5.2.16

EE25BTECH11013 - Bhargav

Question:
Solve the system of equations

$$3x - 5y = 20 \tag{0.1}$$

$$6x - 10y = 40 \tag{0.2}$$

Solution:

The equation of line:

$$\mathbf{n^T x} = c \tag{0.3}$$

Line L:

$$\begin{pmatrix} 3 & -5 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 20 \tag{0.4}$$

Line K:

$$\begin{pmatrix} 6 & -10 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 40 \tag{0.5}$$

These can be combined and written in matrix form:

$$\begin{pmatrix} 3 & -5 \\ 6 & -10 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 20 \\ 40 \end{pmatrix} \tag{0.6}$$

The following augmented matrix can be solved by gaussian elimination

$$\left(\begin{array}{cc|c} 3 & -5 & 20 \\ 6 & -10 & 40 \end{array}\right) \xleftrightarrow{R_2 \leftarrow R_2 - 2R_1} \left(\begin{array}{cc|c} 3 & -5 & 20 \\ 0 & 0 & 0 \end{array}\right) \tag{0.7}$$

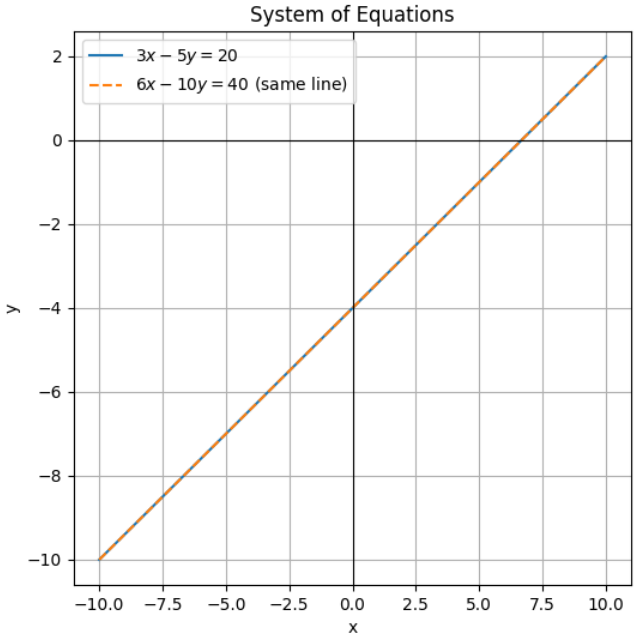
We end up with only one non – zero row (Rank = 1)

$$\begin{pmatrix} 3 & -5 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 20 \tag{0.8}$$

The general solution is

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} t \\ \frac{3t-20}{5} \end{pmatrix} \quad t \in \mathbb{R} \tag{0.9}$$

So there can be infinitely many solutions for this system of equations.


System of Equations
3x − 5y = 20
6x − 10y = 40 (same line)
2
0
−2
−4
−6
−8
−10
y
−10.0
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0
x