EE2703 : Applied Programming Lab
Assignment 4: Fourier Approximations

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Introduction

In this assignment we discussed about approximating a function using the Fourier Series Coefficients or Fourier Expansion. We have explored two ways of generating the Fourier co-efficients, one is the integral method and the other least squares method. Let us discuss about them in detail as we go ahead.

Actual Functions

The two given functions to be approximated using Fourier Series are exp(x) and cos(cos(x)). The original plots of these functions in linear scale are given in Figure 1 and Figure 2 respectively. Since, exp(x) raises rapidly, we have plotted exp(x) in semilog scale in Figure 3.

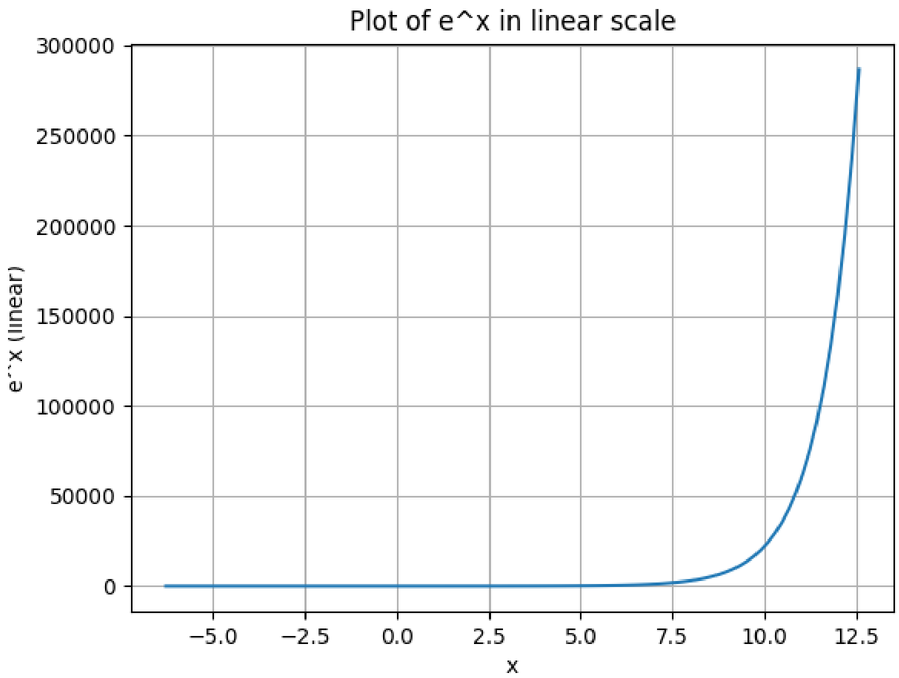

Figure 1: Actual Plot of exp(x) in linear scale

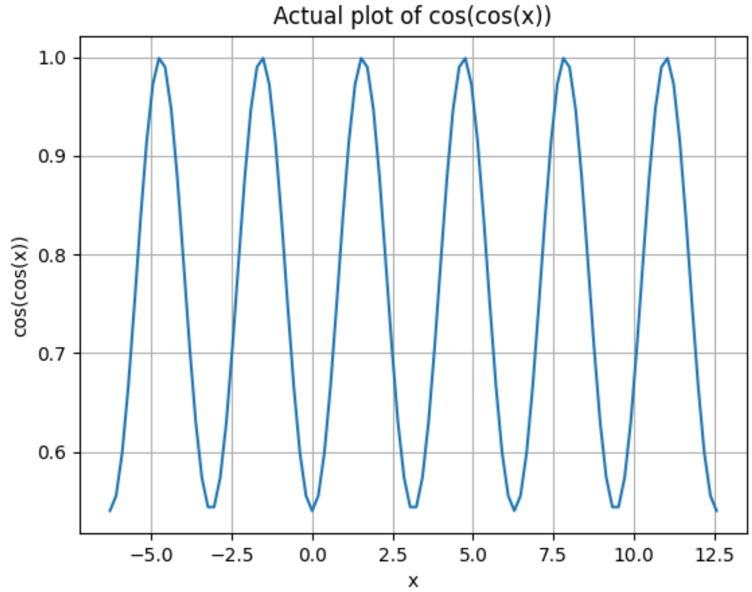

Figure 2: Actual Plot of cos(cos(x)) in linear scale

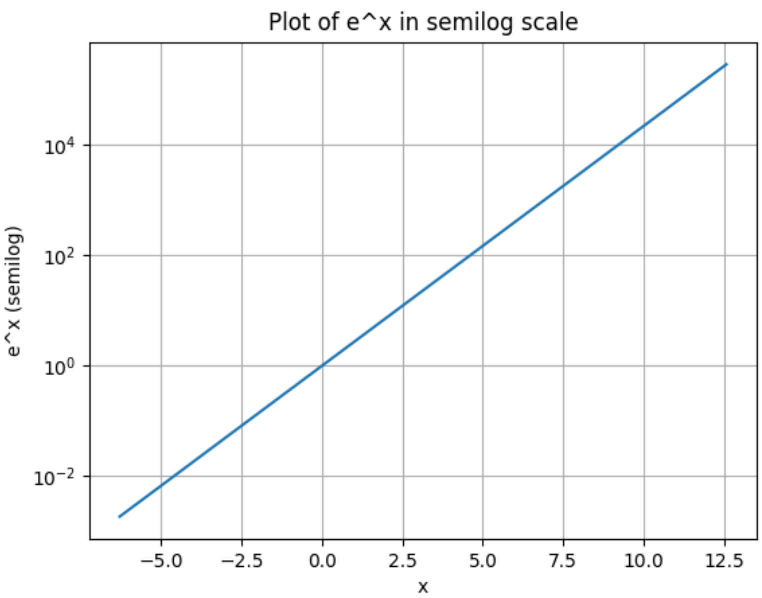

Figure 3: Actual Plot of exp(x) in Semilog scale

Fourier Coefficients by Direct Integration Method

The plots of Fourier Coefficients of exp(x) are given in Figure 4 (in semilog scale) and Figure 5 (in log-log scale) and the plots of Fourier Coefficients of cos(cos(x)) are given in Figure 6 (in semilog scale) and Figure 7 (in log-log scale)

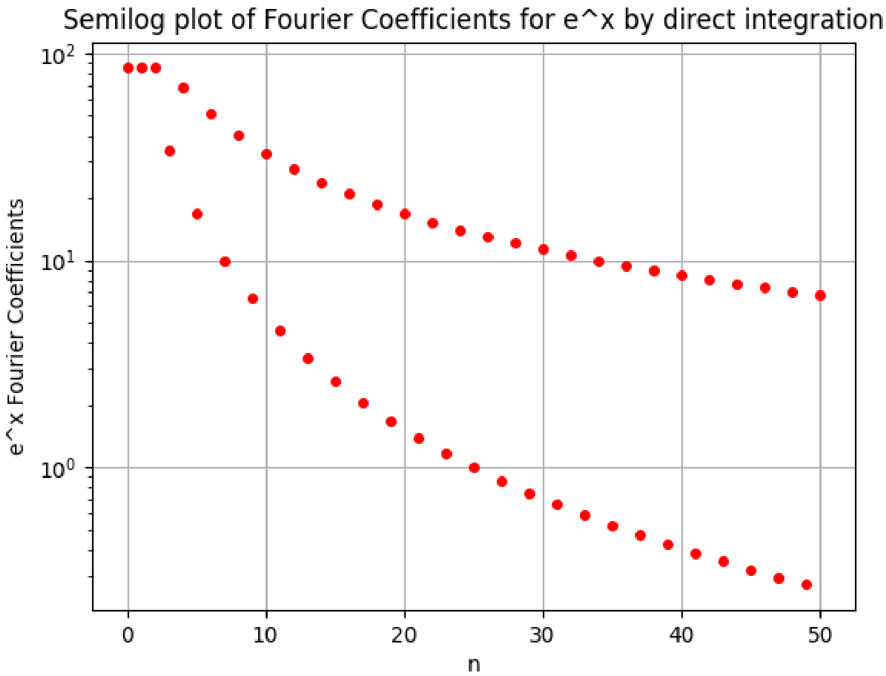

Figure 4: Fourier Coefficients for exp(x) by Direct Integration Method in semilog scale

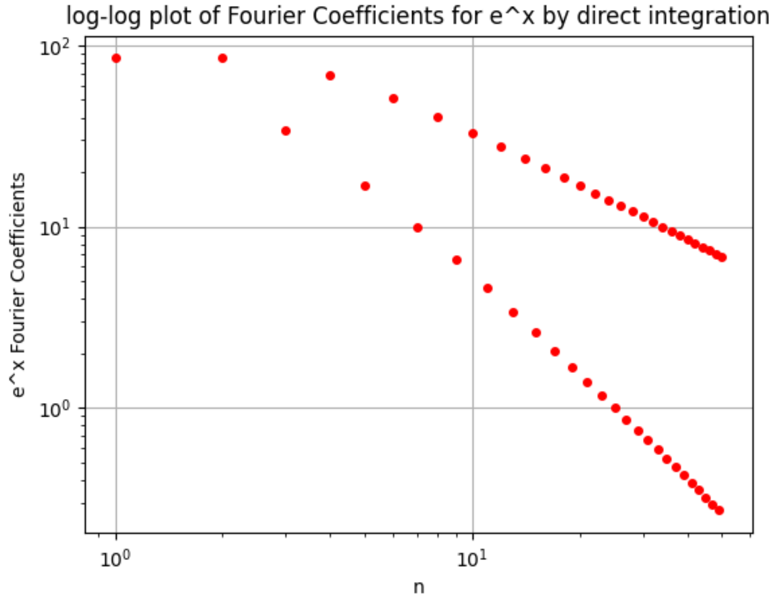

Figure 5: Fourier Coefficients for exp(x) by Direct Integration Method in log-log scale

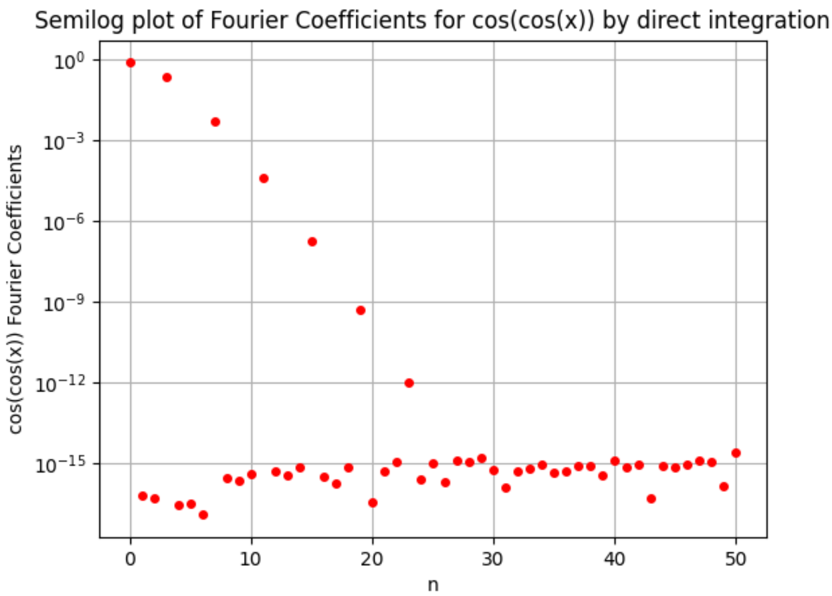

Figure 6: Fourier Coefficients for cos(cos(x)) by Direct Integration Method in semilog scale

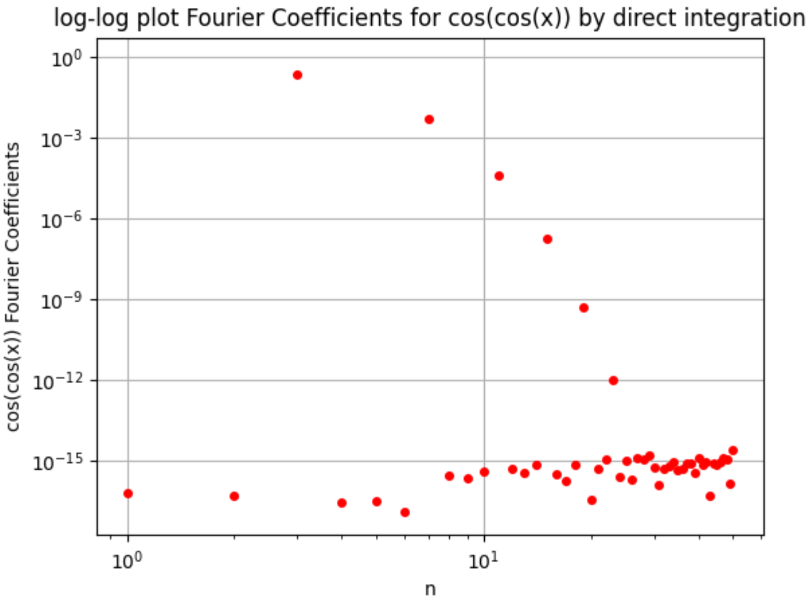

Figure 7: Fourier Coefficients for cos(cos(x)) by Direct Integration Method in log-log scale

Q3.(a)

In the second case i.e., cos(cos(x)), the function itself is an even function. Because of this, the bn values will be zero, since they are the sin components, which is an odd function.

Q3(b)

For exp(x), the higher frequencies also have significant components whereas for cos(cos(x)) the frequency is $1/\pi$ and hence it doesn't have significant components from the higher frequencies. This is the reason the coefficients decay quickly in the second case.

Q3(c)

For the function exp(x), the Fourier coefficients

$$a_n \propto \frac{1}{1+n^2}$$

$$b_n \propto \frac{n}{1+n^2}$$

Since we are considering 'n' to be very large, they can be approximated as,

$$a_n \propto \frac{1}{n^2}$$

$$b_n \propto \frac{1}{n}$$

Taking logarithm gives,

$$\log \frac{1}{1+n^2} \approx -2 \log n$$

$$\log \frac{n}{1+n^2} \approx -\log n$$

Hence, loglog plot is linear in Figure 5. Now, for cos(cos(x)), the fourier coefficients are vary exponentially with n, and hence the semilog plot in Figure 5 looks linear.

Fourier Series Coefficients by Least Square Method

The plot of Fourier Series Coefficients calculated by Least Square Method for functions exp(x) and cos(cos(x)) is plotted in Figure 8 and Figure 9 respectively.

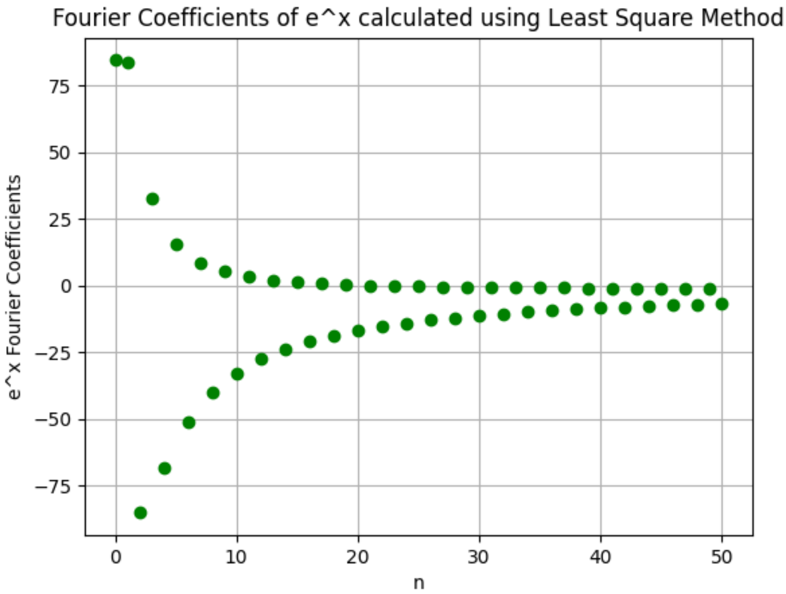

Figure 8: Fourier Series Coefficients if exp(x) calculated using Least Square Method

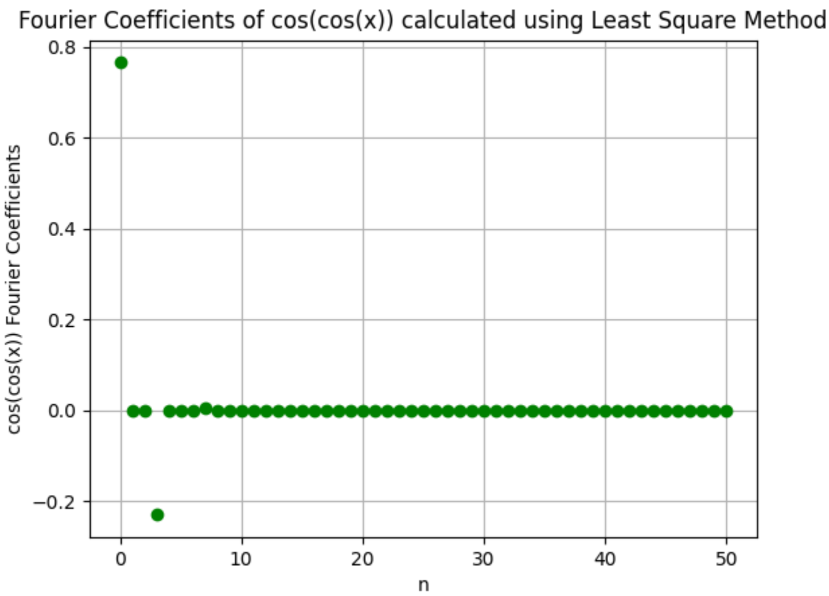

Figure 9: Fourier Series Coefficients of cos(cos(x)) calculated using Least Square Method

The deviation of values calculated by least squares from those calculated by integration are,

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Mean Error in f(x) = exp(x) is 0.6886830611301642
Mean Error in g(x) = cos(cos(x)) is 5.449797451684679e-16
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Comparing Fourier Series approximation plot(by lstsq) with actual plot

Using the Fourier Series coefficients produced by Least Squares method, the function curve is generated and plotted, along with the actual function. exp(x) is plotted in Figure 10 and cos(cos(x)) is plotted in Figure 11. As we see, the Fourier approximation of cos(cos(x)) nearly same with the actual value but the Fourier approximation of exp(x) has a lot of deviation from the actual value. This is because we considered only the first 51 coefficients of the Fourier Series expansion. Since exp(x) has significant components of the higher frequencies as well, we see a large difference, whereas cos(cos(x)) doesn't have significant components at high frequencies, hence it is equivalent to normal graph.

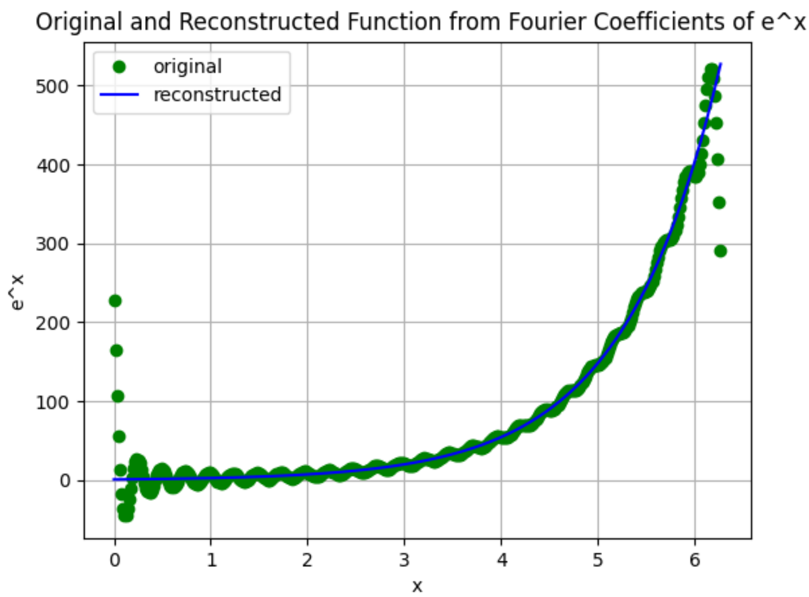

Figure 10: Aerr and Berr for different σ value.

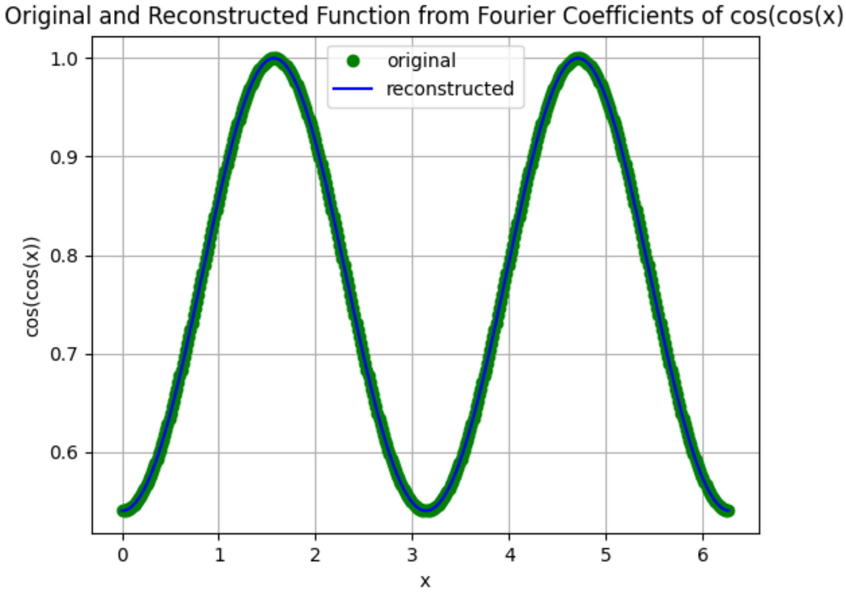

Figure 11: Log-Log Plot

Conclusion

We have seen two ways of calculating the Fourier Coefficients, one by Direct Integration and another by Least Squares method. We have seen that the error in the coefficients when calculated by using least squares is not very large. Hence, it is safe and accurate to use the Least Squares method for these functions and reduce the computation complexity. One more observation we made is compared to exp(x), cos(cos(x)) has very less error because, higher frequencies are significantly dominant in case of exp(x) unlike in the case of cos(cos(x)).