EECS 16A Designing Information Devices and Systems I
Summer 2023 Discussion 4B

1. Mechanical Eigenvalues and Eigenvectors

In each part, find the eigenvalues of the matrix **M** and their associated eigenvectors.

(a) $\mathbf{M} = \begin{bmatrix} 1 & 0 \\ 0 & 9 \end{bmatrix}$
Do you observe anything about the eigenvalues and eigenvectors?

(b) $\mathbf{M} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$

2. Steady and Unsteady States

You're given the matrix $\mathbf{M}$:

$$\mathbf{M} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ 0 & 1 & -2 \\ 0 & 0 & 2 \end{bmatrix}$$

which generates the next state of a physical system from its previous state: $\vec{x}[k+1] = \mathbf{M}\vec{x}[k]$.

(a) The eigenvalues of $\mathbf{M}$ are $\lambda_1 = 1, \lambda_2 = 2, \lambda_3 = \frac{1}{2}$. Define $\vec{x} = \alpha\vec{v}_1 + \beta\vec{v}_2 + \gamma\vec{v}_3$, a linear combination of the eigenvectors corresponding to the eigenvalues. For each of the cases in the table, determine if

$$\lim_{n\to\infty} \mathbf{M}^n\vec{x}$$

converges. If it does, what does it converge to?

α	β	γ	Converges?	$\lim_{n\to\infty} \mathbf{M}^n\vec{x}$
0	0	$\neq 0$		
0	$\neq 0$	0		
0	$\neq 0$	$\neq 0$		
$\neq 0$	0	0		
$\neq 0$	0	$\neq 0$		
$\neq 0$	$\neq 0$	0		
$\neq 0$	$\neq 0$	$\neq 0$		

(b) (**Practice**) Find the eigenspaces associated with the eigenvalues:

i. $\text{span}(\vec{v}_1)$, associated with $\lambda_1 = 1$

ii. $\text{span}(\vec{v}_2)$, associated with $\lambda_2 = 2$

iii. $\text{span}(\vec{v}_3)$, associated with $\lambda_3 = \frac{1}{2}$

3. Are eigenvectors linearly independent?

Suppose we have a square matrix $\mathbf{A}^{n\times n}$ with n distinct eigenvalues $\{\lambda_1, \lambda_2, \ldots, \lambda_n\}$ (meaning that $\lambda_i \neq \lambda_j$ when $i \neq j$) and n corresponding eigenvectors $\{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n\}$. Prove that any two eigenvectors $\vec{v}_i, \vec{v}_j$ (for $i \neq j$) are linearly independent.

HINT: Begin proof by contradiction: Suppose that $\vec{v}_i$ and $\vec{v}_j$ correspond to distinct eigenvalues, so that $(\lambda_i - \lambda_j) \neq 0$, and are linearly dependent. Show this leads to a nonsensical equality after applying $\mathbf{A}$.

If you still feel stuck, apply the definition of linear dependence to $\vec{v}_i$ and $\vec{v}_j$. What happens when we apply $\mathbf{A}$ to eigenvectors, and more importantly to the definition you found in the last sentence? If you need help understanding proof by contradiction, Example 4.4 in Note 4 gives a good explanation and example.