

10.5.10

EE25BTECH11043 - Nishid Khandagre

Question: Construct a tangent to a circle of radius 4 cm from a point which is at a distance of 6 cm from its centre.

Solution: Let the center of the circle be the origin $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$. The equation of the circle with radius $R = 4$ cm is:

$$\mathbf{C} : \mathbf{x}^\top \mathbf{V} \mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 ; \mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, f = -R^2 = -16 \quad (0.1)$$

Let the external point from which the tangent is drawn be \mathbf{h} . Given that the point is at a distance of 6 cm from the center, we can represent it as:

$$\mathbf{h} = \begin{pmatrix} 6 \\ 0 \end{pmatrix} \quad (0.2)$$

Now, calculate the matrix Σ :

$$\Sigma = (\mathbf{V}\mathbf{h} + \mathbf{u})(\mathbf{V}\mathbf{h} + \mathbf{u})^\top - g(\mathbf{h})\mathbf{V} \quad (0.3)$$

$$g(\mathbf{h}) = \mathbf{h}^\top \mathbf{V} \mathbf{h} + 2\mathbf{u}^\top \mathbf{h} + f = \|\mathbf{h}\|^2 + f = 36 - 16 = 20 \quad (0.4)$$

$$\Sigma = \mathbf{h}\mathbf{h}^\top - g(\mathbf{h})\mathbf{V} \quad (0.5)$$

$$= \begin{pmatrix} 6 \\ 0 \end{pmatrix} \begin{pmatrix} 6 & 0 \end{pmatrix} - 20 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad (0.6)$$

$$= \begin{pmatrix} 16 & 0 \\ 0 & -20 \end{pmatrix} \quad (0.7)$$

The eigenvalues of Σ are $\lambda_1 = 16$ and $\lambda_2 = -20$. The normalized eigenvectors form the matrix \mathbf{P} :

$$\mathbf{P} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad (0.8)$$

The direction vectors of the two tangents are given by:

$$\mathbf{m} = \mathbf{P} \begin{pmatrix} \sqrt{|\lambda_2|} \\ \pm \sqrt{|\lambda_1|} \end{pmatrix} = \begin{pmatrix} 2\sqrt{5} \\ \pm 4 \end{pmatrix} \quad (0.9)$$

The length of the tangent is given by

$$\|\mathbf{T} - \mathbf{h}\| = |\mu| \|\mathbf{m}\| \quad (0.10)$$

μ is a parameter

$$\mu = -\frac{\mathbf{m}^T (\mathbf{V}\mathbf{h} + \mathbf{u})}{\|\mathbf{m}\|^2} = -\frac{\begin{pmatrix} 2\sqrt{5} & 4 \end{pmatrix} \begin{pmatrix} 6 \\ 0 \end{pmatrix}}{\left\| \begin{pmatrix} 2\sqrt{5} \\ 4 \end{pmatrix} \right\|^2} = -\frac{\sqrt{5}}{3} \quad (0.11)$$

$$\|\mathbf{T} - \mathbf{h}\| = \frac{\sqrt{5}}{3} \times 6 \quad (0.12)$$

$$= 2\sqrt{5} \approx 4.47 \text{ cm} \quad (0.13)$$

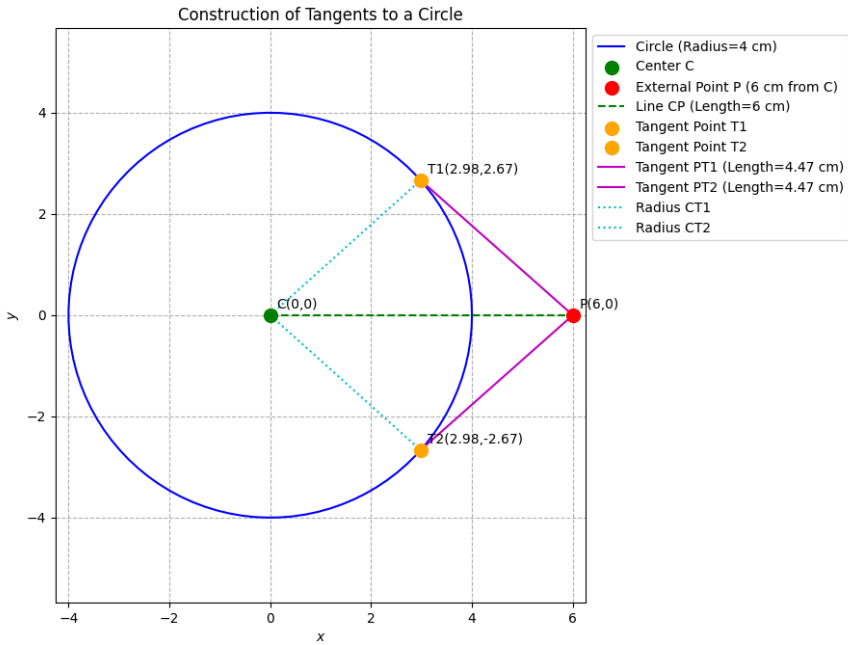


Fig. 0.1