EECS 16A   Designing Information Devices and Systems I
Summer 2023   Discussion 3C

1. Identifying a Subspace: Proof

Is the set

$$V = \left\{ \vec{v} \,\middle|\, \vec{v} = c \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + d \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \text{ where } c, d \in \mathbb{R} \right\}$$

a subspace of $\mathbb{R}^3$? Why or why not?

2. Exploring Column Spaces and Null Spaces

- The **column space** is the **span** of the column vectors of the matrix.
- The **null space** is the set of input vectors that when multiplied with the matrix result in the zero vector.

For the following matrices, answer the following questions:

i. What is the column space of **A**? What is its dimension?

ii. What is the null space of **A**? What is its dimension?

iii. Do the columns of **A** span $\mathbb{R}^2$? Do they form a basis for $\mathbb{R}^2$? Why or why not?

iv. Are the column spaces of the row reduced matrix **A** and the original matrix **A** the same?

(a) $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} -2 & 4 \\ 3 & -6 \end{bmatrix}$

(e) $\begin{bmatrix} 1 & -1 & -2 & -4 \\ 1 & 1 & 3 & -3 \end{bmatrix}$

(f) What do you notice about the relationship between the dimension of the column space, the dimension of the null space, and their sum in all of these matrices?