Probability and Random Processes

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1) In an examination, 20 questions of true-false type are asked. Suppose a student tosses a fair coin to determine his answer to each question. If the coin falls heads, he answers 'true'; if it falls tails, he answers 'false'. Find the probability that he answers at least 12 questions correctly.

   Solution: Let the random variable X denotes true(1) or false(0) for a question with p being the probability that true is answered. Then,

$$X \sim \text{Ber}(p) \tag{1}$$

   Suppose $X_i, 1 \leq i \leq n$ represent each of the n draws. Define Y as

$$Y = \sum_{i=1}^{n} X_i \tag{2}$$

   Then, since the X_i are iid, the pmf of Y is given by

$$Y \sim \text{Bin}(n, p) \tag{3}$$

   The cdf of Y is given by

$$F_Y(k) = p_Y(\leq k) \tag{4}$$

$$= \begin{cases} 0 & k < 0 \\ \sum_{i=1}^{k} \binom{n}{i} p^i (1-p)^{n-i} & 1 \leq k \leq n \\ 1 & k \geq n \end{cases} \tag{5}$$

   In this case,

$$p = \frac{1}{2}, \ n = 20 \tag{6}$$

2) We require $p_Y(\geq 12)$. Since $n = 20$,

$$p_Y(\geq 12) = 1 - p_Y(< 12) \tag{7}$$

$$= F_Y(20) - F_Y(11) \tag{8}$$

$$= \sum_{k=12}^{20} p_Y(k) \tag{9}$$

$$= \sum_{k=12}^{20} \binom{n}{k} p^k (1-p)^{n-k} \tag{10}$$

$$= 0.2517 \tag{11}$$