

Finding maximum value

NCERT-12.6.5.24

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Question:

Our aim is to find the maximum value of the function:

$$f(x) = [x(x-1) + 1]^{1/3} \quad (1)$$

in the interval $0 \leq x \leq 1$ using the gradient ascent method
Simplify the expression

$$f(x) = [x^2 - x + 1]^{1/3} \quad (2)$$

Letting , the derivative of using the chain rule is:

$$f'(x) = \frac{d}{dx} (g(x)^{1/3}) = \frac{1}{3} g(x)^{-2/3} \cdot g'(x), \text{ where: } g'(x) = 2x - 1 \quad (3)$$

The gradient ascent update rule is:

$$x_{n+1} = x_n + \eta \cdot f'(x_n) \quad (4)$$

where:

- 1) x_n is the current estimate.
- 2) η is the learning rate.
- 3) $f'(x)$ is the derivative calculated above.

Implementing gradient ascent:

- 1) We need to Choose a small learning rate η (say, 0.01).
- 2) Choose a starting point x_0 (say 0.0)

Compute the gradient $f'(x_n)$

Update the current point using the formula

$$x_{n+1} = x_n + \eta \cdot f'(x_n) \quad (5)$$

$$x_{n+1} = x_n + \eta \cdot \frac{1}{3} ([x^2 - x + 1]^{-\frac{2}{3}}) \cdot (2x - 1) \quad (6)$$

Behavior in each region:

for $x > 0.5$: $g'(x) = 2x - 1 > 0$

for $x < 0.5$: $g'(x) = 2x - 1 < 0$

Stop when the gradient is close to zero(e.g., $|f'(x_n)| < 10^{-6}$)

Stop if the next step takes x out of the interval $[0,1]$.

Since the interval is restricted to $0 \leq x \leq 1$:

- 1) Compute the function value at the boundaries $f(0)$ and $f(1)$.
- 2) Compare with the value obtained using gradient ascent.

Boundary values:

$$f(0) = [0(0 - 1) + 1]^{\frac{1}{3}} = 1$$

$$f(1) = [1(1 - 1) + 1]^{\frac{1}{3}} = 1$$