

# 16. Applications of derivatives

EE24BTECH11065 - spoorthi

## Section-B JEE Main/AIEEE

- 1) A spherical iron ball 10cm in radius is coated with a layer of ice of uniform thickness that melts at a rate of  $50 \text{ cm}^3 / \text{min}$ . When the thickness of ice is 5cm, then the rate at which the thickness of ice decreases is [2005]
  - a)  $\frac{1}{36\pi} \text{ cm/min}$ .
  - b)  $\frac{1}{18\pi} \text{ cm/min}$ .
  - c)  $\frac{1}{54\pi} \text{ cm/min}$ .
  - d)  $\frac{5}{6\pi} \text{ cm/min}$ .
- 2) If the equation  $a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x = 0$ ,  $a_1 \neq 0, n \geq 2$ , has a positive root  $x = \alpha$ , then the equation  $na_n x^{n-1} + (n-1)a_{n-1} x^{n-2} + \dots + a_1 = 0$  has a positive root, which is [2005]
  - a) greater than  $\alpha$
  - b) smaller than  $\alpha$
  - c) greater than or equal to  $\alpha$
  - d) equal to  $\alpha$
- 3) The function  $f(x) = \frac{x}{2} + \frac{2}{x}$  has a local minimum at [2006]
  - a)  $x = 2$
  - b)  $x = -2$
  - c)  $x = 0$
  - d)  $x = 1$
- 4) A triangular park is enclosed on two sides by a fence and on third side by a straight river bank. The two sides having fence are of same length  $x$ . The maximum area enclosed by the park is [2006]
  - a)  $\frac{3}{2}x^2$
  - b)  $\sqrt{\frac{x^3}{8}}$
  - c)  $\frac{1}{2}x^2$
  - d)  $\pi x^2$
- 5) A value of  $c$  for which conclusion of Mean Value Theorem holds for the function  $f(x) = \log_e x$  on the interval  $[1, 3]$  is [2007]
  - a)  $\log_3 e$
  - b)  $\log_e 3$
  - c)  $2\log_3 e$
  - d)  $\frac{1}{2}\log_3 e$
- 6) The function  $f(x) = \tan^{-1}(\sin x + \cos x)$  is an increasing function in [2007]
  - a)  $(0, \frac{\pi}{2})$
  - b)  $(\frac{\pi}{2}, \frac{3\pi}{2})$
  - c)  $(\frac{\pi}{4}, \frac{5\pi}{4})$
  - d)  $(\frac{\pi}{2}, \frac{3\pi}{4})$
- 7) If  $p$  and  $q$  are positive real numbers such that  $p^2 + q^2 = 1$ , then the maximum value of  $(p+q)$  is [2007]
  - a)  $\frac{1}{2}$
  - b)  $\frac{1}{\sqrt{2}}$
  - c)  $\sqrt{2}$
  - d) 2
- 8) Suppose the cubic  $x^3 - px + q$  has three distinct real roots where  $p > 0$  and  $q > 0$ . Then which one of the following holds ? [2008]
  - a) The cubic has minima at  $\sqrt{\frac{p}{3}}$  and maxima at  $-\sqrt{\frac{p}{3}}$
  - b) The cubic has minima at  $-\sqrt{\frac{p}{3}}$  and maxima at  $\sqrt{\frac{p}{3}}$
  - c) The cubic has minima at both  $\sqrt{\frac{p}{3}}$  and  $-\sqrt{\frac{p}{3}}$
  - d) the cubic has maxima at both  $\sqrt{\frac{p}{3}}$  and  $-\sqrt{\frac{p}{3}}$
- 9) How many real solutions does the equation  $x^7 + 14x^5 + 16x^3 + 30x - 560 = 0$  have ? [2008]
  - a) 7
  - b) 1
  - c) 3
  - d) 5
- 10) Let  $f(x) = x|x|$  and  $g(x) = \sin x$ .
 

**statement-1:**  $g \circ f$  is differentiable at  $x = 0$  and its derivative is continuous at that point.

**statement-2:**  $g \circ f$  is twice differential at  $x = 0$ . [2009]

  - a) Statement-1 is true, Statement-2 is true; statement-2 is not a correct explanation for statement-1.
  - b) Statement-1 is true, Statement-2 is false.

- c) Statement-1 is false, Statement-2 is true.
- d) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
- 11) Given  $P(x) = x^4 + ax^3 + bx^2 + cx + d$  such that  $x = 0$  is the only real root of  $P'(x) = 0$ . If  $P(-1) < P(1)$ , then in the interval  $[-1, 1]$  : [2009]
- $P(-1)$  is not minimum but  $P(1)$  is the maximum of  $P$
  - $P(-1)$  is the minimum but  $P(1)$  is not the maximum of  $P$
  - Neither  $P(-1)$  is the minimum nor  $P(1)$  is the maximum of  $P$
  - $P(-1)$  is the minimum and  $P(1)$  is the maximum of  $P$ .
- 12) The equation of the tangent to the curve  $y = x + \frac{4}{x^2}$ , that is parallel to the  $x$ -axis, is [2010]
- $y = 1$
  - $y = 2$
  - $y = 3$
  - $y = 0$
- 13) Let  $f: R \rightarrow R$  be defined by  $f(x) = \begin{cases} k - 2x & \text{if } x \leq -1 \\ 2x + 3 & \text{if } x > -1 \end{cases}$   
If  $f$  has a local minimum at  $x = -1$ , then a possible value of  $k$  is [2010]
- 0
  - $-\frac{1}{2}$
  - 1
  - 1
- 14) Let  $f: R \rightarrow R$  be a continuous function defined by  $f(x) = \frac{1}{e^x + 2e^{-x}}$  [2010]  
**Statement-1** :  $f(c) = \frac{1}{3}$ , for some  $c \in R$ .  
**Statement-2** :  $0 < f(x) \leq \frac{1}{2\sqrt{2}}$ , for all  $x \in R$
- Statement-1 is true, Statement-2 is true; Statement-2 is **not** a correct explanation for statement-1.
  - Statement-1 is true, Statement-2 is false.
  - Statement-1 is false, Statement-2 is true.
  - Statement-1 is true, Statement 2 is true; Statement-2 is a correct
- 15) The shortest distance between line  $y - x = 1$  and curve  $x = y^2$  is [2011]
- $\frac{3\sqrt{2}}{8}$
  - $\frac{8}{3\sqrt{2}}$
  - $\frac{4}{\sqrt{3}}$
- d)  $\frac{\sqrt{3}}{4}$