EECS 16A Designing Information Devices and Systems I Summer 2023 Discussion 2C

1. Span Basics

(a) What is span
$$\left\{ \begin{bmatrix} 1\\2\\0 \end{bmatrix}, \begin{bmatrix} 2\\1\\0 \end{bmatrix} \right\}$$
?

(b) Is
$$\begin{bmatrix} 5 \\ 5 \\ 0 \end{bmatrix}$$
 in span $\left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} \right\}$?

(c) What is a possible choice for \vec{v} that would make span $\left\{ \begin{bmatrix} 1\\2\\0 \end{bmatrix}, \begin{bmatrix} 2\\1\\0 \end{bmatrix}, \vec{v} \right\} = \mathbb{R}^3$?

(d) For what values of b_1 , b_2 , b_3 is the following system of linear equations consistent? *Note: "Consistent" means there is at least one solution.*

$$\begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 0 & 0 \end{bmatrix} \vec{x} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

2. Span Proofs

Given some set of vectors $\{\vec{v}_1, \vec{v}_2, \dots, \vec{v}_n\}$, show the following:

(a)
$$\mathrm{span}\{\vec{v}_1,\vec{v}_2,\ldots,\vec{v}_n\}=\mathrm{span}\{\alpha\vec{v}_1,\vec{v}_2,\ldots,\vec{v}_n\}, \text{ where } \alpha \text{ is a non-zero scalar}$$

In other words, we can scale our spanning vectors and not change their span.