10.7.91

EE25BTECH11043 - Nishid Khandagre

Question: The point of intersection of the tangents at the ends of the latus rectum of the parabola $y^2 = 4x$ is

Solution: The general form of a conic section is

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{0.1}$$

$$\mathbf{V} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} -2 \\ 0 \end{pmatrix}, f = 0 \tag{0.2}$$

Latus rectum endpoints are $\begin{pmatrix} \text{a} \\ \pm 2\text{a} \end{pmatrix}$

For $y^2 = 4x$, $a = 1$.So the endpoints of the latus rectum are: $\mathbf{q_1} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$,$\mathbf{q_2} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$

The tangent at a point $\mathbf{q}$ on the conic is given by the formula:

$$\left(\mathbf{V}\mathbf{q} + \mathbf{u}\right)^\top \mathbf{x} + \mathbf{u}^\top \mathbf{q} + f = 0 \tag{0.3}$$

For $\mathbf{q_1} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$:

$$\mathbf{V}\mathbf{q_1} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix} \tag{0.4}$$

$$\mathbf{V}\mathbf{q_1} + \mathbf{u} = \begin{pmatrix} 0 \\ 2 \end{pmatrix} + \begin{pmatrix} -2 \\ 0 \end{pmatrix} = \begin{pmatrix} -2 \\ 2 \end{pmatrix} \tag{0.5}$$

$$\mathbf{u}^\top \mathbf{q_1} = \begin{pmatrix} -2 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \end{pmatrix} = -2 \tag{0.6}$$

$$f = 0 \tag{0.7}$$

The tangent at $\mathbf{q_1}$ is:

$$\begin{pmatrix} -2 \\ 2 \end{pmatrix}\mathbf{x} - 2 = 0 \tag{0.8}$$

y = x+1.

For $\mathbf{q_2} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$:

$$\mathbf{V}\mathbf{q_2} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ -2 \end{pmatrix} = \begin{pmatrix} 0 \\ -2 \end{pmatrix} \tag{0.9}$$

$$\mathbf{V}\mathbf{q_2} + \mathbf{u} = \begin{pmatrix} 0 \\ -2 \end{pmatrix} + \begin{pmatrix} -2 \\ 0 \end{pmatrix} = \begin{pmatrix} -2 \\ -2 \end{pmatrix} \tag{0.10}$$

$$\mathbf{u}^\top \mathbf{q_2} = \begin{pmatrix} -2 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ -2 \end{pmatrix} = -2 \tag{0.11}$$

$$f = 0 \tag{0.12}$$

The tangent at $\mathbf{q_2}$ is:

$$\begin{pmatrix} -2 \\ -2 \end{pmatrix} \mathbf{x} - 2 = 0 x + y + 1 = 0 \quad \Rightarrow \quad y = -x - 1 \tag{0.13}$$

Intersection of the two tangents:
We solve the system of linear equations from **??** and **??**:

$$y = x + 1 \tag{0.14}$$

$$y = -x - 1 \tag{0.15}$$

Equating the right-hand sides: $x + 1 = -x - 1$ $2x = -2$ $x = -1$ Substitute $x = -1$ into the first equation: $y = (-1) + 1 = 0$

Therefore, the point of intersection is $\begin{pmatrix} -1 \\ 0 \end{pmatrix}$.

The point of intersection of the tangents at the ends of the latus rectum of the parabola $y^2 = 4x$ is $\boxed{(-1, 0)}$.

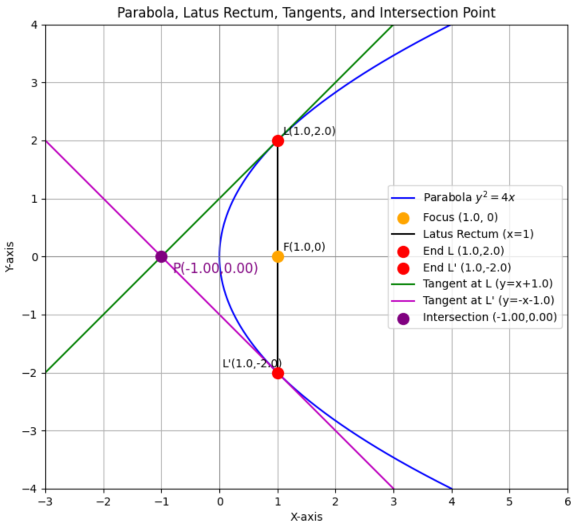
Parabola, Latus Rectum, Tangents, and Intersection Point
Parabola $y^2 = 4x$
Focus (1.0, 0)
Latus Rectum (x=1)
End L (1.0,2.0)
End L' (1.0,-2.0)
Tangent at L (y=x+1.0)
Tangent at L' (y=-x-1.0)
Intersection (-1.00,0.00)
L(1.0,2.0)
F(1.0,0)
L'(1.0,-2.0)
P(-1.00,0.00)
X-axis
Y-axis

Fig. 0.1