12.680

EE25BTECH11043 - Nishid Khandagre

Question: The rank of matrix is

$$\begin{pmatrix} 1 & 2 & 2 & 3 \\ 3 & 4 & 2 & 5 \\ 5 & 6 & 2 & 7 \\ 7 & 8 & 2 & 9 \end{pmatrix}$$

Solution: Let the given matrix be $\mathbf{A}$:

$$\mathbf{A} = \begin{pmatrix} 1 & 2 & 2 & 3 \\ 3 & 4 & 2 & 5 \\ 5 & 6 & 2 & 7 \\ 7 & 8 & 2 & 9 \end{pmatrix} \tag{0.1}$$

$R_2 \to R_2 - 3R_1$, $R_3 \to R_3 - 5R_1$, $R_4 \to R_4 - 7R_1$

$$\begin{pmatrix} 1 & 2 & 2 & 3 \\ 0 & -2 & -4 & -4 \\ 0 & -4 & -8 & -8 \\ 0 & -6 & -12 & -12 \end{pmatrix} \tag{0.2}$$

$R_2 \to -\frac{1}{2}R_2$

$$\begin{pmatrix} 1 & 2 & 2 & 3 \\ 0 & 1 & 2 & 2 \\ 0 & -4 & -8 & -8 \\ 0 & -6 & -12 & -12 \end{pmatrix} \tag{0.3}$$

$R_3 \to R_3 + 4R_2$, $R_4 \to R_4 + 6R_2$, $R_1 \to R_1 - 2R_2$

$$\begin{pmatrix} 1 & 0 & -2 & -1 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{0.4}$$

The number of non-zero rows (pivot rows) in the row-echelon form is 2. Therefore, the rank of the matrix A is 2.