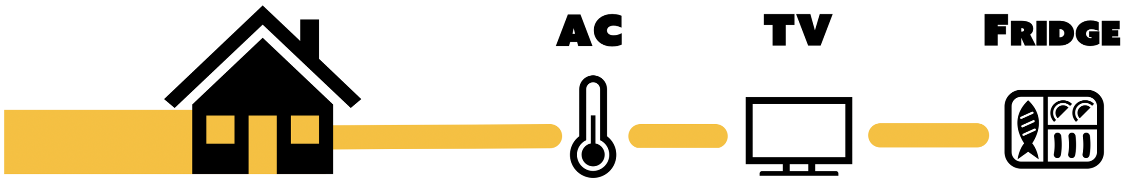
EECS 16A Designing Information Devices and Systems I
Summer 2023 Discussion 1C

1. Energy Disaggregation



Suppose you live in a home with just these **three appliances: an air conditioning unit (AC), a television (TV), and a refrigerator (R)**. Now, say you want to find the amount of electricity these appliances use individually, but the only measurement you can take is of the total power your home draws using your meter outside (this is often mounted on the side of the house and shows a running total of your electricity usage).

To do this, you will turn some appliances on and off and then read different total measurements. You can turn off the TV at any time, but you **can't unplug the fridge** since the food would spoil. We keep the air conditioner off throughout the morning, but then it must stay on during the afternoon. However, the breaker trips (meaning the electricity suddenly shuts off) if all three are running, so **the TV and AC cannot run at the same time**.

(a) Can you design a way to calculate how much power each appliance uses? What type of measurements will you need to make, and how many?

Let x_R be the power consumed by the refrigerator, x_{TV} by the TV, and x_{AC} by the AC, and let m_i represent the power measured in measurement i. To find out the values of three variables, we somehow need three equations/measurements.

Answer:

We must collect 3 different measurements, and each one needs to give us new information. Since we can only toggle (turn on and off) two of the appliances (AC & TV) and we cannot have all 3 appliances running, there are exactly 3 unique measurements we can take.

i. Measure in the morning (AM) with the TV off $\rightarrow \quad m_1 = x_R$
ii. Measure in the morning (AM) with the TV on $\rightarrow \quad m_2 = x_{\text{TV}} + x_R$
iii. Measure in the afternoon (PM) with the TV off $\rightarrow \quad m_3 = x_{\text{AC}} + x_R$

We have three equations and three unknowns and can now try to solve the problem!

(b) Write out a system of equations that describes your measurements. Can you solve this system so that each appliance's power is written in terms of measurements m_i?

For example: if you measure the power m_1 in the afternoon with the AC and refrigerator on but the TV is off, then the equation might look like $x_{\text{AC}} + x_{\text{R}} = m_1$.

Answer: As written above, the unique measurements are

$$\begin{aligned} x_{\mathrm{R}} &= m_1 \\ x_{\mathrm{TV}} + x_{\mathrm{R}} &= m_2 \\ x_{\mathrm{AC}} + x_{\mathrm{R}} &= m_3 \end{aligned}$$

We can solve this by substitution.

$$\begin{aligned} x_{\mathrm{R}} &= m_1 & & \\ x_{\mathrm{TV}} + m_1 &= m_2 & \rightarrow \quad & x_{\mathrm{TV}} = m_2 - m_1 \\ x_{\mathrm{AC}} + m_1 &= m_3 & \rightarrow \quad & x_{\mathrm{AC}} = m_3 - m_1 \end{aligned}$$

(c) Let us say the breaker is fixed, so now we can safely run the AC and TV at the same time. Is there another way (or ways) you could create a new system of equations to solve? If so, see if you can solve your new system!

Answer:
Without the breaker constraint, there are now 3 new possible systems of equations we could solve to determine the power drawn from each appliance.

Two of these systems can be solved via the substitution method like shown above, but for the system omitting the measurement with both AC and TV off we must add/subtract equations to find our result.

$$\begin{aligned} x_{\mathrm{TV}} + x_R &= m_2 \\ x_{\mathrm{AC}} + x_R &= m_3 \\ x_{\mathrm{AC}} + x_{\mathrm{TV}} + x_R &= m_4 \end{aligned}$$

We can identify x_{AC} by subtracting the m_2 equation from the m_4 equation:

$$m_4 - m_2 \quad = \quad x_{\mathrm{AC}} + x_{\mathrm{TV}} + x_R \; - x_{\mathrm{TV}} - x_R \quad = \quad x_{\mathrm{AC}}$$

From this point we can utilize the result via substitution into equations m_3 and m_2.

$$\begin{aligned} (m_4 - m_2) + x_R &= m_3 & \rightarrow \quad & x_R = m_2 + m_3 - m_4 \\ x_{\mathrm{TV}} + (m_2 + m_3 - m_4) &= m_2 & \rightarrow \quad & x_{\mathrm{TV}} = m_4 - m_3 \end{aligned}$$

(d) Lastly, suppose as a busy Berkeley student, you only get a chance to take two measurements. Can you determine how much power each of the three appliances draw? If not, what combinations of power consumption can you find out?

Answer:
As you may expect, you cannot figure out the values of three unknowns from two equations. For three unknowns, we would need a minimum of three equations. But, you can still determine some limited information on the amount of electric power that the appliances draw.

- measurements 1 & 2: You can measure x_R and x_{TV}.
- measurements 1 & 3: You can measure x_R and x_{AC}.
- measurements 1 & 4: You can measure x_R and the combined $(x_{\mathrm{AC}} + x_{\mathrm{TV}})$.
- measurements 2 & 3: You can measure the combined $(x_{\mathrm{TV}} + x_R)$ and $(x_{\mathrm{AC}} + x_R)$. Tricky one!
- measurements 2 & 4: You can measure x_{AC} and the combined $(x_{\mathrm{TV}} + x_R)$.
- measurements 3 & 4: You can measure x_{TV} and the combined $(x_{\mathrm{AC}} + x_R)$.

2. Linear or Nonlinear

Determine whether the following functions (f: $\mathbb{R}^2 \to \mathbb{R}$) are linear or nonlinear.

(a)
$$f(x) = \sin\left(\frac{\pi}{2}\right)x + \cos\left(\frac{\pi}{2}\right)$$

Solution/Answer: First, we can simplify our expression above. Notice that $\sin\left(\frac{\pi}{2}\right) = 1$ and $\cos\left(\frac{\pi}{2}\right) = 0$. With this information we can rewrite the function above as

$$f(x) = x.$$

To check for linearity, check for superposition (additivity) and homogeneity (multiplicative scaling). In other words we must check that:

$$f(\alpha x + \beta y) = \alpha f(x) + \beta f(y) \quad \forall \alpha, \beta, x, y \in \mathbb{R}$$

Linear:

$$\begin{aligned} f(\alpha x + \beta y) &= \alpha x + \beta y \\ &= \alpha f(x) + \beta f(y) \end{aligned}$$

Alternatively, you can state that this function is linear because it is of the form:

$$f(x) = ax$$

where a is a constant.

(b)
$$f(x_1, x_2) = 3x_1 + 4x_2$$

Solution/Answer: To check for linearity, check for superposition (additivity) and homogeneity (multiplicative scaling).
In other words we must check that:

$$f(\alpha x_1 + \beta y_1, \alpha x_2 + \beta y_2) = \alpha f(x_1, x_2) + \beta f(y_1, y_2) \quad \forall \alpha, \beta, x_1, x_2, y_1, y_2 \in \mathbb{R}$$

Linear:

$$\begin{aligned} f(\alpha x_1 + \beta y_1, \alpha x_2 + \beta y_2) &= 3(\alpha x_1 + \beta y_1) + 4(\alpha x_2 + \beta y_2) \\ &= \alpha(3x_1 + 4x_2) + \beta(3y_1 + 4y_2) \\ &= \alpha f(x_1, x_2) + \beta f(y_1, y_2) \end{aligned}$$

Alternatively, you can state that this function is linear because it is of the form:

$$f(x_1, x_2) = a_1 x_1 + a_2 x_2$$

where a_1 and a_2 are constants.

(c)

$$f(x_1,x_2) = e^{x_2} + x_1^2$$

Solution/Answer: To check for linearity, check for additivity and homogeneity (multiplicative scaling).
In other words we must check that:

$$f(\alpha x_1 + \beta y_1, \alpha x_2 + \beta y_2) = \alpha f(x_1,x_2) + \beta f(y_1,y_2) \quad \forall \alpha, \beta, x_1, x_2, y_1, y_2 \in \mathbb{R}$$

Nonlinear:

$$\begin{aligned} f(\alpha x_1 + \beta y_1, \alpha x_2 + \beta y_2) &= e^{\alpha x_2 + \beta y_2} + (\alpha x_1 + \beta y_1)^2 \\ &\neq \alpha e^{x_2} + \alpha x_1^2 + \beta e^{y_2} + \beta y_1^2 \\ &= \alpha f(x_1,x_2) + \beta f(y_1,y_2) \end{aligned}$$

Alternatively, you can state that this function is nonlinear because it is NOT of the form:

$$f(x_1,x_2) = a_1 x_1 + a_2 x_2$$

where a_1 and a_2 are constants.

(d)

$$f(x_1,x_2) = x_2 - x_1 + 3$$

Solution/Answer: To check for linearity, check for additivity and homogeneity (multiplicative scaling).
In other words we must check that:

$$f(\alpha x_1 + \beta y_1, \alpha x_2 + \beta y_2) = \alpha f(x_1,x_2) + \beta f(y_1,y_2) \quad \forall \alpha, \beta, x_1, x_2, y_1, y_2 \in \mathbb{R}$$

Nonlinear (in fact, this function is affine (see notes for more details)):
You may simply state that this function doesn't satisfy homogeneity when scaled by 0.
Alternatively, you can show:

$$\begin{aligned} f(\alpha x_1 + \beta y_1, \alpha x_2 + \beta y_2) &= (\alpha x_2 + \beta y_2) - (\alpha x_1 + \beta y_1) + 3 \\ &\neq \alpha(x_2 - x_1 + 3) + \beta(y_2 - y_1 + 3) \\ &= \alpha f(x_1,x_2) + \beta f(y_1,y_2) \end{aligned}$$

Alternatively, you can state that this function is nonlinear because it is NOT of the form:

$$f(x_1,x_2) = a_1 x_1 + a_2 x_2$$

where a_1 and a_2 are constants.

