9.2.6

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Question:
Area of the region in the first quadrant enclosed by the x-axis, the line $y = x$ and the circle $x^2 + y^2 = 32$ is ______.

Solution:

Variable	Description
P, Q	Points of intersection of line and circle
r	radius of the circle
θ	Angle between line $y = x$ and x-axis
O	Centre of the circle(Origin)
A	Area of the portion

TABLE 0
VARIABLES USED

Given line equation is

$$y = x \tag{1}$$

and circle equation is

$$x^2 + y^2 = 32. \tag{2}$$

By substituting 1 in 2, we get

$$\begin{aligned} \implies x^2 + x^2 &= 32 \\ \implies 2x^2 &= 32 \\ \implies x^2 &= 16 \\ \implies x &= 4, -4 \end{aligned}$$

After substituting the values of x in 1, we get the points of intersection as $P = \left(4, 4\right)$ and $Q = \left(4, 4\right)$.

Hence the area enclosed by the x-axis, the line $y = x$ and the circle $x^2 + y^2 = 32$ is in the shape of a sector.
Area enclosed by the sector is given by

$$A = \frac{1}{2} r^2 \theta \tag{3}$$

to find r, the distance between centre of circle and the point P should be computed.

$$\begin{aligned} \|P\| &= \sqrt{x^2 + y^2} \\ &= \sqrt{4^2 + 4^2} \\ &= \sqrt{32} \\ &= 4\sqrt{2}. \end{aligned}$$

To find the angle θ, slope of the line is required, since the angle is between the given line and $x-$axis. The line $y = x$ passes through the points P and Q. Equation of line can be expressed as:

$$r = (1-t)\left(4,4\right) + t\left(4,4\right), t \in [0,1]$$

Direction vector of r is

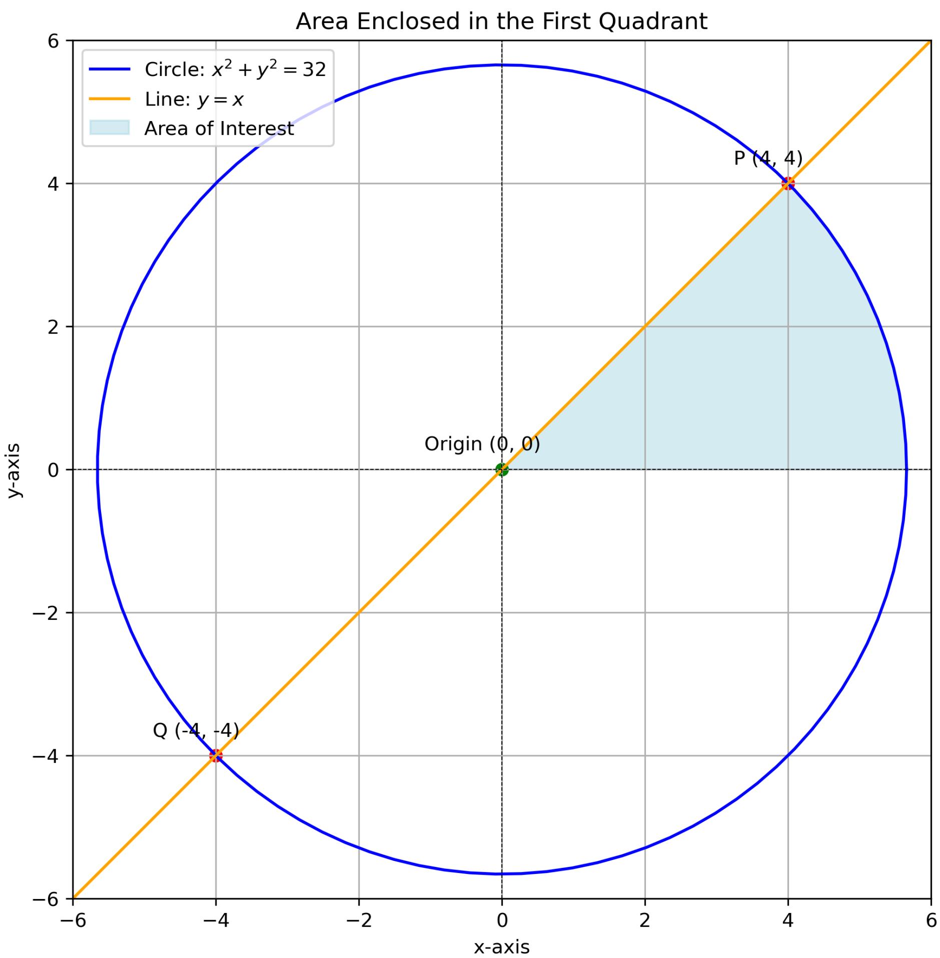
$$\begin{aligned} d &= \left(4-(-4), 4-(-4)\right) \\ &= \left(8,8\right) \\ &= \left(\Delta y, \Delta x\right) \end{aligned}$$

slope of the line=$\tan\theta = \frac{\Delta y}{\Delta x} = \frac{8}{8} = 1$.

$$\implies \theta = \tan^{-1}(1) = \frac{\pi}{4}.$$

Substituting the values in 3, we get

$$A = 4\pi.$$


Area Enclosed in the First Quadrant
Circle: $x^2 + y^2 = 32$
Line: $y = x$
Area of Interest
P (4, 4)
Origin (0, 0)
Q (-4, -4)
x-axis
y-axis
−6
−4
−2
0
2
4
6