EE23BTECH11217 - Prajwal M*

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Exercise 9.5

25) **Find the sum of the following series up to n terms and obtain the Z-transform:**

$$\ldots + 0 + \frac{1^3}{1} + \frac{1^3+2^3}{1+3} + \frac{1^3+2^3+3^3}{1+3+5} + \ldots$$

Solution:

Symbol	Description
$x(n)$	general term of the series
$s(n)$	sum of terms until x(n)
$X(z)$	Z-transform of x(n)
$S(z)$	Z-transform of s(n)

Table 0: Parameters

$$x(n) = \frac{\sum_{i=0}^{n}(i+1)^3}{\sum_{i=0}^{n}(2i+1)} u(n) \tag{1}$$

$$= \frac{x_1(n)}{x_2(n)} u(n) \tag{2}$$

$$x_1(n) = \sum_{i=0}^{n}(i+1)^3 \tag{3}$$

$$= (n+1)^3 * u(n) \tag{4}$$

$$x_1(n) \stackrel{\mathcal{Z}}{\longleftrightarrow} X_1(z) \tag{5}$$

$$X_1(z) = \left(\frac{1+4z^{-1}+z^{-2}}{(1-z^{-1})^4}\right)\left(\frac{1}{1-z^{-1}}\right) \tag{6}$$

$$= \frac{1}{(1-z^{-1})^5} + \frac{4z^{-1}}{(1-z^{-1})^5} + \frac{z^{-2}}{(1-z^{-1})^5} \tag{7}$$

$$x_1(n) = \frac{(n+4)(n+3)(n+2)(n+1)}{24} u(n+4) \tag{8}$$

$$+ \frac{(n+3)(n+2)(n+1)n}{6} u(n+3)$$

$$+ \frac{(n+2)(n+1)n(n-1)}{24} u(n+2)$$

$$= \frac{(n+2)^2(n+1)^2}{4} u(n) \tag{9}$$

$$x_2(n) = \sum_{i=0}^{n} (2i+1) \tag{10}$$

$$= (2n+1) * u(n) \tag{11}$$

$$x_2(n) \overset{Z}{\longleftrightarrow} X_2(z) \tag{12}$$

$$X_2(z) = \left(\frac{1+z^{-1}}{(1-z^{-1})^2}\right)\left(\frac{1}{1-z^{-1}}\right) \tag{13}$$

$$= \frac{1}{(1-z^{-1})^3} + \frac{z^{-1}}{(1-z^{-1})^3} \tag{14}$$

$$x_2(n) = \frac{(n+2)(n+1)}{2} u(n+2) \tag{15}$$

$$+ \frac{(n+1)(n)}{2} u(n+1)$$

$$= (n+1)^2 u(n) \tag{16}$$

using (**??**),

$$x(n) = \frac{(n+2)^2}{4} u(n) \tag{17}$$

$$s(n) = \sum_{r=-\infty}^{n} x(r) \tag{18}$$

using (**??**),

$$s(n) = \sum_{r=-\infty}^{n} \frac{(r+2)^2}{4} u(r) \tag{19}$$

$$= \frac{(n+2)^2}{4} * u(n) \tag{20}$$

$$s(n) \overset{Z}{\longleftrightarrow} S(z) \tag{21}$$

$$S(z) = \left(\frac{4-3z^{-1}+z^{-2}}{4(1-z^{-1})^3}\right)\left(\frac{1}{1-z^{-1}}\right) \tag{22}$$

$$= \frac{1}{(1-z^{-1})^4} - \frac{3z^{-1}}{4(1-z^{-1})^4} \tag{23}$$

$$+ \frac{z^{-2}}{4(1-z^{-1})^4} \tag{24}$$

$$s(n) = \frac{(n+3)(n+2)(n+1)}{6} u(n+3) \tag{25}$$

$$- \frac{(n+2)(n+1)(n)}{8} u(n+2)$$

$$+ \frac{(n+1)(n)(n-1)}{24} u(n+1) \tag{26}$$

$$= \left(1 + \frac{37n}{24} + \frac{5n^2}{8} + \frac{n^3}{12}\right) u(n) \tag{27}$$

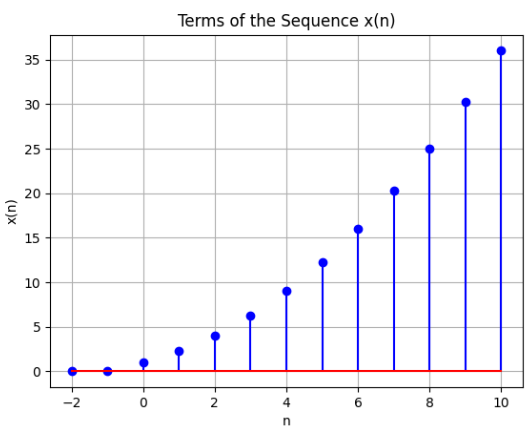

Figure 0: Plot of x(n) vs n