Analysis – Voltage Divider

$$A\vec{x} = \vec{b}$$

Step 5 – Set up the relationship \vec{x} consists of the unknown currents and potentials.

Identify the unknowns.

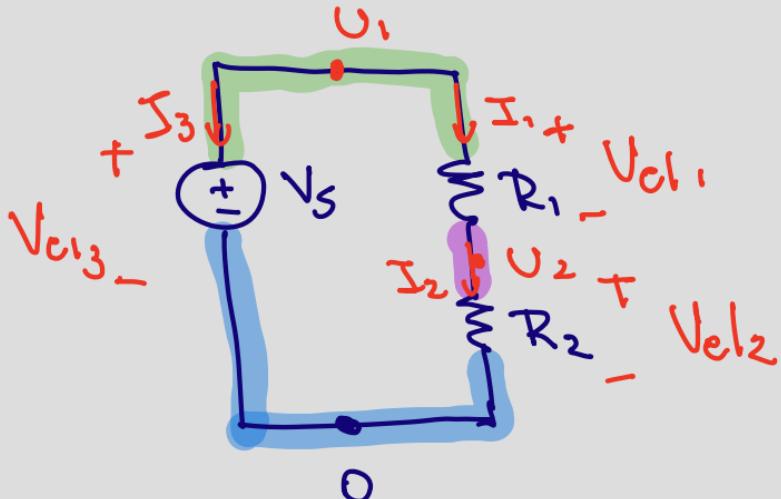


$$\vec{x} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ V_1 \\ V_2 \end{bmatrix}$$

Passive sign convention: the current enters at the positive terminal and exits at the negative terminal.

Node Voltage Analysis – Voltage Divider

Step 6 – Use KCL to fill in as many linearly independent rows in \vec{A} and \vec{b}



$$\text{For } V_{1,0} = I_1 + I_3 \quad (1)$$

$$\text{For } V_2 \Rightarrow I_1 = I_2$$

$$I_1 - I_2 = 0 \quad (2)$$

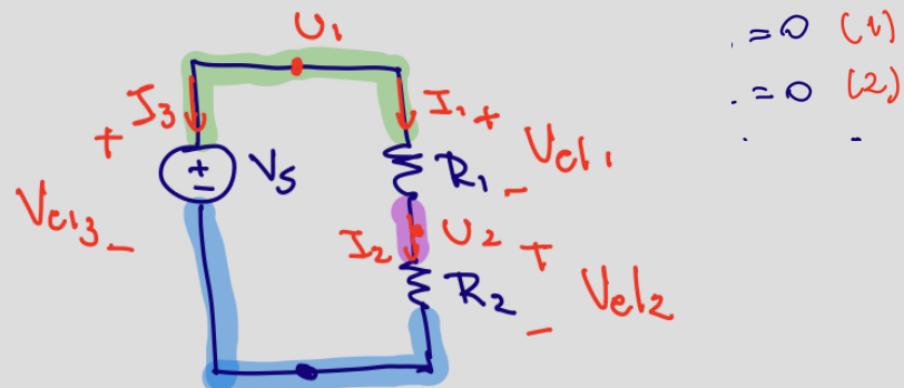
$$\vec{x} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ V_1 \\ V_2 \end{bmatrix}$$

I_2

KCL: the current flowing into a junction must equal the current flowing out of that junction.

Node Voltage Analysis – Voltage Divider

Step 7 – Use the IV relationships of each of the non-wire elements to fill in the remaining rows of A and the remaining rows of A are



$$= 0 \quad (1)$$

$$= 0 \quad (2)$$

$$\dots$$

Voltage Def.

$$V_{el1} = V_1 - V_2$$

$$V_{el2} = V_2 - 0 = V_2$$

$$V_{el3} = V_1 - 0 = V_1$$

Element JV

$$V_{el1} = R_1 I_1$$

$$V_{el2} = R_2 I_2$$

$$V_{el3} = V_s$$

Substitution:

$$El_1 : V_1 - V_2 = R_1 I_1 \Rightarrow R_1 I_1 - V_1 + V_2 = 0 \quad (3)$$

$$El_2 : V_2 = R_2 I_2 \Rightarrow R_2 I_2 - V_2 = 0 \quad (4)$$

$$El_3 : V_1 = V_s \quad (5)$$

Node Voltage Analysis – Voltage Divider

Step 8 – Solve the system of equations to determine values of unknown variables.

$$I_1 + I_3 = 0 \quad (1)$$

$$-I_1 + I_2 = 0 \quad (2)$$

$$R_1 I_1 - U_1 + U_2 = 0 \quad (3)$$

$$R_2 I_2 - U_2 = 0 \quad (4)$$

$$U_1 = V_s \quad (5)$$

$$\begin{matrix} A & \xrightarrow{\text{A}} \\ \left[\begin{array}{ccccc} 1 & 0 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 \\ R_1 & 0 & 0 & -1 & 1 \\ 0 & R_2 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 \end{array} \right] & \left[\begin{array}{c} I_1 \\ I_2 \\ I_3 \\ U_1 \\ U_2 \end{array} \right] = \left[\begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ V_s \end{array} \right] \end{matrix}$$

$$I_1 = \frac{V_s}{R_1 + R_2}$$

$$I_2 = \frac{V_s}{R_1 + R_2}$$

$$I_3 = -\frac{V_s}{R_1 + R_2}$$

$$U_1 = V_s$$

$$U_2 = \frac{R_2}{R_1 + R_2} \cdot V_s$$

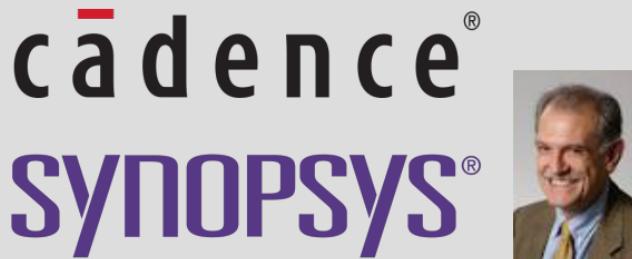
$\hookrightarrow \alpha L_1$

α is an operator

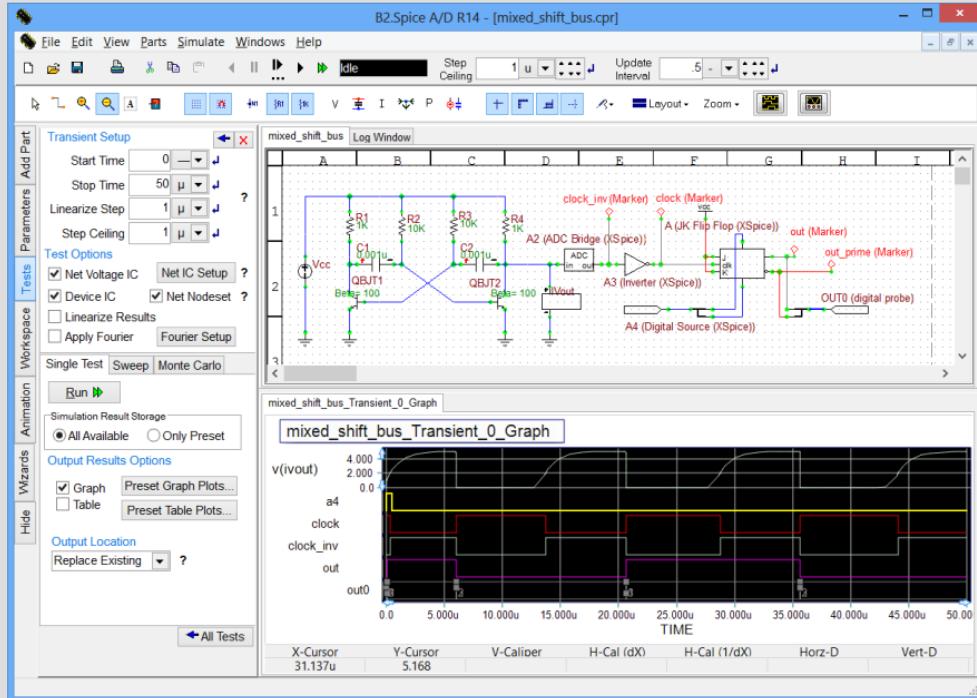
Electrical Circuit Analysis Algorithm (tool)

SPICE (Simulation Program with Integrated Circuit Emphasis): started as a student project at Berkeley!

Now the basis for open-source electronic circuit simulation, to design and model device characteristics and check circuit boards



Prof. Alberto L. Sangiovanni-Vincentelli



Electrical Circuit Analysis Algorithm (tool)

