Lecture 12

Monday, July 10, 2023 3:29 PM

Lecture 4C: (7/12/23)

Announcements:

- Module #2 truly begins!
- Office Hours – Mon/Wed 1-2pm in Cory 144MA
  Today!
- Lab – Please consider moving from afternoon to evening section
- Today's Topics: How to "solve" a circuit
  - Review of circuit concepts (Note 11A)
  - Kirchoff's Laws (Note 11A)
  - Node Voltage Analysis (Note 11B)

Node Voltage Analysis

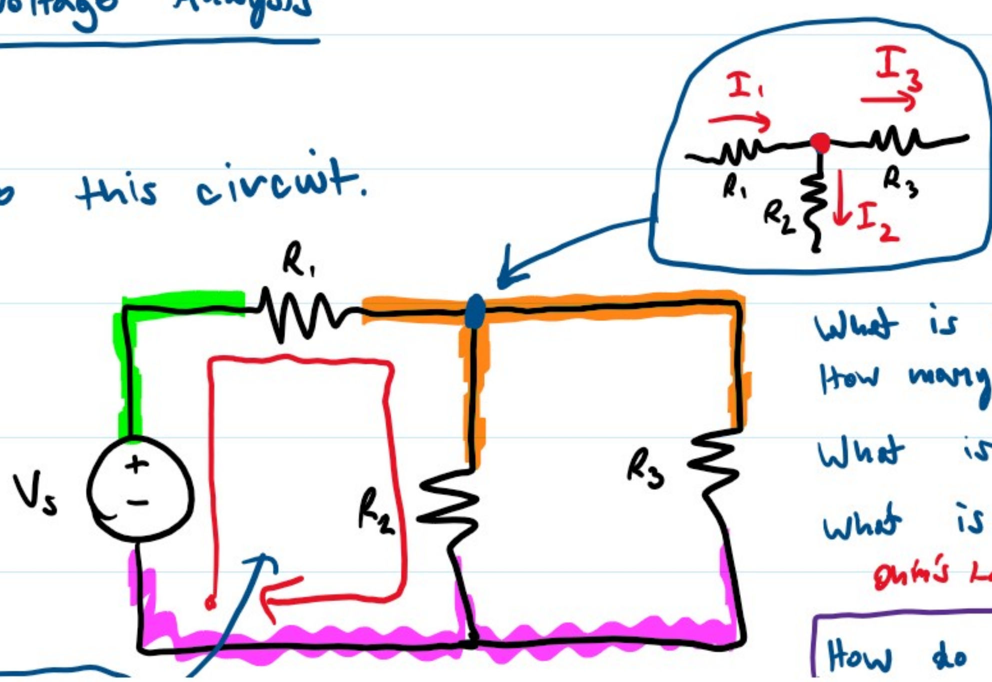
Solve this circuit.

$V_s = 10V$

$R_1 = 1\Omega$

$R_2 = 2\Omega$

$R_3 = 2\Omega$



What is a node?

How many nodes? 3

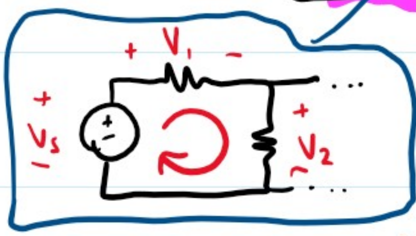
What is (±) V_s?

What is –/\/\/\– R?

Ohm's Law : $V = IR$

$\boxed{\text{How do we "solve"?}}$

How do we "solve"?

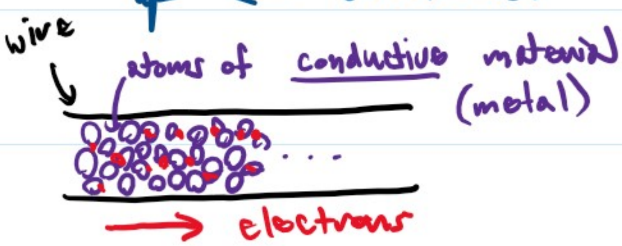


	Variable name	Unit
What is current?	i	A (ampere) ← $\frac{\text{coulomb}}{\text{second}}$

$i = \frac{\partial q}{\partial t}$ ← current is flow of charge

What is charge?	q	C (coulomb)

q ← electrons!



charge of 1 electron is
$q_e = -1.602 \cdot 10^{-19}$ C

1 coulomb is $6.243 \cdot 10^{18}$ electrons

What is voltage?	V	V (Volts)

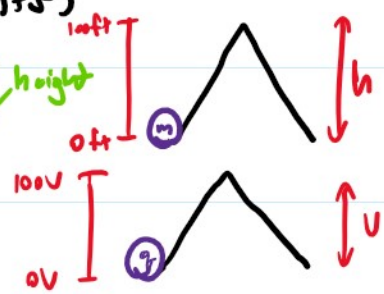
Can make charge move

$E = q \cdot V$ (energy)

potential energy:

$E = mgh$ (mass, $9.80\,\text{m/s}^2$, height)

$E = qV$



- The amount of energy it takes to move a charge to a new voltage
- Voltage is relative (How high is my hand?)

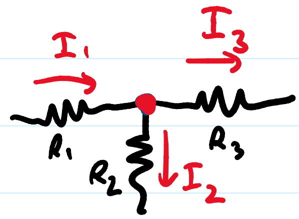
Node:

A point where two or more circuit elements meet.
A region that is equipotential (equal voltage potential throughout)

Kirchoff's Current Law: "KCL"

1. Sum of currents entering and exiting a node is zero

$$\underbrace{+I_1}_{\text{enter}} \underbrace{- I_2 - I_3}_{\text{exit}} = 0$$



2. Sum of entering currents = sum of exiting currents

$$\underbrace{I_1}_{\text{enter}} = \underbrace{I_2 + I_3}_{\text{exit}}$$

- Both definitions are equivalent

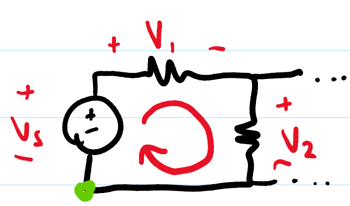
Loop:

A closed path in a circuit. Ends where it starts.

Kirchoff's Voltage Law: "KVL"

1. Sum of voltages in any loop is zero

$$\underbrace{+V_s}_{\text{rise}} \underbrace{- V_1 - V_2}_{\text{drop}} = 0$$



2. Sum of voltage rises = sum of voltage drops

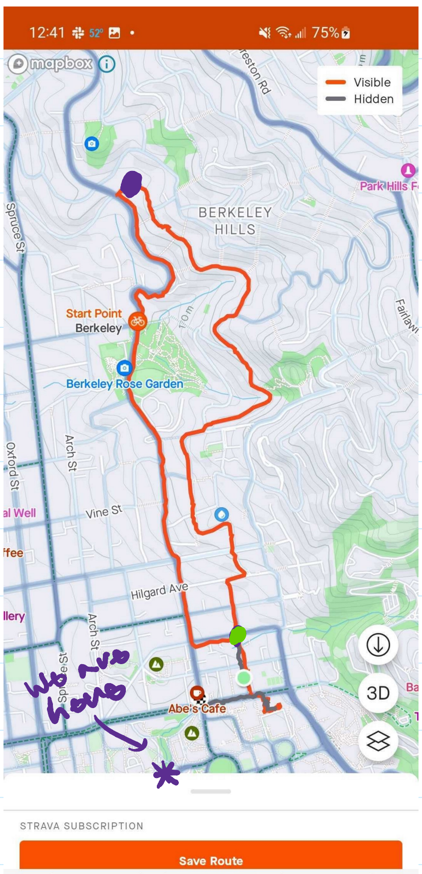
$$\underbrace{V_s}_{\text{rise}} = \underbrace{V_1 + V_2}_{\text{drop}}$$

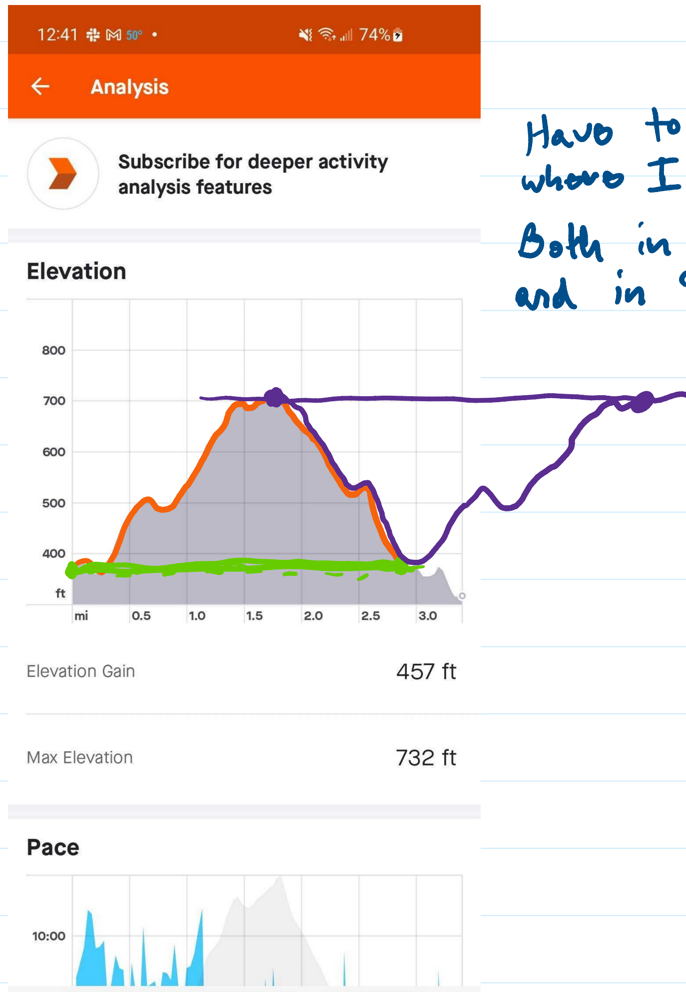
Elevation

$$\underbrace{V_s}_{\text{rise}} = \underbrace{V_1 + V_2}_{\text{drop}}$$

Elevation

Hiking Example:





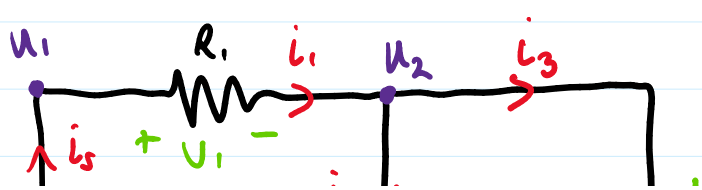
Have to end where I start!

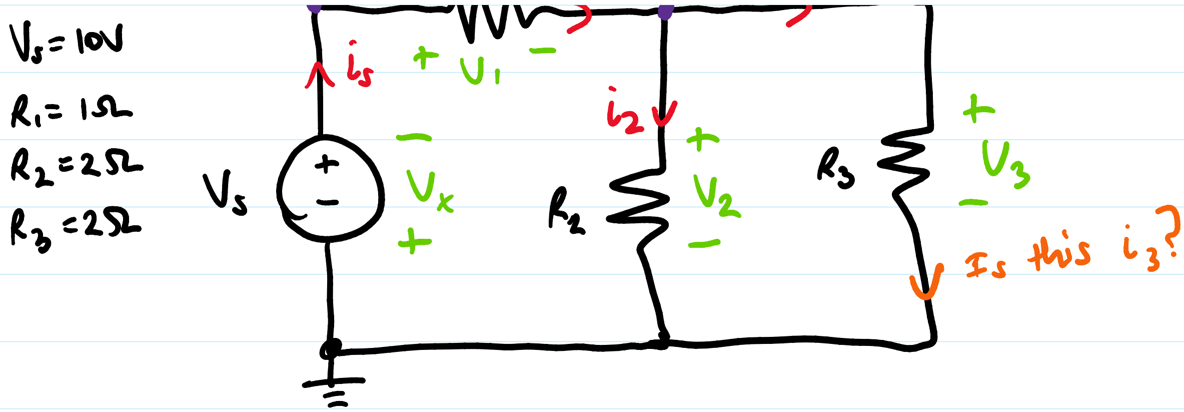
Both in x and y and in "potential"

Let's solve the circuit using "Node Voltage Analysis"

"Write a KCL equation at every node in terms of node voltages and IV characteristics." ~ Nathan

$V_s = 10V$





Step 1: Pick a reference node ("ground node") $\equiv$

– can pick ANY node

Step 2: Label all remaining nodes "u_i"

2 nodes remaining $\rightarrow$ u_1 and u_2 $\leftarrow$ exclude reference node

Step 3: Label the current through every element "i_i"

Does $i_a = i_b$?

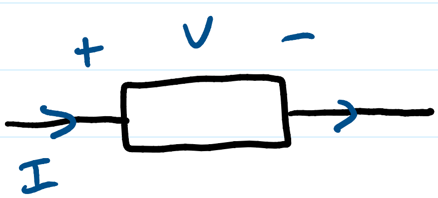
KCL: enter = exit

$i_a = i_b$ ✓

Step 4: Add voltage labels (+/–) across each element

Passive sign convention:

current enters positive terminal and exits negative terminal



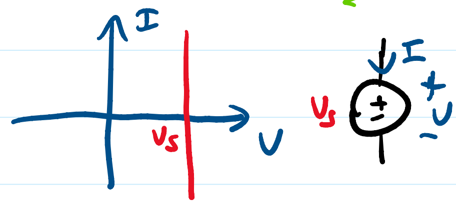
Step 5: Identify all unknowns. Simplify if possible.

node voltages: $\boxed{u_1, u_2}$

these are the important ones

element currents: $i_s, \cancel{i_1, i_2, i_3}$

element voltages: $\cancel{V_x, V_1, V_2, V_3}$



Step 6a: Set up a system of KCL equations

At node u_1: $i_s - i_1 = 0$

At node u_2: $i_1 - i_2 - i_3 = 0$

Step 6b: Use IV relationships (e.g. Ohm's Law)

Element voltages to node voltages $\Rightarrow$ $\Leftarrow$ I-V characteristics

$V_x = 0 - u_1$	$u_1 = V_s$	$V_x = -V_s$
$V_1 = u_1 - u_2$	$u_1 - u_2 = i_1 R_1$	$V_1 = i_1 R_1$
$V_2 = u_2 - 0$	$u_2 = i_2 R_2$	$V_2 = i_2 R_2$
$V_3 = u_2 - 0$	$u_2 = i_3 R_3$	$V_3 = i_3 R_3$

We've eliminated "element voltage" variables!

Step 7: Simplify equations and solve

KCL equations:

$i_s - i_1 = 0$ ①

$i_1 - i_2 - i_3 = 0$ ②

I-V equations:

$u_1 = V_s$

$u_1 - u_2 = i_1 R_1$

$u_2 = i_2 R_2$

$\boxed{u_1 = V_s}$ ③

$i_1 = \frac{u_1 - u_2}{R_1}$

$i_2 = \frac{u_2}{}$

$i_s - i_1 = 0$ (1)

$i_1 - i_2 - i_3 = 0$ (2)

$$\left.\begin{aligned} u_1 - u_2 &= i_1 R_1 \\ u_2 &= i_2 R_2 \\ u_2 &= i_3 R_3 \end{aligned}\right\} \quad \begin{aligned} i_1 &= \frac{u_1 - u_2}{R_1} \\ i_2 &= \frac{u_2}{R_2} \\ i_3 &= \frac{u_2}{R_3} \end{aligned}$$

(1) $i_s - \dfrac{u_1 - u_2}{R_1} = 0$ $\leftarrow$ Eliminated element currents!

(2) $\dfrac{u_1 - u_2}{R_1} - \dfrac{u_2}{R_2} - \dfrac{u_2}{R_3} = 0$

(3) $u_1 = V_s$

$V_s = 10V$

$R_1 = 1\Omega$

$R_2 = 2\Omega$

$R_3 = 2\Omega$

Matrix vector form: $A\vec{x} = \vec{b}$

$$\begin{array}{c} (1) \\ (2) \\ (3) \end{array} \begin{bmatrix} -\frac{1}{R_1} & \frac{1}{R_1} & 1 \\ \frac{1}{R_1} & -\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ i_s \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ V_s \end{bmatrix}$$

This method ALWAYS works. Let's simplify though.

(3) $\rightarrow$ (2) $\dfrac{V_s - u_2}{R_1} - \dfrac{u_2}{R_2} - \dfrac{u_2}{R_3} = 0$

$$\boxed{\begin{aligned} V_s &= 10V \\ R_1 &= 1\Omega \\ R_2 &= 2\Omega \\ R_3 &= 2\Omega \end{aligned}}$$

$$\frac{V_s}{R_1} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) \cdot u_2$$

$$u_2 = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)^{-1} \cdot \frac{1}{R_1} \cdot V_s$$

$$u_2 = \left(\frac{1}{1} + \frac{1}{2} + \frac{1}{2}\right)^{-1} \cdot \frac{1}{1} \cdot 10$$

$$= 2^{-1} \cdot 10$$

$= 2^{-1} \cdot 10$

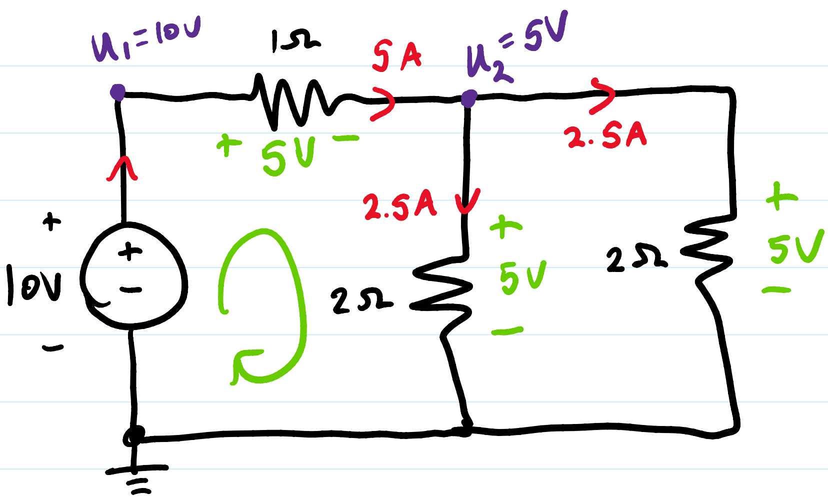
$\boxed{u_2 = 5V}$

3 $\boxed{u_1 = V_s = 10V}$

We've solved the circuit!

Check Results:

$V = iR \rightarrow i = \frac{V}{R}$



Check KCL:

@ u_2: $5A = 2.5A + 2.5A$

Check KVL

$+10V - 5V - 5V = 0V$