

Probability and Random Processes

Gude Pravarsh EE22BTECH11023*

Q) Ten coins are tossed. What is the probability of getting atleast 8 heads?

Solution:

Parameter	Value	Description
n	10	number of tosses
p	$\frac{1}{2}$	Probability for Heads
q	$\frac{1}{2}$	Probability for Tails
$\mu = np$	5	mean of the distribution
$\sigma^2 = npq$	2.5	variance of the distribution
X	$0 \leq X \leq 10$	Number of heads

Gaussian Distribution

$$X \approx Y \sim \mathcal{N}(5, 2.5) \quad (1)$$

1) With a 0.5 correction:

$$\Pr(Y \geq 8) = Q\left(\frac{7.5 - \mu}{\sigma}\right) \quad (2)$$

$$\Rightarrow \Pr(Y \geq 8) = Q(\sqrt{2.5}) = Q(1.5811) \quad (3)$$

$$\Rightarrow \Pr(Y \geq 8) = 0.0569276 \quad (4)$$

2) Without correction:

$$\Pr(Y \geq 8) = Q\left(\frac{8 - \mu}{\sigma}\right) \quad (5)$$

$$\Rightarrow \Pr(Y \geq 8) = Q\left(\frac{3}{\sqrt{2.5}}\right) = Q(1.8973) \quad (6)$$

$$\Rightarrow \Pr(Y \geq 8) = 0.0288898 \quad (7)$$

Binomial Distribution

$$\Pr(X \geq 8) = \sum_{k=8}^{10} \binom{n}{k} p^k (1-p)^{n-k} \quad (8)$$

$$= 0.0546875 \quad (9)$$

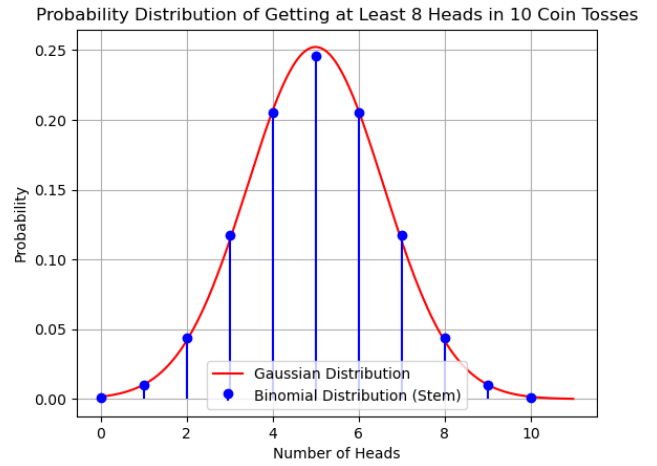


Fig. 1: Binomial vs Gaussian