2.10.33

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Question

Let α, β, γ be distinct real numbers. The points with position vectors $\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$, $\beta\hat{i} + \gamma\hat{j} + \alpha\hat{k}$, $\gamma\hat{i} + \alpha\hat{j} + \beta\hat{k}$:

1. are collinear
2. form an equilateral triangle
3. form a scalene triangle
4. form a right angled triangle

Solution

To answer this question, we need to find the distance between each of these points.

Let $\mathbf{A}$ be $\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix}$, $\mathbf{B}$ be $\begin{pmatrix} \beta \\ \gamma \\ \alpha \end{pmatrix}$, and $\mathbf{C}$ be $\begin{pmatrix} \gamma \\ \alpha \\ \beta \end{pmatrix}$.

First, we need to check when the three points are collinear. We can do this using the collinearity matrix:

$$\begin{pmatrix} \mathbf{C}-\mathbf{A} & \mathbf{B}-\mathbf{A} \end{pmatrix}^T \tag{1}$$

If the rank of the matrix is 1, then the points are collinear.

$$\begin{pmatrix} \gamma-\alpha & \alpha-\beta & \beta-\gamma \\ \beta-\alpha & \gamma-\beta & \alpha-\gamma \end{pmatrix} \tag{2}$$

Solution

The rank of this matrix will be 1 only when all the elements in the bottom row of the matrix are equal to 0. This occurs only when $\alpha = \beta = \gamma$, which contradicts the fact that α, β, γ are distinct.

Therefore the points must be non-collinear and form a triangle.

The sides of the triangle are $\mathbf{A} - \mathbf{B}, \mathbf{B} - \mathbf{C}, \mathbf{C} - \mathbf{A}$.

Solution

$$\mathbf{A} - \mathbf{B} = \begin{pmatrix} \alpha - \beta \\ \beta - \gamma \\ \gamma - \alpha \end{pmatrix} \tag{3}$$

$$\mathbf{B} - \mathbf{C} = \begin{pmatrix} \beta - \gamma \\ \gamma - \alpha \\ \alpha - \beta \end{pmatrix} \tag{4}$$

$$\mathbf{C} - \mathbf{A} = \begin{pmatrix} \gamma - \alpha \\ \alpha - \beta \\ \beta - \gamma \end{pmatrix} \tag{5}$$

$\|\mathbf{A} - \mathbf{B}\| = \|\mathbf{B} - \mathbf{C}\| = \|\mathbf{C} - \mathbf{A}\| = \sqrt{(\alpha - \beta)^2 + (\beta - \gamma)^2 + (\gamma - \alpha)^2}$
The three points therefore form an **equilateral triangle**, so option (2) is correct.

Python Code

```
import numpy as np


vector = np.zeros(3)
vector[0] = input()
vector[1] = input()
vector[2] = input()

print(np.linalg.norm(vector))

A = np.array([vector[0],vector[1],vector[2]])
B = np.array([vector[1],vector[2],vector[0]])
C = np.array([vector[2],vector[0],vector[1]])

fig = plt.figure(figsize = (6,6))
```

Python Code

```
ax = fig.add_subplot(111, projection='3d')

AB = line_gen(A,B)
BC= line_gen(B,C)
CA= line_gen(C,A)

plt.plot(AB[0,:],AB[1,:],AB[2,:],label='$AB$')
plt.plot(BC[0,:],BC[1,:],BC[2,:],label='$BC$')
plt.plot(CA[0,:],CA[1,:],CA[2,:],label='$CA$')

ax = plt.gca()
ax.spines['top'].set_color('none')
ax.spines['left'].set_position('zero')
ax.spines['right'].set_color('none')
ax.spines['bottom'].set_position('zero')

plt.grid()
plt.axis('equal')
plt.show()
```

C Code

```c
#include<stdio.h>
#include<math.h>

float norm(float a, float b, float c){

float answer;
answer = pow(a,2) + pow(b,2) + pow(c,2);
answer = sqrt(answer);

return answer;

}
```

Python and C Code

```
import numpy as np
import ctypes
c_lib=ctypes.CDLL('./5c.so')

c_lib.norm.argtypes = [ctypes.c_float, ctypes.c_float, ctypes.
    c_float]
c_lib.norm.restype = ctypes.c_float

vector = np.zeros(3)
vector[0] = input()
vector[1] = input()
vector[2] = input()

answer = c_lib.norm(
    ctypes.c_float(vector[0]),
    ctypes.c_float(vector[1]),
    ctypes.c_float(vector[2]))
print(answer)
```

Python and C Code

```
ax = fig.add_subplot(111, projection='3d')

AB = line_gen(A,B)
BC= line_gen(B,C)
CA= line_gen(C,A)

plt.plot(AB[0,:],AB[1,:],AB[2,:],label='$AB$')
plt.plot(BC[0,:],BC[1,:],BC[2,:],label='$BC$')
plt.plot(CA[0,:],CA[1,:],CA[2,:],label='$CA$')

ax = plt.gca()
ax.spines['top'].set_color('none')
ax.spines['left'].set_position('zero')
ax.spines['right'].set_color('none')
ax.spines['bottom'].set_position('zero')

plt.grid()
plt.axis('equal')
plt.show()
```

Plot

For example, let us take $\alpha = 2$, $\beta = 1$, $\gamma = 3$. We get an equilateral triangle as shown below: