9.4.18

EE25BTECH11006 - ADUDOTLA SRIVIDYA

Question:
Find the roots of the quadratic equation graphically.

$$4x^2 + 4\sqrt{3}x + 3 = 0 \tag{0.1}$$

Solution:

$$y = 4x^2 + 4\sqrt{3}x + 3 \tag{0.2}$$

This equation can be represented as the conic:

$$\mathbf{x}^T\mathbf{V}\mathbf{x} + 2\mathbf{u}^T\mathbf{x} + f = 0 \tag{0.3}$$

where

$$\mathbf{V} = \begin{pmatrix} 4 & 0 \\ 0 & 0 \end{pmatrix}, \quad \mathbf{u} = \begin{pmatrix} 2\sqrt{3} \\ 0 \end{pmatrix}, \quad f = 3 \tag{0.4}$$

To find the roots, we find the points of intersection of the conic with the x-axis:

$$\mathbf{x} = \mathbf{h} + k_i\mathbf{m} \tag{0.5}$$

$$\mathbf{h} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad \mathbf{m} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{0.6}$$

Substitute into the conic equation:

$$(\mathbf{h} + k_i\mathbf{m})^\top\mathbf{V}(\mathbf{h} + k_i\mathbf{m}) + 2\mathbf{u}^\top(\mathbf{h} + k_i\mathbf{m}) + f = 0 \tag{0.7}$$

This simplifies to a quadratic in k_i:

$$k_i^2\mathbf{m}^T\mathbf{V}\mathbf{m} + 2k_i\mathbf{m}^T(\mathbf{V}\mathbf{h} + \mathbf{u}) + g(\mathbf{h}) = 0 \tag{0.8}$$

Now substituting the values:

$$k_i = \frac{-\mathbf{m}^T(\mathbf{V}\mathbf{h} + \mathbf{u}) \pm \sqrt{[\mathbf{m}^T(\mathbf{V}\mathbf{h} + \mathbf{u})]^2 - \mathbf{m}^T\mathbf{V}\mathbf{m} \cdot g(\mathbf{h})}}{\mathbf{m}^T\mathbf{V}\mathbf{m}} \tag{0.9}$$

$$\Rightarrow k_i = \frac{-2\sqrt{3} \pm \sqrt{(2\sqrt{3})^2 - 4 \cdot 4 \cdot 3}}{4} = \frac{-2\sqrt{3} \pm \sqrt{12 - 12}}{4} = \frac{-2\sqrt{3}}{4} = \frac{-\sqrt{3}}{2} \tag{0.10}$$

Hence, the only real root is:

$$\mathbf{x} = \mathbf{h} + k_i\mathbf{m} = \begin{pmatrix} -\frac{\sqrt{3}}{2} \\ 0 \end{pmatrix} \tag{0.11}$$

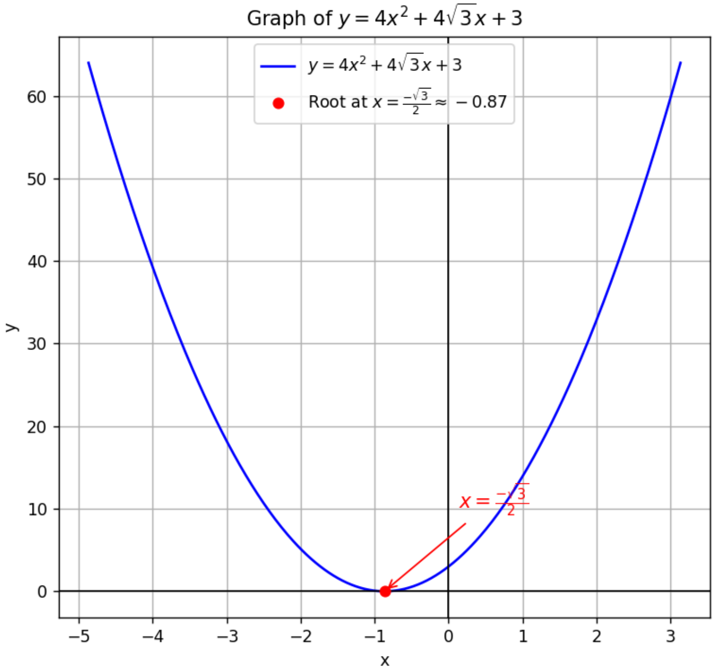

Fig. 0.1: Graph of $y = 4x^2 + 4\sqrt{3}x + 3$ showing a double root