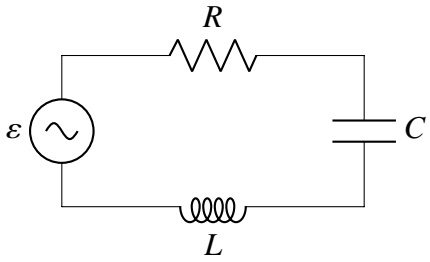
EE23BTECH11217 - Prajwal M*

Exercise 9.1

The given figure shows a series LCR circuit connected to a sinusoidal 230 V source.
L = 5.0 H, C = 80 μF, R = 40 Ω.



1) Determine the source frequency which drives the circuit in resonance.
2) Obtain the impedance of the circuit at the resonating frequency.
3) Determine the rms potential drops across the three elements of the circuit. Show that the potential drop across the LC combination is zero at the resonating frequency.

Solution:

Paramater	Description	Value
ε	Voltage power supply	$Re\left\{230\sqrt{2}e^{j\omega t}\right\}$ V
L	Inductance	5.0 H
C	Capacitance	80 μF
R	Resistance	40 Ω
$\frac{\omega}{2\pi}$	Frequency of voltage source	?
Z	Impedance of circuit	?
V_R	Potential drop across Resistor	?
V_C	Potential drop across Capacitor	?
V_L	Potential drop across Inductor	?

TABLE 3
Parameter description

1) from Fig. 3,

$$Z = R + j\left(\omega L - \frac{1}{\omega C}\right) \tag{1}$$

$$min\left(|Z|\right) = R \text{ at } \omega = \frac{1}{\sqrt{LC}} \tag{2}$$

$$f_{res} = \frac{\omega_{res}}{2\pi} = \frac{1}{2\pi\sqrt{LC}} = 7.958Hz \tag{3}$$

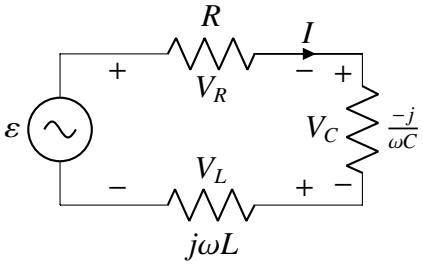


Fig. 3. circuit diagram

2)

$$Z_{res} = R = 40\Omega \tag{4}$$

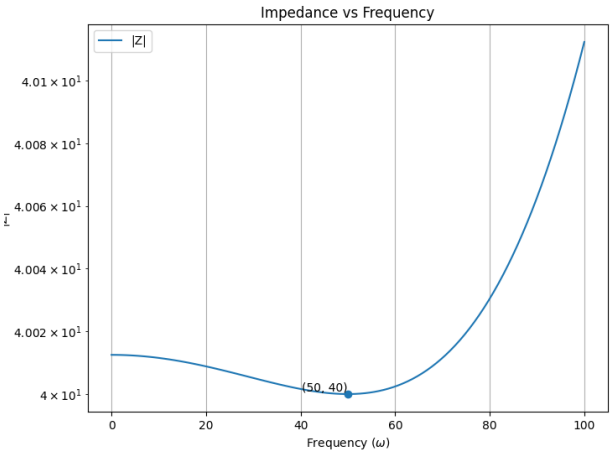


Fig. 2. Impedance vs frequency

3)

$$I_{res} = \frac{\varepsilon}{Z_{res}} \tag{5}$$

$$V_R = Re\{RI_{res}\} \tag{6}$$

$$= Re\left\{R\frac{\varepsilon}{Z_{res}}\right\} \quad \{using\ (5)\} \tag{7}$$

$$= Re\left\{325.28e^{j50t}\right\} \tag{8}$$

$$= 325.28cos(50t) \tag{9}$$

$$V_C = Re\left\{\frac{-j}{\omega_{res}C}I_{res}\right\} \tag{10}$$

$$= Re\left\{\frac{-j}{\omega_{res}C}\frac{\varepsilon}{Z_{res}}\right\} \quad \{using\ (5)\} \tag{11}$$

$$= Re\left\{2032.93e^{j\left(50t-\frac{\pi}{2}\right)}\right\} \tag{12}$$

$$= 2031.93sin(50t) \tag{13}$$

$$V_L = Re\{j\omega_{res}LI_{res}\} \tag{14}$$

$$= Re\left\{j\omega_{res}L\frac{\varepsilon}{Z_{res}}\right\} \quad \{using\ (5)\} \tag{15}$$

$$= Re\left\{2032.93e^{j\left(50t+\frac{\pi}{2}\right)}\right\} \tag{16}$$

$$= -2031.93sin(50t) \tag{17}$$

from (13) and (17), voltage across LC combination is $V_C + V_L = 0V$

Paramater	Description	Value
f_{res}	resonant source frequency	$7.958Hz$
Z_{res}	resonant impedance	40Ω
$rms(V_R)$	rms value of V_R	$230V$
$rms(V_C)$	rms value of V_C	$1437.5V$
$rms(V_L)$	rms value of V_L	$14375V$

TABLE 3
SOLUTION VALUES

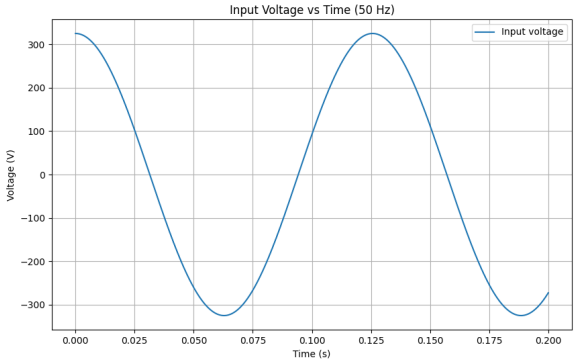


Fig. 3. Input voltage

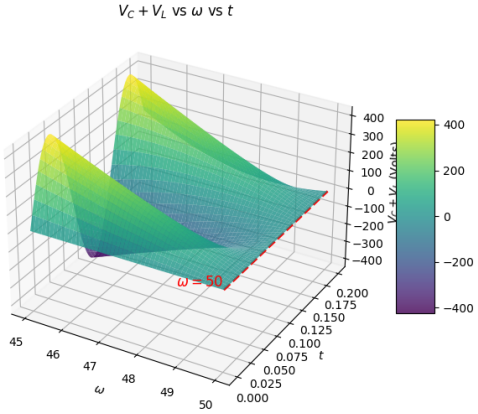


Fig. 3. Voltage across LC combination