10.5.10

EE25BTECH11043 - Nishid Khandagre

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Question

Construct a tangent to a circle of radius 4 cm from a point which is at a distance of 6 cm from its centre.

Theoretical Solution

Let the center of the circle be the origin $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$. The equation of the circle with radius $R = 4$ cm is:

$$\mathbf{C} \;:\; \mathbf{x}^\top \mathbf{V} \mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \;;\; \mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \;,\; \mathbf{u} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \;,\; f = -16 \tag{1}$$

Let the external point from which the tangent is drawn be $\mathbf{h}$. Given that the point is at a distance of 6 cm from the center, we can represent it as:

$$\mathbf{h} = \begin{pmatrix} 6 \\ 0 \end{pmatrix} \tag{2}$$

Theoretical Solution

Now, calculate the matrix Σ:

$$\Sigma = (\mathbf{Vh} + \mathbf{u})(\mathbf{Vh} + \mathbf{u})^\top - g(\mathbf{h})\mathbf{V} \tag{3}$$

$$g(\mathbf{h}) = \mathbf{h}^\top \mathbf{Vh} + 2\mathbf{u}^\top \mathbf{h} + f = \|\mathbf{h}\|^2 + f = 36 - 16 = 20 \tag{4}$$

$$\Sigma = \mathbf{hh}^\top - g(\mathbf{h})\mathbf{V} \tag{5}$$

$$= \begin{pmatrix} 6 \\ 0 \end{pmatrix} \begin{pmatrix} 6 & 0 \end{pmatrix} - 20 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{6}$$

$$= \begin{pmatrix} 16 & 0 \\ 0 & -20 \end{pmatrix} \tag{7}$$

Theoretical Solution

The eigenvalues of the matrix Σ are $\lambda_1 = 16$ and $\lambda_2 = -20$. The normalized eigenvectors form the matrix $\mathbf{P}$:

$$\mathbf{P} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{8}$$

The direction vectors of the two tangents are given by:

$$\mathbf{m} = \mathbf{P} \begin{pmatrix} \sqrt{|\lambda_2|} \\ \pm\sqrt{|\lambda_1|} \end{pmatrix} = \begin{pmatrix} 2\sqrt{5} \\ \pm 4 \end{pmatrix} \tag{9}$$

Theoretical Solution

The length of the tangent is given by

$$\|\mathbf{T} - \mathbf{h}\| = |\mu| \, \|\mathbf{m}\| \tag{10}$$

μ is a parameter

$$\mu = -\frac{\mathbf{m}^\top (\mathbf{V}\mathbf{h} + \mathbf{u})}{\|\mathbf{m}\|^2} = -\frac{\begin{pmatrix} 2\sqrt{5} & 4 \end{pmatrix} \begin{pmatrix} 6 \\ 0 \end{pmatrix}}{\left\| \begin{pmatrix} 2\sqrt{5} \\ 4 \end{pmatrix} \right\|^2} = -\frac{\sqrt{5}}{3} \tag{11}$$

$$\begin{aligned} \|\mathbf{T} - \mathbf{h}\| &= \frac{\sqrt{5}}{3} \times 6 && (12) \\ &= 2\sqrt{5} \approx 4.47 \text{ cm} && (13) \end{aligned}$$

C Code

```c
#include <math.h> // For sqrt

// Function to calculate the length of the tangent
void calculateTangentLength(double radius, double
    distance_from_center, double *tangent_length) {
    if (distance_from_center <= radius) {
        // If the point is inside or on the circle, no tangent
            can be drawn from the outside
        *tangent_length = 0.0; // Or handle as an error, for now
            set to 0
    } else {
        *tangent_length = sqrt(distance_from_center *
            distance_from_center - radius * radius);
    }
}
```

Python Code (using C shared output)

```
import ctypes
import numpy as np
import matplotlib.pyplot as plt

# Load the shared library
lib_tangent = ctypes.CDLL(./code17.so)

# Define argument types and return type for the C function
lib_tangent.calculateTangentLength.argtypes = [
    ctypes.c_double, # radius
    ctypes.c_double, # distance_from_center
    ctypes.POINTER(ctypes.c_double) # tangent_length (output)
]
lib_tangent.calculateTangentLength.restype = None
```

Python Code (using C shared output)

```
# Given values
radius_given = 4.0 # cm
distance_given = 6.0 # cm

# Create a ctypes double to hold the result
tangent_length_result = ctypes.c_double()

# Call the C function
lib_tangent.calculateTangentLength(
    radius_given,
    distance_given,
    ctypes.byref(tangent_length_result)
)
tangent_length = tangent_length_result.value
print(fThe length of the tangent calculated by C function is: {
    tangent_length:.2f} cm)
```

Python Code (using C shared output)

```
# --- Plotting the construction ---
center_x, center_y = 0, 0
point_x, point_y = distance_given, 0
plt.figure(figsize=(8, 8))
theta = np.linspace(0, 2 * np.pi, 200)
circle_x = center_x + radius_given * np.cos(theta)
circle_y = center_y + radius_given * np.sin(theta)
plt.plot(circle_x, circle_y, 'b-', label=f'Circle (Radius = {
    radius_given} cm)')
plt.scatter(center_x, center_y, color='green', s=100, zorder=5,
    label='Center (C)')
plt.annotate('C (0,0)', (center_x, center_y), textcoords=offset
    points, xytext=(5,5), ha='left')

plt.scatter(point_x, point_y, color='red', s=100, zorder=5, label
    =f'External Point (P) - {distance_given} cm from C')
plt.annotate(f'P ({point_x},{point_y})', (point_x, point_y),
    textcoords=offset points, xytext=(5,5), ha='left')
```

Python Code (using C shared output)

```
plt.plot([center_x, point_x], [center_y, point_y], 'g--', label=f
    'CP (Distance = {distance_given} cm)')

angle_CP = np.arctan2(point_y - center_y, point_x - center_x)
angle_CT_offset = np.arcsin(radius_given / distance_given)

T1_angle = angle_CP + angle_CT_offset
T2_angle = angle_CP - angle_CT_offset

T1_x = center_x + radius_given * np.cos(T1_angle)
T1_y = center_y + radius_given * np.sin(T1_angle)
T2_x = center_x + radius_given * np.cos(T2_angle)
T2_y = center_y + radius_given * np.sin(T2_angle)

plt.scatter(T1_x, T1_y, color='orange', s=100, zorder=5, label='
    Tangent Point (T1)')
plt.annotate(f'T1 ({T1_x:.2f},{T1_y:.2f})', (T1_x, T1_y),
    textcoords=offset points, xytext=(5,5), ha='left')
```

Python Code (using C shared output)

```
plt.scatter(T2_x, T2_y, color='orange', s=100, zorder=5, label='Tangent Point (T2)')
plt.annotate(f'T2 ({T2_x:.2f},{T2_y:.2f})', (T2_x, T2_y), textcoords=offset points, xytext=(5,5), ha='left')
plt.plot([point_x, T1_x], [point_y, T1_y], 'm-', label=f'Tangent PT1 (Length = {tangent_length:.2f} cm)')
plt.plot([point_x, T2_x], [point_y, T2_y], 'm-', label=f'Tangent PT2 (Length = {tangent_length:.2f} cm)')
plt.plot([center_x, T1_x], [center_y, T1_y], 'c:', label='Radius CT1 (Perpendicular to Tangent)')
plt.plot([center_x, T2_x], [center_y, T2_y], 'c:', label='Radius CT2 (Perpendicular to Tangent)')
```

Python Code (using C shared output)

```
plt.gca().set_aspect('equal', adjustable='box')
plt.xlabel('X-axis')
plt.ylabel('Y-axis')
plt.title('Construction of Tangents to a Circle (C function for
    length)')
plt.grid(True, linestyle='--')
plt.legend(bbox_to_anchor=(1.05, 1), loc='upper left')
plt.tight_layout()
plt.show()
```

Python Code (Direct)

```
import numpy as np
import numpy.linalg as LA
import matplotlib.pyplot as plt

def line_gen_num(A, B, num_points):
    A = A.flatten()
    B = B.flatten()
    t = np.linspace(0, 1, num_points)
    points = np.array([A[i]*(1-t) + B[i]*t for i in range(len(A))
        ])
    return points

def circ_gen(center, radius, num_points=100):
    center = center.flatten()
    theta = np.linspace(0, 2 * np.pi, num_points)
    x = center[0] + radius * np.cos(theta)
    y = center[1] + radius * np.sin(theta)
    return np.array([x, y])
```

Python Code (Direct)

```
radius = 4
distance_from_center = 6
C = np.array([0, 0]).reshape(-1, 1)
P_ext = np.array([distance_from_center, 0]).reshape(-1, 1)

if distance_from_center <= radius:
    print(Error: The external point is inside or on the circle.)
    tangent_length = 0
else:
    tangent_length = np.sqrt(distance_from_center**2 - radius**2)
    print(fThe length of the tangent is: {tangent_length:.2f} cm)

angle_PCT = np.arcsin(radius / distance_from_center)
vec_CP = P_ext - C
mag_CP = LA.norm(vec_CP)
unit_vec_CP = vec_CP / mag_CP
```

Python Code (Direct)

```
rot_matrix_pos = np.array([
    [np.cos(angle_PCT), -np.sin(angle_PCT)],
    [np.sin(angle_PCT), np.cos(angle_PCT)]
])
vec_CT1_unit = rot_matrix_pos @ unit_vec_CP
T1 = C + radius * vec_CT1_unit

rot_matrix_neg = np.array([
    [np.cos(-angle_PCT), -np.sin(-angle_PCT)],
    [np.sin(-angle_PCT), np.cos(-angle_PCT)]
])
vec_CT2_unit = rot_matrix_neg @ unit_vec_CP
T2 = C + radius * vec_CT2_unit
plt.figure(figsize=(9, 9))
x_circ = circ_gen(C, radius)
plt.plot(x_circ[0,:], x_circ[1,:], b-, label=fCircle (Radius={
    radius} cm))
```

Python Code (Direct)

```
plt.scatter(C[0], C[1], color='green', s=100, zorder=5, label='Center C')
plt.annotate(f'C({C[0,0]:.0f},{C[1,0]:.0f})', (C[0,0], C[1,0]), textcoords=offset points, xytext=(5,5), ha='left')
plt.scatter(P_ext[0], P_ext[1], color='red', s=100, zorder=5, label=f'External Point P ({distance_from_center} cm from C)')
plt.annotate(f'P({P_ext[0,0]:.0f},{P_ext[1,0]:.0f})', (P_ext[0,0], P_ext[1,0]), textcoords=offset points, xytext=(5,5), ha='left')

x_CP = line_gen_num(C, P_ext, 20)
plt.plot(x_CP[0,:], x_CP[1,:], g--, label=fLine CP (Length={distance_from_center} cm))

plt.scatter(T1[0], T1[1], color='orange', s=100, zorder=5, label='Tangent Point T1')
plt.annotate(f'T1({T1[0,0]:.2f},{T1[1,0]:.2f})', (T1[0,0], T1[1,0]), textcoords=offset points, xytext=(5,5), ha='left')
```

Python Code (Direct)

```
plt.scatter(T2[0], T2[1], color='orange', s=100, zorder=5, label=
    'Tangent Point T2')
plt.annotate(f'T2({T2[0,0]:.2f},{T2[1,0]:.2f})', (T2[0,0], T2
    [1,0]), textcoords=offset points, xytext=(5,5), ha='left')

x_PT1 = line_gen_num(P_ext, T1, 20)
plt.plot(x_PT1[0,:], x_PT1[1,:], m-, label=fTangent PT1 (Length={
    tangent_length:.2f} cm))

x_PT2 = line_gen_num(P_ext, T2, 20)
plt.plot(x_PT2[0,:], x_PT2[1,:], m-, label=fTangent PT2 (Length={
    tangent_length:.2f} cm))

x_CT1 = line_gen_num(C, T1, 20)
plt.plot(x_CT1[0,:], x_CT1[1,:], c:, label=Radius CT1)
```

Python Code (Direct)

```
x_CT2 = line_gen_num(C, T2, 20)
plt.plot(x_CT2[0,:], x_CT2[1,:], c:, label=Radius CT2)

plt.xlabel('$x$')
plt.ylabel('$y$')
plt.legend(loc='upper left', bbox_to_anchor=(1, 1))
plt.grid(True, linestyle='--')
plt.title(Construction of Tangents to a Circle)
plt.axis('equal')
plt.tight_layout()
plt.show()
```

Plot by Python using shared output from C

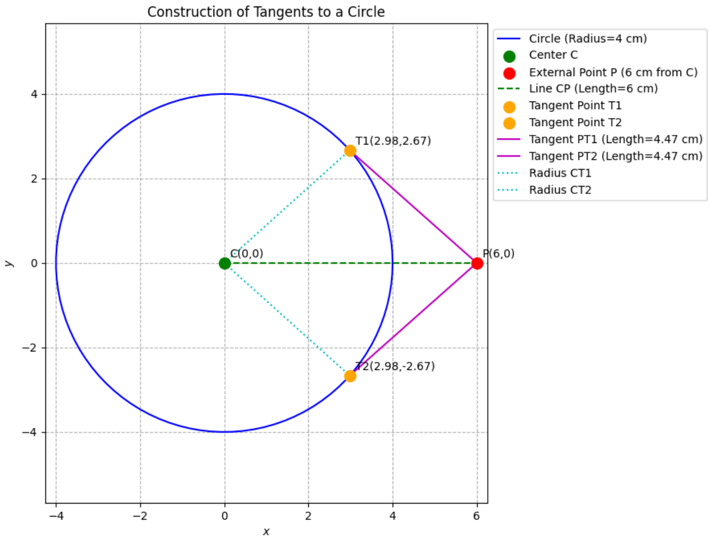
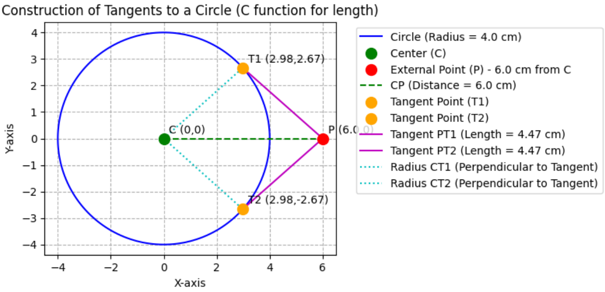


Figure:


Construction of Tangents to a Circle (C function for length)
T1 (2.98,2.67)
C (0,0)
P (6.0,0)
T2 (2.98,-2.67)
X-axis
Y-axis
Circle (Radius = 4.0 cm)
Center (C)
External Point (P) - 6.0 cm from C
CP (Distance = 6.0 cm)
Tangent Point (T1)
Tangent Point (T2)
Tangent PT1 (Length = 4.47 cm)
Tangent PT2 (Length = 4.47 cm)
Radius CT1 (Perpendicular to Tangent)
Radius CT2 (Perpendicular to Tangent)