Lecture 13

Monday, July 10, 2023 7:16 PM

Lecture 4D: (7/13/23)

Announcements:

- Lab - First Buffer Lab ← come if you've missed or not finished a lab so far
- Today's Topics:
  - Review of NVA
  - Physical Resistor (Note 12)
  - Power (Note 13)
  - Voltage/Current Measurement (Note 13)

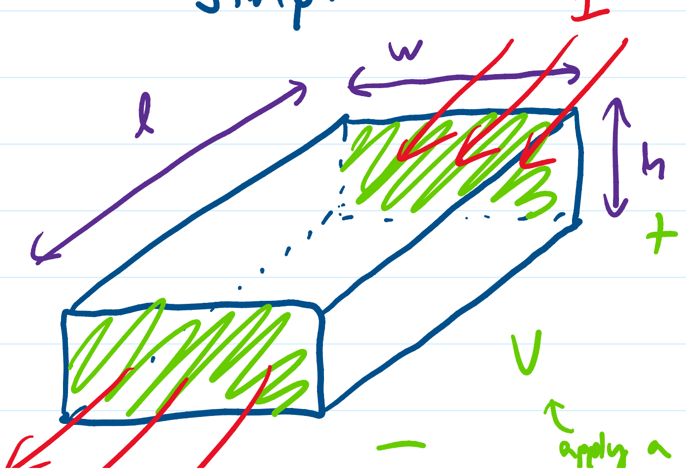
We are working towards a resistive touchscreen

1) Physics of Materials
2) Measurement

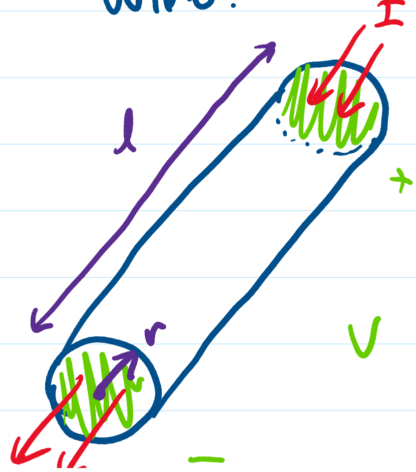
Touch Labs

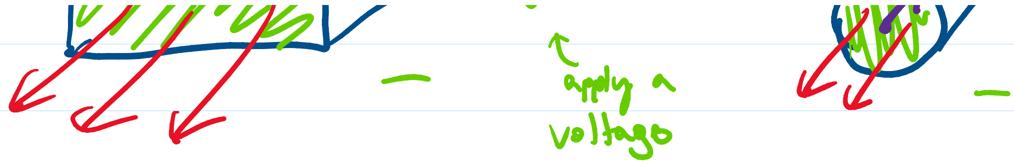
Physical Resistance:

Strip:



Wire:





$$R = \rho \frac{l}{wh} \longleftarrow R = \rho \cdot \frac{l}{A} \longrightarrow R = \rho \cdot \frac{l}{\pi r^2}$$

l: length (in direction of current flow)

ρ: resistivity

A: cross-sectional area (perpendicular to current flow)

$$[\Omega] = [\Omega \cdot m] \cdot \frac{[m]}{[m^2]}$$

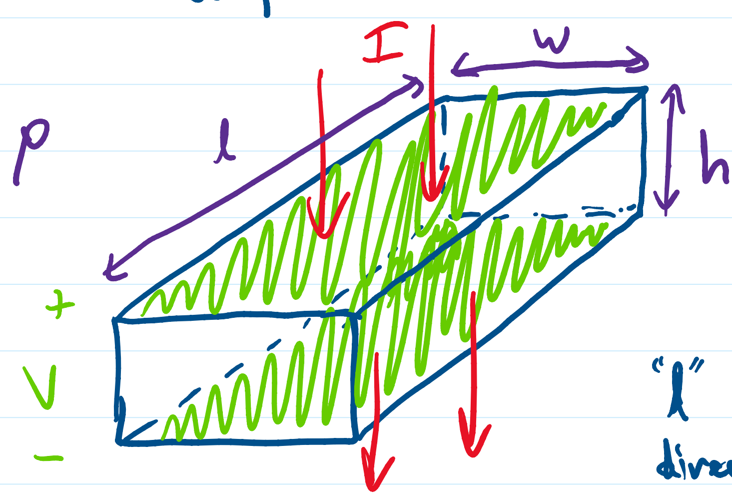
What is resistivity?

- Intrinsic material property
- Variable "ρ" (Greek Letter "rho")
- $\rho = \frac{1}{\sigma}$ ← conductivity (Greek Letter "sigma")

- More conductive ⟷ less resistive
- More resistive ⟷ less conductive

https://en.wikipedia.org/wiki/Electrical_resistivity_and_conductivity

Another Strip...



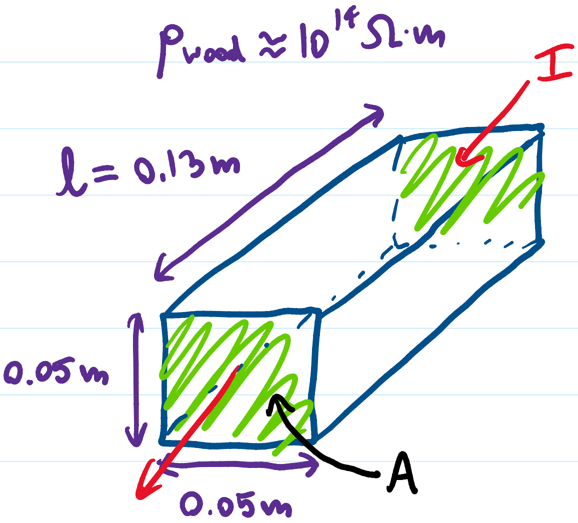
$R = ?$

$= \rho \cdot \frac{\text{"}l\text{"}}{\text{"}A\text{"}}$

$R = \rho \cdot \frac{h}{lw}$

"l" and "A" depend on direction of current flow

Demo with wooden blocks

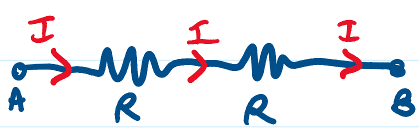


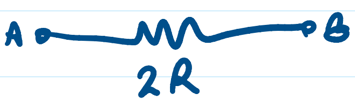
$$R = \rho \frac{l}{A} = (10^{14}\,\Omega m)\cdot\frac{(0.13m)}{(0.0025\,m^2)}$$
$$= 5.2\cdot 10^{15}\,\Omega$$

Back-to-back?

$$R_{tot} = \rho\frac{(2l)}{A} = 2R$$

Model:



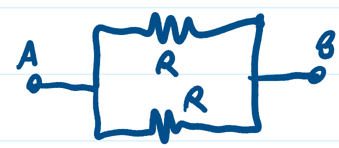


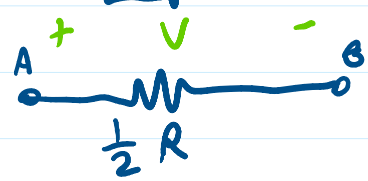
"series"-connected
(share the same current)

Side-to-side?

$$R_{tot} = \rho\frac{l}{(2A)} = \frac{1}{2}R$$

Model:

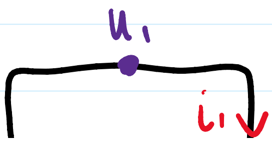




"parallel"-connected
(share the same voltage)

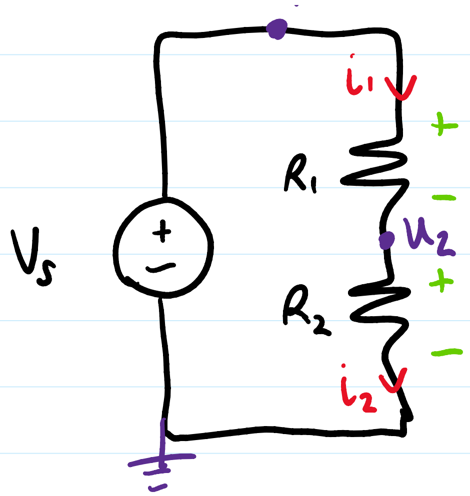
NVA Review: $\boxed{\text{Find the node voltages}}$

Ex). Voltage Divider



Step 1: Label reference node

Step 2: Label unknown node voltages



$$\boxed{\begin{aligned} u_1 &= V_s \\ u_2 &= \frac{R_2}{R_1+R_2}\cdot V_s \end{aligned}}$$

Step 1: Label reference node

Step 2: Label unknown node voltages

Step 3: Label currents

key unknowns

Step 4: Add +/− labels

Step 5: Identify unknowns (u_1, u_2, i_1, i_2)

Step 6a: KCL equations at unknown nodes

$u_1 = V_s$ (known) $\quad u_2 = ? \rightarrow i_1 - i_2 = 0$

Step 6b: IV characteristics

$u_1 - u_2 = i_1 R_1 \qquad u_2 - 0 = i_2 R_2$

Step 7: Substitute and solve

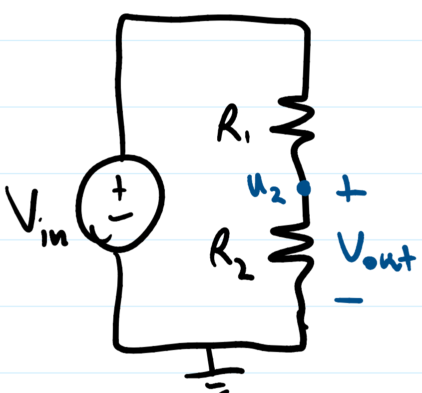
$$i_1 - i_2 = 0 \longrightarrow \frac{u_1 - u_2}{R_1} - \frac{u_2}{R_2} = 0$$

Solve equations:

$$u_1 = V_s$$

$$\frac{u_1 - u_2}{R_1} - \frac{u_2}{R_2} = 0 \rightarrow \frac{V_s}{R_1} - \frac{1}{R_1}u_2 - \frac{1}{R_2}u_2 = 0$$

Multiply by R_1 and R_2



$$V_s \cdot R_2 - R_2 u_2 - R_1 u_2 = 0$$

$$u_2 \cdot (R_1 + R_2) = R_2 \cdot V_s$$

$$u_2 = \underbrace{\frac{R_2}{R_1+R_2}}_{0 < x < 1} \cdot V_s$$

Why is it called a voltage divider?

Pick: $R_1 = 1\Omega$, $R_2 = 3\Omega$

$$V_{out} = \frac{R_2}{\underbrace{R_1 + R_2}_{\text{total resis}}} \cdot V_{in} = \frac{3}{1+3} \cdot V_{in} = \frac{3}{4} \cdot V_{in}$$

R_2

$v_{out} = \frac{}{\underbrace{R_1+R_2}} v_{in} = \frac{}{1+3} v_{in} = \frac{}{4} v_{in}$

↖ total series resistance

Power

Electricity performs two basic functions:

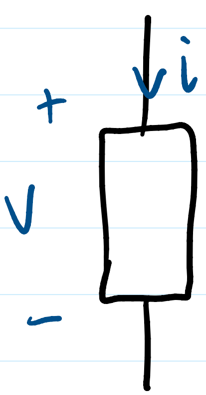
1) Transfer information
* 2) Transfer energy

It's really hard to store electrical energy

What is power? $P = \frac{\partial E}{\partial t}$ ← flow of energy

watt $[W] = \frac{[J]}{[s]}$ $\frac{\text{joule}}{\text{second}}$

Electrical power:

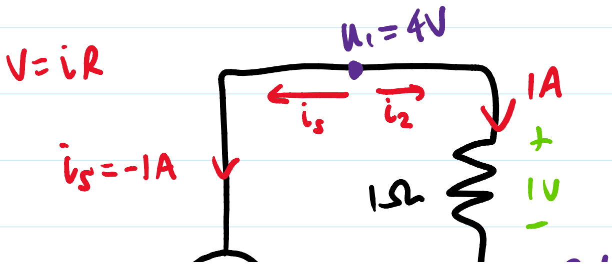


$$\boxed{P = v \cdot i} = \frac{\partial E}{\cancel{\partial q}} \cdot \frac{\cancel{\partial q}}{\partial t} = \frac{\partial E}{\partial t} \checkmark$$

↑ ALWAYS TRUE

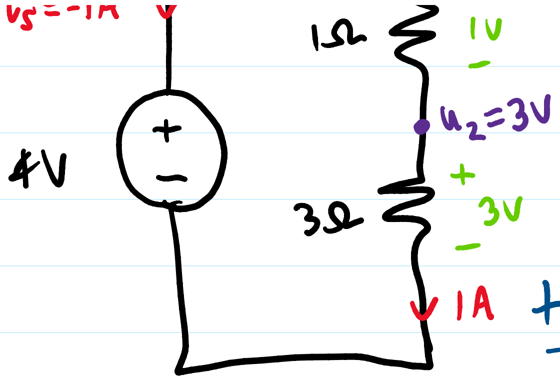
(passive sign convention) ↔ $\begin{cases} P > 0 \rightarrow \text{dissipates power} \\ P < 0 \rightarrow \text{delivers power} \end{cases}$

Let's consider our voltage divider example again



$V_s = 4V$, $R_1 = 1\Omega$, $R_2 = 3\Omega$

$u_2 = \frac{R_2}{R_1+R_2} V_s = \frac{3\Omega}{1\Omega + 3\Omega} \cdot 4V = 3V$



$v_2 = \frac{R_2}{R_1+R_2} V_s = \frac{3\Omega}{1\Omega + 3\Omega} \cdot 4V = 3V$

$P_s = V_s \cdot i_s = (4V)\cdot(-1A) = -4W$

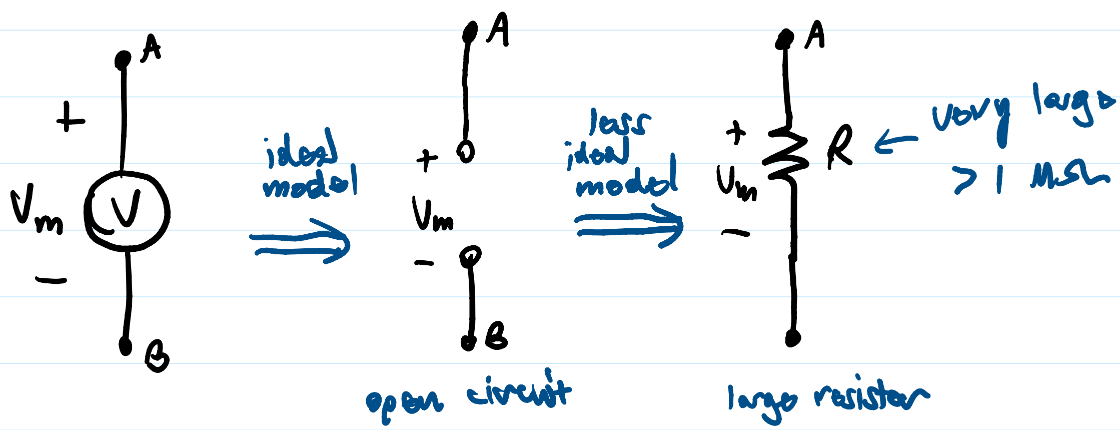
$P_{1\Omega} = (1V)\cdot(1A) = +1W$

$+\ P_{3\Omega} = (3V)(1A) = +3W$

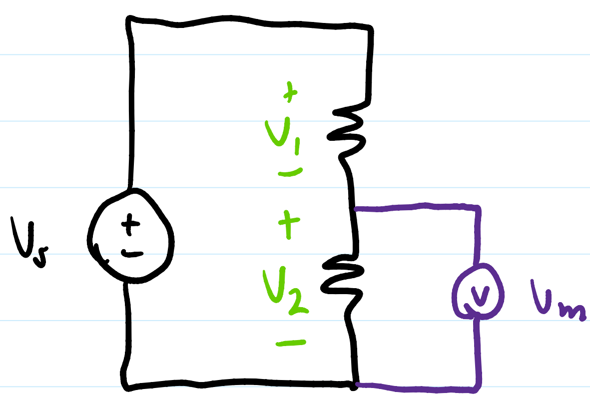
Conservation of energy/power: $\boxed{P_{total} = 0W}$

Measuring Voltage and Current

voltmeter: device which measures voltage <u>across</u> it



Ex).



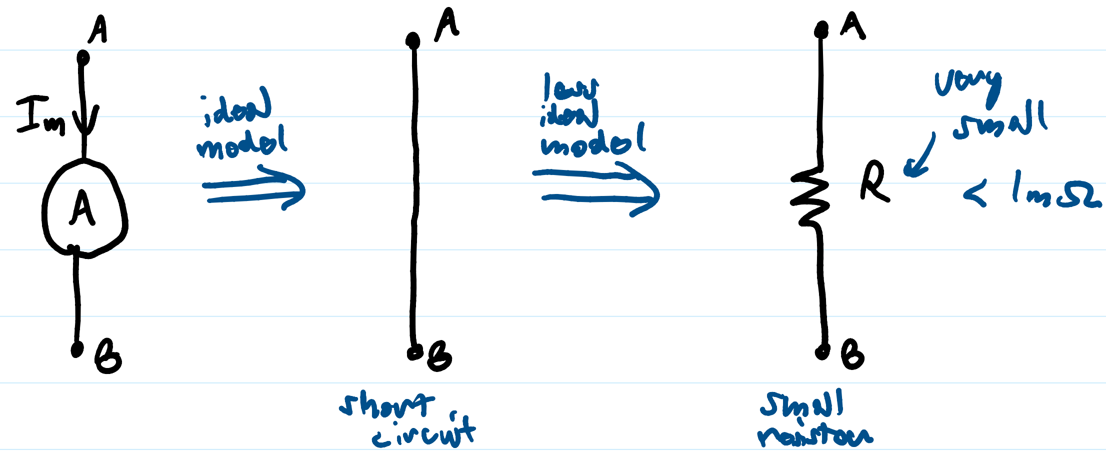
$V_m = V_2$

If voltmeter is open circuit then

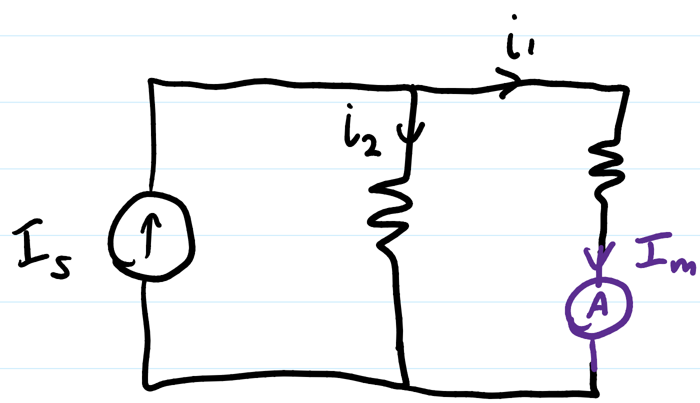
$p = v \cdot \underset{=0}{i} = 0$ ← no energy loss

$= 0$ energy loss

ammeter: device which measures current (amps) <u>through</u> it



Ex).



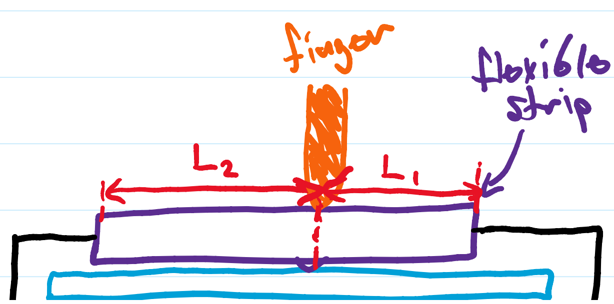
$I_m = i_1$

If ammeter is short circuit, then

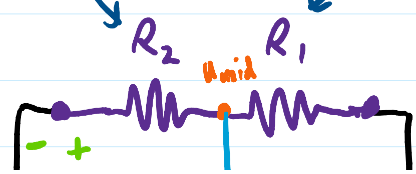
$p = v \cdot i = 0$ ← no energy loss

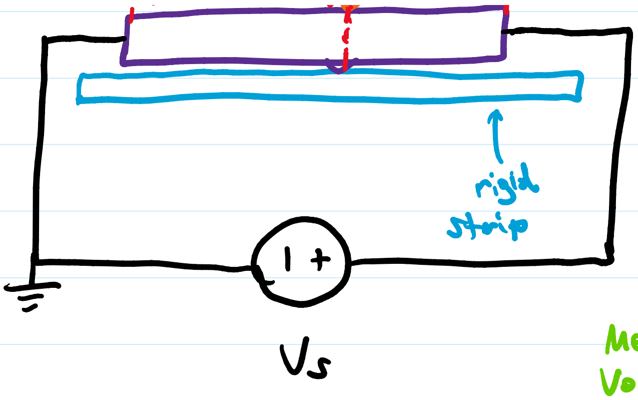
$v = 0$

1D Resistive Touch

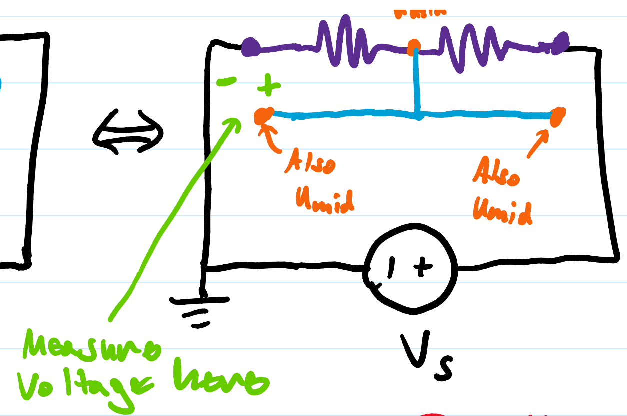


$R_2 = \rho \frac{L_2}{A}$ $R_1 = \rho \frac{L_1}{A}$





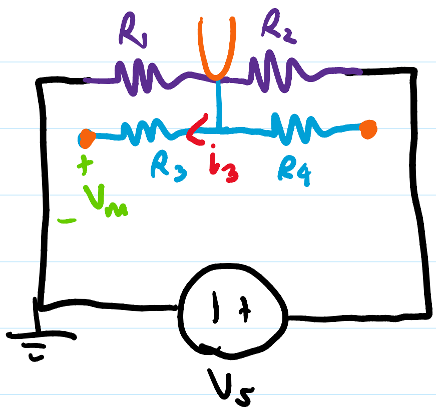
$\Leftrightarrow$



$$u_{mid} = \frac{R_2}{R_1+R_2} V_s = \frac{\left(\rho \frac{L_2}{A}\right)}{\left(\rho \frac{L_1}{A}\right)+\left(\rho \frac{L_2}{A}\right)} V_s = \frac{L_2}{L_1+L_2} V_s = \underbrace{\frac{L_2}{L_{total}}}_{\text{relative position}} \cdot V_s$$

How can we actually measure u_{mid}?

Use rigid layer ← Doesn't this have resistance too? :(

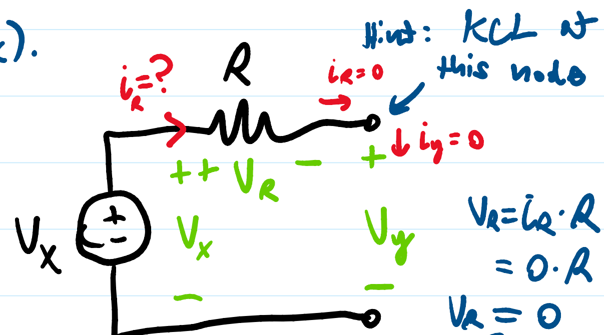


Does measured V_m change because of R_3 and R_4?

How much current flows through R_3? $i_3 = 0$!

$\boxed{\text{Current flow requires closed loops}}$

Ex).

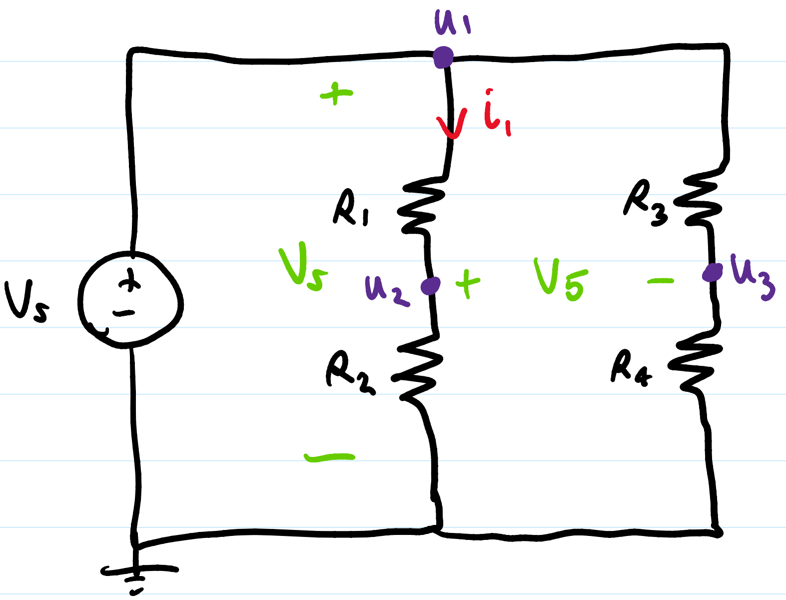


$V_R = i_R \cdot R$
$= 0 \cdot R$
$V_R = 0$

no voltage drop!

The resistance of the rigid layer is inconsequential

Interesting Circuit:



What is u_2?

$$u_2 = \frac{R_2}{R_1+R_2} V_s$$

Add second branch:

What is u_3?

$$u_3 = \frac{R_4}{R_3+R_4} V_s$$

Does u_2 change?

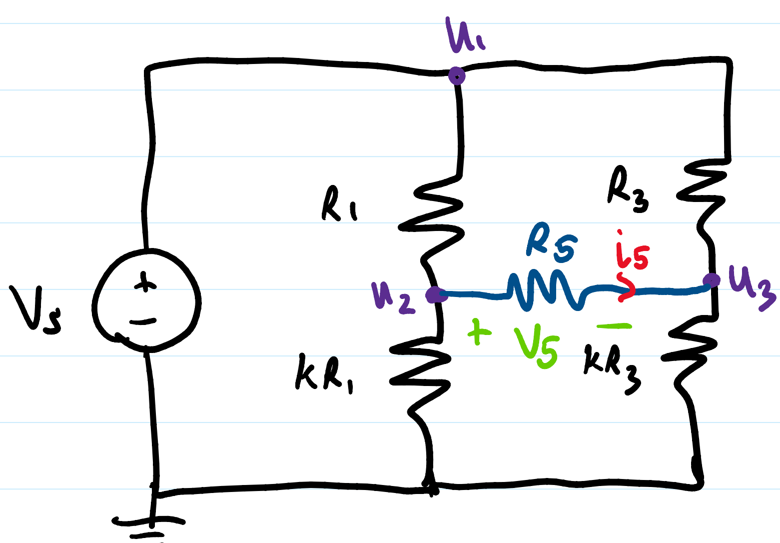
(Does i_1 change?)

No

$$i_1 = \frac{V_s}{R_1+R_2} \qquad u_2 = i_1 \cdot R_2 = \frac{R_2}{R_1+R_2} V_s$$

If you add (or subtract) to the circuit, the circuit voltages and currents likely change. ← Need to check

What if $R_2 = k \cdot R_1$ and $R_4 = k \cdot R_3$?



$$u_2 = \frac{kR_1}{R_1+kR_1} V_s \; \boxed{= \frac{k}{1+k} V_s}$$

$$u_3 = \frac{kR_3}{R_3+kR_3} V_s \; \boxed{= \frac{k}{1+k} V_s}$$

The same!

$$\boxed{u_2 = u_3}$$

$$i_5 = \frac{V_5}{R_5} = \frac{(u_2 - u_3)}{R_5} = 0$$

Let's add a resistor R_5 between u_2 and u_3.

What's the current through this resistor?

Foolproof way $\Rightarrow$ Perform full NVA

Sneaky way $\Rightarrow$ Do u_2 and u_3 change?

Before: $V_5 = u_2 - u_3 = 0$

After: $V_5 = u_2 - u_3 = 0$

Trick: If you connect a resistor between two nodes with the same voltage, then it does not change the circuit $\leftarrow$ "virtual open circuit"

since $\underbrace{i=0}_{\text{I-V characteristic of open circuit}}$ ALWAYS