EECS 16A   Designing Information Devices and Systems I
Summer 2023   Final Review

**1. Some Proofs (Spring 2023 Midterm 1)**

(a) Prove that if there are two unique solutions $\vec{x}_1$ and $\vec{x}_2$ to the system $A\vec{x} = \vec{b}$, then there are infinitely many solutions $\vec{x}$ to this system.

(b) A matrix $\mathbf{A} \in \mathbb{R}^{N \times N}$ has eigenvalues λ_1, λ_2, ... λ_n and corresponding eigenvectors $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n$. Let $\lambda_1 = 0$, $\lambda_2 = 0$ and λ_3, λ_4, ... $\lambda_n \neq 0$. Prove that $\vec{x} \in Null(A)$ for any $\vec{x} \in span\{\vec{v}_1, \vec{v}_2\}$.
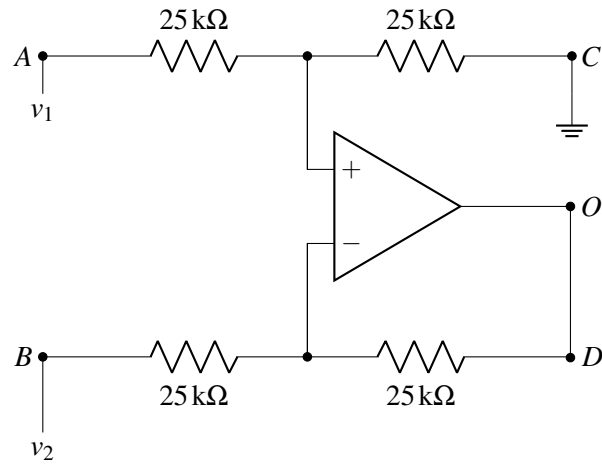
2. A Versatile Opamp Circuit

For each circuit configuration, determine the voltage at O, given that v_1 and v_2 are voltage sources. All circuit configurations are in negative feedback.
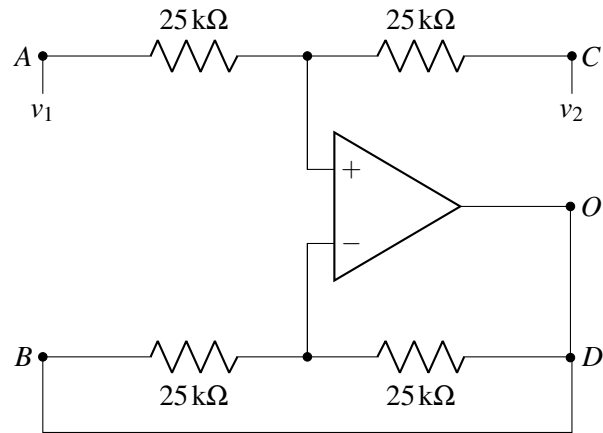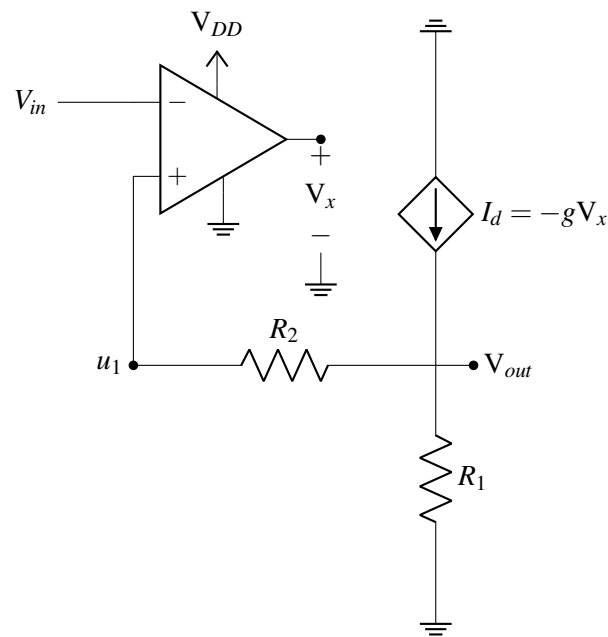
(a) Configuration 1:
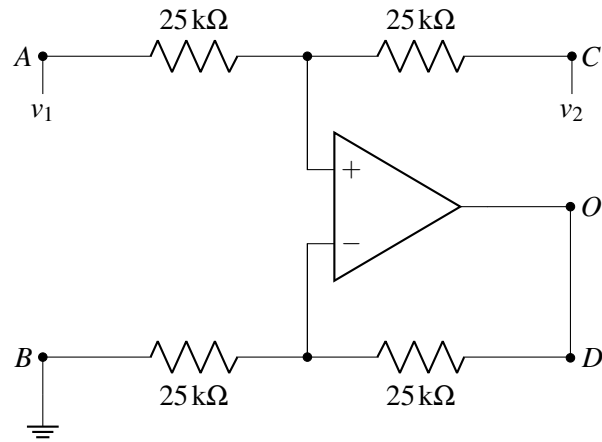
(b) Configuration 2:

(c) Configuration 3:

3. Can I Give You Some Feedback? (Spring 2023 Final Question 5)

The following circuit is a linear voltage regulator.

V_{DD} and V_{in} are both connected to ideal voltage sources. g is the gain factor of the dependent current source. The opamp has finite gain A.

Using the method for negative feedback analysis, if V_{out} **increases**, determine what happens to the following values. Circle one of the two options for each line below. *Note that if a quantity is getting more negative, that means it is decreasing.*

Voltage at u_1 will: Increase   Decrease
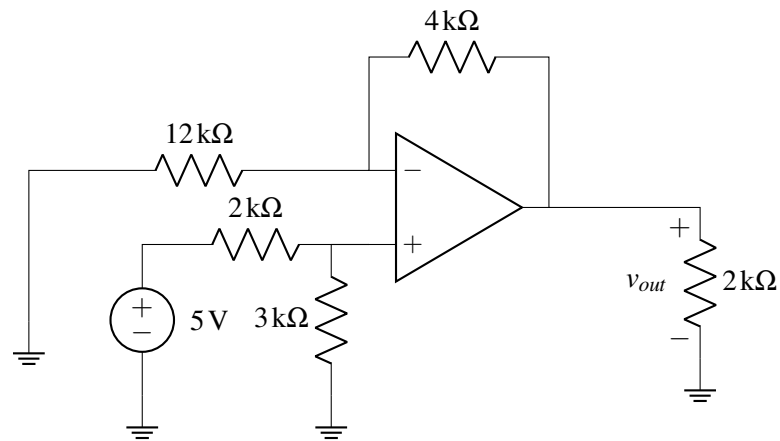
V_x will: Increase   Decrease

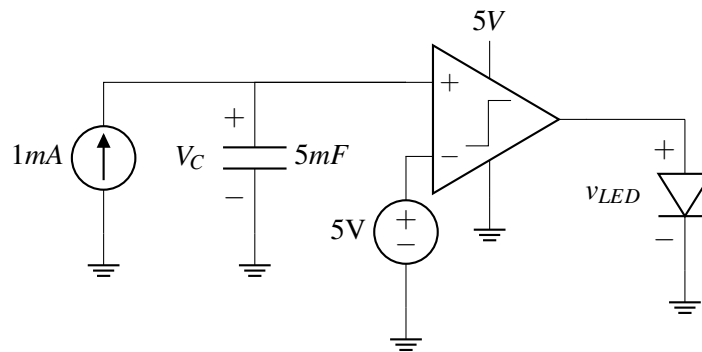Dependent current I_d will: Increase   Decrease

The circuit is in: Negative Feedback   Positive Feedback

4. Op-amps and Comparators (Spring 2022 Midterm 2 Question 10)

(a) You are given the following op-amp in negative feedback. Find v_{out}.

(b) You are given the circuit below. The capacitor is initially uncharged. At time $t = 0$, the current source is turned on. Find $V_c(t)$.

(c) The LED turns on when the voltage across it is greater than 3.3V. Using the same setup as part (b), at what time t does the LED turn on?

5. **An Easier Way To Do Math Homework (Spring 2023 Final Question 13)**

You're working on your Math 1B homework and you don't know how to calculate an integral. Instead, you decide to put your circuit skills to use to solve this problem!
The integral that you're trying to solve is of the form

$$-\frac{1}{5}\int_0^\tau x(t)\,dt$$

Your helpful lab TA, Raghav, gives you several circuit elements that you can use.

These elements are:

- A current source $I_s = x(t)$ amps in parallel with a capacitance of 1F.
- **Two** op-amps (assume that the supply voltages to the op-amps are provided).
- A resistor $R_{\text{fixed}} = 1\text{k}\Omega$.
- One additional resistor R_{extra} that can have **any value**. Be sure to specify the resistance you use.
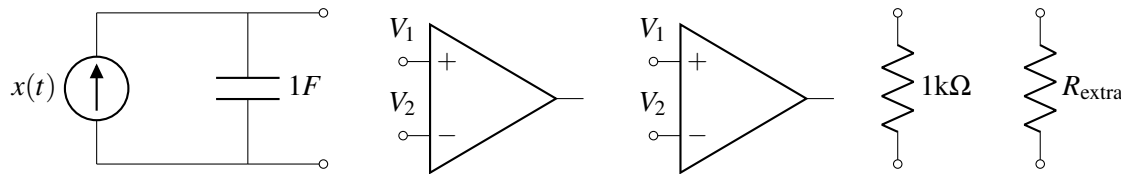
Design a circuit to have the above output using the provided elements. Clearly specify the resistance of R_{extra} if used.

6. Least Squares (Fall 2022 Final Question 3)

(a) Consider the system of equations $\vec{a}x = \vec{b}$ where $\vec{a}, \vec{b} \in \mathbb{R}^2$ and $x \in \mathbb{R}$. When applying least squares, we want to find the $\vec{v} \in \text{span}(\vec{a})$ that is closest to $\vec{b}$ in Euclidean distance.

Hint: It might be helpful to draw the vectors.

   i. When solving for vector $\vec{v}$, which of the following operations are required?

- ◯ Projecting $\vec{b}$ onto $\vec{a}$
- ◯ Projecting $\vec{a}$ onto $\vec{b}$
- ◯ Subtracting $\vec{b}$ from $\vec{a}$
- ◯ Subtracting $\vec{a}$ from $\vec{b}$
- ◯ None of the above

   ii. The vector $\vec{v}$ can also be determined by minimizing the length of the error vector, represented as

- ◯ $\vec{v} = \underset{\vec{b}}{\text{argmin}} \|\vec{a} - \vec{b}\|$
- ◯ $\vec{v} = \underset{\vec{v}}{\text{argmin}} \|\vec{a} - \vec{v}\|$
- ◯ $\vec{v} = \underset{\vec{b}}{\text{argmin}} \|\vec{b} - \vec{v}\|$
- ◯ $\vec{v} = \underset{\vec{v}}{\text{argmin}} \|\vec{b} - \vec{v}\|$

(b) For the following systems of $A\vec{x} = \vec{b}$, determine if they have a unique least squares solution.

   i. $A = \begin{bmatrix} 1 & 1 \\ 3 & 4 \\ 0 & 0 \end{bmatrix}, \quad \vec{b} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

- ◯ Yes
- ◯ No

   ii. $A = \begin{bmatrix} 1 & 4 \\ 3 & 12 \\ 2 & 8 \end{bmatrix}, \quad \vec{b} = \begin{bmatrix} 2 \\ 5 \\ 6 \end{bmatrix}$

- ◯ Yes
- ◯ No

(c) For the following three questions, consider the system of $A\vec{x} = \vec{b}$ with $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$

    i. Can we apply the least squares formula?

       ○ Yes

       ○ No

    ii. What is the determinant of $A^T A$?

$$\det(A^T A) = \boxed{}$$

   iii. (1 point) Does $A\vec{x} = \vec{b}$ have zero, one, or more than one solution for $\vec{x}$?

       ○ No solutions

       ○ One unique solution

       ○ More than one solution

(d) Find the best approximation $x = \hat{x}$ to this system of equations:

$$a_1 x = b_1$$
$$a_2 x = b_2$$

i. Write the problem into $A\vec{x} \approx \vec{b}$ format and solve for $\hat{x}$ using least squares. Choose the correct $\hat{x}$.

○ $\hat{x} = \dfrac{a_1 b_1 + a_2 b_2}{a_1^2 + a_2^2}$

○ $\hat{x} = \dfrac{a_1 b_1 - a_2 b_2}{a_1^2 + a_2^2}$

○ $\hat{x} = \dfrac{a_1 b_2 + a_2 b_1}{a_1^2 + a_2^2}$

○ $\hat{x} = \dfrac{a_1 b_2 - a_2 b_1}{a_1^2 + a_2^2}$

○ None of the above

ii. Suppose the inner product is defined instead as a non-Euclidean $\langle x, y \rangle = x^T \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} y$. Which of the following expressions must be true with respect to the minimized least squares error vector, $\vec{e}$?

○ $\vec{e}^T A = \vec{0}$

○ $A^T \vec{e} = \vec{0}$

○ $A^T \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \vec{e} = \vec{0}$

○ $\left(A^T \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} A \right)^{-1} \vec{e} = \vec{0}$

○ None of the above