

# EE23BTECH11217 - Prajwal M\*

## EXERCISE 9.5

25) Find the sum of the following series up to  $n$  terms and obtain the Z-transform:

$$\dots + 0 + \frac{1^3}{1} + \frac{1^3 + 2^3}{1 + 3} + \frac{1^3 + 2^3 + 3^3}{1 + 3 + 5} + \dots$$

Solution:

$$x(n) = \frac{\sum_{i=0}^n (i+1)^3}{\sum_{j=0}^n (2j+1)} u(n) \quad (1)$$

$$= \frac{(n+2)^2}{4} u(n) \quad (2)$$

$$S(n) = \sum_{r=-\infty}^n x(r) \quad (3)$$

using (2),

$$= \sum_{r=-\infty}^n \frac{(r+2)^2}{4} u(r) \quad (4)$$

$$= \sum_{r=0}^n \frac{r^2 + 4r + 4}{4} \quad (5)$$

$$= 1 + \frac{37n}{24} + \frac{5n^2}{8} + \frac{n^3}{12} \quad (6)$$

$$x(n) \xleftrightarrow{Z} X(z) \quad (7)$$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n} \quad (8)$$

using (2),

$$= \sum_{n=-\infty}^{\infty} \frac{(n+2)^2}{4} u(n) z^{-n} \quad (9)$$

$$= \sum_{n=0}^{\infty} \frac{(n+2)^2}{4} z^{-n} \quad (10)$$

$$= \sum_{n=0}^{\infty} \frac{n^2}{4} z^{-n} + \sum_{n=0}^{\infty} n z^{-n} \quad (11)$$

$$+ \sum_{n=0}^{\infty} z^{-n} = \frac{z(4z^2 - 3z + 1)}{4(z-1)^3} \quad \{z \in \mathbb{C} : |z| > 1\} \quad (12)$$

Symbol	Value	Description
$x(n)$	$\frac{(n+2)^2}{4}$	general term of the series
$S(n)$	$1 + \frac{37n}{24} + \frac{5n^2}{8} + \frac{n^3}{12}$	sum of terms until $x(n)$
$X(z)$	$\frac{z(4z^2 - 3z + 1)}{4(z-1)^3}$	Z-transform of $x(n)$
$u(n)$		unit step function

TABLE 0  
PARAMETERS

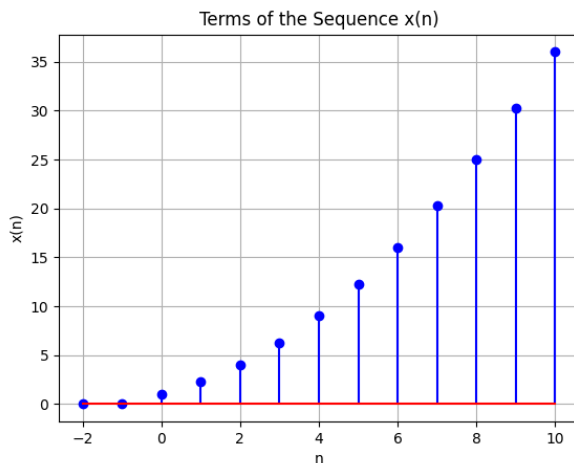


Fig. 0. Plot of  $x(n)$  vs  $n$