4.5.8

EE25BTECH11006 - ADUDOTLA SRIVIDYA

Question: The value of λ for which the vectors $3i-6j+k$ and $2i-4j+\lambda k$ are parallel is
a) 2/3 b) 3/2 c) 5/2 d) 2/5

Solution:
Given,

$$\begin{pmatrix} 2 \\ -4 \\ \lambda \end{pmatrix} \text{ is parallel to } \begin{pmatrix} 3 \\ -6 \\ 1 \end{pmatrix} \tag{1}$$

If the vectors are parallel, then they are linearly dependent. Hence,

$$\begin{pmatrix} 3 & 2 \\ -6 & -4 \\ 1 & \lambda \end{pmatrix} \mathbf{x} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{2}$$

$$\begin{pmatrix} 3 & 2 \\ -6 & -4 \\ 1 & \lambda \end{pmatrix} \xrightarrow{R_1 \to \frac{R_1}{3}} \begin{pmatrix} 1 & \frac{2}{3} \\ -6 & -4 \\ 1 & \lambda \end{pmatrix} \xrightarrow{R_2 \to R_2 + 6R_1} \begin{pmatrix} 1 & \frac{2}{3} \\ 0 & 0 \\ 1 & \lambda \end{pmatrix} \xrightarrow{R_3 \to R_3 - R_1} \begin{pmatrix} 1 & \frac{2}{3} \\ 0 & 0 \\ 0 & \lambda - \frac{2}{3} \end{pmatrix} \tag{3}$$

Swap R_2 and R_3

$$\begin{pmatrix} 1 & \frac{2}{3} \\ 0 & \lambda - \frac{2}{3} \\ 0 & 0 \end{pmatrix} \tag{4}$$

Gauss-Jordan elimination depending on λ:

If $\lambda = \frac{2}{3}$.
Then R_2 becomes zero and we have

$$\begin{pmatrix} 1 & \frac{2}{3} \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \tag{5}$$

Rank $= 1 \Rightarrow$ columns dependent $\Rightarrow$ vectors parallel. Therefore,

$$\lambda = \frac{2}{3} \tag{6}$$

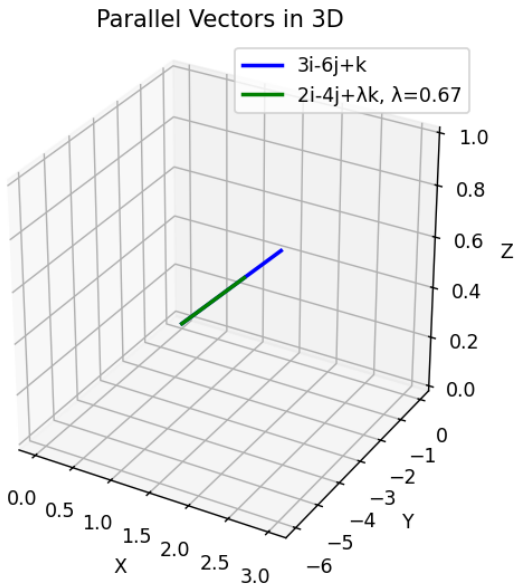

Fig. 0: Vectors $3i - 6j + k$ and $2i - 4j + \lambda k$ (parallel in 3D)