2020-ST

EE24BTECH11056 - S.Kavya Anvitha

1) Consider the linear transformation $T : \mathbb{C}^3 \text{to} \mathbb{C}^3$ defined by

$$T\left((x, y, z)\right) = \left(x, \frac{\sqrt{3}}{2}y - \frac{1}{2}z, \frac{1}{2}y + \frac{\sqrt{3}}{2}z\right),$$

where $\mathbb{C}$ is the set of all complex numbers and $\mathbb{C}^3 = \mathbb{C} \times \mathbb{C} \times \mathbb{C}$. Which of the following statements is TRUE?

2020-ST

a) There exists a non-zero vector X such that $T(X) = -X$
b) There exist a non-zero vector Y and a real number $\lambda \neq 1$ such that $T(Y) = \lambda Y$
c) T is diagonalizable
d) $T^2 = I_3$, where I_3 is the 3×3 identity matrix

2) For real numbers a, b and c, let

$$M = \begin{bmatrix} a & ac & 0 \\ b & bc & 1 \\ 1 & c & 0 \end{bmatrix}.$$

Then, which of the following statements is TRUE? 2020-ST

a) Rank$(M) = 3$ for every $a, b, c \in \mathbb{R}$
b) If $a + c = 0$ then M is diagonalizable for every $b \in \mathbb{R}$
c) M has a pair of orthogonal eigenvectors for every $a, b, c \in \mathbb{R}$
d) If $b = 0$ and $a + c = 1$ then M is NOT idempotent

3) Let M be a 4×4 matrix with $(x-1)^2(x-3)^2$ as its minimal polynomial. Then, which of the following statements is FALSE? 2020-ST

a) The eigenvalues of M are 1 and 3
b) The algebraic multiplicity of the eigenvalue 1 is 3
c) M is NOT diagonalizable
d) Trace$(M) = 8$

4) Let $f : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ be defined by

$$f(x, y) = |y - 2| \sqrt{|x - 1|}, \left(x, y\right) \in \mathbb{R} \times \mathbb{R},$$

where $\mathbb{R}$ denotes the set of all real numbers. Then which of the following statements is TRUE? 2020-ST

a) f is differentiable at $\left(1, 2\right)$
b) f is continuous at $\left(1, 2\right)$ but NOT differentiable at $\left(1, 2\right)$
c) The partial derivative of f, with respect to x, at $\left(1, 2\right)$ does NOT exist
d) The directional derivative of f at $\left(1, 2\right)$ along $\mathbf{u} = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ equals 1

5) Which of the following functions is uniformly continuous on the specified domain? 2020-ST

a) $f_1(x) = e^{x^2}, -\infty < x < \infty$
b) $f_2(x) = \begin{cases} \frac{1}{x}, & 0 < x \leq 1 \\ 0, & x = 0 \end{cases}$
c) $f_3(x) = \frac{x^2}{1+x^2}, |x| \leq 1$
d) $f_4(x) = \begin{cases} x, & |x| \leq 1 \\ \frac{x}{2}, & |x| > 1 \end{cases}$

6) Let the random vector $X = (X_1, X_2, X_3)$ have the joint probability density function

$$f_X(x_1, x_2, x_3) = \begin{cases} \frac{1-\sin x_1 \sin x_2 \sin x_3}{8\pi^3}, & 0 \leq x_1, x_2, x_3 \leq 2\pi, \\ 0, & \text{otherwise.} \end{cases}$$

Which of the following statements is TRUE? 2020-ST

a) X_1, X_2 and X_3 are mutually independent
b) X_1, X_2 and X_3 are pairwise independent
c) (X_1, X_2) and X_3 are independently distributed
d) Variance of $X_1 + X_2$ is π^2

7) Suppose that P_1 and P_2 are two populations having bivariate normal distributions with mean vectors $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$, respectively, and the same variance-covariance matrix $\begin{pmatrix} 1 & 0.5 \\ 0.5 & 1 \end{pmatrix}$. Let $Z_1 = (0.75, 0.75)$ and $Z_2 = (0.25, 0.25)$ be two new observations. If the prior probabilities for P_1 and P_2 are assumed to be equal and the misclassification costs are also assumed to be equal then, according to the linear discriminant rule, 2020-ST

a) Z_1 is assigned to P_1 and Z_2 is assigned to P_2
b) Z_1 is assigned to P_2 and Z_2 is assigned to P_1
c) Both Z_1 and Z_2 are assigned to P_1
d) Both Z_1 and Z_2 are assigned to P_2

8) Let $X_1, \ldots, X_n$ be a random sample of size $n(\geq 2)$ from an exponential distribution with the probability density function

$$f(x; \theta) = \begin{cases} \frac{1}{\theta} e^{-(x-\theta)/\theta}, & x > \theta, \\ 0, & \text{otherwise,} \end{cases}$$

where $\theta \in (0, \infty)$. Which of the following statements is TRUE? 2020-ST

a) $\min_{1 \leq i \leq n} X_i$ is a minimal sufficient statistic
b) $\sum_{i=1}^{n} X_i$ is a minimal sufficient statistic
c) Any minimal sufficient statistic is complete
d) $\left(\min_{1 \leq i \leq n} X_i, \sum_{i=1}^{n} X_i\right)$ is minimal sufficient statistic

9) Let the joint distribution of (X, Y) be bivariate normal with mean vector $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ and

variance-covariance matrix $\begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}$, where $-1 < \rho < 1$. Then $E\left[\max(X, Y)\right]$ equals 2020-ST

a) $\frac{\sqrt{1-\rho}}{\pi}$
b) $\sqrt{\frac{1-\rho}{\pi}}$
c) 0
d) $\frac{1}{2}$

10) Let $X_1, X_2, \ldots, X_{10}$ be independent and identically distributed $N_3(0, I_3)$ random vectors, where I_3 is the 3×3 identity matrix. Let

$$T = \sum_{i=1}^{10} \left(X_i^t \left(I_3 - \frac{1}{3} J_3 \right) X_i \right),$$

where J_3 is the 3×3 matrix with each entry 1 and for any column vector U, U^t denotes its transpose. Then the distribution of T is 2020-ST

a) central chi-square with 5 degrees of freedom
b) central chi-square with 10 degrees of freedom
c) central chi-square with 20 degrees of freedom
d) central chi-square with 30 degrees of freedom

11) Let X_1, X_2 and X_3 be independent and identically distributed $N_4(0, \Sigma)$ random vectors, where Σ is a positive definite matrix. Further, let

$$X = \begin{pmatrix} X_1^t \\ X_2^t \\ X_3^t \end{pmatrix}$$

be a 3×4 matrix, where for any matrix M, M^t denotes its transpose. If $W_m(n, \Sigma)$ denotes a Wishart distribution of order m with n degrees of freedom and variance-covariance matrix Σ, then which of the following statements is TRUE? 2020-ST

a) $\Sigma^{-1/2} X^t X \Sigma^{-1/2}$ follows $W_4(3, I_4)$ distribution
b) $\Sigma^{-1/2} X^t X \Sigma^{-1/2}$ follows $W_3(4, I_3)$ distribution
c) Trace$(X \Sigma^{-1} X^t)$ follows χ^2_3 distribution
d) $X^t X$ follows $W_3(4, \Sigma)$ distribution

12) Let the joint distribution of the random variables X_1, X_2 and X_3 be $N_3(\mu, \Sigma)$, where

$$\mu = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \quad \text{and} \quad \Sigma = \begin{pmatrix} 1 & 0.5 & 0 \\ 0.5 & 1 & 0 \\ 0 & 0 & 5 \end{pmatrix}.$$

Then which of the following statements is TRUE? 2020-ST

a) $X_1 - X_2 + X_3$ and X_1 are independent
b) $X_1 + X_2$ and $X_3 - X_1$ are independent
c) $X_1 + X_2 + X_3$ and $X_1 + X_2$ are independent
d) $X_1 - 2X_2$ and $2X_1 + X_2$ are independent

13) Consider the following one-way fixed effects analysis of variance model

$$Y_{ij} = \mu + \tau_i + \epsilon_{ij}, \quad i = 1, 2, 3;\ j = 1, 2, 3, 4;$$

where ϵ_{ij} are independent and identically distributed $N(0, \sigma^2)$ random variables, $\sigma \in (0, \infty)$ and $\tau_1 + \tau_2 + \tau_3 = 0$. Let MST and MSE denote the mean sum of squares due to treatment and the mean sum of squares due to error, respectively. For testing $H_0 : \tau_1 = \tau_2 = \tau_3 = 0$ against $H_1 : \tau_i \neq 0$, for some $i = 1, 2, 3$, consider the test based on the statistic $\frac{MST}{MSE}$. For positive integers ν_1 and ν_2, let F_{ν_1,ν_2} be a random variable having the central F-distribution with ν_1 and ν_2 degrees of freedom. If the observed value of $\frac{MST}{MSE}$ is given to be 104.45, then the p-value of this test equals 2020-ST

a) $P(F_{2,9} > 104.45)$
b) $P(F_{9,2} < 104.45)$
c) $P(F_{3,11} < 104.45)$
d) $P(F_{2,6} > 104.45)$