# EECS 16A Designing Information Devices and Systems I
# Summer 2023 Homework 6

**This homework is due Friday, July 28th, 2023, at 23:59.**
**Self-grades are due Friday, August 4th, 2023, at 23:59.**

**Submission Format**
Your homework submission should consist of **one** file.

- `hw6.pdf`: A single PDF file that contains all of your answers (any handwritten answers should be scanned).

Submit the file to the appropriate assignment on Gradescope.

**Mid-Semester Survey**

Please fill out the mid-semester survey: https://forms.gle/XKNPXWDidcsoM7LB9.

We highly value and appreciate your feedback!

**1. Reading Assignment**

For this homework, please read Note 17, 18, and 19. Note 17 introduces comparators and how we can use them to detect touch in a capacitive touchscreen. Note 18 and 19 provide an overview of op-amps, including the "golden rules" of op-amps, and various op-amp configurations. You are always encouraged to read beyond this as well.

(a) Consider the capacitive touchscreen. Briefly describe how it works: what quantity changes when your finger touches it? Compare and contrast it to the resistive touchscreens we have seen in previous lectures and homeworks.

(b) What is the purpose of a comparator? How can we use a comparator circuit to detect a touch for a capacitive touchscreen?

(c) If the op-amp supply voltages are $V_{DD} = 5\,\text{V}$ and $V_{SS} = 0\,\text{V}$, then what are the minimum and maximum value of $V_{out}$?

(d) What does the internal gain of an op-amp, $A$, mean? What is its value for an ideal op-amp? What about for a non-ideal one?

(e) What are the two "golden rules" of ideal op-amps? When do these rules hold true?

(f) What is the effect of "loading" and how can op-amps be used to mitigate this effect?

**2. Pre-Lab Questions**

These questions pertain to the Pre-Lab reading for the APS lab. You can find the reading under the APS Lab section on the 'Schedule' page of the website.

(a) What two devices do we use in the APS setup as signal emitter and receiver?

(b) What is the formula for the time delay of arrival of the signal emitted from a speaker? Provide an expression in terms of the number of samples and sampling frequency ($f_s$)

(c) What value of $\theta$ maximizes the dot product $\mathbf{a} \cdot \mathbf{b}$? HINT: Think of what value $\theta$ maximizes the function $\cos\theta$

**3. Digital to Analog Converter (DAC)**

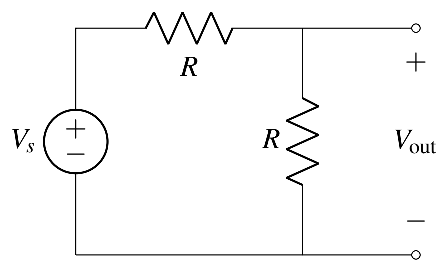
For some outputs, such as audio applications, we need to produce an analog output, or a continuous voltage from 0 to $V_s$. These analog voltages must be produced from digital voltages, that is sources, that can only be $V_s$ or 0. A circuit that does this is known as a Digital to Analog Converter. It takes a binary representation of a number and turns it into an analog voltage.

The output of a DAC can be represented with the equation shown below:

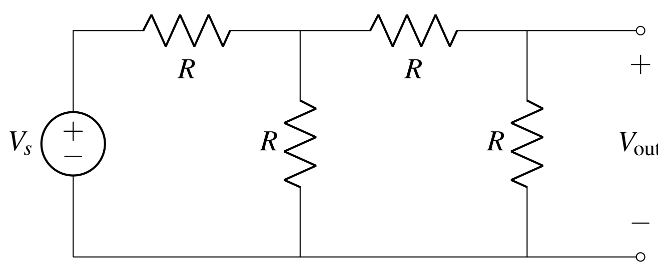
$$V_{\text{out}} = V_s \sum_{n=0}^{N} \frac{1}{2^n} \cdot b_n$$

where each binary digit $b_n$ is multiplied by $\frac{1}{2^n}$.

(a) We know how to take an input voltage and divide it by 2:



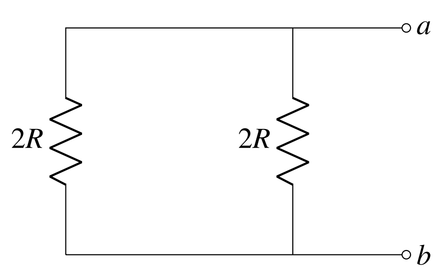

To divide by larger powers of two, we might hope to just "cascade" the above voltage divider. For example, consider:



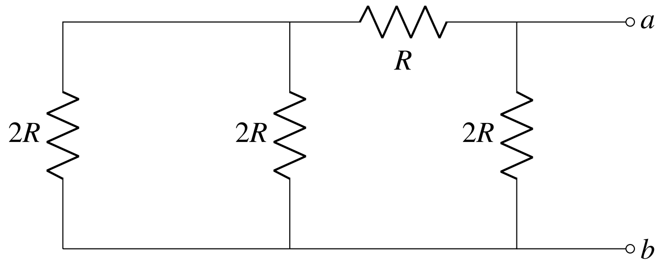

Calculate $V_{\text{out}}$ in the above circut. Is $V_{\text{out}} = \frac{1}{4}V_s$?

(b) The $R$-$2R$ ladder, shown below, has a very nice property. For each of the circuits shown below, find the equivalent resistance looking in from points $a$ and $b$. Do you see a pattern?

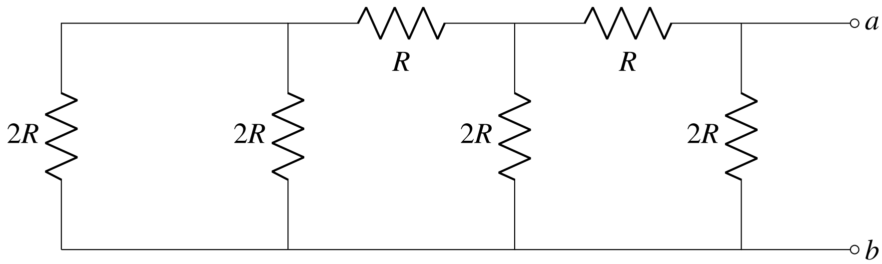
i.



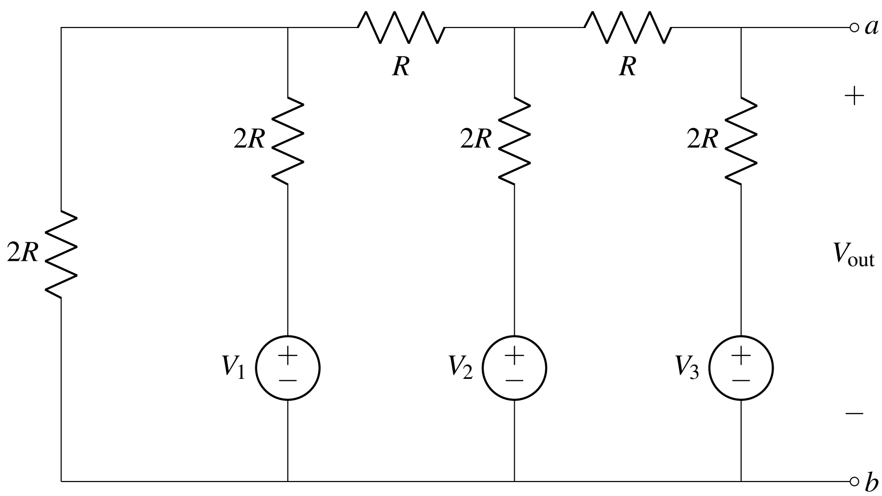
ii.




iii.



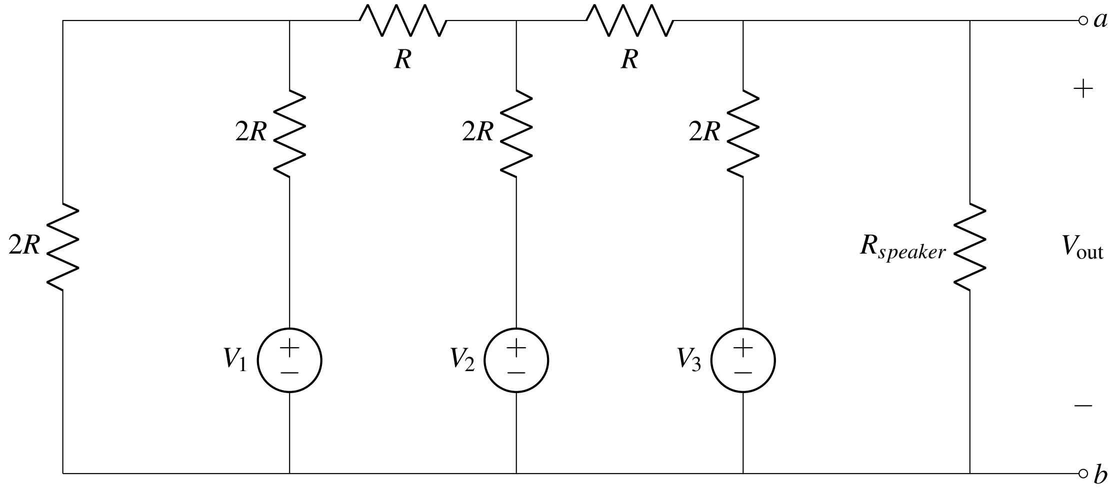

(c) The following circuit is an $R$-$2R$ DAC. To understand its functionality, use superposition to find $V_{\text{out}}$ in terms of each $V_k$ in the circuit.




(d) We've now designed a 3-bit $R$-$2R$ DAC. What is the output voltage $V_{\text{out}}$ if $V_2 = 1\,\text{V}$ and $V_1 = V_3 = 0\,\text{V}$?

(e) Draw the Thévenin equivalent of the above circuit, looking in from the terminals $a$ and $b$ with $V_2 = 1\,\text{V}$ and $V_1 = V_3 = 0\,\text{V}$.

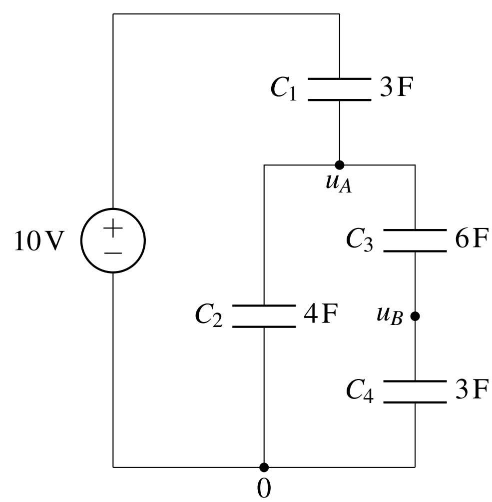
(f) Suppose that we now attach a speaker to the DAC with a resistance of $R_{speaker} = 50\Omega$ as shown in the figure below. Assume also, that the value of R is $50\Omega$ as well. What is the voltage across the speaker and the power dissipated by the speaker? *Hint:* Use the Thevenin equivalent circuit to calculate the above.




(g) Repeat part (f) now assuming that the speaker resistance is 100Ω. The value of R remains 50Ω. How do the power and voltage values you found in the two parts compare? Why is the voltage across the speaker in both cases lower than $V_{th}$?

## 4. Circuit with Capacitors

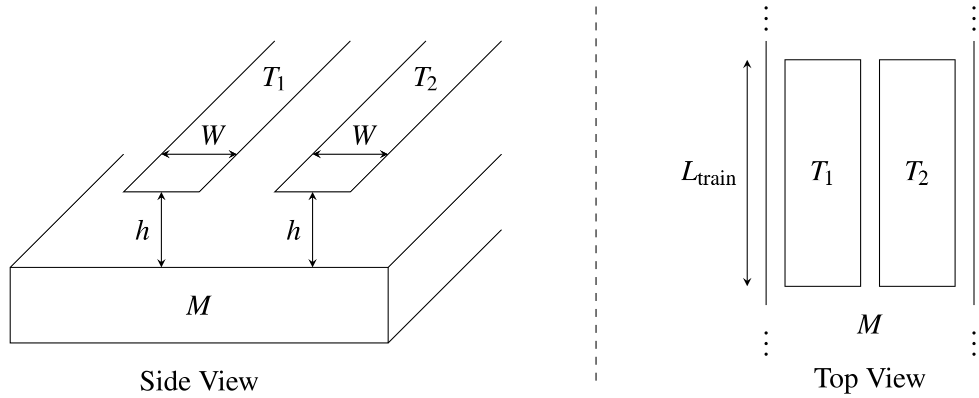
Find the voltages at nodes $u_A$ and $u_B$, and currents flowing through all of the capacitors at steady state. Assume that before the voltage source is applied, the capacitors all initially have a charge of 0 Coulombs.




## 5. Maglev Train Height Control System

One of the fastest forms of land transportation are trains that actually travel slightly elevated from the ground using magnetic levitation (or "maglev" for short). Ensuring that the train stays at a relatively constant height above its "tracks" (the tracks in this case are what provide the force to levitate the train and propel it forward) is critical to both the safety and fuel efficiency of the train. In this problem, we'll explore how maglev trains use capacitors to stay elevated. (Note that real maglev trains may use completely different and much more sophisticated techniques to perform this function, so if you get a contract to build such a train, you'll probably want to do more research on the subject.)

(a) As shown below, we put two parallel strips of metal ($T_1$, $T_2$) along the bottom of the train and we have one solid piece of metal (M) on the ground below the train (perhaps as part of the track).




We assume that the entire train is at a uniform height above the track, so we can use the simple equations developed in lecture to model the capacitance.

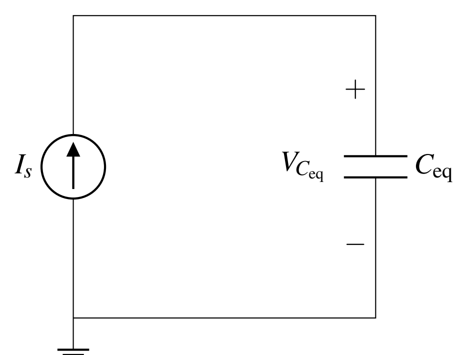
As a function of $L_{\text{train}}$ (the length of the train), $W$ (the width of $T_1$ and $T_2$), and $h$ (the height of the train above the track), determine the capacitances between $T_1$ and M and between $T_2$ and M. *Hint: Note that the area of a capacitor is given by the overlap area between its two plates.*

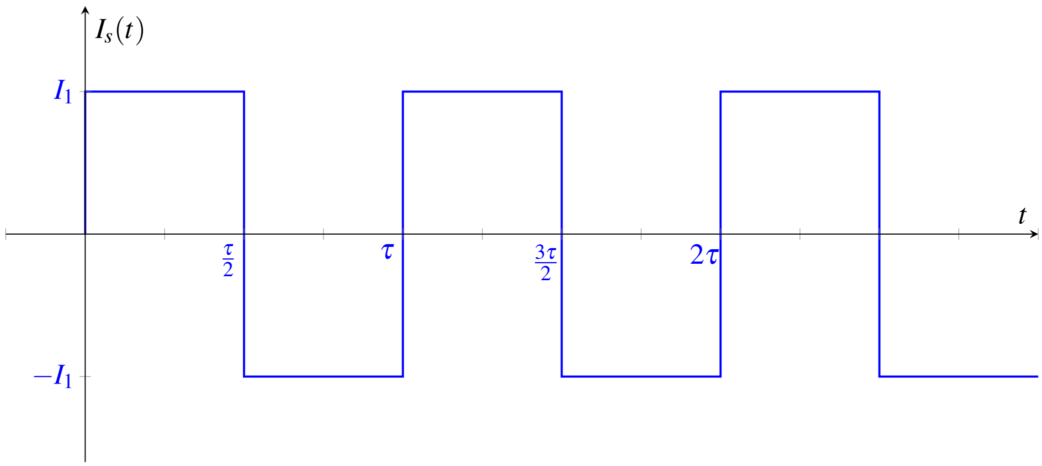
(b) Any circuit on the train can only make direct contact at $T_1$ and $T_2$. Thus, you can only measure the equivalent capacitance between $T_1$ and $T_2$. Draw a circuit model showing how the capacitors between $T_1$ and M and between $T_2$ and M are connected to each other.

(c) Using the same parameters as in part (a), provide an expression for the equivalent capacitance measured between $T_1$ and $T_2$.

(d) We want to build a circuit that creates a voltage waveform with an amplitude that changes based on the height of the train. Your colleague recommends you start with the circuit as shown below, where $I_s$ is a periodic current source, and $C_{\text{eq}}$ is the equivalent capacitance between $T_1$ and $T_2$. The graph below shows $I_s$, a square wave with period $\tau$ and amplitude $I_1$, as a function of time $t$.

**Find an equation for and draw the voltage $V_{C_{\text{eq}}}(t)$ as a function of time $t$.** Assume the capacitor $C_{\text{eq}}$ is discharged at time $t = 0$, so $V_{C_{\text{eq}}}(0) = 0\,\text{V}$.






(e) We now want to develop an indicator that alerts us when the train is too high above the tracks. We want to have an output of 5 V (to trigger the alert) when the height of the train $h$ is above 1 cm, and an output of 0 V when $h$ is below 1 cm.

We will assume the train has length $L_{\text{train}} = 100\,\text{m}$ and that the metals, $T_1$ and $T_2$, have width $W = 1\,\text{cm}$ and permitivity $\varepsilon = 8.85\text{e} - 12\text{F}\,\text{m}^{-1}$.

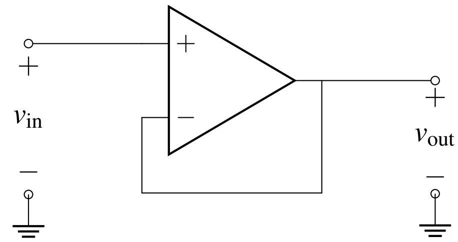
Design a circuit using **a square wave current source (i.e. $I_s$ in part (d)) with period $\tau = 1\,\mu\text{s}$ and pulses of amplitude $I_1 = 1\,\text{mA}$, a comparator, and any number of voltage sources** to implement this function. *Hint: You should use the circuit you analyzed in part (d).*

## 6. Testing for Negative Feedback

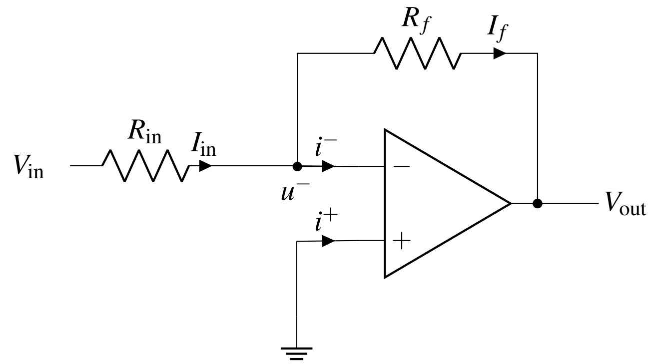
While it is tempting to say "if the feedback voltage is connected to the negative op-amp terminal, then we have negative feedback", this is not always true. Here is a two-step procedure for determining if a circuit is in negative feedback:

- **Step 1: Zero out all independent sources**, replacing voltage sources with wires and current sources with opens as we did in superposition. You do not need to zero out the voltage sources that serve as the power supplies to the op-amp, since they are not treated as signals and are not considered part of the op-amp.
- **Step 2: Wiggle the output and check the loop**. Assume that the output increases slightly. Check the direction of change of the feedback signal and the error signal from the circuit. Any change in the error signal will cause a new change in the output. This change is the feedback loop's response to the initial change.
  - If the error signal decreases, then the output must also decrease. This is the *opposite direction* we initially assumed, i.e. the loop is trying to correct for the change. So the circuit is in negative feedback.
  - If the error signal instead increased, then the output would also increase. This is the *same direction* we initially assume, i.e. the initial increase lead to further increase. We call this positive feedback.

(a) Show that the voltage buffer circuit is in negative feedback. Note that here $v_{\text{in}}$ is acting as a voltage source.

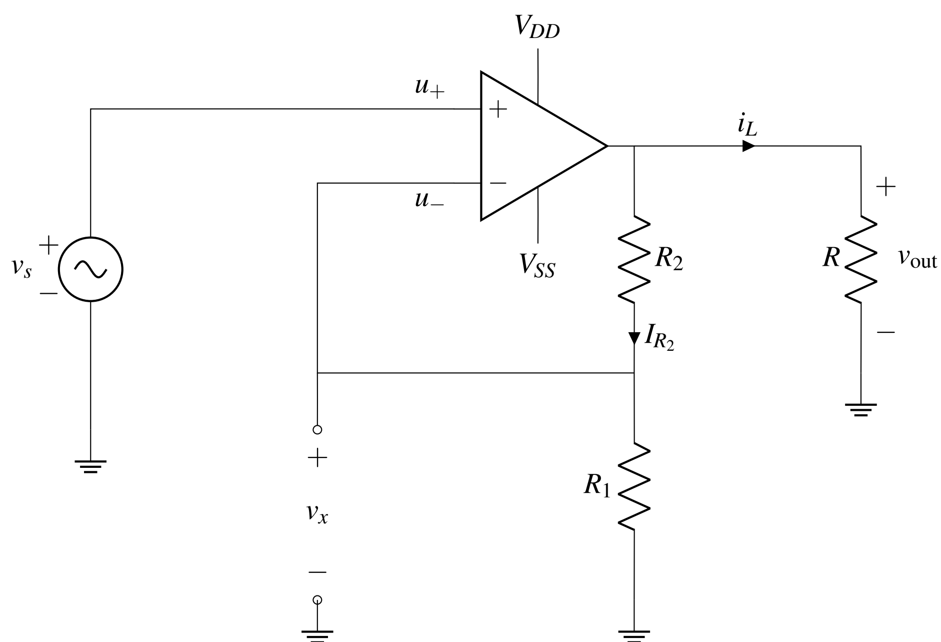



(b) Show that the inverting amplifier circuit is in negative feedback.




## 7. Op-Amp in Negative Feedback

In this question, we analyze op-amp circuits that have finite op-amp gain $A$. We replace the op-amp with an equivalent circuit model with parameterized gain, $A$, and observe the gain's effect on the terminal and output voltages as the gain approaches infinity. **Note here that $V_{SS} = -V_{DD}$.**




**For parts (a) - (e) only, assume that the op-amp is ideal (i.e., $A \to \infty$).** We will consider the case of finite gain $A$ in parts (f) - (h).

(a) Consider the circuit shown above and $V_{SS} = -V_{DD}$. What is $u_+ - u_-$?

(b) Find $v_x$ as a function of $v_{out}$. *Hint: What is the current into the negative terminal $u_-$ of the op-amp?*

(c) What is $I_{R_2}$, i.e. the current flowing through $R_2$ as a function of $v_s$? *Hint: Find the current through $R_1$ first.*

(d) Find $v_{out}$ as a function of $v_s$.

(e) What is the current $i_L$ through the load resistor $R$? Give your answer in terms of $v_{out}$.

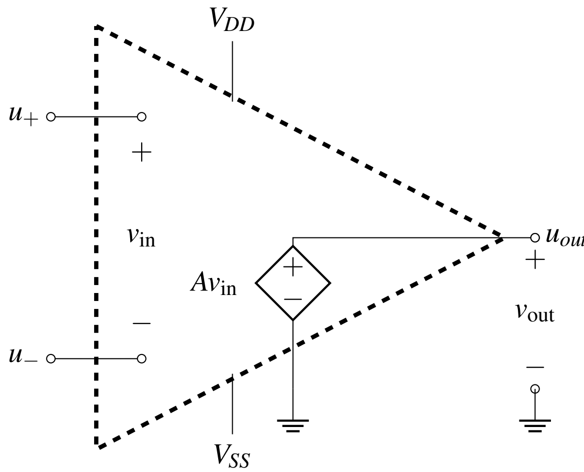



Figure 1: Op-amp model with finite gain, $A$

(f) We will now examine what happens when $A$ is not $\infty$. To understand what happens in this case, first draw an equivalent circuit for the first op amp circuit, **by replacing the ideal op-amp in the non-inverting amplifier with the op-amp model shown above.**

Now, using this setup, calculate $v_{\text{out}}$ and $v_x$ in terms of $A$, $v_s$, $R_1$, $R_2$, and $R$. Is the magnitude of $v_x$ larger or smaller than the magnitude of $v_s$? Do these values depend on $R$? *Hint: Note that the first golden rule still applies, i.e. the currents through the input terminals are zero.*

(g) Using your solution to the previous part, calculate the limits of $v_{\text{out}}$ and $v_x$ as $A \to \infty$. You should get the same answer as in part (d) for $v_{\text{out}}$.

## 8. Transresistance Amplifier

A common use of an op-amp is to convert a current signal into a voltage signal. This configuration is called a *transresistance amplifier*, as shown in Figure 2. (Note: In the real world, we call this a trans*impedance* amplifier. Impedance is just a fancy word to describe resistance as a function of frequency.) Assume that $V_{SS} = -V_{DD}$ for all the parts of this problem.

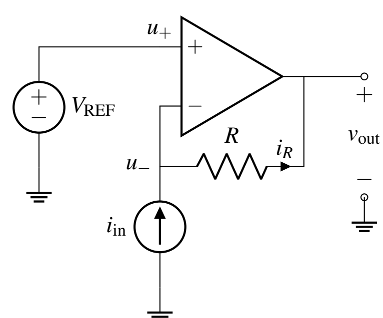



Figure 2: Transresistance amplifier

(a) What is the value of the current $i_R$ in Figure 2? *Hint: Your answer should be in terms of $i_{in}$.*

(b) What is the voltage at the negative terminal of the op-amp $u_-$ in terms of $V_{\text{REF}}$.

(c) Using the results from parts (a) and (b), find an expression for $v_{\text{out}}$ in terms of $V_{\text{REF}}$ and $i_{\text{in}}$.

(d) If we set $V_{\text{REF}} = 0\,\text{V}$, calculate the gain of the overall circuit $G = \frac{v_{\text{out}}}{i_{\text{in}}}$.

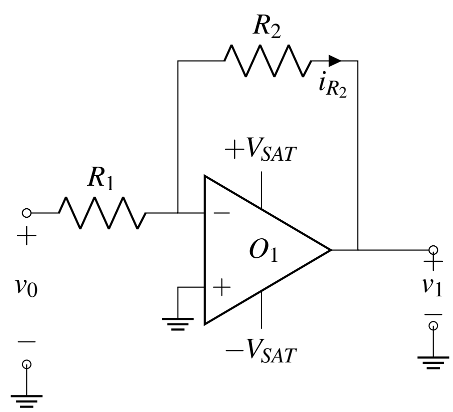
**9. Integration using Op-amps**

Analog circuits can be used to implement many different mathematical functions. In this problem, we will see how we can use an op-amp to create an integrator. An integrator circuit takes a time-varying voltage input $v_0(t)$ and integrates it over a time period. In other words, we want to build a circuit where the output is of the form

$$v_1(t) = K \int_0^t v_0(\tau) d\tau$$

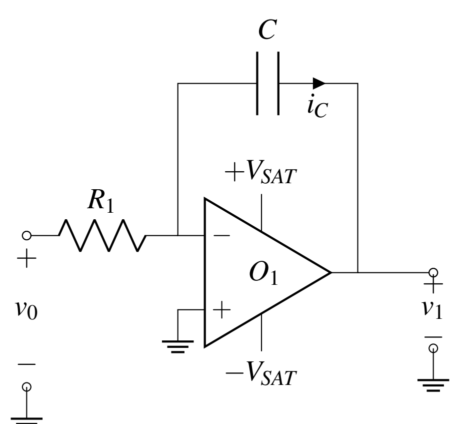
for some constant $K$.

(a) Let's analyze the inverting op-amp configuration shown below. For this problem, we will assume that the op-amp is ideal and apply the op-amp golden rules.



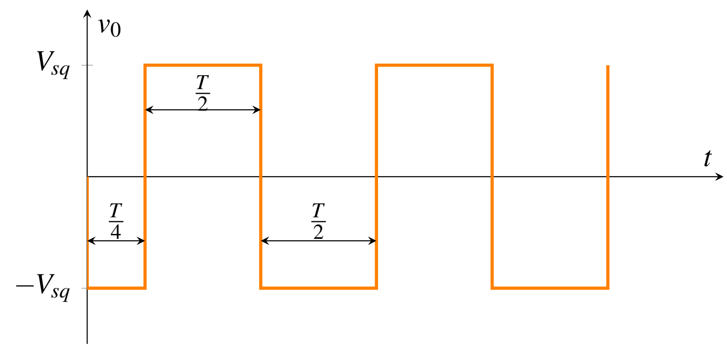

What is the current $i_{R_2}$ flowing through resistor $R_2$? Write your answer in terms of $v_0$, $R_1$, and $R_2$.

(b) What happens if we replace the resistor in feedback $R_2$ with a capacitor $C$ instead? Analyze the circuit to find the current through the capacitor $i_C$ and express your answer in terms of $v_0$, $R_1$, and $C$. How does this current differ from the previous part?



(c) Assume that the capacitor starts uncharged at $t = 0$ and that $v_0(t)$ varies with time. Solve for the output voltage $v_1(t)$ as a function of time $t$. Express your answer in terms of $v_0(t)$, $R_1$, and $C$.
*Hint: You may leave your answer as an integral of $v_0(t)$ as shown in the initial problem statement.*

(d) If $v_0$ varies with time as shown in the following diagram, plot $v_1$ for $t = 0$ to $t = 1.5T$. In your plot indicate an algebraic expression for the slope (as a function of $R_1$, $C$ and $V_{sq}$) and add tick marks on the x and y axis indicating the time and voltage values where the ramp slope changes. You may assume again that capacitor $C$ has 0V across it at time $t = 0$.



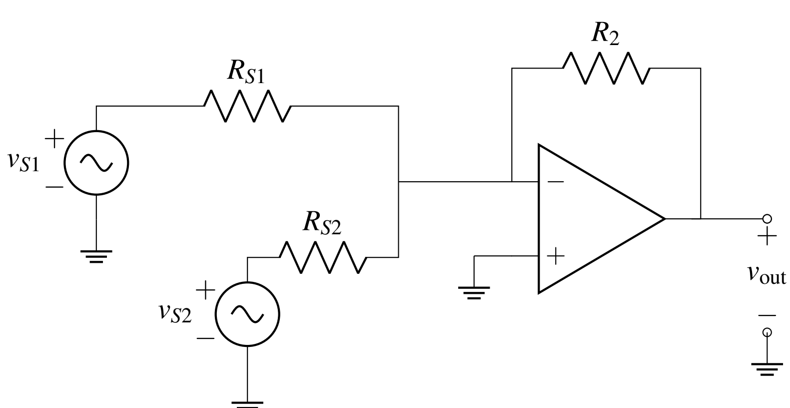

## 10. Cool For The Summer

You and a friend want to make a box that helps control an air conditioning unit based on both your inputs. You both have individual dials which you can use to control the voltage. An input of 0 V means that you want to leave the temperature as is. A **negative voltage input** means that you want to **reduce** the temperature. (It's hot out, so we will assume that you never want to increase the temperature – so no, we're not talking about a Berkeley summer...)

Your air conditioning unit, however, responds only to **positive voltages**. The higher the magnitude of the voltage, the stronger it runs. At zero, it is off. You also need a system that **sums up** both you and your friend's control inputs.

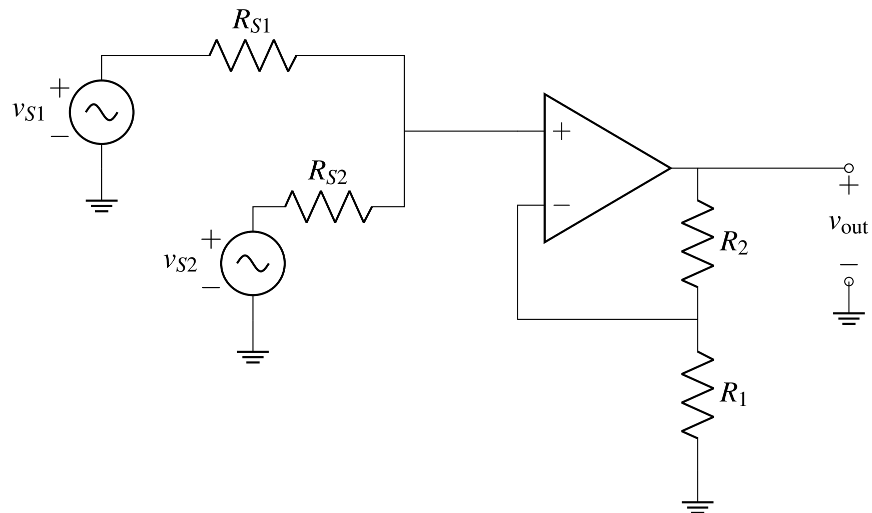
Therefore, you need a box that acts as an **an inverting summer** – *it outputs a weighted sum of two voltages where the weights are both negative*. The sum is weighted because one room is bigger, so you need to compensate for this.

(a) You suggest the circuit below, essentially an inverting amplifier with two inputs. Find $v_{\text{out}}$ in terms of $v_{S1}$, $v_{S2}$, $R_{S1}$, $R_{S2}$ and $R_2$.
*Hint: You can solve this problem using either superposition or our tried-and-true KCL analysis.*



(b) Let's suppose that you want $v_{\text{out}} = -\left(\frac{1}{4}v_{S1} + 2v_{S2}\right)$ where again $v_{S1}$ and $v_{S2}$ represent the input voltages from you and your friend's control knobs. Select resistor values such that the circuit from part (b) implements this desired relationship.

(c) Your friend has a different circuit idea. He proposes the following circuit below.




Find $v_{\text{out}}$ in terms of $v_{S1}$, $v_{S2}$, $R_{S1}$, $R_{S2}$, $R_1$, and $R_2$. Can we also use this circuit to control our AC system? Why or why not?
*Hint: How does this circuit relate to the one in question 2?*

**11. Homework Process and Study Group**

Who did you work with on this homework? List names and student ID's. (In case you met people at homework party or in office hours, you can also just describe the group.) How did you work on this homework? If you worked in your study group, explain what role each student played for the meetings this week.