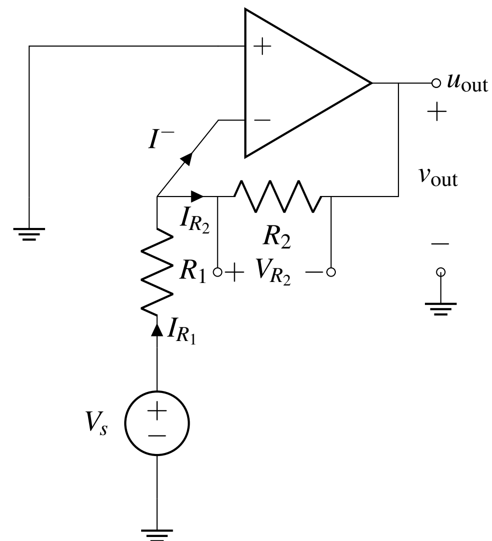
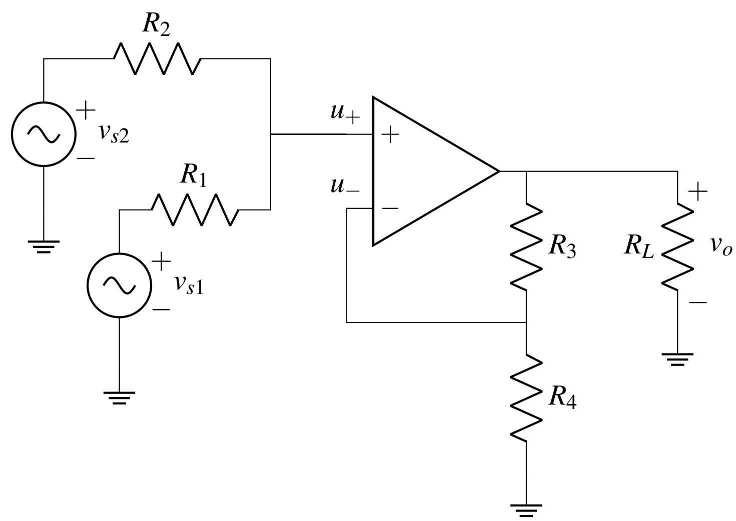
EECS 16A Designing Information Devices and Systems I
Summer 2023 Discussion 06C

1. An Inverting Amplifier



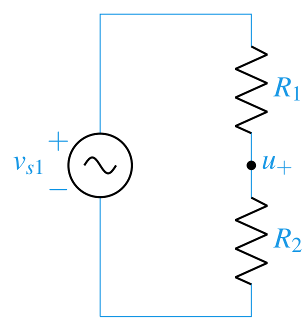
Calculate v_{out} as a function of V_s and R_1 and R_2.

2. Multiple Inputs To One Op-Amp



(a) First, let's focus on the left part of the circuit containing the voltage sources v_{s1} and v_{s2}, and resistances R_1 and R_2. Solve for u_+ in the circuit above. *(Hint: Use superposition.)*

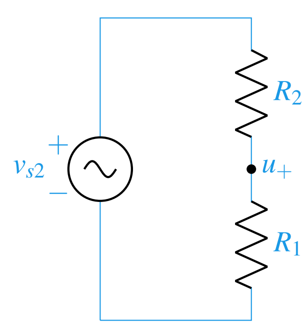
Answer: Let's call the potential at the positive input of the op-amp u_+. Using superposition, we first turn off v_{s2} and find u_+. The circuit then looks like:



We recognize the above circuit as a voltage divider. Thus,

$$u_{+,vs1} = \frac{R_2}{R_1 + R_2} v_{s1}$$

By symmetry, we expect v_{s2} to have a similar circuit and expression. The circuit for v_{s2} looks like:



The expression for u_+ with v_{s2} is then:

$$u_{+,vs2} = \frac{R_1}{R_1+R_2} v_{s2}$$

From superposition, we know the output must be the sum of these.

$$u_+ = \frac{R_2}{R_1+R_2} v_{s1} + \frac{R_1}{R_1+R_2} v_{s2}$$

(b) How would you choose R_1 and R_2 that produce a voltage $u_+ = \frac{1}{2}V_{s1} + \frac{1}{2}V_{s2}$? Could you also achieve $u_+ = \frac{1}{3}V_{s1} + \frac{2}{3}V_{s2}$?

Answer:

We found that the output voltage for any two resistors R_1 and R_2 is:

$$v_+ = \frac{R_2}{R_1+R_2} V_1 + \frac{R_1}{R_1+R_2} V_2$$

Thus, to create the $\frac{1}{2}$ - $\frac{1}{2}$ ratio, we can use any nonzero resistances with value R such that:

$$R_1 = R \qquad R_2 = R$$

Similarly, to create the $\frac{1}{3}$ - $\frac{2}{3}$ ratio, we can use:

$$R_1 = 2R \qquad R_2 = R$$

In general, you can achieve anything of the form $u_+ = kV_1 + (1-k)V_1$ with $k \in (0,1)$! This is a very useful topology to combine two voltages.

(c) Now, for the whole circuit, find an expression for v_o.

Answer:

With u_+ determined, we can find the output voltage directly from the formula for a non-inverting amplifier. We can also derive it using the process below.

From the negative feedback rule, $u_+ = u_-$. Using voltage dividers, we can express u_- in terms of v_o:

$$u_- = \frac{R_4}{R_3+R_4} v_o$$

$$v_o = \left(1 + \frac{R_3}{R_4}\right) u_- = \left(1 + \frac{R_3}{R_4}\right) u_+$$

Now, to find the final output, we can set u_+ to our earlier expression.

$$v_o = \left(1 + \frac{R_3}{R_4}\right)\left(\frac{R_2}{R_1+R_2} v_{s1} + \frac{R_1}{R_1+R_2} v_{s2}\right)$$

(d) How should we select our values R_1, R_2, R_3, R_4 to find the sum of different signals, i.e. $V_{s1} + V_{s2}$? What about taking the sum and multiplying by 2, i.e. $2(V_{s1} + V_{s2})$?

Answer:

The circuit already finds the weighted sum of two inputs. By setting $R_1 = R_2$ and $R_3 = R_4$, we can take the exact sum of two inputs.

$$v_o = \left(1 + \frac{R_3}{R_4}\right)\left(\frac{R_2}{R_1+R_2} v_{s1} + \frac{R_1}{R_1+R_2} v_{s2}\right) = (1+1)\left(\frac{1}{2} v_{s1} + \frac{1}{2} v_{s2}\right) = v_{s1} + v_{s2}$$

Notice that the first half of this circuit (R_1 and R_2) form a voltage summer with coefficients less than one; the second half is just a non-inverting amplifier. Thus we can always use R_1 and R_2 to take an equally weighted sum of the inputs and then multiply greater than 1 using the non-inverting amplifier. If we set $R_1 = R_2$, we get $\left(\frac{1}{2}v_{s1} + \frac{1}{2}v_{s2}\right)$ into the op-amp. To get a total gain of 2, then the non-inverting op-amp needs a gain of 4, so we can pick $R_3 = 3R_4$.