Assignment 5
The Resistor Problem

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1 Abstract

In this assignment we are going to see how to obtain the potential as a function of space with some initial conditions.As Current depends on the Geometry of Conductor,We need some Boundary conditions to obtain the exact Current profile.In this case,There is a wire Soldered to a Plate of Dimensions (Nx X Ny) and one of the side is grounded and the remaining 3 are floating.We would like to get Potential Profile as well as Current profile.

2 Assignment

2.1 Introduction

To find the Potential as a function of space we are going to use these equations.

$$\Delta^2\phi = 0 \tag{1}$$

And to compute this in the computer using the data at discrete points we are going to use the following relation

$$\phi_{i,j} = \frac{\phi_{i,j-1} + \phi_{i,j+1} + \phi_{i-1,j} + \phi_{i+1,j}}{4} \tag{2}$$

For The Current Profile,

$$j_x = -\frac{\partial\phi}{\partial x} \tag{3}$$

$$j_y = -\frac{\partial\phi}{\partial y} \tag{4}$$

It can be approximated as,

$$J_{x,ij} = \frac{1}{2}\phi_{i,j-1} - \phi_{i,j+1} \tag{5}$$

$$J_{y,ij} = \frac{1}{2}\phi_{i-1,j} - \phi_{i+1,j} \tag{6}$$

2.2 Getting the Parameters

By default we choose 25x25 grid with circle of radius 8 centrally located at V=1V also to run 1500 iterations. Relevant Code is as follows

```
if(len(sys.argv)==5):
   Nx=int(sys.argv[1])      #  X length
   Ny=int(sys.argv[2])      #  Y length
   radius=int(sys.argv[3])  #radius of Wire
   Iter=int(sys.argv[4])    #number of iterations to perform
elif(len(sys.argv)==1):
        Nx=25
   Ny=25                    #Default Values
   radius=8
   Iter=1500
else:
    print("Give proper arguments")
    exit()
```

2.3 Initializing the Potential

We initialize a zero 2D array of size Nx,Ny and then update 1V to all points lying within radius of central portion(Inside the Wire's Boundary).And the corresponding code is given below.

```
Old_Phi=zeros((Nx,Ny),dtype=float) # Creating Matrices for Potentials
New_Phi=zeros(Old_Phi.shape)

X=array([i for i in range(Nx)])  # For Creating Meshgrid
Y=array([i for i in range(Ny)])
x,y=meshgrid(X,Y)
i=where((x-Nx/2)**2+(y-Ny/2)**2<radius**2)
New_Phi[i]=1                     #Intializing the Wire's Potential
```

2.4 Updating the Potential

In a for loop we perform iterations by updating potential using the above mentioned equation and then applying boundary conditions and calculate error in each loop by subtracting phi's from Updated Potential and previous Potential.

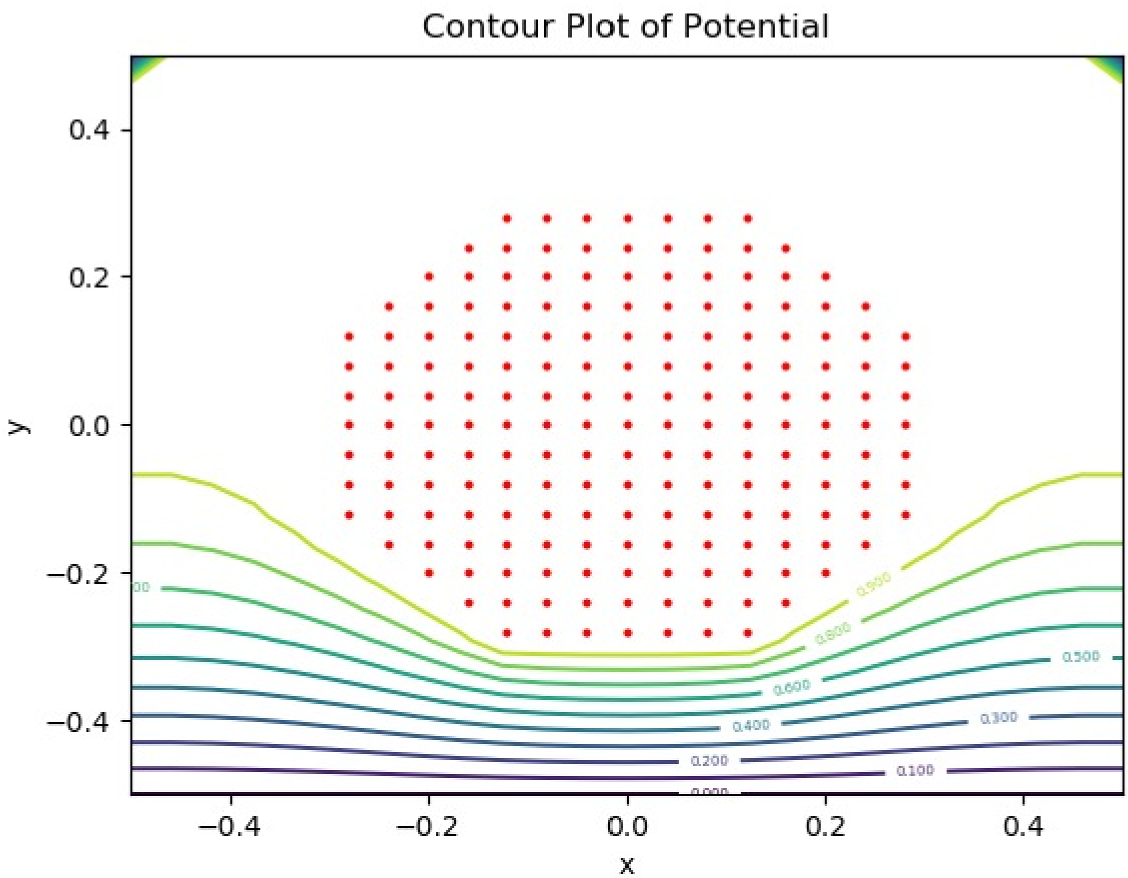
```
for k in range(Iter):
    Old_Phi=New_Phi.copy()
```

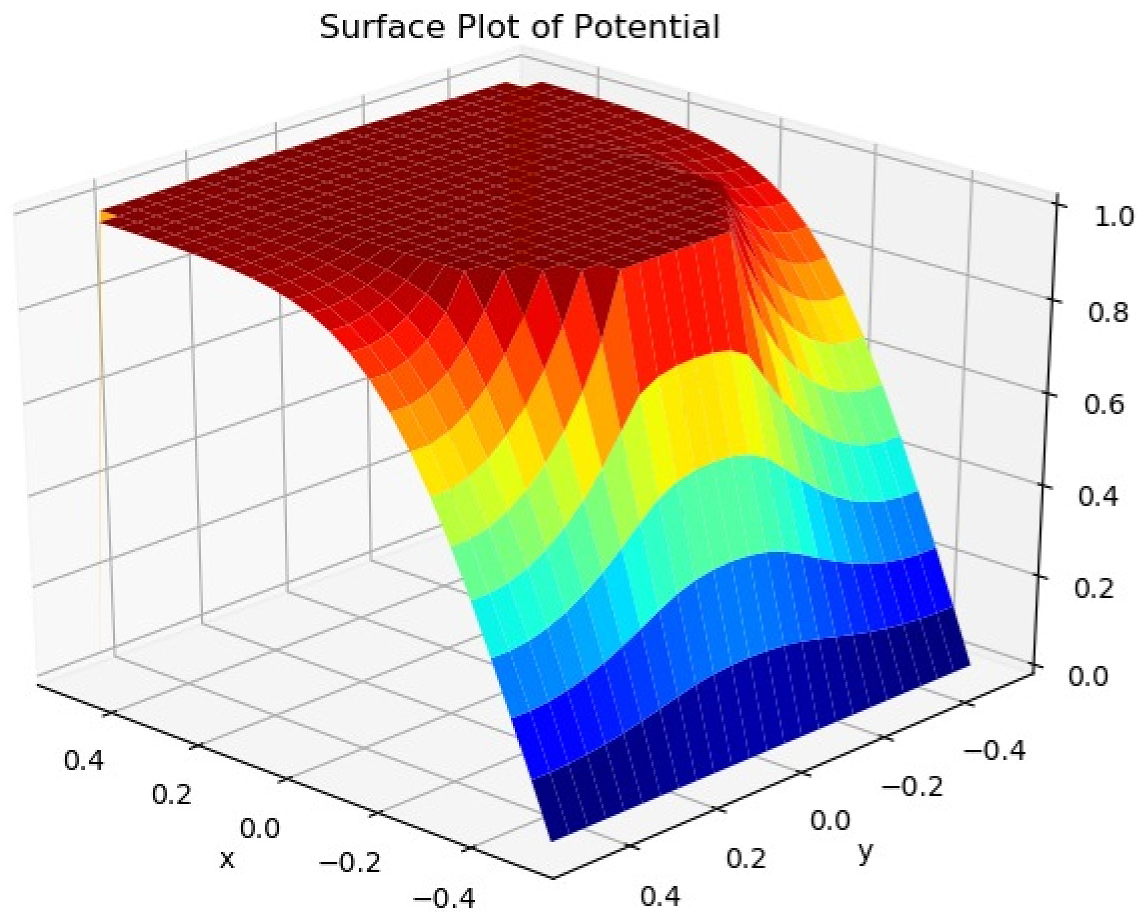
```
New_Phi[1:-1,1:-1]=0.25*(Old_Phi[0:-2,1:-1]+Old_Phi[2:,1:-1]+Old_Phi[1:-1,0:-2]+Old_Phi[1:-1,2:])
    #Top,Bottom,Left,Right members subarray Respectively
New_Phi[i]=1
New_Phi[1:-1,-1]=New_Phi[1:-1,-2]    #Top Boundary
New_Phi[0,1:-1]=New_Phi[1,1:-1]     #Right Boundary
New_Phi[-1,1:-1]=New_Phi[-2,1:-1]   #Left Boundary
error.append((New_Phi-Old_Phi).max())
```

2.5 Potential Curves

After Updating the Potential with Boundary value we approximated it to some Iterations so now we can plot the contour of the potential(Function of space(x,y)) also similarly we plot the Surface plot of the Potential.

```
fig1=contour(x,y,New_Phi,10)
clabel(fig1,fontsize=5)                   #Contour Plot
plot(i[0],i[1],'ro',markersize=2)
title('Contour Plot of Potential')
xlabel('x')
ylabel('y')
show()

fig2=figure(1)                            #Surface Plot
ax=p3.Axes3D(fig2)
surf=ax.plot_surface(y,x,New_Phi.T,rstride=1,cstride=1,cmap=cm.jet)
title('Surface Plot of Potential')
xlabel('x')
ylabel('y')
show()
```


Contour Plot of Potential
x
y
0.4
0.2
0.0
−0.2
−0.4
0.900
0.800
0.600
0.500
0.400
0.300
0.200
0.100



2.6 Fitting the Error

Whatever Error we got is like an exponentially decaying function for the initial values of the iteration .So we will try fit this data to an exponential curve in two ways.First ,We consider only the elements from $\frac{1}{3^{rd}}$ iterations to Complete Iterations and Fit it.Second we will consider all iterations and Fit the data. And again plot the Error using this fitted curves.The code for this is given below.

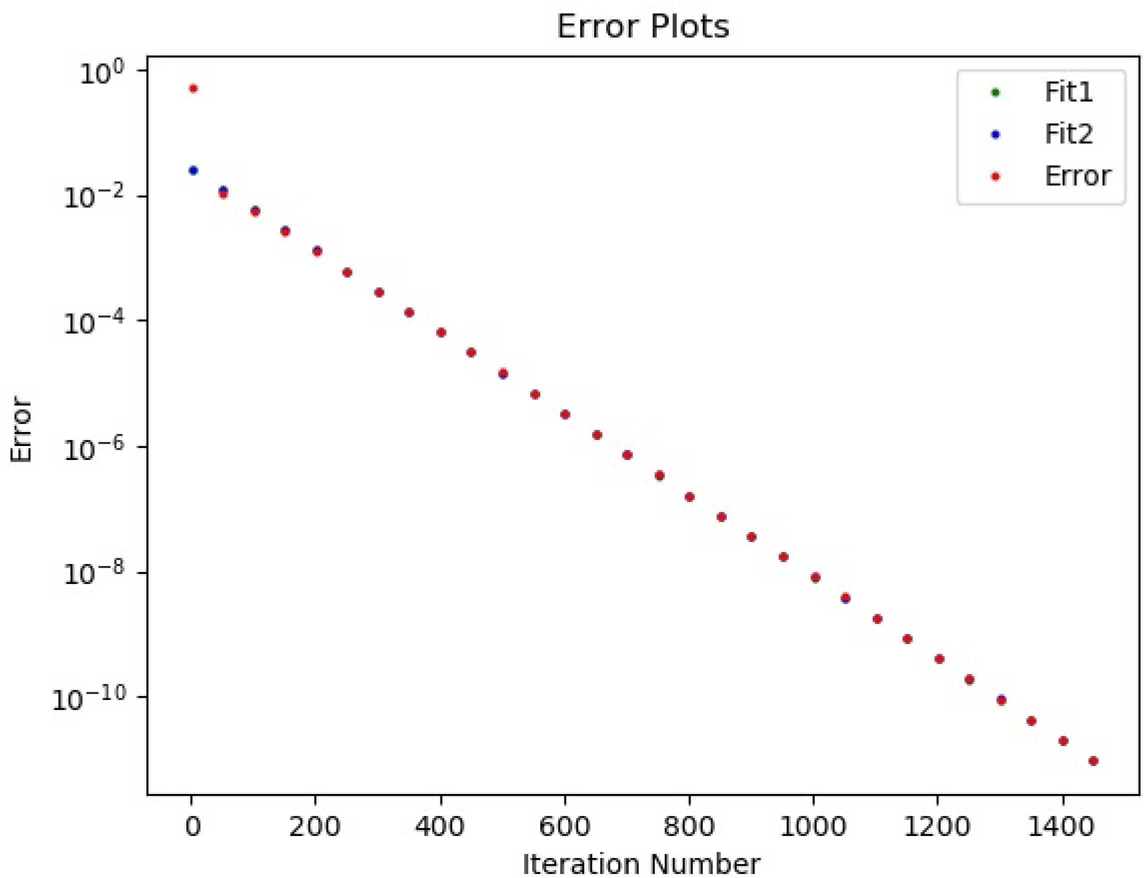
```
Mat1=zeros((int(2*Iter/3),2),dtype=float)    # Coefficient Matrices For Fitting Mat1 for 500 Iter, Mat2
    for 1500 iter
Mat2=zeros((int(Iter),2))

Mat1[:,0]=1
try:
   Mat1[:,1]=array(range(int(Iter/3),int(Iter)) )      #Intializing them
except Exception as e:
```

```
    print("Give iteration number as multiple of 3",e)
    exit()

Mat2[:,0]=1
Mat2[:,1]=array(range(1,Iter+1))

error1=array(error[int(Iter/3):])
error2=array(error[:Iter])

Sol1=lstsq(Mat1,log(error1),rcond=None)[0]     #Solution for the fitted data
Sol2=lstsq(Mat2,log(error2),rcond=None)[0]
```

2.7 Plotting Error curves

We plot the Errors(Actual),Fitted error using limited Iterations,Fitted error using Complete Iterations.Corresponding code for the above is given below

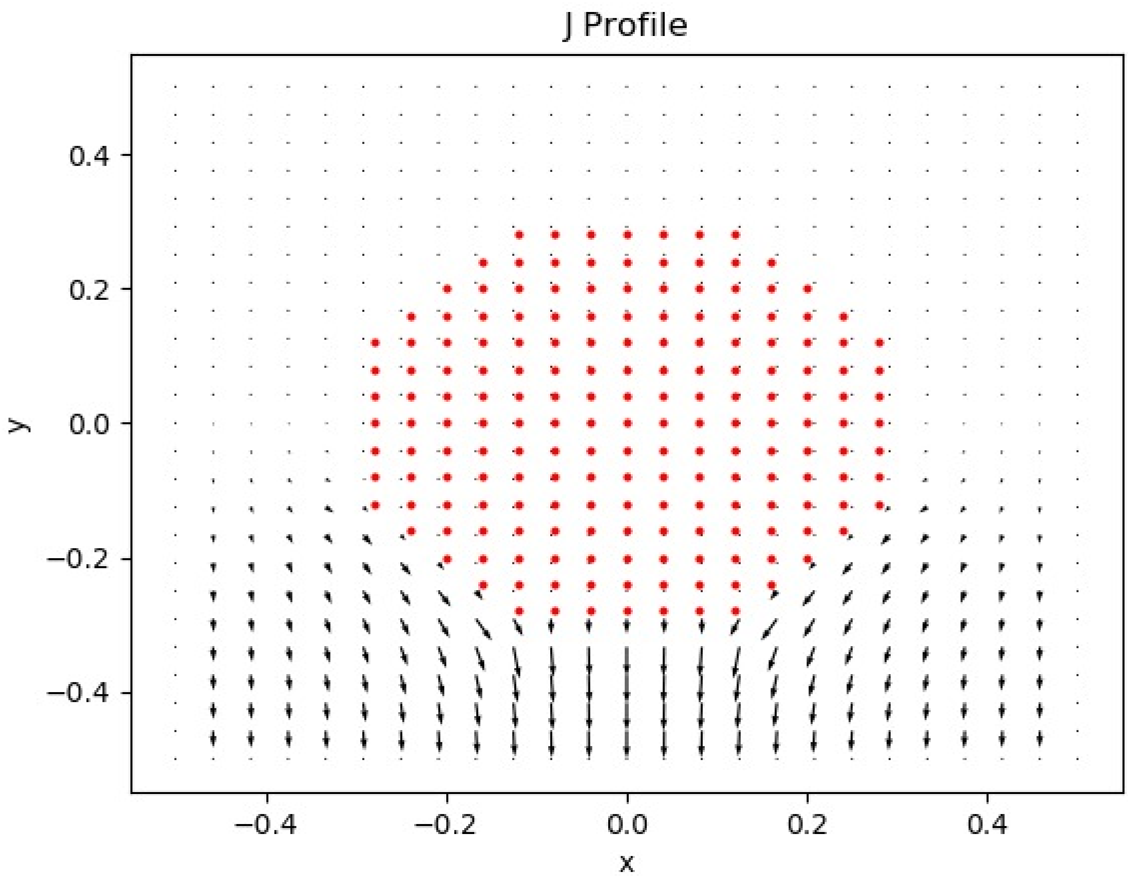
```
semilogy(range(1,int(Iter)+1,50),(e**Sol1[0])*e**(Sol1[1]*range(1,int(Iter),50)),'go',markersize=2,label='Fit1')
semilogy(range(1,Iter+1,50),(e**Sol2[0])*e**(Sol2[1]*range(1,Iter,50)),'bo',markersize=2,label='Fit2')
semilogy(range(1,Iter+1,50),error[::50],'ro',markersize=2,label='Error')
title('Error Plots')
xlabel('Iteration Number')
ylabel('Error')
legend()
show()
```



2.8 Forming J Profile

The following code is used to create J Profile.

```
Jx=zeros(New_Phi.shape)                                    #Current plotting
Jy=zeros(New_Phi.shape)
Jx[1:-1,1:-1]=0.5*(New_Phi[1:-1,0:-2]-New_Phi[1:-1,2:])
Jy[1:-1,1:-1]=0.5*(New_Phi[0:-2,1:-1]-New_Phi[2:,1:-1])
quiver(y,x,Jy[::-1,:],Jx[::-1,:],scale=7)
plot(i[0],i[1],'ro',markersize=2)                          #Dots for Central Wire
title('J Profile')
xlabel('x')
ylabel('y')
show()
```



3 Conclusion

In this Assignment we learned how to find the Laplace approximations using vectorised code and also we learnt about sub arrays.We have used discrete differentiation to solve Laplace equations and this method is known to be the worst available because of slow rate of reduction of bounding error.Analysing quiver plots for current densities , we understand that current is normal to both wire and metal plate at the Grounded side.