

EE23BTECH11217 - Prajwal M*

EXERCISE 9.1

12 Write the five terms at $n = 1, 2, 3, 4, 5$ of the sequence and obtain the corresponding series

$$x(n) = \begin{cases} -1 & n = 1 \\ \frac{x(n-1)}{n} & n \geq 2 \\ 0 & n \leq 0 \end{cases}$$

Solution:

$$x(1) = -1$$

$$x(2) = \frac{x(1)}{2} = -\frac{1}{2}$$

$$x(3) = \frac{x(2)}{3} = -\frac{1}{23} = -\frac{1}{6}$$

$$x(4) = \frac{x(3)}{4} = -\frac{1}{234} = -\frac{1}{24}$$

$$x(5) = \frac{x(4)}{5} = -\frac{1}{2345} = -\frac{1}{120}$$

The corresponding series:

$$\begin{aligned} \sum_{n=-\infty}^{\infty} x(n) &= \dots + 0 + x(1) + x(2) + x(3) + \dots \\ &= \dots + 0 - 1 + \left(-\frac{1}{2}\right) + \left(-\frac{1}{6}\right) + \dots \end{aligned}$$

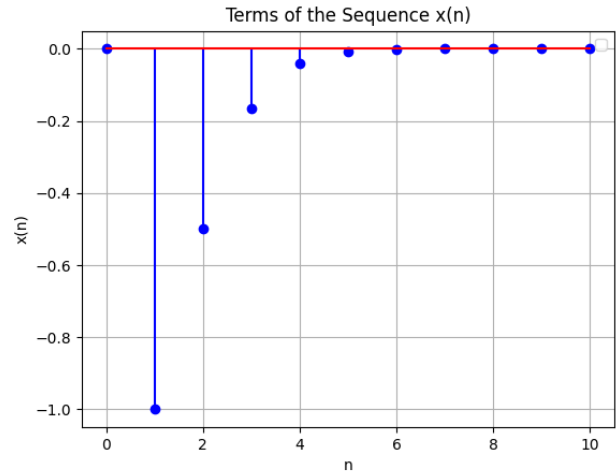
The n th term of the series is,

$$x(n) = \frac{-1}{n!} (u(n))$$

The Z-transform of $x(n)$ is given by:

$$x(n) \stackrel{\mathcal{Z}}{\rightleftharpoons} F(z)$$

$$\begin{aligned} F(z) &= \sum_{n=-\infty}^{\infty} x(n) z^{-n} \\ &= \sum_{n=-\infty}^{\infty} \frac{-1}{n!} u(n) z^{-n} \\ &= \sum_{n=1}^{\infty} \frac{-1}{n!} z^{-n} \\ &= -(e^{z^{-1}} - 1) \\ &= 1 - e^{z^{-1}} \end{aligned}$$



So, the Z-transform of the given series is $1 - e^{z^{-1}}$.

For the series to converge, the ratio test must be satisfied for $n > 0$

$$\begin{aligned} \lim_{n \rightarrow \infty} \left| \frac{x(n+1) z^{-(n+1)}}{x(n) z^{-n}} \right| &< 1 \\ \lim_{n \rightarrow \infty} \left| \frac{-z^{-n-1} n!}{-z^{-n} (n+1)!} \right| &< 1 \\ \lim_{n \rightarrow \infty} \left| \frac{z^{-1}}{n+1} \right| &< 1 \end{aligned}$$

The condition is satisfied for $z \neq 0$

Hence, ROC of Z transform is

$$z \in \mathbb{C} : z \neq 0$$