EE23BTECH11217 - Prajwal M*

Exercise 9.1

12) Write the first five terms of each of the sequences and obtain the corresponding series $x(1) = -1,\ x(n) = \dfrac{x(n-1)}{n},\ n \geq 2$

Solution:
Given: $x(1) = -1,\ x(n) = \dfrac{x(n-1)}{n},\ n \geq 2.$

First five terms of the sequence:

$$\begin{aligned}
x(1) &= -1 \\
x(2) &= \frac{x(1)}{2} = -\frac{1}{2} \\
x(3) &= \frac{x(2)}{3} = -\frac{1}{2 \cdot 3} = -\frac{1}{6} \\
x(4) &= \frac{x(3)}{4} = -\frac{1}{2 \cdot 3 \cdot 4} = -\frac{1}{24} \\
x(5) &= \frac{x(4)}{5} = -\frac{1}{2 \cdot 3 \cdot 4 \cdot 5} = -\frac{1}{120}
\end{aligned}$$

So the first five terms of the series are:

$$-1, -\frac{1}{2}, -\frac{1}{6}, -\frac{1}{24}, -\frac{1}{120}$$

The corresponding series:

$$\begin{aligned}
\sum_{n=1}^{\infty} x(n) &= x(1) + x(2) + x(3) + x(4) + x(5) + \dots \\
&= -1 + \left(-\frac{1}{2}\right) + \left(-\frac{1}{6}\right) + \left(-\frac{1}{24}\right) + \dots
\end{aligned}$$

The nth term of the series is,

$$x(n) = \frac{-1}{n!}(u(n))$$

The Z-transform of $x(n)$ is given by:

$$x(n) \overset{\mathcal{Z}}{\rightleftharpoons} F(z)$$

$$\begin{aligned}
F(z) &= \sum_{n=-\infty}^{\infty} x(n) \cdot z^{-n} \\
&= \sum_{n=-\infty}^{\infty} \frac{-1}{n!} \cdot u(n) \cdot z^{-n} \\
&= \sum_{n=1}^{\infty} \frac{-1}{n!} \cdot z^{-n} \\
&= -(e^{z^{-1}} - 1) \\
&= 1 - e^{z^{-1}}
\end{aligned}$$

So, the Z-transform of the given series is $1 - e^{z^{-1}}$.