16. Applications of derivatives

EE24BTECH11065 - spoorthi

Section-B JEE Main/AIEEE

1) A spherical iron ball $10cm$ in radius is coated with a layer of ice of uniform thickness that melts at a rate of 50 cm^3 /min. When the thickness of ice is $5cm$,then the rate at which the thickness of ice decreases is [2005]

   a) $\frac{1}{36\pi}$ $cm/min.$ b) $\frac{1}{18\pi}$ $cm/min.$ c) $\frac{1}{54\pi}$ $cm/min.$ d) $\frac{5}{6\pi}$ $cm/min.$

2) If the equation $a_n x^n + a_{n-1}x^{n-1}$ + ... + $a_1 x = 0$, $a_1 \neq 0$,$n \geq 2$, has a positive root $x = \alpha$, then the equation $na_n x^{n-1}$ + $(n_1)a_{n-1}x^{n-2}$ +....... + a_1 =0 has a positive root, which is [2005]
   a) greater than α
   b) smaller than α
   c) greater than or equal to α
   d) equal to α

3) The function $f(x) = \frac{x}{2} + \frac{2}{x}$ has a local minimum at [2006]

   a) $x = 2$ b) $x =$ -2 c) $x = 0$ d) $x = 1$

4) A triangular park is enclosed on two sides by a fence and on third side by a straight river bank. The two sides having fence are of same length x. The maximum area enclosed by the park is [2006]

   a) $\frac{3}{2}x^2$ b) $\sqrt{\frac{x^3}{8}}$ c) $\frac{1}{2}x^2$ d) πx^2

5) A value of c for which conclusion of Mean Value Theorem holds for the function $f(x) = \log_e x$ on the interval $[1, 3]$ is [2007]

   a) $\log_3 e$ b) $\log_e 3$ c) $2\log_3 e$ d) $\frac{1}{2}\log_3 e$

6) The function $f(x) = \tan^{-1}(\sin x + \cos x)$is an increasing function in [2007]

   a) $\left(0, \frac{\pi}{2}\right)$ b) $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ c) $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ d) $\left(\frac{-\pi}{2}, \frac{\pi}{4}\right)$

7) If p and q are positive real numbers such that $p^2 + q^2 = 1$, then the maximum value of $(p + q)$ is [2007]

   a) $\frac{1}{2}$ b) $\frac{1}{\sqrt{2}}$ c) $\sqrt{2}$ d) 2

8) Suppose the cubic x^3 - $px + q$ has three distinct real roots where $p > 0$ and $q > 0$. Then which one of the following holds ? [2008]
   a) The cubic has minima at $\sqrt{\frac{p}{3}}$ and maxima at -$\sqrt{\frac{p}{3}}$
   b) The cubic has minima at -$\sqrt{\frac{p}{3}}$ and maxima at $\sqrt{\frac{p}{3}}$
   c) The cubic has minima at both $\sqrt{\frac{p}{3}}$ and -$\sqrt{\frac{p}{3}}$
   d) the cubic has maxima at both $\sqrt{\frac{p}{3}}$ and -$\sqrt{\frac{p}{3}}$

9) How many real solutions does the equation $x^7 + 14x^5 + 16x^3 + 30x - 560 = 0$ have ? [2008]

a) 7 b) 1 c) 3 d) 5

10) Let $f(x) = x|x|$ and $g(x) = \sin x$.
statement-1: gof is differentiable at $x = 0$ and its derivative is continuous at that point. **statement-2:** gof is twice differential at $x = 0$. [2009]
a) Statement-1 is true, Statement-2 is true; statement-2 is not a correct explanation for statement-1.
b) Statement-1 is true, Statement-2 is false.
c) Statement-1 is false,Statement-2 is true.
d) Statement-1 is true, Statement-2 is true;Statement-2 is a correct explanation for Statement-1.

11) Given $P(x) = x^4 + ax^3 + bx^2 + cx + d$ such that $x = 0$ is the only real root of $P'(x) = 0$. If $P(-1) < P(1)$, then in the interval $[-1, 1]$: [2009]
a) $P(-1)$ is not minimum but $P(1)$ is the maximum of P.
b) $P(-1)$ is the minimum but $P(1)$ is not the maximum of P.
c) Neither $P(-1)$ is the minimum nor $P(1)$ is the maximum of P.
d) $P(-1)$ is the minimum and $P(1)$ is the maximum of P.

12) The equation of the tangent to the curve $y = x + \frac{4}{x^2}$, that is parallel to the x-axis, is [2010]

a) $y = 1$ b) $y = 2$ c) $y = 3$ d) $y = 0$

13) Let $f : R \to R$ be defined by $f(x) =$
$$\begin{cases} k - 2x & \text{if} \quad x \leq -1 \\ 2x + 3 & \text{if} \quad x > -1 \end{cases}$$
If f has a local minimum at $x = -1$,then a possible value of k is [2010]

a) 0 b) -$\frac{1}{2}$ c) -1 d) 1

14) Let $f : R \to R$ be a continuous function defined by $f(x) = \frac{1}{e^x + 2e^{-x}}$ [2010]
Statement-1 : $f(c) = \frac{1}{3}$, for some $c \in R$. **Statement-2**: $0 < f(x) \leq \frac{1}{2\sqrt{2}}$, for all $x \in R$
a) Statement-1 is true, Statement-2 is true;Statement-2 is **not** a correct explanation for statement-1.
b) Statement-1 is true, Statement-2 is false.
c) Statement-1 is false, Statement-2 is true.
d) Statement-1 is true,Statement 2 is true; Statement-2 is a correct explanation for statement-1.

15) The shortest distance between line $y - x = 1$ and curve $x = y^2$ is [2011]

a) $\frac{3\sqrt{2}}{8}$ b) $\frac{8}{3\sqrt{2}}$ c) $\frac{4}{\sqrt{3}}$ d) $\frac{\sqrt{3}}{4}$