

Probability and Random Processes

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Q)A die is loaded in such a way that each odd number is twice as likely to occur as each even number. Find $P(G)$, where G is the event that a number greater than 3 occurs on a single roll of the die.

Solution: Let the random variable be X and p is the probability that rolled number is odd. The PMF of X in terms of p is given by:

$$p_X(k) = \begin{cases} 2p & \text{if } k \text{ is odd} \\ p & \text{if } k \text{ is even} \end{cases}$$

To find Probability that rolled number is odd: We know,

$$\sum_{k=1}^6 p_X(k) = 1 \quad (1)$$

$$\Rightarrow p + \frac{p}{2} + p + \frac{p}{2} + p + \frac{p}{2} = 1 \quad (2)$$

$$\Rightarrow \frac{9p}{2} = 1 \quad (3)$$

$$\Rightarrow p = \frac{2}{9} \quad (4)$$

$$p_X(k) = \begin{cases} \frac{2}{9} & \text{if } k \text{ is odd} \\ \frac{1}{9} & \text{if } k \text{ is even} \end{cases} \quad (5)$$

The cdf of X is given by

$$F_X(k) = \Pr(X \leq k) \quad (6)$$

The cdf of X in terms of k is given by:

$$F_X(k) = \begin{cases} \frac{k+1}{9} & \text{if } k \text{ is odd} \\ \frac{k}{6} & \text{if } k \text{ is even} \end{cases} \quad (7)$$

1) We require $\Pr(X > 3)$.

$$\Pr(X > 3) = 1 - \Pr(X \leq 3) \quad (8)$$

$$= 1 - F_X(3) \quad (9)$$

$$= 1 - \frac{5}{9} \quad (10)$$

$$= \frac{4}{9} \quad (11)$$

$$= 0.44 \quad (12)$$

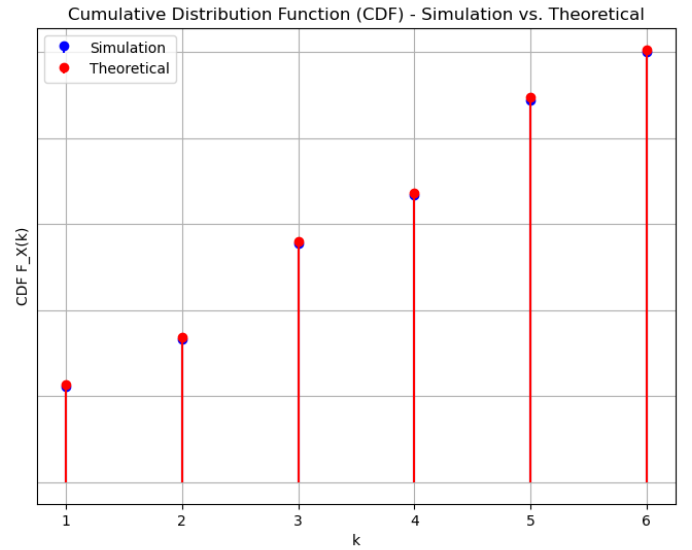


Fig. 1: CDF plot - Simulation and theoretical.