EE23BTECH11217 - Prajwal M*

Exercise 9.1

12 Write the five terms at n = 1, 2, 3, 4, 5 of the sequence and obtain the Z-transform of the series

$$x(n) = -1, \qquad n = 0 \tag{1}$$
$$= \frac{x(n-1)}{n}, \qquad n > 0 \tag{2}$$
$$= 0, \qquad n < 0 \tag{3}$$

Solution:

$$x(1) = \frac{x(0)}{1} = -1 \tag{4}$$
$$x(2) = \frac{x(1)}{2} = -\frac{1}{2} \tag{5}$$
$$x(3) = \frac{x(2)}{3} = -\frac{1}{23} = -\frac{1}{6} \tag{6}$$
$$x(4) = \frac{x(3)}{4} = -\frac{1}{234} = -\frac{1}{24} \tag{7}$$
$$x(5) = \frac{x(4)}{5} = -\frac{1}{2345} = -\frac{1}{120} \tag{8}$$
$$x(n) = \frac{-1}{n!}\left(u(n)\right) \tag{9}$$

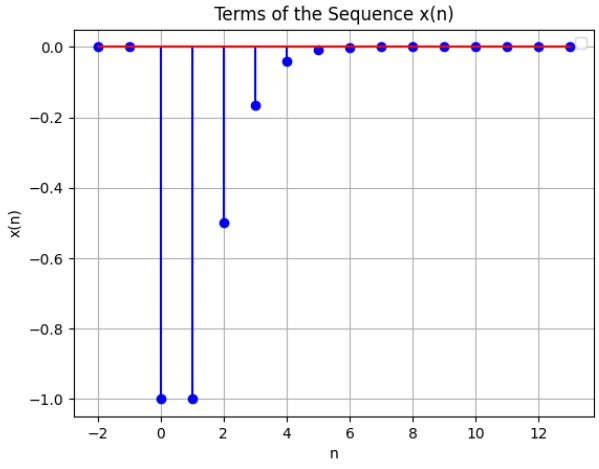

Fig. 1. Plot of x(n) vs n

$$x(n) \overset{\mathcal{Z}}{\longleftrightarrow} X(z) \tag{10}$$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n} \tag{11}$$
$$= \sum_{n=-\infty}^{\infty} \frac{-1}{n!} u(n) z^{-n} \qquad \text{using (9)} \tag{12}$$
$$= \sum_{n=0}^{\infty} \frac{-1}{n!} z^{-n} \tag{13}$$
$$= -e^{z^{-1}} \qquad \{z \in \mathbb{C} : z \neq 0\} \tag{14}$$

So, the Z-transform of the given series is $-e^{z^{-1}}$.

Symbol	Parameters
x(n)	general term of the series
X(z)	Z-transform of x(n)
u(n)	unit step function

TABLE I
Parameters