Lecture 13 -- Post

Monday, July 10, 2023 7:16 PM

Lecture 4D: (7/13/23)

Announcements:

- Quest Redo is available! Due Sunday night
- Lab - First Buffer Lab ← come if you've missed or not finished a lab so far
  - Touch 1 and 2 pre-labs on Ed
- Today's Topics:
  - Review of NVA
  - Physical Resistor (Note 12)
  - Power (Note 13)

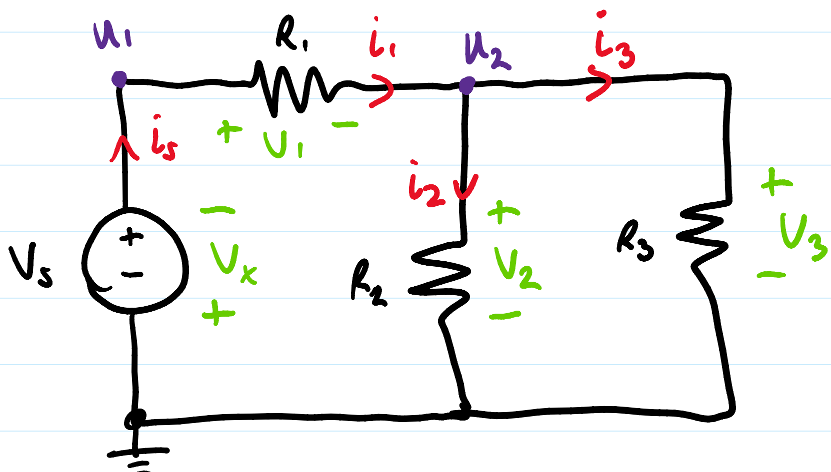
Circuit from yesterday's lecture: Let's finish solving!

$V_s = 10V$

$R_1 = 1\Omega$

$R_2 = 2\Omega$

$R_3 = 2\Omega$



At node u_1: $i_s - \frac{u_1 - u_2}{\ } = 0$ 1

At node u_1: $i_s - \dfrac{u_1 - u_2}{R_1} = 0$ 1

At node u_2: $\dfrac{u_1 - u_2}{R_1} - \dfrac{u_2}{R_2} - \dfrac{u_2}{R_3} = 0$ 2

$u_1 = V_s$ 3

Matrix vector form: $A\vec{x} = \vec{b} \longrightarrow \vec{x} = A^{-1}\vec{b}$

$$\begin{array}{c c} 1 & \\ 2 & \\ 3 & \end{array}\begin{bmatrix} -\frac{1}{R_1} & \frac{1}{R_1} & 1 \\ \frac{1}{R_1} & -\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right) & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ i_s \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ V_s \end{bmatrix}$$

Solve using substitution instead:

3 → 2 $\dfrac{V_s - u_2}{R_1} - \dfrac{u_2}{R_2} - \dfrac{u_2}{R_3} = 0$

$$\cancel{R_1} R_2 \cdot R_3 \cdot \frac{(V_s - u_2)}{\cancel{R_1}} - R_1 \cancel{R_2} R_3 \frac{u_2}{\cancel{R_2}} - R_1 R_2 \cancel{R_3} \frac{u_2}{\cancel{R_3}} = 0$$

$\boxed{\begin{aligned} V_s &= 10V \\ R_1 &= 1\Omega \\ R_2 &= 2\Omega \\ R_3 &= 2\Omega \end{aligned}}$

$$R_2 R_3 \cdot V_s = (R_2 R_3 + R_1 R_3 + R_1 R_2) \cdot u_2$$

$$u_2 = \frac{R_2 R_3}{(R_2 R_3 + R_1 R_3 + R_1 R_2)} V_s$$

$$= \frac{(2)\cdot(2)}{(2\cdot 2 + 1\cdot 2 + 1\cdot 2)} \cdot (10V) = \frac{4}{8} 10V$$

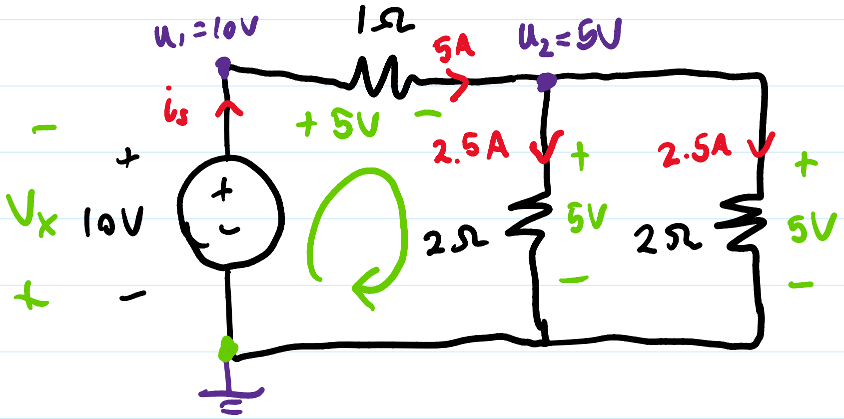
$\boxed{\begin{aligned} u_2 &= 5V \\ u_1 &= 10V \end{aligned}}$

We've solved the circuit!

$\boxed{u_1 = 10V}$

Check Results:

$V = iR \rightarrow i = \frac{V}{R}$



Check KCL:

@ u_2: $5A = 2.5A + 2.5A$

Check KVL:

$+10V - 5V - 5V = 0$

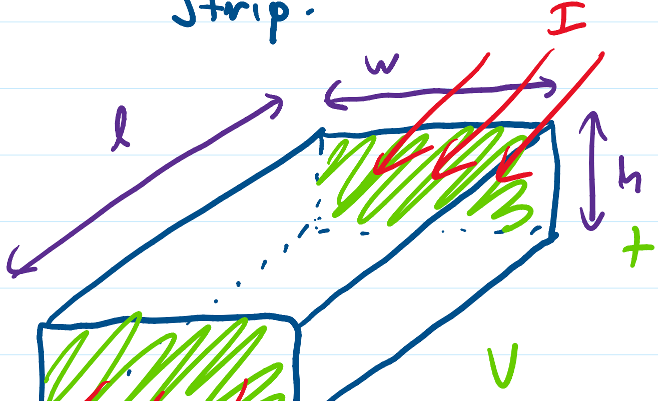
We are working towards a resistive touchscreen

1) Physics of Materials
2) Measurement

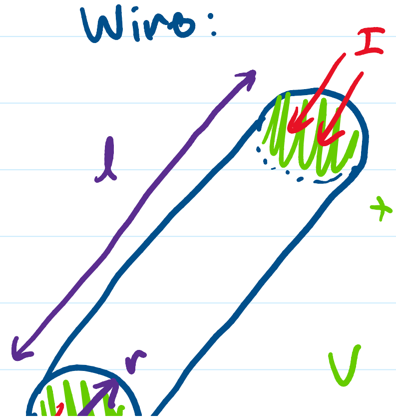
Touch Labs

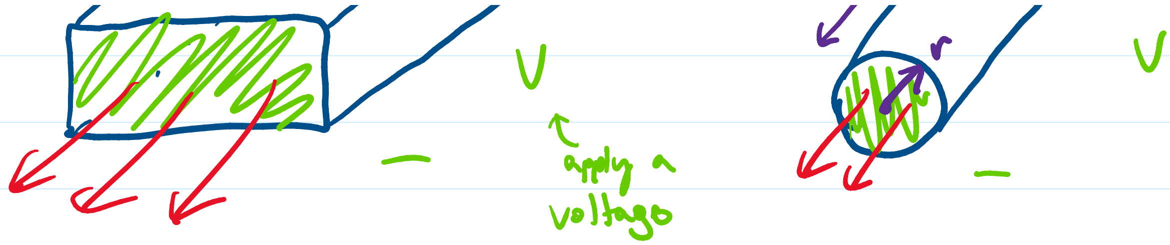
Physical Resistance:

Strip:



Wire:





$$R = \rho \frac{l}{wh} \longleftarrow R = \rho \cdot \frac{l}{A} \longrightarrow R = \rho \cdot \frac{l}{\pi r^2}$$

length (in direction of current flow)

cross-sectional area (perpendicular to current flow)

resistivity

$$[\Omega] = [\Omega \cdot m] \cdot \frac{[m]}{[m^2]}$$

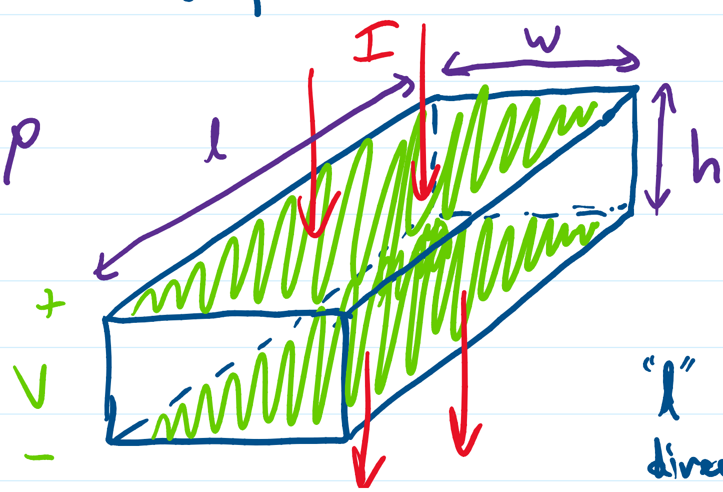
What is resistivity?

- Intrinsic material property
- Variable "ρ" (Greek Letter "rho")
- $\rho = \frac{1}{\sigma}$ conductivity (Greek Letter "sigma")

- More conductive $\longleftrightarrow$ less resistive
- More resistive $\longleftrightarrow$ less conductive

https://en.wikipedia.org/wiki/Electrical_resistivity_and_conductivity

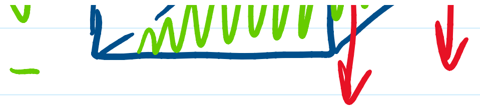
Another Strip...



$$R = ? = \rho \cdot \frac{"l"}{"A"}$$

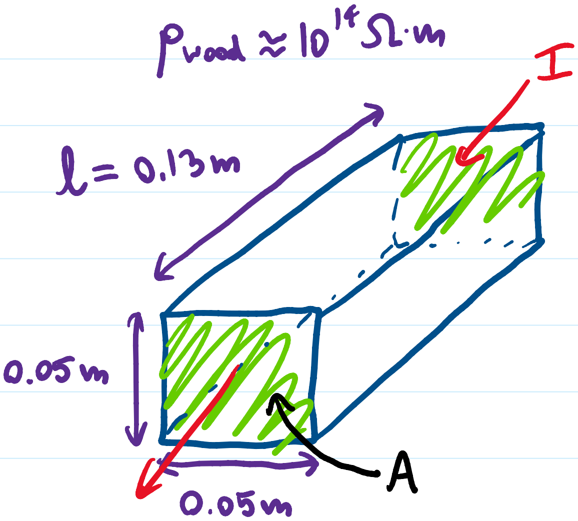
$$R = \rho \cdot \frac{h}{lw}$$

"l" and "A" depend on direction of current flow



$+$ and $-$ depend on direction of current flow

Demo with wooden blocks

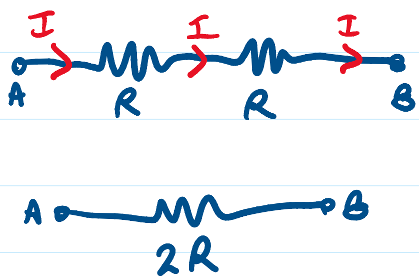


$$R = \rho\frac{l}{A} = (10^{14}\,\Omega m)\cdot\frac{(0.13m)}{(0.0025\ m^2)}$$
$$= 5.2\cdot 10^{15}\,\Omega$$

Back-to-back?

$$R_{tot} = \rho\frac{(2l)}{A} = 2R$$

Model:

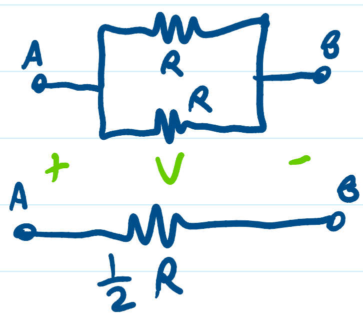


"series"-connected
(share the same current)

Side-to-side?

$$R_{tot} = \rho\frac{l}{(2A)} = \frac{1}{2}R$$

Model:



"parallel"-connected
(share the same voltage)

NVA Review: $\boxed{\text{Find the node voltages}}$

Ex). Voltage Divider

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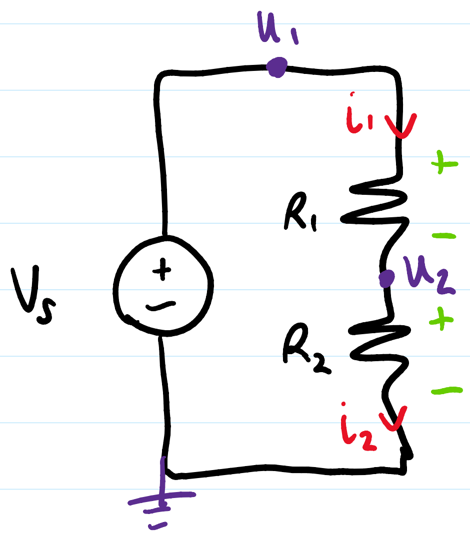


Step 1: Label reference node
Step 2: Label unknown node voltages
Step 3: Label currents
Step 4: Add +/- labels
Step 5: Identify unknowns $\left(\boxed{u_1, u_2,} \; i_1, i_2\right)$ key unknowns
Step 6a: KCL equations at unknown nodes

$\underset{\text{known}}{u_1 = V_s}$ $\quad u_2 = ? \rightarrow i_1 - i_2 = 0$

Step 6b: IV characteristics

$u_1 - u_2 = i_1 R_1 \qquad u_2 - 0 = i_2 R_2$

Step 7: substitute and solve

$i_1 - i_2 = 0 \longrightarrow \frac{u_1 - u_2}{R_1} - \frac{u_2}{R_2} = 0$

$$\boxed{\begin{aligned} u_1 &= V_s \\ u_2 &= \frac{R_2}{R_1 + R_2} \cdot V_s \end{aligned}}$$

Solve equations:

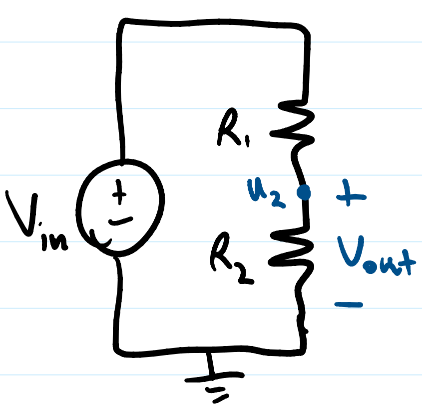
$u_1 = V_s$

$\frac{u_1 - u_2}{R_1} - \frac{u_2}{R_2} = 0 \rightarrow \frac{V_s}{R_1} - \frac{1}{R_1} u_2 - \frac{1}{R_2} u_2 = 0$ multiply by R_1 and R_2

$V_s \cdot R_2 - R_2 u_2 - R_1 u_2 = 0$

$u_2 \cdot (R_1 + R_2) = R_2 \cdot V_s$

$u_2 = \underbrace{\frac{R_2}{R_1 + R_2}}_{0 < x < 1} \cdot V_s$



Why is it called a voltage divider?

Pick: $R_1 = 1\Omega$, $R_2 = 3\Omega$

R_2

Pick: $R_1 = 1\Omega$, $R_2 = 3\Omega$

$$V_{out} = \frac{R_2}{\underbrace{R_1+R_2}_{\text{total series resistance}}}\cdot V_{in} = \frac{3}{1+3}\cdot V_{in} = \frac{\overset{R_2}{3}}{4}\cdot V_{in}$$

Power.

Electricity performs two basic functions:

1) Transfer information

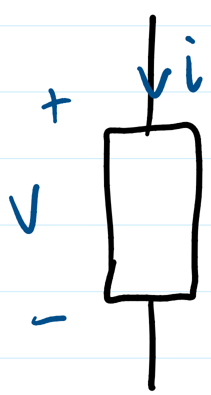
* 2) Transfer energy

It's really hard to store electrical energy

What is power? $P = \frac{\partial E}{\partial t}$ ← flow of energy

watt $[W] = \frac{[J]}{[s]}$ $\frac{\text{joule}}{\text{second}}$

Electrical power:



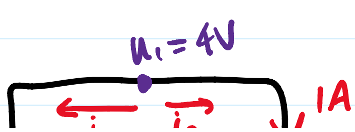
$$\boxed{P = v \cdot i} = \frac{\partial E}{\cancel{\partial q}} \cdot \frac{\cancel{\partial q}}{\partial t} = \frac{\partial E}{\partial t} \checkmark$$

↑ ALWAYS TRUE

(passive sign convention) $\begin{cases} p > 0 \rightarrow \text{dissipates power} \\ p < 0 \rightarrow \text{delivers power} \end{cases}$

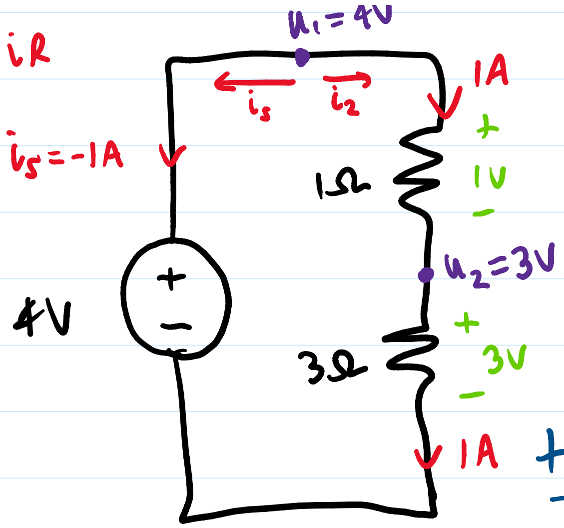
Let's consider our voltage divider example again

$V = iR$



$V_s = 4V$, $R_1 = 1\Omega$, $R_2 = 3\Omega$

$V = iR$



$V_s = 4V, \quad R_1 = 1\Omega, \quad R_2 = 3\Omega$

$$u_2 = \frac{R_2}{R_1 + R_2} V_s = \frac{3\Omega}{1\Omega + 3\Omega} \cdot 4V = 3V$$

$$P_s = V_s \cdot i_s = (4V) \cdot (-1A) = -4W$$

$$P_{1\Omega} = (1V) \cdot (1A) = +1W$$

$$+ \; P_{3\Omega} = (3V)(1A) = +3W$$

Conservation of energy/power: $\boxed{P_{total} = 0W}$