9.4.18

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Question:
Find the roots of the quadratic equation graphically.

$$4x^2 + 4\sqrt{3}x + 3 = 0 \tag{1}$$

Solution

$$y = 4x^2 + 4\sqrt{3}x + 3 = 0 \tag{2}$$

This equation can be represented as the conic

$$\mathbf{x}^{\mathsf{T}}\mathbf{V}\mathbf{x} + 2\mathbf{u}^{\mathsf{T}}\mathbf{x} + f = 0 \tag{3}$$

$$\mathbf{V} = \begin{pmatrix} 4 & 0 \\ 0 & 0 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} 2\sqrt{3} \\ 0 \end{pmatrix}, f = 3 \tag{4}$$

Solution

To find the roots, we find the points of intersection of the conic with the x-axis.

$$\mathbf{x} = \mathbf{h} + k_i\mathbf{m} \tag{5}$$

$$\mathbf{h} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{m} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{6}$$

Solution

The value of k_i can be found out by solving the line and conic equation

$$(\mathbf{h} + k_i\mathbf{m})^\top \mathbf{V}(\mathbf{h} + k_i\mathbf{m}) + 2\mathbf{u}^\top(\mathbf{h} + k_i\mathbf{m}) + f = 0 \tag{7}$$

$$\implies k_i^2\mathbf{m}^\top\mathbf{V}\mathbf{m} + 2k_i\mathbf{m}^\top(\mathbf{V}\mathbf{h} + \mathbf{u}) + \mathbf{h}^\top\mathbf{V}\mathbf{h} + 2\mathbf{u}^\top\mathbf{h} + f = 0 \tag{8}$$

$$\text{or, } k_i^2\mathbf{m}^\top\mathbf{V}\mathbf{m} + 2k_i\mathbf{m}^\top(\mathbf{V}\mathbf{h} + \mathbf{u}) + g(\mathbf{h}) = 0 \tag{9}$$

Solution

Solving the above quadratic gives the equation

$$k_i = \frac{1}{\mathbf{m}^\top \mathbf{V}\mathbf{m}} \left(-\mathbf{m}^\top (\mathbf{V}\mathbf{h} + \mathbf{u}) \pm \sqrt{[\mathbf{m}^\top (\mathbf{V}\mathbf{h} + \mathbf{u})]^2 - g(\mathbf{h}) (\mathbf{m}^\top \mathbf{V}\mathbf{m})} \right) \tag{10}$$

$$\therefore k_i = \frac{-2\sqrt{3}}{4} = \frac{-\sqrt{3}}{2} \tag{11}$$

Solution

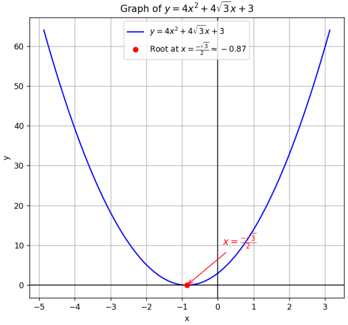
$$\implies k_1 = k_2 = \frac{-\sqrt{3}}{2} \tag{12}$$

$$\therefore \mathbf{x} = \mathbf{h} + k_i \mathbf{m} = \begin{pmatrix} -\frac{\sqrt{3}}{2} \\ 0 \end{pmatrix} \tag{13}$$

Python,C,Python+C codes

codes permalink

Plot



Graph of $y = 4x^2 + 4\sqrt{3}x + 3$
$y = 4x^2 + 4\sqrt{3}x + 3$
Root at $x = \frac{-\sqrt{3}}{2} \approx -0.87$
$x = \frac{-\sqrt{3}}{2}$
60
50
40
30
20
10
0
y
−5
−4
−3
−2
−1
0
1
2
3
x