12.368

EE25BTECH11043 - Nishid Khandagre

Question: If the nullity of the matrix $\begin{pmatrix} k & 1 & 2 \\ 1 & -1 & -2 \\ 1 & 1 & 4 \end{pmatrix}$ is 1, then the value of k is ?

Solution:

$$\mathbf{A} = \begin{pmatrix} k & 1 & 2 \\ 1 & -1 & -2 \\ 1 & 1 & 4 \end{pmatrix} \tag{0.1}$$

Nullity = 1 for a 3×3 matrix means rank$(A) = 2$.

Perform row operations:
Swap R_1 and R_2:

$$\begin{pmatrix} 1 & -1 & -2 \\ k & 1 & 2 \\ 1 & 1 & 4 \end{pmatrix} \tag{0.2}$$

$R_2 \to R_2 - kR_1$, $R_3 \to R_3 - R_1$:

$$\begin{pmatrix} 1 & -1 & -2 \\ 0 & 1+k & 2+2k \\ 0 & 2 & 6 \end{pmatrix} \tag{0.3}$$

For the rank to be 2, the last two rows must be linearly dependent. Thus, there exists a scalar t such that:

$$1 + k = 2t \tag{0.4}$$
$$2 + 2k = 6t \tag{0.5}$$

$$2 + 2k = 3 + 3k \tag{0.6}$$
$$k = -1 \tag{0.7}$$

Thus, the value of k is -1.