4.13.51

EE25BTECH11047 - RAVULA SHASHANK REDDY

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Question:

One of the diameters of the circle circumscribing the rectangle $ABCD$ is given by

$$4y = x + 7.$$

If $\mathbf{A} = (-3, 4)$ and $\mathbf{B} = (5, 4)$, find the area of the rectangle.

Solution:

Circle equation :   Centre $\mathbf{O} = -\mathbf{u} = -\begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$

$$\|\mathbf{x}\|^2 + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{1}$$

$$\|\mathbf{A}\|^2 + 2\mathbf{u}^\top \mathbf{A} + f = 0 \tag{2}$$

$$\|\mathbf{B}\|^2 + 2\mathbf{u}^\top \mathbf{B} + f = 0 \tag{3}$$

Diameter equation:(Centre lies on diameter)

$$\begin{pmatrix} 1 \\ -4 \end{pmatrix}^T \mathbf{x} = -7 \tag{4}$$

$$\mathbf{n}^T \mathbf{u} = c \tag{5}$$

$$\begin{pmatrix} 2\mathbf{A} & 2\mathbf{B} & n \\ 1 & 1 & 0 \end{pmatrix}^T \begin{pmatrix} \mathbf{u} \\ f \end{pmatrix} = -\begin{pmatrix} \|\mathbf{A}\|^2 \\ \|\mathbf{B}\|^2 \\ c \end{pmatrix} \tag{6}$$

$$\begin{pmatrix} -6 & 10 & 1 \\ 8 & 8 & -4 \\ 1 & 1 & 0 \end{pmatrix}^T \begin{pmatrix} \mathbf{u} \\ f \end{pmatrix} = -\begin{pmatrix} 25 \\ 41 \\ -7 \end{pmatrix} \tag{7}$$

$$\begin{pmatrix} -6 & 8 & 1 & -25 \\ 10 & 8 & 1 & -41 \\ 1 & -4 & 0 & 7 \end{pmatrix} \xrightarrow{R_1 \leftrightarrow R_3} \begin{pmatrix} 1 & -4 & 0 & 7 \\ 10 & 8 & 1 & -41 \\ -6 & 8 & 1 & -25 \end{pmatrix} \tag{8}$$

$$R_2 \to R_2 - 10R_1, \; R_3 \to R_3 + 6R_1 : \begin{pmatrix} 1 & -4 & 0 & 7 \\ 0 & 48 & 1 & -111 \\ 0 & -16 & 1 & 17 \end{pmatrix} \tag{9}$$

$$R_3 \to R_3 + \frac{1}{3}R_2 : \begin{pmatrix} 1 & -4 & 0 & 7 \\ 0 & 48 & 1 & -111 \\ 0 & 0 & 4/3 & -20 \end{pmatrix} \tag{10}$$

$$\frac{4}{3}f = -20 \implies f = -15 \tag{11}$$

$$48u_2 + f = -111 \implies 48u_2 - 15 = -111 \implies u_2 = -2 \tag{12}$$

$$u_1 - 4u_2 = 7 \implies u_1 = -1 \tag{13}$$

$$\mathbf{u} = \begin{pmatrix} -1 \\ -2 \end{pmatrix}, \quad f = -15 \tag{14}$$

$$\mathbf{O} = -\mathbf{u} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \tag{15}$$

Equation of the Circumcircle:

$$\boxed{\|\mathbf{x}\|^2 + 2\begin{pmatrix} -1 & -2 \end{pmatrix}\mathbf{x} - 15 = 0} \tag{16}$$

$$\mathbf{C} = 2\mathbf{O} - \mathbf{A} = 2\begin{pmatrix} 1 \\ 2 \end{pmatrix} - \begin{pmatrix} -3 \\ 4 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \end{pmatrix} \tag{17}$$

$$\mathbf{D} = 2\mathbf{O} - \mathbf{B} = 2\begin{pmatrix} 1 \\ 2 \end{pmatrix} - \begin{pmatrix} 5 \\ 4 \end{pmatrix} = \begin{pmatrix} -3 \\ 0 \end{pmatrix} \tag{18}$$

$$\text{Area} = |(\mathbf{B} - \mathbf{A}) \times (\mathbf{D} - \mathbf{A})| = \begin{vmatrix} 8 & 0 \\ 0 & -4 \end{vmatrix} = 32 \tag{19}$$
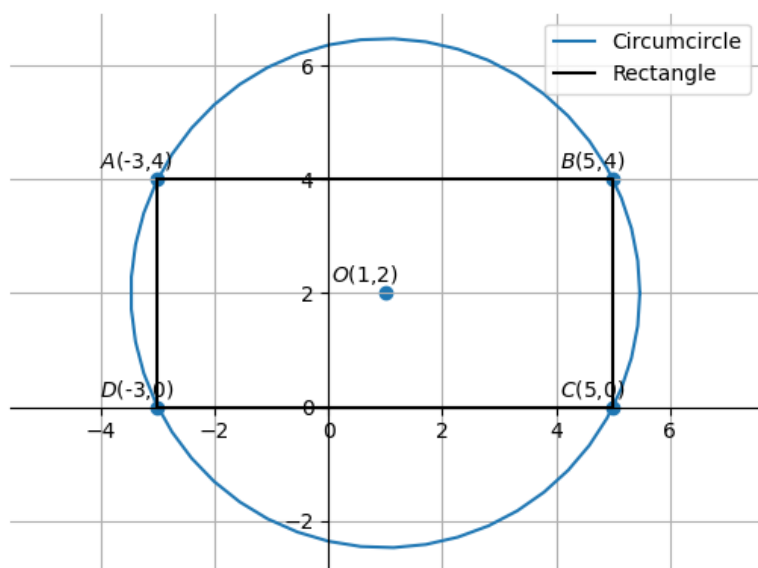
Figure 1