9.2.3

EE25BTECH11043 - Nishid Khandagre

Question: Draw a rough sketch of the given curve $y = 1 + |x + 1|$, $x = -3$, $x = 3$, $y = 0$, and find the area of the region bounded by them, using integration.

Solution: Given the curve $y = 1 + |x + 1|$.

- For $x < -1$: $|x + 1| = -(x + 1)$.

$$y = 1 - (x + 1) \tag{0.1}$$
$$y = -x \tag{0.2}$$

$$\begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 0 \tag{0.3}$$

- For $x \geq -1$: $|x + 1| = x + 1$.

$$y = 1 + (x + 1) \tag{0.4}$$
$$y = x + 2 \tag{0.5}$$

$$\begin{pmatrix} 1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = -2 \tag{0.6}$$

At $x = -3$: $y = -(-3) = 3$.

At $x = -1$: For $y = -x$, $y = 1$. For $y = x + 2$, $y = (-1) + 2 = 1$.
Both pieces meet at $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$.

At $x = 3$: $y = 3 + 2 = 5$.

The region is bounded by $y = -x$ from $\begin{pmatrix} -3 \\ 3 \end{pmatrix}$ to $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$ and $y = x + 2$ from $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$ to $\begin{pmatrix} 3 \\ 5 \end{pmatrix}$ and by the lines $x = -3$, $x = 3$, and $y = 0$.

Area calculation for the left piece: For $y = -x$

$$\text{Area}_1 = \int_{-3}^{-1} -x\,dx \tag{0.7}$$
$$= \begin{pmatrix} -1 & 0 \end{pmatrix} \left(\begin{pmatrix} \frac{x^2}{2} \\ x \end{pmatrix} \right) \Big|_{-3}^{-1} \tag{0.8}$$
$$= \left[-\frac{x^2}{2} \right]_{-3}^{-1} \tag{0.9}$$
$$= -\frac{1}{2} + \frac{9}{2} \tag{0.10}$$
$$= 4 \tag{0.11}$$

Area calculation for the right piece: For $y = x + 2$

$$\text{Area}_2 = \int_{-1}^{3} (x+2)\,dx \tag{0.12}$$

$$= \begin{pmatrix} 1 & 2 \end{pmatrix} \left(\begin{pmatrix} \frac{x^2}{2} \\ x \end{pmatrix} \right) \Big|_{-1}^{3} \tag{0.13}$$

$$= \left[\frac{x^2}{2} + 2x \right]_{-1}^{3} \tag{0.14}$$

$$= \left(\frac{9}{2} + 6 \right) - \left(\frac{1}{2} - 2 \right) \tag{0.15}$$

$$= \frac{21}{2} + \frac{3}{2} \tag{0.16}$$

$$= 12 \tag{0.17}$$

The total area is the sum of the areas of the two pieces.

$$\text{Total Area} = \text{Area}_1 + \text{Area}_2 \tag{0.18}$$

$$= 4 + 12 \tag{0.19}$$

$$= 16 \tag{0.20}$$

Thus, the total area of the region bounded by the curves is 16 square units.

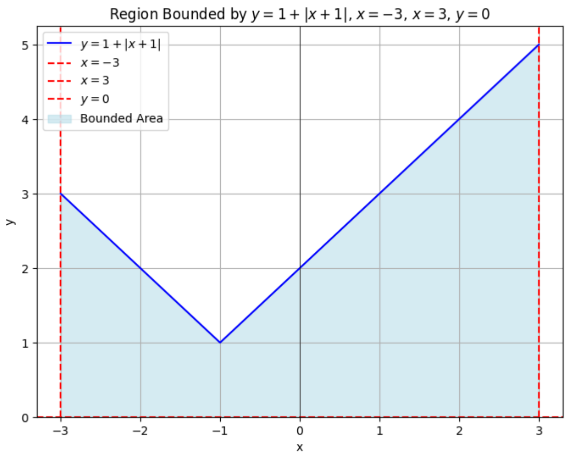

Fig. 0.1