EE23BTECH11217 - Prajwal M*

Exercise 9.1

12) Write the five terms at n = 1, 2, 3, 4, 5 of the sequence and obtain the corresponding series

$$x(n) = \begin{cases} -1 & n = 1 \\ \frac{x(n-1)}{n} & n \geq 2 \\ 0 & n \leq 0 \end{cases}$$

Solution:

$$\begin{aligned} x(1) &= -1 \\ x(2) &= \frac{x(1)}{2} = -\frac{1}{2} \\ x(3) &= \frac{x(2)}{3} = -\frac{1}{23} = -\frac{1}{6} \\ x(4) &= \frac{x(3)}{4} = -\frac{1}{234} = -\frac{1}{24} \\ x(5) &= \frac{x(4)}{5} = -\frac{1}{2345} = -\frac{1}{120} \end{aligned}$$

The corresponding series:

$$\begin{aligned} \sum_{n=-\infty}^{\infty} x(n) &= \ldots + 0 + x(1) + x(2) + x(3) + x(4) + \ldots \\ &= \ldots + 0 - 1 + \left(-\frac{1}{2}\right) + \left(-\frac{1}{6}\right) + \left(-\frac{1}{24}\right) + \ldots \end{aligned}$$

The nth term of the series is,

$$x(n) = \frac{-1}{n!}(u(n))$$

The Z-transform of $x(n)$ is given by:

$$x(n) \stackrel{\mathcal{Z}}{\rightleftharpoons} F(z)$$

$$\begin{aligned} F(z) &= \sum_{n=-\infty}^{\infty} x(n)z^{-n} \\ &= \sum_{n=-\infty}^{\infty} \frac{-1}{n!}u(n)z^{-n} \\ &= \sum_{n=1}^{\infty} \frac{-1}{n!}z^{-n} \\ &= -(e^{z^{-1}} - 1) \\ &= 1 - e^{z^{-1}} \end{aligned}$$

So, the Z-transform of the given series is $1 - e^{z^{-1}}$.

For the series to converge, the ratio test must be satisfied for n > 0

$$\lim_{n\to\infty} \left| \frac{x(n+1)z^{-(n+1)}}{x(n)z^{-n}} \right| < 1$$

$$\lim_{n\to\infty} \left| \frac{-z^{-n-1}n!}{-z^{-n}(n+1)!} \right| < 1$$

$$\lim_{n\to\infty} \left| \frac{z^{-1}}{n+1} \right| < 1$$

The condition is satisfied for $\text{Re}(z) \neq 0$

Hence, ROC of Z transform is $z \in \mathbb{C} : Re(z) \neq 0$.