4.2.7

EE25BTECH11004 - Aditya Appana

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Question

Find the direction and normal vectors of the line $y - 2 = 0$

Solution

A line can be expressed in two forms:

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ c \end{pmatrix} + x \begin{pmatrix} 1 \\ m \end{pmatrix} \tag{1}$$

where $\begin{pmatrix} 1 \\ m \end{pmatrix}$ is the direction vector of the line and m is the **slope** of the line.

$$\mathbf{n}^T x = c \tag{2}$$

where **n** is the normal vector of the line.

$$\mathbf{n}^T \begin{pmatrix} 1 \\ m \end{pmatrix} = 0 \tag{3}$$

The slope of the line $y - 2 = 0$ is 0, therefore it can be expressed in the first form as:

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix} + x \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{4}$$

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Let $\begin{pmatrix} x \\ y \end{pmatrix}$ be normal vector. Therefore

$$\begin{pmatrix} x \\ y \end{pmatrix}^T \begin{pmatrix} 1 \\ 0 \end{pmatrix} = 0 \tag{5}$$

$$x = 0 \tag{6}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{7}$$

Therefore the line can be expressed as

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix}^T x = 2 \tag{8}$$

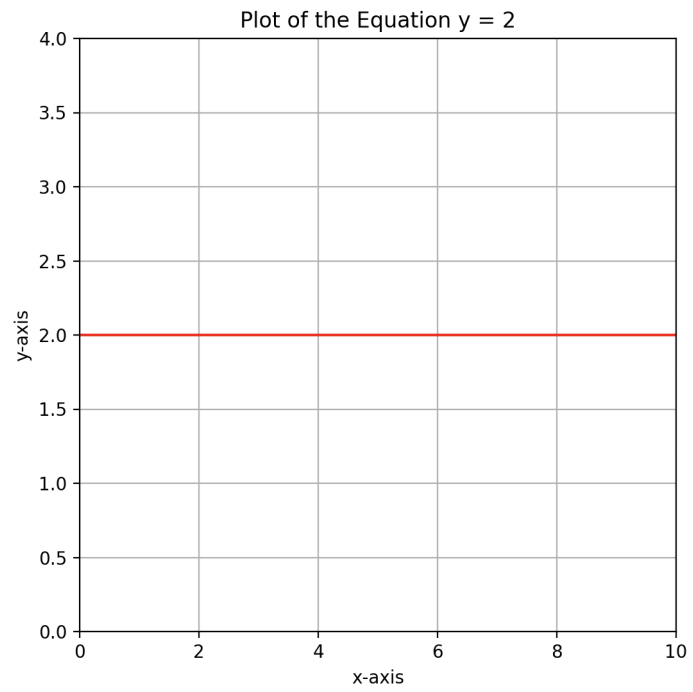Therefore, the direction vector is $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$, and the normal vector is $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

Figure 1: Equilateral Triangle