10.7.91

EE25BTECH11043 - Nishid Khandagre

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Question

The point of intersection of the tangents at the ends of the latus rectum of the parabola $y^2 = 4x$ is

Solution

The general form of a conic section is:

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{1}$$

For the parabola $y^2 = 4x$, we can identify the matrices and vectors:

$$\mathbf{V} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \tag{2}$$

$$\mathbf{u} = \begin{pmatrix} -2 \\ 0 \end{pmatrix} \tag{3}$$

$$f = 0 \tag{4}$$

Solution

For a parabola $y^2 = 4ax$, the latus rectum endpoints are $(a, 2a)$ and $(a, -2a)$.
Given parabola is $y^2 = 4x$. Comparing with $y^2 = 4ax$, we find $4a = 4$, which means $a = 1$.
So, the endpoints of the latus rectum are:

$$\mathbf{q_1} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \tag{5}$$

$$\mathbf{q_2} = \begin{pmatrix} 1 \\ -2 \end{pmatrix} \tag{6}$$

Solution

The tangent at a point $\mathbf{q}$ on the conic is given by the formula:

$$\left(\mathbf{V}\mathbf{q} + \mathbf{u}\right)^\top \mathbf{x} + \mathbf{u}^\top \mathbf{q} + f = 0 \tag{7}$$

Let's apply this for the first endpoint $\mathbf{q_1} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$.

Solution

For $\mathbf{q_1} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$:

$$\mathbf{V}\mathbf{q_1} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix} \tag{8}$$

$$\mathbf{V}\mathbf{q_1} + \mathbf{u} = \begin{pmatrix} 0 \\ 2 \end{pmatrix} + \begin{pmatrix} -2 \\ 0 \end{pmatrix} = \begin{pmatrix} -2 \\ 2 \end{pmatrix} \tag{9}$$

$$\mathbf{u}^\top \mathbf{q_1} = \begin{pmatrix} -2 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix} = -2 \tag{10}$$

$$f = 0 \tag{11}$$

The tangent at $\mathbf{q_1}$ is:

$$\begin{pmatrix} -2 \\ 2 \end{pmatrix}^\top \mathbf{x} - 2 = 0 \tag{12}$$

$$\begin{pmatrix} -1 \\ 1 \end{pmatrix}^\top \mathbf{x} = 1 \tag{13}$$

Solution

For $\mathbf{q_2} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$:

$$\mathbf{V}\mathbf{q_2} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ -2 \end{pmatrix} = \begin{pmatrix} 0 \\ -2 \end{pmatrix} \tag{14}$$

$$\mathbf{V}\mathbf{q_2} + \mathbf{u} = \begin{pmatrix} 0 \\ -2 \end{pmatrix} + \begin{pmatrix} -2 \\ 0 \end{pmatrix} = \begin{pmatrix} -2 \\ -2 \end{pmatrix} \tag{15}$$

$$\mathbf{u}^\top \mathbf{q_2} = \begin{pmatrix} -2 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ -2 \end{pmatrix} = -2 \tag{16}$$

$$f = 0 \tag{17}$$

The tangent at $\mathbf{q_2}$ is:

$$\begin{pmatrix} -2 \\ -2 \end{pmatrix}^\top \mathbf{x} - 2 = 0 \tag{18}$$

$$\begin{pmatrix} 1 \\ 1 \end{pmatrix}^\top \mathbf{x} = -1 \tag{19}$$

Solution

Intersection of the two tangents:

$$\begin{pmatrix} -1 \\ 1 \end{pmatrix}^\top \mathbf{x} = 1 \tag{20}$$

$$\begin{pmatrix} 1 \\ 1 \end{pmatrix}^\top \mathbf{x} = -1 \tag{21}$$

$$\begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{22}$$

$$\left(\begin{array}{cc|c} -1 & 1 & 1 \\ 1 & 1 & -1 \end{array}\right) \tag{23}$$

$R_1 \to -R_1$

$$\left(\begin{array}{cc|c} 1 & -1 & -1 \\ 1 & 1 & -1 \end{array}\right) \tag{24}$$

Solution

$R_2 \to R_2 - R_1$

$$\left(\begin{array}{cc|c} 1 & -1 & -1 \\ 0 & 2 & 0 \end{array}\right) \tag{25}$$

$R_2 \to R_2/2,\ R_1 \to R_1 + R_2$

$$\left(\begin{array}{cc|c} 1 & 0 & -1 \\ 0 & 1 & 0 \end{array}\right) \tag{26}$$

$$\mathbf{x} = \begin{pmatrix} -1 \\ 0 \end{pmatrix} \tag{27}$$

Therefore, the point of intersection is $\begin{pmatrix} -1 \\ 0 \end{pmatrix}$.

C Code

```c
#include <stdio.h>

// Function to find the point of intersection of tangents at the
    ends of the latus rectum
// For y^2 = 4ax, the ends of the latus rectum are (a, 2a) and (a
    , -2a)
// The tangents are:
// at (a, 2a): y(2a) = 2a(x + a) => 2ay = 2ax + 2a^2 => y = x + a
// at (a, -2a): y(-2a) = 2a(x + a) => -2ay = 2ax + 2a^2 => -y = x
     + a => y = -x - a
// x + a = -x - a => 2x = -2a => x = -a
// Substitute x = -a into y = x + a => y = -a + a => y = 0
void findIntersectionOfTangents(double a_param, double *
    intersect_x, double *intersect_y) {
    *intersect_x = -a_param;
    *intersect_y = 0.0;
}
```

Python Code through shared output

```
import ctypes
import numpy as np
import matplotlib.pyplot as plt

# Load the shared library
lib_code = ctypes.CDLL(/Users/nishidkhandagre/matgeo/venv/bin/
    code18.so)
# Define the argument types and return type for the C function
lib_code.findIntersectionOfTangents.argtypes = [
    ctypes.c_double, # a_param
    ctypes.POINTER(ctypes.c_double), # intersect_x
    ctypes.POINTER(ctypes.c_double) # intersect_y
]
lib_code.findIntersectionOfTangents.restype = None

# Given parabola: y^2 = 4x
# Comparing with y^2 = 4ax, we get 4a = 4, so a = 1
a_value = 1.0
```

Python Code through shared output

```python
# Create ctypes doubles to hold the results
intersect_x_result = ctypes.c_double()
intersect_y_result = ctypes.c_double()

# Call the C function to find the point of intersection
lib_code.findIntersectionOfTangents(
    a_value,
    ctypes.byref(intersect_x_result),
    ctypes.byref(intersect_y_result)
)

intersection_x = intersect_x_result.value
intersection_y = intersect_y_result.value

print(fFor the parabola y^2 = 4x (where a = {a_value}):)
print(fThe point of intersection of the tangents at the ends of
    the latus rectum is ({intersection_x:.2f}, {intersection_y:.2
    f}))
```

Python Code through shared output

```
# --- Plotting Section ---
# Generate points for the parabola y^2 = 4x
y_parabola = np.linspace(-4, 4, 400)
x_parabola = (y_parabola**2) / 4

# Ends of the latus rectum
latus_rectum_end1_x, latus_rectum_end1_y = a_value, 2 * a_value
latus_rectum_end2_x, latus_rectum_end2_y = a_value, -2 * a_value

# Tangent equations derived from the problem (y = x + a and y = -x - a)
# For y = x + a (tangent at (a, 2a))
x_tangent1 = np.linspace(-3, 3, 100)
y_tangent1 = x_tangent1 + a_value

# For y = -x - a (tangent at (a, -2a))
x_tangent2 = np.linspace(-3, 3, 100)
y_tangent2 = -x_tangent2 - a_value
```

Python Code through shared output

```
plt.figure(figsize=(10, 8))

# Plot the parabola
plt.plot(x_parabola, y_parabola, 'b-', label='Parabola $y^2 = 4x$
    ')
# Plot the focus
plt.scatter(a_value, 0, color='orange', s=100, zorder=5, label=f'
    Focus ({a_value}, 0)')
plt.annotate(f'F({a_value},0)', (a_value, 0), textcoords=offset
    points, xytext=(5,5), ha='left')
# Plot the latus rectum line
plt.plot([a_value, a_value], [-2*a_value, 2*a_value], 'k-', label
    ='Latus Rectum (x=1)')

# Plot the ends of the latus rectum
plt.scatter(latus_rectum_end1_x, latus_rectum_end1_y, color='red'
    , s=100, zorder=5, label=f'End L ({latus_rectum_end1_x},{
    latus_rectum_end1_y})')
```

```
plt.annotate(f'L({latus_rectum_end1_x},{latus_rectum_end1_y})', (
    latus_rectum_end1_x, latus_rectum_end1_y), textcoords=offset
    points, xytext=(5,5), ha='left')
plt.scatter(latus_rectum_end2_x, latus_rectum_end2_y, color='red'
    , s=100, zorder=5, label=f'End L\' ({latus_rectum_end2_x},{
    latus_rectum_end2_y})')
plt.annotate(f'L\'({latus_rectum_end2_x},{latus_rectum_end2_y})',
     (latus_rectum_end2_x, latus_rectum_end2_y), textcoords=
    offset points, xytext=(5,5), ha='right')

# Plot the tangents
plt.plot(x_tangent1, y_tangent1, 'g-', label=f'Tangent at L (y=x
    +{a_value})')
plt.plot(x_tangent2, y_tangent2, 'm-', label=f'Tangent at L\' (y
    =-x-{a_value})')
```

Python Code through shared output

```python
# Plot the intersection point
plt.scatter(intersection_x, intersection_y, color='purple', s=100, zorder=6, label=f'Intersection ({intersection_x:.2f},{intersection_y:.2f})')
plt.annotate(f'P({intersection_x:.2f},{intersection_y:.2f})', (intersection_x, intersection_y), textcoords=offset points, xytext=(10, -15), ha='left', color='purple', fontsize=12)
plt.xlim(-3, 6)
plt.ylim(-4, 4)
plt.gca().set_aspect('equal', adjustable='box')
plt.xlabel('X-axis')
plt.ylabel('Y-axis')
plt.title('Parabola, Latus Rectum, Tangents, and Intersection Point')
plt.grid(True)
plt.legend()
plt.axhline(0, color='gray', linewidth=0.5)
plt.axvline(0, color='gray', linewidth=0.5)
plt.show()
```

```
import numpy as np
import matplotlib.pyplot as plt

def find_intersection_of_tangents_at_latus_rectum_ends(a_param):
    #the ends of the latus rectum are:
    # L1 = (a, 2a)
    # L2 = (a, -2a)
    # 2. Determine the equations of the tangents at these points
    # The general equation of a tangent to y^2 = 4ax at a point (
        x1, y1) is:
    # y * y1 = 2a * (x + x1)
    # Tangent at L1 (a, 2a):
    # y * (2a) = 2a * (x + a)
    # y = x + a (Equation for Tangent 1)

    # Tangent at L2 (a, -2a):
    # y * (-2a) = 2a * (x + a)
    # y = -x - a (Equation for Tangent 2)
```

Python Code : Direct

```
    # Solving the system:
    # x + a = -x - a
    # x = -a

    # Substitute x = -a
    # y = (-a) + a
    # y = 0

    x_intersect = -a_param
    y_intersect = 0.0
    return x_intersect, y_intersect

# --- Main execution and plotting ---
# Given parabola: y^2 = 4x
# 4a = 4 => a = 1
a_value = 1.0
```

Python Code : Direct

```python
# Calculate the intersection point using the Python function
intersection_x, intersection_y = find_intersection_of_tangents_at_latus_rectum_ends(a_value)

print(fFor the parabola y^2 = 4x (where a = {a_value}):)
print(fThe point of intersection of the tangents at the ends of the latus rectum is ({intersection_x:.2f}, {intersection_y:.2f}))

# --- Plotting Section ---
plt.figure(figsize=(10, 8))

# 1. Plot the parabola y^2 = 4x
y_parabola = np.linspace(-4, 4, 400)
x_parabola = (y_parabola**2) / (4 * a_value)
plt.plot(x_parabola, y_parabola, 'b-', label=f'Parabola $y^2 = 4x$')
```

Python Code : Direct

```python
# 2. Plot the focus
focus_x, focus_y = a_value, 0
plt.scatter(focus_x, focus_y, color='orange', s=100, zorder=5, label=f'Focus ({focus_x}, {focus_y})')
plt.annotate(f'F({focus_x},{focus_y})', (focus_x, focus_y), textcoords=offset points, xytext=(5,5), ha='left')

# 3. Plot the latus rectum line
latus_rectum_x_val = a_value
latus_rectum_y_min, latus_rectum_y_max = -2 * a_value, 2 * a_value
plt.plot([latus_rectum_x_val, latus_rectum_x_val], [latus_rectum_y_min, latus_rectum_y_max], 'k-', label='Latus Rectum (x=1)')
```

Python Code : Direct

```
# 4. Plot the ends of the latus rectum
latus_rectum_end1 = (a_value, 2 * a_value)
latus_rectum_end2 = (a_value, -2 * a_value)
plt.scatter(latus_rectum_end1[0], latus_rectum_end1[1], color='red', s=100, zorder=5, label=f'End L ({latus_rectum_end1[0]},{latus_rectum_end1[1]})')
plt.annotate(f'L({latus_rectum_end1[0]},{latus_rectum_end1[1]})', latus_rectum_end1, textcoords=offset points, xytext=(5,5), ha='left')
plt.scatter(latus_rectum_end2[0], latus_rectum_end2[1], color='red', s=100, zorder=5, label=f'End L\' ({latus_rectum_end2[0]},{latus_rectum_end2[1]})')
plt.annotate(f'L\'({latus_rectum_end2[0]},{latus_rectum_end2[1]})', latus_rectum_end2, textcoords=offset points, xytext=(5,-5), ha='right')
```

Python Code : Direct

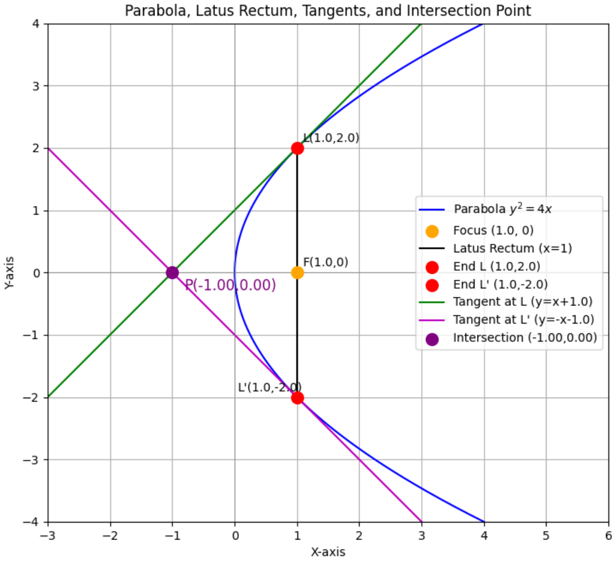
```
# 5. Plot the tangents
x_tangent_range = np.linspace(-3, 3, 100)
y_tangent1 = x_tangent_range + a_value
y_tangent2 = -x_tangent_range - a_value

plt.plot(x_tangent_range, y_tangent1, 'g-', label=f'Tangent at L (y=x+{a_value})')
plt.plot(x_tangent_range, y_tangent2, 'm-', label=f'Tangent at L\' (y=-x-{a_value})')

# 6. Plot the intersection point
plt.scatter(intersection_x, intersection_y, color='purple', s=100, zorder=6, label=f'Intersection ({intersection_x:.2f},{intersection_y:.2f})')
plt.annotate(f'P({intersection_x:.2f},{intersection_y:.2f})', (intersection_x, intersection_y), textcoords=offset points, xytext=(10, -15), ha='left', color='purple', fontsize=12)
```

Python Code : Direct

```python
# --- Plot Aesthetics ---
plt.xlim(-3, 6)
plt.ylim(-4, 4)
plt.gca().set_aspect('equal', adjustable='box')
plt.xlabel('X-axis')
plt.ylabel('Y-axis')
plt.title('Parabola, Latus Rectum, Tangents, and Intersection
    Point')
plt.grid(True)
plt.legend()
plt.axhline(0, color='gray', linewidth=0.5)
plt.axvline(0, color='gray', linewidth=0.5)
plt.show()
```


Parabola, Latus Rectum, Tangents, and Intersection Point
L(1.0,2.0)
F(1.0,0)
P(-1.00,0.00)
L'(1.0,-2.0)
Parabola $y^2 = 4x$
Focus (1.0, 0)
Latus Rectum (x=1)
End L (1.0,2.0)
End L' (1.0,-2.0)
Tangent at L (y=x+1.0)
Tangent at L' (y=-x-1.0)
Intersection (-1.00,0.00)
X-axis
Y-axis

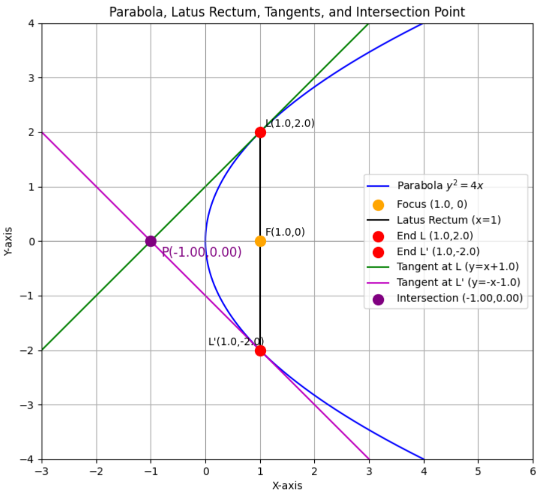

Parabola, Latus Rectum, Tangents, and Intersection Point
L(1.0,2.0)
F(1.0,0)
P(-1.00,0.00)
L'(1.0,-2.0)
Parabola $y^2 = 4x$
Focus (1.0, 0)
Latus Rectum (x=1)
End L (1.0,2.0)
End L' (1.0,-2.0)
Tangent at L (y=x+1.0)
Tangent at L' (y=-x-1.0)
Intersection (-1.00,0.00)
X-axis
Y-axis

Figure: