EECS 16A Designing Information Devices and Systems I
Summer 2023 Homework 5

This homework is due Friday, July 21, 2023 at 23:59.
Self-grades are due Friday, July 28, 2023 at 23:59.

Submission Format

Your homework submission should consist of **one** file.

We strongly recommended that you submit your self-grades PRIOR to taking the midterm on July 24, 2023, since looking at the solutions earlier will help you to study for the midterm.

Mid Semester Survey

Please fill out the mid semester survey: https://forms.gle/XKNPXWDidcsoM7LB9.

We highly appreciate your feedback!

1. **Reading Assignment**

For this homework, please read Note 12, Note 13, Note 14, Note 15, and Note 16. Notes 12 and 13 cover voltage dividers, how a simple 1-D resistive touchscreen works, the physics of circuits, and introduces the notion of power in electric circuits. Note 14 introduces better, but more complex models for the resistive touchscreen. Note 15 covers superposition and equivalence, two techniques to simplify circuit analysis. Note 16 will provide an introduction to capacitors (a circuit element which stores charge), capacitive equivalence, and the underlying physics behind them.

(a) Describe the key ideas behind how the 1D touchscreen works. In general, why is it useful to be able to convert a "physical" quantity like the position of your finger to an electronic signal (i.e. voltage)?

(b) For the touch screen model introduced in Note 14, why can't we simultaneously get the horizontal and vertical position of the touch with a single measurement? *Think about how many unknowns there are.*

(c) Explain the connection between the "superposition" you learned about in Note 15 with the "superposition" you learned back in module 1 in the context of linear functions.

(d) Describe the short-circuit test to find the Norton equivalent circuit. This test allows us to determine what voltage/current?

(e) How do we calculate the equivalent capacitance of series and parallel capacitors? Compare this with how we calculate resistor equivalences.

Solution:

a) The 1D touchscreen works because of the bottom plate that is touched by the top plate when a finger touches the top plate, creating another node. Converting a physical quantity into an electronic signal helps us model phenomena in the real, physical world.

b) Since we have two unknowns—vertical and horizontal positions—we also need two measurements/equations. Thus, for every touch we need to measure the voltage, change the voltage/ground configuration, and measure the voltage again (not super convenient :( ).

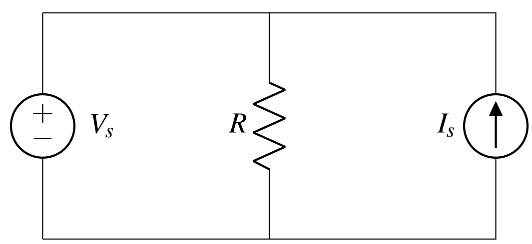
c) They are the same idea! We can view the circuits we have seen thus far as linear functions where the inputs are the sources, and the outputs are the unknown circuit quantities. With this view, we know by superposition that we can construct the output for a particular set of input sources by getting the output for each individual source first, then linearly recombining these outputs. That is exactly the superposition introduced in Note 15!

d) The short circuit test connects a short circuit across the terminals of a circuit. The current through this short circuit will be the Norton current I_{no}.

d) Capacitors in parallel can be combined into an equivalent capacitance that is the sum of the individual capacitance (just like resistors in **series**). Capacitors C_1, C_2 in series can be combined into an equivalent capacitance of $\frac{C_1 C_2}{C_1+C_2}$ (just like resistors in **parallel**).

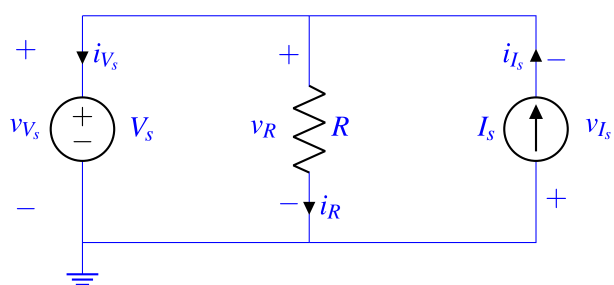
2. Power Analysis

Learning Goal: *This problem aims to help you practice calculating power dissipation in different circuit elements. It will also give you insights into how power is conserved in a circuit.*



(a) Find expressions for power dissipated by the voltage source (P_{V_s}), the current source (P_{I_s}), and the resistor (P_R) in the circuit above. Remember to label voltage-current pairs using passive sign convention.

Solution: We label a reference node, and then solve for the currents i_V, i_R and the voltages V_R, V_I.



Solving the above circuit using nodal analysis, we get

$$i_R = \frac{V_s}{R}$$

$$i_{V_s} = I_s - \frac{V_s}{R}$$

$$v_{I_s} = -V_s$$

$$v_R = V_s$$

Using this we can calculate

$$P_{V_s} = V_{V_s} \cdot i_V = I_s \cdot V_s - \frac{{V_s}^2}{R}$$

$$P_{I_s} = i_{I_s} \cdot v_{I_s} = -I_s \cdot V_s$$

$$P_R = i_R \cdot v_R = \frac{V_s{}^2}{R}$$

Note that $P_{V_s} + P_I + P_R = 0$, i.e. energy provided is energy dissipated, which verifies our intuition about conservation of energy.

(b) Use $R = 5$ kΩ, $V_s = 5$ V, and $I_s = 10$ mA. Calculate the power dissipated by each element. What does a negative value of dissipated power represent? Additionally compute the total power dissipated in all elements.

Solution:

$P_{V_S} = (0.01\text{A})(5\text{V}) - \frac{(5\text{V})^2}{5000\Omega} = 0.045W$
$P_{I_s} = -(0.01\text{A})(5\text{V}) = -0.05\text{W}$
$P_R = \frac{(5\text{V})^2}{5000\Omega} = 0.005\text{W}$

A negative value of dissipated power means the element is *delivering* power.

The total power dissipated in all elements is $P_{V_S} + P_I + P_R = 0$.

(c) Once again, let $R = 5$ kΩ, $V_s = 5$ V. What does the value I_s of the current source have to be such that the current source **dissipates** 40 mW? Note that it is possible for a current source to *dissipate* power, i.e. under passive sign convention, $P_{I_s} = +40\text{mW}$. For this value of I_s, compute P_{V_s} and P_R as well.

Hint: If the current source were delivering *power, under passive sign convention the computed power would have been* $P_{I_s} = -40\text{mW}$*, but this is NOT what the question is asking.*

Solution:

Remember that using passive sign convention, an element whose power is negative is supplying power, and an element whose power is positive is dissipating power. Therefore, we want $P_{I_s} = 40\text{mW}$. We know that $P_{I_s} = -I_s \cdot V_s$. Therefore, $I_s = -\frac{0.04\text{W}}{5\text{V}} = -0.008\text{A}$.

$P_{V_s} = (-0.008\text{A})(5\text{V}) - \frac{(5\text{V})^2}{5000\Omega} = -0.045W$
$P_{I_s} = -(-0.008\text{A})(5\text{V}) = 0.04\text{W}$
$P_R = \frac{(5\text{V})^2}{5000\Omega} = 0.005\text{W}$

Note that still $P_{V_S} + P_I + P_R = 0$.

3. Cell Phone Battery

As great as smartphones are, one of their drawbacks is that their batteries don't last a long time. For example, a typical smartphone, under average usage conditions (internet, a few cat videos, etc.) uses 0.3W of power. We will model the battery as an ideal voltage source (which maintains a constant voltage across its terminals regardless of current) except that we assume that the voltage drops abruptly to zero when the battery is discharged (in reality, the voltage drops gradually, but let's keep things simple).

Battery capacity is specified in mAh (this is a unit of charge), which indicates how many mA of current the battery can supply for one hour before it needs to be recharged. Suppose the phone's battery has a capacity of 2770mAh at 3.8V. For example, this battery could provide 1000mA (or $P = 1000\,\text{mA} \cdot 3.8\,\text{V} = 3.8\,\text{W}$) for $\frac{2770\text{mAh}}{1000\,\text{mA}} = 2.77$ hours before the voltage abruptly drops from 3.8V to zero.

(a) How long will the phone's full battery last assuming an average power usage of 300 mW?

Solution:

Using out power relation $P = IV$ we see that 300 mW of power at 3.8 V is about 79 mA of current. Our 2770 mA h battery can supply 79 mA for $\frac{2770\,\text{mA h}}{79\,\text{mA}} = 35\,\text{h}$, or about a day and a half.

(b) Suppose the cell phone battery is completely discharged and you want to recharge it completely. How much energy (in J) is this? Recall that a J is equivalent to a W s.

Solution:

The battery capacity is 2770 mA h at 3.8 V. Using $E = Pt = IVt = V(It)$ we see that the battery has a total stored energy of $3.8\,\text{V} \cdot 2770\,\text{mA h} = 10.5\,\text{W h} = 10.5\text{Wh} \cdot \frac{3600\,\text{s}}{1\,\text{h}} = 37.9\text{kJ}$.

(c) The battery has internal circuitry that prevents it from getting overcharged (and possibly exploding!). We will model the battery and its internal circuitry as a resistor R_{bat}. We now wish to charge the battery by plugging it into a wall plug. The wall plug can be modeled as a 5 V voltage source and 200 mΩ resistor, as pictured in Figure 1. What is the power dissipated across R_{bat} for $R_{\text{bat}} = 1\,\Omega$ (i.e. how much power is being supplied to the phone battery as it is charging) and how long will the battery take to charge?

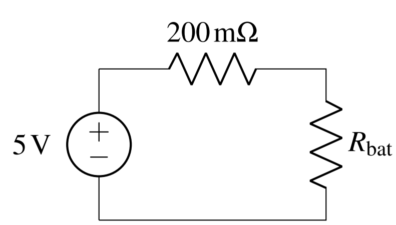


Figure 1: Model of wall plug, wire, and battery.

Solution:

As per the last part, the energy stored in the battery is 2770 mA h at 3.8 V, which is $2.77\,\text{A h} \cdot 3.8\,\text{V} = 10.5\,\text{W h}$. We can find the time to charge by dividing this energy by power dissipated across R_{bat} (in W) to get time in hours. To find the dissipated power, we first need to find the voltage across and current through R_{bat}. We can recognize this circuit as a voltage divider and so we can find the voltage across R_{bat} using our voltage divider equation:

$$V_{bat} = \frac{R_{bat}}{200\,\text{m}\Omega + R_{bat}} \cdot 5\,\text{V} = \frac{1\,\Omega}{1.2\,\Omega} \cdot (5\,\text{V}) = 4.167\,\text{V}$$

and the current via Ohm's law

$$I_{bat} = 4.167\,\text{V}/1\,\Omega = 4.167\,\text{A}$$

. With these we can use $P = IV$ to get the dissipated power across R_{bat}.
This gives us

$$P_{bat} = (4.167\,\text{V}) \cdot (4.167\,\text{A}) = 17.36\,\text{W}$$

and finally the time

$$t = \frac{E}{P_{bat}} = \frac{10.5\,\text{W h}}{17.36\,\text{W}} = 0.6\,\text{h}$$

or about 36 min.

4. Volt and ammeter

Learning Goal: *This problem helps you explore what happens to voltages and currents in a circuit when you connect voltmeters and ammeters in different configurations.*

Use the following numerical values in your calculations: $R_1 = 1\,\text{k}\Omega$, $R_2 = 2\,\text{k}\Omega$, $R_3 = 3\,\text{k}\Omega$, $R_4 = 4\,\text{k}\Omega$, $R_5 = 5\,\text{k}\Omega$, $V_s = 8V$.

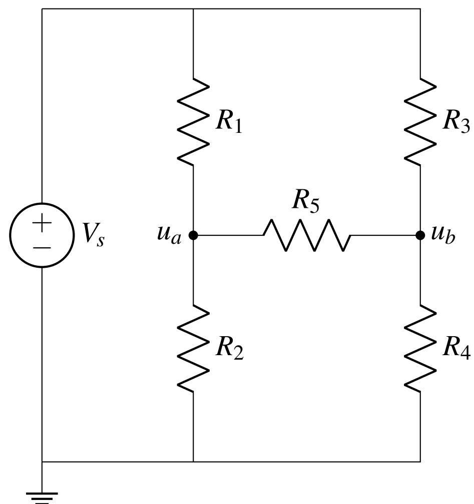
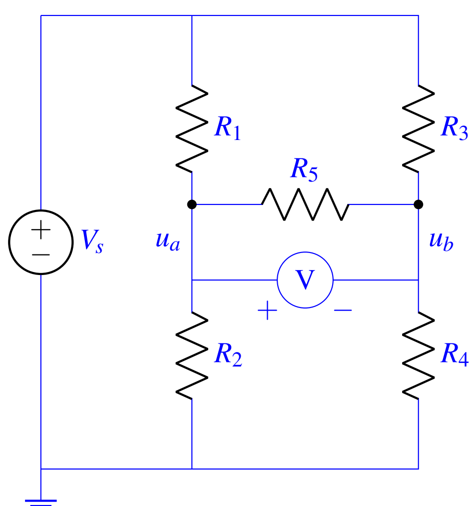


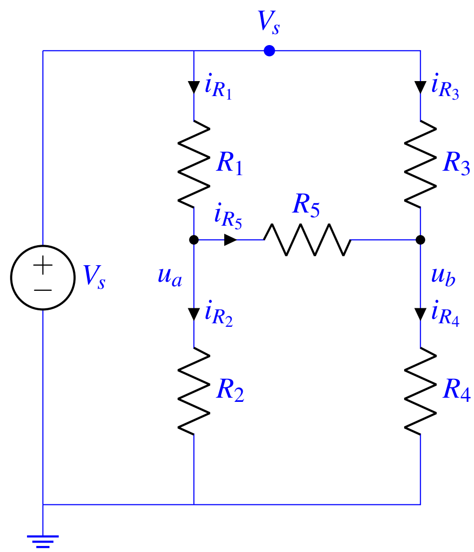
Figure 2: Circuit consisting of a voltage source V_s and five resistors R_1 to R_5

(a) Redraw the circuit diagram shown in Figure 2 by adding a voltmeter (letter V in a circle and plus and minus signs indicating direction) to measure voltage V_{ab} from node u_a (positive) to node u_b (negative). Calculate the value of V_{ab}. You may use a numerical tool such as IPython to solve the final system of linear equations.

Solution: Below is the redrawn circuit with the voltmeter. Note that it is also correct to have the voltmeter above R_5, as it will still be connected to the same nodes.



Using NVA analysis we need to label our nodes. u_a and u_b are already labelled. The topmost node has voltage V_s and the bottom most node is our reference. We also label the currents in each element.



Using KCL at node u_a and u_b, we find:

$$i_{R_1} - i_{R_5} - i_{R_2} = 0$$

$$i_{R_5} + i_{R_3} - i_{R_4} = 0$$

Let's substitute IV relationships into the previous equations.

$$\frac{V_s - u_a}{R_1} - \frac{u_a - u_b}{R_5} - \frac{u_a}{R_2} = 0$$

$$\frac{u_a - u_b}{R_5} + \frac{V_s - u_b}{R_3} - \frac{u_b}{R_4} = 0$$

Gathering the u_a and u_b terms:

$$\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_5}\right) u_a - \left(\frac{1}{R_5}\right) u_b = \frac{V_s}{R_1}.$$

$$-\left(\frac{1}{R_5}\right) u_a + \left(\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}\right) u_b = \frac{V_s}{R_3}.$$

Notice that we wrote our unknowns (u_a and u_b) on the left side of the equation. We can then represent this in matrix form as:

$$\begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_5} & -\frac{1}{R_5} \\ -\frac{1}{R_5} & \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5} \end{bmatrix} \begin{bmatrix} u_a \\ u_b \end{bmatrix} = \begin{bmatrix} \frac{V_s}{R_1} \\ \frac{V_s}{R_3} \end{bmatrix}$$

Plugging in the values we were given into the matrix above and using Gaussian elimination we can find the vector of unknowns.

$$\begin{bmatrix} u_a \\ u_b \end{bmatrix} = \begin{bmatrix} 5.265V \\ 4.748V \end{bmatrix}$$

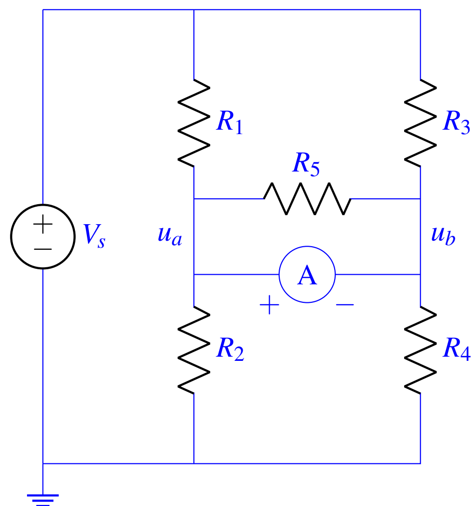
From these node voltages, the voltage V_{ab} can be calculated.

$$V_{ab} = u_a - u_b = 0.516V$$

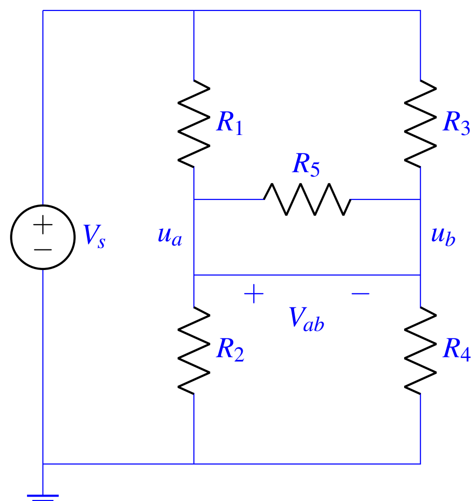
You should give yourself full-credit if your answer is off by a rounding error.

(b) Suppose you accidentally connect an ammeter in part (a) instead of a voltmeter. Calculate the value of V_{ab} with the ammeter connected.

Solution: While you did not have to redraw the circuit, it is depicted below.

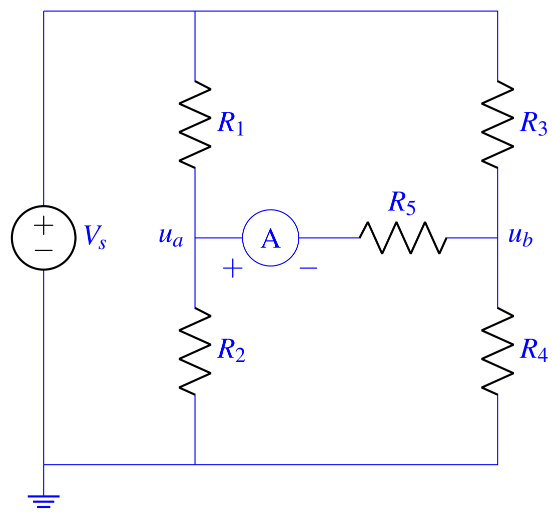


If we assume that the internal resistance of an ammeter is ideally zero, placing it across the nodes u_a and u_b will short them. So $u_a = u_b$. Thus $V_{ab} = 0$. The circuit below shows how the ammeter behaves as a short that unifies the previously separate nodes.



(c) Redraw the circuit diagram shown in Figure 2 by adding an ammeter (letter A in a circle and plus and minus signs indicating direction) in series with resistor R_5. This will measure the current I_{R_5} through R_5. Calculate the value of I_{R_5}.

Solution: The redrawn circuit with the ammeter measuring the current through R_5 is shown in the following circuit. It is also correct to draw the ammeter to the right of R_5 with the orientation of the meter remaining the same: the plus sign should be most proximal to the node labeled u_a, and the minus sign is most proximal to the node labeled u_b.



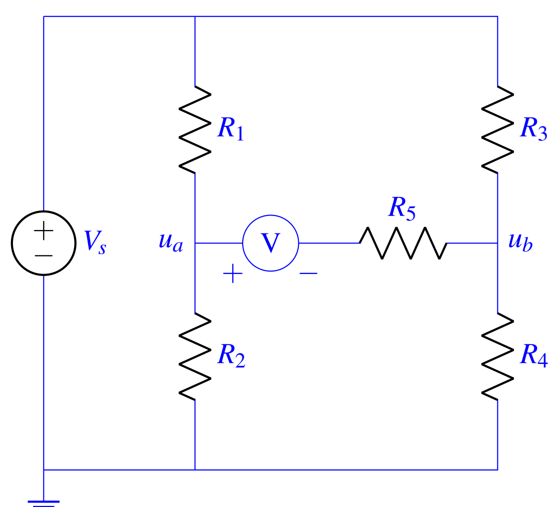
After calculating the node voltages u_a and u_b from part a, we can write:

$$I_{R_5} = \frac{u_a - u_b}{R_5} = 103.2\mu A$$

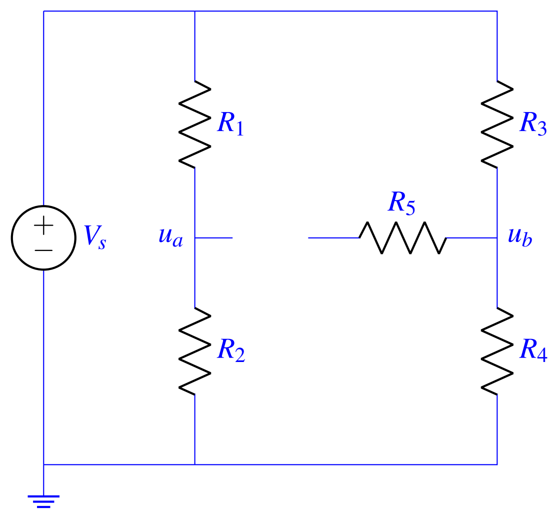
You should give yourself full-credit if your answer is off by a rounding error.

(d) Your friend accidentally connects a voltmeter in part (c) above, rather than an ammeter. Calculate the value of I_{R_5} with the voltmeter connected.

Solution: While you were not required to redraw the new circuit, the circuit is shown below.



The resistance of a voltmeter is infinite and it behaves as an open circuit. There will be no current flowing through R_5. Therefore, $I_{R_5} = 0$. The circuit below depicts how the voltmeter behaves as an open that prevents any current through R_5.


R_1
R_3
R_5
V_s
u_a
u_b
R_2
R_4

5. Resistive Touchscreen

Learning Goal: *The objective of this problem is to provide insight into modeling of resistive elements. This will also help to apply the concepts from resistive touchscreen.*

In this problem, we will investigate how a resistive touchscreen with a defined thickness, width, and length can actually be modeled as a series combination of resistors. As we know the value of a resistor depends on its length.

Figure 3 shows the top view of a resistive touchscreen consisting of a conductive layer with resistivity ρ_1, thickness t, width W, and length L. At the top and bottom it is connected through perfect conductors ($\rho = 0$) to the rest of the circuit. The touchscreen is wired to voltage source V_s.

Use the following numerical values in your calculations: $W = 50$ mm, $L = 80$ mm, $t = 1$ mm, $\rho_1 = 2\,\Omega\,\mathrm{m}$, $V_s = 5\mathrm{V}$, $x_1 = 20$ mm, $x_2 = 45$ mm, $y_1 = 30$ mm, $y_2 = 60$ mm.

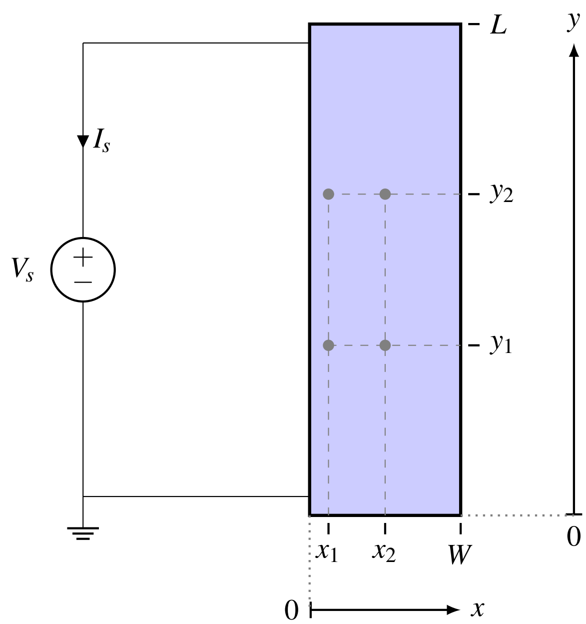
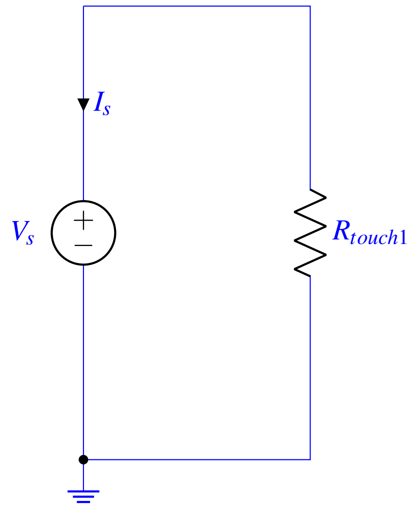


Figure 3: Top view of resistive touchscreen (not to scale). z axis i.e. the thickness not shown (into the page).

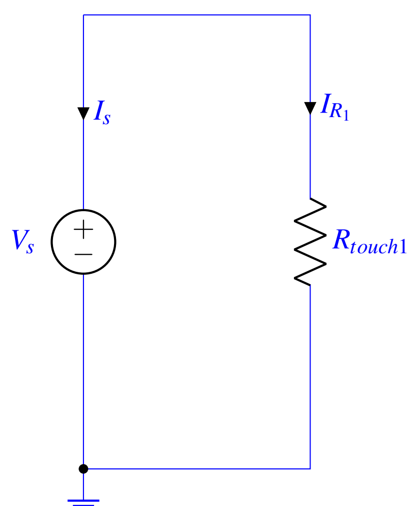
(a) Draw a circuit diagram representing **Figure 3**, where the entire touchscreen is represented as *a single resistor*. **Note that no touch is occurring in this scenario.** Remember that circuit diagrams in general consist of only circuit elements (resistors, sources, etc) represented by symbols, connecting wires, and the reference/ ground symbol. Calculate the value of current I_S based on the circuit diagram you drew. *Do not forget to specify the correct unit as always, and double check the direction of I_s!*

Solution:



The touchscreen resistance can be found from the following expression:

$$
\begin{aligned}
R_{touch1} &= \rho_1 \cdot \frac{L}{A} \\
&= \rho_1 \cdot \frac{L}{W \cdot t} \\
&= 2\,\Omega\,\text{m}\left(\frac{80\,\text{mm}}{50\,\text{mm} \cdot 1\,\text{mm}}\right) \\
R_{touch1} &= 3200\,\Omega = 3.2\,\text{k}\Omega
\end{aligned}
$$



From KCL, we can write:

$$I_s + I_{R_1} = 0 \tag{1}$$
$$I_s = -I_{R_1} \tag{2}$$

Therefore, the current I_{R_1} is equal to:

$$I_{R_1} = \frac{V_s}{R_{touch1}} = \frac{5}{3200}\,\text{A} = 1.56\,\text{mA}$$

And the current I_s is equal to:

$$I_s = -I_{R_1} = -1.56\,\text{mA}$$

(b) Let us assume u_{12} is the node voltage at the node represented by coordinates (x_1, y_2) of the touchscreen, as shown in **Figure 4**. What is the value of u_{12}? You should first draw a circuit diagram representing Figure 4, which includes node u_{12}. Specify all resistance values in the diagram. Does the value of u_{12} change based on the value of the x-coordinate x_1?

Hint: You will need more than one resistor to represent this scenario.

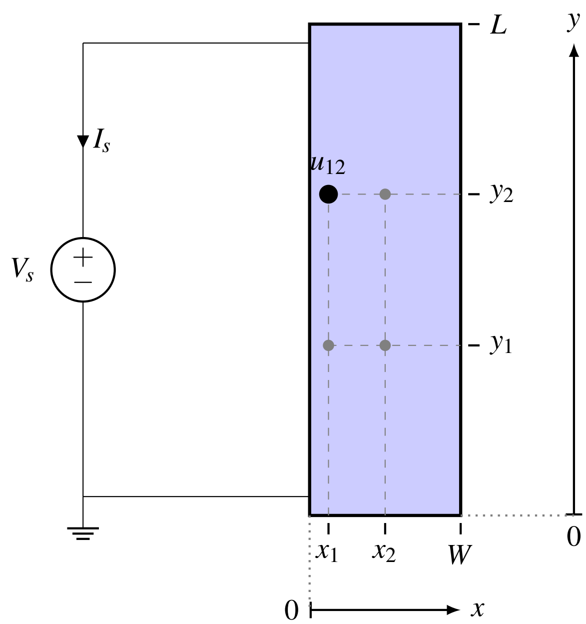
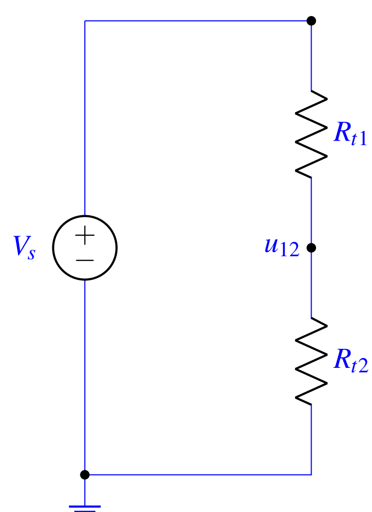


Figure 4: Top view of resistive touchscreen showing node u_{12}.

Solution:

We can represent this setup with the circuit shown below.



Using voltage division, u_{12} can be found from the following expression:

$$u_{12} = V_s \frac{R_{t2}}{R_{t1} + R_{t2}}$$

We know $R_{t1} = \rho_1 \cdot \frac{L-y_2}{W \cdot t}$ and $R_{t2} = \rho_1 \cdot \frac{y_2}{W \cdot t}$. We also know the $\frac{\rho_1}{W \cdot t}$ is common to both R_{t1} and R_{t2}, so those terms will cancel out when we them in.

$$u_{12} = V_s \frac{y_2}{L - y_2 + y_2}$$

$$u_{12} = V_s \frac{y_2}{L}$$

$$u_{12} = 5V \cdot \frac{60\,\text{mm}}{80\,\text{mm}} = 3.75\,\text{V}$$

The value of u_{12} would not change based on the value of the x coordinate, because in our setup the current is flowing from the top to the bottom of the screen. This means that voltage is only dissipated in the y direction, so we can only measure the difference in the y coordinate. We would need another closed circuit where current could flow along the width W to determine where the finger touched in the x direction.

(c) Assume V_{ab} is the voltage measured between the nodes represented by touchscreen coordinates (x_1, y_1) and coordinates (x_1, y_2), as shown in **Figure 5**. Calculate the absolute value of V_{ab}. As with the previous part, you should first draw the circuit diagram representing Figure 5, which includes V_{ab}. Calculate all resistor values in the circuit. *Hint: Try representing the segment of the touchscreen between these two coordinates as a separate resistor itself.*

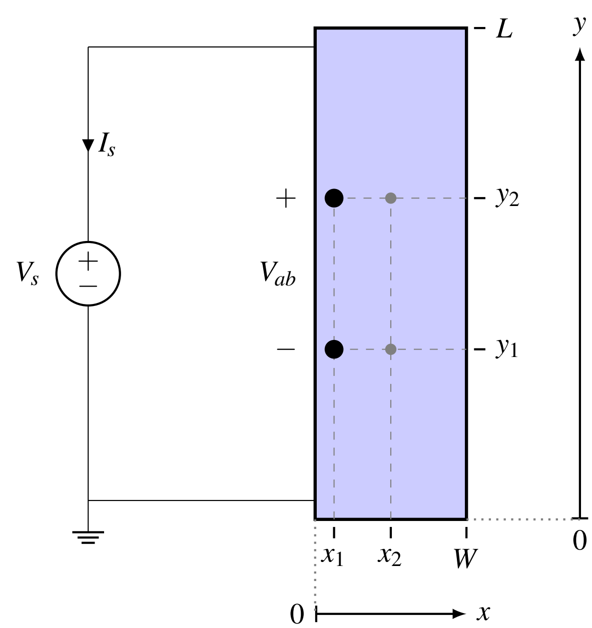
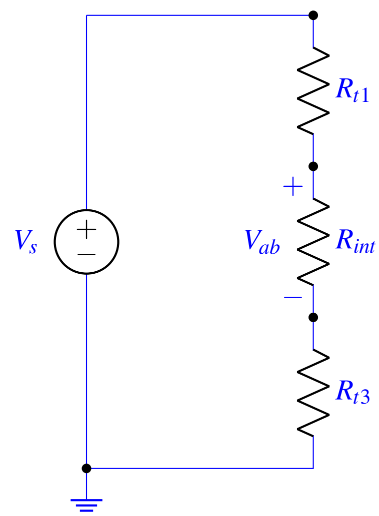
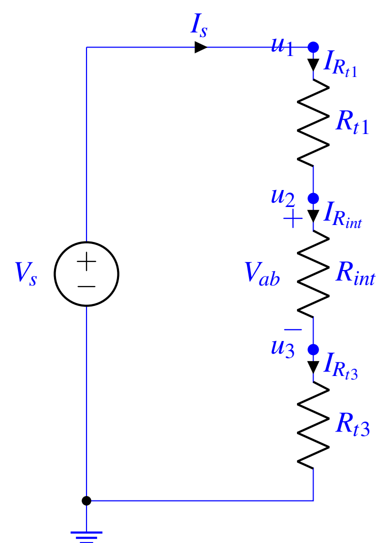


Figure 5: Top view of resistive touchscreen showing voltage V_{ab}.

Solution:



We can use node voltage analysis to find V_{ab}.



Using KCL at the three labelled nodes:

$$I_s = I_{R_{t1}}$$

$$I_{R_{t1}} = I_{R_{t3}}$$

$$I_{R_{t3}} = I_{R_{int}}$$

We see that there is only one current, I_s, going through all resistor elements. Writing the IV relationships for each element:

$$u_1 - u_2 = I_s R_{t1}$$

$$u_2 - u_3 = I_s R_{int}$$

$$u_3 = I_s R_{t3}$$

Knowing that $V_{ab} = u_2 - u_3$, we can write:

$$V_{ab} = u_2 - u_3 = I_s R_{int}$$

Now we just need to find I_s. Looking at the IV relationship equations and using back substitution, we can write:

$$u_1 = V_s = I_{R_{t1}} R_{t1} + I_{R_{int}} R_{int} + I_{R_{t3}} R_{t3}$$
$$I_s = \frac{V_s}{R_{t1} + R_{int} + R_{t3}}$$

Finally, we get:

$$V_{ab} = V_s \frac{R_{int}}{R_{t1} + R_{int} + R_{t3}}$$

Each of the resistances can be calculated as $R_{t1} = \rho_1 \cdot \frac{L-y_2}{W \cdot t}$, $R_{int} = \rho_1 \cdot \frac{y_2-y_1}{W \cdot t}$ and $R_{t3} = \rho_1 \cdot \frac{y_1}{W \cdot t}$. This gives for V_{ab}:

$$V_{ab} = \frac{y_2 - y_1}{L} V_s = \frac{3}{8} \cdot 5\,\text{V} = 1.875\,\text{V}$$

(d) Calculate (the absolute value of) the voltage between the nodes represented by touchscreen coordinates (x_1, y_1) and coordinates (x_2, y_1) in **Figure 5**.

Solution:

The two points have the same y coordinate, therefore they have the same potential in our touchscreen model. Again, this is because the current is flowing from the top to the bottom of the screen, so the x position does not make a difference. Recall that the touchscreen is effectively being modeled as a single vertical resistor which can be considered as several resistors of varying lengths, all totaling to L. Hence, we do not consider the effect of the x-coordinate on potential – we just need to consider the difference in the y-coordinate between two points. Thus, $\Delta V = 0$.

(e) Calculate (the absolute value of) the voltage between the nodes represented by touchscreen coordinates (x_1, y_1) and coordinates (x_2, y_2) in **Figure 5**.

Solution:

The two points have different x and y coordinates. However, the potential is the same across the x-axis for a fixed y coordinate, as was explained in the solution for part (d). Therefore, the problem is similar to part (c), since the potential is only determined by the y-coordinate of a point. Using the same equivalent circuit in part (c) we have:

$$\Delta V = V_s \frac{R_{int}}{R_{t1} + R_{int} + R_{t3}} = 1.875\,\text{V}$$

(f) **Figure 6** shows a new arrangement with two touchscreens. The two touchscreens are next to each other and are connected to the voltage source in the same way. The second touchscreen (the one on the right) is identical to the one shown in Figure 3, except for different width, W_2, and resistivity, ρ_2.

Use the following numerical values in your calculations: $W_1 = 50$ mm, $L = 80$ mm, $t = 1$ mm, $\rho_1 = 2\,\Omega\text{m}$, $V_s = 5\text{V}$, $x_1 = 20$ mm, $x_2 = 45$ mm, $y_1 = 30$ mm, $y_2 = 60$ mm, which are the same values as before. The new touchscreen has the following numerical values which are different: $W_2 = 85$ mm, $\rho_2 = 1.5\,\Omega\text{m}$.

Draw a circuit diagram representing **Figure 6**, where the two touchscreens are represented as *two separate resistors*. **Note that no touch is occurring in this scenario.**

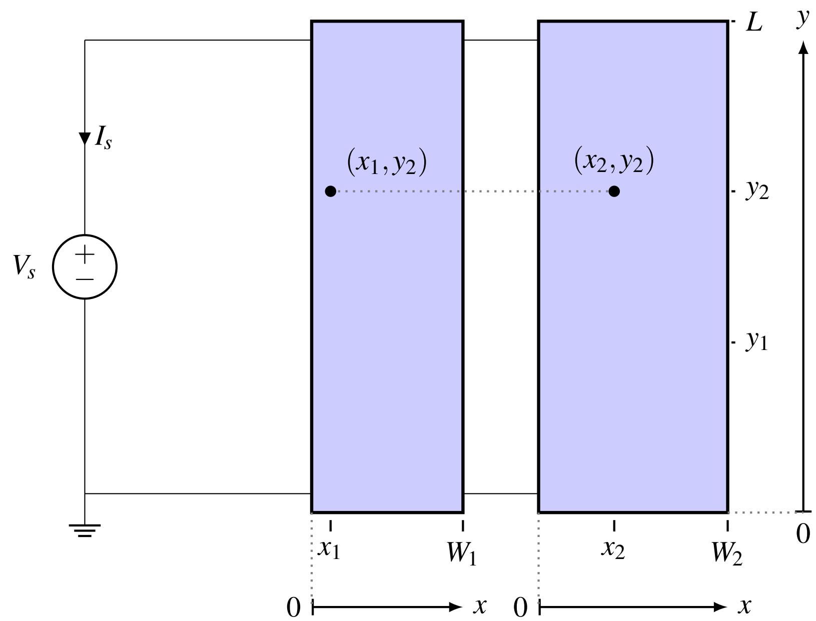
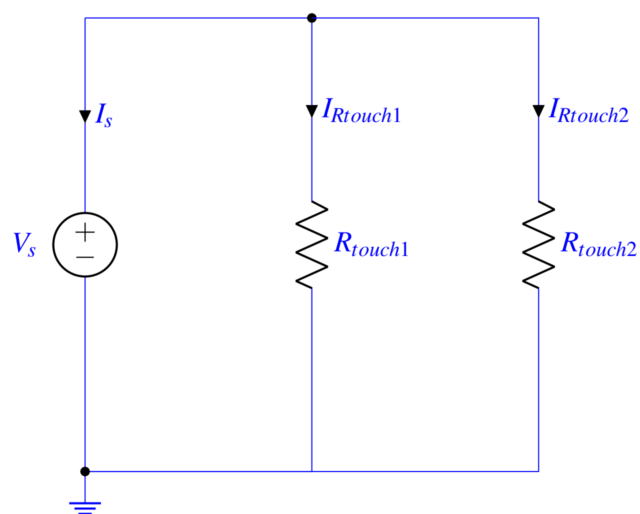


Figure 6: Top view of two touchscreens wired in parallel (not to scale). z axis not shown (into the page).

Solution:



(g) Calculate the value of current I_s for the two touchscreen arrangement based on the circuit diagram you drew in the last part.

Solution:

From KCL, we can write:

$$I_s + I_{Rtouch1} + I_{Rtouch2} = 0 \quad (3)$$

$$I_s = -(I_{Rtouch1} + I_{Rtouch2}) \quad (4)$$

Using Ohm's Law for each element:

$$I_s = -\left(\frac{V_s}{R_{touch1}} + \frac{V_s}{R_{touch2}}\right)$$

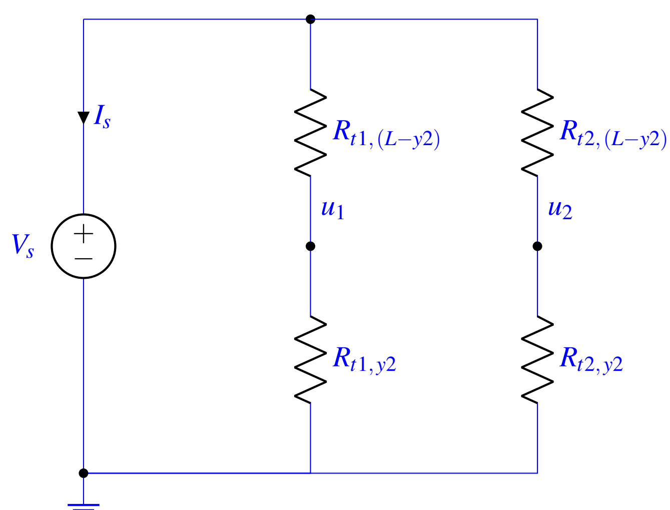
However, the resistance of the second touchscreen can be given by:

$$R_{touch2} = \rho_2 \cdot \frac{L}{W_2 \cdot t} = 1.5\,\Omega\,\text{m}\left(\frac{80\,\text{mm}}{85\,\text{mm} \cdot 1\,\text{mm}}\right)$$
$$R_{touch2} = 1411.8\,\Omega$$

Therefore, using the resistance values for the first and second touchscreen and the applied voltage source, we have:

$$I_s \approx -(1.563\text{mA} + 3.542\text{mA}) = -5.1\text{mA}$$

(h) Consider the two points: (x_1, y_2) in the touchscreen on the left, and (x_2, y_2) in the touchscreen on the right in **Figure 6**. Show that the node voltage at (x_1, y_2) is the same that at (x_2, y_2), i.e. the potential difference between the two points is 0. You can show this without explicitly calculating the node voltages at the two points.
If you were to connect a wire between the two coordinates (x_1, y_2) in the touchscreen on the left, and (x_2, y_2) in the touchscreen on the right, would any current flow through this wire?
Solution:
It will be helpful to first consider the circuit representation of this scenario to understand why the node voltages at the two points on each of the touch screens should be same (and therefore that the potential difference between points (x_1, y_2) and (x_2, y_2) is 0).



Without calculating the node voltages, note that the ratio of the value of $R_{t1,(L-y_2)}$ to $R_{t1,y2}$ is the same as the ratio of the value of $R_{t2,(L-y2)}$ to $R_{t2,y2}$:

$$\frac{R_{t1,y2}}{R_{t1,(L-y2)}} = \frac{\rho_1(y_2)/(W_1 \cdot t)}{\rho_1(L-y_2)/(W_1 \cdot t)} = \frac{y_2}{L-y_2}$$
$$\frac{R_{t2,y2}}{R_{t2,(L-y2)}} = \frac{\rho_2(y_2)/(W_2 \cdot t)}{\rho_2(L-y_2)/(W_2 \cdot t)} = \frac{y_2}{L-y_2}$$

Also note that the ratio of the resistors used in the voltage divider equations can be written as:

$$\frac{R_{t1,y2}}{R_{t1,(L-y2)}+R_{t1,y2}} = \frac{\rho_1(y_2)/(W_1\cdot t)}{\rho_1(L-y_2)/(W_1\cdot t)+\rho_1(y_2)/(W_1\cdot t)} = \frac{y_2}{L}$$
$$\frac{R_{t2,y2}}{R_{t2,(L-y2)}+R_{t2,y2}} = \frac{\rho_2(y_2)/(W_2\cdot t)}{\rho_2(L-y_2)/(W_2\cdot t)+\rho_2(y_2)/(W_2\cdot t)} = \frac{y_2}{L}$$

Beacuse the voltage across the entirety of each of the individual touchscreens is the same: it is $V_s - 0$ or just V_s. The voltage V_s is therefore *divided* between $R_{t1,(L-y2)}$ and $R_{t1,y2}$ exactly the same as it is divided between $R_{t_2,(L-y2)}$ and $R_{t_2,y2}$ because of the ratio argument presented above.

Therefore, the potential difference between u_1 and u_2 will be 0, so long as the y-coordinate value is chosen to be the same.

This also means that there is no current flowing through the wire, since the points in the two touchscreens have the same potential. Therefore,

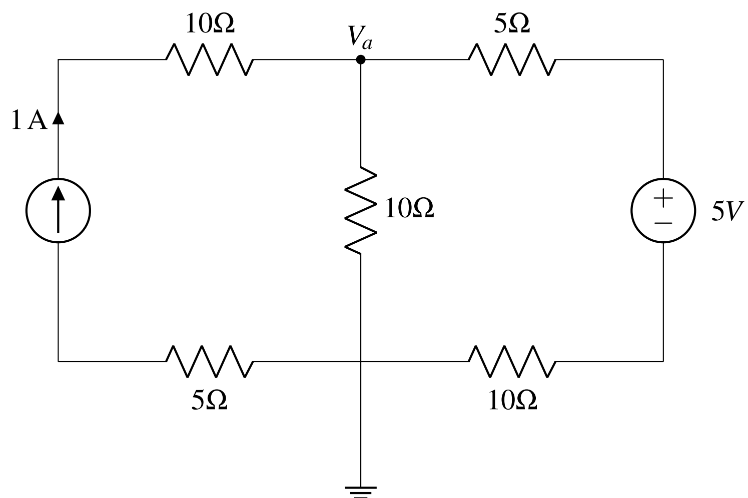
$$I_{12} = \frac{u_1-u_2}{R_{wire}} = 0$$

We should note that what causes current to flow is a voltage difference. Here, we have a voltage difference of zero, so the current flowing through the wire is zero.

6. Superposition

Learning Goal: *The objective of this problem is to help you practice solving circuits using the principles of superposition.*

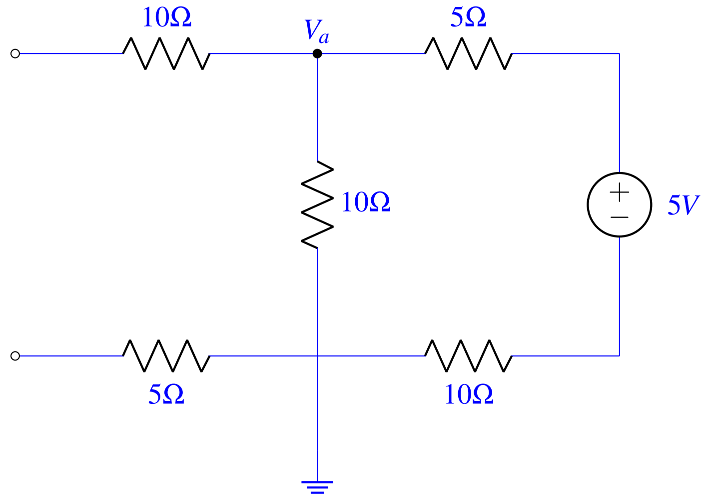
Find the node potential V_a indicated in the diagram using superposition. Be careful when solving to take into account where the reference potential is.



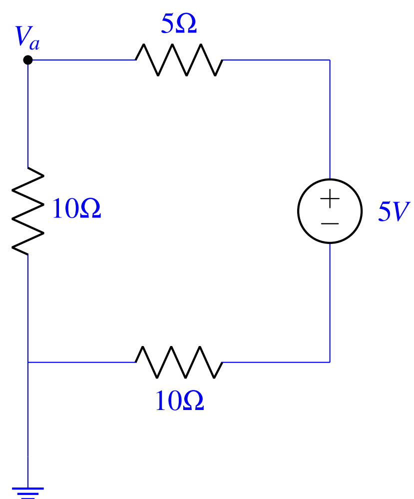
Solution:

Consider the circuits obtained by:

(a) Zeroing out the 1 A current source:

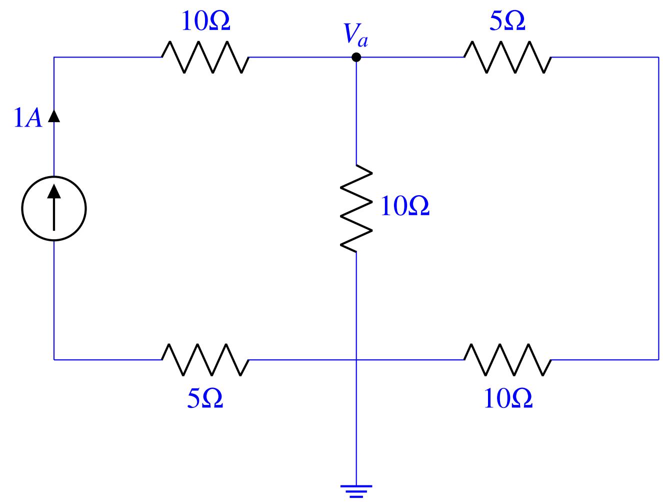


In the above circuit, no current flows through the two resistors in the top left and bottom left, so we can remove them and get the following circuit:

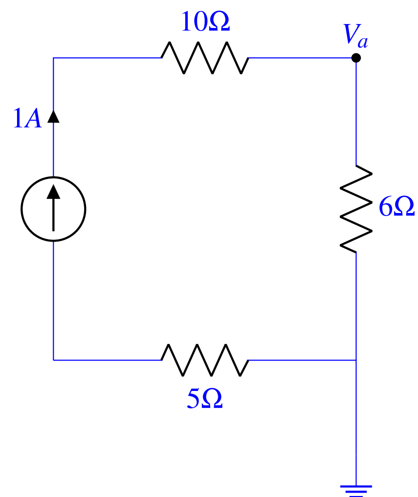


Using NVA, we can find that the voltage drop across the 5 Ω resistor is 1V and the voltage drop across the 10 Ω resistor is 2V. Note that the reference node is not right under the voltage source like we typically see, so V_a is just the voltage drop across the 10 Ω resistor which is 2V. (You can also make this easier by combining resistors and applying the voltage divider, but you still need to take account for where the reference potential is).

(b) Zeroing out the 5 V voltage source:



We can reduce this circuit using resistor equivalences to make it easier to solve. We note that the top right and bottom right resistors are in series, so combined we get 15Ω. Then we can combine this 15Ω resistor with the 10Ω resistor in the middle as they are in parallel to get $\frac{(10\Omega)(15\Omega)}{10\Omega+15\Omega} = 6\Omega$. Our reduced circuit then looks like this:

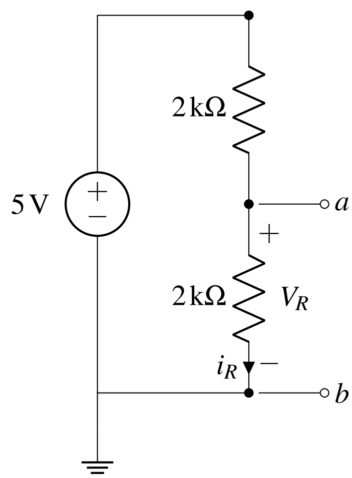


We can then just use Ohm's law to find the voltage drop across the 6Ω to find our node potential at V_a, which is just $(1A)(6\Omega) = 6V$

Now, applying the principle of superposition, we have $V_a = 2V + 6V = 8V$

7. Why Bother With Thévenin Anyway?

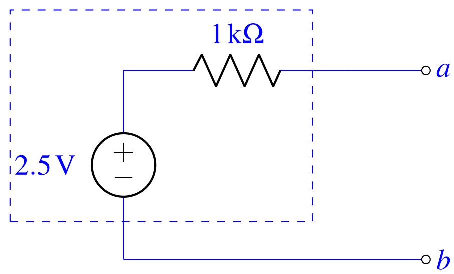
(a) Find a Thévenin eqiuvalent for the circuit shown below looking from the terminals a and b. (*Hint: That is, find the open circuit voltage V_R across the terminals a and b. Also, find the equivalent resistance looking from the terminals a and b when the input voltage source is zeroed.*)



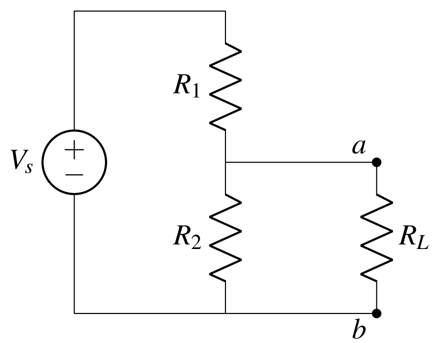
Solution: To find the voltage across terminals a and b, we notice that the circuit is a voltage divider. Therefore, we can use the voltage divider formula to find the voltage across a and b. Then for the equivalent resistance, we zero out the voltage source and notice that the resistors are in parallel with respect to the terminals a and b so we can use the parallel resistor equation to find the equivalent resistance. Be careful! It looks like the the resistors are in series but if we combine them that way, we would be destroying node a!

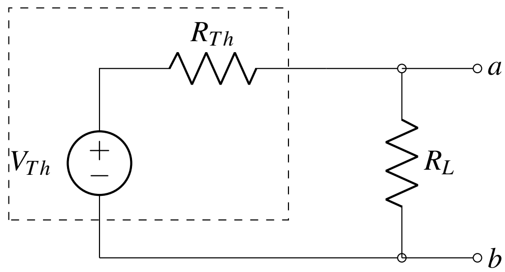
$$V_{Th} = \frac{2\,\text{k}\Omega}{2\,\text{k}\Omega + 2\,\text{k}\Omega} \cdot 5\,\text{V} = 2.5\,\text{V}$$

$$R_{Th} = 2\,\text{k}\Omega \parallel 2\,\text{k}\Omega = 1\,\text{k}\Omega$$

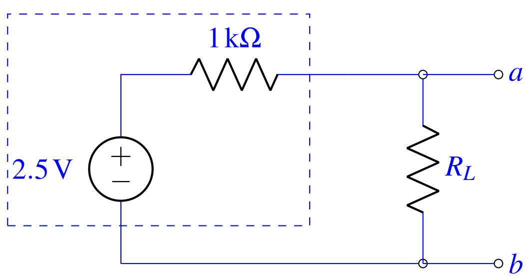


(b) Now consider the circuit shown below where a load resistor of resistance R_L is attached across the terminals a and b. Such a load resistor is often used to a model a device that we want to plug our circuit into, like an audio speaker. Compute the voltage drop V_R across the terminals a and b in this new circuit with the attached load. Express your answer in terms of R_L. *Hint: We have already computed the Thévenin equivalent of the unloaded circuit in part (a). To analyze the new circuit, attach R_L as the load resistance across the Thévenin equivalent computed in part (a), as shown in the figure below. One of the main advantages of using Thévenin (and Norton) equivalents is to avoid re-analyzing different circuits which differ only by the amount of loading (which depends on the device we are connecting!).*





Solution:
We just attach the R_L resistor to our Thévenin equivalent circuit that we found part (a) and calculate the voltage across it.



$$V_R = \frac{R_L}{1\,\mathrm{k\Omega} + R_L} \cdot 2.5\,\mathrm{V}$$

(c) Now compute the voltage drop V_R for three different values of R_L equal to $5/3\,\mathrm{k\Omega}$, $5\,\mathrm{k\Omega}$, and $50\,\mathrm{k\Omega}$? What can you comment on the value of R_L needed to ensure that the loading does not reduce the voltage drop V_R compared to the unloaded voltage V_R computed in part (a)? **Solution:**

$R_L = \frac{5}{3}\mathrm{k\Omega}$:

$$V_R = \frac{\frac{5}{3}\mathrm{k\Omega}}{1\,\mathrm{k\Omega} + \frac{5}{3}\mathrm{k\Omega}} \cdot 2.5\,\mathrm{V} = 1.56\,\mathrm{V}$$

$R_L = 5\,\mathrm{k\Omega}$:

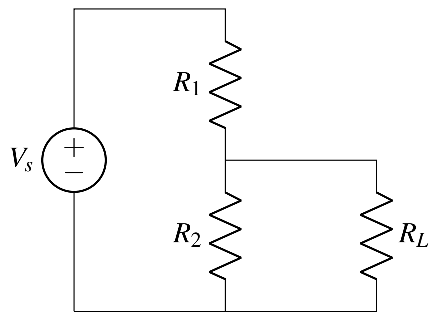
$$V_R = \frac{5\,\mathrm{k\Omega}}{1\,\mathrm{k\Omega} + 5\,\mathrm{k\Omega}} \cdot 2.5\,\mathrm{V} = 2.08\,\mathrm{V}$$

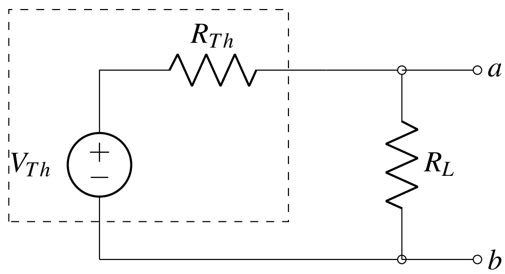
$R_L = 50\,\mathrm{k\Omega}$:

$$V_R = \frac{50\,\text{k}\Omega}{1\,\text{k}\Omega + 50\,\text{k}\Omega} \cdot 2.5\,\text{V} = 2.45\,\text{V}$$

As the value of R_L is increased, the voltage drop V_R approaches the unloaded Thévenin voltage computed in part (a).

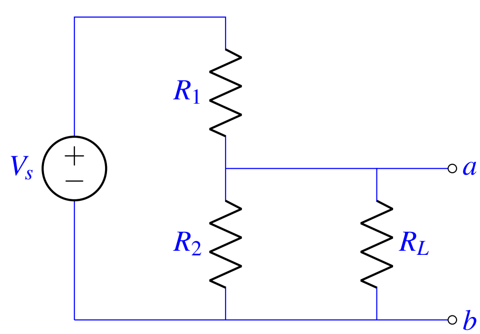
(d) Thus far, we have seen how to use Thévenin equivalents to compute the voltage drop across a load without re-analyzing the entire circuit. We would like to see if we can use the Thévenin equivalent for power computations. Consider the case where the load resistance $R_L = 8\text{k}\Omega$, $V_S = 5V$, $R_1 = R_2 = 2\text{k}\Omega$. Compute the power dissipated across the load resistor R_L both using the original circuit and the Thévenin equivalent. Are they equal? Now, compute the power dissipated by the voltage source V_s in the original circuit. Also, compute the power dissipated by the Thévenin voltage source V_{Th} in the Thévenin equivalent circuit. Is the power dissipated by the two sources equal?



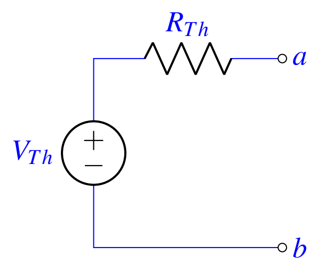


Solution:

We will compare the power dissipation in V_s vs. V_{Th} and R_L in either case. This could be done for the specific example above (with $R_L = 8\text{k}\Omega$), but it's more useful to go through this exercise generally. Thus, we will use the circuit shown below:



Recall that the Thévenin equivalent for the circuit above looks as follows:



where $R_{Th} = \frac{R_1 R_2}{R_1+R_2}$ and $V_{Th} = \frac{R_2}{R_1+R_2} V_s$.

Because we are going to end up writing a few expressions multiple times, we are going to define a new variable:

$$\beta = R_1 R_2 + R_L R_1 + R_L R_2$$

Let's start with our equivalent circuit. In the equivalent circuit, the current through the load resistor and equivalently every other element in the circuit is:

$$I = \frac{V_R}{R_L} = \frac{V_{Th}}{R_L + R_{Th}}$$

With this current, we find the power dissapated across the source and the load resistor.

$$P_{V_{Th}} = -IV = -\frac{V_{Th}^2}{R_L + R_{Th}} = -\frac{V_{Th}^2 (R_1 + R_2)}{\beta} = -\frac{V_s^2 R_2^2}{\beta (R_1 + R_2)} = -0.694\,\text{mW}$$

$$P_{R_L} = I^2 R = \frac{V_{Th}^2}{(R_L + R_{Th})^2} \cdot R_L = \frac{V_{Th}^2 (R_1 + R_2)^2}{\beta^2} \cdot R_L = \frac{V_s^2 R_2^2}{\beta^2} \cdot R_L = 0.617\,\text{mW}$$

Let's try to find the answer from the original circuit. We will begin by calculating the current through the source.

$$I_s = \frac{V_s}{R_{eq}} = \frac{V_s}{R_1 + R_2 \parallel R_L} = \frac{V_s (R_1 + R_2)}{\beta}$$

Now, we can calulate the power through the source.

$$P_{V_s} = -I_s V_s = -\frac{V_s^2 (R_2 + R_L)}{\beta} = -6.94\,\text{mW}$$

The power dissipated by the source in the original circuit is not the same as the power dissipated in the new circuit. What about the load resistor? We will first calculate the voltage across the load resistor.

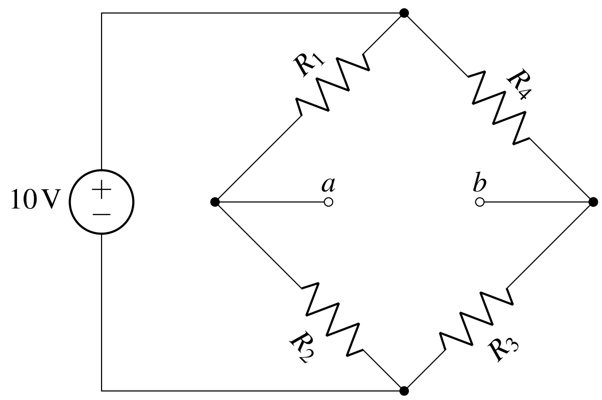
$$V_L = \frac{R_2 \parallel R_L}{R_1 + R_2 \parallel R_L} \cdot V_s = \frac{\frac{R_2 R_L}{R_2 + R_L}}{R_1 + \frac{R_2 R_L}{R_2 + R_L}} \cdot V_s = \frac{R_2 R_L}{\beta} \cdot V_s$$

$$P_L = \frac{V_L^2}{R_L} = \frac{V_s^2 R_2^2}{\beta^2} R_L = 0.617\,\text{mW}$$

The power through the load is the same! Thévenin equivalents can be used to calculate the power through elements that are not part of the circuit that was transformed.

8. Wheatstone Bridge

Thévenin equivalence is a powerful technique we can use to analyze the Wheatstone bridge circuit shown below. This circuit is used in many sensor applications where resistors R_1, R_2, R_3, R_4 are varying with respect to some external actuation. For example, it can be used to build a strain gauge or a scale. In that case the resistors R_1, R_2, R_3, R_4 would vary with respect to a strain caused by a force, and the Wheatstone Bridge circuit would translate that variation into a voltage difference across the “bridge" terminals a and b. Assume that $R_1 = 2\,\mathrm{k\Omega}$, $R_2 = 2\,\mathrm{k\Omega}$, $R_3 = 1\,\mathrm{k\Omega}$, $R_4 = 3\,\mathrm{k\Omega}$



(a) Calculate the voltage V_{ab} between the two terminals a and b.

Solution:

Notice in the above circuit that there are two voltage dividers, so we can calculate v_a and v_b quickly.

$$v_a = \frac{R_2}{R_1 + R_2} \cdot 10\,\mathrm{V} = 5V$$

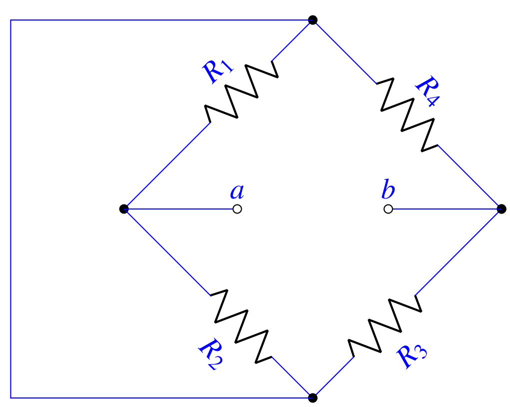
$$v_b = \frac{R_3}{R_3 + R_4} \cdot 10\,\mathrm{V} = 2.5V$$

Thus, the voltage difference between the two terminals a and b is: $V_{ab} = v_a - v_b = 2.5V$.

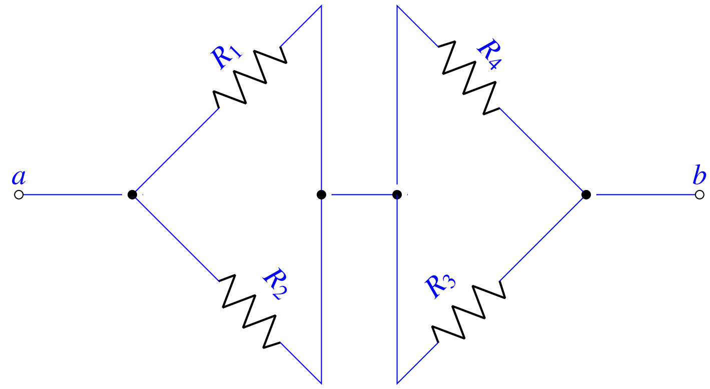
(b) Next, draw the Thévenin equivalent of the Wheatstone bridge circuit.

Solution:

We find the Thévenin resistance by replacing the voltage source with a short and calculating the resistance between the two terminals a and b. The circuit now looks like:



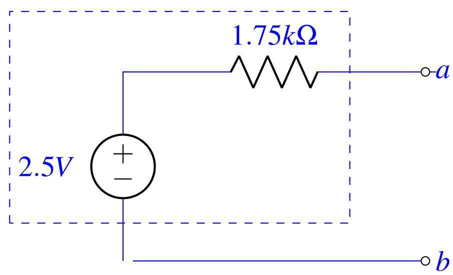
Notice that because the top and bottom node are shorted, we have $R_1 \parallel R_2$ in series with $R_3 \parallel R_4$ between nodes "a" and "b".



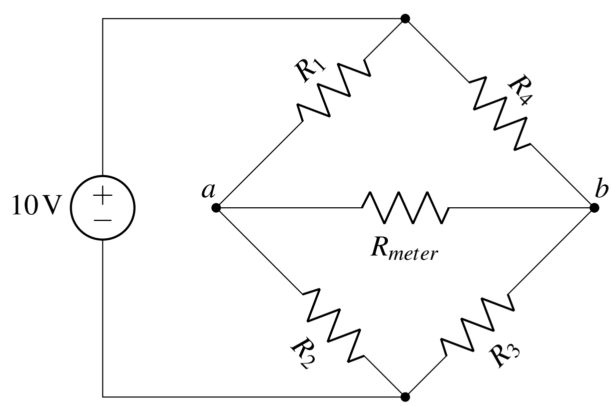
It follows that R_{th} is:

$$\begin{aligned} R_{\text{Th}} &= (R_1 \parallel R_2) + (R_3 \parallel R_4), \text{ where } \parallel \text{ denotes the parallel operator.} \\ &= \frac{R_1 R_2}{R_1 + R_2} + \frac{R_3 R_4}{R_3 + R_4} \\ &= 1.75k\Omega \end{aligned}$$

Using $V_{\text{Th}} = V_{ab} = 2.5V$ from part (a), we can construct the Thévenin equivalent circuit:

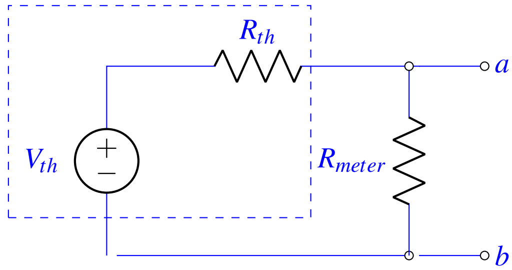


(c) Now assume that you are trying to measure the voltage V_{ab} using a voltmeter, whose resistance is R_{meter}, so you end up with the circuit below.



Unfortunately, you voltmeter is far from ideal, so $R_{meter} = 4k\Omega$. Is the voltage V_{ab} you found in part (a) equal to the new voltage $V_{R_{meter}}$ across the voltmeter resistor? Why or why not? Calculate the current $I_{R_{meter}}$ through the voltmeter resistor and the voltage $V_{R_{meter}}$ across the meter resistor.

Solution:
No, the Thévenin voltage we found in part (a) is the open-circuit voltage. If we add R_{meter} back into the original circuit, R_{meter} would load the other resistors (or, equivalently, the Thévenin resistance), so the Thévenin voltage is not equal to the actual voltage across the meter resistor.
Having derived the Thévenin equivalent circuit, we can now draw the following:

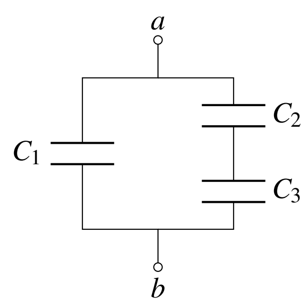


Using the facts that, $R_{meter} = 4k\Omega$, $R_{th} = 1.75k\Omega$, $V_{th} = 2.5V$ we can write:

$$I_{R_{meter}} = \frac{2.5V}{1.75k\Omega + 4k\Omega} \approx 0.43\text{mA}$$
$$V_{R_{meter}} = I_{R_{meter}} R_{meter} \approx 1.74V$$

9. Equivalent Capacitance

(a) Find the equivalent capacitance between terminals a and b of the following circuit in terms of the given capacitors C_1, C_2, and C_3. Leave your answer in terms of the addition, subtraction, multiplication, and division operators **only**.



Solution:

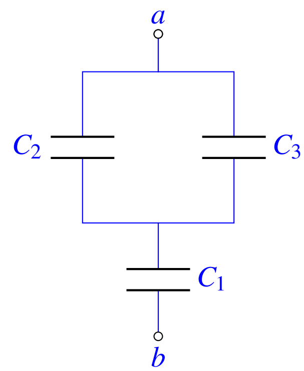
$$C_{eq} = C_1 + (C_2 || C_3)$$
$$C_{eq} = C_1 + \frac{C_2 C_3}{C_2 + C_3}$$

Here, $||$ represents the mathematical parallel operator ($a||b = \frac{ab}{a+b}$).

(b) Find and draw a capacitive circuit using three capacitors, C_1, C_2, and C_3, that has equivalent capacitance of

$$\frac{C_1(C_2 + C_3)}{C_1 + C_2 + C_3}$$

Solution: This expression is the same as $C_1 || (C_2 + C_3)$, so C_2 and C_3 are in parallel with each other, and C_1 is series with both of them:



10. It's finally raining!

A lettuce farmer in the Salinas Valley has grown tired of imprecise online rainfall forecasts. They decide to take matters into their own hands by building a rain sensor. They place a square tank outside and attach two metal plates to two opposite sides in an effort to make a capacitor whose capacitance varies with the amount of water inside.

Note: In practice, water is conductive. However for this problem, assume the metal plates are properly insulated so that no current flows through the water and we can treat it like a dielectric material. In other words, the electric circuit is better modeled as a capacitance and not a resistance.

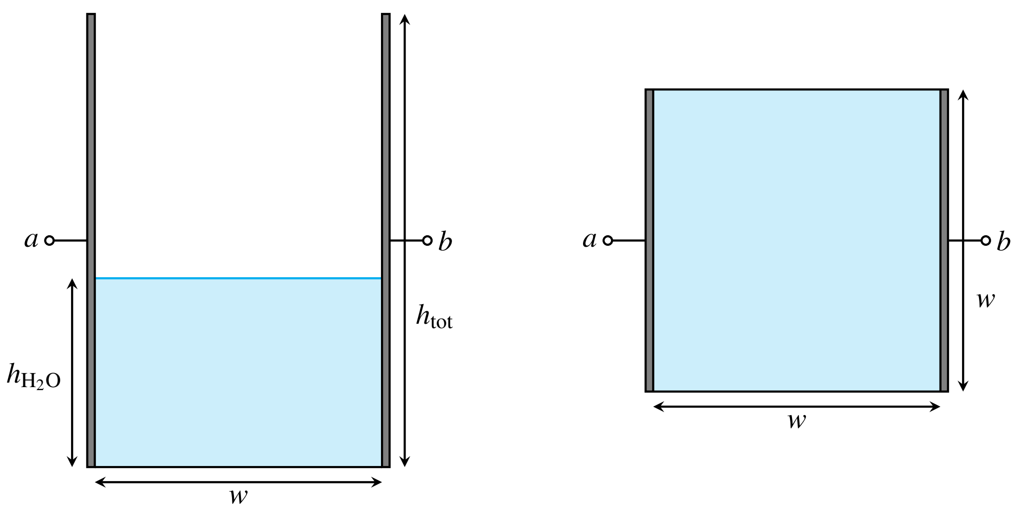


Figure 7: Tank side view (left) and top view (right).

The width and length of the tank are both w (i.e., the base is square) and the height of the tank is h_{tot}.

(a) What is the capacitance between terminals a and b when the tank is full? What about when it is empty? The permittivity of air is $\varepsilon_{air} = \varepsilon_0$, and the permittivity of rainwater is $\varepsilon_{H_2O} = 75\varepsilon_0$.

Solution:

Capacitance of parallel plates is governed by the equation:

$$C = \frac{\varepsilon A}{d},$$

where ε is the *permittivity* of the dielectric material, A is the area of the plates, and d is the distance between the plates. If we apply this to our physical structure, we find that the area of the plates is $h_{\text{tot}} \cdot w$, and the distance between the plates is w. The only difference here between a full and empty tank is the permittivity of the material between the two plates.

$$C_{\text{empty}} = \frac{\varepsilon_{\text{air}} h_{\text{tot}} w}{w} = \varepsilon_0 h_{\text{tot}}$$

$$C_{\text{full}} = \frac{\varepsilon_{\text{H}_2\text{O}} h_{\text{tot}} w}{w} = 75\varepsilon_0 h_{\text{tot}}$$

(b) Suppose the height of the water in the tank is $h_{\text{H}_2\text{O}}$. Model the tank as a pair of capacitors in parallel, where one capacitor has a dielectric of air, and one capacitor has a dielectric of water. Find the total capacitance C_{tank} between the two metal walls/plates using circuit equivalence.

Solution:

We can break the total capacitance into two parts. First, let's calculate the capacitance of the two plates separated by water:

$$C_{\text{water}} = \frac{\varepsilon_{\text{H}_2\text{O}} h_{\text{H}_2\text{O}} w}{w} = 75\varepsilon_0 h_{\text{H}_2\text{O}}$$

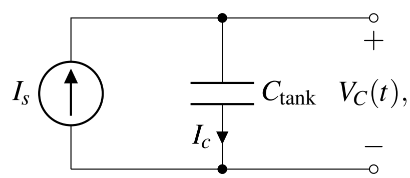
And now we can calculate the capacitance of the two plates separated by air:

$$C_{\text{air}} = \frac{\varepsilon_{\text{air}} (h_{\text{tot}} - h_{\text{H}_2\text{O}}) w}{w} = \varepsilon_0 \cdot (h_{\text{tot}} - h_{\text{H}_2\text{O}})$$

These two capacitors appear in parallel, as the result from the layer of water at the bottom of the tank, and the air above the water. Because these two capacitors appear in parallel, we can simply add our two previous results to find the total equivalent capacitance:

$$C_{\text{tank}} = C_{\text{water}} + C_{\text{air}} = \varepsilon_0 \cdot (h_{\text{tot}} + 74 h_{\text{H}_2\text{O}})$$

(c) After building this tank, the farmer consults the internet to assist them with a capacitance-measuring circuit. A fellow internet user recommends building the following circuit:



where C_{tank} is the total tank capacitance between terminals a and b calculated in part (b), and I_s is a known current supplied by a current source.

The user suggests measuring $V_C(t)$ for a brief interval of time, compute the rate of change of V_C, and determine C_{tank}.

Determine $V_C(t)$, where t is the number of seconds elapsed since the start of the measurement. You should assume that before any measurements are taken, the voltage across C_{tank} is initialized to 0 V, i.e. $V_C(0) = 0$.

Solution:

The element equation for the capacitor is:

$$I_C = C_{\text{tank}} \frac{dV_C}{dt}$$

We also know from KCL that:

$$I_C = I_s$$

Thus, we get the following differential equation for V_C:

$$\frac{dV_C}{dt} = \frac{I_s}{C_{\text{tank}}}$$

We recall that I_s and C_{tank} are constant values and the initial value of V_C is zero ($V_C(0) = 0$). Applying these facts and integrating the differential equation, we get the following equation for V_C:

$$V_C(t) = \frac{I_s}{C_{\text{tank}}} t$$

(d) Using the equation you derived for $V_C(t)$, describe how you can use this circuit to determine C_{tank} and $h_{\text{H}_2\text{O}}$.

Solution:

We connect the current source providing I_s to the capacitor C_{tank}. After a known amount time, t_f, passes, we measure the capacitor voltage, $V_C(t_f)$, and plug it into the following equation (assuming, as before, that $V_C(0) = 0$):

$$C_{\text{tank}} = \frac{I_s}{V_C(t_f)} t_f$$

If we know C_{tank}, we can determine $h_{\text{H}_2\text{O}}$. Using the equation derived in part (b), we see that

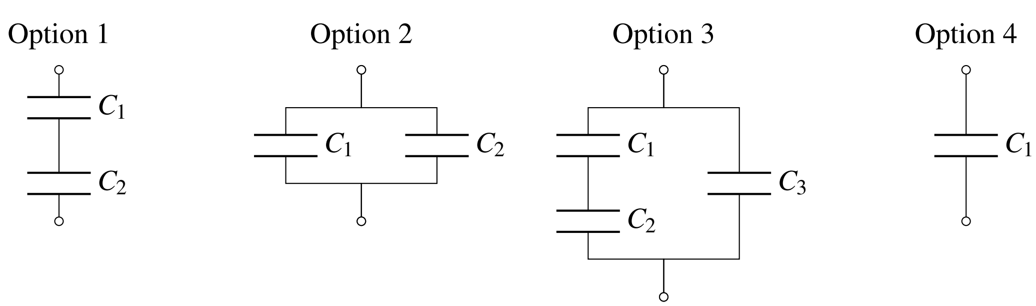
$$h_{\text{H}_2\text{O}} = \frac{C_{\text{tank}} - h_{tot}\varepsilon}{74\varepsilon}$$

11. Modeling Weird Capacitors

For each part of this problem,

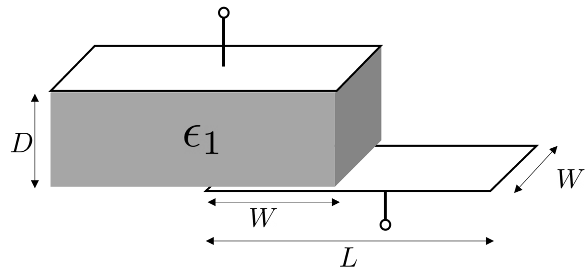
i. Pick the circuit option from below that *best* models the given physical capacitor.

ii. Calculate the total equivalent capacitance of the circuit in terms of the given quantities (e.g. $\varepsilon_1, \varepsilon_2, \varepsilon_3, L, W, D$).



(a) A parallel plate capacitor with plate dimensions L and W, separated by a gap D, is filled with an insulator of permittivity ε_1, with the bottom plate displaced with overlap W as shown below. You can assume $W < L$ and $D << W$.

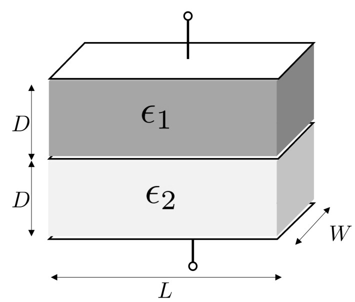
(i) Pick the circuit option from above that best models this physical capacitor, and (ii) calculate the total equivalent capacitance of the circuit in terms of L, W, D, ε_1.



Solution: Option 4, where

$$C = C_1 = \varepsilon_1 \frac{W \cdot W}{D}$$

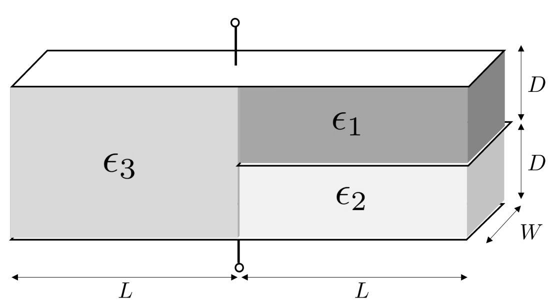
(b) A parallel plate capacitor with plate dimensions L and W, separated by a gap $2 \cdot D$, is filled with two insulators of permittivities ε_1 and ε_2 as shown below. You can assume there's a plate between the two dielectrics.
(i) Pick the circuit option from above that best models this physical capacitor, and (ii) calculate the total equivalent capacitance of the circuit in terms of $L, W, D, \varepsilon_1, \varepsilon_2$.



Solution: Option 1, where

$$C = C_1 || C_2 = \frac{L \cdot W}{D} \frac{\varepsilon_1 \varepsilon_2}{\varepsilon_1 + \varepsilon_2}$$

(c) A parallel plate capacitor with plate dimensions L and W, separated by a gap $2 \cdot D$, is filled with three different materials with permittivities ε_1, ε_2, and ε_3 as shown in the figure below. You can assume there's a plate between the two dielectrics on the right.
(i) Pick the circuit option from above that best models this physical capacitor, and (ii) calculate the total equivalent capacitance of the circuit in terms of $L, W, D, \varepsilon_1, \varepsilon_2, \varepsilon_3$.



Solution: Option 3, where

$$C = (C_1 || C_2) + C_3 = \frac{L \cdot W}{D} \frac{\varepsilon_1 \varepsilon_2}{\varepsilon_1 + \varepsilon_2} + \frac{L \cdot W \varepsilon_3}{2D} = \frac{L \cdot W}{D} \left(\frac{\varepsilon_1 \varepsilon_2}{\varepsilon_1 + \varepsilon_2} + \frac{\varepsilon_3}{2} \right)$$

12. Prelab Questions

These questions pertain to the prelab reading for the Touch 3A lab. You can find the reading under the Touch 3A Lab section on the 'Schedule' page of the website. We do not expect in-depth answers for the questions. Please limit your answers to a maximum of 2 sentences.

(a) What is the equation for the relationship between the current and voltage of a capacitor? Write out the steps of your derivation. (Hint: Start from the charge equation, and see how you can plug in $I = \frac{dQ}{dt}$)

(b) What is the equation for the C_{eq} of capacitors in series? Write your answer in the form: C_{eq} = .

(c) Does adding another capacitor in series increase or decrease the total capacitance C_{eq}?

(d) Why does touching our touchscreen cause a change in capacitance?

(e) What is the purpose of using a comparator in the capacitive touchscreen that we'll be building?

Solution:

(a) $Q = CV, I = \frac{dQ}{dt} \rightarrow \frac{d}{dt} = CV\frac{d}{dt} \rightarrow I = CV\frac{d}{dt} \rightarrow I = C\frac{dV}{dt}$

(b) $C_{eq} = \dfrac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \ldots + \frac{1}{C_n}}$

(c) Decreases, since the value of the denominator gets larger as we add more $\frac{1}{C_i}$ terms to it.

(d) Because our fingers have a capacitance, so touching our touchscreen is essentially adding a capacitor to our system.

(e) We can use our comparator to determine if a touch took place on the touchscreen. (We will use the values that the comparator outputs, either $+V_{CC}$ or $-V_{CC}$, to visualize the difference in voltage between $Vtouch$ and $Vno-touch$.)

13. Homework Process and Study Group

Who did you work with on this homework? List names and student ID's. (In case you met people at homework party or in office hours, you can also just describe the group.) How did you work on this homework? If you worked in your study group, explain what role each student played for the meetings this week.

Solution:

I first worked by myself for 2 hours, but got stuck on problem 5. Then I met with my study group.

XYZ played the role of facilitator ... etc. We were still stuck on problem 5 so we went to office hours to talk about the problem.

Then I went to homework party for a few hours, where I finished the homework.