EECS 16A Designing Information Devices and Systems I
Summer 2023 Pre-Lab Reading APS

1 APS Overview

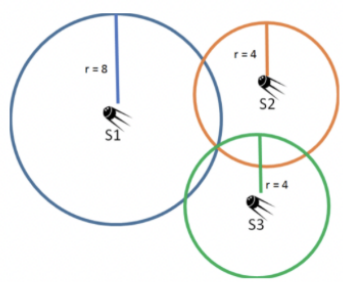
1.1 Introduction

We use various positioning systems every day. The most commonly used system is the Global Positioning System (GPS). GPS is used in cars, Google Maps, cell phones, and even rockets and spaceships. GPS is a technology that allows you to determine your precise location anywhere on Earth. GPS receivers make use of the fact that light propagates at a known speed: $3 \times 10^8 \frac{m}{s^2}$. A receiver can therefore compute its distance to the satellite by measuring how long it takes the GPS signal - a radio wave - to travel from the satellite to the receiver, using the speed of light and the known time that the signal was broadcasted from the satellite.

In this lab, we will explore a similar idea - the Acoustic Positioning System (APS). APS uses sound waves instead of radio waves as signals. In the lab, we will be using speakers as signal emitters to emit the sound waves, and microphones as receivers. We can determine the distance to each emitter based on the delay between the times at which we receive each signal. We know the location of each satellite and what beacon signal each satellite is playing. The goal of the lab is to figure out the **location of receiver**!

1.2 Trilateration

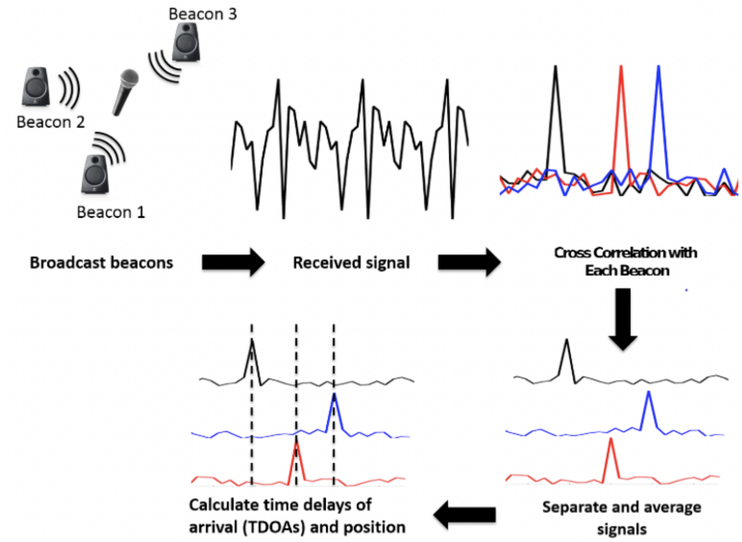
Trilateration is a technique used to determine the location of an object by measuring the distances between that object and three or more reference points.



The time (called time delay) of arrival of the signal from an emitter is given by the following formula: time $= \frac{\#samples}{f_s} = \frac{\#samples}{\frac{\#samples}{seconds}} =$ seconds, where f_s is the sampling frequency of the receiver

1.3 System Overview

The block diagram below describes the system at a high level. First, each speaker transmits a unique signal. The combined signal from all speakers is then recorded by the microphone and converted to a format that is convenient for further processing. Next, the component signal from each speaker is identified from the received total signal using cross-correlation analysis, and the corresponding time differences are used to determine the receiver's location.



1.4 Math Review

1.4.1 Inner (Dot) product

The dot product of two vectors computes how similar two vectors are. The inner product of two vectors is given by the following two equivalent formulas:

$$\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + \cdots + a_n b_n = \sum_{i=1}^{n} a_i b_i$$

$$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos\theta$$

From the above formula, we can see that the similarity (dot product) of two vectors is maximized when $\cos\theta$ is maximized.

1.4.2 Cross-correlation

Cross-correlation is a mathematical tool for finding similarities between signals. Computes dot product between r and signal BA shifted by k samples:

$$corr_r(B_A)[k] = \sum_{i=-\infty}^{\infty} r[i] B_A[i-k] \Leftrightarrow$$

In Python:
`cross_correlation(r, B_A)[k]`

We can use cross-correlation by comparing each **known** signal with the **received** signal along the time axis to figure out when each signal arrived.

1.5 Least Squares

Least squares is used to find the best-fit line or curve that describes the relationship between two or more variables. It's used when there is no exact solution to a system of equations, or when there is more data than unknowns, and it minimizes the sum of the squared errors between the predicted values and the actual values. $\hat{\mathbf{x}} = (\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\mathbf{b}$, where $\hat{\mathbf{x}}$ is the vector of coefficients that minimizes the sum of the squared errors, $\mathbf{A}$ is the overdetermined matrix of predictor variables, and $\mathbf{b}$ is the vector of response variables.
Please refer to Note 23 for more detailed information about least squares.

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