

Probability and Random Processes

Gude Pravash EE22BTECH11023*

Q) Verify that

$$\frac{BG}{GE} = \frac{CG}{GF} = \frac{AG}{GD} = 2$$

Solution: Three vertices of the triangle are:

$$\mathbf{A} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad (1)$$

$$\mathbf{B} = \begin{pmatrix} -4 \\ 6 \end{pmatrix} \quad (2)$$

$$\mathbf{C} = \begin{pmatrix} -3 \\ -5 \end{pmatrix} \quad (3)$$

If \mathbf{D} divides \mathbf{BC} in the ratio $k : 1$

$$\mathbf{D} = \frac{k\mathbf{C} + \mathbf{B}}{k + 1}$$

Since D is midpoint of BC ,

$$\begin{aligned} \mathbf{D} &= \frac{\mathbf{C} + \mathbf{B}}{2} \\ &= \frac{\begin{pmatrix} -3 \\ -5 \end{pmatrix} + \begin{pmatrix} -4 \\ 6 \end{pmatrix}}{2} \\ \mathbf{D} &= \begin{pmatrix} -3.5 \\ 0.5 \end{pmatrix} \end{aligned}$$

Since E is midpoint of CA ,

$$\begin{aligned} \mathbf{E} &= \frac{\mathbf{A} + \mathbf{C}}{2} \\ &= \frac{\begin{pmatrix} 1 \\ -1 \end{pmatrix} + \begin{pmatrix} -3 \\ -5 \end{pmatrix}}{2} \\ \mathbf{E} &= \begin{pmatrix} -1 \\ -3 \end{pmatrix} \end{aligned}$$

Since F is midpoint of AB ,

$$\begin{aligned} \mathbf{F} &= \frac{\mathbf{B} + \mathbf{A}}{2} \\ &= \frac{\begin{pmatrix} -4 \\ 6 \end{pmatrix} + \begin{pmatrix} 1 \\ -1 \end{pmatrix}}{2} \\ \mathbf{F} &= \begin{pmatrix} -1.5 \\ 2.5 \end{pmatrix} \end{aligned}$$

For equation of AD we have point A and direction vector \mathbf{AD} .

$$\mathbf{A} = \begin{pmatrix} 1 & -1 \end{pmatrix}$$

The direction vector \mathbf{AD} is given by

$$\begin{aligned} &= \mathbf{D} - \mathbf{A} \\ &= \begin{pmatrix} -3.5 \\ 0.5 \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \end{pmatrix} \\ &= \begin{pmatrix} -4.5 \\ 1.5 \end{pmatrix} \end{aligned}$$

The equation of AD is then given by:

$$\mathbf{AD} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} + k \begin{pmatrix} -4.5 \\ 1.5 \end{pmatrix} \quad (4)$$

For equation of BE we have point B and direction vector \mathbf{BE} .

$$\mathbf{B} = \begin{pmatrix} -4 & 6 \end{pmatrix}$$

The direction vector \mathbf{BE} is given by

$$\begin{aligned} &= \mathbf{E} - \mathbf{B} \\ &= \begin{pmatrix} -1 \\ -3 \end{pmatrix} - \begin{pmatrix} -4 \\ 6 \end{pmatrix} \\ &= \begin{pmatrix} 3 \\ -9 \end{pmatrix} \end{aligned}$$

The equation of BE is then given by:

$$\mathbf{BE} = \begin{pmatrix} -4 \\ 6 \end{pmatrix} + k \begin{pmatrix} 3 \\ -9 \end{pmatrix}$$

The augmented matrix is:

$$\begin{bmatrix} 1 & 3 & -2 \\ 3 & 1 & -6 \end{bmatrix}$$

The row reduction steps are:

$$\begin{bmatrix} 1 & 3 & -2 \\ 3 & 1 & -6 \end{bmatrix} \xrightarrow{R_2 \rightarrow R_2 - 3R_1} \begin{bmatrix} 1 & 3 & -2 \\ 0 & -8 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & -8 & 0 \end{bmatrix} \xrightarrow{R_2 \rightarrow R_2/8} \begin{bmatrix} 1 & 3 & -2 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & 1 & 0 \end{bmatrix} \xrightarrow{R_1 \rightarrow R_1 - 3R_2} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \end{bmatrix}$$

The centroid G is $\mathbf{G} = \begin{pmatrix} -2 \\ 0 \end{pmatrix}$

We have to find AG and GD .

$$\|\mathbf{AG}\| = \|\mathbf{G} - \mathbf{A}\|$$

$$= \left\| \begin{pmatrix} -2 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right\| = \left\| \begin{pmatrix} -3 \\ 1 \end{pmatrix} \right\| = \sqrt{10}$$

$$\|\mathbf{GD}\| = \|\mathbf{D} - \mathbf{G}\|$$

$$= \left\| \begin{pmatrix} -3.5 \\ -0.5 \end{pmatrix} - \begin{pmatrix} -2 \\ 0 \end{pmatrix} \right\| = \left\| \begin{pmatrix} -1.5 \\ -0.5 \end{pmatrix} \right\| = \sqrt{2.5}$$

The ratio $AG : GD$ is $\frac{\sqrt{10}}{\sqrt{2.5}} = 2:1$

We have to find BG and GE .

$$\|\mathbf{BG}\| = \|\mathbf{G} - \mathbf{B}\|$$

$$= \left\| \begin{pmatrix} -2 \\ 0 \end{pmatrix} - \begin{pmatrix} -4 \\ 6 \end{pmatrix} \right\| = \left\| \begin{pmatrix} 2 \\ -6 \end{pmatrix} \right\| = \sqrt{40}$$

$$\|\mathbf{GE}\| = \|\mathbf{E} - \mathbf{G}\|$$

$$= \left\| \begin{pmatrix} -1 \\ -3 \end{pmatrix} - \begin{pmatrix} -2 \\ 0 \end{pmatrix} \right\| = \left\| \begin{pmatrix} 1 \\ -3 \end{pmatrix} \right\| = \sqrt{10}$$

(5) The ratio $BG : GE$ is $\frac{\sqrt{40}}{\sqrt{10}} = 2:1$

We have to find CG and GF .

$$\|\mathbf{CG}\| = \|\mathbf{G} - \mathbf{C}\|$$

$$= \left\| \begin{pmatrix} -2 \\ 0 \end{pmatrix} - \begin{pmatrix} -3 \\ -5 \end{pmatrix} \right\| = \left\| \begin{pmatrix} 1 \\ 5 \end{pmatrix} \right\| = \sqrt{26}$$

To find GF we find $\|\mathbf{GF}\| = \|\mathbf{F} - \mathbf{G}\|$

$$= \left\| \begin{pmatrix} -1.5 \\ 2.5 \end{pmatrix} - \begin{pmatrix} -2 \\ 0 \end{pmatrix} \right\| = \left\| \begin{pmatrix} 0.5 \\ 2.5 \end{pmatrix} \right\| = \sqrt{6.5}$$

The ratio $CG : GF$ is $\frac{\sqrt{26}}{\sqrt{6.5}} = 2:1$

$$\therefore \frac{BG}{GE} = \frac{CG}{GF} = \frac{AG}{GD} = 2$$