Assignment 9

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May 2021

1 Introduction

In this assignment, we continue our analysis of signals using Fourier Transforms. This time, we focus on finding transforms of non periodic functions. These functions have a discontinuity when periodically extended. The discontinuity causes fourier components in frequencies other than the sinusoids frequency which decay as $\frac{1}{\omega}$, due to Gibbs phenomenon. We resolve this problem using a hamming window in the case of this assignment. We use this windowed transform to analyse signals known to contain a sinusoid of unknown frequencies and extract its phase and frequency. We also perform a sliding DFT on a chirped signal and plot the results
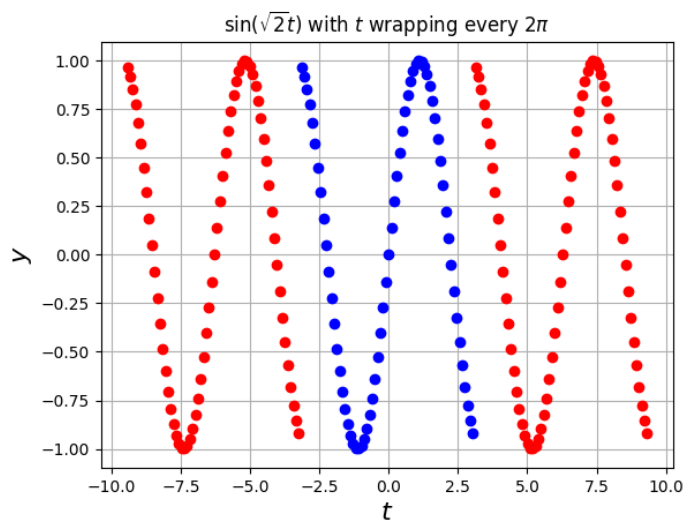
2 Worked Examples

The worked examples in the assignment are given below: Spectrum of $sin(\sqrt{2}t)$ is given below
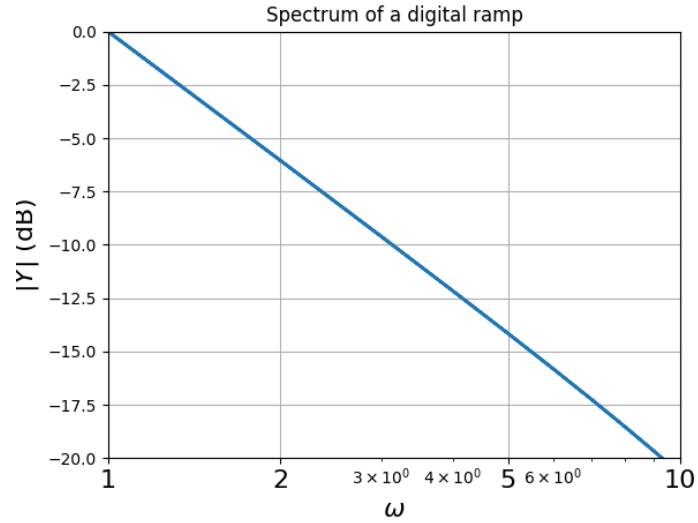
Spectrum of $\sin(\sqrt{2}t)$
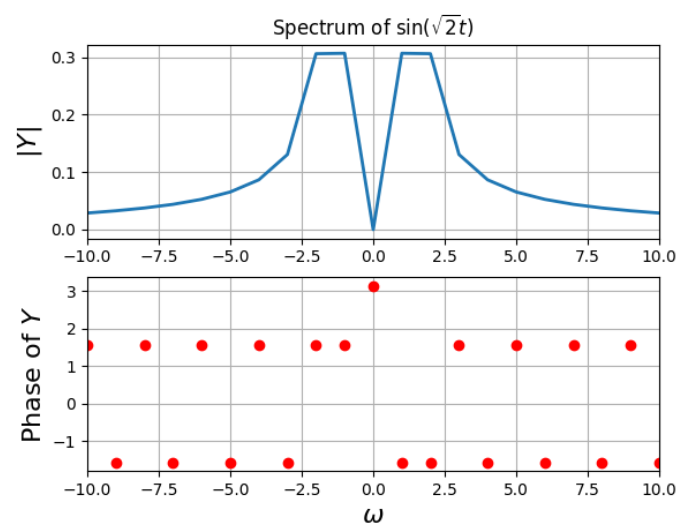
Original function for which we want the DFT:
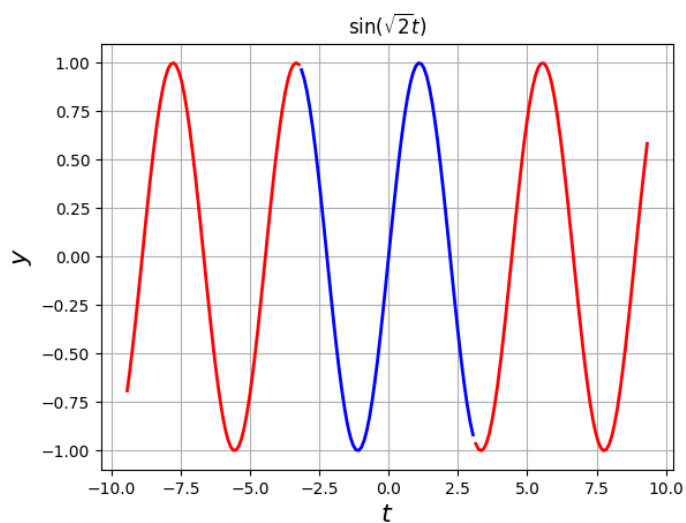
Since the DFT is computed over a finite time interval, We actually plotted the DFT for this function

These discontinuities lead to non harmonic components in the FFT which decay as $\frac{1}{\omega}$. To confirm this, we plot the spectrum of the periodic ramp below:

Spectrum of a digital ramp

2.1   Hamming Window

The hamming window removes discontinuities by attenuating the high frequency components that cause the discontinuities. The hamming window function is given by
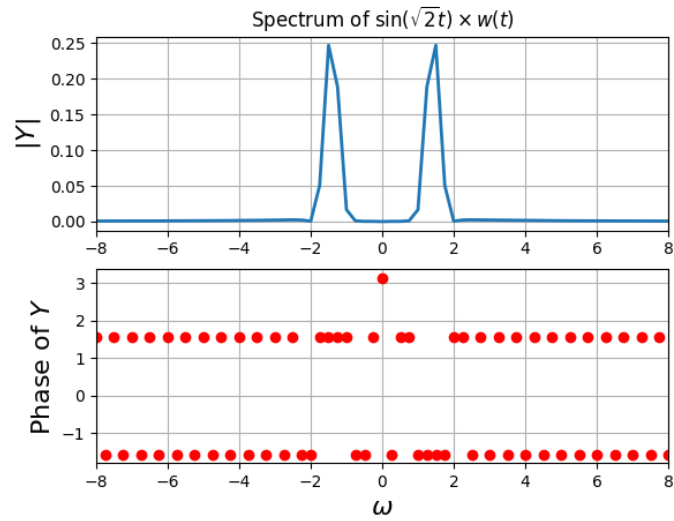
$$x[n] = 0.54 + 0.46 cos(\frac{2\pi n}{N-1}) \tag{1}$$

We now multiply our signal with the hamming window and periodically extend it. Note that the discontinuities nearly vanish

$\sin(\sqrt{2}t) \times w(t)$ with t wrapping every 2π
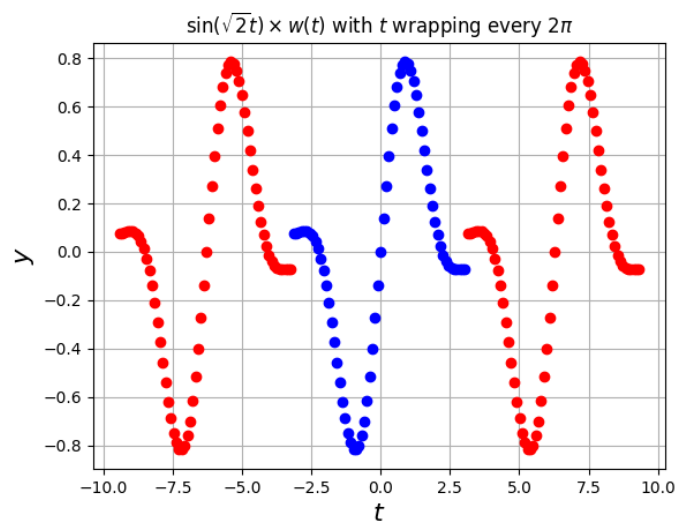
The spectrum that is obtained with a time period 2π is given below:

Spectrum of $\sin(\sqrt{2}t) \times w(t)$
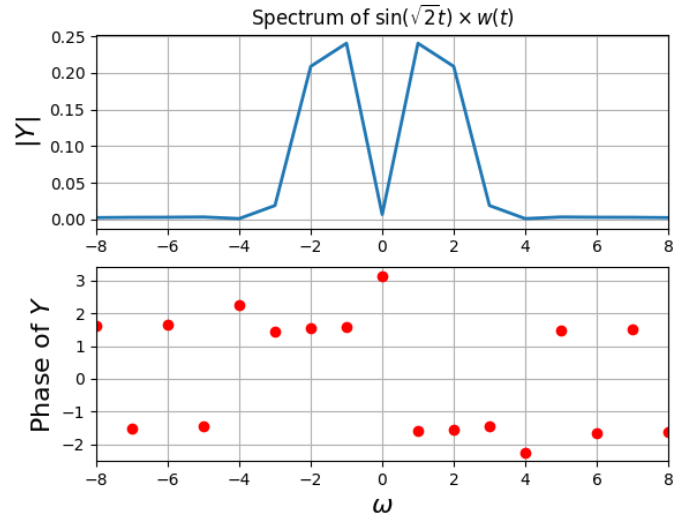
The spectrum that is obtained with a time period 8π has a slightly sharper peak and is given below:

Spectrum of $\sin(\sqrt{2}t) \times w(t)$

3 Assignment questions

3.1 Helper Functions

Given below is a helper function that plots and returns the DFT of an arbitrary function.

```
def spectrum(lim,n,f,t_=0,show_ = True,t_lims = False,windowing= False,
xlim1=10,title1 = r"Spectrum_of_$\sin\left(\sqrt{2}t\right)$",
xlabel1 = r"$\omega$",ylabel1= r"$|Y|$", ylabel2 = r"Phase_of_$Y$",
savename = "abc.png"):
    if(t_lims):
        t = t_
    else:
        t=linspace(-lim,lim,n+1)[:-1]
    dt=t[1]-t[0];
    fmax=1/dt
    y = f(t)
    if (windowing):
        m=arange(n)
        wnd=fftshift(0.54+0.46*cos(2*pi*m/n))
        y = y*wnd
    y[0]=0 # the sample corresponding to -tmax should be set zeroo
    y=fftshift(y) # make y start with y(t=0)
    Y=fftshift(fft(y))/float(n)
    w=linspace(-pi*fmax,pi*fmax,n+1)[:-1]

    mag = abs(Y)
    ph = angle(Y)
    if(show_):
        figure()
        subplot(2,1,1)
        plot(w,mag,lw=2)
        xlim([-xlim1,xlim1])
        ylabel(ylabel1,size=16)
        title(title1)
        grid(True)
        subplot(2,1,2)
        ph[where(mag<3e-3)] = 0
        plot(w,ph,'ro',lw=2)
        xlim([-xlim1,xlim1])
        ylabel(ylabel2,size=16)
        xlabel(xlabel1,size=16)
        grid(True)
        savefig(savename)
        show()
    return w,Y
```
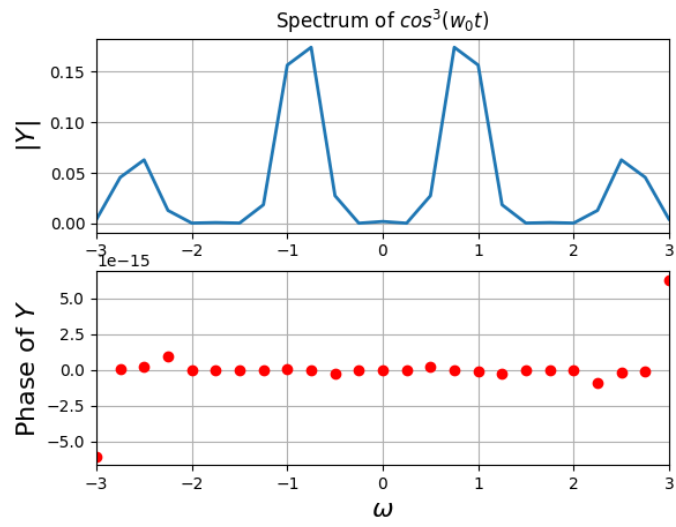
3.2 Question 2

In this question, we shall plot the FFT of $cos^3(0.86t)$ The FFT without the hamming Window:

Spectrum of $cos^3(w_0 t)$
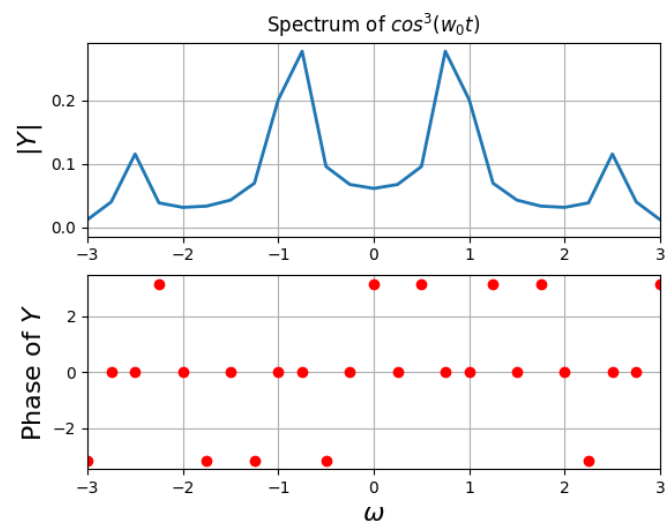
The FFT with the hamming Window:

Spectrum of $cos^3(w_0t)$

We notice that a lot of the energy is stored in frequencies that aren't a part of the signal. After windowing, these frequencies are attenuated and hence the peaks are sharper in the windowed function. It is still not an impulse because the convolution with the Fourier transform of the windowed function smears out the peak

3.3   Question 3

We need to estimate ω and δ for a signal $\cos(\omega t + \delta)$ for 128 samples between $[-\pi, \pi)$. We estimate omega using a weighted average. We have to extract the digital spectrum of the signal and find the two peaks at $\pm\omega_0$, and estimate ω and δ.
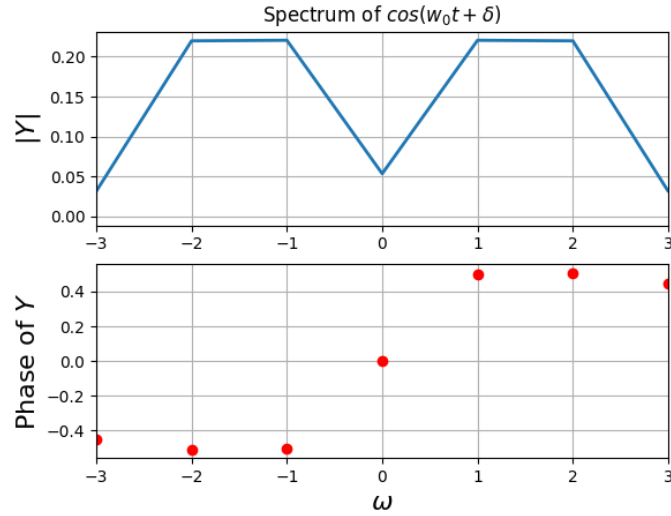

Figure 8: Fourier transform of $cos(1.5t + 0.5)$

We estimate omega by performing a Mean average of ω over the magnitude of $|Y(j\omega)|$.

For delta we consider a widow on each half of ω (split into positive and negative values) and extract their mean slope. The intuition behind this is that, a circular shift in the time domain of a sequence results in the linear phase of the spectra.

3.4   Question 4

We repeat the exact same process as question 3 but with noise added to the original signal.

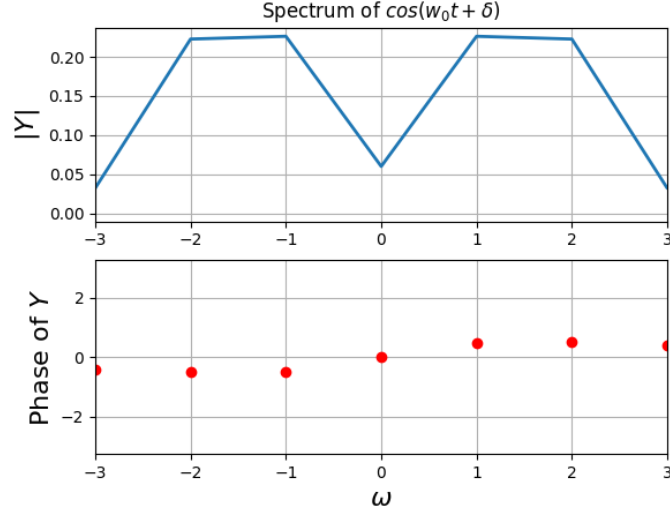For true value of ω and $\delta = 1.5$ and 0.5 respectively I got :

Figure 9: Fourier transform of noise $+ \ cos(1.5t + 0.5)$

omega = 1.5163179648582414
delta = 0.506776265719626 in no noise case
omega = 2.078915947064576
delta = 0.5006782553206393 in the noisy case

3.5   Question 5

In this question we analyze a chirp signal which is an FM signal where frequency is directly proportional to time. A chirp signal we shall consider is given by

$$f(t) = cos(16t(1.5 + \frac{t}{2\pi})) \tag{2}$$

The FFT of the chirp is given by: We note that the frequency response is spread between 5-50 rad/s. A large section of this range apears due to Gibbs phenomenon. On windowing, only frequencies between 16 and 32 rad/s remain.

3.6   Question 6

For the same chirped signal, we break the 1024 vector into pieces that are 64 samples wide. Extract the DFT of each and store as a column in a 2D array. Then plot the array as a surface plot to show how the frequency of the signal varies with time. This is new. So far we worked either in time or in frequency. But this is a "time- frequency" plot, where we get localized DFTs and show
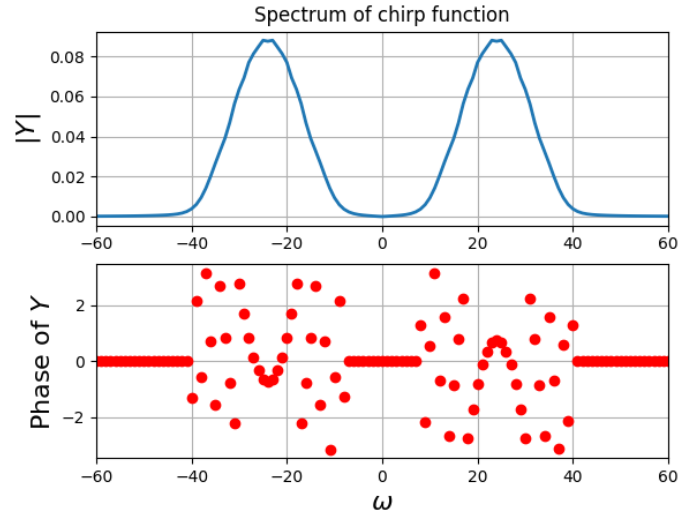
11

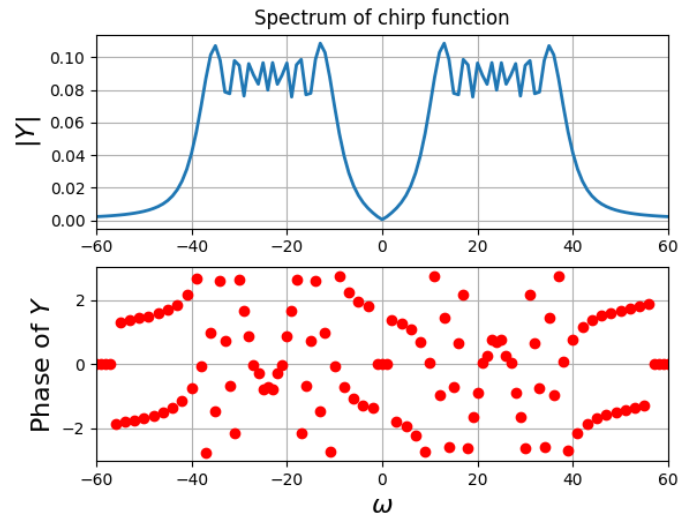Figure 10: Chirp function fourier transform, windowed

Figure 11: Chirp function fourier transform

how the spectrum evolves in time. We do this for both phase and magnitude. Let us explore their surface plots.
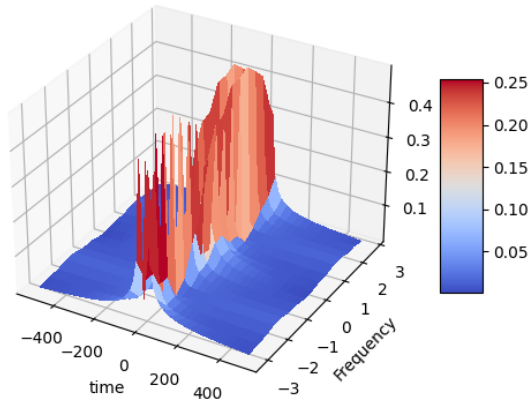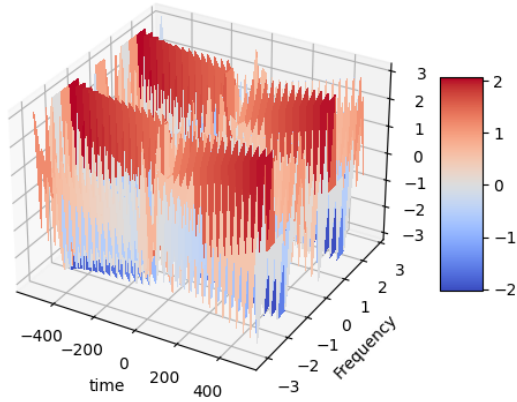
Figure 12: Time Frequency Plot of DFT

Figure 13: Variation of Frequency contained with time

4   Conclusion

In this assignment we have covered the requirement of windowing in the case of non-periodic series in DFT's. In particular this is to mitigate the effect of

Gibbs phenomena owing to the discontinuous nature of the series $\tilde{x}[n]$ realised by a discrete fourier transform.

The last question addresses the time varying spectra for a chirped signal, where we plot fourier spectra for different time slices of a signal. We noted the case of sparse number of slices and hence took more closely spaced slices.

The general properties of a fourier spectra for a chirped signal are observable in the time avrying plots , ie..., existence of two peaks (slow growth), vanishing of chirp effects in case of a windowed transform, and a phase plot that periodically varies with reduced phase near maximum values.