Solving differential equation NCERT-9.5.4

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simuluation method
Given:

$$\frac{dy}{dx} + \sec(x) \cdot y = \tan(x) \tag{1}$$

Rewriting:

$$\frac{dy}{dx} = \tan(x) - y \cdot \sec(x) \tag{2}$$

Using the definition of derivative:

$$\lim_{h \to 0} \frac{y(x+h) - y(x)}{h} = \tan(x) - y \cdot \sec(x) \tag{3}$$

Approximating for small h:

$$\frac{y_{n+1} - y_n}{h} \approx \tan(x) - y \cdot \sec(x) \tag{4}$$

Reorganizing:

$$y_{n+1} - y_n = h \cdot \frac{dy}{dx} \tag{5}$$

Therefore:

$$y_{n+1} = y_n + h \cdot \frac{dy}{dx} \tag{6}$$

$$y_{n+1} = y_n + h \cdot (\tan(x_n) - y_n \cdot \sec(x_n)) \tag{7}$$

By initializing the values of x and y and iterating the process several times and plotting them gives the curve for solution of the differential equation

Theoretical solution:
Consider the differential equation:

$$\frac{dy}{dx} + \sec(x)y = \tan(x) \tag{8}$$

where $0 \leq x < \frac{\pi}{2}$.
This is a first-order linear differential equation of the form:

$$\frac{dy}{dx} + P(x)y = Q(x) \tag{9}$$

where $P(x) = \sec(x)$ and $Q(x) = \tan(x)$.
The integrating factor (IF) is given by:

$$I(x) = e^{\int P(x)\,dx} = e^{\int \sec(x),dx}. \tag{10}$$

The integral of $\sec(x)$ is:

$$\int \sec(x)\,dx = \ln\left|\sec(x) + \tan(x)\right|. \tag{11}$$

Thus, the integrating factor becomes:

$$I(x) = e^{\ln|\sec(x)+\tan(x)|} = \left|\sec(x) + \tan(x)\right|. \tag{12}$$

Applying the integrating factor to the differential equation:

$$\frac{d}{dx}\left(y \cdot |\sec(x) + \tan(x)|\right) = \tan(x)\left|\sec(x) + \tan(x)\right|. \tag{13}$$

Integrate both sides:

$$y \cdot |\sec(x) + \tan(x)| = \int \tan(x)\left|\sec(x) + \tan(x)\right|\,dx. \tag{14}$$

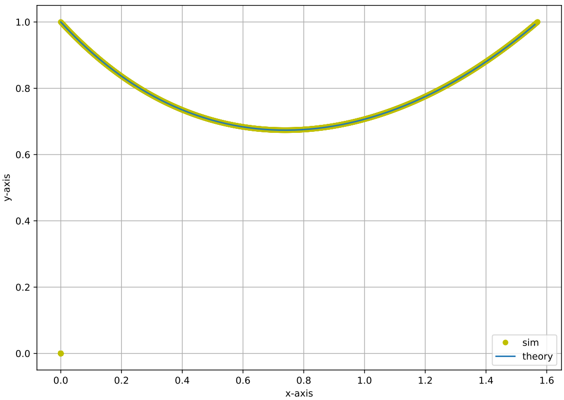
Observe that the derivative of $\sec(x) + \tan(x)$ is $\sec(x)\tan(x) + \sec^2(x)$, so the integral simplifies to:

$$y \cdot |\sec(x) + \tan(x)| = |\sec(x) + \tan(x)| + C. \tag{15}$$

Therefore, the general solution is:

$$y = 1 + \frac{C}{|\sec(x) + \tan(x)|} \tag{16}$$

where C is the constant of integration.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