Probability and Random Processes

Gude Pravarsh EE22BTECH11023*

Q) Verify that

$$\frac{BG}{GE} = \frac{CG}{GF} = \frac{AG}{GD} = 2$$

Solution: Three vertices of the triangle are:

$$\mathbf{A} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$
$$\mathbf{B} = \begin{pmatrix} -4 \\ 6 \end{pmatrix}$$
$$\mathbf{C} = \begin{pmatrix} -3 \\ -5 \end{pmatrix}$$

If $\mathbf{D}$ divides $\mathbf{BC}$ in the ratio $k : 1$

$$\mathbf{D} = \frac{k\mathbf{C} + \mathbf{B}}{k+1}$$

Since D is midpoint of BC,

$$\mathbf{D} = \frac{\mathbf{C} + \mathbf{B}}{2}$$
$$\mathbf{D} = \frac{\begin{pmatrix} -3 \\ -5 \end{pmatrix} + \begin{pmatrix} -4 \\ 6 \end{pmatrix}}{2}$$
$$\mathbf{D} = \begin{pmatrix} -3.5 \\ 0.5 \end{pmatrix}$$

Since E is midpoint of CA,

$$\mathbf{E} = \frac{\mathbf{A} + \mathbf{C}}{2}$$
$$\mathbf{E} = \frac{\begin{pmatrix} 1 \\ -1 \end{pmatrix} + \begin{pmatrix} -3 \\ -5 \end{pmatrix}}{2}$$
$$\mathbf{E} = \begin{pmatrix} -1 \\ -3 \end{pmatrix}$$

Since F is midpoint of AB,

$$\mathbf{F} = \frac{\mathbf{B} + \mathbf{A}}{2}$$
$$\mathbf{F} = \frac{\begin{pmatrix} -4 \\ 6 \end{pmatrix} + \begin{pmatrix} 1 \\ -1 \end{pmatrix}}{2}$$
$$\mathbf{F} = \begin{pmatrix} -1.5 \\ 2.5 \end{pmatrix}$$

For equation of AD we have point A and direction vector $\mathbf{AD}$.

$$\mathbf{A} = \begin{pmatrix} 1 & -1 \end{pmatrix}$$

The direction vector $\mathbf{AD}$ is given by

$$= \mathbf{D} - \mathbf{A}$$
$$= \begin{pmatrix} -3.5 \\ 0.5 \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$
$$= \begin{pmatrix} -4.5 \\ 1.5 \end{pmatrix}$$

The equation of AD is then given by:

$$\mathbf{AD} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} + k\begin{pmatrix} -4.5 \\ 1.5 \end{pmatrix}$$

For equation of BE we have point B and direction vector $\mathbf{BE}$.

$$\mathbf{B} = \begin{pmatrix} -4 & 6 \end{pmatrix}$$

The direction vector $\mathbf{BE}$ is given by

$$= \mathbf{E} - \mathbf{B}$$
$$= \begin{pmatrix} -1 \\ -3 \end{pmatrix} - \begin{pmatrix} -4 \\ 6 \end{pmatrix}$$
$$= \begin{pmatrix} 3 \\ -9 \end{pmatrix}$$

The equation of BE is then given by:

$$\mathbf{BE} = \begin{pmatrix}-4\\6\end{pmatrix} + k\begin{pmatrix}3\\-9\end{pmatrix}$$

The pair of equations are:

1) $x + 3y = -2$
2) $3x + y = -6$

Therefore, the augmented matrix is:

$$\begin{bmatrix}1 & 3 & -2\\3 & 1 & -6\end{bmatrix}$$

The row reduction steps are:

$$\begin{bmatrix}1 & 3 & -2\\3 & 1 & -6\end{bmatrix} \xrightarrow{R_2 \longrightarrow R_2 - 3R_1} \begin{bmatrix}1 & 3 & -2\\0 & -8 & 0\end{bmatrix}$$

$$\begin{bmatrix}1 & 3 & -2\\0 & -8 & 0\end{bmatrix} \xrightarrow{R_2 \longrightarrow R_2/8} \begin{bmatrix}1 & 3 & -2\\0 & 1 & 0\end{bmatrix}$$

$$\begin{bmatrix}1 & 3 & -2\\0 & 1 & 0\end{bmatrix} \xrightarrow{R_1 \longrightarrow R_1 - 3R_2} \begin{bmatrix}1 & 0 & -2\\0 & 1 & 0\end{bmatrix}$$

The centroid G is $\mathbf{G} = \begin{pmatrix}-2\\0\end{pmatrix}$

We have to find AG and GD.

$||\mathbf{AG}|| = ||\mathbf{G} - \mathbf{A}||$

$$= \left\|\begin{pmatrix}-2\\0\end{pmatrix} - \begin{pmatrix}1\\-1\end{pmatrix}\right\| = \left\|\begin{pmatrix}-3\\1\end{pmatrix}\right\| = \sqrt{10}$$

$||\mathbf{GD}|| = ||\mathbf{D} - \mathbf{G}||$

$$= \left\|\begin{pmatrix}-3.5\\-0.5\end{pmatrix} - \begin{pmatrix}-2\\0\end{pmatrix}\right\| = \left\|\begin{pmatrix}-1.5\\-0.5\end{pmatrix}\right\| = \sqrt{2.5}$$

The ratio $AG : GD$ is $\frac{\sqrt{10}}{\sqrt{2.5}} = 2{:}1$

We have to find BG and GE.

$||\mathbf{BG}|| = ||\mathbf{G} - \mathbf{B}||$

$$= \left\|\begin{pmatrix}-2\\0\end{pmatrix} - \begin{pmatrix}-4\\6\end{pmatrix}\right\| = \left\|\begin{pmatrix}2\\-6\end{pmatrix}\right\| = \sqrt{40}$$

$||\mathbf{GE}|| = ||\mathbf{E} - \mathbf{G}||$

$$= \left\|\begin{pmatrix}-1\\-3\end{pmatrix} - \begin{pmatrix}-2\\0\end{pmatrix}\right\| = \left\|\begin{pmatrix}1\\-3\end{pmatrix}\right\| = \sqrt{10}$$

The ratio $BG : GE$ is $\frac{\sqrt{40}}{\sqrt{10}} = 2{:}1$

We have to find CG and GF.

$||\mathbf{CG}|| = ||\mathbf{G} - \mathbf{C}||$

$$= \left\|\begin{pmatrix}-2\\0\end{pmatrix} - \begin{pmatrix}-3\\-5\end{pmatrix}\right\| = \left\|\begin{pmatrix}1\\5\end{pmatrix}\right\| = \sqrt{26}$$

To find GF we find $||\mathbf{GF}|| = ||\mathbf{F} - \mathbf{G}||$

$$= \left\|\begin{pmatrix}-1.5\\2.5\end{pmatrix} - \begin{pmatrix}-2\\0\end{pmatrix}\right\| = \left\|\begin{pmatrix}0.5\\2.5\end{pmatrix}\right\| = \sqrt{6.5}$$

The ratio $CG : GF$ is $\frac{\sqrt{26}}{\sqrt{6.5}} = 2{:}1$

$$\therefore \frac{BG}{GE} = \frac{CG}{GF} = \frac{AG}{GD} = 2$$