

EE23BTECH11217 - Prajwal M*

EXERCISE 9.1

The given figure shows a series LCR circuit connected to a sinusoidal 230 V source.

$L = 5.0 \text{ H}$, $C = 80 \mu\text{F}$, $R = 40 \Omega$.

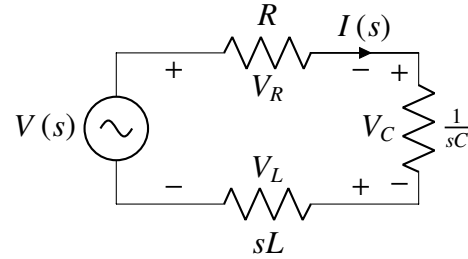
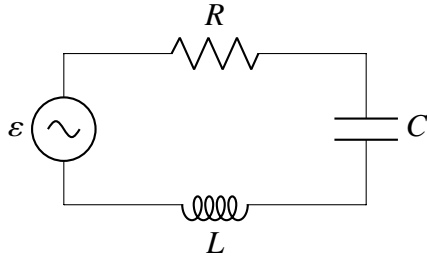


Fig. 3. s-domain circuit diagram

- 1) Determine the source frequency which drives the circuit in resonance.
- 2) Obtain the impedance of the circuit at the resonating frequency.
- 3) Determine the rms potential drops across the three elements of the circuit. Show that the potential drop across the LC combination is zero at the resonating frequency.

using KVL,

$$V(t) \xleftrightarrow{Z} V(s) \quad (1)$$

$$230 \sqrt{2} \cos(2\pi f t) \xleftrightarrow{Z} V(s) \quad (2)$$

$$V(s) = \frac{230 \sqrt{2} s}{s^2 + (2\pi f)^2} \quad (3)$$

$$V(s) - I(s)R - \frac{I(s)}{sC} - I(s)sL = 0 \quad (4)$$

$$Z = \frac{V(s)}{I(s)} = \frac{sCR + s^2LC + 1}{sC} \quad (5)$$

Solution:

replacing $s = j2\pi f$ in (5),

$$Z = R + j(1 - (2\pi f)^2 LC) \quad (6)$$

$$\min(|Z|) = R \text{ at } f = \frac{1}{2\pi \sqrt{LC}} \quad (7)$$

$$f_{res} = \frac{1}{\sqrt{LC}} = 7.958 \text{ Hz} \quad (8)$$

$$Z_{res} = R = 40 \Omega \quad (9)$$

substituting (8) and (9) in (3), (5),

$$I(s) = \left(\frac{1}{40} \right) \frac{230 \sqrt{2} s}{s^2 + 50^2} \quad (10)$$

$$I(s) \xleftrightarrow{\mathcal{L}^{-\infty}} I(t) \quad (11)$$

$$I(t) = (8.132) \cos(50t) \quad (12)$$

Paramater	Description	Value
$V(t)$	Voltage power supply	$230 \sqrt{2} \cos(2\pi f t) \text{ V}$
$V(s)$	Laplace transform of $V(t)$?
L	Inductance	5.0 H
C	Capacitance	$80 \mu\text{F}$
R	Resistance	40Ω
f	Frequency of voltage source	?
Z	Impedance of circuit	?
$V_R(t)$	Potential drop across Resistor	?
$V_R(s)$	Laplace transform of $V_R(s)$?
$V_C(t)$	Potential drop across Capacitor	?
$V_C(s)$	Laplace transform of $V_C(s)$?
$V_L(t)$	Potential drop across Inductor	?
$V_L(s)$	Laplace transform of $V_L(s)$?

TABLE 3
PARAMETER DESCRIPTION

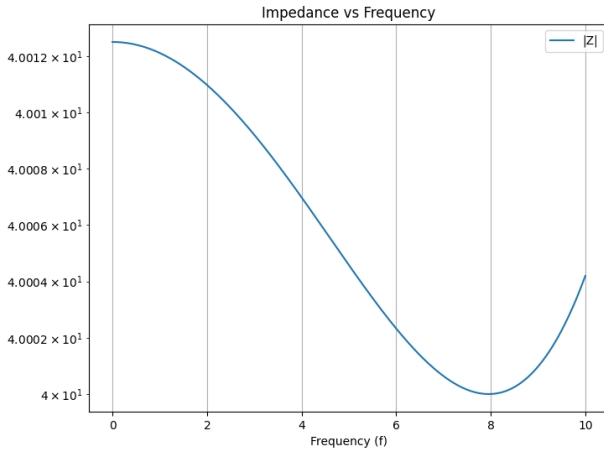


Fig. 3. Impedance vs frequency

R:

$$V_R(s) = RI(s) \quad (13)$$

$$V_R(s) \xrightarrow{\mathcal{L}^{-\infty}} V_R(t) \quad (14)$$

$$V_R(t) = RI(t) \quad (15)$$

$$= 325.28 \cos(50t) \quad \{\text{using (12)}\} \quad (16)$$

$$(17)$$

C:

$$V_C(s) = \frac{I(s)}{sC} \quad (18)$$

$$= \frac{10^4}{32} \frac{230 \sqrt{2}}{s^2 + 50^2} \quad \{\text{using (10)}\} \quad (19)$$

$$V_C(s) \xrightarrow{\mathcal{L}^{-\infty}} V_C(t) \quad (20)$$

$$V_C(t) = 2032.93 \sin(50t) \quad (21)$$

L:

$$V_L(s) = sLI(s) \quad (22)$$

$$= \frac{1}{8} \frac{230 \sqrt{2} s^2}{s^2 + 50^2} \quad \{\text{using (10)}\} \quad (23)$$

$$V_L(s) \xrightarrow{\mathcal{L}^{-\infty}} V_L(t) \quad (24)$$

$$V_L(t) = 40.658 (\delta(t) - 50 \sin(50t)) \quad (25)$$

$$\text{for } t > 0, \quad (26)$$

$$= -2032.93 \sin(50t) \quad (27)$$

from (21) and (27), voltage across LC combination is $V_C + V_L = 0V$

Paramater	Description	Value
f_{res}	resonant source frequency	$7.958Hz$
Z_{res}	resonant impedance	40Ω
V_R	rms value of $V_R(t)$	$230V$
V_C	rms value of $V_C(t)$	$1437.5V$
V_L	rms value of $V_L(t)$	$14375V$

TABLE 0
SOLUTION VALUES