11. Limits, Continuity and Differentiability

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Section-B JEE Main/AIEEE

1) Let f be differentiable for all x. If $f(1) = -2$ and $f'(x) \geq 2$ for $x \in [1, 6]$, then [2005]
   a) $f(6) \geq 8$
   b) $f(6) < 8$
   c) $f(6) < 5$
   d) $f(6) = 5$

2) If f is a real valued differentiable function satisfying $|f(x) - f(y)| \leq (x - y)^2$, $x, y \in R$ and $f(0) = 0$, then $f(1)$ equals [2005]

   a) -1 b) 0 c) 2 d) 1

3) Let $f : R \to R$ be a function defined by
   $f(x)$=min $\{x + 1, |x|+1\}$, Then which of the following is true? [2007]
   a) $f(x)$ is differentiable everywhere.
   b) $f(x)$ is not differentiable at $x = 0$.
   c) $f(x) \geq 1$ for all $x \in R$.
   d) $f(x)$ is not differentiable at $x = 1$.

4) The function $f : R - \{0\} \to R$ given by

$$f(x) = \frac{1}{x} - \frac{2}{e^{2x} - 1}$$

   can be made continuous at $x = 0$ by defining $f(0)$ as [2007]

   a) 0 b) 1 c) 2 d) -1

5) Let

$$f(x) = \begin{cases} (x-1)\sin\frac{1}{x-1} & \text{if } x \neq 1 \\ 0 & \text{if } x = 1 \end{cases}$$

   Then which of the following is true? [2008]
   a) f is neither differentiable at $x = 0$ nor at $x = 1$.
   b) f is differentiable at $x = 0$ and $x = 1$.
   c) f is differentiable at $x = 0$ but not at $x = 1$.
   d) f is differentiable at $x = 1$ but not at $x = 0$.

6) Let $f : R \to R$ be a positive increasing function with $\lim_{x\to\infty} \frac{f(3x)}{f(x)} = 1$. Then $\lim_{x\to\infty} \frac{f(2x)}{f(x)} =$ [2010]

   a) $\frac{2}{3}$ b) $\frac{3}{2}$ c) 3 d) 1

7)

$$\lim_{x\to 2}\left(\frac{\sqrt{1 - \cos(2(x-2))}}{x-2}\right)$$

[2011]

a) equals $\sqrt{2}$
b) equals $-\sqrt{2}$
c) equals $\frac{1}{\sqrt{2}}$
d) does not exist

8) The values of p and q for which the function

$$f(x)=\begin{cases}\frac{\sin(p+1)x+\sin x}{x} & \text{if } x<0\\ q & \text{if } x=0\\ \frac{\sqrt{x+x^2}-\sqrt{x}}{x^{3/2}} & \text{if } x>0\end{cases}$$

is continuous for all x in R,are [2011]

a) $p=\frac{5}{2}, q=\frac{1}{2}$
b) $p=-\frac{3}{2}, q=\frac{1}{2}$
c) $p=\frac{1}{2}, q=\frac{3}{2}$
d) $p=\frac{1}{2}, q=-\frac{3}{2}$

9) Let $f: R \to [0,\infty)$ be such that $\lim_{x\to 5} f(x)$ exists and

$$\lim_{x\to 5}\frac{(f(x))^2-9}{\sqrt{|x-5|}}=0.$$

Then $\lim_{x\to 5} f(x)$ equals:

a) 0 b) 1 c) 2 d) 3

10) If $f: R \to R$ is a function defined by $f(x) = [x]\cos(\frac{2x-1}{2})\pi$, where $[x]$ denotes greatest integer function,then f is [2012]

a) continuous for every real x.
b) discontinuous only at $x = 0$.
c) discontinuous only at non-zero integral values of x.
d) continuous only at $x = 0$.

11) Consider the function $f(x) = |x-2|+|x-5|, x \in R$.
Statement-1: $f'(4) = 0$
Statement-2: f is continuous in [2,5], differentiable in (2,5) and $f(2) = f(5)$. [2012]

a) Statement-1 is false, Statement-2 is true.
b) Statement-1 is true,Statement-2 is true; Statement-2 is correct explanation for Statement-1.
c) Statement-1 is true, Statement-2 is true;Statement-2 is **not** a correct explanation for Statement-1.
d) Statement-1 is true, Statement-2 is false.

12)

$$\lim_{x\to 0}\frac{(1-\cos 2x)(3+\cos x)}{x\tan 4x}$$

is equal to: [JEE M 2013]

a) -$\frac{1}{4}$ b) $\frac{1}{2}$ c) 1 d) 2

13)

$$\lim_{x\to 0}\frac{\sin\left(\pi\cos^2 x\right)}{x^2}$$

is equal to: [JEE M 2014]

a) $-\pi$ b) π c) $\frac{\pi}{2}$ d) 1

14)

$$\lim_{x\to 0}\frac{(1-\cos 2x)(3+\cos x)}{x\tan 4x}$$

is equal to: [JEE M 2015]

a) 2 b) $\frac{1}{2}$ c) 4 d) 3

15) If the function

$$g(x)=\begin{cases}x & ,0\le x\le 3\\ mx+2 & ,3<x\le 5\end{cases}$$

is differentiable, then the value of k+m is : [JEE M 2015]

a) $\frac{10}{3}$ b) 4 c) 2 d) $\frac{16}{5}$