5.8.4

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Question

Half the perimeter of a rectangular garden, whose length is 4m more than its width, is 36m. Find the dimensions of the garden.

Formulating the Equations

Let length be l, breadth be b.

$$\text{Half perimeter:} \quad l + b = 18 \tag{1}$$

$$\text{Length exceeds breadth by 4m:} \quad l - b = 4 \tag{2}$$

Matrix form:

$$\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} l \\ b \end{pmatrix} = \begin{pmatrix} 18 \\ 4 \end{pmatrix} \tag{3}$$

Solving Using Orthogonal Matrix

Let

$$\mathbf{A} = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \quad \mathbf{X} = \begin{pmatrix} I \\ b \end{pmatrix}, \quad \mathbf{Y} = \begin{pmatrix} 18 \\ 4 \end{pmatrix} \tag{4}$$

$$\mathbf{A}^T\mathbf{A} = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}^T \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = 2I \tag{5}$$

Since $\mathbf{A}^T\mathbf{A} = 2I$, we have

$$\mathbf{A}^{-1} = \frac{1}{2}\mathbf{A}^T \tag{6}$$

Solving Using Orthogonal Matrix

$$\mathbf{X} = \mathbf{A}^{-1}\mathbf{Y} \tag{7}$$

$$= \frac{1}{2}\mathbf{A}^{T}\mathbf{Y} \tag{8}$$

$$= \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 18 \\ 4 \end{pmatrix} \tag{9}$$

$$= \frac{1}{2}\begin{pmatrix} 22 \\ 14 \end{pmatrix} \tag{10}$$

$$= \begin{pmatrix} 11 \\ 7 \end{pmatrix} \tag{11}$$

Final Answer

Solution:

$$\mathbf{X} = \begin{pmatrix} 11 \\ 7 \end{pmatrix} \tag{12}$$

Therefore:

$$Length = 11m \qquad Breadth = 7m \tag{13}$$

Python,C,Python+C codes

codes permalink

Plot