2.2.10

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Question

The vectors $\mathbf{A} = 3\hat{i} - 2\hat{j} + 2\hat{k}$ and $\mathbf{B} = \hat{i} - 2\hat{k}$ are the adjancent sides of a parallelogram.
The acute angle between its diagonals is ___________.

Solution:

The diagonals of the parallelogram are given by

$$\mathbf{A}+\mathbf{B} = \begin{pmatrix} 4 \\ -2 \\ 0 \end{pmatrix}, \mathbf{A}-\mathbf{B} = \begin{pmatrix} 2 \\ -2 \\ 4 \end{pmatrix} \tag{1}$$

The angle θ between them satisfies
$\cos\theta = \frac{\mathbf{d}_1^T \mathbf{d}_2}{\|\mathbf{d}_1\| \, \|\mathbf{d}_2\|} = \frac{(\mathbf{A}+\mathbf{B})^T(\mathbf{A}-\mathbf{B})}{\|\mathbf{A}+\mathbf{B}\| \, \|\mathbf{A}-\mathbf{B}\|} = \frac{\|\mathbf{A}\|^2-\|\mathbf{B}\|^2}{\|\mathbf{A}+\mathbf{B}\| \, \|\mathbf{A}-\mathbf{B}\|}.$
Now compute:

$$\|\mathbf{A}\|^2 = 3^2 + (-2)^2 + 2^2 = 17, \qquad \|\mathbf{B}\|^2 = 1^2 + 0^2 + (-2)^2 = 5 \tag{2}$$

Theoretical Solution

$$\mathbf{A} + \mathbf{B} = \langle 4, -2, 0 \rangle, \quad \|\mathbf{A} + \mathbf{B}\| = \sqrt{20} = 2\sqrt{5}, \tag{3}$$

$$\mathbf{A} - \mathbf{B} = \langle 2, -2, 4 \rangle, \quad \|\mathbf{A} - \mathbf{B}\| = \sqrt{24} = 2\sqrt{6}. \tag{4}$$

Hence

$$\cos\theta = \frac{17 - 5}{(2\sqrt{5})(2\sqrt{6})} = \frac{12}{4\sqrt{30}} = \frac{3}{\sqrt{30}}. \tag{5}$$

Therefore, the acute angle between the diagonals is $\theta = \cos^{-1}\left(\frac{3}{\sqrt{30}}\right) \approx 56.7^\circ$.

C Code

```
#include <stdio.h>
#include <math.h>

// Function to compute dot product of two vectors
double dot(double v1[3], double v2[3]) {
    return v1[0]*v2[0] + v1[1]*v2[1] + v1[2]*v2[2];
}

// Function to compute magnitude of vector
double magnitude(double v[3]) {
    return sqrt(dot(v, v));
}

int main() {
    // Given vectors A and B
    double A[3] = {3, -2, 2};
    double B[3] = {1, 0, -2};
    double d1[3], d2[3];
    double cos_theta, theta;
```

C Code

```c
// Compute diagonals
   for (int i = 0; i < 3; i++) {
       d1[i] = A[i] + B[i]; // A + B
       d2[i] = A[i] - B[i]; // A - B
   }

   // Compute cosine of angle
   cos_theta = dot(d1, d2) / (magnitude(d1) * magnitude(d2));

   // Clamp value to [-1, 1] for numerical stability
   if (cos_theta > 1.0) cos_theta = 1.0;
   if (cos_theta < -1.0) cos_theta = -1.0;

   // Angle in radians
   theta = acos(cos_theta);
```

C Code

```c
  // Convert to degrees
    theta = theta * 180.0 / M_PI;

    // Ensure acute angle
    if (theta > 90.0) {
        theta = 180.0 - theta;
    }

    printf("The acute angle between diagonals is: %.2f degrees\n"
        , theta);

    return 0;
}
```

Python Code

```
import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D

# Define vectors
A = np.array([3, -2, 2])
B = np.array([1, 0, -2])

# Diagonals of parallelogram
diag1 = A + B # A+B
diag2 = A - B # AB

fig = plt.figure(figsize=(7,7))
ax = fig.add_subplot(111, projection='3d')

# Plot A
ax.quiver(0, 0, 0, A, A[1], A[asset:1], color='blue', label='A',
    arrow_length_ratio=0.1)
```

Python Code

```
# Plot B
ax.quiver(0, 0, 0, B, B[1], B[asset:1], color='green', label='B',
    arrow_length_ratio=0.1)
# Plot first diagonal
ax.quiver(0, 0, 0, diag1, diag1[1], diag1[asset:1], color='red',
    label='A+B', arrow_length_ratio=0.1)
# Plot second diagonal
ax.quiver(0, 0, 0, diag2, diag2[1], diag2[asset:1], color='purple
    ', label='A-B', arrow_length_ratio=0.1)

ax.set_xlabel('X')
ax.set_ylabel('Y')
ax.set_zlabel('Z')
ax.set_title('3D Vectors: Sides and Diagonals of Parallelogram')
ax.legend()
plt.tight_layout()
plt.savefig('parallelogram_diagonals.png', dpi=200)
plt.close()
```

Plot

Beamer/figs/parallelogram_diagonals.png

Figure: