

Assignment 4

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Abstract—This document explains the concept of the equation of a circle passing through three points.

Download the python code from

https://github.com/ee17btech11034/AI5106/blob/main/Assignment_4/AI_assignment_4.py

and latex-tikz codes from

https://github.com/ee17btech11034/AI5106/blob/main/Assignment_4/assignment_4.tex

1 PROBLEM

Find the equation to the circle which has its centre at $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$ and touches the straight line $\begin{pmatrix} 5 & 12 \end{pmatrix} \mathbf{x} = 1$.

2 EXPLANATION

General equation of line and circle are given by:

$$\mathbf{n}^T \mathbf{x} - d = 0 \quad (2.0.1)$$

$$\mathbf{x}^T \mathbf{x} - 2\mathbf{c}^T \mathbf{x} + f = 0 \quad (2.0.2)$$

The straight line touches the circle so the radius of circle:

$$r = \left| \frac{\mathbf{n}^T \mathbf{c} - d}{\|\mathbf{n}\|} \right| \quad (2.0.3)$$

$$f = \|\mathbf{c}\|^2 - r^2 \quad (2.0.4)$$

3 SOLUTION

So, the radius of circle is :

$$r = \left| \frac{\begin{pmatrix} 5 & 12 \end{pmatrix} \begin{pmatrix} 3 \\ 4 \end{pmatrix} - 1}{\sqrt{(5)^2 + (12)^2}} \right| \quad (3.0.1)$$

$$r = \frac{62}{13} \quad (3.0.2)$$

$$f = (3)^2 + (4)^2 - \left(\frac{62}{13}\right)^2 \quad (3.0.3)$$

$$f = \frac{381}{169} \quad (3.0.4)$$

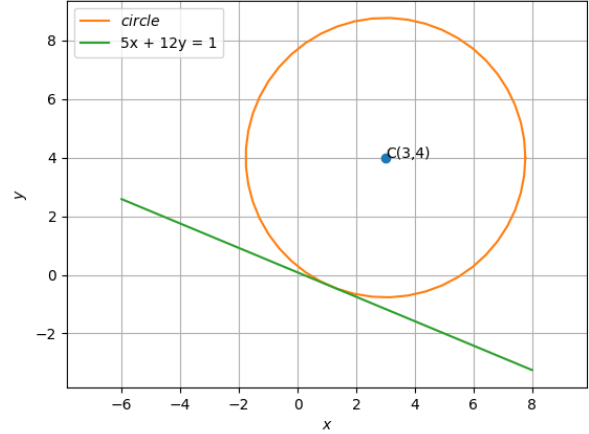


Fig. 0: Circle touches a line

Equation of circle is:

$$\mathbf{x}^T \mathbf{x} - 2 \begin{pmatrix} 3 & 4 \end{pmatrix} \mathbf{x} + \frac{381}{169} = 0 \quad (3.0.5)$$