
EECS 16A Designing Information Devices and Systems I

Summer 2023 Discussion 1D

1. Gaussian Elimination

Use Gaussian elimination to solve the following systems. Does a solution exist? Is it unique?

(a)

$$\left[\begin{array}{ccc|c} 2 & 0 & 4 & 6 \\ 0 & 1 & 2 & -3 \\ 1 & 2 & 0 & 3 \end{array} \right]$$

(b)

$$\left[\begin{array}{ccc|c} 1 & 4 & 2 & 2 \\ 1 & 2 & 8 & 0 \\ 1 & 3 & 5 & 3 \end{array} \right]$$

(c)

$$\left[\begin{array}{ccc|c} 2 & 2 & 3 & 7 \\ 0 & 1 & 1 & 3 \\ 2 & 0 & 1 & 1 \end{array} \right]$$

- (d) True or False: A system of equations with more equations than unknowns will always have either infinite solutions or no solutions.

(e) **(Practice)**

$$\left[\begin{array}{ccc|c} 3 & -1 & 2 & 1 \\ 0 & 0 & 2 & 1 \end{array} \right]$$

(f) **(Practice)**

$$\left[\begin{array}{ccccccc} 2x & + & 4y & + & 2z & = & 8 \\ x & + & y & + & z & = & 6 \\ x & - & y & - & z & = & 4 \end{array} \right]$$

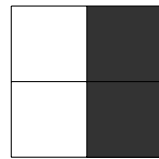
2. Finding The Bright Cave

Nara the one-handed druid and Kody the one-handed ranger find themselves in dire straits. Before them is a cliff with four cave entrances arranged in a square: two upper caves and two lower caves. Each entrance emits a certain amount of light, and the two wish to find exactly the amount of light coming from each cave. Here's the catch: after contracting a particularly potent strain of ghoul fever, our intrepid heroes are only able to see the total intensity of light before them (so their eyes operate like a single-pixel camera). Kody and Nara are capable adventurers, but they don't know any linear algebra – and they need your help.

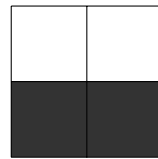
Kody proposes an imaging strategy where he uses his hand to completely block the light from two caves at a time. He is able to take measurements using the following four masks (black means the light is blocked from that cave):

Cave Labels

x_1	x_2
x_3	x_4



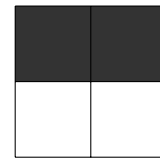
Measurement 1



Measurement 2



Measurement 3



Measurement 4

Figure 1: Four image masks.

- (a) Let x_1 , x_2 , x_3 , and x_4 represent the magnitude of light emanating from the four cave entrances shown in the image above. Write an equation for each masking process in Figure 1 which results in the four measurements of total light: m_1 , m_2 , m_3 , and m_4 . Then, create an augmented matrix that represents this system.
- (b) Does Kody's set of masks give us a unique solution for all four caves' light intensities? Why or why not?
- (c) Nara, in her infinite wisdom, places her one hand diagonally across the entrances, covering two of the cave entrances. However, her hand is not wide enough, letting in 50% of the light from the caves covered and 100% of the light from the caves not covered. The following diagram shows the percentage of light let through from each cave:

50%	100%
100%	50%

Does this additional measurement give them enough information to solve the problem? Why or why not?