

12.264

EE25BTECH11043 - Nishid Khandagre

Question: Consider the system of equations

$$2x_1 + x_2 + x_3 = 0$$

$$x_2 - x_3 = 0$$

$$x_1 + x_2 = 0$$

This system has

- a) a unique solution
- b) no solution
- c) infinite number of solutions
- d) five solutions

Solution:

$$\mathbf{Ax} = \mathbf{0} \tag{0.1}$$

$$\mathbf{A} = \begin{pmatrix} 2 & 1 & 1 \\ 0 & 1 & -1 \\ 1 & 1 & 0 \end{pmatrix} \tag{0.2}$$

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \tag{0.3}$$

Augmented matrix:

$$\left(\begin{array}{ccc|c} 2 & 1 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 1 & 1 & 0 & 0 \end{array} \right) \tag{0.4}$$

$$R_1 \rightarrow \frac{1}{2}R_1$$

$$\left(\begin{array}{ccc|c} 1 & 0.5 & 0.5 & 0 \\ 0 & 1 & -1 & 0 \\ 1 & 1 & 0 & 0 \end{array} \right) \tag{0.5}$$

$$R_3 \rightarrow R_3 - R_1$$

$$\left(\begin{array}{ccc|c} 1 & 0.5 & 0.5 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0.5 & -0.5 & 0 \end{array} \right) \tag{0.6}$$

$$R_3 \rightarrow R_3 - 0.5R_2$$

$$\left(\begin{array}{ccc|c} 1 & 0.5 & 0.5 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right) \quad (0.7)$$

$$x_1 + 0.5x_2 + 0.5x_3 = 0 \quad (0.8)$$

$$x_2 - x_3 = 0 \quad (0.9)$$

$$x_2 = x_3 \quad (0.10)$$

Substitute into the first equation:

$$x_1 + 0.5x_2 + 0.5x_2 = 0 \quad (0.11)$$

$$x_1 + x_2 = 0 \quad (0.12)$$

$$x_1 = -x_2 \quad (0.13)$$

Let $x_2 = t$. Then $x_3 = t$, and $x_1 = -t$.

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = t \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}, \quad t \in \mathbb{R} \quad (0.14)$$

Since there is one free parameter t , the system has infinitely many solutions.

Also $\text{rank}(A) < 3$, and the system is consistent (it has solutions) therefore the system has infinitely many solutions. The answer is option(c).

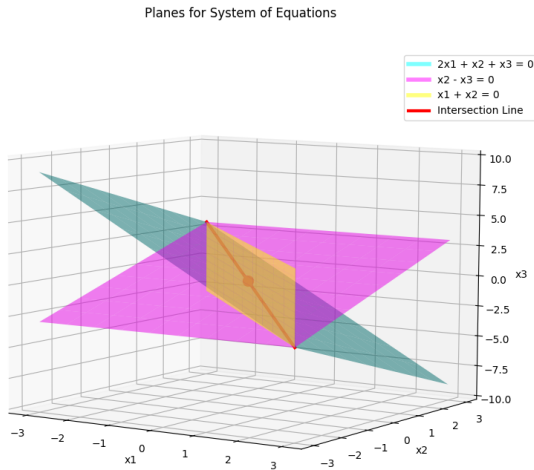


Fig. 0.1