Assignment 3

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***Abstract*—This document explains the concept of the equation of a circle passing through three points.**

Download the python code from

https://github.com/ee17btech11034/AI5106/blob/main/Assignment_3/AI_assignment_3.py

and latex-tikz codes from

https://github.com/ee17btech11034/AI5106/blob/main/Assignment_3/assignment_3.tex

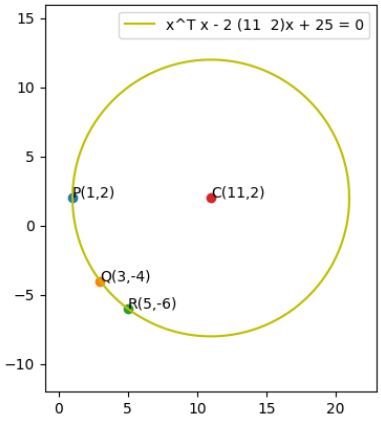

Fig. 0: Circle passing through three points

1 Problem

Find the equation of a circle that passes through the points $\begin{pmatrix}1\\2\end{pmatrix}$, $\begin{pmatrix}3\\-4\end{pmatrix}$ and $\begin{pmatrix}5\\-6\end{pmatrix}$.

2 Explanation

General equation of circle is given by:

$$\mathbf{x}^T\mathbf{x} - 2\mathbf{C}^T\mathbf{x} + c = 0 \tag{2.0.1}$$

Circle is passing through points $\mathbf{P}, \mathbf{Q}, \mathbf{R}$. So, these points will satisfy the circle equation. We can create a determinant :

$$\begin{vmatrix} x^2+y^2 & x & y & 1 \\ x_1^2+y_1^2 & x_1 & y_1 & 1 \\ x_2^2+y_2^2 & x_2 & y_2 & 1 \\ x_3^2+y_3^2 & x_3 & y_3 & 1 \end{vmatrix} \tag{2.0.2}$$

M_{ij} is the minor i^{th} row and j^{th} column of above determinant. $\mathbf{C}$ is the centre of circle:

$$\mathbf{C} = \frac{1}{2M_{11}}\begin{pmatrix} M_{12} \\ -M_{13} \end{pmatrix} \tag{2.0.3}$$

$$c = \frac{-M_{14}}{M_{11}} \tag{2.0.4}$$

3 Solution

So, the centre of circle is :

$$\mathbf{C} = \frac{1}{2*8}\begin{pmatrix}176\\-32\end{pmatrix} = \begin{pmatrix}11\\2\end{pmatrix} \tag{3.0.1}$$

$$c = \frac{-1*(-200)}{8} = 25 \tag{3.0.2}$$

Equation of circle is:

$$\mathbf{x}^T\mathbf{x} - 2\begin{pmatrix}11 & 2\end{pmatrix}\mathbf{x} + 25 = 0 \tag{3.0.3}$$