16. Applications of derivatives

EE24BTECH11065 - spoorthi

Section-B JEE Main/AIEEE

10. A spherical iron ball $10cm$ in radius is coated with a layer of ice of uniform thickness that melts at a rate of 50 cm^3 /min. When the thickness of ice is $5cm$,then the rate at which the thickness of ice decreases is [2005]
    a) $\frac{1}{36\pi}$ $cm/min.$
    b) $\frac{1}{18\pi}$ $cm/min.$
    c) $\frac{1}{54\pi}$ $cm/min.$
    d) $\frac{5}{6\pi}$ $cm/min.$
11. If the equation $a_nx^n + a_{n-1}x^{n-1} + ... + a_1x = 0$, $a_1 \neq 0$,$n \geq 2$, has a positive root $x = \alpha$, then the equation na_nx^{n-1}

    +

    $(n_1)a_{n-1}x^{n-2}$ +....... + a_1 =0 has a positive root, which is [2005]
    a) greater than α
    b) smaller than α
    c) greater than or equal to α
    d) equal to α
12. The function $f(x) = \frac{x}{2} + \frac{2}{x}$ has a local minimum at [2006]
    a) $x = 2$
    b) $x = -2$
    c) $x = 0$
    d) $x = 1$
13. A triangular park is enclosed on two sides by a fence and on third side by a straight river bank. The two sides having fence are of same length x. The maximum area enclosed by the park is [2006]
    a) $\frac{3}{2}x^2$
    b) $\sqrt{\frac{x^3}{8}}$
    c) $\frac{1}{2}x^2$
    d) πx^2
14. A value of c for which conclusion of Mean Value Theorem holds for the function $f(x) = \log_e x$ on the interval $[1, 3]$ is [2007]
    a) $\log_3 e$
    b) $\log_e 3$
    c) $2\log_3 e$
    d) $\frac{1}{2}\log_3 e$
15. The function $f(x) = \tan^{-1}(\sin x + \cos x)$is an increasing function in [2007] explanation for Statement-1.
    a) $(0,\frac{\pi}{2})$
    b) $(\frac{-\pi}{2},\frac{\pi}{2})$
    c) $(\frac{\pi}{4},\frac{\pi}{2})$
    d) $(\frac{-\pi}{2},\frac{\pi}{4})$
16. If p and q are positive real numbers such that $p^2 + q^2 = 1$,then the maximum value of $(p+q)$ is [2007]
    a) $\frac{1}{2}$
    b) $\frac{1}{\sqrt{2}}$
    c) $\sqrt{2}$
    d) 2
17. Suppose the cubic x^3 - $px + q$ has three distinct real roots where $p > 0$ and $q > 0$. Then which one of the following holds ? [2008]
    a) The cubic has minima at $\sqrt{\frac{p}{3}}$ and maxima at - $\sqrt{\frac{p}{3}}$
    b) The cubic has minima at - $\sqrt{\frac{p}{3}}$ and maxima at $\sqrt{\frac{p}{3}}$
    c) The cubic has minima at both $\sqrt{\frac{p}{3}}$ and - $\sqrt{\frac{p}{3}}$
    d) the cubic has maxima at both $\sqrt{\frac{p}{3}}$ and - $\sqrt{\frac{p}{3}}$
18. How many real solutions does the equation $x^7 + 14x^5 + 16x^3 + 30x - 560 = 0$ have ? [2008]
    a) 7
    b) 1
    c) 3
    d) 5
19. Let $f(x)$= $x|x|$ and $g(x) = \sin x$.
    statement-1: gof is differentiable at $x = 0$ and its derivative is continuous at that point. **statement-2:** gof is twice differential at $x = 0$. [2009]
    a) Statement-1 is true, Statement-2 is true;

statement-2 is not a correct explanation for statement-1.

b) Statement-1 is true, Statement-2 is false.
c) Statement-1 is false,Statement-2 is true.
d) Statement-1 is true, Statement-2 is true;Statement-2 is a correct explanation for Statement-1.

20. Given $P(x) = x^4 + ax^3 + bx^2 + cx + d$ such that $x = 0$ is the only real root of $P^1(x) = 0$. If $P(-1) < P(1)$, then in the interval $[-1, 1]$: [2009]
    a) $P(-1)$ is not minimum but $P(1)$. is the maximum of P
    b) $P(-1)$ is the minimum but $P(1)$ is not the maximum of P
    c) Neither $P(-1)$ is the minimum nor $P(1)$ is the maximum of P
    d) $P(-1)$ is the minimum and $P(1)$ is the maximum of P.

21. The equation of the tangent to the curve $y = x + \frac{4}{x^2}$,that is parallel to the x-axis, is [2010]
    a) $y = 1$
    b) $y = 2$
    c) $y = 3$
    d) $y = 0$

22. Let $f: R \to R$ be defined by $f(x) = \begin{cases} k - 2x & \text{if} \quad x \leq -1 \\ 2x + 3 & \text{if} \quad x > -1 \end{cases}$
    If f has a local minimum at $x = -1$,then a possible value of k is [2010]
    a) 0
    b) $-\frac{1}{2}$
    c) -1
    d) 1

23. Let $f: R \to R$ be a continuous function defined by $f(x) = \frac{1}{e^x+2e^{-x}}$ [2010]
    Statement-1 : $f(c) = \frac{1}{3}$, for some $c \in R$.
    Statement-2: $0 < f(x) \leq \frac{1}{2\sqrt{2}}$, for all $x \in R$
    a) Statement-1 is true, Statement-2 is true;Statement-2 is **not** a correct explanation for statement-1.
    b) Statement-1 is true, Statement-2 is false.
    c) Statement-1 is false, Statement-2 is true.
    d) Statement-1 is true,Statement 2 is true; Statement-2 is a correct

24. The shortest distance between line $y - x = 1$ and curve $x=y^2$ is [2011]
    a) $\frac{3\sqrt{2}}{8}$
    b) $\frac{8}{3\sqrt{2}}$
    c) $\frac{4}{\sqrt{3}}$
    d) $\frac{\sqrt{3}}{4}$