Probability and Random Processes

Gude Pravarsh EE22BTECH11023*

Q)Find the probability distribution of the maximum of the two scores obtained when a die is thrown twice. Determine also the mean of the distribution.
Solution: Let us define two random variables X and Y which represent the scores of the two dices which are rolled and random variable Z which represents the maximum of the two scores.

Random variables	value
X	$1 \leq X \leq 6$
Y	$1 \leq Y \leq 6$

$$Z = \max(X, Y) = \begin{cases} X & \text{if } X > Y \\ Y & \text{if } Y \geq X \end{cases} \tag{1}$$

$$\begin{aligned} F_Z(z) &= \Pr\left(\{\max(X, Y) \leq Z\}\right) \quad (2) \\ &= \Pr\left(\{(X \leq Z, X > Y) \cup (Y \leq Z, X \leq Y)\}\right) \quad (3) \\ &= \Pr\left(\{X \leq Z, X > Y\}\right) + \Pr\left(\{Y \leq Z, X \leq Y\}\right) \quad (4) \end{aligned}$$

Since $\{X > Y\}$ and $\{X \leq Y\}$ are mutually exclusive sets that form a partition.

$$F_Z(z) = P\{X \leq Z, Y \leq Z\} = F_{XY}(Z, Z) \tag{5}$$

if X,Y are independent, then

$$F_Z(z) = F_X(z) \cdot F_Y(z) \tag{6}$$

Finding $F_X(k)$ and $F_Y(k)$ for some random k.

$$F_X(k) = \begin{cases} 0, & \text{if } k < 1 \\ \frac{k}{6} & 1 \leq k \leq 6 \\ 1, & \text{if } k > 6 \end{cases} \tag{7}$$

$$F_Y(k) = \begin{cases} 0, & \text{if } k < 1 \\ \frac{k}{6}, & \text{if } 1 \leq k \leq 6 \\ 1, & \text{if } k > 6 \end{cases} \tag{8}$$

Finding $F_Z(k)$ for some random k.

$$F_Z(k) = \begin{cases} 0, & \text{if } k < 1 \\ \frac{k^2}{6}, & 1 \leq k \leq 6 \\ 1, & \text{if } k > 6 \end{cases} \tag{9}$$

$$p_Z(k) = F_Z(k) - F_Z(k-1) = \frac{2k-1}{36} \tag{10}$$

$$p_Z(k) = \begin{cases} \frac{2k-1}{36} & \text{if } 1 \leq k \leq 6 \\ 0 & \text{otherwise} \end{cases} \tag{11}$$

The mean of the distribution is given by:

$$\begin{aligned} \mu &= \sum_{k=1}^{6} k \cdot p_Z(k) \quad (12) \\ &= \sum_{k=1}^{6} k \cdot \frac{2k-1}{36} \quad (13) \\ &= \sum_{k=1}^{6} \frac{2k^2-k}{36} \quad (14) \\ &= \sum_{k=1}^{6} \frac{2k^2}{36} - \sum_{k=1}^{6} \frac{k}{36} \quad (15) \\ &= \frac{161}{36} \quad (16) \end{aligned}$$

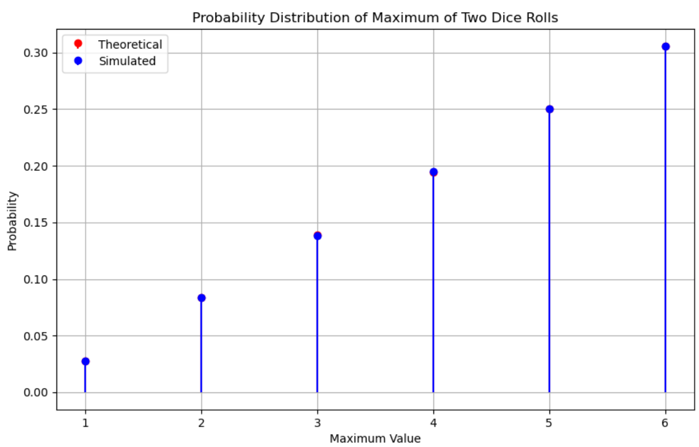

Fig. 1: Probabilities - Simulation and theoretical.