16. Applications of derivatives

EE24BTECH11065 - spoorthi

Section-B JEE Main/AIEEE

10. A spherical iron ball 10cm in radius is coated with a layer of ice of uniform thickness that melts at a rate of 50 cm^3 /min. When the thickness of ice is 5cm,then the rate at which the thickness of ice decreases is [2005]
    a) $\frac{1}{36\pi}$ cm/min.
    b) $\frac{1}{18\pi}$ cm/min.
    c) $\frac{1}{54\pi}$ cm/min.
    d) $\frac{5}{6\pi}$ cm/min.
11. If the equation $a_n x^n$ + $a_{n-1}x^{n-1}$ + ... + a_1x = 0, $a_1 \neq 0$,n≥2, has a positive root x = α, then the equation n$a_n x^{n-1}$ + (n-1)$a_{n-1}x^{n-2}$ +....... + a_1 =0 has a positive root, which is [2005]
    a) greater than α
    b) smaller than α
    c) greater than or equal to α
    d) equal to α
12. The function f(x) = $\frac{x}{2} + \frac{2}{x}$ has a local minimum at [2006]
    a) x = 2
    b) x = -2
    c) x = 0
    d) x = 1
13. A triangular park is enclosed on two sides by a fence and on third side by a straight river bank. The two sides having fence are of same length x. The maximum area enclosed by the park is [2006]
    a) $\frac{3}{2}x^2$
    b) $\sqrt{\frac{x^3}{8}}$
    c) $\frac{1}{2}x^2$
    d) πx^2
14. A value of c for which conclusion of Mean Value Theorem holds for the function f(x) = log_ex on the interval [1,3] is [2007]
    a) log_3e
    b) $log_e 3$
    c) 2log_3e
    d) $\frac{1}{2}log_3$e
15. The function f(x) = tan^{-1}(sinx + cosx) is an increasing function in [2007] explanation for Statement-1.
    a) $(0,\frac{\pi}{2})$
    b) $(\frac{-\pi}{2},\frac{\pi}{2})$
    c) $(\frac{\pi}{4},\frac{\pi}{2})$
    d) $(\frac{-\pi}{2},\frac{\pi}{4})$
16. If p and q are positive real numbers such that $p^2 + q^2 = 1$,then the maximum value of (p + q) is [2007]
    a) $\frac{1}{2}$
    b) $\frac{1}{\sqrt{2}}$
    c) $\sqrt{2}$
    d) 2
17. Suppose the cubic x^3 - px + q has three distinct real roots where p > 0 and q > 0. Then which one of the following holds ? [2008]
    a) The cubic has minima at $\sqrt{\frac{p}{3}}$ and maxima at -$\sqrt{\frac{p}{3}}$
    b) The cubic has minima at -$\sqrt{\frac{p}{3}}$ and maxima at $\sqrt{\frac{p}{3}}$
    c) The cubic has minima at both $\sqrt{\frac{p}{3}}$ and -$\sqrt{\frac{p}{3}}$
    d) the cubic has maxima at both $\sqrt{\frac{p}{3}}$ and -$\sqrt{\frac{p}{3}}$
18. How many real solutions does the equation $x^7 + 14x^5 + 16x^3 + 30x - 560 = 0$ have ? [2008]
    a) 7
    b) 1
    c) 3
    d) 5
19. Let f(x) = x$|x|$ and g(x) = sinx.
    statement-1: gof is differentiable at x=0 and its derivative is continuous at that point. **statement-2:** gof is twice differential at x = 0. [2009]
    a) Statement-1 is true, Statement-2 is true; statement-2 is not a correct explanation for statement-1.

b) Statement-1 is true, Statement-2 is false.
c) Statement-1 is false,Statement-2 is true.
d) Statement-1 is true, Statement-2 is true;Statement-2 is a correct explanation for Statement-1.

20. Given P(x)=$x^4+ax^3+bx^2+cx+d$ such that x = 0 is the only real root of P^1(x) = 0. If P(-1) < P(1), then in the interval [-1,1]: [2009]
a) P(-1) is not minimum but P(1). is the maximum of P
b) P(-1) is the minimum but P(1) is not the maximum of P
c) Neither P(-1) is the minimum nor P(1) is the maximum of P
d) P(-1) is the minimum and P(1) is the maximum of P.

21. The equation of the tangent to the curve y = x + $\frac{4}{x^2}$,that is parallel to the x-axis, is [2010]
a) y=1
b) y=2
c) y=3
d) y=0

22. Let f: R $\rightarrow$ R be defined by f(x) =
$$\begin{cases} k-2x & \text{if} \quad x \le -1 \\ 2x+3 & \text{if} \quad x > -1 \end{cases}$$
If f has a local minimum at x = -1,then a possible value of k is [2010]
a) 0
b) -$\frac{1}{2}$
c) -1
d) 1

23. Let f: R $\rightarrow$ R be a continuous function defined by f(x) = $\frac{1}{e^x+2e^{-x}}$ [2010]
Statement-1 : f(c) = $\frac{1}{3}$, for some c$\in$R.
Statement-2: 0<f(x)$\le\frac{1}{2\sqrt{2}}$, for all x$\in$ R
a) Statement-1 is true, Statement-2 is true;Statement-2 is **not** a correct explanation for statement-1.
b) Statement-1 is true, Statement-2 is false.
c) Statement-1 is false, Statement-2 is true.
d) Statement-1 is true,Statement 2 is true; Statement-2 is a correct

24. The shortest distance between line y-x=1 and curve x=y^2 is [2011]
a) $\frac{3\sqrt{2}}{8}$
b) $\frac{8}{3\sqrt{2}}$
c) $\frac{4}{\sqrt{3}}$
d) $\frac{\sqrt{3}}{4}$