EE23BTECH11217 - Prajwal M*

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Exercise 9.5

25) **Find the sum of the following series up to n terms and obtain the Z-transform:**

$$\ldots + 0 + \frac{1^3}{1} + \frac{1^3+2^3}{1+3} + \frac{1^3+2^3+3^3}{1+3+5} + \ldots$$

Solution:

Symbol	Description
$s(n)$	sum of n terms
$S(z)$	Z-transform of s(n)
$s_1(n)$	$(n+1)^3 * u(n)$
$S_1(z)$	Z-transform of $s_1(n)$
$s_2(n)$	$(2n+1) * u(n)$
$S_2(z)$	Z-transform of $s_2(n)$

Table 0: Parameters

$$s(n) = \sum_{r=0}^{n} \left(\frac{\sum_{i=0}^{r} (i+1)^3}{\sum_{i=0}^{r} (2i+1)} \right) \tag{1}$$

$$= \frac{(n+1)^3 * u(n)}{(2n+1) * u(n)} * u(n) \tag{2}$$

$$= \frac{s_1(n)}{s_2(n)} * u(n) \tag{3}$$

$$s_1(n) = (n+1)^3 * u(n) \tag{4}$$

$$s_1(n) \overset{\mathcal{Z}}{\longleftrightarrow} S_1(z) \tag{5}$$

$$S_1(z) = \left(\frac{1+4z^{-1}+z^{-2}}{(1-z^{-1})^4} \right) \left(\frac{1}{1-z^{-1}} \right) \quad \{|z| > 1\} \tag{6}$$

$$= \frac{1}{(1-z^{-1})^5} + \frac{4z^{-1}}{(1-z^{-1})^5} + \frac{z^{-2}}{(1-z^{-1})^5} \tag{7}$$

using (**??**) and (**??**)

$$s_1(n) = \frac{(n+4)(n+3)(n+2)(n+1)}{24} u(n+4) + \frac{(n+3)(n+2)(n+1)n}{6} u(n+3) + \frac{(n+2)(n+1)n(n-1)}{24} u(n+2) \tag{8}$$

$$= \frac{(n+2)^2(n+1)^2}{4} \quad \{n \geq 0\} \tag{9}$$

$$s_2(n) = (2n+1) * u(n) \tag{10}$$

$$s_2(n) \overset{\mathcal{Z}}{\longleftrightarrow} S_2(z) \tag{11}$$

$$S_2(z) = \left(\frac{1+z^{-1}}{(1-z^{-1})^2} \right) \left(\frac{1}{1-z^{-1}} \right) \quad \{|z| > 1\} \tag{12}$$

$$= \frac{1}{(1-z^{-1})^3} + \frac{z^{-1}}{(1-z^{-1})^3} \tag{13}$$

using (**??**) and (**??**)

$$s_2(n) = \frac{(n+2)(n+1)}{2} u(n+2) + \frac{(n+1)(n)}{2} u(n+1) \tag{14}$$

$$= (n+1)^2 \qquad \{n \geq 0\} \tag{15}$$

replacing (9) and (15) in (3)

$$s(n) = \frac{(n+2)^2}{4} * u(n) \tag{16}$$

$$s(n) \overset{\mathcal{Z}}{\longleftrightarrow} S(z) \tag{17}$$

$$S(z) = \left(\frac{4 - 3z^{-1} + z^{-2}}{4(1-z^{-1})^3}\right)\left(\frac{1}{1-z^{-1}}\right) \qquad \{|z| > 1\} \tag{18}$$

$$= \frac{1}{(1-z^{-1})^4} - \frac{3z^{-1}}{4(1-z^{-1})^4} + \frac{z^{-2}}{4(1-z^{-1})^4} \tag{19}$$

$$s(n) = \frac{(n+3)(n+2)(n+1)}{6} u(n+3) - \frac{(n+2)(n+1)(n)}{8} u(n+2) + \frac{(n+1)(n)(n-1)}{24} u(n+1) \tag{20}$$

$$= \left(1 + \frac{37n}{24} + \frac{5n^2}{8} + \frac{n^3}{12}\right) u(n) \tag{21}$$

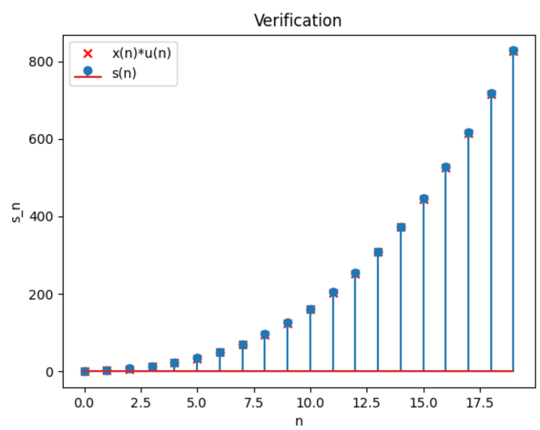

Figure 0: Plot of x(n) vs n