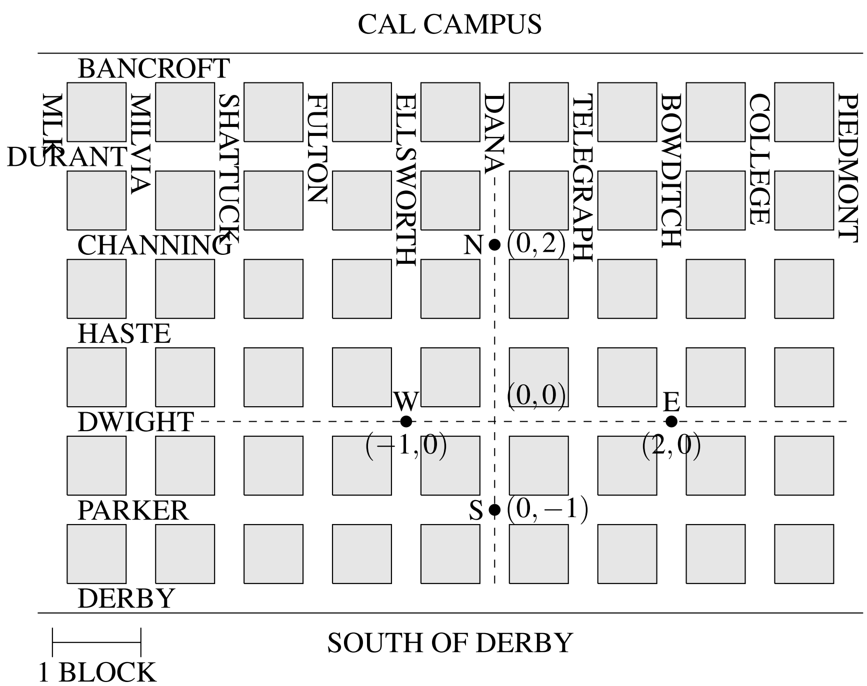
EECS 16A Designing Information Devices and Systems I
Summer 2023 Discussion 07B

1. Search and Rescue Cats

Berkeley's Kitten Shelter needs your help! While S'more the Kitten was being walked, the volunteer let go of her leash and she is now running wild in the streets of Berkeley (which are quite dangerous)! Thankfully, all of the kittens at the shelter have a collar that sends a Bluetooth signal to receiver towers, which are spread throughout the streets (pictured below). If the kitten/collar is within range of the receiver tower, the collar will send the tower a message: the distance of the collar to the tower. Each cell tower has a range of five city blocks. Can you help the shelter locate their lost kitten?

Note: A city block is defined as the middle of an intersection to the middle of an adjacent intersection (scale provided on map). S'more is constrained to running wild in the streets, meaning she won't be found in any buildings. If your TA asks 'Where is S'more?' it is sufficient to answer with her intersection or 'between these two intersections.'



(a) You check the logs of the cell towers, and they have received the following messages:

Sensor	Distance
N	5
W	$\sqrt{20}$
E	$\sqrt{5}$
S	$\sqrt{10}$

On the map provided above, identify where S'more is!

(b) Can you set this up as a system of equations? Are these equations linear? If not, can these equations be linearized? If you can linearize these equations, **write down a simplified form of your set of equations.**

Hint: Set $(0,0)$ to be Dwight and Dana.

Hint 2: Distance $= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

Hint 3: You don't need all 4 equations. You have two unknowns, x and y. You know from lecture that you need at least three circles to uniquely find a point on a 2D plane. How can you use the third circle/equation to get two equations and two unknowns?

Note: Remember to check for consistency for all nonlinear equations after finding the coordinates.

(c) Suppose S'more is moving fast, and by the time you get to destination in part (a) she's already run off! You check the logs of the cell towers again, and see the following updated messages:

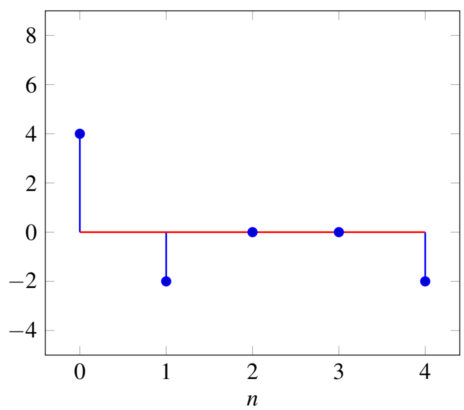
Sensor	Distance
N	5
W	$\sqrt{20}$
E	Out of Range
S	Out of Range

Can you find S'more? With a system of linear equations? Other methods? If so, on the map provided above, identify where S'more is!

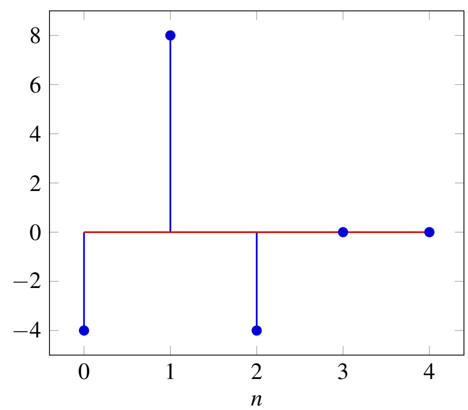
2. Correlation

(a) Sketch the linear cross-correlation of Signal 1 with Signal 2. That is, find: $\text{corr}(\vec{s}_1, \vec{s}_2)[k]$ for $k = 0, 1, .., 4$. Do not assume the signals are periodic.

Signal 1, $\vec{s}_1[n]$



Signal 2, $\vec{s}_2[n]$



$k = 0$:

n	0	1	2	3	4	5	6	7	8
$\vec{s}_1[n]$									
$\vec{s}_2[n]$									
$\langle \vec{s}_1[n], \vec{s}_2[n] \rangle$	+	+	+	+	+	+	+	+	=

$k = 1$:

$\vec{s}_1[n]$									
$\vec{s}_2[n-1]$									
$\langle \vec{s}_1[n], \vec{s}_2[n-1] \rangle$	+	+	+	+	+	+	+	+	=

$k = 2$:

$\vec{s}_1[n]$									
$\vec{s}_2[n-2]$									
$\langle \vec{s}_1[n], \vec{s}_2[n-2] \rangle$	+	+	+	+	+	+	+	+	=

$k = 3$:

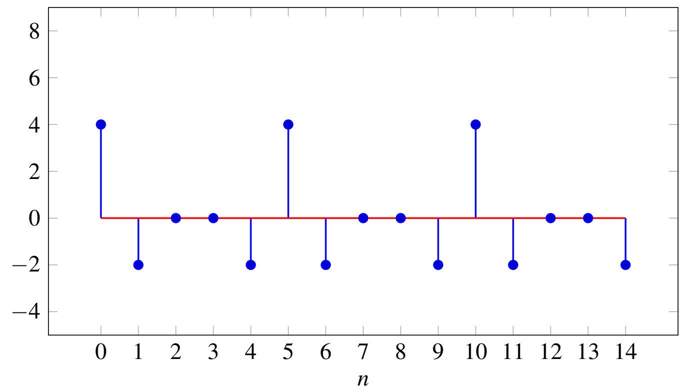
$\vec{s}_1[n]$									
$\vec{s}_2[n-3]$									
$\langle \vec{s}_1[n], \vec{s}_2[n-3] \rangle$	+	+	+	+	+	+	+	+	=

$k = 4$:

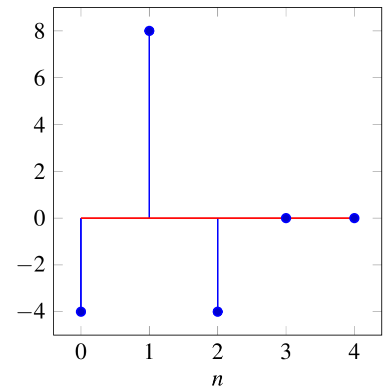
$\vec{s}_1[n]$									
$\vec{s}_2[n-4]$									
$\langle \vec{s}_1[n], \vec{s}_2[n-4] \rangle$	+	+	+	+	+	+	+	+	=

(b) Now, the pattern in $\vec{s}_1$ is repeated three times and we call this Signal 3

Signal 3, $\vec{s}_3[n]$



Signal 2, $\vec{s}_2[n]$



Sketch the linear cross-correlation of Signal 3 with Signal 2, $\mathrm{corr}(\vec{s}_3, \vec{s}_2)[k]$, for $k = 0, 1, .., 4$.

$k = 0$:

n	0	1	2	3	4	5	6	7	8	9	
$\vec{s}_3[n]$											
$\vec{s}_2[n]$											
$\langle \vec{s}_3[n], \vec{s}_2[n] \rangle$		+	+	+	+	+	+	+	+	+	=

$k = 1$:

$\vec{s}_3[n]$											
$\vec{s}_2[n-1]$											
$\langle \vec{s}_3[n], \vec{s}_2[n-1] \rangle$		+	+	+	+	+	+	+	+	+	=

$k = 2$:

$\vec{s}_3[n]$											
$\vec{s}_2[n-2]$											
$\langle \vec{s}_3[n], \vec{s}_2[n-2] \rangle$		+	+	+	+	+	+	+	+	+	=

$k = 3$:

$\vec{s}_3[n]$											
$\vec{s}_2[n-3]$											
$\langle \vec{s}_3[n], \vec{s}_2[n-3] \rangle$		+	+	+	+	+	+	+	+	+	=

$k = 4$:

$\vec{s}_3[n]$											
$\vec{s}_2[n-4]$											
$\langle \vec{s}_3[n], \vec{s}_2[n-4] \rangle$		+	+	+	+	+	+	+	+	+	=