Lecture 5B: (7/18/23)

Announcements:

- Lab – Touch 1

Today's Topics:

- Superposition
- Circuit Equivalence
  - Equivalent Resistance
  - Thevenin/Norton Equivalents

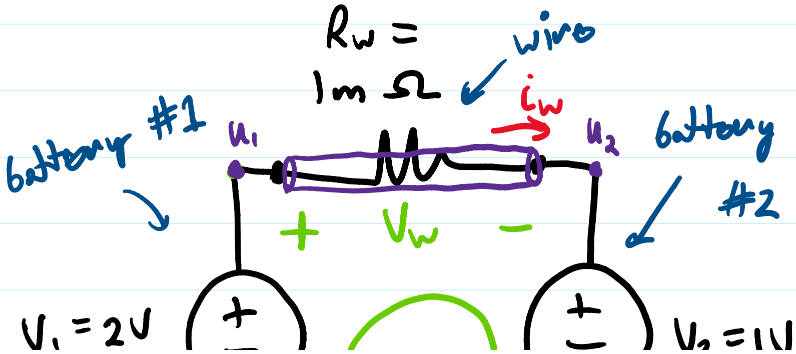
} Note 15

Superposition

NVA always works, however can we linearly decompose a circuit into simpler circuits? YES!

Ex). Connecting two batteries

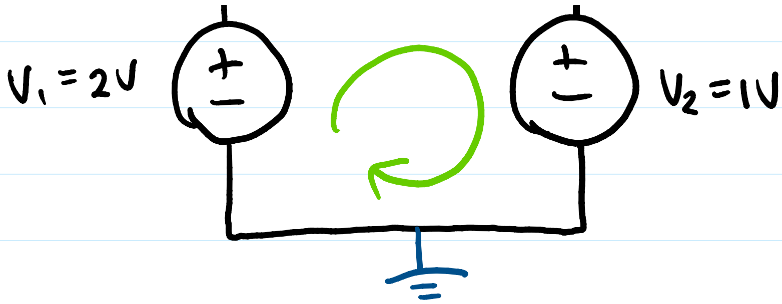
What's the current i_w through the wire?



Option 1: NVA

$u_1 = 2V \quad u_2 = 1V$

$i_w = \frac{u_1 - u_2}{[illegible]} = 1000A$



$v_1 = 2V \quad v_2 = 1V$

$i_w = \frac{v_1 - v_2}{1m\Omega} = 1000A$

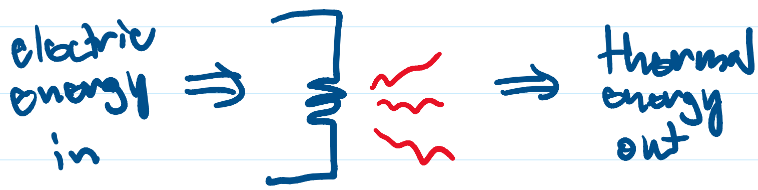
Option 2: KVL

$2V - V_w - 1V = 0$

$V_w = 1V$

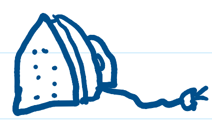
$i_w = \frac{V_w}{1m\Omega} = 1000A$

This much current would melt the wire



power: $P = V_w \cdot i_w = (1V)(1000A) = 1000W$

enough energy/second to power a steam iron

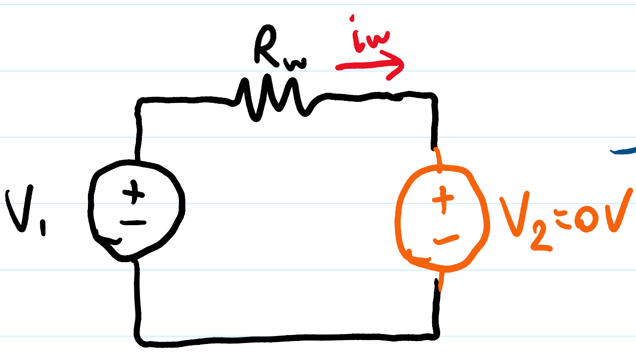


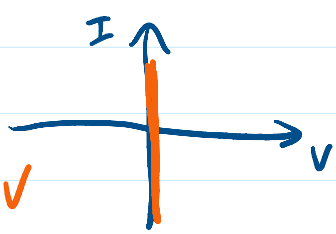
Maybe we shouldn't connect two batteries together with a wire...

Back to superposition

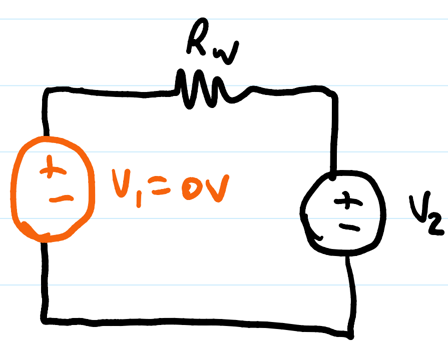
→ what if we "nulled" one of the voltage sources?

Null V_2



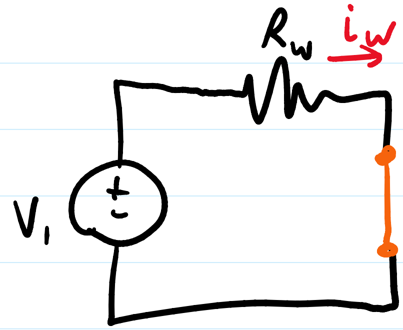


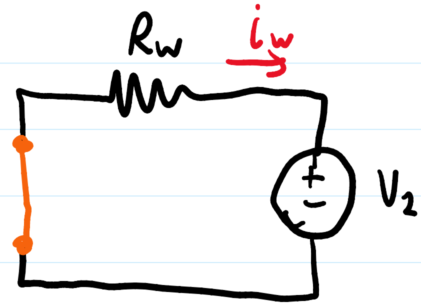
Null V_1



⇓ equivalent to a short circuit! ⇓

short circuit.





$$i_w\Big|_{V_2=0} = \frac{V_1}{R_w} \qquad\qquad i_w\Big|_{V_1=0} = -\frac{V_2}{R_w}$$

What if?

$$i_w = i_w\Big|_{V_2=0} + i_w\Big|_{V_1=0}$$

i_w from V_1 i_w from V_2

$$= \frac{V_1}{R_w} - \frac{V_2}{R_w} = \frac{2V}{1m\Omega} - \frac{1V}{1m\Omega} = 2000A - 1000A$$

$$= 1000A$$ ← same as before!

This is superposition! Application of the same linearity property from Module #1

$$f(x+y) = f(x) + f(y)$$

$f(\cdot)$ is our LINEAR circuit

$$f(\vec{x}) = \vec{b}$$

linear

$$A\vec{x} = \vec{b}$$

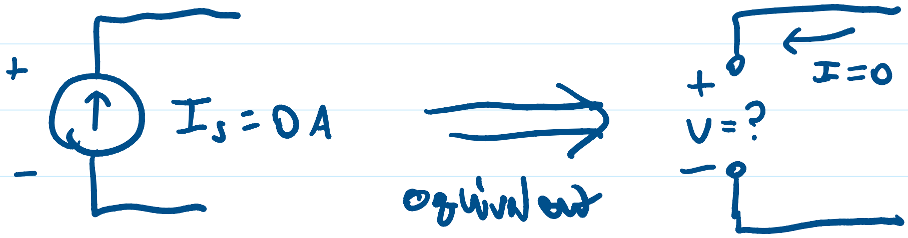
unique solution

node voltages   voltage/current sources

$$\vec{x} = A^{-1}\vec{b} \rightarrow \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix} = A^{-1}\begin{bmatrix} I_{s_1} \\ I_{s_2} \\ V_{s_1} \\ \vdots \end{bmatrix} \rightarrow u_1 = \alpha_1 I_{s_1} + \beta_1 \cdot I_{s_2} + \gamma_1 V_{s_1} + \dots$$

Node voltage is linear combination of sources

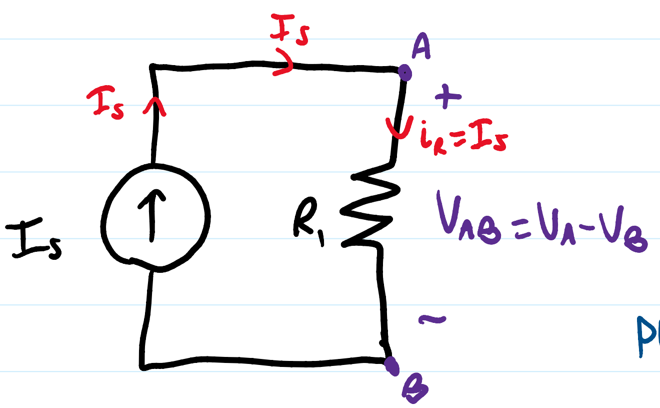
What about nulling current sources?



equivalent to an open circuit

Circuit Equivalence:

We have a circuit and we want to connect another resistor to it ← connect to two nodes A, B

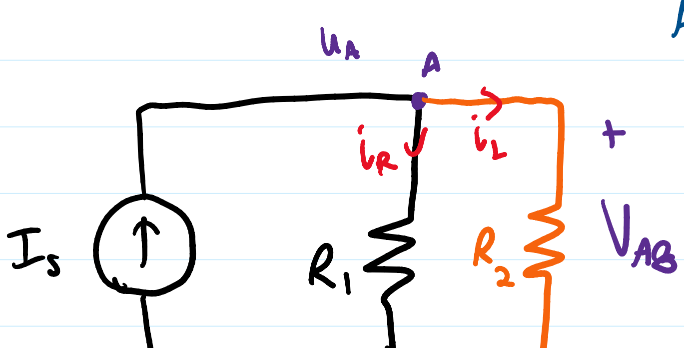


$i_R = I_s$ ✓

$V_{ab} = i_R \cdot R_1 = I_s \cdot R_1 = 3V$

Plug in some values! $I_s = 3A$, $R_1 = 1\Omega$

Does V_{ab} change when we add resistor R_2 between nodes A and B?

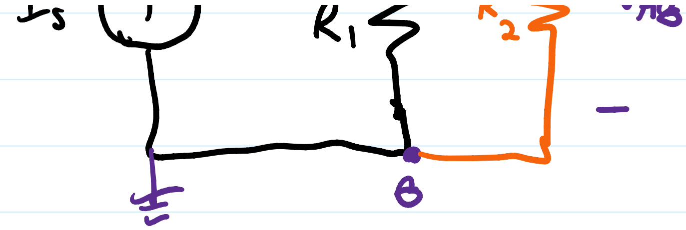


Apply NVA:

KCL @ A: $I_s - i_R - i_L = 0$

$$I_s - \frac{u_A}{R_1} - \frac{u_A}{R_2} = 0$$

Simplify: $I_s - \left(\frac{1}{R_1} + \frac{1}{R_2}\right) \cdot u_A = 0$



Simplify: $I_S - \left(\frac{1}{R_1}+\frac{1}{R_2}\right)\cdot u_A = 0$

$$u_A = I_S \cdot \left(\frac{1}{R_1}+\frac{1}{R_2}\right)^{-1}$$

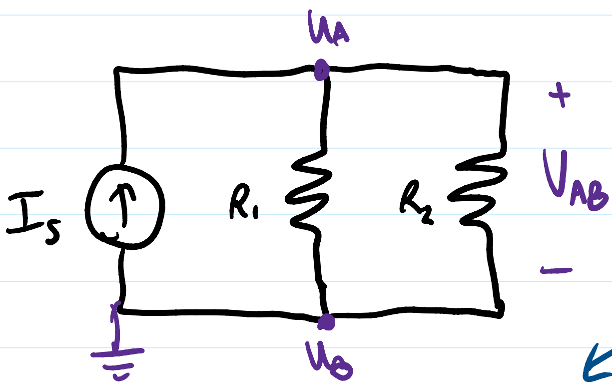
Plug in same numbers! $I_S = 3A$, $R_1 = 1\Omega$, $R_2 = 2\Omega$ → $u_A = 3\cdot\left(\frac{1}{1}+\frac{1}{2}\right)^{-1} = 3\cdot\frac{2}{3}$ $= 2V$

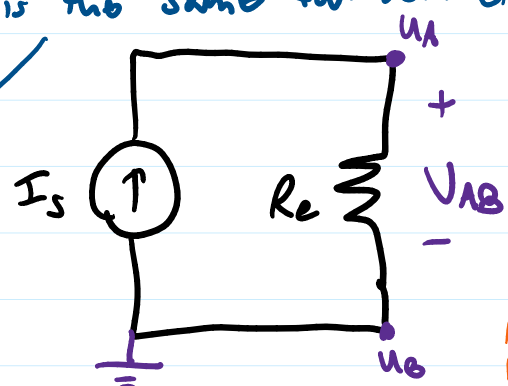
The voltage decreased from 3V to 2V after adding R_2

Can this circuit be equivalently represented with other circuits?

Equivalent Resistance

Can we pick R_e so that V_{AB} is the same for both circuits?





$$u_A - u_B = V_{AB} = \underbrace{I_S\cdot\left(\frac{1}{R_1}+\frac{1}{R_2}\right)^{-1}}_{}$$

Looks a lot like ohm's Law: $V = I\cdot R$

Hmmm...

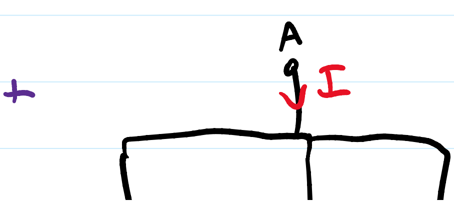
$I_S = 3A$
$R_1 = 1\Omega$
$R_2 = 2\Omega$

$$R_e = \left(\frac{1}{R_1}+\frac{1}{R_2}\right)^{-1} = \left(\frac{1}{1}+\frac{1}{2}\right)^{-1} = \frac{2}{3}\Omega$$

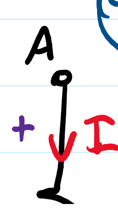
$$V_{AB} = I_S \cdot R_e = (3A)\cdot\left(\tfrac{2}{3}\Omega\right) = 2V$$

Same as before!

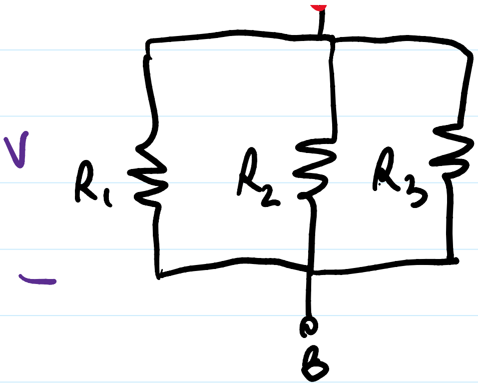
In this example, R_1 and R_2 are connected in "parallel." (share same voltage across)



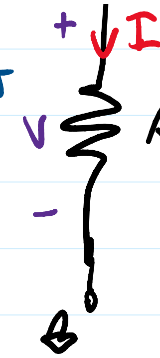
Equivalent



$\left(\frac{1}{\quad}+\frac{1}{\quad}+\frac{1}{\quad}\right)^{-1}$



Equivalent $\Longrightarrow$



$$R_e = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)^{-1}$$
$$= R_1 \| R_2 \| R_3$$

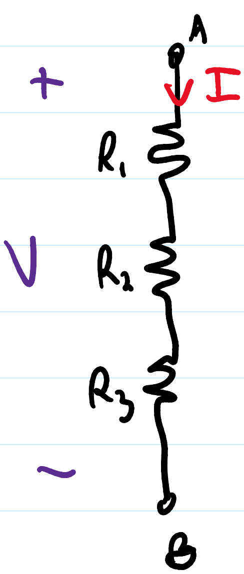
"parallel operator"

What information do we lose by making this substitution?
current through each resistor

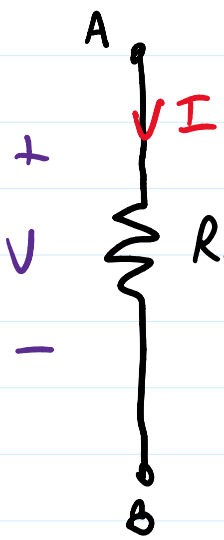
For the two resistor case:

$$\left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1} = \frac{R_1 R_2}{R_1 + R_2}$$

Series Resistances ← share the same current



Equivalent $\Longrightarrow$

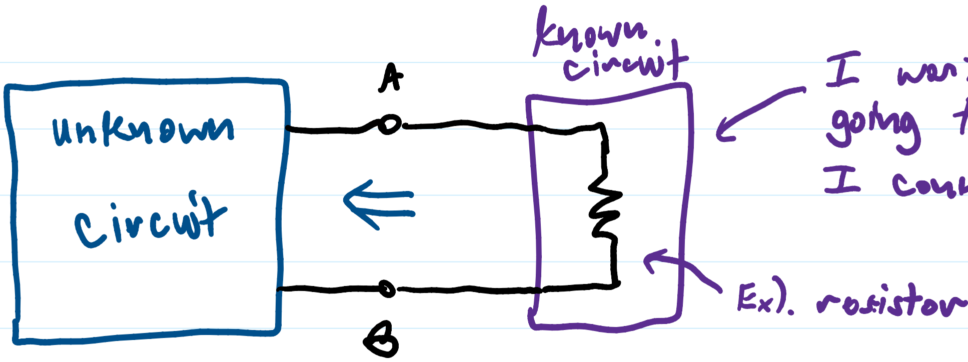


$$R_{eq} = R_1 + R_2 + R_3$$

Thévenin and Norton Equivalent Circuits:

Let us determine a reduced equivalent circuit for larger circuits with resistors + voltage sources & current sources

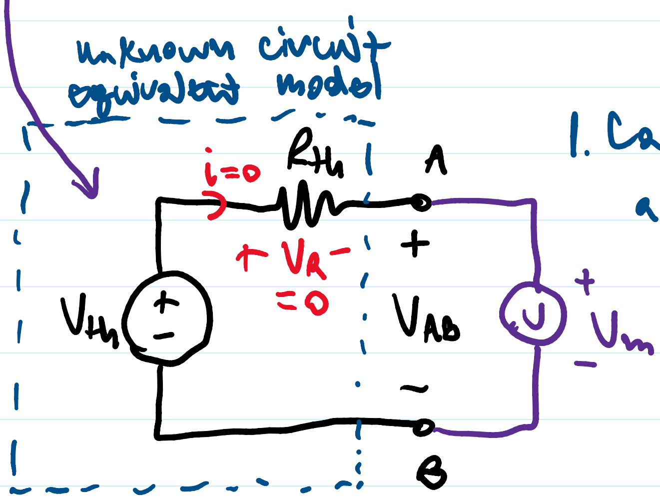
known circuit I want to know what's



I want to know what's going to happen when I connect this.

Can we determine how this unknown circuit behaves at the port AB without knowing what's inside?

Assume unknown circuit is equivalently modeled with this circuit called a **Thévenin equivalent circuit.**



1. Can we determine V_{Th} using a voltmeter? ✓

$$V_m = V_{AB} = V_{Th} + \cancel{V_R}^{\,0}$$
$$= V_{Th}$$

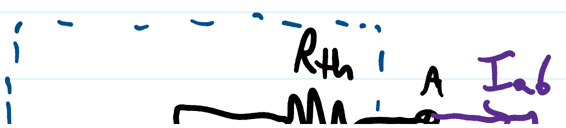
* Open-circuit Test:

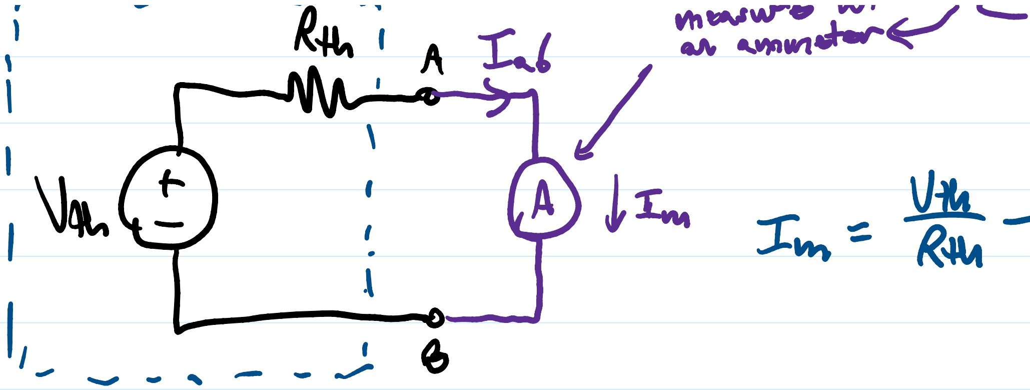
Measure the voltage between nodes A and B when connecting an external open circuit. $V_{AB,oc} = V_{Th}$ ← Thévenin Voltage

2. Can we determine R_{Th} as well? ✓

(equivalent short circuit)

measure with an ammeter





known from OC Test

$$I_m = \frac{V_{th}}{R_{th}} \rightarrow R_{th} = \frac{V_{th}}{I_m}$$

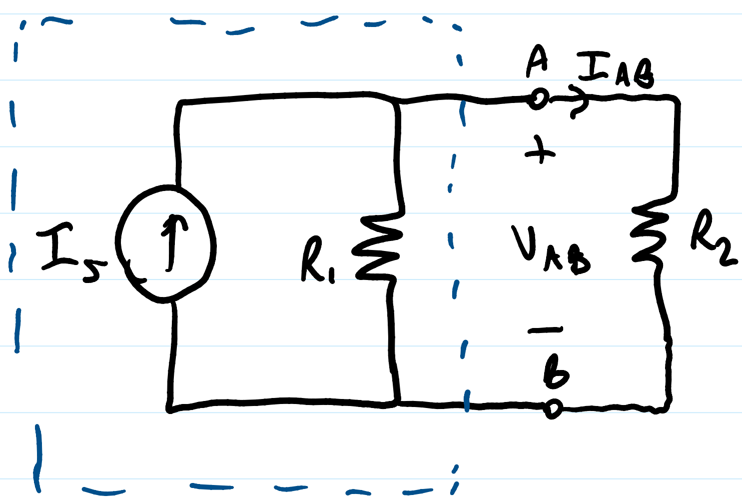
* Short-circuit Test:

Measure the current between nodes A and B when connecting an external short circuit. $I_m = \frac{V_{th}}{R_{th}} = I_{no}$

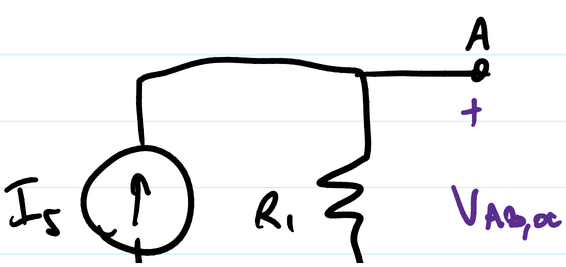
Thevenin resistance

Norton current

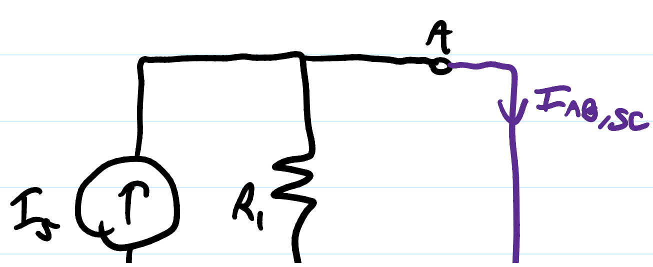
Consider the earlier circuit:

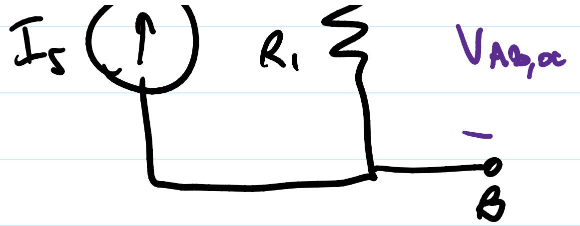


Open-circuit test:
(measure V_{AB})



Short-circuit test:
(measure I_{AB})



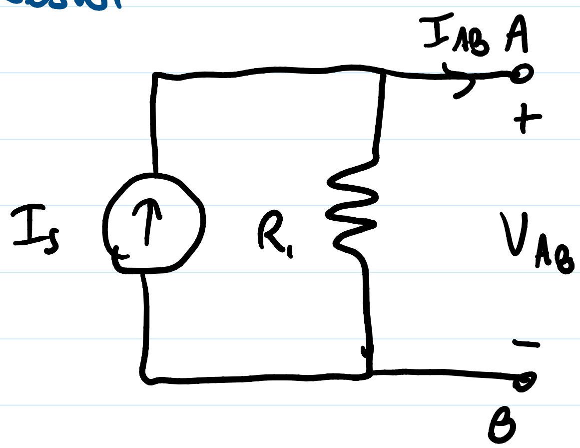


$$V_{th} = V_{AB,oc} = I_s \cdot R_1$$

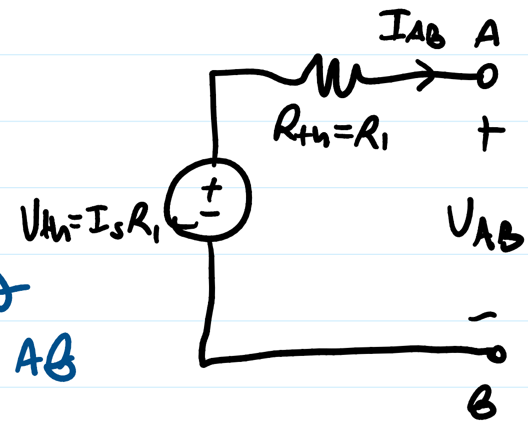
$$I_{no} = I_{AB,sc} = I_s$$

$$R_{th} = \frac{V_{th}}{I_{no}} = \frac{I_s R_1}{I_s} = R_1$$

Result

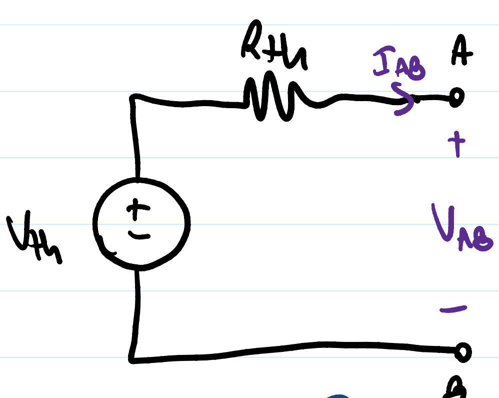


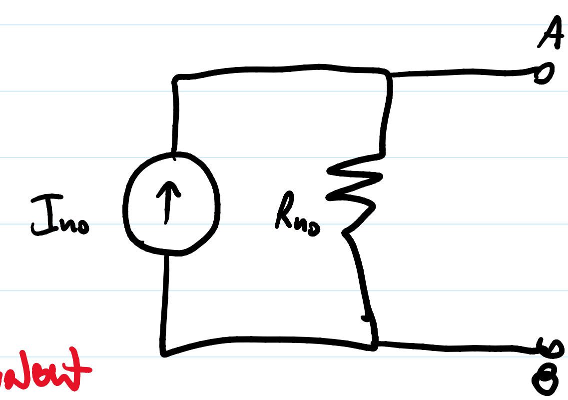
Equivalent at port AB



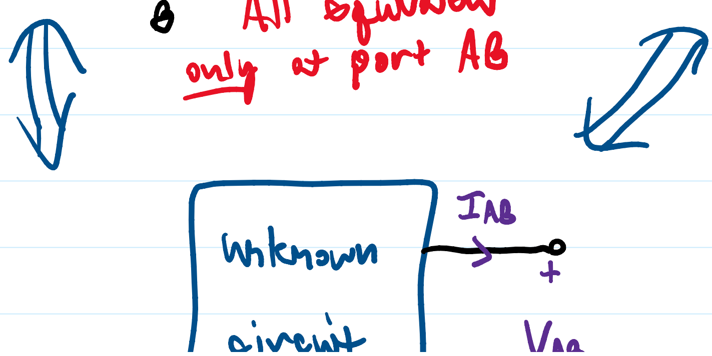
Thevenin equivalent circuit:

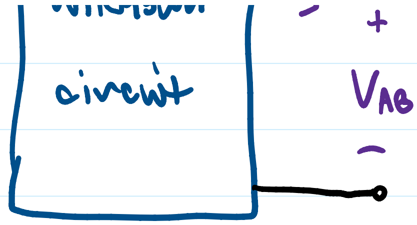
Norton equivalent circuit:
($R_{no} = R_{th}$)





All equivalent
only at port AB




circuit
+
V_{AB}
−