

# ASSIGNMENT 7: CIRCUIT ANALYSIS USING SYMPY AND LAPLACE TRANSFORMS

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## Abstract

The goal of this assignment is the following:

- To analyze Filters using Laplace Transform.
- To see how python can be used for symbolic Algebra.
- To plot graphs to understand high pass and low pass analog filters.

## Low Pass Filter

The low pass filter that we use gives the following matrix equation after simplification of the modified nodal equations.

$$\begin{pmatrix} 0 & 0 & 1 & -\frac{1}{G} \\ -\frac{1}{1+sR_2C_2} & 1 & 0 & 0 \\ 0 & -G & G & 1 \\ -\frac{1}{R_1} - \frac{1}{R_2} - sC_1 & \frac{1}{R_2} & 0 & sC_1 \end{pmatrix} \begin{pmatrix} V_1 \\ V_p \\ V_m \\ V_o \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ -V_i(s)/R_1 \end{pmatrix}$$

The python code snippet that declares the low pass function and solves the matrix equation to get the V matrix is as shown below:

```
#Low Pass Filter
def lowpass(R1,R2,C1,C2,G,Vi):
    s=symbols('s')
    A=Matrix([[0,0,1,-1/G],[-1/(1+s*R2*C2),1,0,0],[0,-G,G,1],[-1/R1-1/R2-s*C1,1/R2,0,0]])
    b=Matrix([0,0,0,-Vi/R1])
    V = A.inv()*b
    return (A,b,V)
```

The plot for the magnitude of the transfer function (magnitude bode plot) is as shown in Figure 1.

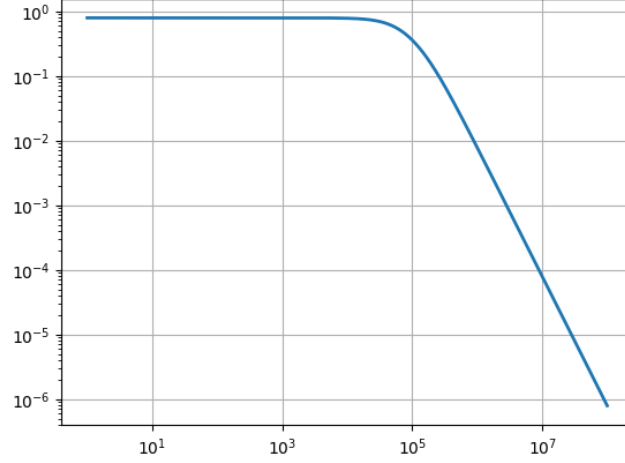


Figure 1: Magnitude bode plot of the low pass filter

## High Pass Filter

The high pass filter we use gives the following matrix equations after simplification of the modified nodal equations

$$\begin{pmatrix} 0 & 0 & 1 & -\frac{1}{G} \\ -\frac{sR_3C_2}{1+sR_3C_2} & 1 & 0 & 0 \\ 0 & -G & G & 1 \\ -1 - (sR_1C_1) - (sR_3C_2) & sC_2R_1 & 0 & 1 \end{pmatrix} \begin{pmatrix} V_1 \\ V_p \\ V_m \\ V_o \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ -V_i(s)sR_1C_1 \end{pmatrix}$$

The python code snippet that declares the high pass function and solves the matrix equation to get the V matrix is as shown below:

```
#High Pass Filter
def highpass(R1,R3,C1,C2,G,Vi):
    s=symbols('s')
    A=Matrix([[0,0,1,-1/G], [-1/(1+1/(s*R3*C2)), 1,0,0], [0,-G,G,1], [-s*C1-s*C2-1/R1,s],
    b=Matrix([0,0,0,-Vi*s*C1])
    V = A.inv()*b
    return (A,b,V)
```

The plot for the magnitude of the transfer function (magnitude bode plot) is as shown in Figure 2.

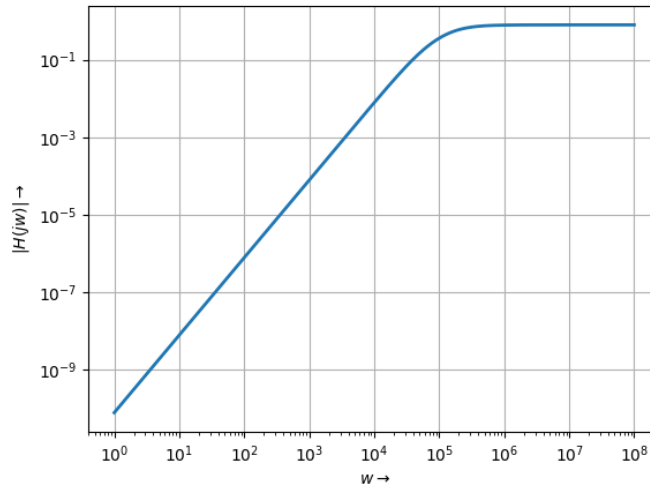


Figure 2: Magnitude bode plot of the high pass filter

## Format Change of Sympy Functions

The sympy functions, that are expressed in terms of 's', must be converted to another form that is understood by sp.signal. This is done using the following function.

The python code snippet is as shown:

```
# Convert Sympy transfer function polynomial to Scipy LTI
def sympy_to_lti(xpr, s=symbols('s')):

    num, den = simplify(xpr).as_numer_denom() # returns the expressions
    p_num_den = poly(num, s), poly(den, s)
    c_num_den = [p.all_coeffs() for p in p_num_den] # returns the coefficients
    l_num, l_den = [lambdify((), c)() for c in c_num_den] # convert to floats
    return sp.lti(l_num, l_den)
```

## Step Response Of the Low Pass Filter

In order to find the step response of the low pass filter, we need to assign  $V_i(s) = 1/s$ . The python code below explains it.

```
# Step response
t = np.linspace(0,0.001,1000)
Vo = sp.step(H,T=t)
display(4,Vo[0],Vo[1],r't$\rightarrow$',r'$V_{o}$\rightarrow$')
```

The unit step response, as expected is high at  $t=0$  when there is an abrupt change in the input. Since there is no other change at large time values outside the neighbourhood of 0, the Fourier transform of the unit step has high values near 0 frequency, which the high pass filter attenuates.

The plot for the step response of the low pass circuit is as shown in Figure 3.

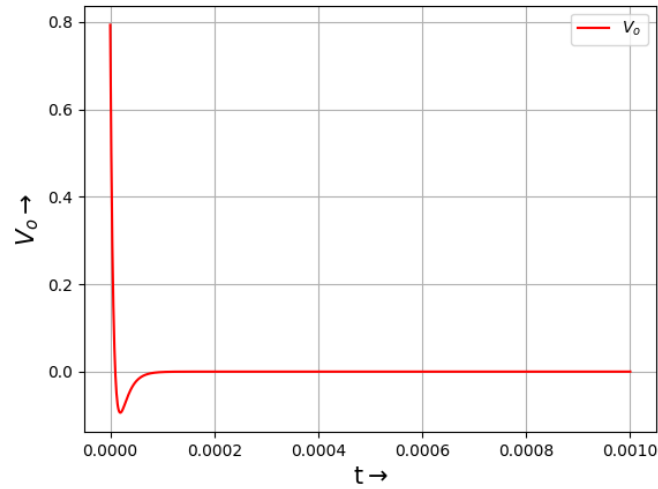


Figure 3: Step response of low pass filter.

## Response to Mixed Frequency Input

When the input is a sum of sinusoids like,

$$V_i(t) = ( \sin(2000\pi t) + \cos(2 * 10^6 \pi t) ) u_o(t) \text{ Volts}$$

Then the output response for the lowpass filter can easily be found as shown in the code snippet.

```
# Obtaining the response for mixed frequency sinusoid
t = np.linspace(0,0.01,100000)
Vi = np.multiply((np.sin(2000*np.pi*t)+np.cos(2000000*np.pi*t)),np.heaviside(t,0.5))
Vo = sp.lsim(H,Vi,T=t)
pylab.figure(1)
pylab.plot(Vo[0],Vi,label=r'$V_{in}$')
display(1,V0[0],Vo[1],r't$\rightarrow$',r'$V\rightarrow$')
```

The output response to the sum of sinusoids for the low pass filter is as shown:

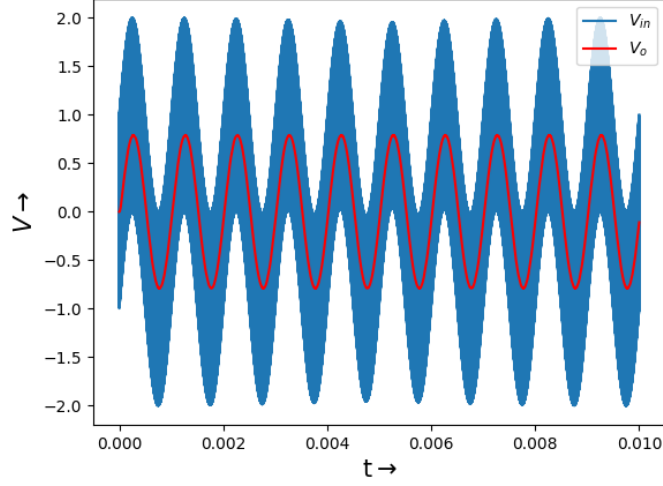


Figure 4: Output response to the sum of sinusoids.

## Response to Damped Sinusoids

In this case we assign the input voltage as a damped sinusoid like,  
Low frequency,

$$V_i(t) = e^{-500t} ( \cos(2000\pi t) ) u_o(t) \text{ Volts}$$

High frequency,

$$V_i(t) = e^{-500t} ( \cos(2 * 10^6 \pi t) ) u_o(t) \text{ Volts}$$

The high frequency and low frequency plots of the input damped sinusoids are as shown:

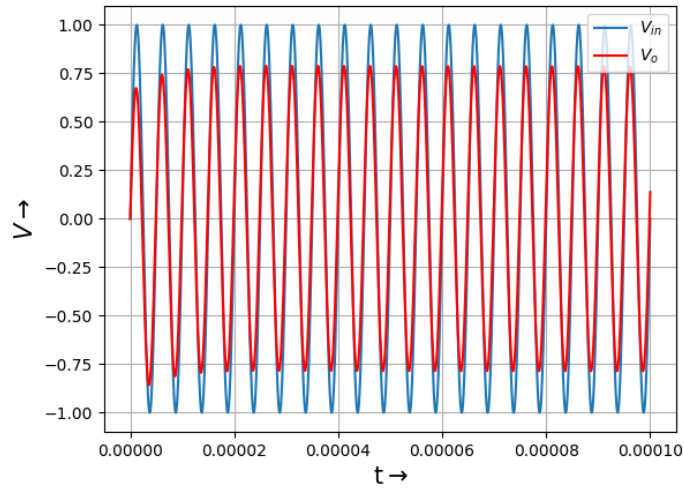


Figure 5: High frequency damped sinusoid

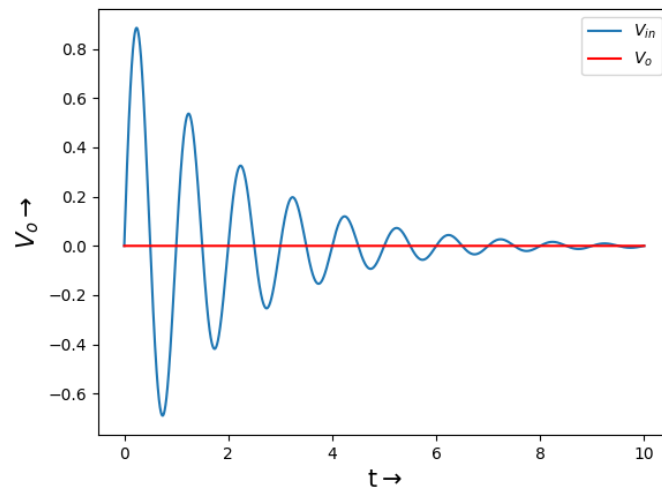


Figure 6: Low frequency damped sinusoid

The python code snippet to execute the above is as shown:

```
# response for damped sinusoids
t = np.linspace(0,10,1000)
Vi = np.multiply(np.multiply(np.exp(-0.5*t),np.sin(2*np.pi*t)),np.heaviside(t,0.5))
Vo = sp.lsim(H,Vi,T=t)
pylab.figure(2)
```

```

pylab.plot(Vo[0],Vi,label=r'$V_{in}$')
display(2,Vo[0],Vo[1],r't$\rightarrow$,r'$V\rightarrow$')

t = np.linspace(0,0.0001,10000)
Vi = np.multiply(np.multiply(np.exp(-0.5*t),np.sin(2*np.pi*200000*t)),np.heaviside(t))
Vo = sp.lsim(H,Vi,T=t)
pylab.figure(2)
pylab.plot(Vo[0],Vi,label=r'$V_{in}$')
display(3,Vo[0],Vo[1],r't$\rightarrow$,r'$V\rightarrow$')

```

## Conclusion

In conclusion, the sympy module has allowed us to analyse quite complicated circuits by analytically solving their node equations. We then interpreted the solutions by plotting time domain responses using the signals toolbox. Thus, sympy combined with the scipy.signal module is a very useful toolbox for analyzing complicated systems like the active filters in this assignment.