Probability and Random Processes

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Q)One million random numbers are generated from a statistically stationary process with a Gaussian distribution with mean zero and standard deviation σ_0.
The σ_0 is estimated by randomly drawing out 10,000 number of samples (x_n). The estimates $\hat{\sigma}_1^2$ and $\hat{\sigma}_2^2$ are computed in the following two ways:

$$\hat{\sigma}_1^2 = \frac{1}{10000}\sum_{n=1}^{10000} x_n^2$$

$$\hat{\sigma}_2^2 = \frac{1}{9999}\sum_{n=1}^{10000} x_n^2$$

Which of the following statements is true?

1) $E(\hat{\sigma}_2^2) = \sigma_0^2$
2) $E(\hat{\sigma}_2) = \sigma_0$
3) $E(\hat{\sigma}_1^2) = \sigma_0^2$
4) $E(\hat{\sigma}_1) = E(\hat{\sigma}_2)$

(GATE EE 2023)

Solution:

$$\hat{\sigma}_1^2 = \frac{1}{10000}\sum_{n=1}^{10000} x_n^2 \tag{1}$$

$$\hat{\sigma}_2^2 = \frac{1}{9999}\sum_{n=1}^{10000} x_n^2 \tag{2}$$

We know,

$$\begin{aligned} \text{Var}(x_n) &= E(x_n^2) - [E(x_n)]^2 && (3)\\ &= E(x_n^2) - 0 && (4)\\ &= E(x_n^2) && (5)\\ &= \sigma_0^2 && (6)\\ \implies E(x_n^2) &= \sigma_0^2 && (7) \end{aligned}$$

$$\text{E}[\hat{\sigma}_1^2] = \frac{1}{10000}\sum_{n=1}^{10000} \text{E}[x_n^2] \tag{8}$$

$$\text{E}[\hat{\sigma}_2^2] = \frac{1}{9999}\sum_{n=1}^{10000} \text{E}[x_n^2] \tag{9}$$

Using Equation 7, we get:

$$\text{E}[\hat{\sigma}_1^2] = \sigma_0^2 \tag{10}$$

$$\text{E}[\hat{\sigma}_2^2] = \frac{10000}{9999}\sigma_0^2 \tag{11}$$

Hence, the correct answer is 3

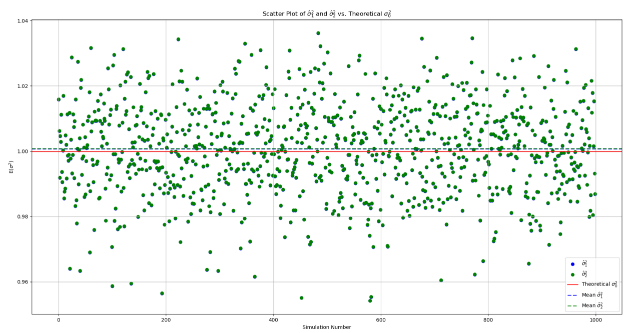

Fig. 1: Scatter plot of $\hat{\sigma}_1^2$ and $\hat{\sigma}_2^2$ vs σ_0^2