2.3.13

EE25BTECH11010 - Arsh Dhoke

Question:
Find the angle which the line $\frac{x}{1} = \frac{y}{-1} = \frac{z}{2}$ makes with the positive direction of the Y axis.

Solution:

The line can be represented as $k\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$

Hence its direction vector is

$$\mathbf{v} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} \tag{0.1}$$

$$\mathbf{e_2} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \tag{0.2}$$

$$\mathbf{v}^T \mathbf{e_2} = \begin{pmatrix} 1 & -1 & 2 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = -1 \tag{0.3}$$

$$\|\mathbf{v}\| = \sqrt{\mathbf{v}^T \mathbf{v}} = \sqrt{\begin{pmatrix} 1 & -1 & 2 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}} = \sqrt{6} \tag{0.4}$$

$$\|\mathbf{e_2}\| = 1 \tag{0.5}$$

$$\cos\theta = \frac{\mathbf{v}^T \mathbf{e_2}}{\|\mathbf{v}\|\|\mathbf{e_2}\|} = \frac{-1}{\sqrt{6}} \tag{0.6}$$

$$\theta = \cos^{-1}\left(-\frac{1}{\sqrt{6}}\right) \tag{0.7}$$

Therefore, $\boxed{\theta = \cos^{-1}\left(-\frac{1}{\sqrt{6}}\right) \approx 114.09°}$

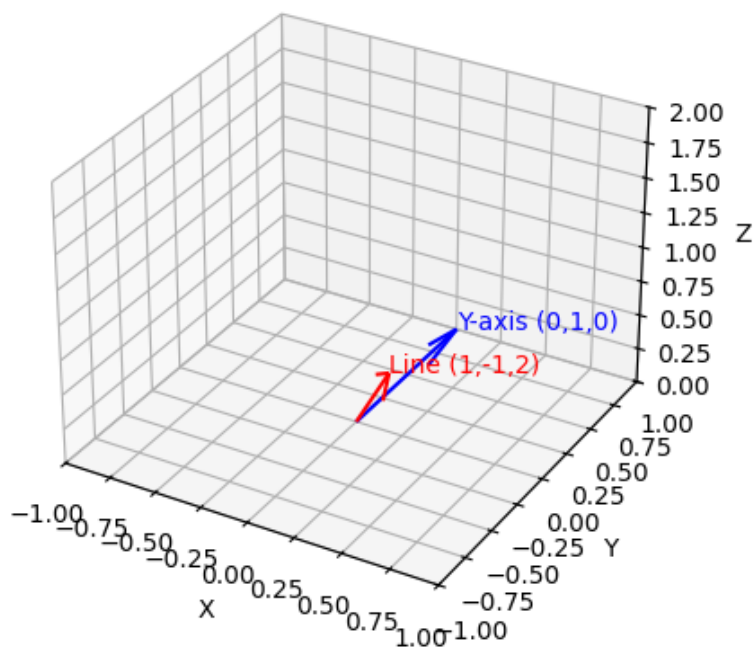Fig. 0.1: Graph