Probability

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Contents

1 Uniform Random Numbers

Let U be a uniform random variable between 0 and 1.

1.1 Generate 10^6 samples of U using a C program and save into a file called uni.dat .
Solution: Result of the following are provided in the link.

```
https://github.com/ee22mtech
    11017/assingment/blob/main/
    variance%20(1).c
```

1.2 Load the uni.dat file into python and plot the empirical CDF of U using the samples in uni.dat. The CDF is defined as

$$F_U(x) = \Pr(U \le x)$$

Solution: The following code plots Fig. 1.2

```
https://github.com/ee22mtech
    11017/assingment/blob/main/
    cdfplot.py
```

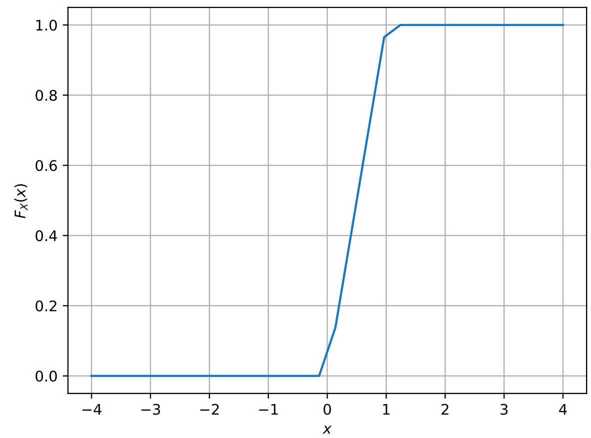

Fig. 1.2. The CDF of U

1.3 Find a theoretical expression for $F_U(x)$.
Solution: cummulative density function:

$$F_U(x) = \int_{-\infty}^{x} f(x)\, dt$$

$$f(x) = \begin{cases} 1, & \text{for } 0 < \ x \le 1 \\ 0, & \text{otherwise} \end{cases}$$

$$F_U(x) = \begin{cases} 0, & \text{for } x \le 0 \\ (x-0)/(1-0), & \text{for } 0 < x \le 1 \\ 1, & \text{for } x \ge 1 \end{cases}$$

The mean of U is defined as

$$E[U] = \frac{1}{N}\sum_{i=1}^{N} U_i$$

and its variance as

$$\text{var}[U] = E[U - E[U]]^2$$

1.4 Write a C program to find the mean and variance of U.
Solution: Results are stored in the link.

```
https://github.com/ee22mtech
    11017/assingment/blob/main/
    variance%20(1).c
```

1.5 Verify your result theoretically given that

$$E\left[U^k\right] = \int_{-\infty}^{\infty} x^k dF_U(x)$$

Solution:

$$k = 1$$

$$E\left[U^1\right] = \int_{-\infty}^{\infty} x^1 dF_U(x)$$

$$E\left[U\right] = \int_0^1 x dF_U(x)$$

$$dF_U = dx/(1-0)$$

$$E\left[U\right] = \left[x^2/2\right]_0^1$$

$$E\left[U\right] = 0.5$$

$$E\left[U^2\right] = \int_0^1 x^2 dF_U(x)$$

$$E\left[U^2\right] = [1/3]$$

$$Variance = E\left[U^2\right] - \left[E\left[U\right]\right]^2$$

$$variance = 0.0833$$

The result has been verified