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EE25BTECH11043 - Nishid Khandagre

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Question

If **A** is square symmetric real valued matrix of dimension 2n, the eigenvalues of **A** are

- a) 2n distinct real values numbers
- b) 2n real values, not necessarily distinct
- c) n distinct pairs of complex conjugate numbers
- d) n pairs of complex conjugate numbers, not necessarily distinct

Theoretical Solution

Solution: Let $\mathbf{A}$ be a real symmetric matrix of size $2n \times 2n$.

$$\mathbf{A} = \mathbf{A}^T = \mathbf{A}^* \tag{1}$$

Let $\mathbf{v}$ be an eigenvector and λ its eigenvalue:

$$\mathbf{A}\mathbf{v} = \lambda\mathbf{v} \tag{2}$$

Take Hermitian inner product of both sides with $\mathbf{v}$:

$$\mathbf{v}^*\mathbf{A}\mathbf{v} = \lambda\mathbf{v}^*\mathbf{v} \tag{3}$$

Theoretical Solution

Since $\mathbf{A} = \mathbf{A}^*$:

$$\begin{aligned} \mathbf{v}^*\mathbf{A}\mathbf{v} &= (\mathbf{A}\mathbf{v})^*\mathbf{v} && (4) \\ &= (\lambda\mathbf{v})^*\mathbf{v} && (5) \\ &= \overline{\lambda}\mathbf{v}^*\mathbf{v} && (6) \end{aligned}$$

Equating both expressions for $\mathbf{v}^*\mathbf{A}\mathbf{v}$:

$$\lambda\mathbf{v}^*\mathbf{v} = \overline{\lambda}\mathbf{v}^*\mathbf{v} \tag{7}$$

Since $\mathbf{v}^*\mathbf{v} > 0$ (as $\mathbf{v}$ is an eigenvector, it must be non-zero):

$$\begin{aligned} \lambda &= \overline{\lambda} && (8) \\ \implies \lambda &\text{ is real.} && (9) \end{aligned}$$

Theoretical Solution

Therefore, All eigenvalues are real.

A has 2n eigenvalues since its dimension is $2n \times 2n$. The eigenvalues may be repeated (not necessarily distinct).

Therefore, the correct option is (b).