2.3.9

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Question

If vectors $\mathbf{a}$ and $\mathbf{b}$ are such that

$$|\mathbf{a}| = \frac{1}{2}, \quad |\mathbf{b}| = \frac{4}{\sqrt{3}}, \quad |\mathbf{a} \times \mathbf{b}| = \frac{1}{\sqrt{3}},$$

then find $\mathbf{a} \cdot \mathbf{b}$.

Formula

We know that

$$|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}| \sin\theta \tag{1}$$

$$\mathbf{a}^{\mathrm{T}}\mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos\theta \tag{2}$$

where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$.

Solution

Substitute values:

$$\frac{1}{\sqrt{3}} = \left(\frac{1}{2}\right)\left(\frac{4}{\sqrt{3}}\right)\sin\theta \tag{3}$$

$$\sin\theta = \frac{1}{2} \implies \theta = 30^\circ \text{ or } 150^\circ \tag{4}$$

Now,

$$\mathbf{a}^{\mathrm{T}}\mathbf{b} = \left(\frac{1}{2}\right)\left(\frac{4}{\sqrt{3}}\right)\cos\theta = \frac{2}{\sqrt{3}}\cos\theta \tag{5}$$

Final Result

For $\theta = 30°$:

$$\mathbf{a}^{\mathrm{T}}\mathbf{b} = 1$$

For $\theta = 150°$:

$$\mathbf{a}^{\mathrm{T}}\mathbf{b} = -1$$

Therefore, $\mathbf{a} \cdot \mathbf{b} = \pm 1$.

Python,C,Python+C codes

codes permalink

Vector Plot

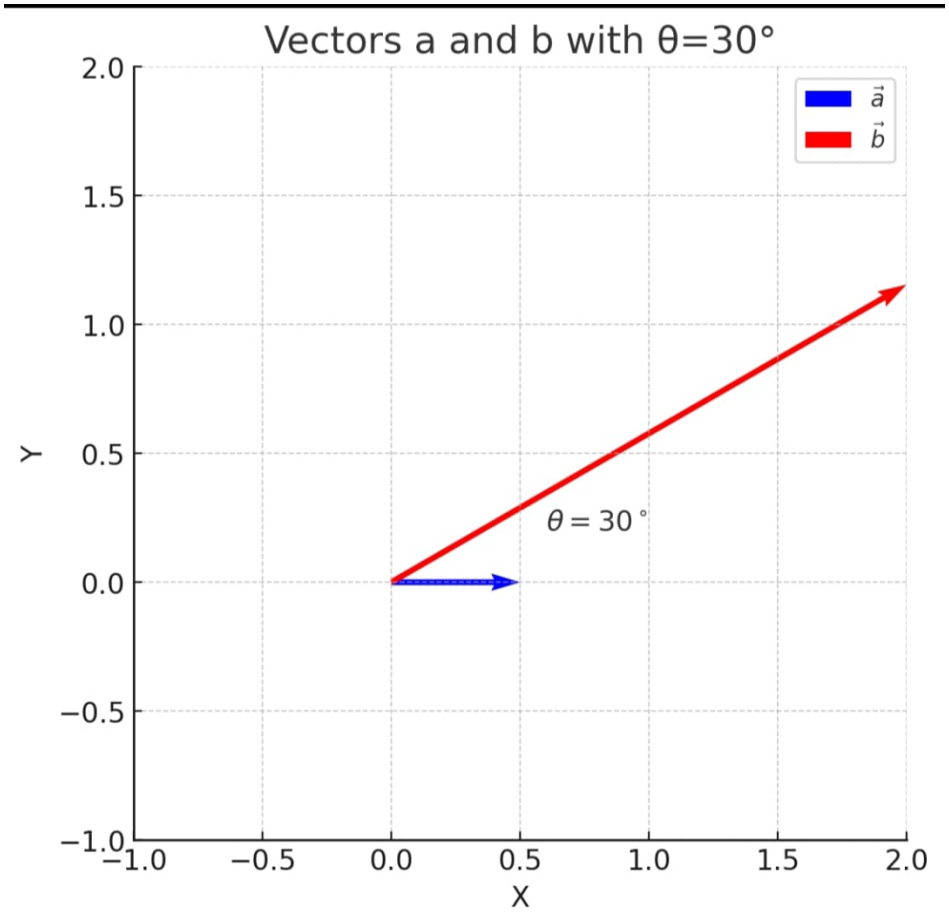

Figure: Vectors **a** and **b** with θ