Assignment-4

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1 Triangular Distribution 1

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1.1 Generate

$$T = U_1 + U_2 \tag{1.1.1}$$

Solution: Samples generated using the code

```
https://github.com/ee22
    mtech11017/
    assingment/blob/main
    /txrand.py
```

1.2 Find the CDF of T.

Solution: The CDF is plotted using the code

```
https://github.com/ee22mtech
    11017/assingment/blob/main/
    txrand.py
```

1.3 Find the PDF of T.

Solution: The PDF of T is plotted using the code

```
https://github.com/ee22mtech
    11017/assingment/blob/main/
    pdf.py
```

1.4 Find the theoretical expressions for the PDF and CDF of T.

Solution: $\mathrm{U}_2(t-x)f_{U_1}(x)dx$

where U_1 and U_2 are uniform random variables in [0, 1]. Clearly, $0 \le U_1 + U_2 \le 2$.
Again we know that

$$f_{U_1}(x) = \begin{cases} 0 & \text{for x } < 0 \\ 1 & \text{for x } \geq 0 \end{cases}$$

$$F_{U_1}(x) = \Pr(U_1 \le x) = \begin{cases} 0 & \text{for x } < 0 \\ x & \text{for } x \in [0,1] \\ 1 & \text{for x } > 1 \end{cases}$$

and

$$f_{U_2}(x) = \begin{cases} 0 & \text{for x } < 0 \\ 1 & \text{for x } \geq 0 \end{cases}$$

$$F_{U_2}(x) = \Pr(U_2 \le x) = \begin{cases} 0 & \text{for x } < 0 \\ x & \text{for } x \in [0,1] \\ 1 & \text{for x } > 1 \end{cases}$$

Now to deal with the interval from 0 to 2, it is useful to break this down into four cases:

(i) $t < 0$: In this case, it is clear that $F_T(t) = 0$.

(ii) $0 \le t < 1$: We need $t - x \ge 0$ and thus $x \le t$ and therefore, we integrate from $x = 0$ and $x = t$.

$$\begin{aligned} F_T(t) &= \int_0^t F_{U_2}(t-x) f_{U_1}(x) dx \\ &= \int_0^t (t-x) dx \\ &= \left[tx - \frac{x^2}{2} \right]_0^t \\ &= \frac{t^2}{2} \end{aligned}$$

(iii) $1 \le t < 2$: Here, we need $t - x \le 1$ and thus $x \le t - 1$ and therefore, we integrate from 0 to $x = t - 1$ and $x = t - 1$ to $x = 1$.

$$\begin{aligned} F_T(t) &= \int_0^{t-1} F_{U_2}(t-x) f_{U_1}(x) dx + \\ & \int_{t-1}^{1} F_{U_2}(t-x) F_{U_1}(x) dx \\ &= \int_0^{t-1} dx + \int_{t-1}^{1} (t-x) dx \\ &= t - 1 + t(2-t) - \frac{1-(t-1)^2}{2} \\ &= -\frac{t^2}{2} + 2t - 1 \end{aligned}$$

(iv) $t \ge 2$: Also $F_T(t) = 1$.

So, the CDF for T is given by

$$F_T(t) = \Pr(T \le t) = \begin{cases} 0 & x < 0 \\ \frac{t^2}{2} & 0 \le t < 1 \\ 2t - \frac{t^2}{2} - 1 & 1 \le t < 2 \\ 1 & x \ge 2 \end{cases} \tag{1.4.1}$$

We can take the derivative with respect to t of the CDF to get the PDF and thus

$$f_T(t) = \frac{d}{dz} F_t(t) = \int_0^1 f_{U_2}(t-x) f_{U_1}(x) dx$$

Thus, the PDF for T is given by

$$f_T(t) = \begin{cases} t & 0 \le t < 1 \\ 2 - t & 1 \le t < 2 \\ 0 & \text{Elsewhere} \end{cases} \tag{1.4.2}$$

We know that $T = U_1 + U_2$. Accordingly,

1.5 Verify your results through a plot.

Solution: The CDF and PDF figures plotted above serve our purpose.

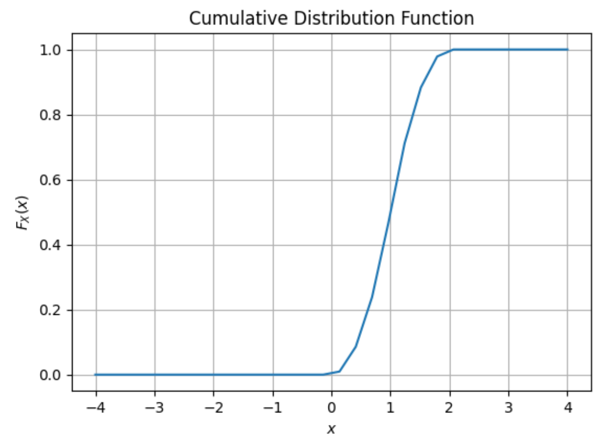

Fig. 1.5. The PDF of T

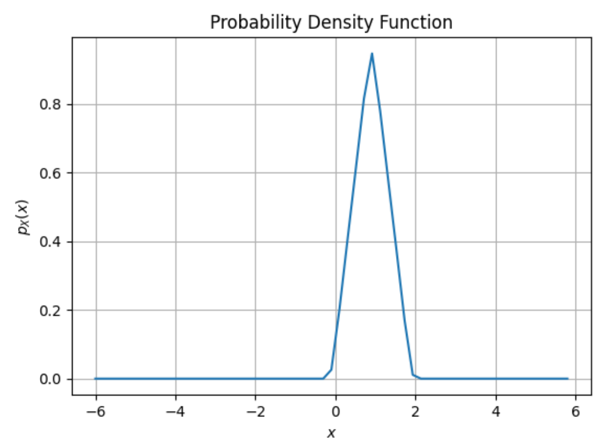

Fig. 1.5. The PDF of T