4.10.4

ADUDOTLA SRIVIDYA -EE25BTECH11006

Oct 4,2025

Question

Find the coordinates of the point where the line through the points $A(3, 4, 1)$ and $B(5, 1, 6)$ crosses the XY plane.

Solution

The equation of the line passing through:

$$\mathbf{A} = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 5 \\ 1 \\ 6 \end{pmatrix} \tag{1}$$

The direction vector of the line

$$\mathbf{m} = \mathbf{A} - \mathbf{B} \tag{2}$$

$$= \begin{pmatrix} -2 \\ 3 \\ -5 \end{pmatrix} \tag{3}$$

Solution

vector equation of the line is

$$\mathbf{x} = \mathbf{A} + \lambda \mathbf{m} \tag{4}$$

equation of XY plane is

$$\begin{pmatrix} 0 & 0 & 1 \end{pmatrix} \mathbf{x} = 0 \tag{5}$$

Solving the equation of plane ($\mathbf{n}^T \mathbf{x} = 0$) and the line ($\mathbf{x} = \mathbf{A} + \lambda \mathbf{m}$),

$$\mathbf{n}^T (\mathbf{A} + \lambda \mathbf{m}) = 0 \tag{6}$$

$$\mathbf{n}^T \mathbf{A} + \lambda \mathbf{n}^T \mathbf{m} = 0 \tag{7}$$

$$\lambda = \frac{-\mathbf{n}^T \mathbf{A}}{\mathbf{n}^T \mathbf{m}} \tag{8}$$

$$\tag{9}$$

Solution

Substituting the $\mathbf{n}, \mathbf{A}, \mathbf{m}$

$$\lambda = \frac{-\begin{pmatrix}0\\0\\1\end{pmatrix}^{T}\begin{pmatrix}3\\4\\1\end{pmatrix}}{\begin{pmatrix}0\\0\\1\end{pmatrix}^{T}\begin{pmatrix}-2\\3\\-5\end{pmatrix}} \tag{10}$$

$$= \frac{-(0+0+1)}{0+0-5} \tag{11}$$

$$= \frac{-1}{-5} \tag{12}$$

$$\lambda = \frac{1}{5} \tag{13}$$

Solution

Therefore,

$$\mathbf{x} = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} + \begin{pmatrix} \frac{-2}{5} \\ \frac{3}{5} \\ -1 \end{pmatrix} \tag{14}$$

$$= \begin{pmatrix} \frac{13}{5} \\ \frac{23}{5} \\ 0 \end{pmatrix} \tag{15}$$

Solution

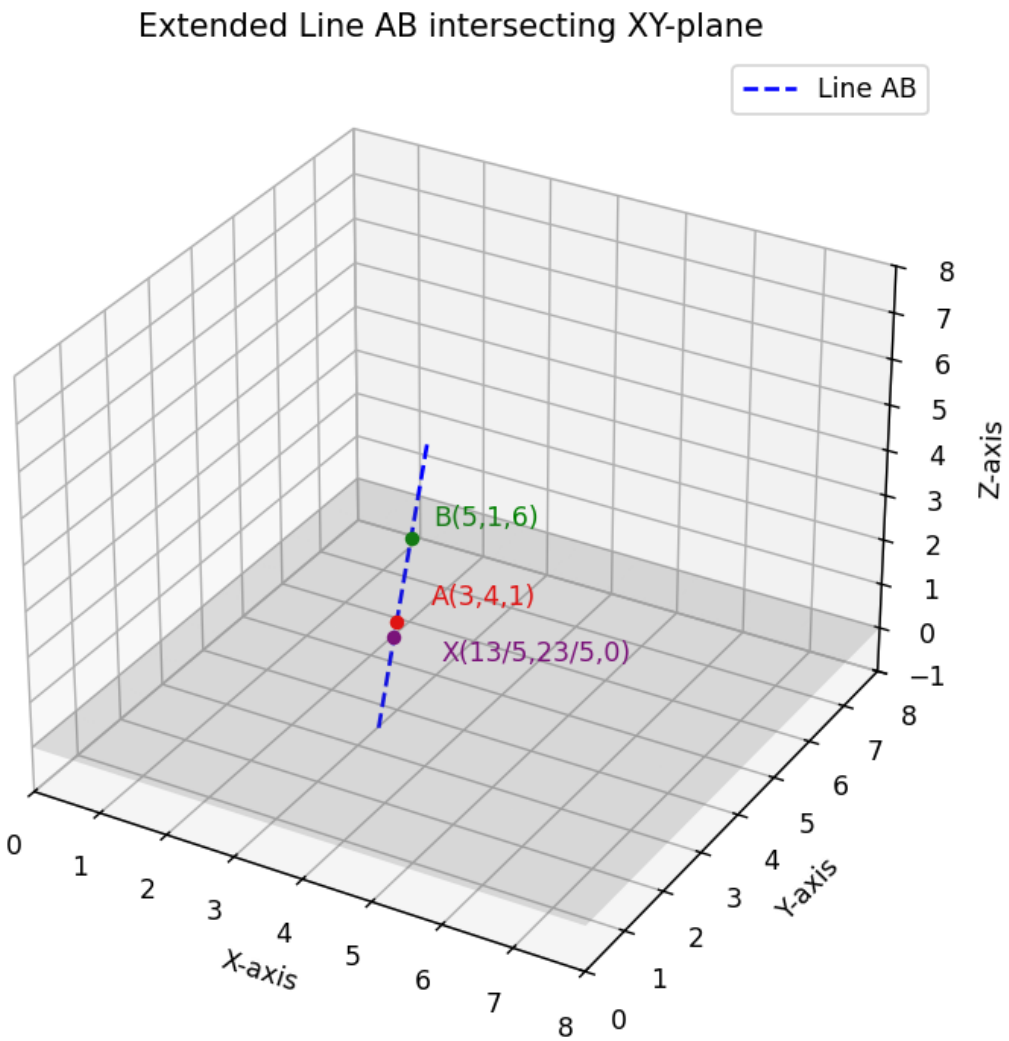
The point where the line crosses the XY plane is :

$$\mathbf{x} = \begin{pmatrix} \frac{13}{5} \\ \frac{23}{5} \\ 0 \end{pmatrix} \tag{16}$$

Python,C,Python+C codes

codes permalink

Plot


Extended Line AB intersecting XY-plane
Line AB
B(5,1,6)
A(3,4,1)
X(13/5,23/5,0)
X-axis
Y-axis
Z-axis