EE23BTECH11217 - Prajwal M*

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# Exercise 9.5

25) **Find the sum of the following series up to n terms and obtain the Z-transform:**

$$\frac{1^3}{1} + \frac{1^3+2^3}{1+3} + \frac{1^3+2^3+3^3}{1+3+5} + \dots$$

Solution:

| Symbol | Description |
|---|---|
| $s(n)$ | sum of n terms |
| $S(z)$ | Z-transform of s(n) |
| $s_1(n)$ | $(n+1)^3 * u(n)$ |
| $S_1(z)$ | Z-transform of $s_1(n)$ |
| $s_2(n)$ | $(2n+1) * u(n)$ |
| $S_2(z)$ | Z-transform of $s_2(n)$ |

Table 0: Parameters

$$s(n) = \sum_{r=0}^{n} \left( \frac{\sum_{i=0}^{r} (i+1)^3}{\sum_{i=0}^{r} (2i+1)} \right) \tag{1}$$

$$= \frac{(n+1)^3 * u(n)}{(2n+1) * u(n)} * u(n) \tag{2}$$

$$= \frac{s_1(n)}{s_2(n)} * u(n) \tag{3}$$

$$s_1(n) = (n+1)^3 * u(n) \tag{4}$$

$$s_1(n) \xleftrightarrow{\mathcal{Z}} S_1(z) \tag{5}$$

$$(n+1)^3 \xleftrightarrow{\mathcal{Z}} \frac{1+4z^{-1}+z^{-2}}{(1-z^{-1})^4} \tag{6}$$

$$S_1(z) = \left( \frac{1+4z^{-1}+z^{-2}}{(1-z^{-1})^4} \right) \left( \frac{1}{1-z^{-1}} \right) \quad \{|z| > 1\} \tag{7}$$

$$= \frac{1}{(1-z^{-1})^5} + \frac{4z^{-1}}{(1-z^{-1})^5} + \frac{z^{-2}}{(1-z^{-1})^5} \tag{8}$$

using (**??**) and (**??**)

$$s_1(n) = \frac{(n+4)(n+3)(n+2)(n+1)}{24} u(n+4) + \frac{(n+3)(n+2)(n+1)n}{6} u(n+3) + \frac{(n+2)(n+1)n(n-1)}{24} u(n+2) \tag{9}$$

$$= \frac{(n+2)^2(n+1)^2}{4} \quad \{n \geq 0\} \tag{10}$$

$$s_2(n) = (2n+1) * u(n) \tag{11}$$

$$s_2(n) \overset{z}{\longleftrightarrow} S_2(z) \tag{12}$$

$$2n+1 \overset{z}{\longleftrightarrow} \left(\frac{1+z^{-1}}{(1-z^{-1})^2}\right) \tag{13}$$

$$S_2(z) = \left(\frac{1+z^{-1}}{(1-z^{-1})^2}\right)\left(\frac{1}{1-z^{-1}}\right) \qquad \{|z| > 1\} \tag{14}$$

$$= \frac{1}{(1-z^{-1})^3} + \frac{z^{-1}}{(1-z^{-1})^3} \tag{15}$$

using (**??**) and (**??**)

$$s_2(n) = \frac{(n+2)(n+1)}{2} u(n+2) + \frac{(n+1)(n)}{2} u(n+1) \tag{16}$$

$$= (n+1)^2 \qquad \{n \geq 0\} \tag{17}$$

replacing (10) and (17) in (3)

$$s(n) = \frac{(n+2)^2}{4} * u(n) \tag{18}$$

$$s(n) \overset{z}{\longleftrightarrow} S(z) \tag{19}$$

$$\frac{(n+2)^2}{4} \overset{z}{\longleftrightarrow} \left(\frac{4-3z^{-1}+z^{-2}}{4(1-z^{-1})^3}\right) \tag{20}$$

$$S(z) = \left(\frac{4-3z^{-1}+z^{-2}}{4(1-z^{-1})^3}\right)\left(\frac{1}{1-z^{-1}}\right) \qquad \{|z| > 1\} \tag{21}$$

$$= \frac{1}{(1-z^{-1})^4} - \frac{3z^{-1}}{4(1-z^{-1})^4} + \frac{z^{-2}}{4(1-z^{-1})^4} \tag{22}$$

$$s(n) = \frac{(n+3)(n+2)(n+1)}{6} u(n+3) - \frac{(n+2)(n+1)(n)}{8} u(n+2) + \frac{(n+1)(n)(n-1)}{24} u(n+1) \tag{23}$$

$$= \left(1 + \frac{37n}{24} + \frac{5n^2}{8} + \frac{n^3}{12}\right) u(n) \tag{24}$$

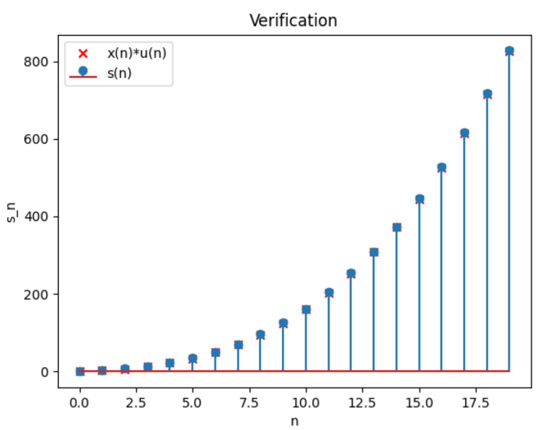


Figure 0: Plot of s(n) vs n