Solving differential equation

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Question:

$$\frac{dy}{dx} + \frac{y}{x} = x^2 \tag{1}$$

Theoretical Solution:

$$\frac{dy}{dx} + \frac{y}{x} = x^2 \tag{2}$$

$$\text{Integrating Factor: } e^{\int \frac{1}{x}\,dx} = e^{\ln x} = x \tag{3}$$

$$y \cdot x = \int x^2 \cdot x\,dx \tag{4}$$

$$y \cdot x = \frac{x^4}{4} + c \tag{5}$$

$$\text{Therefore:} \tag{6}$$

$$y = \frac{x^3}{4} + \frac{c}{x}. \tag{7}$$

Taking C as 1 we get a solution i.e.,

$$y = \frac{x^3}{4} + \frac{1}{x}. \tag{8}$$

Method of finite differences:

$$\frac{dy}{dx} = x^2 - \frac{y}{x} \tag{9}$$

$$\text{where } \lim_{h\to 0} \frac{y(x+h) - y(x)}{h} = x^2 - \frac{y}{x} \tag{10}$$

$$\tag{11}$$

Approximaing for small h:

$$y_{n+1} - y_n = h \cdot \left(x_n^2 - \frac{y_n}{x_n}\right) \tag{12}$$

$$\tag{13}$$

Therefore:

$$y_{n+1} = y_n + h \cdot \left(x_n^2 - \frac{y_n}{x_n}\right) \tag{14}$$

$$x_{n+1} = x_n + h \tag{15}$$