2.3.9

EE25BTECH11006 - ADUDOTLA SRIVIDYA

Question

If vectors $\mathbf{a}$ and $\mathbf{b}$ are such that $|\mathbf{a}| = \frac{1}{2}$, $|\mathbf{b}| = \frac{4}{\sqrt{3}}$ and $|\mathbf{a} \times \mathbf{b}| = \frac{1}{\sqrt{3}}$, then find $\mathbf{a} \cdot \mathbf{b}$.

Solution

We know that

$$|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}| \sin\theta \tag{1}$$

$$\mathbf{a}^{\mathrm{T}}\mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos\theta \tag{2}$$

where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$.
Substitute the values:

$$\frac{1}{\sqrt{3}} = \left(\frac{1}{2}\right)\left(\frac{4}{\sqrt{3}}\right)\sin\theta \tag{3}$$

$$\sin\theta = \frac{1}{2} \tag{4}$$

Thus,

$$\theta = 30^\circ \quad \text{or} \quad 150^\circ \tag{5}$$

Now,

$$\mathbf{a}^{\mathrm{T}}\mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos\theta \tag{6}$$

$$= \left(\frac{1}{2}\right)\left(\frac{4}{\sqrt{3}}\right)\cos\theta \tag{7}$$

$$= \frac{2}{\sqrt{3}}\cos\theta \tag{8}$$

So, the possible values are:

$$\mathbf{a}^{\mathrm{T}}\mathbf{b} = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{2} = 1 \quad (\theta = 30^\circ) \tag{9}$$

$$\mathbf{a}^{\mathrm{T}}\mathbf{b} = \frac{2}{\sqrt{3}} \cdot \left(-\frac{\sqrt{3}}{2}\right) = -1 \quad (\theta = 150^\circ) \tag{10}$$

Therefore, $\mathbf{a} \cdot \mathbf{b} = \pm 1$.

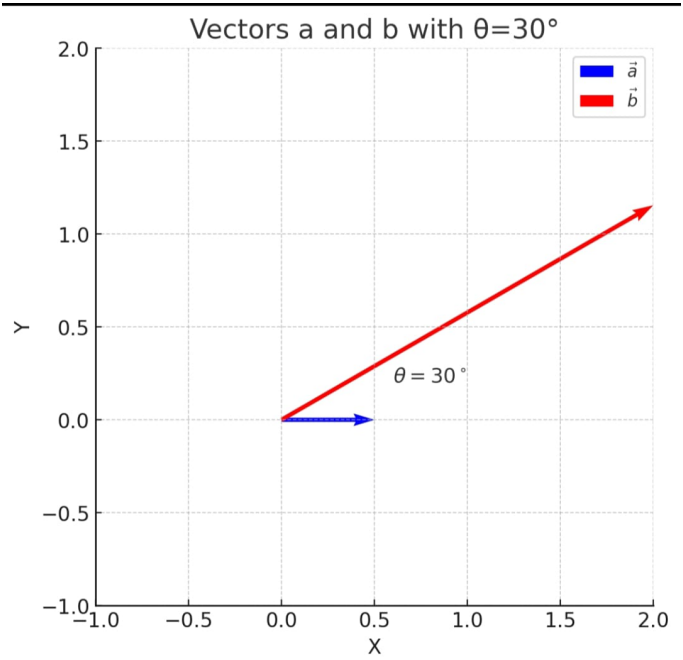

Fig. 1: Vectors **a** and **b** with angle θ