EE23BTECH11217 - Prajwal M*

Exercise 9.1

12 Write the five terms at n = 1, 2, 3, 4, 5 of the sequence and obtain the corresponding series

$$x(n) = \begin{cases} -1 & n = 0 \\ \frac{x(n-1)}{n} & n > 0 \\ 0 & n < 0 \end{cases}$$

Solution:

$$x(1) = \frac{x(0)}{1} = -1 \tag{1}$$

$$x(2) = \frac{x(1)}{2} = -\frac{1}{2} \tag{2}$$

$$x(3) = \frac{x(2)}{3} = -\frac{1}{23} = -\frac{1}{6} \tag{3}$$

$$x(4) = \frac{x(3)}{4} = -\frac{1}{234} = -\frac{1}{24} \tag{4}$$

$$x(5) = \frac{x(4)}{5} = -\frac{1}{2345} = -\frac{1}{120} \tag{5}$$

The corresponding series:

$$\sum_{n=-\infty}^{\infty} x(n) = \ldots + 0 + x(0) + x(1) + x(2) + \ldots \tag{6}$$

$$= \ldots + 0 + (-1) + (-1) + \left(-\frac{1}{2}\right) + \ldots \tag{7}$$

The nth term of the series is,

$$x(n) = \frac{-1}{n!}\left(u(n)\right) \tag{8}$$

The Z-transform of $x(n)$ is given by:

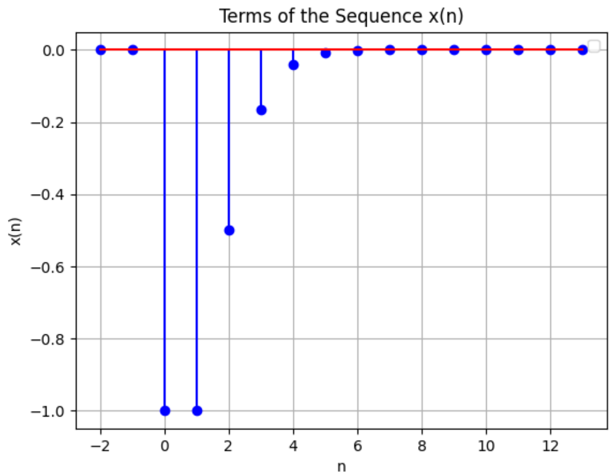
$$x(n) \overset{\mathcal{Z}}{\rightleftharpoons} X(z) \tag{9}$$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n} \tag{10}$$

$$= \sum_{n=-\infty}^{\infty} \frac{-1}{n!} u(n) z^{-n} \tag{11}$$

$$= \sum_{n=0}^{\infty} \frac{-1}{n!} z^{-n} \tag{12}$$

$$= -e^{z^{-1}} \tag{13}$$



So, the Z-transform of the given series is $-e^{z^{-1}}$.

For the series to converge, the ratio test must be satisfied for n $\geq$ 0

$$\lim_{n\to\infty} \left| \frac{x(n+1) z^{-(n+1)}}{x(n) z^{-n}} \right| < 1 \tag{14}$$

$$\lim_{n\to\infty} \left| \frac{-z^{-n-1} n!}{-z^{-n}(n+1)!} \right| < 1 \tag{15}$$

$$\lim_{n\to\infty} \left| \frac{z^{-1}}{n+1} \right| < 1 \tag{16}$$

The condition is satisfied for $z \neq 0$

Hence, ROC of Z transform is

$$z \in \mathbb{C} : z \neq 0. \tag{17}$$

Symbol	**Parameters**
x(n)	general term of the series
X(z)	Z-transform of x(n)
u(n)	unit step function

TABLE I
Parameters