Assignment 3

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***Abstract*—This document explains the concept of the equation of a circle passing through three points.**

Download the python code from

https://github.com/ee17btech11034/AI5106/blob/main/Assignment_3/AI_assignment_3.py

and latex-tikz codes from

https://github.com/ee17btech11034/AI5106/blob/main/Assignment_3/assignment_3.tex

1 Problem

Find the equation of a circle that passes through the points $\begin{pmatrix}1\\2\end{pmatrix}$, $\begin{pmatrix}3\\-4\end{pmatrix}$ and $\begin{pmatrix}5\\-6\end{pmatrix}$.

2 Explanation

General equation of circle is given by:

$$\mathbf{x}^T\mathbf{x} - 2\mathbf{c}^T\mathbf{x} + f = 0 \tag{2.0.1}$$

$\mathbf{c}$ is the centre. Circle is passing through points $\mathbf{P}, \mathbf{Q}, \mathbf{R}$, so substituting these in circle equation

$$2\begin{pmatrix}1 & 2\end{pmatrix}\mathbf{c} - f = 5 \tag{2.0.2}$$

$$2\begin{pmatrix}3 & -4\end{pmatrix}\mathbf{c} - f = 25 \tag{2.0.3}$$

$$2\begin{pmatrix}5 & -6\end{pmatrix}\mathbf{c} - f = 61 \tag{2.0.4}$$

can be expressed as

$$\begin{pmatrix}2 & 4 & -1\\6 & -8 & -1\\10 & -12 & -1\end{pmatrix}\begin{pmatrix}\mathbf{c}\\f\end{pmatrix} = \begin{pmatrix}5\\25\\61\end{pmatrix} \tag{2.0.5}$$

Row reducing the augmented matrix

$$\begin{pmatrix}2 & 4 & -1 & 5\\6 & -8 & -1 & 25\\10 & -12 & -1 & 61\end{pmatrix} \tag{2.0.6}$$

$$\begin{matrix}R_2 \to R_2 - 3R_1\\R_3 \to R_3 - 5R_1\\\longleftrightarrow\end{matrix}\begin{pmatrix}2 & 4 & -1 & 5\\0 & -20 & 2 & 10\\0 & -32 & 4 & 36\end{pmatrix} \tag{2.0.7}$$

$$\begin{matrix}R_1 \to R_1 + \frac{1}{5}R_2\\R_3 \to R_3 - \frac{8}{5}R_2\\\longleftrightarrow\end{matrix}\begin{pmatrix}2 & 0 & -\frac{3}{5} & 7\\0 & -20 & 2 & 10\\0 & 0 & \frac{4}{5} & 20\end{pmatrix} \tag{2.0.8}$$

$$\begin{matrix}R_1 \to R_1 + \frac{3}{4}R_3\\R_2 \to R_2 - \frac{5}{2}R_3\\\longleftrightarrow\end{matrix}\begin{pmatrix}2 & 0 & 0 & 22\\0 & -20 & 0 & -40\\0 & 0 & \frac{4}{5} & 20\end{pmatrix} \tag{2.0.9}$$

3 Solution

So, the centre of circle is :

$$\mathbf{c} = \begin{pmatrix}\frac{22}{2}\\\frac{-40}{-20}\end{pmatrix} = \begin{pmatrix}11\\2\end{pmatrix} \tag{3.0.1}$$

$$f = \frac{20}{\frac{4}{5}} = 25 \tag{3.0.2}$$

Equation of circle is:

$$\mathbf{x}^T\mathbf{x} - 2\begin{pmatrix}11 & 2\end{pmatrix}\mathbf{x} + 25 = 0 \tag{3.0.3}$$

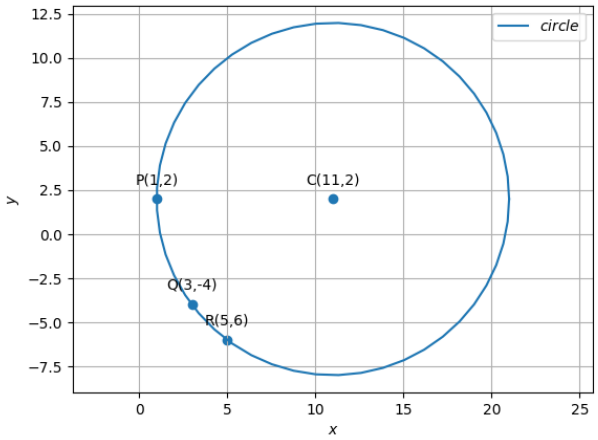

Fig. 0: Circle passing through three points