Probability and Random Processes

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Q)Find the probability distribution of the maximum of the two scores obtained when a die is thrown twice. Determine also the mean of the distribution.
Solution: Let us define two random variables X and Y which represent the scores of the two dices which are rolled and random variable Z which represents the maximum of the two scores.

Random variables	value
X	$1 \le X \le 6$
Y	$1 \le Y \le 6$

$$Z = \max(X, Y) = \begin{cases} X & \text{if } X > Y \\ Y & \text{if } Y \ge X \end{cases} \quad (1)$$

$$F_Z(z) = \Pr\left(\{\max(X, Y) \le z\}\right) \quad (2)$$
$$= \Pr\left(\{(X \le z, X > Y) \cup (Y \le z, X \le Y)\}\right) \quad (3)$$
$$= \Pr\left(\{X \le z, X > Y\}\right) + \Pr\left(\{Y \le z, X \le Y\}\right) \quad (4)$$

Since $\{X > Y\}$ and $\{X \le Y\}$ are mutually exclusive sets that form a partition.

$$F_Z(z) = \Pr\left(\{X \le z, Y \le z\}\right) = F_{XY}(z, z) \quad (5)$$

if X,Y are independent, then

$$F_Z(z) = F_X(z) \cdot F_Y(z) \quad (6)$$

Finding $F_X(z)$ and $F_Y(z)$ for some random z.

$$F_X(z) = \begin{cases} 0, & \text{if } z < 1 \\ \frac{z}{6} & 1 \le z \le 6 \\ 1, & \text{if } z > 6 \end{cases} \quad (7)$$

$$F_Y(z) = \begin{cases} 0, & \text{if } z < 1 \\ \frac{z}{6}, & \text{if } 1 \le z \le 6 \\ 1, & \text{if } z > 6 \end{cases} \quad (8)$$

Finding $F_Z(z)$ for some random z.

$$F_Z(z) = \begin{cases} 0, & \text{if } z < 1 \\ \frac{z^2}{6}, & 1 \le z \le 6 \\ 1, & \text{if } z > 6 \end{cases} \quad (9)$$

$$p_Z(z) = F_Z(z) - F_Z(z-1) = \frac{2z-1}{36} \quad (10)$$

$$p_Z(z) = \begin{cases} \frac{2z-1}{36} & \text{if } 1 \le z \le 6 \\ 0 & \text{otherwise} \end{cases} \quad (11)$$

The mean of the distribution is given by:

$$\mu = \sum_{z=1}^{6} z \cdot p_Z(z) \quad (12)$$

$$= \sum_{z=1}^{6} z \cdot \frac{2z-1}{36} \quad (13)$$

$$= \sum_{z=1}^{6} \frac{2z^2 - z}{36} \quad (14)$$

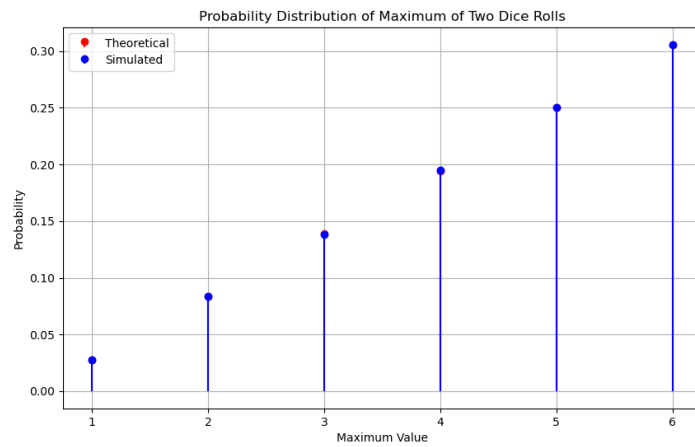$$= \sum_{z=1}^{6} \frac{2z^2}{36} - \sum_{z=1}^{6} \frac{z}{36} \quad (15)$$

$$= \frac{161}{36} \quad (16)$$

Fig. 1: Probabilities - Simulation and theoretical.