Probability and Random Processes

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Q) Verify that

$$\frac{BG}{GE} = \frac{CG}{GF} = \frac{AG}{GD} = 2$$

Solution: Three vertices of the triangle and midpoints are:

$$\mathbf{A} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \qquad \mathbf{B} = \begin{pmatrix} -4 \\ 6 \end{pmatrix} \tag{1}$$

$$\mathbf{C} = \begin{pmatrix} -3 \\ -5 \end{pmatrix}, \qquad \mathbf{D} = \begin{pmatrix} -3.5 \\ 0.5 \end{pmatrix} \tag{2}$$

$$\mathbf{E} = \begin{pmatrix} -1 \\ -3 \end{pmatrix}, \qquad \mathbf{F} = \begin{pmatrix} -1.5 \\ 2.5 \end{pmatrix} \tag{3}$$

$$\mathbf{G} = \begin{pmatrix} -2 \\ 0 \end{pmatrix} \tag{4}$$

1) Calculate AG:

$$\begin{aligned} AG &= \|\mathbf{G} - \mathbf{A}\| \\ &= \left\| \begin{pmatrix} -2 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right\| \\ &= \left\| \begin{pmatrix} -3 \\ 1 \end{pmatrix} \right\| = \sqrt{10} \end{aligned}$$

2) Calculate GD:

$$\begin{aligned} GD &= \|\mathbf{D} - \mathbf{G}\| \\ &= \left\| \begin{pmatrix} -3.5 \\ 0.5 \end{pmatrix} - \begin{pmatrix} -2 \\ 0 \end{pmatrix} \right\| \\ &= \left\| \begin{pmatrix} -1.5 \\ 0.5 \end{pmatrix} \right\| = \sqrt{2.5} \end{aligned}$$

3) The ratio of $AG : GD = \frac{\sqrt{10}}{\sqrt{2.5}} = 2{:}1$

4) Calculate BG:

$$\begin{aligned} BG &= \|\mathbf{G} - \mathbf{B}\| \\ &= \left\| \begin{pmatrix} -2 \\ 0 \end{pmatrix} - \begin{pmatrix} -4 \\ 6 \end{pmatrix} \right\| \\ &= \left\| \begin{pmatrix} 2 \\ -6 \end{pmatrix} \right\| = \sqrt{40} \end{aligned}$$

5) Calculate GE:

$$\begin{aligned} GE &= \|\mathbf{E} - \mathbf{G}\| \\ &= \left\| \begin{pmatrix} -1 \\ -3 \end{pmatrix} - \begin{pmatrix} -2 \\ 0 \end{pmatrix} \right\| \\ &= \left\| \begin{pmatrix} 1 \\ -3 \end{pmatrix} \right\| = \sqrt{10} \end{aligned}$$

6) The ratio of $BG : GE = \frac{\sqrt{40}}{\sqrt{10}} = 2{:}1$

7) Calculate CG:

$$\begin{aligned} CG &= \|\mathbf{G} - \mathbf{C}\| \\ &= \left\| \begin{pmatrix} -2 \\ 0 \end{pmatrix} - \begin{pmatrix} -3 \\ -5 \end{pmatrix} \right\| \\ &= \left\| \begin{pmatrix} 1 \\ 5 \end{pmatrix} \right\| = \sqrt{26} \end{aligned}$$

8) Calculate GF:

$$\begin{aligned} GF &= \|\mathbf{F} - \mathbf{G}\| \\ &= \left\| \begin{pmatrix} -1.5 \\ 2.5 \end{pmatrix} - \begin{pmatrix} -2 \\ 0 \end{pmatrix} \right\| \\ &= \left\| \begin{pmatrix} 0.5 \\ 2.5 \end{pmatrix} \right\| = \sqrt{6.5} \end{aligned}$$

9) The ratio of $CG : GF = \frac{\sqrt{26}}{\sqrt{6.5}} = 2{:}1$

10) Therefore $\frac{BG}{GE} = \frac{CG}{GF} = \frac{AG}{GD} = 2$