EE23BTECH11217 - Prajwal M*

Exercise 9.1

12) Write the first five terms of each of the sequences and obtain the corresponding series

$$a_1 = -1,\ a_n = \frac{a_{n-1}}{n},\ n \geq 2$$

Solution:
Given: $a_1 = -1,\ a_n = \dfrac{a_{n-1}}{n},\ n \geq 2.$

First five terms of the sequence:

$$\begin{aligned}
a_1 &= -1\\
a_2 &= \frac{a_1}{2} = -\frac{1}{2}\\
a_3 &= \frac{a_2}{3} = -\frac{1}{2\cdot 3} = -\frac{1}{6}\\
a_4 &= \frac{a_3}{4} = -\frac{1}{2\cdot 3\cdot 4} = -\frac{1}{24}\\
a_5 &= \frac{a_4}{5} = -\frac{1}{2\cdot 3\cdot 4\cdot 5} = -\frac{1}{120}
\end{aligned}$$

So the first five terms of the series are:

$$-1, -\frac{1}{2}, -\frac{1}{6}, -\frac{1}{24}, -\frac{1}{120}$$

The corresponding series:

$$\begin{aligned}
\sum_{n=1}^{\infty} a_n &= a_1 + a_2 + a_3 + a_4 + a_5 + \dots\\
&= -1 + \left(-\frac{1}{2}\right) + \left(-\frac{1}{6}\right) + \left(-\frac{1}{24}\right) + \dots
\end{aligned}$$

The general term of the series,

$$f(n) = \frac{-1}{n!}$$

The Z-transform of $f(n)$ is given by:

$$\begin{aligned}
F(z) &= \sum_{n=1}^{\infty} f(n)\cdot z^{-n}\\
&= \sum_{n=1}^{\infty} \frac{-1}{n!}\cdot z^{-n}\\
&= -(e^{z^{-1}} - 1)\\
&= 1 - e^{z^{-1}}
\end{aligned}$$

So, the Z-transform of the given series is $1 - e^{z^{-1}}$.