Probability

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Contents

1 Central Limit Theorem

1.1 Generate 10^6 samples of the random variable

$$X = \sum_{i=1}^{12} U_i - 6 \tag{1.1.1}$$

using a C program, where $U_i, i = 1, 2, \ldots, 12$ are a set of independent uniform random variables between 0 and 1 and save in a file called gau.dat

Solution: The results are stored in the following links.

```
https://github.com/ee22mtech
    11017/assingment/blob/main/
    gauss.c
```

1.2 Load gau.dat in python and plot the empirical CDF of X using the samples in gau.dat.

Solution: The CDF of X is plotted in using the code below.

```
https://github.com/ee22mtech
    11017/assingment/blob/main/
    cdfplot.py
```

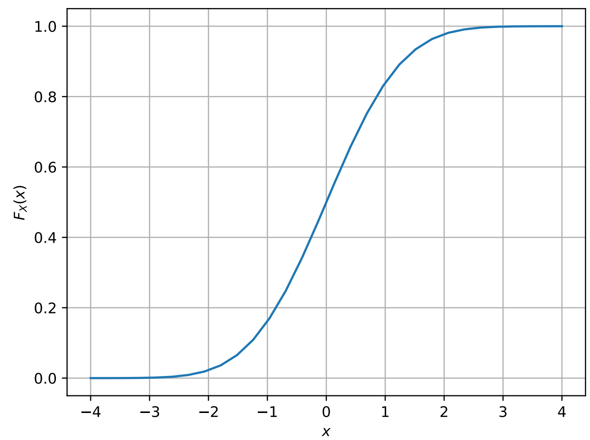

Fig. 1.3. The CDF of X

1.3 Load gau.dat in python and plot the empirical PDF of X using the samples in gau.dat. The PDF of X is defined as

$$p_X(x) = \frac{d}{dx} F_X(x) \tag{1.3.1}$$

What properties does the PDF have?

Solution: The PDF of X is plotted in using the code below

```
https://github.com/ee22mtech
    11017/assingment/blob/main/
    pdf_plot.py
```

properties of pdf are as follows:

1.The probability funtion is never negative or cannot be less than zero.

2.The probability density function is always greater than or equal to zero for all real numbers.

$$0 \leq f(x)$$

3.The total area under probability density curve is always equal to one.

$$\int_{-\infty}^{\infty} f(x)dx = 1$$

2.The probability density function curve is continuous over the entire range due to the property of continuous random variables. This also defines itself over a range of continuous values, or the variable's domain.

1.4 Find the mean and variance of X by writing a C program.

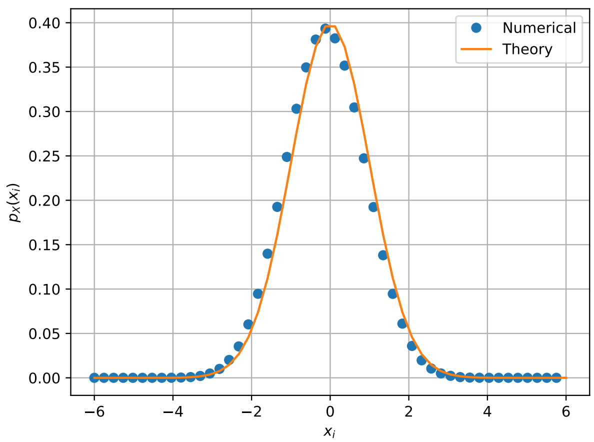

Fig. 1.5. The PDF of X

```
https://github.com/ee22mtech
    11017/assingment/blob/main/
    gauss.c
```

1.5 Given that

$$p_X(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right), -\infty < x < \infty, \tag{1.5.1}$$

repeat the above exercise theoretically.
Solution:

$$\int_{-\infty}^{\infty} p_x(x)(dx) \tag{1.5.2}$$

The mean of $p_x(x)dx$ is given by:

$$= \int_{-\infty}^{\infty} x.p_x(x)dx \tag{1.5.3}$$

$$= \int_{-\infty}^{0} x.p_x(x)dx + \int_{0}^{\infty} x.p_x(x)dx \tag{1.5.4}$$

$$= -0.5 + 0.5 = 0. \tag{1.5.5}$$

$$Mean = 0.$$

$$Variance = E\left[X^2\right] - [E[X]]^2 \tag{1.5.6}$$

We know that mean is zero so $E[X]^2$ is also zero.

$$Var(x) = E\left[X^2\right] \tag{1.5.7}$$

$$= \int_{-\infty}^{\infty} (x)^2.p_x(x)dx \tag{1.5.8}$$

$$= \int_{-\infty}^{\infty} \frac{(x)^2}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right) dx \tag{1.5.9}$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} (x)^2 \exp\left(-\frac{x^2}{2}\right) dx \tag{1.5.10}$$

We can rewrite the above integral as,

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} x \exp\left(-\frac{x^2}{2}\right) xdx \tag{1.5.11}$$

Let us assume y = $\frac{1}{2}x^2$

$$\frac{dy}{dx} = \frac{2x}{2}$$
$$dy = xdx$$
$$x = \sqrt{2y}$$

Let us substitute,

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \sqrt{2y} \exp(-y)\, dy \tag{1.5.12}$$

$$= \frac{\sqrt{2}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \sqrt{y} \exp(-y)\, dy \tag{1.5.13}$$

Since it is an even function we know

$\int_{-\infty}^{\infty} F(x)(dx) = 2 \int_{0}^{\infty} F(x)dx$

$$= \frac{2}{\sqrt{\pi}} \int_{0}^{\infty} \sqrt{y} \exp(-y)\, dy$$

We can see that, $\int_{0}^{\infty} \sqrt{y} \exp(-y)\, dy$ is a Gamma function of the form

$$\Gamma(a) = \int_{0}^{\infty} y^{a-1} \exp(-y)\, dy \tag{1.5.14}$$

$$E[x^2] = \frac{2}{\sqrt{\pi}} \int_{0}^{\infty} y^{(0.5+1)-1} \exp(-y)\, dy \tag{1.5.15}$$

$$E[x^2] = \frac{2}{\sqrt{\pi}} \Gamma(0.5 + 1) \tag{1.5.16}$$

$$As\Gamma(\frac{1}{2}) = \sqrt{\pi} \tag{1.5.17}$$

$$E[x^2] = 1$$

(1.5.18)

$$var[x] = 1$$

we theoritically proved the mean and variance values.