Lecture 6B: (7/25/23)

Announcements:

- Midterm ← Congrats! :)
  - Midterm Redo due Sunday (7/23) at midnight. (Will be released Wednesday)
  - 2 weeks of lecture left!
  - I will review Midterm on Friday from noon-2pm in Cory 144MA

- Final Exam
  - Wednesday (8/9) from 6-9pm in 2040 VLSB
  - Lab Practical on Tuesday (8/8) during normal lab times: 2-5pm and 5-8pm
    - 15% of final exam grade
    - Allotted 30-40 minutes to complete
    - Sign-up for slots will open next week

- Lab - Touch Button today
  - For Touch 1 and 2
  - Sign-up on Ed

Today's Topics:

- Comparators (Note 17)

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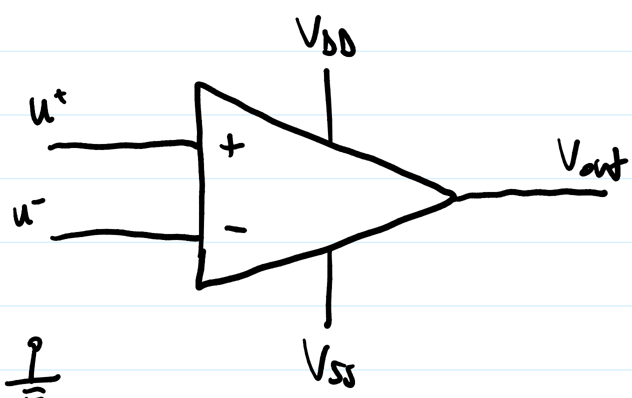
- Comparators (Note 17)
- Circuits as Systems
- Dependent Sources (Note 17.3.1)
- Op-amps (Note 17/18)
- Feedback (Note 18)

Operational Amplifier: "op-amp"

5-terminal circuit element

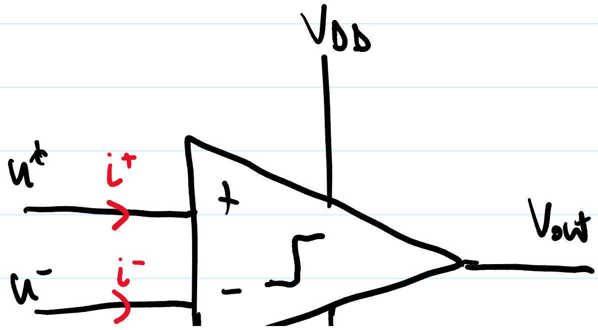


Inputs: u^+ and u^-
Output: V_{out}
Voltage Supply/Rails: V_{DD}, V_{SS}

(all are node voltages relative to the reference/zero node)

We will discuss op-amps later
For now we will use as a <u>comparator</u>.

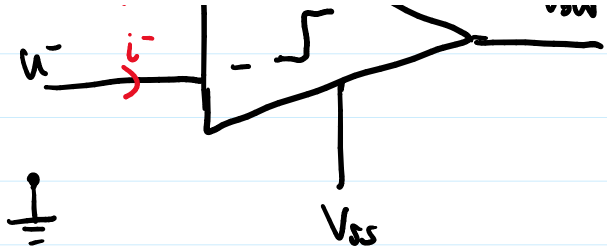
Comparator:



Is input #1 is greater than input #2?

Rules:

1. If $u^+ > u^-$ then $V_{out} = V$

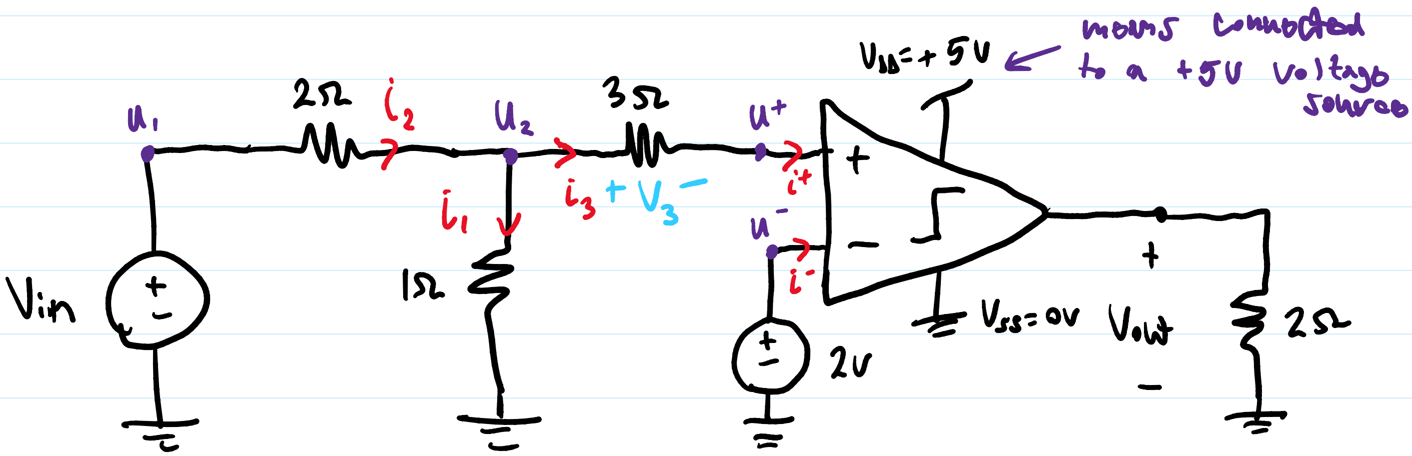


1. If $u^+ > u^-$, then $V_{out} = V_{DD}$
   $(u^+ - u^- > 0)$

2. If $u^+ < u^-$, then $V_{out} = V_{SS}$
   $(u^+ - u^- < 0)$

3. $i^+ = i^- = 0$ ALWAYS

Ex). Find V_{out} as a function of V_{in}



$i^+ = i^- = 0 \rightarrow i_3 = 0 \rightarrow V_3 = 0 \rightarrow u_2 = u^+$

$u_1 = V_{in}$

NVA: KCL @ node u_2: $i_2 - i_1 - \cancel{i_3}^{\,0} = 0$

$$\frac{u_1 - u_2}{2\Omega} - \frac{u_2 - 0}{1\Omega} - 0 = 0 \longrightarrow \frac{1}{2}V_{in} - \frac{3}{2}u_2 = 0 \rightarrow u_2 = \frac{1}{3}V_{in}$$

For a comparator we need to focus on when $u^+ > u^-$ and $u^+ < u^- \longrightarrow u^+ = u^-$

If $u^+ > u^- \longrightarrow V_{out} = V_{DD} = +5V$

$u^+ = \frac{1}{3}V_{in}$ ↘ $V_{in} = 6V$

If $u^+ > u^- \rightarrow V_{out} = V_{DD} = +5V$

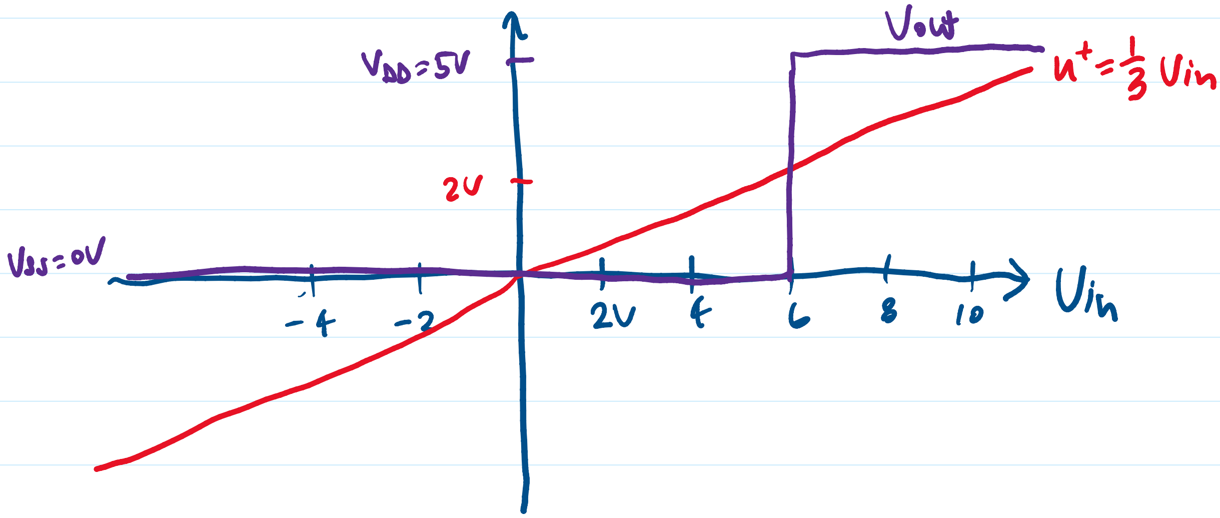
If $u^+ < u^- \rightarrow V_{out} = V_{SS} = 0V$

$u^+ = \frac{1}{3} V_{in}$

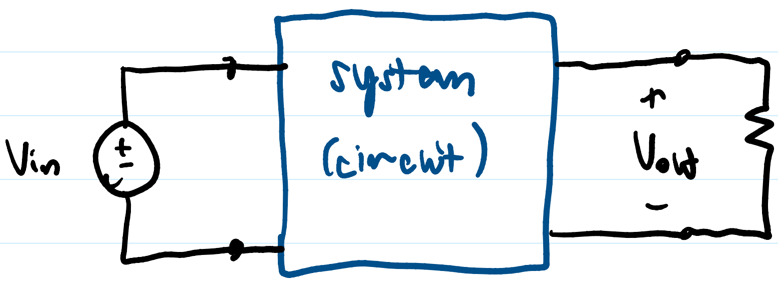
$u^- = 2V$

$\rightarrow V_{in} = 6V$

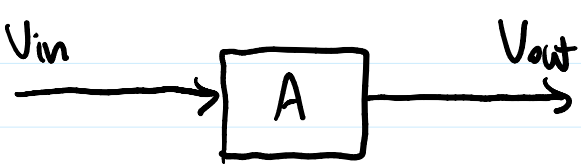
↑ transition point



Circuit as a system: (with input and output)



Block Diagram



A is scalar

↓

$V_{out} = A \cdot V_{in}$

output = gain · input

$\boxed{\text{LINEAR}}$

Ex). Gain of system → $A = \frac{1}{4}$

What if we want $V_{out} = 2V$?

$V_{in} = 8V$

What about $V_{out} = 1V$?

$V_{in} = 4V$

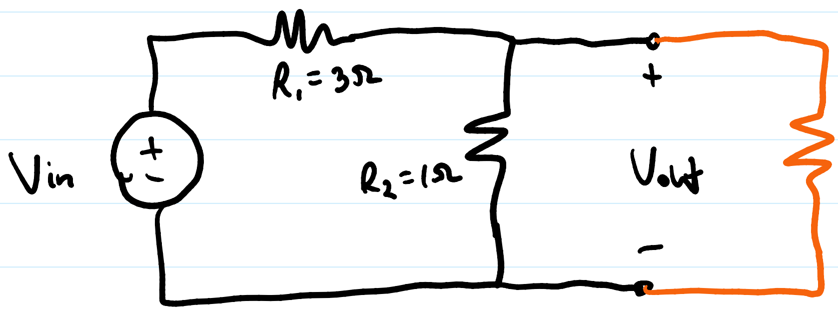
What if we want $V_{out} = 2V$?
- We would choose $V_{in} = 8V$

What about $V_{out} = 1V$?
$V_{in} = 4V$

Can we design a circuit with a gain of $A = \frac{1}{4}$?

Ex). Voltage Divider

$$V_{out} = \underbrace{\frac{R_2}{R_1+R_2}}_{A} \cdot V_{in} = \frac{1}{4} V_{in}$$

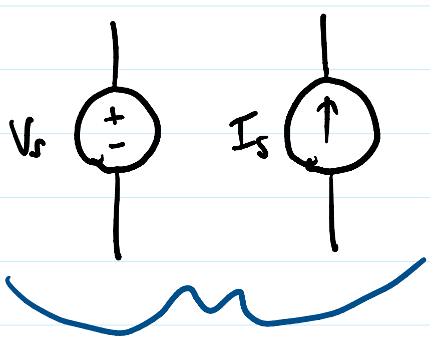


Load resistance
- CPU
- lightbulb
- audio speaker (8Ω)

What's the problem? Adding a load (or more load) makes V_{out} drop

We need a circuit where $A = \frac{V_{out}}{V_{in}}$ does not depend on the amount of loading.

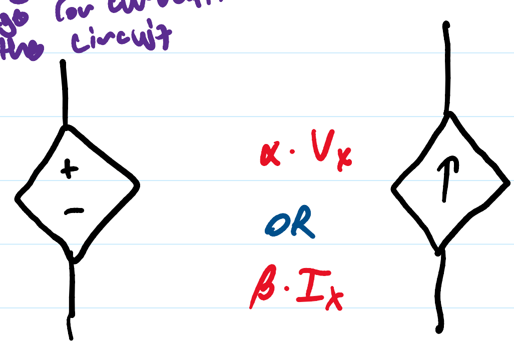
Dependent Sources: (Note 17.3.1)



scaling of a voltage (or current) in the circuit

$\alpha \cdot V_x$ OR $\beta \cdot I_x$

$\alpha \cdot V_x$ OR $\beta \cdot I_x$



Dependent

Dependent

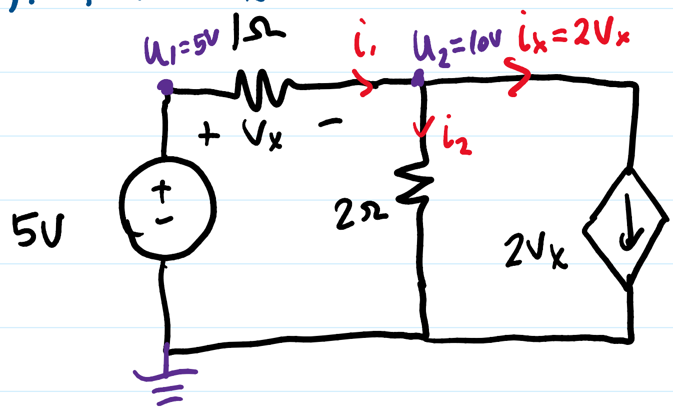
Independent sources

V_s and I_s are independent of the rest of the circuit

Dependent voltage source / Dependent current source

Dependent on the rest of the circuit

Ex). Find V_x



KCL @ u_2: $i_1 - i_2 - i_x = 0$

(1) $\frac{u_1 - u_2}{1\Omega} - \frac{u_2 - 0}{2\Omega} - 2V_x = 0$

$u_1 = 5V$ (2) $\qquad V_x = u_1 - u_2$ (3)

$V_x = 5V - 10V$

$\boxed{V_x = -5V}$

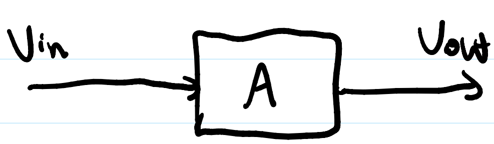
Substitute (2) and (3) into (1)

$$\frac{5-u_2}{1} - \frac{u_2}{2} - 2\cdot(5-u_2) = 0 \rightarrow 5 - 10 = \left(1 + \tfrac{1}{2} - 2\right)\cdot u_2$$

$$u_2 = \frac{-5}{-0.5} = 10V$$

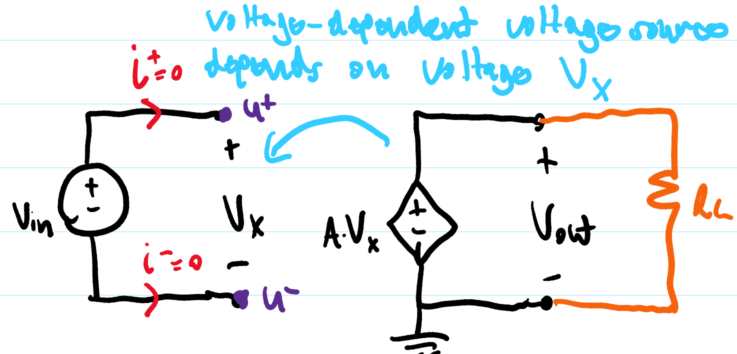
Return to our earlier system:

How could we implement this with a dependent voltage source?



$A = \frac{V_{out}}{V_{in}}$ (Gain)

$\Longrightarrow$



Does R_L affect V_{out}? No!

$V_{out} = A\cdot V_x = A\cdot(u^+ - u^-)$

$= A\cdot V_{in}$ ✓

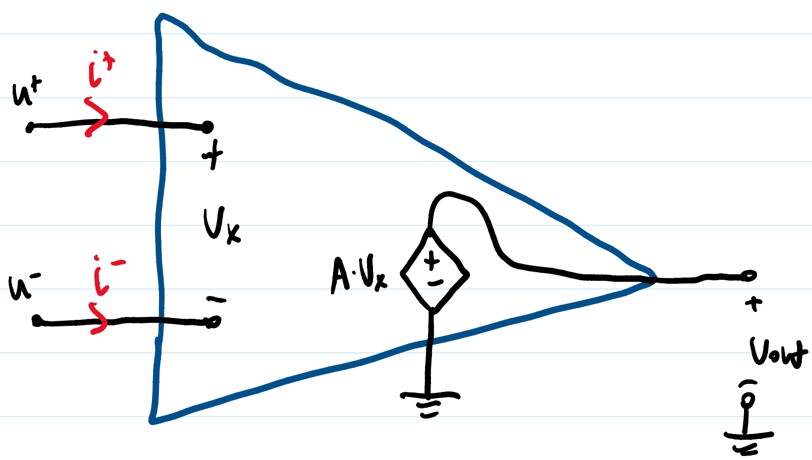
Operational Amplifier: "op-amp"

Internal op-amp model:

V_x is difference of inputs

$\downarrow$

$V_{out} = A\cdot V_x = A\cdot(u^+ - u^-)$



$\boxed{\text{Golden Rule \#1: } i^+ = i^- = 0}$

It doesn't matter what is connected to u^+ and u^- externally. i^+ and i^- are ALWAYS zero.

We did it! Unfortunately, no :(

All op-amps have internal values of gain "A" which are:

1) very large $\longrightarrow A > 1000 \rightarrow A > 1\cdot10^6$
2) are imprecise $\rightarrow$ Two op-amps in the batch could have "A" $= 1\cdot10^6$ and "A" $= 1.1\cdot10^6$

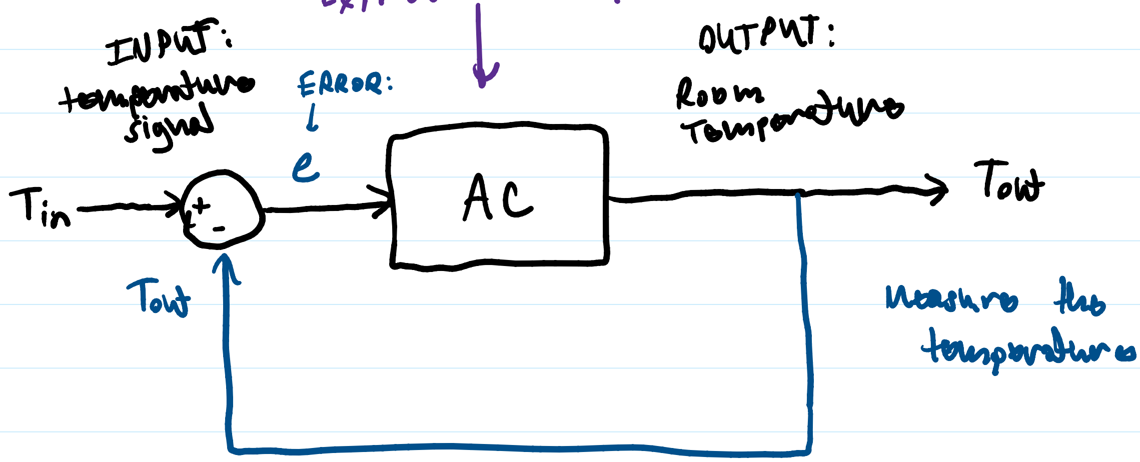
How do we achieve the V_{out} we want when we can't know the exact value of the gain "A"?

Feedback:

Use information about the output of a system to "feed" back into the input of a system.

Ex). Thermostat ← control the air conditioning (AC) (temperature)

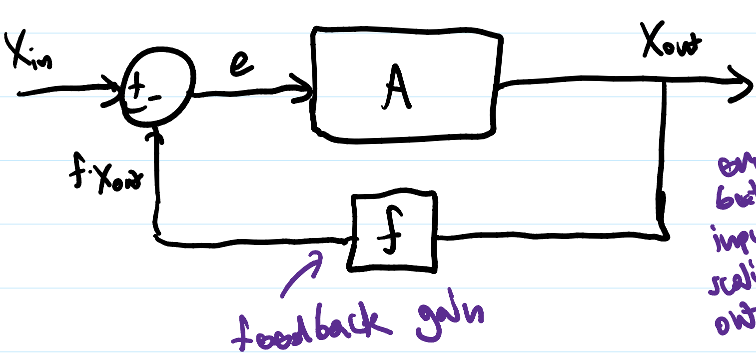
The "gain" can change
Ex). outside temp., windows open



Feedback the temperature to correct if too hot or too cold

What's the math behind feedback? → Control Theory

Before: $\frac{X_{out}}{X_{in}} = A$



After (feedback):

error between input and scaling of output → $e = X_{in} - f X_{out}$ substitute

$X_{out} = A \cdot e$

$X_{out} = A \cdot (X_{in} - f X_{out})$

| Move X_{out} to

feedback gai… outp…

$X_{out} = A \cdot (X_{in} - f X_{out})$

↓ Move X_{out} to one side

$(1 + Af) \cdot X_{out} = A \cdot X_{in}$

$$\boxed{\frac{X_{out}}{X_{in}} = \frac{A}{1+Af}}$$ Black's Formula

(We changed the system gain from "A" to "$\frac{A}{1+Af}$". So what?

What if A is very large? $(A \gg 1)$

Ex). $A = 1000$

$$\frac{X_{out}}{X_{in}} = \frac{1000}{1 + 1000f} \approx \frac{1000}{1000f} = \frac{1}{f}$$

As $A \to \infty$

$$\lim_{A\to\infty} \frac{X_{out}}{X_{in}} = \lim_{A\to\infty} \frac{A}{1+Af} = \lim_{A\to\infty} \frac{A}{Af}$$

$$\boxed{\lim_{A\to\infty} \frac{X_{out}}{X_{in}} = \frac{1}{f}}$$

This is the essence of feedback. "A" is unknown, but we can control the input "X_{in}" and feedback gain "f" to control the overall gain and ultimately set the desired output "X_{out}".

How do we implement feedback gain "f"? Using circuits