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EE25BTECH11043 - Nishid Khandagre

October 10, 2025

Question

If the characteristic polynomial and minimal polynomial of a square matrix $\mathbf{A}$ are $(\lambda - 1)(\lambda + 1)^4(\lambda - 2)^5$ and $(\lambda - 1)(\lambda + 1)(\lambda - 2)$, respectively, then the rank of the matrix $\mathbf{A} + \mathbf{I}$ is?

Solution

Given:

$$\chi_A(\lambda) = (\lambda - 1)(\lambda + 1)^4(\lambda - 2)^5 \tag{1}$$

$$m_A(\lambda) = (\lambda - 1)(\lambda + 1)(\lambda - 2) \tag{2}$$

Size of **A**=degree of χ_A

$$\deg \chi_A = 1 + 4 + 5 = 10 \tag{3}$$

Thus, **A** is a 10×10 matrix.

Solution

The minimal polynomial $m_A(\lambda)$ has simple roots (all linear factors with exponent 1).

$$m_A(\lambda) = (\lambda - 1)(\lambda + 1)(\lambda - 2) \tag{4}$$

Since all roots are distinct, the matrix $\mathbf{A}$ is diagonalizable.

Solution

Eigenvalues of $\mathbf{A}+\mathbf{I}$ and the zero-eigenspace:

If λ is an eigenvalue of $\mathbf{A}$, then $\lambda+1$ is an eigenvalue of $\mathbf{A}+\mathbf{I}$.

The eigenvalue 0 of $\mathbf{A}+\mathbf{I}$ corresponds to the eigenvalue -1 of $\mathbf{A}$.
From $\chi_A(\lambda)$, the algebraic multiplicity of $\lambda=-1$ is 4.

Since $\mathbf{A}$ is diagonalizable, the geometric multiplicity of $\lambda=-1$ is equal to its algebraic multiplicity, which is 4.

Therefore, the geometric multiplicity of 0 for $\mathbf{A}+\mathbf{I}$ is 4.

$$\text{nullity}(\mathbf{A}+\mathbf{I}) = \dim\ker(\mathbf{A}+\mathbf{I}) = 4 \tag{5}$$

Solution

Rank-nullity theorem:

$$\text{rank}(\mathbf{A}+\mathbf{I}) + \text{nullity}(\mathbf{A}+\mathbf{I}) = n \tag{6}$$

Here, $n = 10$ and $\text{nullity}(\mathbf{A}+\mathbf{I}) = 4$.

$$\text{rank}(\mathbf{A}+\mathbf{I}) = 10 - 4 \tag{7}$$

$$= 6 \tag{8}$$

Thus, the rank of the matrix $\mathbf{A}+\mathbf{I}$ is 6.