1.8.23

EE25BTECH11006 - ADUDOTLA SRIVIDYA

Question:
If the point $\mathbf{A}(2, -4)$ is equidistant from $\mathbf{P}(3, 8)$ and $\mathbf{Q}(-10, y)$, find the values of y. Also find distance $\mathbf{PQ}$.

Solution: :
The input parameters for this problem are available in Table

Symbol	Value	Description
A	$\begin{pmatrix} 2 \\ -4 \end{pmatrix}$	equidistant point
P	$\begin{pmatrix} 3 \\ 8 \end{pmatrix}$	First point
Q	$\begin{pmatrix} -10 \\ y \end{pmatrix}$	Second point

TABLE I: Parameters for the problem

Since $\mathbf{A}$ is equidistant from $\mathbf{P}$ and $\mathbf{Q}$,

$$\left\| (\mathbf{A} - \mathbf{P}) \right\| = \left\| (\mathbf{A} - \mathbf{Q}) \right\| \tag{1}$$

$$\left\| (\mathbf{A} - \mathbf{P}) \right\|^2 = \left\| (\mathbf{A} - \mathbf{Q}) \right\|^2 \tag{2}$$

$$(\mathbf{A} - \mathbf{P})^\top (\mathbf{A} - \mathbf{P}) = (\mathbf{A} - \mathbf{Q})^\top (\mathbf{A} - \mathbf{Q}) \tag{3}$$

$$\|\mathbf{A}\|^2 - 2\mathbf{A}^\top\mathbf{P} + \|\mathbf{P}\|^2 = \|\mathbf{A}\|^2 - 2\mathbf{A}^\top\mathbf{Q} + \|\mathbf{Q}\|^2 \tag{4}$$

$$(\mathbf{P} - \mathbf{Q})^\top \mathbf{A} = \frac{\|\mathbf{P}\|^2 - \|\mathbf{Q}\|^2}{2} \tag{5}$$

After substituting the values,

$$\begin{pmatrix} 3 - (-10) \\ 8 - y \end{pmatrix}^\top \begin{pmatrix} 2 \\ -4 \end{pmatrix} = \frac{73 - (-10)^2 - y^2}{2} \tag{6}$$

$$y^2 + 8y + 15 = 0 \tag{7}$$

Therefore,

$$y = -5, -3 \tag{8}$$

$$\mathbf{Q_1} = \begin{pmatrix} -10 \\ -5 \end{pmatrix}, \quad \mathbf{Q_2} = \begin{pmatrix} -10 \\ -3 \end{pmatrix} \tag{9}$$

$$\left\| (\mathbf{P} - \mathbf{Q_1}) \right\| = \left\| \begin{pmatrix} 3 \\ 8 \end{pmatrix} - \begin{pmatrix} -10 \\ -5 \end{pmatrix} \right\| \tag{10}$$
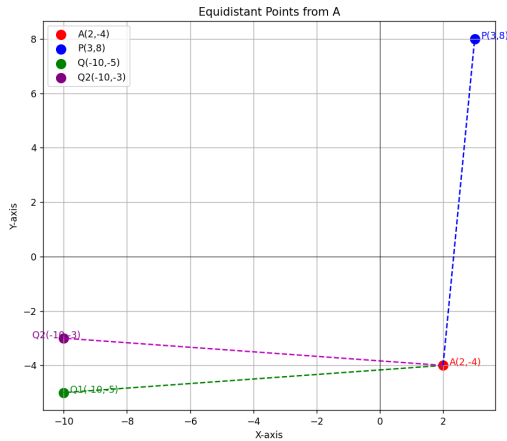
$$= \left\| \begin{pmatrix} 13 \\ 13 \end{pmatrix} \right\| \tag{11}$$

$$= 13\sqrt{2} \tag{12}$$

$$\left\| (\mathbf{P} - \mathbf{Q_2}) \right\| = \left\| \begin{pmatrix} 3 \\ 8 \end{pmatrix} - \begin{pmatrix} -10 \\ -3 \end{pmatrix} \right\| \tag{13}$$

$$= \left\| \begin{pmatrix} 13 \\ 11 \end{pmatrix} \right\| \tag{14}$$

$$= \sqrt{290} \tag{15}$$

Equidistant Points from $\mathbf{A}$ with Distances