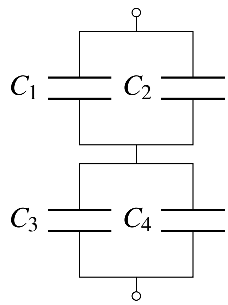
EECS 16A Designing Information Devices and Systems I
Summer 2023 Discussion 5C

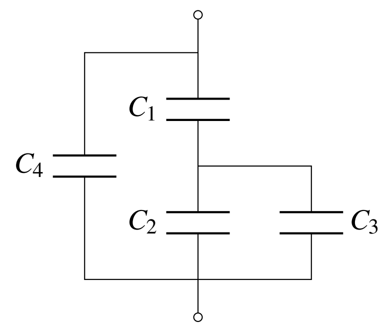
1. Series And Parallel Capacitors

Derive C_{eq} for the following circuits.

(a)



(b)



(a) Given that we know what the relationship for capacitors in series and parallel are we can write:

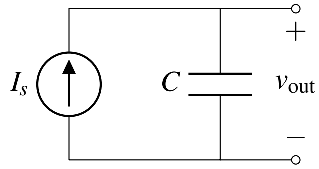
$$C_{eq} = ((C_1 + C_2) \parallel (C_3 + C_4)) = \frac{(C_1 + C_2)(C_3 + C_4)}{C_1 + C_2 + C_3 + C_4}$$

(b)

$$C_{eq} = (C_4 + (C_1 \parallel (C_2 + C_3))) = \frac{C_4(C_1 + C_2 + C_3) + C_1(C_2 + C_3)}{C_1 + C_2 + C_3}$$

2. Current Sources And Capacitors

Given the circuit below, find an expression for $v_{\text{out}}(t)$ in terms of I_s, C, V_0, and t, where V_0 is the initial voltage across the capacitor at $t = 0$.



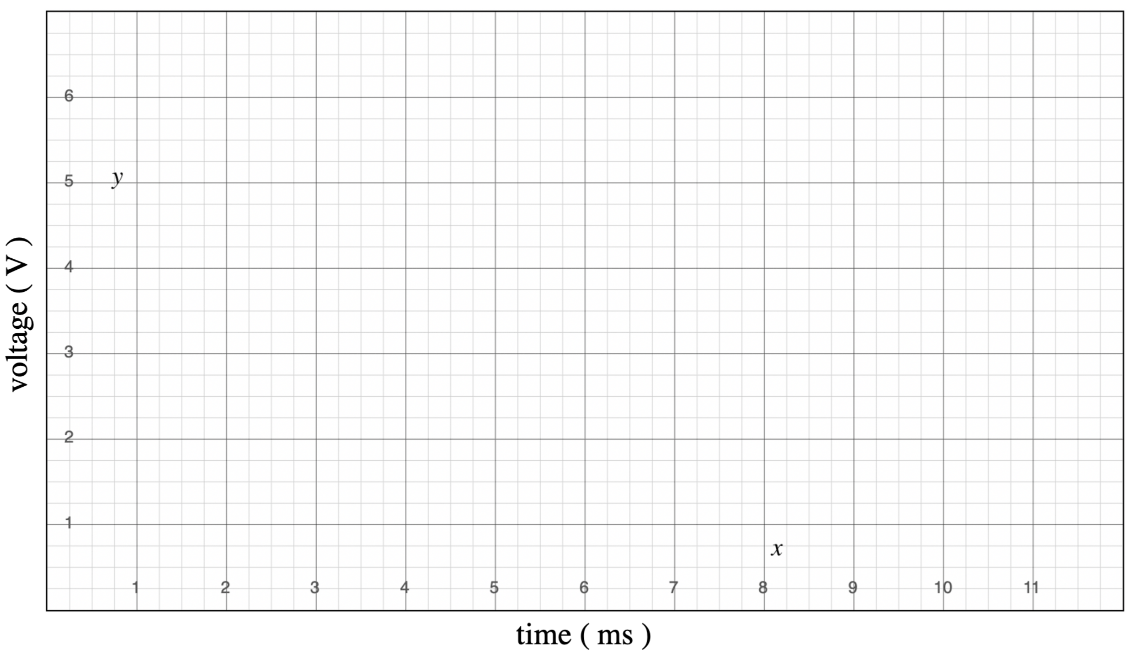
Then plot the function $v_{\text{out}}(t)$ over time on the graph below for the following conditions detailed below. Use the values $I_s = 1\text{mA}$ and $C = 2\,\mu\text{F}$.

(a) Capacitor is initially uncharged, with $V_0 = 0$ at $t = 0$.

(b) Capacitor has been charged with $V_0 = +1.5\text{V}$ at $t = 0$.

(c) **Practice:** Swap this capacitor for one with half the capacitance $C = 1\,\mu\text{F}$, which is initially uncharged, with $V_0 = 0$ at $t = 0$.

HINT: Recall the calculus identity $\int_a^b f'(x)dx = f(b) - f(a)$, where $f'(x) = \frac{df}{dx}$.



Answer:

The key here is to exploit the capacitor equation by taking its time-derivative

$$Q = C\,V_{out} \quad \longrightarrow \quad \frac{dQ}{dt} \equiv I_s = C\frac{dV_{out}}{dt}\,.$$

From here we can rearrange and show that $\frac{dV_{out}}{dt} = \frac{I_s}{C}$.

Thus the voltage has a constant slope!

Our solution is

$$V_{out}(t) = V_0 + \left(\frac{I_s}{C}\right) t \qquad \square$$

To be more mathematically formal, we are solving a differential equation that happens to return a linear function for $v_{out}(t)$:

$$\frac{dV_{out}}{dt} = \frac{I_s}{C} \quad \longrightarrow \quad \int_0^t \frac{dV_{out}}{dt} dt \equiv V_{out}(t) - V_{out}(0) \quad = \quad \int_0^t \frac{I_s}{C} dt \equiv \frac{I_s}{C} \int_0^t 1 dt \equiv \frac{I_s}{C} t$$

Thus we arrive at the same statement as seen earlier $V_{out}(t) = V_{out}(0) + \left(\frac{I_s}{C}\right) t$.

For all parts, we have $I_s = 1\text{mA}$. For part (a), we have $V_{out}(0) = 0\text{V}$ and $C = 2\mu\text{F}$. Plugging this into our equation, we get:

$$V_{out}(t) = 0\text{V} + \frac{1\text{mA}}{2\mu\text{F}} = \left(\frac{1}{2} \frac{\text{V}}{\text{ms}}\right) t$$

.

For part (b), this only changes by an intercept / initial condition $V_{out}(0) = 1.5\text{V}$:

$$V_{out}(t) = 1.5\text{V} + \frac{1\text{mA}}{2\mu\text{F}} = 1.5\text{V} + \left(\frac{1}{2} \frac{\text{V}}{\text{ms}}\right) t$$

.

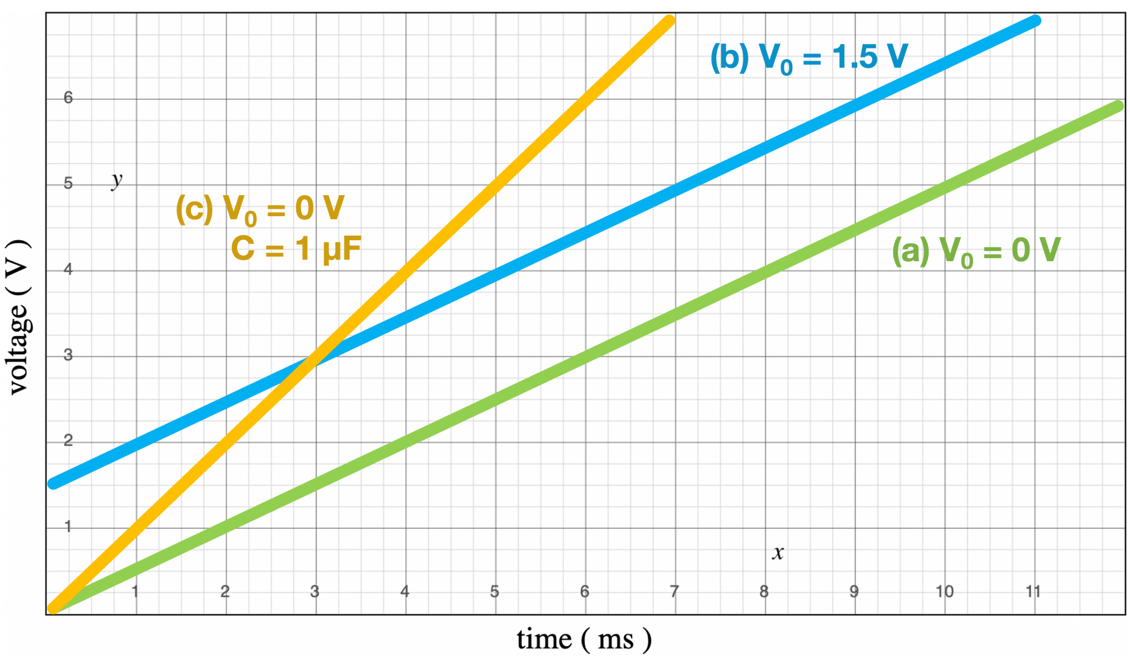
For part (c), we have $V_{out}(0) = 0\text{V}$ and $C = 1\mu\text{F}$.

$$V_{out}(t) = 0\text{V} + \frac{1\text{mA}}{1\mu\text{F}} = \left(1 \frac{\text{V}}{\text{ms}}\right) t$$

.

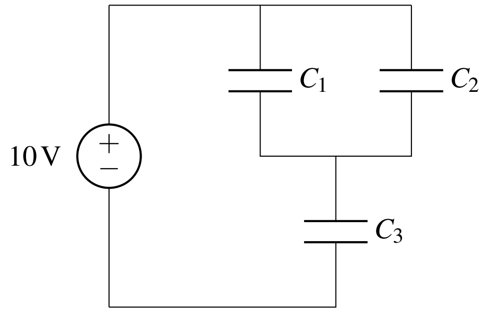
With half the capacitance and the same current, we get that the slope is twice as large. Some physical intuition: capacitors are like buckets, and charge is like the total water in the bucket. Voltage is a measure of the height of the water in the bucket. Capacitance is the cross-sectional area of the bucket, i.e. the area of the top hole. If the bucket's cross-sectional area (capacitance) is halved, then a hose filling the bucket (current source) would cause the water level (voltage) to rise twice as fast.

When plotting, make sure to recall (a) and (c) start at the origin, while (b) has initially charged plates by $V_0 = 1.5V$. Results are shown below



3. Series And Parallel Capacitors

(a) Consider the following circuit with $C_1 = 1\,\text{F}$, $C_2 = 3\,\text{F}$ and $C_3 = 4\,\text{F}$ Assume that both capacitors are initially uncharged before voltage is applied.



What are the voltages across each capacitor? Assume that we are in steady state.

Answer:

Let $+Q_1$ charge be on the top plate of C_1, $+Q_2$ charge be on top of C_2, and $+Q_3$ charge be on top of C_3.

We will label the voltages as V_{C_1}, V_{C_2} and V_{C_3} accordingly. By KVL, we have $V_{C_1} = V_{C_2}$ and $10 = V_{C_1} + V_{C_3}$. Using the charge equation $Q = CV$, we can substitute our KVL equations with Q and C, which we get:

$$\frac{Q_1}{C_1} = \frac{Q_2}{C_2} \implies 3Q_1 = Q_2$$
$$10 = \frac{Q_1}{C_1} + \frac{Q_3}{C_3} \implies 10 = Q_1 + \frac{Q_3}{4}$$

With charge conservation, we have $Q_1 + Q_2 = Q_3$. This gives us enough equations to solve Q_1, Q_2, Q_3, which we get:

$$Q_1 = 5\text{C}, Q_2 = 15\text{C}, Q_3 = 20\text{C} \tag{1}$$

Therefore, the voltages are $V_1 = 5V, V_2 = 5V, V_3 = 5V$

