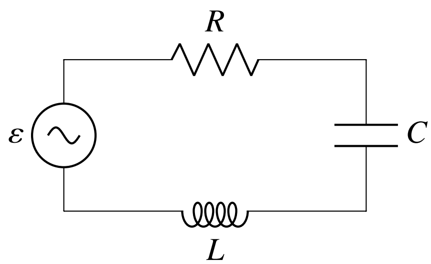
# EE23BTECH11217 - Prajwal M*

## Exercise 9.1

The given figure shows a series LCR circuit connected to a sinusoidal 230 V source.
L = 5.0 H, C = 80 $\mu$F, R = 40 $\Omega$.




1) Determine the source frequency which drives the circuit in resonance.
2) Obtain the impedance of the circuit at the resonating frequency.
3) Determine the rms potential drops across the three elements of the circuit. Show that the potential drop across the LC combination is zero at the resonating frequency.

Solution:

| Paramater | Description | Value |
|---|---|---|
| $V(t)$ | Voltage power supply | $230\sqrt{2}cos(2\pi ft)$ V |
| $V(s)$ | Laplace transform of $V(t)$ | ? |
| L | Inductance | 5.0 H |
| C | Capacitance | 80 $\mu$F |
| R | Resistance | 40 $\Omega$ |
| $f$ | Frequency of voltage source | ? |
| $Z$ | Impedance of circuit | ? |
| $V_R(t)$ | Potential drop across Resistor | ? |
| $V_R(s)$ | Laplace transform of $V_R(s)$ | ? |
| $V_C(t)$ | Potential drop across Capacitor | ? |
| $V_C(s)$ | Laplace transform of $V_C(s)$ | ? |
| $V_L(t)$ | Potential drop across Inductor | ? |
| $V_L(s)$ | Laplace transform of $V_L(s)$ | ? |

TABLE 3
Parameter description

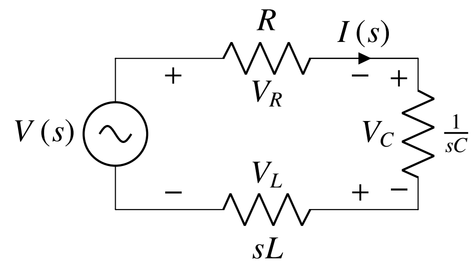



Fig. 3. s-domain circuit diagram

$$V(t) \xleftrightarrow{\mathcal{L}} V(s) \tag{1}$$

$$230\sqrt{2}cos(2\pi ft) \xleftrightarrow{\mathcal{L}} V(s) \tag{2}$$

$$V(s) = \frac{230\sqrt{2}s}{s^2 + (2\pi f)^2} \tag{3}$$

using KVL,

$$V(s) - I(s)R - \frac{I(s)}{sC} - I(s)sL = 0 \tag{4}$$

$$Z = \frac{V(s)}{I(s)} = \frac{sCR + s^2LC + 1}{sC} \tag{5}$$

replacing $s = j2\pi f$ in (5),

$$Z = R + j\left(1 - (2\pi f)^2 LC\right) \tag{6}$$

$$min(|Z|) = R \text{ at } f = \frac{1}{2\pi\sqrt{LC}} \tag{7}$$

$$f_{res} = \frac{1}{\sqrt{LC}} = 7.958Hz \tag{8}$$

$$Z_{res} = R = 40\Omega \tag{9}$$

substituting (8) and (9) in (3), (5),

$$I(s) = \left(\frac{1}{40}\right)\frac{230\sqrt{2}s}{s^2 + 50^2} \tag{10}$$

$$I(s) \xleftrightarrow{\mathcal{L}^{-\infty}} I(t) \tag{11}$$

$$I(t) = (8.132)\,cos(50t) \tag{12}$$

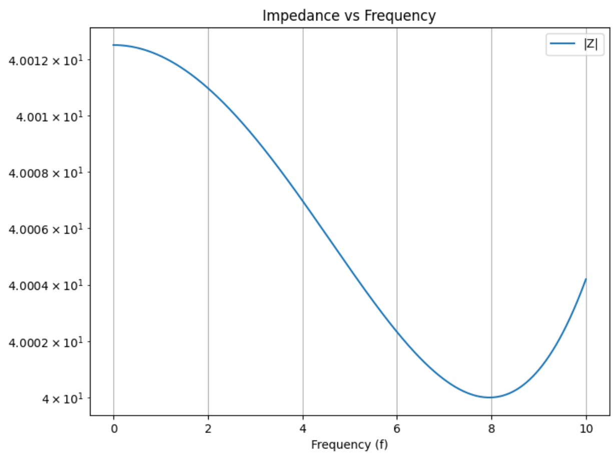



Fig. 3. Impedance vs frequency

R:

$$V_R(s) = RI(s) \tag{13}$$
$$V_R(s) \xleftrightarrow{\mathcal{L}^{-\infty}} V_R(t) \tag{14}$$
$$V_R(t) = RI(t) \tag{15}$$
$$= 325.28\, cos(50t) \quad \{\text{using (12)}\} \tag{16}$$
$$\tag{17}$$

C:

$$V_C(s) = \frac{I(s)}{sC} \tag{18}$$
$$= \frac{10^4}{32}\frac{230\sqrt{2}}{s^2+50^2} \qquad \{\text{using (10)}\} \tag{19}$$
$$V_C(s) \xleftrightarrow{\mathcal{L}^{-\infty}} V_C(t) \tag{20}$$
$$V_C(t) = 2032.93\, sin(50t) \tag{21}$$

L:

$$V_L(s) = sLI(s) \tag{22}$$
$$= \frac{1}{8}\frac{230\sqrt{2}s^2}{s^2+50^2} \qquad \{\text{using (10)}\} \tag{23}$$
$$V_L(s) \xleftrightarrow{\mathcal{L}^{-\infty}} V_L(t) \tag{24}$$
$$V_L(t) = 40.658\,(\delta(t) - 50 sin(50t)) \tag{25}$$
$$\text{for } t > 0, \tag{26}$$
$$= -2032.93 sin(50t) \tag{27}$$

from (21) and (27), voltage across LC combination is $V_C + V_L = 0V$

| Paramater | Description | Value |
|---|---|---|
| $f_{res}$ | resonant source frequency | $7.958Hz$ |
| $Z_{res}$ | resonant impedance | $40\Omega$ |
| $V_R$ | rms value of $V_R(t)$ | $230V$ |
| $V_C$ | rms value of $V_C(t)$ | $1437.5V$ |
| $V_L$ | rms value of $V_L(t)$ | $14375V$ |

TABLE 0
SOLUTION VALUES