EECS 16A Designing Information Devices and Systems I
Summer 2023 Discussion 3D

1. Matrix Multiplication Proof

(a) Given that matrix A is square and has linearly independent columns, which of the following are true? (You do not need to prove everything)

i. A is full rank
ii. A has a trivial nullspace
iii. $A\vec{x} = \vec{b}$ has a unique solution for all $\vec{b}$
iv. A is invertible
v. The determinant of A is non-zero

(b) Let two square matrices $\mathbf{M}_1, \mathbf{M}_2 \in \mathbb{R}^{2x2}$ each have linearly independent columns. Prove that $\mathbf{G} = \mathbf{M}_1\mathbf{M}_2$ also has linearly independent columns.

2. Exploring Dimension and Linear Independence

In this problem, we are going to talk about the connections between several concepts we have learned about in linear algebra – linear independence and dimension of a vector space/subspace.

Let's consider the vector space $\mathbb{R}^k$ (the k-dimensional real-world) and a set of n vectors $\{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n\}$ in $\mathbb{R}^k$.

(a) For the first part of the problem, let $k > n$. Can $\{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n\}$ span the full $\mathbb{R}^k$ space? If so, prove it. If not, what conditions does it violate/what is missing?

(b) Let $k = n$. Can $\{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n\}$ span the full $\mathbb{R}^k$ space? Why/why not? What conditions would we need?

(c) Finally, let $k < n$. Can $\{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n\}$ span the full $\mathbb{R}^k$ space?
Hint: Think about whether the vectors can be linearly independent.

3. Row Space

Consider:

$$\mathbf{V} = \begin{bmatrix} 2 & 4 & 6 \\ 4 & 0 & 4 \\ 6 & 4 & 10 \\ -2 & 4 & 2 \end{bmatrix}$$

Row reducing this matrix yields:

$$\mathbf{U} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(a) Show that the row spaces of $\mathbf{U}$ and $\mathbf{V}$ are the same. Argue that in general, Gaussian elimination preserves the row space.

(b) Show that the null spaces of $\mathbf{U}$ and $\mathbf{V}$ are the same. Argue that in general, Gaussian elimination preserves the null space.

