Lecture 16 -- Post

Saturday, July 15, 2023 1:02 PM

Lecture 5C: (7/19/23)

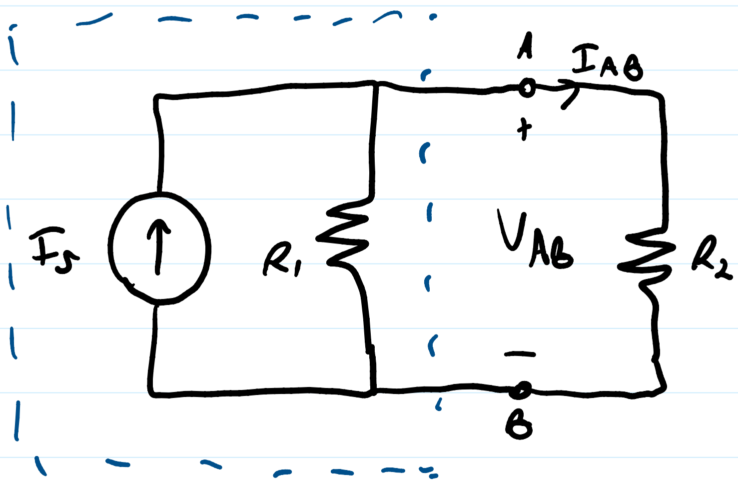
Announcements:

- Quest Regrade Request due tonight
- Today's content (including discussion) is end of scope for the Midterm (Monday night!)

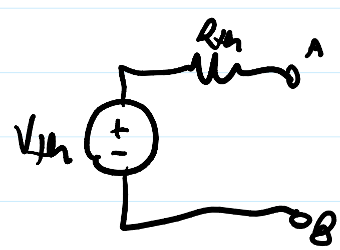
Today's Topics:

- Capacitors (Note 16)
  - IV Characteristics
  - Capacitor Circuits and Charge Sharing
  - Capacitor Equivalence
  - Physical Capacitance

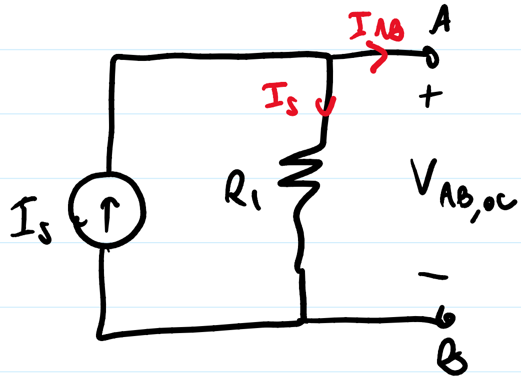
Consider yesterday's circuit



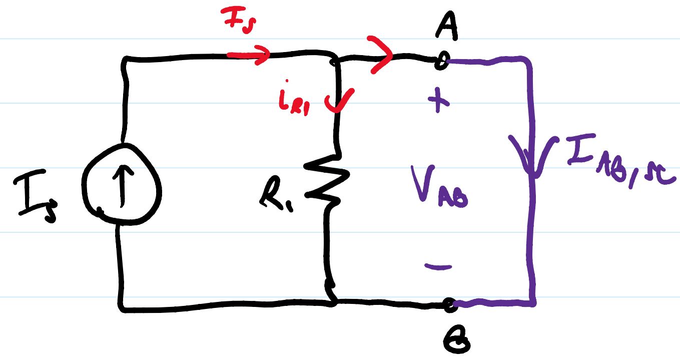
Represent as this



Open-circuit test:
(measure V_{AB})



Short-circuit test:
(measure I_{AB})



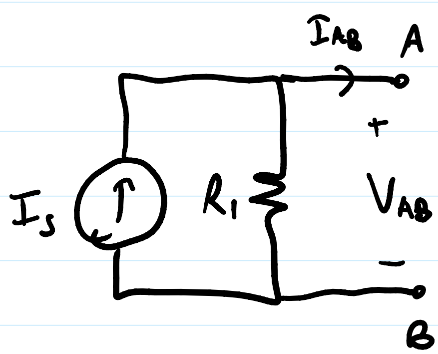
$V_{AB}=0 \rightarrow i_{R_1}=0$

$V_{th} = V_{AB,oc} = I_s R_1$

$I_{no} = I_{AB,sc} = I_s$

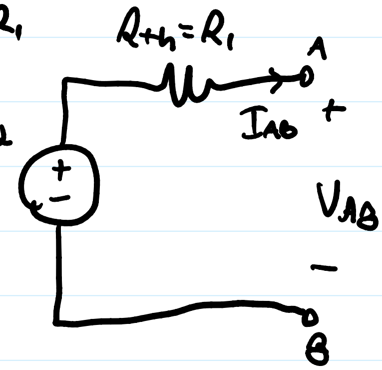
$R_{th} = \dfrac{V_{th}}{I_{no}} = \dfrac{I_s R_1}{I_s} = R_1$

Result:

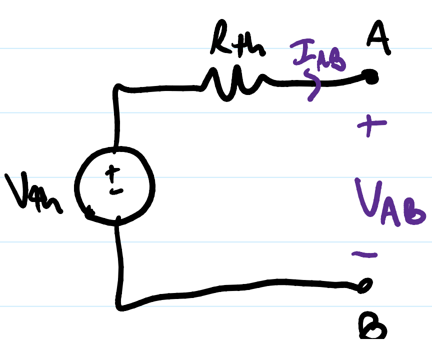


$\Longleftrightarrow$

Equivalent at port AB



Thevenin equivalent circuit:

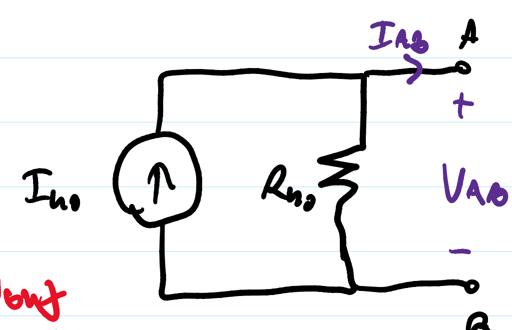


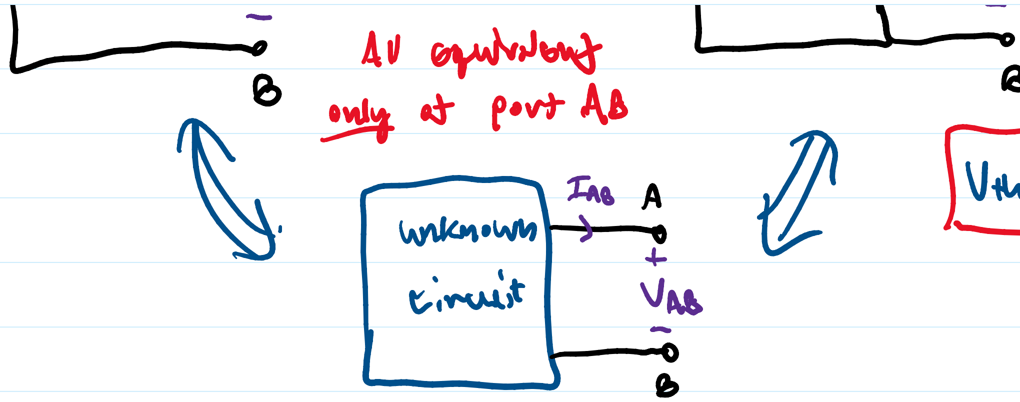
$\Longleftrightarrow$

All equivalent

Norton equivalent circuit

($R_{no}=R_{th}$)



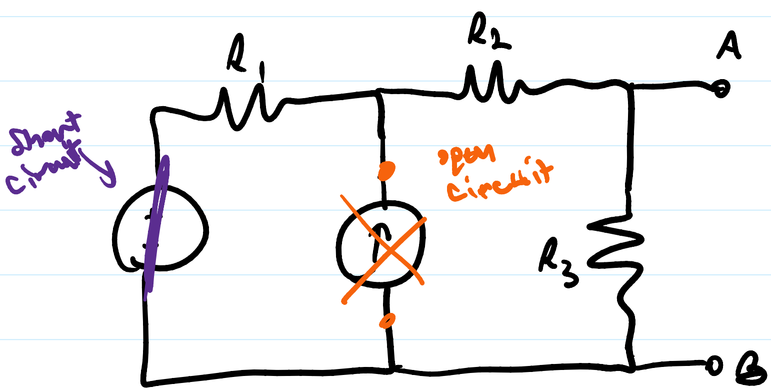


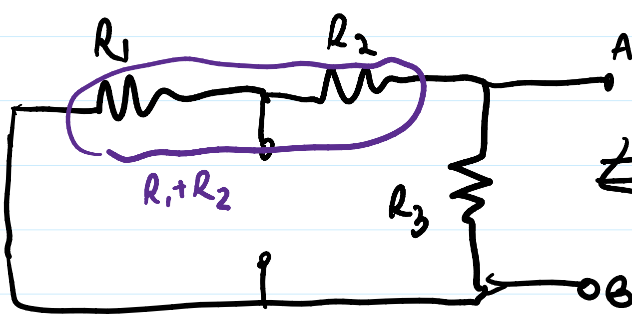
AV equivalent
only at port AB

$$\boxed{V_{th} = I_{no} \cdot R_{th}}$$

Another method to find R_{th}:

* Turn off all independent voltage and current sources and find equivalent resistance between A and B

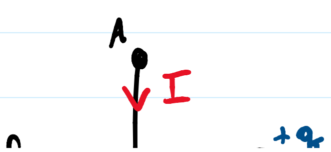




$\Leftarrow R_{eq} = R_{th}$

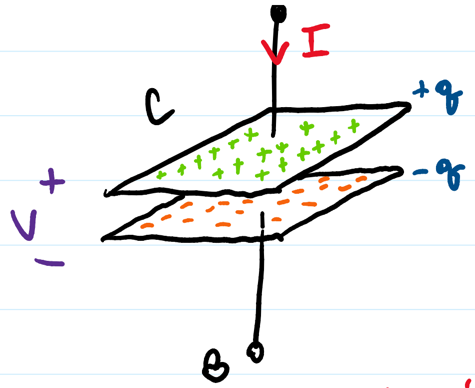
$= (R_1+R_2) \| R_3$

What is a capacitor?



A capacitor...

- Stores and releases energy (does not dissipate energy)

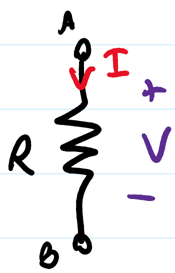


A capacitor...

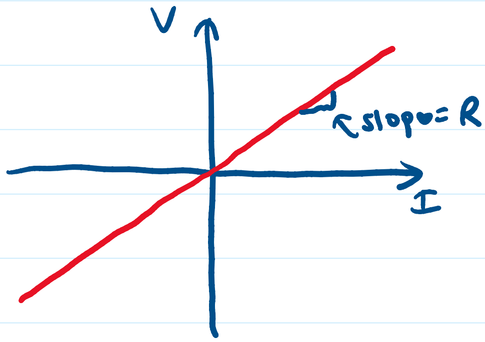
- Stores and releases energy (does not dissipate energy like a resistor)
- Separates charge
- Has capacitance — variable: "C", unit: [F] (Farad)
- Has a linear IV characteristic

How is this an "IV" characteristic? $q = C\,V$ — q: charge [C]; C: capacitance [F]; V: voltage [V]

Resistor:

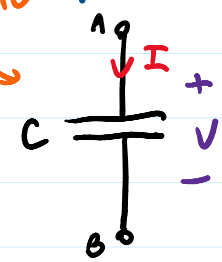


$V = IR$



Capacitor:

schematic symbol

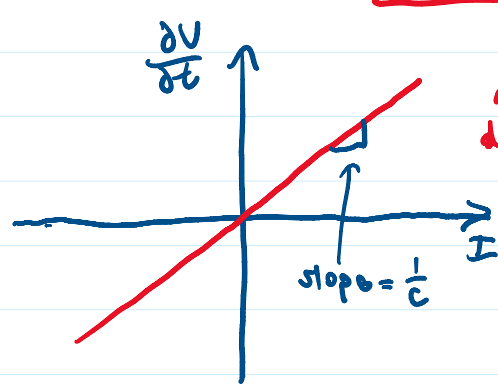


$$q = CV$$

$$\frac{\partial}{\partial t} q = \frac{\partial}{\partial t}(CV)$$

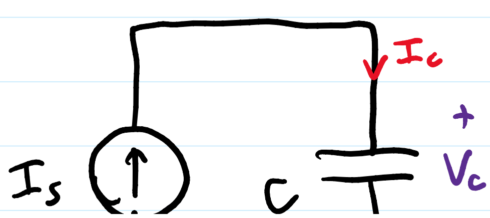
$$\boxed{I = C\frac{\partial V}{\partial t}}$$

This is a linear differential equation



A capacitor is a linear passive circuit element
→ many of our linear circuit analysis techniques apply

Ex). Find V_c



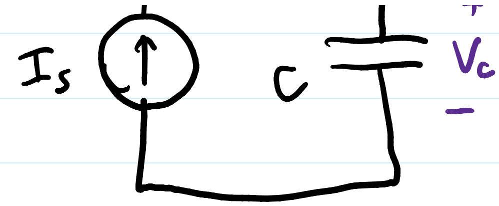
$$q = CV_c \rightarrow I_c = C\frac{\partial V_c}{\partial t}$$

Separate and integrate!

$I_c = I_s$ constant; C is constant

$$\int_{t_0}^{t} I_c\, dt = \int_{V_c(t_0)}^{V_c(t)} C\, dV_c$$

t | $V_c(t)$



$$\int_{t_0} I_c \, dt = \int_{V_c(t_0)} C \, dV_c$$

$$I_s \cdot t \Big|_{t_0}^{t} = C \cdot V_c(t) \Big|_{V_c(t_0)}^{V_c(t)}$$

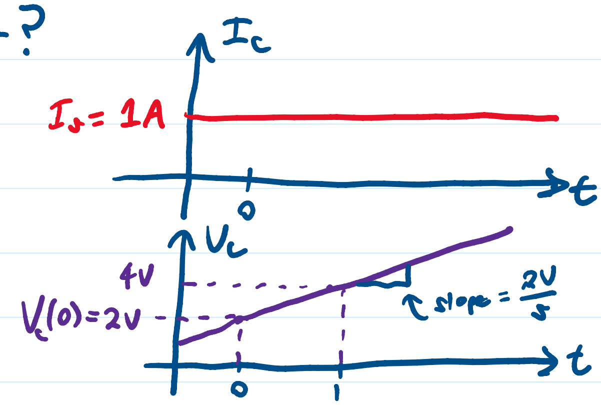
$$I_s \cdot t - I_s \cdot t_0 = C \cdot V_c(t) - C \cdot V_c(t_0)$$

$$* \quad V_c(t) = \frac{1}{C} I_s \cdot (t - \underset{\text{initial time}}{t_0}) + \underbrace{V_c(t_0)}_{\text{initial voltage across C}} \leftarrow \text{"initial condition"}$$

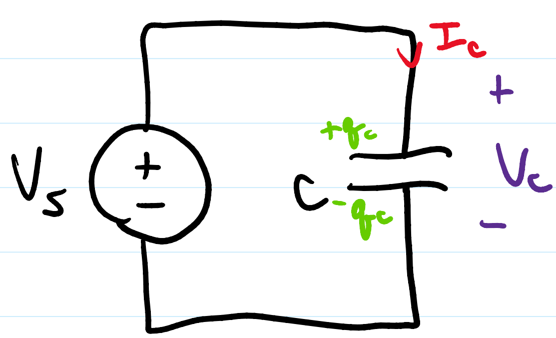
What if we know $I_s = 1A$, $C = 0.5F$ and $V_c(t_0) = 2V$ at time $t_0 = \emptyset\ s$?

$$V_c(t) = \frac{1}{(0.5F)} \cdot (1A) \cdot (t - 0) + (2V)$$

$$= \underset{\text{volt per second}}{\left(2 \tfrac{V}{s}\right)} \cdot t + 2V$$



Ex). Find charge across the capacitor, q_c



$$q_c = C V_c = C V_s$$

capacitor voltage is static (i.e., constant) with time

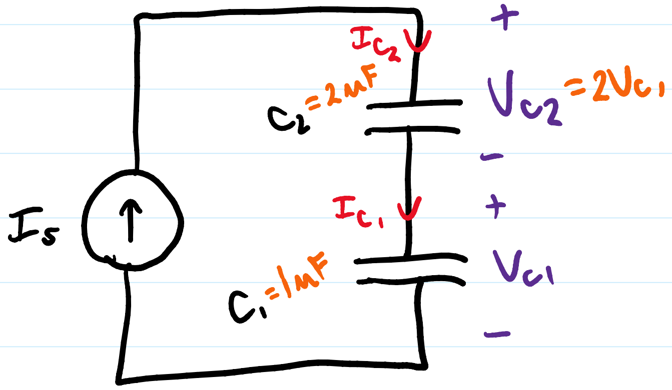
What about current through the capacitor, I_c?

Capacitor has nonzero charge and voltage across it, but no current flowing through it.

$$I_c = C \cancel{\frac{\partial V_c}{\partial t}}^{\,0} = C \cdot \emptyset = \emptyset A$$

Ex). Capacitors in series. Find V_{c1}. Assume $V_{c1}(0) = V_{c2}(0) = 0V$

$I_c = C\frac{\partial V_c}{\partial t}$



$I_{c1} = I_{c2} = I_s$

$$C_1 \frac{\partial V_{c1}}{\partial t} = C_2 \frac{\partial V_{c2}}{\partial t} = I_s$$

$$C_1 \int_{V_{c1}(t_0)}^{V_{c1}(t)} dV_{c1} = C_2 \int_{V_{c2}(t_0)}^{V_{c2}(t)} dV_{c2} = I_s \int_{t_0}^{t} dt$$

$$C_1 V_{c1}(t) - C_1 \cancelto{0}{V_{c1}(t_0)} = C_2 V_{c2}(t) - C_2 \cancelto{0}{V_{c2}(t_0)} = I_s \cdot (t - \cancelto{0}{t_0})$$

Pick $t_0 = 0$

$$C_1 V_{c1}(t) = C_2 V_{c2}(t) = I_s t$$

$$q_{c1}(t) = q_{c2}(t) = I_s t$$

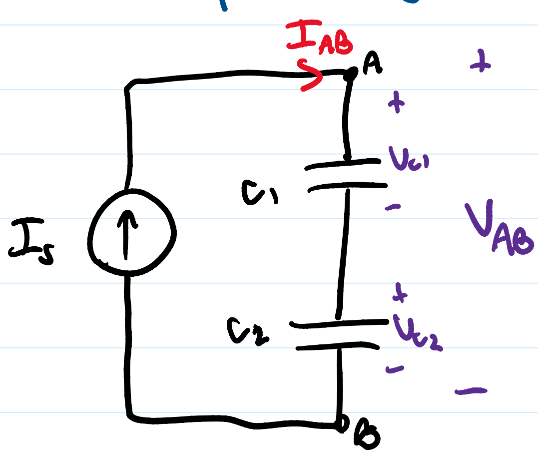
The accumulated charge in series capacitors is equal since they share the same current $I = \frac{\partial q}{\partial t}$.

However, they have different voltages

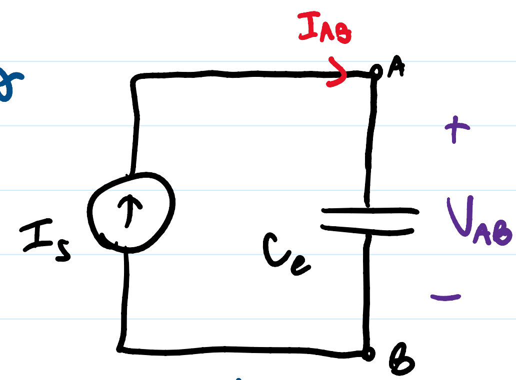
Pick $C_1 = 1mF$, $C_2 = 2mF$,
then $V_{c_2}(t) = \frac{C_2}{C_1} \cdot V_{c1}(t) = 2 \cdot V_{c1}(t)$

Equivalent Capacitance

Series Capacitances ← share the same current



Equivalent $\Longrightarrow$



$V_{AB} = V_{c1} + V_{c2}$

$\frac{\partial V_{AB}}{\partial t} = \frac{\partial V_{c1}}{\partial t} + \frac{\partial V_{c2}}{\partial t}$ $\quad I_{c1} = I_{c2} = I_{AB}$

$V_{AB} \xrightarrow{\text{Take derivative}} \frac{\partial V_{AB}}{\partial t}$

$\frac{\partial V_{AB}}{\partial t} = \frac{1}{\ } I$

$$\frac{\partial V_{AB}}{\partial t} = \frac{\partial V_{C1}}{\partial t} + \frac{\partial V_{C2}}{\partial t} \qquad I_{C1} = I_{C2} = I_{AB}$$

$$= \frac{1}{C_1} I_{C1} + \frac{1}{C_2} I_{C2} = \left(\frac{1}{C_1} + \frac{1}{C_2}\right) I_{AB}$$

$$\downarrow$$

$$I_{AB} = \underbrace{\left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1}}_{C_e = C_1 \| C_2 = \frac{C_1 C_2}{C_1 + C_2}} \cdot \frac{\partial V_{AB}}{\partial t}$$

parallel operator for series capacitors

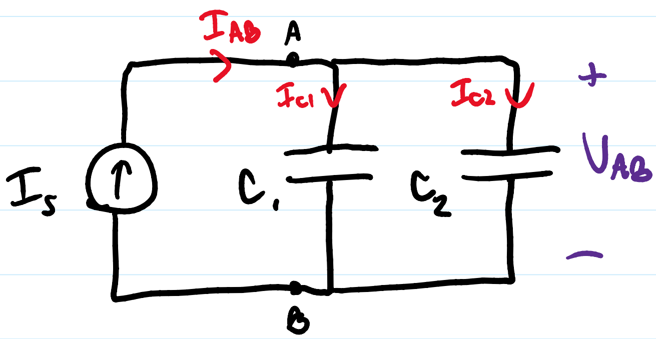
$\Longrightarrow$ Equivalent

$$\frac{\partial V_{AB}}{\partial t} = \frac{1}{C_e} I_{AB}$$

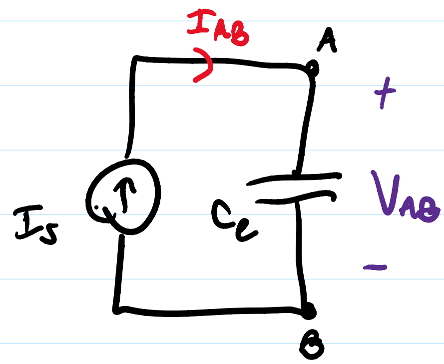
$$\downarrow$$

$$I_{AB} = C_e \cdot \frac{\partial V_{AB}}{\partial t}$$

Parallel Capacitances ← share the same voltage



Equivalent $\Longrightarrow$



$$I_{AB} = I_{C1} + I_{C2} \quad \text{(KCL)}$$

$$= C_1 \frac{\partial V_{C1}}{\partial t} + C_2 \frac{\partial V_{C2}}{\partial t}$$

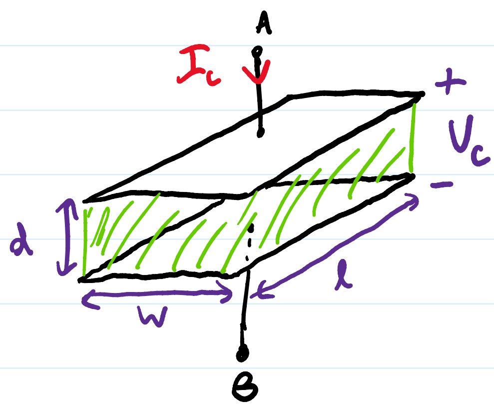
$$I_{AB} = \underbrace{(C_1 + C_2)}_{C_e = C_1 + C_2} \cdot \frac{\partial V_{AB}}{\partial t}$$

$$\left(\begin{aligned} V_{AB} &= V_{C1} = V_{C2} \\ \frac{\partial V_{AB}}{\partial t} &= \frac{\partial V_{C1}}{\partial t} = \frac{\partial V_{C2}}{\partial t} \end{aligned} \right)$$

"series" operator for parallel capacitances

$$I_{AB} = C_e \cdot \frac{\partial V_{AB}}{\partial t}$$

Physical Capacitor:



Capacitance forms between overlapping parallel plates of metal.

$$C = \varepsilon \frac{A}{d} = \underbrace{\varepsilon \frac{l \cdot w}{d}}_{\text{for this example}}$$

A ← overlapping plate area

d ← plate separation distance

ε ← permittivity

$$[F] = \left[\frac{F}{m}\right] \cdot \frac{[m^2]}{[m]}$$

What is permittivity?
- Intrinsic material property
- Variable "ε" (Greek Letter "Epsilon")
- Often expressed as: $\varepsilon = \varepsilon_r \cdot \varepsilon_0$ ← "permittivity of free space" (vacuum or air)

  ε_r ← relative permittivity

  $\varepsilon_0 = 8.85 \cdot 10^{-12} \frac{F}{m}$
- Dielectric material has $\varepsilon_r > 1$

Demo with copper plates:

Two 15 × 15 cm plates held 1 cm apart in air

$$C = \varepsilon \cdot \frac{l \cdot w}{d} = \frac{(15\,cm) \cdot (15\,cm)}{(1\,cm)} = \left(8.85 \cdot 10^{-12} \frac{F}{m}\right) \cdot \frac{(0.15\,m)^2}{(0.01\,m)}$$

$$= 1.99 \cdot 10^{-11}\,F$$

$$= 19.9\,pF$$ ← Really small!

$\varepsilon \approx \varepsilon_0$ for air

What if area doubles? → C doubles

What if separation doubles? → C halves