12.888

EE25BTECH11043 - Nishid Khandagre

Question: Which one of the following matrices has eigenvalues 1 and 6?

- a) $\begin{pmatrix} 5 & -2 \\ -2 & 2 \end{pmatrix}$
- b) $\begin{pmatrix} 3 & -1 \\ -2 & 2 \end{pmatrix}$
- c) $\begin{pmatrix} 3 & -1 \\ -1 & 2 \end{pmatrix}$
- d) $\begin{pmatrix} 2 & -1 \\ -1 & 3 \end{pmatrix}$

Solution:

To find the eigenvalues λ of a matrix $\mathbf{M}$, we solve the characteristic equation

$$\det\left(\mathbf{M} - \lambda\mathbf{I}\right) = 0 \tag{0.1}$$

For $\mathbf{A} = \begin{pmatrix} 5 & -2 \\ -2 & 2 \end{pmatrix}$

$$\det\left(\mathbf{A} - \lambda\mathbf{I}\right) = \begin{vmatrix} 5 - \lambda & -2 \\ -2 & 2 - \lambda \end{vmatrix} \tag{0.2}$$

$$= (5 - \lambda)(2 - \lambda) - (-2)(-2) \tag{0.3}$$

$$= \lambda^2 - 7\lambda + 6 \tag{0.4}$$

$$\tag{0.5}$$

$$\lambda^2 - 7\lambda + 6 = 0 \tag{0.6}$$

$$(\lambda - 1)(\lambda - 6) = 0 \tag{0.7}$$

$$\lambda = 1, 6 \tag{0.8}$$

The eigenvalues are 1 and 6. This matches the requirement.

For $\mathbf{B} = \begin{pmatrix} 3 & -1 \\ -2 & 2 \end{pmatrix}$

$$\det\left(\mathbf{B} - \lambda\mathbf{I}\right) = \begin{vmatrix} 3 - \lambda & -1 \\ -2 & 2 - \lambda \end{vmatrix} \tag{0.9}$$

$$= (3 - \lambda)(2 - \lambda) - (-1)(-2) \tag{0.10}$$

$$= \lambda^2 - 5\lambda + 4 \tag{0.11}$$

$$\tag{0.12}$$

$$\lambda^2 - 5\lambda + 4 = 0 \tag{0.13}$$

$$(\lambda - 1)(\lambda - 4) = 0 \tag{0.14}$$

$$\lambda = 1, 4 \tag{0.15}$$

The eigenvalues are 1 and 4. This does not match the requirement.

For $\mathbf{C} = \begin{pmatrix} 3 & -1 \\ -1 & 2 \end{pmatrix}$

$$\det\left(\mathbf{C} - \lambda\mathbf{I}\right) = \begin{vmatrix} 3 - \lambda & -1 \\ -1 & 2 - \lambda \end{vmatrix} \tag{0.16}$$

$$= (3 - \lambda)(2 - \lambda) - (-1)(-1) \tag{0.17}$$

$$= \lambda^2 - 5\lambda + 5 \tag{0.18}$$

$$\tag{0.19}$$

$$\lambda^2 - 5\lambda + 5 = 0 \tag{0.20}$$

$$\lambda = \frac{5 \pm \sqrt{5}}{2} \tag{0.21}$$

The eigenvalues are $\frac{5+\sqrt{5}}{2}$ and $\frac{5-\sqrt{5}}{2}$. This does not match the requirement.

For $\mathbf{D} = \begin{pmatrix} 2 & -1 \\ -1 & 3 \end{pmatrix}$

$$\det\left(\mathbf{D} - \lambda\mathbf{I}\right) = \begin{vmatrix} 2 - \lambda & -1 \\ -1 & 3 - \lambda \end{vmatrix} \tag{0.22}$$

$$= (2 - \lambda)(3 - \lambda) - (-1)(-1) \tag{0.23}$$

$$= \lambda^2 - 5\lambda + 5 \tag{0.24}$$

$$\tag{0.25}$$

$$\lambda^2 - 5\lambda + 5 = 0 \tag{0.26}$$

$$\lambda = \frac{5 \pm \sqrt{5}}{2} \tag{0.27}$$

The eigenvalues are $\frac{5+\sqrt{5}}{2}$ and $\frac{5-\sqrt{5}}{2}$. This does not match the requirement.

Only matrix $\mathbf{A}$ has eigenvalues 1 and 6.