5.8.4

EE25BTECH11006 - ADUDOTLA SRIVIDYA

Question:
Half the perimeter of a rectangular garden, whose length is $4m$ more than its width, is $36m$. Find the dimensions of the garden.

Solution:

$$\text{Perimeter} = 2(l + b) \tag{1}$$
$$\implies l + b = 18 \tag{2}$$

Also given,

$$l - b = 4 \tag{3}$$

$$\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} l \\ b \end{pmatrix} = \begin{pmatrix} 18 \\ 4 \end{pmatrix} \tag{4}$$

Let

$$\mathbf{A} = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \quad \mathbf{X} = \begin{pmatrix} l \\ b \end{pmatrix}, \quad \mathbf{Y} = \begin{pmatrix} 18 \\ 4 \end{pmatrix} \tag{5}$$

$$\mathbf{A}^T\mathbf{A} = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}^T \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = 2I \tag{6}$$

Since $\mathbf{A}^T\mathbf{A} = 2I$, we have

$$\mathbf{A}^{-1} = \frac{1}{2}\mathbf{A}^T \tag{7}$$

$$\mathbf{X} = \mathbf{A}^{-1}\mathbf{Y} \tag{8}$$
$$= \frac{1}{2}\mathbf{A}^T\mathbf{Y} \tag{9}$$
$$= \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 18 \\ 4 \end{pmatrix} \tag{10}$$
$$= \frac{1}{2}\begin{pmatrix} 22 \\ 14 \end{pmatrix} \tag{11}$$
$$= \begin{pmatrix} 11 \\ 7 \end{pmatrix} \tag{12}$$

Therefore,

$$l = 11, \quad b = 7 \tag{13}$$