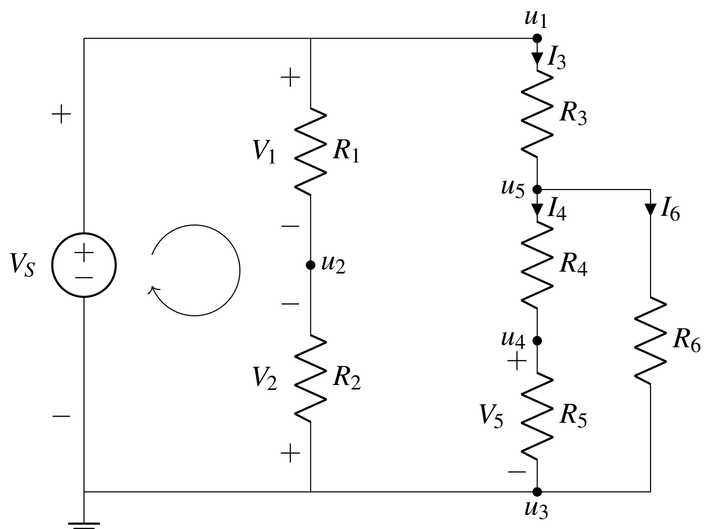
EECS 16A Designing Information Devices and Systems I
Summer 2023 Discussion 4C

1. Passive Sign Convention and NVA Basics

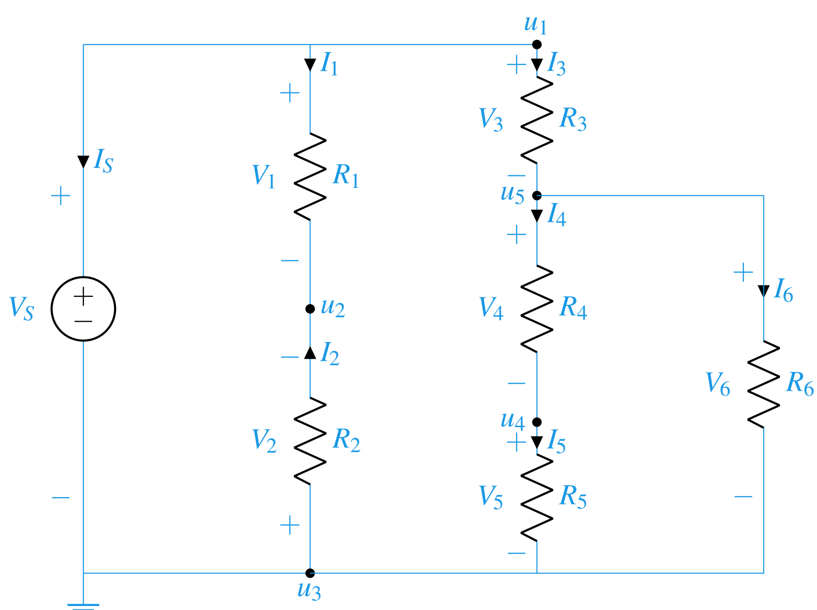
The following question is a modified version of Spring 2022 Midterm 2 Question 1

Suppose we have the following circuit:



(a) Following passive sign convention, **label** the missing currents and the missing voltages for each element in the circuit, including the voltage source.

Answer: Following the passive sign convention (current flows into the terminal with a positive voltage), we have the missing labels:



(b) **Write the KCL expression** at node u_5 in terms of currents I_3, I_4, and I_6 as labeled in the circuit diagram.

Answer: I_3 flows into the node, and I_4, I_6 flow out of the node, so the KCL expression is

$$-I_3 + I_4 + I_6 = 0.$$

Any equivalent expressions (for example, $I_3 - I_4 - I_6 = 0$, $I_3 = I_4 + I_6$, etc.) are acceptable.

(c) Find the voltage across R_4, R_5, and R_6 in terms of the node voltages u_3, u_4, and u_5. Then use Ohm's law to express the currents across R_4, R_5, and R_6 in terms of node voltages and resistances.

Answer:

Solving for the voltages in terms of node voltages, we have:

$$\begin{aligned} V_4 &= u_5 - u_4 \\ V_5 &= u_4 - u_3 = u_4 \\ V_6 &= u_5 - u_3 = u_5 \end{aligned}$$

Now for Ohm's law, we have the following:

$$\begin{aligned} I_4 &= \frac{V_4}{R_4} \\ I_5 &= \frac{V_5}{R_5} \\ I_6 &= \frac{V_6}{R_6} \end{aligned}$$

Combining the equations in terms of node voltages, we get the new set of equations in terms of node voltages and resistors:

$$\begin{aligned} I_4 &= \frac{V_4}{R_4} = \frac{u_5 - u_4}{R_4} \\ I_5 &= \frac{V_5}{R_5} = \frac{u_4}{R_5} \\ I_6 &= \frac{V_6}{R_6} = \frac{u_5}{R_6} \end{aligned}$$

(d) **Write the KVL expression** for the loop drawn in the circuit diagram in terms of voltages V_S, V_1, and V_2.

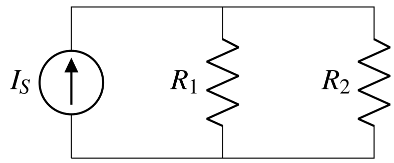
Answer: If we travel in the loop, we will first meet the negative terminal of V_S, the positive terminal of V_1, and the negative terminal of V_2, respectively. So the KVL expression is

$$-V_S + V_1 - V_2 = 0.$$

Any equivalent expressions (for example, $V_S - V_1 + V_2 = 0$, $V_S + V_2 = V_1$, etc.) are acceptable.

2. A Simple Current Circuit

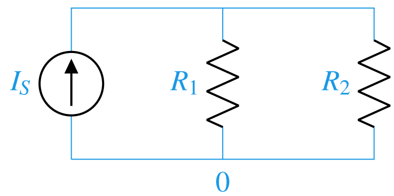
For the circuit shown below, find the voltages across all the elements and the currents through all the elements.



(a) In the above circuit, pick a reference node. Does your choice of reference matter?

Answer:

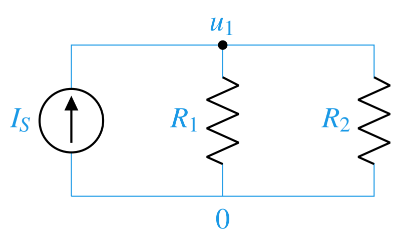
There are two nodes in this circuit and thus two choices for the reference node. The choice of reference does not matter. We will use the reference node shown below:



(b) With your choice of reference, label the node potentials for every node in the circuit.

Answer:

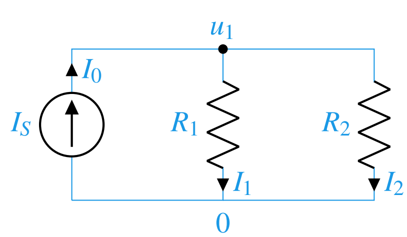
Since this circuit only has two nodes, there will only be one additional node potential.



(c) Label all of the branch currents. Does the direction you pick matter?

Answer:

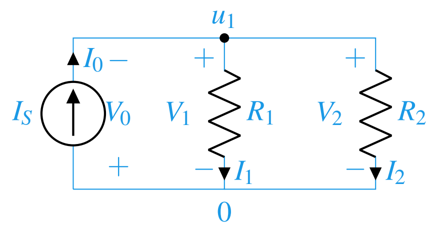
When labeling the currents through branches, the direction you pick does not matter.



(d) Draw the $+/-$ labels on every element. What convention must you follow?

Answer:

When drawing the $+/-$ labels, you must follow the passive sign convention. That is, current flows into the $+$ terminal of every element.



(e) Use KCL to find as many equations as you can.

Answer:

KCL gives us one equation for the node at the top, namely that $I_0 - I_1 - I_2 = 0$.

(f) Use KVL and Ohm's law to find the remaining equations to solve the circuit.

Answer: We know that the current through the current source must be the value of the current source, i.e.

$$I_0 = I_S \tag{1}$$

We also know that the voltage across the resistor is equal to the current times the resistance, i.e.

$$\begin{aligned} V_1 &= I_1 R_1 \\ V_2 &= I_2 R_2 \end{aligned} \tag{2}$$

Writing the equations for node potentials we have:

$$\begin{aligned} 0 - u_1 &= V_0 \\ u_1 - 0 &= V_1 \\ u_1 - 0 &= V_2 \end{aligned} \tag{3}$$

Using Equation (5) and substituting expressions from Equation (6) into Equation (7), we have:

$$\begin{aligned} I_0 &= I_S \\ V_1 = I_1 R_1 \implies I_1 &= \frac{u_1}{R_1} \\ V_2 = I_2 R_2 \implies I_2 &= \frac{u_1}{R_2} \end{aligned} \tag{4}$$

(g) Solve for the voltages across both resistors and the currents going through them if $I_S = 5\,\text{A}$, $R_1 = 5\,\Omega$, and $R_2 = 10\,\Omega$.

Answer: Substituting what we found from the last two parts, we have:

$$\begin{aligned} I_0 - I_1 - I_2 &= 0 \\ I_S - \frac{u_1}{R_1} - \frac{u_1}{R_2} &= 0 \\ I_S &= u_1\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \\ 5 &= u_1\left(\frac{1}{5} + \frac{1}{10}\right) \\ u_1 &= \frac{5}{\frac{1}{5} + \frac{1}{10}} \\ u_1 &= 16.67V \end{aligned} \tag{5}$$

Therefore $V_1 = V_2 = u_1 = 16.67$ V. Now to solve for currents going through each resistor, we just plug in u_1 back into equations we found, so:

$$\begin{aligned} I_1 &= \frac{u_1}{R_1} = 3.33A \\ I_2 &= \frac{u_1}{R_2} = 1.67A \end{aligned} \tag{6}$$

(h) (OPTIONAL) Rather than solve for the system using substitution, we can also use matrices! Set up a matrix equation in the form $\mathbf{A}\vec{x} = \vec{b}$ to solve for the unknown node potentials and currents, which are I_0, I_1, I_2 and u_1. Then use part (e) and (f) to fill in the entries of $\mathbf{A}$ and $\vec{b}$, and solve for the unknowns.

Answer:

$$\begin{bmatrix} ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \end{bmatrix} \begin{bmatrix} I_0 \\ I_1 \\ I_2 \\ u_1 \end{bmatrix} = \begin{bmatrix} ? \\ ? \\ ? \\ ? \end{bmatrix}$$

$\mathbf{A}$ will be a 4×4 matrix since there are four unknowns in the circuit, the currents I_0, I_1, and I_2 and the one potential u_1.

We then fill in the KCL equations into the matrix as follows:

$$\begin{bmatrix} 1 & -1 & -1 & 0 \\ ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \end{bmatrix} \begin{bmatrix} I_0 \\ I_1 \\ I_2 \\ u_1 \end{bmatrix} = \begin{bmatrix} 0 \\ ? \\ ? \\ ? \end{bmatrix}$$

Then we fill in the rest of the matrix with Ohm's Law equations. Note that we need to rearrange the equations, so $I_1 = \frac{u_1}{R_1} \implies I_1 R_1 - u_1 = 0$ and $I_2 = \frac{u_1}{R_2} \implies I_2 R_2 - u_1 = 0$.

$$\begin{bmatrix} 1 & -1 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & R_1 & 0 & -1 \\ 0 & 0 & R_2 & -1 \end{bmatrix} \begin{bmatrix} I_0 \\ I_1 \\ I_2 \\ u_1 \end{bmatrix} = \begin{bmatrix} 0 \\ I_S \\ 0 \\ 0 \end{bmatrix}$$

By plugging in the values we are given into the system of equations, we get:

$$\begin{bmatrix} I_0 \\ I_1 \\ I_2 \\ u_1 \end{bmatrix} = \begin{bmatrix} 5 \\ 3.33 \\ 1.67 \\ 16.67 \end{bmatrix}$$