EE23BTECH11217 - Prajwal M*

Exercise 9.1

12 Write the five terms at n = 1, 2, 3, 4, 5 of the sequence and obtain the corresponding series

$$x(n) = \begin{cases} -1 & n = 1 \\ \frac{x(n-1)}{n} & n \geq 2 \\ 0 & n \leq 0 \end{cases}$$

Solution:

$$x(1) = -1 \tag{1}$$
$$x(2) = \frac{x(1)}{2} = -\frac{1}{2} \tag{2}$$
$$x(3) = \frac{x(2)}{3} = -\frac{1}{23} = -\frac{1}{6} \tag{3}$$
$$x(4) = \frac{x(3)}{4} = -\frac{1}{234} = -\frac{1}{24} \tag{4}$$
$$x(5) = \frac{x(4)}{5} = -\frac{1}{2345} = -\frac{1}{120} \tag{5}$$

The corresponding series:

$$\sum_{n=-\infty}^{\infty} x(n) = \ldots + 0 + x(1) + x(2) + x(3) + \ldots \tag{6}$$
$$= \ldots + 0 - 1 + \left(-\frac{1}{2}\right) + \left(-\frac{1}{6}\right) + \ldots \tag{7}$$

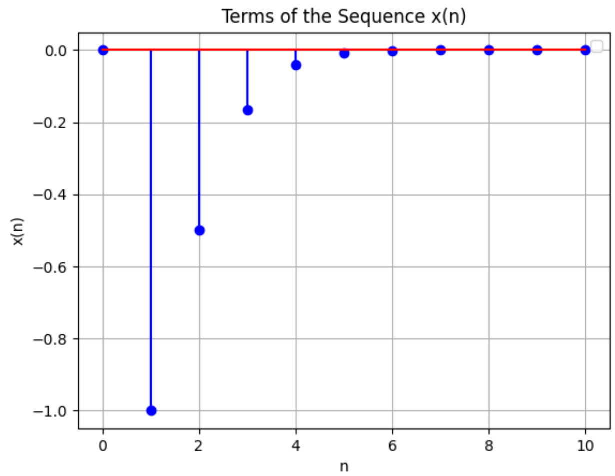
The nth term of the series is,

$$x(n) = \frac{-1}{n!}\left(u(n-1)\right) \tag{8}$$

The Z-transform of $x(n)$ is given by:

$$x(n) \overset{\mathcal{Z}}{\rightleftharpoons} F(z) \tag{9}$$

$$F(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n} \tag{10}$$
$$= \sum_{n=-\infty}^{\infty} \frac{-1}{n!} u(n-1) z^{-n} \tag{11}$$
$$= \sum_{n=1}^{\infty} \frac{-1}{n!} z^{-n} \tag{12}$$
$$= -\left(e^{z^{-1}} - 1\right) \tag{13}$$
$$= 1 - e^{z^{-1}} \tag{14}$$



So, the Z-transform of the given series is $1 - e^{z^{-1}}$.

For the series to converge, the ratio test must be satisfied for n > 0

$$\lim_{n\to\infty} \left| \frac{x(n+1) z^{-(n+1)}}{x(n) z^{-n}} \right| < 1 \tag{15}$$
$$\lim_{n\to\infty} \left| \frac{-z^{-n-1} n!}{-z^{-n}(n+1)!} \right| < 1 \tag{16}$$
$$\lim_{n\to\infty} \left| \frac{z^{-1}}{n+1} \right| < 1 \tag{17}$$

The condition is satisfied for $z \neq 0$

Hence, ROC of Z transform is

$$z \in \mathbb{C} : z \neq 0. \tag{18}$$