12.472

EE25BTECH11043 - Nishid Khandagre

Question: A rigid ball of weight 100 N is suspended with the help of a string. The ball is pulled by a horizontal force F such that the string makes an angle of 30° with the vertical. The magnitude of force F (in N) is

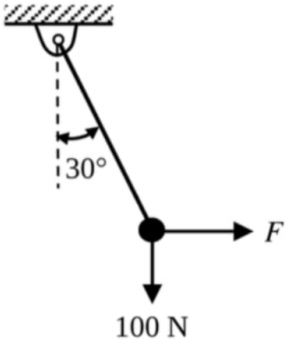

Fig. 0.1

Solution: Let T be the tension in the string and F the horizontal force. Equilibrium of the ball gives the linear system:

$$T\sin 30^\circ - F = 0, \tag{0.1}$$

$$T\cos 30^\circ - 100 = 0. \tag{0.2}$$

$$\frac{1}{2}T - F = 0, \tag{0.3}$$

$$\frac{\sqrt{3}}{2}T = 100. \tag{0.4}$$

Writing this in matrix form $\mathbf{Ax} = \mathbf{b}$ with $\mathbf{x} = \begin{pmatrix} T \\ F \end{pmatrix}$:

$$\begin{pmatrix} \frac{1}{2} & -1 \\ \frac{\sqrt{3}}{2} & 0 \end{pmatrix} \begin{pmatrix} T \\ F \end{pmatrix} = \begin{pmatrix} 0 \\ 100 \end{pmatrix} \tag{0.5}$$

Augmented matrix:

$$\left(\begin{array}{cc|c} \frac{1}{2} & -1 & 0 \\ \frac{\sqrt{3}}{2} & 0 & 100 \end{array}\right) \tag{0.6}$$

$R_1 \to 2R_1, R_2 \to 2R_2$

$$\left(\begin{array}{cc|c} 1 & -2 & 0 \\ \sqrt{3} & 0 & 200 \end{array}\right) \tag{0.7}$$

$R_2 \to R_2 - \sqrt{3}R_1$:

$$\left(\begin{array}{cc|c} 1 & -2 & 0 \\ 0 & 2\sqrt{3} & 200 \end{array}\right) \tag{0.8}$$

From the second row:

$$2\sqrt{3}F = 200 \tag{0.9}$$

$$F = \frac{100}{\sqrt{3}}. \tag{0.10}$$

Thus, the magnitude of force F is:

$$F = \frac{100}{\sqrt{3}} \text{ N}. \tag{0.11}$$

Numerically, $F \approx 57.7$ N.