5.8.4

ADUDOTLA SRIVIDYA -EE25BTECH11006

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Question

Half the perimeter of a rectangular garden, whose length is 4m more than its width, is 36m. Find the dimensions of the garden.

Formulating the Equations

Let length be I, breadth be b.

Half perimeter:
$$I + b = 18$$
 (1)

Length exceeds breadth by 4m:
$$l-b=4$$
 (2)

Matrix form:

$$\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} l \\ b \end{pmatrix} = \begin{pmatrix} 18 \\ 4 \end{pmatrix} \tag{3}$$

Solving Using Orthogonal Matrix

Let

$$\mathbf{A} = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \quad \mathbf{X} = \begin{pmatrix} l \\ b \end{pmatrix}, \quad \mathbf{Y} = \begin{pmatrix} 18 \\ 4 \end{pmatrix} \tag{4}$$

$$\mathbf{A}^{T}\mathbf{A} = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}^{I} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = 2I \tag{5}$$

Since $\mathbf{A}^T \mathbf{A} = 2I$, we have

$$\mathbf{A}^{-1} = \frac{1}{2}\mathbf{A}^{T} \tag{6}$$

Solving Using Orthogonal Matrix

$$\mathbf{X} = \mathbf{A}^{-1}\mathbf{Y} \tag{7}$$

$$=\frac{1}{2}\mathbf{A}^{T}\mathbf{Y}$$
 (8)

$$=\frac{1}{2}\begin{pmatrix}1&1\\1&-1\end{pmatrix}\begin{pmatrix}18\\4\end{pmatrix}\tag{9}$$

$$=\frac{1}{2}\begin{pmatrix}22\\14\end{pmatrix}\tag{10}$$

$$= \begin{pmatrix} 11 \\ 7 \end{pmatrix} \tag{11}$$

Final Answer

Solution:

$$\mathbf{X} = \begin{pmatrix} 11 \\ 7 \end{pmatrix} \tag{12}$$

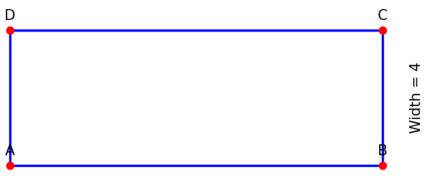
Therefore:

$$Length = 11m Breadth = 7m (13)$$

Python, C, Python + C codes

codes permalink

Plot



Length = 11