Probability and Random Processes

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Q) Verify that

$$\frac{BG}{GE} = \frac{CG}{GF} = \frac{AG}{GD} = 2$$

Solution: Three vertices of the triangle and midpoints are:

- $\mathbf{A} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \qquad \mathbf{B} = \begin{pmatrix} -4 \\ 6 \end{pmatrix} \tag{1}$
- $\mathbf{C} = \begin{pmatrix} -3 \\ -5 \end{pmatrix}, \qquad \mathbf{D} = \begin{pmatrix} -3.5 \\ 0.5 \end{pmatrix} \tag{2}$
- $\mathbf{E} = \begin{pmatrix} -1 \\ -3 \end{pmatrix}, \qquad \mathbf{F} = \begin{pmatrix} -1.5 \\ 2.5 \end{pmatrix} \tag{3}$
- $\mathbf{G} = \begin{pmatrix} -2\\0 \end{pmatrix} \tag{4}$

1) Calculate AG:

$$AG = \|\mathbf{G} - \mathbf{A}\| \tag{5}$$

$$= \left\| \begin{pmatrix} -2\\0 \end{pmatrix} - \begin{pmatrix} 1\\-1 \end{pmatrix} \right\| \tag{6}$$

$$= \left\| \begin{pmatrix} -3\\1 \end{pmatrix} \right\| = \sqrt{10} \tag{7}$$

2) Calculate *GD*:

$$GD = ||\mathbf{D} - \mathbf{G}|| \tag{8}$$

$$= \left\| \begin{pmatrix} -3.5 \\ 0.5 \end{pmatrix} - \begin{pmatrix} -2 \\ 0 \end{pmatrix} \right\| \tag{9}$$

$$= \left\| \begin{pmatrix} -1.5 \\ 0.5 \end{pmatrix} \right\| = \sqrt{2.5} \tag{10}$$

3) The ratio of $AG : GD = \frac{\sqrt{10}}{\sqrt{2.5}} = 2:1$

4) Calculate BG:

$$BG = \|\mathbf{G} - \mathbf{B}\| \tag{11}$$

$$= \left\| \begin{pmatrix} -2\\0 \end{pmatrix} - \begin{pmatrix} -4\\6 \end{pmatrix} \right\| \tag{12}$$

$$= \left\| \begin{pmatrix} 2 \\ -6 \end{pmatrix} \right\| = \sqrt{40} \tag{13}$$

- (14)
- 5) Calculate GE:

$$GE = ||\mathbf{E} - \mathbf{G}|| \tag{15}$$

$$= \left\| \begin{pmatrix} -1 \\ -3 \end{pmatrix} - \begin{pmatrix} -2 \\ 0 \end{pmatrix} \right\| \tag{16}$$

$$= \left\| \begin{pmatrix} 1 \\ -3 \end{pmatrix} \right\| = \sqrt{10} \tag{17}$$

6) The ratio of $BG : GE = \frac{\sqrt{40}}{\sqrt{10}} = 2:1$

7) Calculate *CG*:

$$CG = ||\mathbf{G} - \mathbf{C}|| \tag{18}$$

$$= \left\| \begin{pmatrix} -2\\0 \end{pmatrix} - \begin{pmatrix} -3\\-5 \end{pmatrix} \right\| \tag{19}$$

$$= \left\| \begin{pmatrix} 1 \\ 5 \end{pmatrix} \right\| = \sqrt{26} \tag{20}$$

(21)

8) Calculate *GF*:

$$GF = ||\mathbf{F} - \mathbf{G}|| \tag{22}$$

$$= \left\| \begin{pmatrix} -1.5 \\ 2.5 \end{pmatrix} - \begin{pmatrix} -2 \\ 0 \end{pmatrix} \right\| \tag{23}$$

$$= \left\| \begin{pmatrix} 0.5 \\ 2.5 \end{pmatrix} \right\| = \sqrt{6.5} \tag{24}$$

9) The ratio of
$$CG : GF = \frac{\sqrt{26}}{\sqrt{6.5}} = 2:1$$

10) Therefore
$$\frac{BG}{GE} = \frac{CG}{GF} = \frac{AG}{GD} = 2$$