EE23BTECH11217 - Prajwal M*

Exercise 9.5

25) **Find the sum of the following series up to n terms and obtain the Z-transform:**

$$\ldots + 0 + \frac{1^3}{1} + \frac{1^3+2^3}{1+3} + \frac{1^3+2^3+3^3}{1+3+5} + \ldots$$

Solution:

$$x(n) = \frac{\sum_{i=0}^{n}(i+1)^3}{\sum_{j=0}^{n}(2j+1)}u(n) \tag{1}$$

$$= \frac{(n+2)^2}{4}u(n) \tag{2}$$

$$S(n) = \sum_{r=-\infty}^{n} x(r) \tag{3}$$

using (2),

$$= \sum_{r=-\infty}^{n} \frac{(r+2)^2}{4}u(r) \tag{4}$$

$$= \sum_{r=0}^{n} \frac{r^2+4r+4}{4} \tag{5}$$

$$= 1 + \frac{37n}{24} + \frac{5n^2}{8} + \frac{n^3}{12} \tag{6}$$

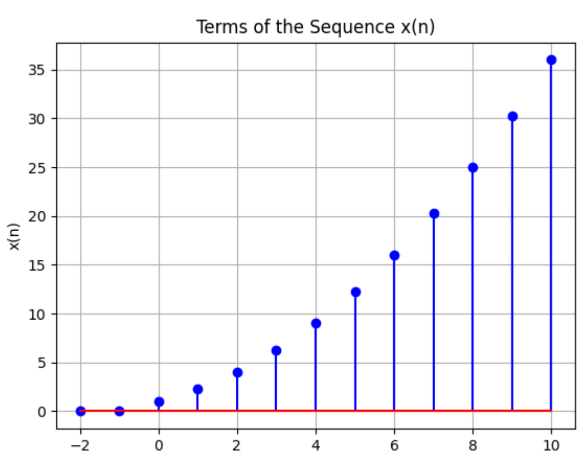

Fig. 0. Plot of x(n) vs n

$$x(n) \overset{\mathcal{Z}}{\longleftrightarrow} X(z) \tag{7}$$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n} \tag{8}$$

using (2),

$$= \sum_{n=-\infty}^{\infty} \frac{(n+2)^2}{4}u(n)z^{-n} \tag{9}$$

$$= \sum_{n=0}^{\infty} \frac{(n+2)^2}{4}z^{-n} \tag{10}$$

$$= \sum_{n=0}^{\infty} \frac{n^2}{4}z^{-n} + \sum_{n=0}^{\infty} nz^{-n} + \sum_{n=0}^{\infty} z^{-n} \tag{11}$$

$$= \frac{z(4z^2-3z+1)}{4(z-1)^3} \qquad \{z \in \mathbb{C} : |z| > 1\} \tag{12}$$

Symbol	Value	Description
$x(n)$	$\frac{-1}{n!}$	general term of the series
$S(n)$	$1 + \frac{37n}{24} + \frac{5n^2}{8} + \frac{n^3}{12}$	sum of terms until x(n)
$X(z)$	$-e^{z^{-1}}$	Z-transform of x(n)
$u(n)$		unit step function

TABLE 0
Parameters