## 16. Applications of derivatives

## EE24BTECH11065 - spoorthi

1) A spherical iron ball 10cm in radius is coated with a layer of ice of uniform thickness that melts at

## **Section-B JEE Main/AIEEE**

a) $\frac{1}{36\pi}$ cm/min. b) $\frac{1}{18\pi}$ cm/min. c) $\frac{1}{54\pi}$ cm/min. d) $\frac{5}{6\pi}$ cm/min.  2) If the equation $a_nx^n + a_{n-1}x^{n-1} + \dots + a_{1}x = 0$ , $a_1 \neq 0$ , $n \geq 2$ , has a positive root $x = \alpha$ , then the equation $a_nx^{n-1} + (n_1)a_{n-1}x^{n-2} + \dots + a_1 = 0$ has a positive root, which is a) greater than $\alpha$ b) smaller than $\alpha$ c) greater than $\alpha$ c) greater than $\alpha$ corrected and or equal to $\alpha$ d) equal to $\alpha$ 3) The function $f(x) = \frac{x}{2} + \frac{2}{x}$ has a local minimum at [2006] a) $x = 2$ b) $x = -2$ c) $x = 0$ d) $x = 1$ 4) A triangular park is enclosed on two sides by a fence and on third side by a straight river bank. The two sides having fence are of same length $x$ . The maximum area enclosed by the park is [2006] a) $\frac{3}{2}x^2$ b) $\sqrt{\frac{x^3}{8}}$ c) $\frac{1}{2}x^2$ d) $\pi x^2$ 5) A value of $c$ for which conclusion of Mean Value Theorem holds for the function $f(x) = \log_e x$ on the interval $[1,3]$ is [2007] a) $\log_3 e$ b) $\log_e 3$ c) $2\log_3 e$ d) $\frac{1}{2}\log_3 e$ 6) The function $f(x) = \tan^{-1}(\sin x + \cos x)$ is an increasing function in [2007] a) $(0, \frac{\pi}{2})$ b) $(\frac{-\pi}{2}, \frac{\pi}{2})$ c) $(\frac{\pi}{4}, \frac{\pi}{2})$ d) $(\frac{-\pi}{2}, \frac{\pi}{4})$ 7) If $p$ and $q$ are positive real numbers such that $p^2 + q^2 = 1$ , then the maximum value of $(p+q)$ is [2007] a) $\frac{1}{2}$ b) $\frac{1}{\sqrt{2}}$ c) $\sqrt{2}$ d) 2  8) Suppose the cubic $x^3 - px + q$ has three distinct real roots where $p > 0$ and $q > 0$ . Then which one of the following holds? [2008] a) The cubic has minima at $\sqrt{\frac{p}{3}}$ and maxima at $-\sqrt{\frac{p}{3}}$ and maxima at $-\sqrt{\frac{p}{3}}$ d) the cubic has minima at both $\sqrt{\frac{p}{3}}$ and maxima at $\sqrt{\frac{p}{3}}$ and $-\sqrt{\frac{p}{3}}$ d) the cubic has minima at both $\sqrt{\frac{p}{3}}$ and $-\sqrt{\frac{p}{3}}$ g) How many real solutions does the equation $x^7 + 14x^5 + 16x^3 + 30x - 560 = 0$ have? [2008]	a rate of 50 cm <sup>3</sup> /m decreases is	nin. When the thickness o	of ice is $5cm$ , then the rate	at which the thickn	ess of ice [2005]
equation $na_n x^{n-1} + (n_1)a_{n-1} x^{n-2} + \dots + a_1 = 0$ has a positive root, which is a greater than $\alpha$ b) smaller than $\alpha$ c) greater than or equal to $\alpha$ d) equal to $\alpha$ 3) The function $f(x) = \frac{x}{2} + \frac{2}{x}$ has a local minimum at [2006] a) $x = 2$ b) $x = -2$ c) $x = 0$ d) $x = 1$ 4) A triangular park is enclosed on two sides by a fence and on third side by a straight river bank. The two sides having fence are of same length $x$ . The maximum area enclosed by the park is [2006] a) $\frac{3}{2}x^2$ b) $\sqrt{\frac{x^3}{8}}$ c) $\frac{1}{2}x^2$ d) $\pi x^2$ 5) A value of $c$ for which conclusion of Mean Value Theorem holds for the function $f(x) = \log_e x$ on the interval $[1,3]$ is [2007] a) $\log_3 e$ b) $\log_e 3$ c) $2\log_3 e$ d) $\frac{1}{2}\log_3 e$ 6) The function $f(x) = \tan^{-1}(\sin x + \cos x)$ is an increasing function in [2007] a) $(0, \frac{\pi}{2})$ b) $(-\frac{\pi}{2}, \frac{\pi}{2})$ c) $(\frac{\pi}{4}, \frac{\pi}{2})$ d) $(-\frac{\pi}{2}, \frac{\pi}{4})$ 7) If $p$ and $q$ are positive real numbers such that $p^2 + q^2 = 1$ , then the maximum value of $(p+q)$ is [2007] a) $\frac{1}{2}$ b) $\frac{1}{\sqrt{2}}$ c) $\sqrt{2}$ d) 2  8) Suppose the cubic $x^3 - px + q$ has three distinct real roots where $p > 0$ and $q > 0$ . Then which one of the following holds?  a) The cubic has minima at $\sqrt{\frac{p}{3}}$ and maxima at $-\sqrt{\frac{p}{3}}$ and maxima at $-\sqrt{\frac{p}{3}}$ and maxima at $-\sqrt{\frac{p}{3}}$ and the cubic has minima at both $\sqrt{\frac{p}{3}}$ and $-\sqrt{\frac{p}{3}}$ and $-\sqrt{\frac{p}{3}}$ and the cubic has maxima at both $\sqrt{\frac{p}{3}}$ and $-\sqrt{\frac{p}{3}}$ and $-\sqrt{\frac{p}{3}}$ and the cubic has maxima at both $\sqrt{\frac{p}{3}}$ and $-\sqrt{\frac{p}{3}}$ a	a) $\frac{1}{36\pi}$ cm/min.	b) $\frac{1}{18\pi} \ cm/min$ .	c) $\frac{1}{54\pi}$ cm/min.	d) $\frac{5}{6\pi}$ cm/min.	
a) $x=2$ b) $x=-2$ c) $x=0$ d) $x=1$ 4) A triangular park is enclosed on two sides by a fence and on third side by a straight river bank. The two sides having fence are of same length $x$ . The maximum area enclosed by the park is [2006] a) $\frac{3}{2}x^2$ b) $\sqrt{\frac{x^3}{8}}$ c) $\frac{1}{2}x^2$ d) $\pi x^2$ 5) A value of $c$ for which conclusion of Mean Value Theorem holds for the function $f(x) = \log_e x$ on the interval $[1,3]$ is [2007] a) $\log_3 e$ b) $\log_e 3$ c) $2\log_3 e$ d) $\frac{1}{2}\log_3 e$ 6) The function $f(x) = \tan^{-1}(\sin x + \cos x)$ is an increasing function in [2007] a) $(0, \frac{\pi}{2})$ b) $(-\frac{\pi}{2}, \frac{\pi}{2})$ c) $(\frac{\pi}{4}, \frac{\pi}{2})$ d) $(-\frac{\pi}{2}, \frac{\pi}{4})$ 7) If $p$ and $q$ are positive real numbers such that $p^2 + q^2 = 1$ , then the maximum value of $(p+q)$ is $[2007]$ a) $\frac{1}{2}$ b) $\frac{1}{\sqrt{2}}$ c) $\sqrt{2}$ d) $2$ 8) Suppose the cubic $x^3 - px + q$ has three distinct real roots where $p > 0$ and $q > 0$ . Then which one of the following holds? [2008] a) The cubic has minima at $\sqrt{\frac{p}{3}}$ and maxima at $-\sqrt{\frac{p}{3}}$ and maxima at $-\sqrt{\frac{p}{3}}$ or The cubic has minima at both $\sqrt{\frac{p}{3}}$ and $-\sqrt{\frac{p}{3}}$ and $-\sqrt{\frac{p}{3}}$ d) the cubic has maxima at both $\sqrt{\frac{p}{3}}$ and $-\sqrt{\frac{p}{3}}$	equation $na_n x^{n-1}$ + a) greater than $\alpha$ b) smaller than $\alpha$ c) greater than or edd) equal to $\alpha$	$(n_1)a_{n-1}x^{n-2} + \dots + a_1 =$ qual to $\alpha$	=0 has a positive root, when	a positive root $x = \alpha$ hich is	[2005]
4) A triangular park is enclosed on two sides by a fence and on third side by a straight river bank. The two sides having fence are of same length $x$ . The maximum area enclosed by the park is [2006] a) $\frac{3}{2}x^2$ b) $\sqrt{\frac{x^3}{8}}$ c) $\frac{1}{2}x^2$ d) $\pi x^2$ 5) A value of $c$ for which conclusion of Mean Value Theorem holds for the function $f(x) = \log_e x$ on the interval $[1,3]$ is [2007] a) $\log_3 e$ b) $\log_e 3$ c) $2\log_3 e$ d) $\frac{1}{2}\log_3 e$ 6) The function $f(x) = \tan^{-1}(\sin x + \cos x)$ is an increasing function in [2007] a) $\left(0, \frac{\pi}{2}\right)$ b) $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ c) $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ d) $\left(\frac{-\pi}{2}, \frac{\pi}{4}\right)$ 7) If $p$ and $q$ are positive real numbers such that $p^2 + q^2 = 1$ , then the maximum value of $(p+q)$ is $[2007]$ a) $\frac{1}{2}$ b) $\frac{1}{\sqrt{2}}$ c) $\sqrt{2}$ d) 2  8) Suppose the cubic $x^3 - px + q$ has three distinct real roots where $p > 0$ and $q > 0$ . Then which one of the following holds? [2008] a) The cubic has minima at $\sqrt{\frac{p}{3}}$ and maxima at $\sqrt{\frac{p}{3}}$ b) The cubic has minima at both $\sqrt{\frac{p}{3}}$ and maxima at $\sqrt{\frac{p}{3}}$ c) The cubic has minima at both $\sqrt{\frac{p}{3}}$ and $-\sqrt{\frac{p}{3}}$ d) the cubic has maxima at both $\sqrt{\frac{p}{3}}$ and $-\sqrt{\frac{p}{3}}$				d) $y = 1$	[2000]
5) A value of $c$ for which conclusion of Mean Value Theorem holds for the function $f(x) = \log_e x$ on the interval $[1,3]$ is $[2007]$ a) $\log_3 e$ b) $\log_e 3$ c) $2\log_3 e$ d) $\frac{1}{2}\log_3 e$ 6) The function $f(x) = \tan^{-1}(\sin x + \cos x)$ is an increasing function in $[2007]$ a) $\left(0,\frac{\pi}{2}\right)$ b) $\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$ c) $\left(\frac{\pi}{4},\frac{\pi}{2}\right)$ d) $\left(\frac{-\pi}{2},\frac{\pi}{4}\right)$ 7) If $p$ and $q$ are positive real numbers such that $p^2 + q^2 = 1$ , then the maximum value of $(p+q)$ is $[2007]$ a) $\frac{1}{2}$ b) $\frac{1}{\sqrt{2}}$ c) $\sqrt{2}$ d) 2  8) Suppose the cubic $x^3 - px + q$ has three distinct real roots where $p > 0$ and $q > 0$ . Then which one of the following holds?  a) The cubic has minima at $\sqrt{\frac{p}{3}}$ and maxima at $-\sqrt{\frac{p}{3}}$ b) The cubic has minima at both $\sqrt{\frac{p}{3}}$ and maxima at $\sqrt{\frac{p}{3}}$ and $-\sqrt{\frac{p}{3}}$ d) the cubic has maxima at both $\sqrt{\frac{p}{3}}$ and $-\sqrt{\frac{p}{3}}$	4) A triangular park is	enclosed on two sides by	y a fence and on third side	e by a straight river	
the interval $[1,3]$ is $[2007]$ a) $\log_3 e$ b) $\log_e 3$ c) $2\log_3 e$ d) $\frac{1}{2}\log_3 e$ 6) The function $f(x) = \tan^{-1}(\sin x + \cos x)$ is an increasing function in $[2007]$ a) $\left(0,\frac{\pi}{2}\right)$ b) $\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$ c) $\left(\frac{\pi}{4},\frac{\pi}{2}\right)$ d) $\left(\frac{-\pi}{2},\frac{\pi}{4}\right)$ 7) If $p$ and $q$ are positive real numbers such that $p^2 + q^2 = 1$ , then the maximum value of $(p+q)$ is $[2007]$ a) $\frac{1}{2}$ b) $\frac{1}{\sqrt{2}}$ c) $\sqrt{2}$ d) 2  8) Suppose the cubic $x^3 - px + q$ has three distinct real roots where $p > 0$ and $q > 0$ . Then which one of the following holds?  a) The cubic has minima at $\sqrt{\frac{p}{3}}$ and maxima at $-\sqrt{\frac{p}{3}}$ b) The cubic has minima at both $\sqrt{\frac{p}{3}}$ and maxima at $\sqrt{\frac{p}{3}}$ c) The cubic has minima at both $\sqrt{\frac{p}{3}}$ and $-\sqrt{\frac{p}{3}}$ d) the cubic has maxima at both $\sqrt{\frac{p}{3}}$ and $-\sqrt{\frac{p}{3}}$	a) $\frac{3}{2}x^2$	b) $\sqrt{\frac{x^3}{8}}$	c) $\frac{1}{2}x^2$	d) $\pi x^2$	
6) The function $f(x) = \tan^{-1}(\sin x + \cos x)$ is an increasing function in [2007] a) $\left(0, \frac{\pi}{2}\right)$ b) $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ c) $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ d) $\left(\frac{-\pi}{2}, \frac{\pi}{4}\right)$ 7) If $p$ and $q$ are positive real numbers such that $p^2 + q^2 = 1$ , then the maximum value of $(p+q)$ is [2007] a) $\frac{1}{2}$ b) $\frac{1}{\sqrt{2}}$ c) $\sqrt{2}$ d) 2 8) Suppose the cubic $x^3 - px + q$ has three distinct real roots where $p > 0$ and $q > 0$ . Then which one of the following holds? [2008] a) The cubic has minima at $\sqrt{\frac{p}{3}}$ and maxima at $-\sqrt{\frac{p}{3}}$ b) The cubic has minima at both $\sqrt{\frac{p}{3}}$ and maxima at $\sqrt{\frac{p}{3}}$ c) The cubic has minima at both $\sqrt{\frac{p}{3}}$ and $-\sqrt{\frac{p}{3}}$ d) the cubic has maxima at both $\sqrt{\frac{p}{3}}$ and $-\sqrt{\frac{p}{3}}$			Value Theorem holds for	the function $f(x) =$	
a) $\left(0,\frac{\pi}{2}\right)$ b) $\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$ c) $\left(\frac{\pi}{4},\frac{\pi}{2}\right)$ d) $\left(\frac{-\pi}{2},\frac{\pi}{4}\right)$ 7) If $p$ and $q$ are positive real numbers such that $p^2+q^2=1$ , then the maximum value of $(p+q)$ is [2007]  a) $\frac{1}{2}$ b) $\frac{1}{\sqrt{2}}$ c) $\sqrt{2}$ d) 2  8) Suppose the cubic $x^3-px+q$ has three distinct real roots where $p>0$ and $q>0$ . Then which one of the following holds? [2008]  a) The cubic has minima at $\sqrt{\frac{p}{3}}$ and maxima at $-\sqrt{\frac{p}{3}}$ b) The cubic has minima at $-\sqrt{\frac{p}{3}}$ and maxima at $\sqrt{\frac{p}{3}}$ c) The cubic has minima at both $\sqrt{\frac{p}{3}}$ and $-\sqrt{\frac{p}{3}}$ d) the cubic has maxima at both $\sqrt{\frac{p}{3}}$ and $-\sqrt{\frac{p}{3}}$	a) $\log_3 e$	b) $\log_e 3$	c) $2\log_3 e$	d) $\frac{1}{2}\log_3 e$	
7) If $p$ and $q$ are positive real numbers such that $p^2+q^2=1$ , then the maximum value of $(p+q)$ is [2007]  a) $\frac{1}{2}$ b) $\frac{1}{\sqrt{2}}$ c) $\sqrt{2}$ d) 2  8) Suppose the cubic $x^3-px+q$ has three distinct real roots where $p>0$ and $q>0$ . Then which one of the following holds? [2008]  a) The cubic has minima at $\sqrt{\frac{p}{3}}$ and maxima at $-\sqrt{\frac{p}{3}}$ b) The cubic has minima at $-\sqrt{\frac{p}{3}}$ and maxima at $\sqrt{\frac{p}{3}}$ and compare $-\sqrt{\frac{p}{3}}$ and $-\sqrt{\frac{p}{3}}$ d) the cubic has maxima at both $\sqrt{\frac{p}{3}}$ and $-\sqrt{\frac{p}{3}}$	6) The function $f(x) =$	$= \tan^{-1} (\sin x + \cos x) $ is an	increasing function in		[2007]
<ul> <li>a) ½</li> <li>b) ½</li> <li>c) √2</li> <li>d) 2</li> <li>8) Suppose the cubic x³ - px + q has three distinct real roots where p &gt; 0 and q &gt; 0. Then which one of the following holds? [2008]</li> <li>a) The cubic has minima at √p/3 and maxima at -√p/3</li> <li>b) The cubic has minima at -√p/3 and maxima at √p/3</li> <li>c) The cubic has minima at both √p/3 and -√p/3</li> <li>d) the cubic has maxima at both √p/3 and -√p/3</li> </ul>	a) $\left(0,\frac{\pi}{2}\right)$	b) $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$	c) $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$	d) $\left(\frac{-\pi}{2}, \frac{\pi}{4}\right)$	
<ul> <li>8) Suppose the cubic x³ - px + q has three distinct real roots where p &gt; 0 and q &gt; 0. Then which one of the following holds? [2008]</li> <li>a) The cubic has minima at √p/3 and maxima at -√p/3</li> <li>b) The cubic has minima at -√p/3 and maxima at √p/3</li> <li>c) The cubic has minima at both √p/3 and -√p/3</li> <li>d) the cubic has maxima at both √p/3 and -√p/3</li> </ul>		tive real numbers such th	$aat p^2 + q^2 = 1, then the$	maximum value of	(p+q) is
of the following holds? [2008]  a) The cubic has minima at $\sqrt{\frac{p}{3}}$ and maxima at $-\sqrt{\frac{p}{3}}$ b) The cubic has minima at $-\sqrt{\frac{p}{3}}$ and maxima at $\sqrt{\frac{p}{3}}$ c) The cubic has minima at both $\sqrt{\frac{p}{3}}$ and $-\sqrt{\frac{p}{3}}$ d) the cubic has maxima at both $\sqrt{\frac{p}{3}}$ and $-\sqrt{\frac{p}{3}}$	a) $\frac{1}{2}$	b) $\frac{1}{\sqrt{2}}$	c) $\sqrt{2}$	d) 2	
	of the following ho a) The cubic has m b) The cubic has m c) The cubic has m	lds? inima at $\sqrt{\frac{p}{3}}$ and maxima inima at $-\sqrt{\frac{p}{3}}$ and maxima inima at both $\sqrt{\frac{p}{3}}$ and $-\sqrt{\frac{p}{3}}$	a at $-\sqrt{\frac{p}{3}}$ a at $\sqrt{\frac{p}{3}}$	0 and $q > 0$ . Then v	
->,,,				560 = 0  have  ?	[2008]

[2009]

d) 5

11)	Given $P(x) = x^4 + ax^3 - P(1)$ , then in the interv	$+bx^{2}+cx+d$ such that x al [-1, 1]:	r = 0 is the only real root		1) < 009]			
1	a) $P(-1)$ is not minimum b) $P(-1)$ is the minimum b) Neither $P(-1)$ is the	Im but $P(1)$ is the maxim m but $P(1)$ is not the ma minimum nor $P(1)$ is the m and $P(1)$ is the maxim	ximum of <i>P</i> . e maximum of <i>P</i> .					
12)	The equation of the tan	gent to the curve $y = x +$	$\frac{4}{x^2}$ , that is parallel to th	e <i>x</i> -axis, is [20]	010]			
;	a) $y = 1$	b) $y = 2$	c) $y = 3$	d) $y = 0$				
13) Let $f: R \to R$ be defined by $f(x) = \begin{cases} k - 2x & \text{if } x \le -1 \\ 2x + 3 & \text{if } x > -1 \end{cases}$ If $f$ has a local minimum at $x = -1$ , then a possible value of $k$ is								
;	a) 0	b) $-\frac{1}{2}$	c) -1	d) 1				
<ul> <li>14) Let f: R → R be a continuous function defined by f(x) = 1/(e<sup>x</sup>+2e<sup>-x</sup> [2010]</li> <li>Statement-1: f(c) = 1/3, for some c∈R. Statement-2: 0<f(x)≤1 (√2),="" all="" for="" li="" r<="" x∈=""> <li>a) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for statement-1.</li> <li>b) Statement-1 is true, Statement-2 is false.</li> <li>c) Statement-1 is false, Statement-2 is true.</li> <li>d) Statement-1 is true, Statement 2 is true; Statement-2 is a correct explanation for statement-1.</li> </f(x)≤1></li></ul>								
		etween line $y - x = 1$ and			011]			
;	a) $\frac{3\sqrt{2}}{8}$	b) $\frac{8}{3\sqrt{2}}$	c) $\frac{4}{\sqrt{3}}$	d) $\frac{\sqrt{3}}{4}$				

c) 3

**statement-1:**  $g \circ f$  is differentiable at x = 0 and its derivative is continuous at that point. **statement-2:** 

a) Statement-1 is true, Statement-2 is true; statement-2 is not a correct explanation for statement-1.

d) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

b) 1

a) 7

10) Let f(x) = x|x| and  $g(x) = \sin x$ .

gof is twice differential at x = 0.

b) Statement-1 is true, Statement-2 is false.c) Statement-1 is false, Statement-2 is true.