Probability and Random Processes

Gude Pravarsh EE22BTECH11023*

Q)Ten coins are tossed. What is the probability of getting atleast 8 heads?
Solution:

TABLE 0: Random Variables

Variable	Value	Description
X	$0 \le X \le 10$	Number of Heads

Gaussian Distribution
Here $n = 10$ and $p = 0.5$
The mean μ of X

$$\mu = np = 5 \tag{1}$$

The variance σ^2 of X

$$\sigma^2 = np(1-p) = 2.5 \tag{2}$$

Let

$$Z \approx \frac{X-\mu}{\sigma} \tag{3}$$

Here, Z is a random variable with $\mathcal{N}(0,1)$
For $X \ge 8$

1) With a 0.5 correction:

$$\Pr(X \ge 8) = 1 - \Pr(X < 7.5) \tag{4}$$
$$X < 7.5 \tag{5}$$
$$\implies Z < \frac{7.5-\mu}{\sigma} = Z < 1.5811 \tag{6}$$
$$\Pr(X \ge 8) = 1 - \Pr(Z < 1.5811) \tag{7}$$

$$\Pr(Z < 1.5811) = 0.94308 \tag{8}$$
$$\implies \Pr(X \ge 8) = 0.056923 \tag{9}$$

2) Without correction:

$$X \ge 8 \tag{10}$$
$$Z \ge \frac{8-\mu}{\sigma} = Z \ge 1.8973 \tag{11}$$
$$\Pr(X \ge 8) = \Pr(Z \ge 1.8973) \tag{12}$$
$$= 0.02889 \tag{13}$$

Binomial Distribution

$$\Pr(X \ge 8) = 1 - \Pr(X < 8) \tag{14}$$
$$= \sum_{k=8}^{10} \binom{n}{k} p^k (1-p)^{n-k} \tag{15}$$
$$= 0.0546875 \tag{16}$$

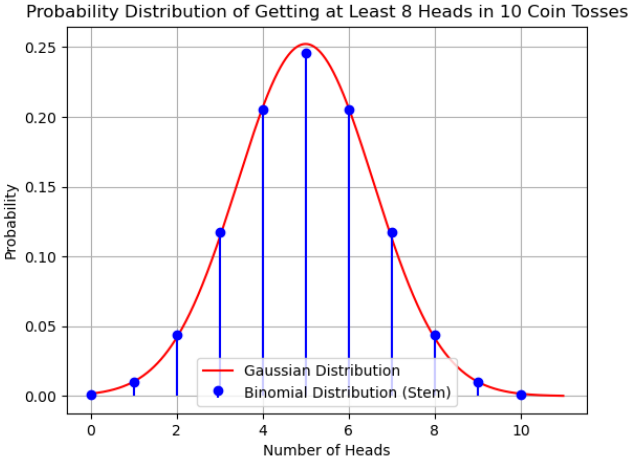

Fig. 1: Binomial vs Guassian