EE25BTECH11006 - ADUDOTLA SRIVIDYA

Question: The value of λ for which the vectors 3i - 6j + k and $2i - 4j + \lambda k$ are parallel is

a) 2/3 b) 3/2 c) 5/2 d) 2/5

Solution:

Given,

$$\begin{pmatrix} 2 \\ -4 \\ \lambda \end{pmatrix} \text{ is parallel to } \begin{pmatrix} 3 \\ -6 \\ 1 \end{pmatrix} \tag{1}$$

If the vectors are parallel, then they are linearly dependent. Hence,

$$\begin{pmatrix} 3 & 2 \\ -6 & -4 \\ 1 & \lambda \end{pmatrix} \mathbf{x} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{2}$$

To find the condition for non-trivial solutions, perform row reduction on the coefficient matrix:

$$\begin{pmatrix} 3 & 2 \\ -6 & -4 \\ 1 & \lambda \end{pmatrix} \xrightarrow{R_2 \to R_2 + 2R_1} \begin{pmatrix} 3 & 2 \\ 0 & 0 \\ 1 & \lambda \end{pmatrix}$$
 (3)

$$R_1 \to \frac{R_1}{3} \implies \begin{pmatrix} 1 & \frac{2}{3} \\ 0 & 0 \\ 1 & \lambda \end{pmatrix} \tag{4}$$

$$R_3 \to R_3 - R_1 \implies \begin{pmatrix} 1 & \frac{2}{3} \\ 0 & 0 \\ 0 & \lambda - \frac{2}{3} \end{pmatrix}$$
 (5)

For non-trivial solutions

$$\lambda - \frac{2}{3} = 0 \tag{6}$$

$$\therefore \lambda = \frac{2}{3} \tag{7}$$

Hence, the vectors are parallel when $\lambda = \frac{2}{3}$.

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Parallel Vectors in 3D

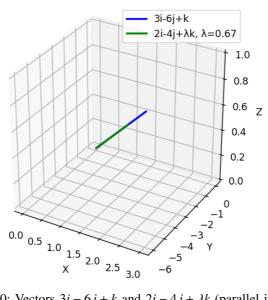


Fig. 0: Vectors 3i - 6j + k and $2i - 4j + \lambda k$ (parallel in 3D)