Probability and Random Processes

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Q)Ten coins are tossed. What is the probability of getting atleast 8 heads?

Solution:

Parameter	Value	Description
n	10	number of tosses
p	$\frac{1}{2}$	Probability for Heads
q	$\frac{1}{2}$	Probability for Tails
$\mu = np$	5	mean of the distribution
$\sigma^2 = npq$	2.5	variance of the distribution

Gaussian Distribution

Let us define a random variable X which represents the number of Heads.

$$X = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} \tag{1}$$

For $X \geq 8$

1) With a 0.5 correction:

$$\Pr(X \geq 8) = 1 - \Pr(X < 7.5) \tag{2}$$

$$\Pr(X \geq 8) = Q\left(\frac{7.5 - \mu}{\sigma}\right) \tag{3}$$

$$\Pr(X \geq 8) = Q\left(\sqrt{2.5}\right) = Q(1.5811) \tag{4}$$

$$\implies \Pr(X \geq 8) = 0.0569276 \tag{5}$$

2) Without correction:

$$\Pr(X \geq 8) = 1 - \Pr(X < 8) \tag{6}$$

$$\Pr(X \geq 8) = Q\left(\frac{8 - \mu}{\sigma}\right) \tag{7}$$

$$\Pr(X \geq 8) = Q\left(\frac{3}{\sqrt{2.5}}\right) = Q(1.8973) \tag{8}$$

$$\implies \Pr(X \geq 8) = 0.0288898 \tag{9}$$

Binomial Distribution

$$\Pr(X \geq 8) = 1 - \Pr(X < 8) \tag{10}$$

$$= \sum_{k=8}^{10} \binom{n}{k} p^k (1-p)^{n-k} \tag{11}$$

$$= 0.0546875 \tag{12}$$

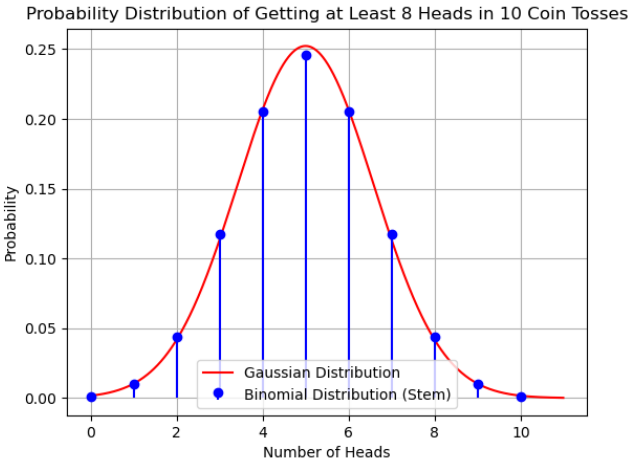

Fig. 1: Binomial vs Guassian