EE23BTECH11217 - Prajwal M*

EXERCISE 9.5

25) **Find the sum of the following series up to n terms and obtain the Z-transform:**

$$\ldots + 0 + \frac{1^3}{1} + \frac{1^3+2^3}{1+3} + \frac{1^3+2^3+3^3}{1+3+5} + \ldots$$

Solution:

$$x(n) = \frac{\sum_{i=0}^{n}(i+1)^3}{\sum_{j=0}^{n}(2j+1)}u(n) \tag{1}$$

$$= \frac{(n+1)^3 * u(n)}{(2n+1) * u(n)}u(n) \tag{2}$$

$$= \frac{\mathcal{Z}^{-1}\left\{\mathcal{Z}\left\{(n+1)^3\right\}\mathcal{Z}\{u(n)\}\right\}}{\mathcal{Z}^{-1}\{\mathcal{Z}\{2n+1\}\mathcal{Z}\{u(n)\}\}}u(n) \tag{3}$$

$$= \frac{\mathcal{Z}^{-1}\left\{\left(\frac{z^4+4z^3+z^2}{(z-1)^4}\right)\left(\frac{z}{z-1}\right)\right\}}{\mathcal{Z}^{-1}\left\{\left(\frac{z^2+z}{(z-1)^2}\right)\left(\frac{z}{z-1}\right)\right\}}u(n) \tag{4}$$

$$= \frac{\frac{1}{2\pi j}\oint_C\left\{\left(\frac{z^5+4z^4+z^3}{(z-1)^5}\right)z^{n-1}\right\}dz}{\frac{1}{2\pi j}\oint_C\left\{\left(\frac{z^3+z^2}{(z-1)^3}\right)z^{n-1}\right\}dz}u(n) \tag{5}$$

$$= \frac{\frac{1}{4!}\lim_{n\to1}\frac{d^4}{dz^4}\left\{z^{n+4}+4z^{n+3}+z^{n+2}\right\}}{\frac{1}{2!}\lim_{n\to1}\frac{d^2}{dz^2}\left\{z^{n+2}+z^{n+1}\right\}}u(n) \tag{6}$$

$$= \frac{(n+2)^2}{4}u(n) \tag{7}$$

$$s(n) = \sum_{r=-\infty}^{n} x(r) \tag{8}$$

using (7),

$$s(n) = \sum_{r=-\infty}^{n}\frac{(r+2)^2}{4}u(r) \tag{9}$$

$$= \sum_{r=0}^{n}\frac{r^2+4r+4}{4} \tag{10}$$

$$= 1 + \frac{37n}{24} + \frac{5n^2}{8} + \frac{n^3}{12} \tag{11}$$

$$= \frac{(n+2)^2}{4}u(n) * u(n) \tag{12}$$

$$= \mathcal{Z}^{-1}\left\{\frac{z\left(1-3z+4z^2\right)}{4(z-1)^3}\frac{z}{z-1}\right\} \tag{13}$$

$$= \frac{1}{2\pi j}\oint_C\left\{\left(\frac{4z^4-3z^3+z^2}{4(z-1)^4}\right)z^{n-1}\right\}dz \tag{14}$$

$$= \frac{1}{3!}\lim_{n\to1}\frac{d^3}{dz^3}\left\{z^{n+3} - \frac{3z^{n+2}}{4} + \frac{z^{n+1}}{4}\right\} \tag{15}$$

$$= 1 + \frac{37n}{24} + \frac{5n^2}{8} + \frac{n^3}{12} \tag{16}$$

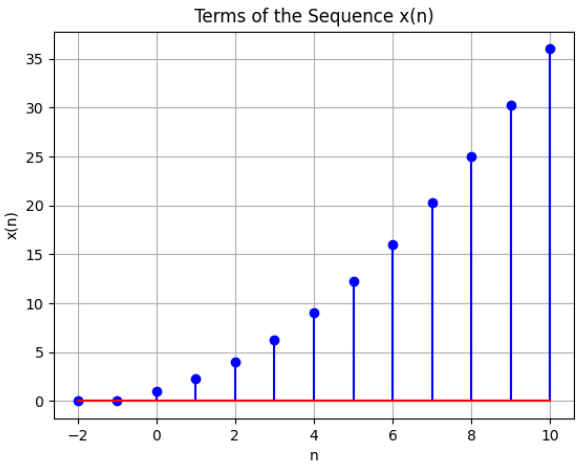

Fig. 0. Plot of x(n) vs n

Symbol	Value	Description
$x(n)$	$\frac{(n+2)^2}{4}$	general term of the series
$s(n)$	$1 + \frac{37n}{24} + \frac{5n^2}{8} + \frac{n^3}{12}$	sum of terms until x(n)
$u(n)$		unit step function

TABLE 0
PARAMETERS