Assignment 3

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Abstract—**This document explains the concept of the equation of a circle passing through three points.**

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1 Problem

Find the equation of a circle that passes through the points $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$, $\begin{pmatrix} 3 \\ -4 \end{pmatrix}$ and $\begin{pmatrix} 5 \\ -6 \end{pmatrix}$.

2 Explanation

General equation of circle is given by:

$$\mathbf{x}^T\mathbf{x} - 2\mathbf{c}^T\mathbf{x} + f = 0 \qquad (2.0.1)$$

$\mathbf{c}$ is the centre. Circle is passing through points $\mathbf{P, Q, R}$, so substituting these in circle equation

$$2\begin{pmatrix} 1 & 2 \end{pmatrix}\mathbf{c} - f = 5 \qquad (2.0.2)$$

$$2\begin{pmatrix} 3 & -4 \end{pmatrix}\mathbf{c} - f = 25 \qquad (2.0.3)$$

$$2\begin{pmatrix} 5 & -6 \end{pmatrix}\mathbf{c} - f = 61 \qquad (2.0.4)$$

can be expressed as

$$\begin{pmatrix} 2 & 4 & -1 \\ 6 & -8 & -1 \\ 10 & -12 & -1 \end{pmatrix}\begin{pmatrix} \mathbf{c} \\ f \end{pmatrix} = \begin{pmatrix} 5 \\ 25 \\ 61 \end{pmatrix} \qquad (2.0.5)$$

Row reducing the augmented matrix

$$\begin{pmatrix} 2 & 4 & -1 & 5 \\ 6 & -8 & -1 & 25 \\ 10 & -12 & -1 & 61 \end{pmatrix} \qquad (2.0.6)$$

$$\begin{matrix} R_2 \to R_2 - 3R_1 \\ R_3 \to R_3 - 5R_1 \\ \longleftrightarrow \end{matrix} \begin{pmatrix} 2 & 4 & -1 & 5 \\ 0 & -20 & 2 & 10 \\ 0 & -32 & 4 & 36 \end{pmatrix} \qquad (2.0.7)$$

$$\begin{matrix} R_1 \to R_1 + \frac{1}{5}R_2 \\ R_3 \to R_3 - \frac{8}{5}R_2 \\ \longleftrightarrow \end{matrix} \begin{pmatrix} 2 & 0 & -\frac{3}{5} & 7 \\ 0 & -20 & 2 & 10 \\ 0 & 0 & \frac{4}{5} & 20 \end{pmatrix} \qquad (2.0.8)$$

$$\begin{matrix} R_1 \to R_1 + \frac{3}{4}R_3 \\ R_2 \to R_2 - \frac{5}{2}R_3 \\ \longleftrightarrow \end{matrix} \begin{pmatrix} 2 & 0 & 0 & 22 \\ 0 & -20 & 0 & -40 \\ 0 & 0 & \frac{4}{5} & 20 \end{pmatrix} \qquad (2.0.9)$$

3 Solution

So, the centre of circle is :

$$\mathbf{c} = \begin{pmatrix} \frac{22}{2} \\ \frac{-40}{-20} \end{pmatrix} = \begin{pmatrix} 11 \\ 2 \end{pmatrix} \qquad (3.0.1)$$

$$f = \frac{20}{\frac{4}{5}} = 25 \qquad (3.0.2)$$

Equation of circle is:

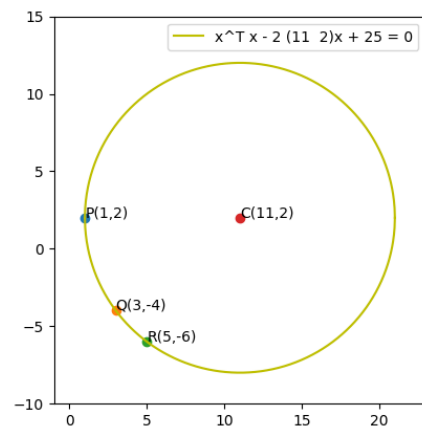$$\mathbf{x}^T\mathbf{x} - 2\begin{pmatrix} 11 & 2 \end{pmatrix}\mathbf{x} + 25 = 0 \qquad (3.0.3)$$

Fig. 0: Circle passing through three points