Probability and Random Processes

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Q)Find the probability distribution of the maximum of the two scores obtained when a die is thrown twice. Determine also the mean of the distribution.
Solution: Let us define two random variables X and Y which represent the scores of the two dices which are rolled and random variable Z which represents the maximum of the two scores.

Random variables	**value**
X	$1 \le X \le 6$
Y	$1 \le Y \le 6$

$$Z = \max(X, Y) = \begin{cases} X & \text{if } X > Y \\ Y & \text{if } Y \ge X \end{cases} \tag{1}$$

$$F_Z(z) = \Pr\left(\{\max(X, Y) \le Z\}\right) \tag{2}$$
$$= \Pr\left(\{(X \le Z, X > Y) \cup (Y \le Z, X \le Y)\}\right) \tag{3}$$
$$= \Pr\left(\{X \le Z, X > Y\}\right) + \Pr\left(\{Y \le Z, X \le Y\}\right) \tag{4}$$

Since $\{X > Y\}$ and $\{X \le Y\}$ are mutually exclusive sets that form a partition.

$$F_Z(z) = \Pr\left(\{X \le Z, Y \le Z\}\right) = F_{XY}(Z, Z) \tag{5}$$

if X,Y are independent, then

$$F_Z(z) = F_X(z) \cdot F_Y(z) \tag{6}$$

Finding $F_X(k)$ and $F_Y(k)$ for some random k.

$$F_X(k) = \begin{cases} 0, & \text{if } k < 1 \\ \frac{k}{6} & 1 \le k \le 6 \\ 1, & \text{if } k > 6 \end{cases} \tag{7}$$

$$F_Y(k) = \begin{cases} 0, & \text{if } k < 1 \\ \frac{k}{6}, & \text{if } 1 \le k \le 6 \\ 1, & \text{if } k > 6 \end{cases} \tag{8}$$

Finding $F_Z(k)$ for some random k.

$$F_Z(k) = \begin{cases} 0, & \text{if } k < 1 \\ \frac{k^2}{6}, & 1 \le k \le 6 \\ 1, & \text{if } k > 6 \end{cases} \tag{9}$$

$$p_Z(k) = F_Z(k) - F_Z(k-1) = \frac{2k-1}{36} \tag{10}$$

$$p_Z(k) = \begin{cases} \frac{2k-1}{36} & \text{if } 1 \le k \le 6 \\ 0 & \text{otherwise} \end{cases} \tag{11}$$

The mean of the distribution is given by:

$$\mu = \sum_{k=1}^{6} k \cdot p_Z(k) \tag{12}$$
$$= \sum_{k=1}^{6} k \cdot \frac{2k-1}{36} \tag{13}$$
$$= \sum_{k=1}^{6} \frac{2k^2 - k}{36} \tag{14}$$
$$= \sum_{k=1}^{6} \frac{2k^2}{36} - \sum_{k=1}^{6} \frac{k}{36} \tag{15}$$
$$= \frac{161}{36} \tag{16}$$

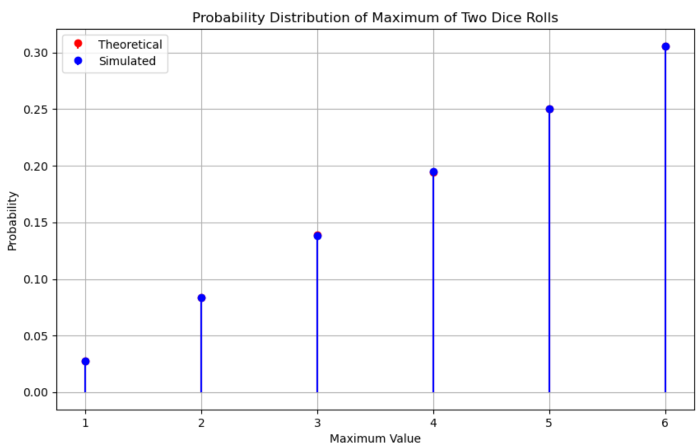

Fig. 1: Probabilities - Simulation and theoretical.