Probability and Random Processes

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Q) Verify that

$$\frac{BG}{GE} = \frac{CG}{GF} = \frac{AG}{GD} = 2$$

Solution: Three vertices of the triangle and midpoints are:

$$\mathbf{A} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \qquad \mathbf{B} = \begin{pmatrix} -4 \\ 6 \end{pmatrix} \tag{1}$$

$$\mathbf{C} = \begin{pmatrix} -3 \\ -5 \end{pmatrix}, \qquad \mathbf{D} = \begin{pmatrix} -3.5 \\ 0.5 \end{pmatrix} \tag{2}$$

$$\mathbf{E} = \begin{pmatrix} -1 \\ -3 \end{pmatrix}, \qquad \mathbf{F} = \begin{pmatrix} -1.5 \\ 2.5 \end{pmatrix} \tag{3}$$

$$\mathbf{G} = \begin{pmatrix} -2 \\ 0 \end{pmatrix} \tag{4}$$

1) Calculate AG and GD:

$$AG = \|\mathbf{G} - \mathbf{A}\| \tag{5}$$

$$= \left\| \begin{pmatrix} -2 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right\| \tag{6}$$

$$= \left\| \begin{pmatrix} -3 \\ 1 \end{pmatrix} \right\| = \sqrt{10} \tag{7}$$

$$GD = \|\mathbf{D} - \mathbf{G}\| \tag{8}$$

$$= \left\| \begin{pmatrix} -3.5 \\ 0.5 \end{pmatrix} - \begin{pmatrix} -2 \\ 0 \end{pmatrix} \right\| \tag{9}$$

$$= \left\| \begin{pmatrix} -1.5 \\ 0.5 \end{pmatrix} \right\| = \sqrt{2.5} \tag{10}$$

$$The ratio of AG : GD = \frac{\sqrt{10}}{\sqrt{2.5}} = 2 : 1 \tag{11}$$

(12)

2) Calculate BG and GE:

$$BG = \|\mathbf{G} - \mathbf{B}\| \tag{13}$$
$$= \left\| \begin{pmatrix} -2 \\ 0 \end{pmatrix} - \begin{pmatrix} -4 \\ 6 \end{pmatrix} \right\| \tag{14}$$
$$= \left\| \begin{pmatrix} 2 \\ -6 \end{pmatrix} \right\| = \sqrt{40} \tag{15}$$
$$GE = \|\mathbf{E} - \mathbf{G}\| \tag{16}$$
$$= \left\| \begin{pmatrix} -1 \\ -3 \end{pmatrix} - \begin{pmatrix} -2 \\ 0 \end{pmatrix} \right\| \tag{17}$$
$$= \left\| \begin{pmatrix} 1 \\ -3 \end{pmatrix} \right\| = \sqrt{10} \tag{18}$$

3) The ratio of $BG : GE = \frac{\sqrt{40}}{\sqrt{10}} = 2{:}1$

4) Calculate CG and GF:

$$CG = \|\mathbf{G} - \mathbf{C}\| \tag{19}$$
$$= \left\| \begin{pmatrix} -2 \\ 0 \end{pmatrix} - \begin{pmatrix} -3 \\ -5 \end{pmatrix} \right\| \tag{20}$$
$$= \left\| \begin{pmatrix} 1 \\ 5 \end{pmatrix} \right\| = \sqrt{26} \tag{21}$$
$$GF = \|\mathbf{F} - \mathbf{G}\| \tag{22}$$
$$= \left\| \begin{pmatrix} -1.5 \\ 2.5 \end{pmatrix} - \begin{pmatrix} -2 \\ 0 \end{pmatrix} \right\| \tag{23}$$
$$= \left\| \begin{pmatrix} 0.5 \\ 2.5 \end{pmatrix} \right\| = \sqrt{6.5} \tag{24}$$

5) The ratio of $CG : GF = \frac{\sqrt{26}}{\sqrt{6.5}} = 2{:}1$

6) Therefore $\frac{BG}{GE} = \frac{CG}{GF} = \frac{AG}{GD} = 2$