

4.10.4

EE25BTECH11006 - ADUDOTLA SRIVIDYA

Question:

Find the coordinates of the point where the line through the points $A(3, 4, 1)$ and $B(5, 1, 6)$ crosses the XY plane.

Solution: :

The equation of the line passing through:

$$\mathbf{A} = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 5 \\ 1 \\ 6 \end{pmatrix} \quad (1)$$

The direction vector of the line

$$\mathbf{m} = \mathbf{A} - \mathbf{B} \quad (2)$$

$$= \begin{pmatrix} -2 \\ 3 \\ -5 \end{pmatrix} \quad (3)$$

vector equation of the line is

$$\mathbf{x} = \mathbf{A} + \lambda \mathbf{m} \quad (4)$$

equation of XY plane is

$$\begin{pmatrix} 0 & 0 & 1 \end{pmatrix} \mathbf{x} = 0 \quad (5)$$

Solving the equation of plane ($\mathbf{n}^T \mathbf{x} = 0$) and the line ($\mathbf{x} = \mathbf{A} + \lambda \mathbf{m}$),

$$\mathbf{n}^T (\mathbf{A} + \lambda \mathbf{m}) = 0 \quad (6)$$

$$\mathbf{n}^T \mathbf{A} + \lambda \mathbf{n}^T \mathbf{m} = 0 \quad (7)$$

$$\lambda = \frac{-\mathbf{n}^T \mathbf{A}}{\mathbf{n}^T \mathbf{m}} \quad (8)$$

$$(9)$$

Substituting the $\mathbf{n}, \mathbf{A}, \mathbf{m}$

$$\lambda = \frac{-\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}^T \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix}}{\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}^T \begin{pmatrix} -2 \\ 3 \\ -5 \end{pmatrix}} \quad (10)$$

$$= \frac{-(0+0+1)}{0+0-5} \quad (11)$$

$$= \frac{-1}{-5} \quad (12)$$

$$\lambda = \frac{1}{5} \quad (13)$$

Therefore,

$$\mathbf{x} = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} + \begin{pmatrix} \frac{-2}{5} \\ \frac{3}{5} \\ -1 \end{pmatrix} \quad (14)$$

$$= \begin{pmatrix} \frac{13}{5} \\ \frac{23}{5} \\ 0 \end{pmatrix} \quad (15)$$

The point where the line crosses the XY plane is :

$$\mathbf{x} = \begin{pmatrix} \frac{13}{5} \\ \frac{23}{5} \\ 0 \end{pmatrix} \quad (16)$$

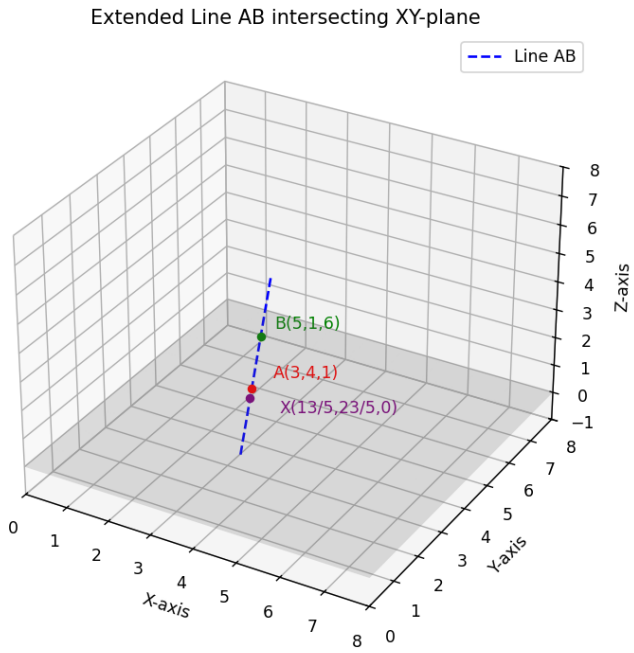


Fig. 1: Caption