5.4.41

RAVULA SHASHANK REDDY - EE25BTECH11047

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Question

Using elementary transformations, find the inverse of the matrix

$$\mathbf{A} = \begin{pmatrix} 2 & 1 & 3 \\ 4 & -1 & 0 \\ -7 & 2 & 1 \end{pmatrix}.$$

Solution

$$\mathbf{A}.\mathbf{A}^{-1} = \mathbf{I} \tag{1}$$

$$[\mathbf{A} \mid \mathbf{I}] = \begin{pmatrix} 2 & 1 & 3 & 1 & 0 & 0 \\ 4 & -1 & 0 & 0 & 1 & 0 \\ -7 & 2 & 1 & 0 & 0 & 1 \end{pmatrix}$$
 (2)

$$\frac{R_1 \to \frac{1}{2}R_1}{\longrightarrow} \begin{pmatrix}
1 & \frac{1}{2} & \frac{3}{2} & \frac{1}{2} & 0 & 0 \\
4 & -1 & 0 & 0 & 1 & 0 \\
-7 & 2 & 1 & 0 & 0 & 1
\end{pmatrix}$$
(3)

$$\xrightarrow{R_2 \to R_2 - 4R_1, R_3 \to R_3 + 7R_1} \begin{pmatrix} 1 & \frac{1}{2} & \frac{3}{2} & \frac{1}{2} & 0 & 0 \\ 0 & -3 & -6 & -2 & 1 & 0 \\ 0 & \frac{11}{2} & \frac{23}{2} & \frac{7}{2} & 0 & 1 \end{pmatrix} \tag{4}$$

Solution

$$\xrightarrow{R_2 \to -\frac{1}{3}R_2} \begin{pmatrix} 1 & \frac{1}{2} & \frac{3}{2} & \frac{1}{2} & 0 & 0\\ 0 & 1 & 2 & \frac{2}{3} & -\frac{1}{3} & 0\\ 0 & \frac{11}{2} & \frac{23}{2} & \frac{7}{2} & 0 & 1 \end{pmatrix}$$
 (5)

$$\frac{R_1 \to R_1 - \frac{1}{2}R_2, R_3 \to R_3 - \frac{11}{2}R_2}{0 \quad 1 \quad 2 \quad \frac{1}{2} \quad \frac{1}{6} \quad \frac{1}{6} \quad 0}$$

$$0 \quad 1 \quad 2 \quad \frac{2}{3} \quad -\frac{1}{3} \quad 0$$

$$0 \quad 0 \quad \frac{1}{2} \quad -\frac{1}{6} \quad \frac{11}{6} \quad 1$$
(6)

$$\xrightarrow{R_3 \to 2R_3} \begin{pmatrix} 1 & 0 & \frac{1}{2} & \frac{1}{6} & \frac{1}{6} & 0\\ 0 & 1 & 2 & \frac{2}{3} & -\frac{1}{3} & 0\\ 0 & 0 & 1 & -\frac{1}{3} & \frac{11}{3} & 2 \end{pmatrix}$$
 (7)

Solution

$$\frac{R_1 \to R_1 - \frac{1}{2}R_3, R_2 \to R_2 - 2R_3}{\longrightarrow} \begin{pmatrix} 1 & 0 & 0 & \frac{1}{3} & -\frac{5}{3} & -1\\ 0 & 1 & 0 & \frac{4}{3} & -\frac{23}{3} & -4\\ 0 & 0 & 1 & -\frac{1}{3} & \frac{11}{3} & 2 \end{pmatrix}$$
(8)

$$\mathbf{A}^{-1} = \begin{pmatrix} \frac{1}{3} & -\frac{5}{3} & -1\\ \frac{4}{3} & -\frac{23}{3} & -4\\ -\frac{1}{3} & \frac{11}{3} & 2 \end{pmatrix} \tag{9}$$



C Code

```
#include <stdio.h>
int main() {
    int i, j, k;
   double A[3][3] = {
       \{2, 1, 3\},\
       \{4, -1, 0\},\
       \{-7, 2, 1\}
   };
   double I[3][3] = {
        \{1, 0, 0\},\
       \{0, 1, 0\},\
       \{0, 0, 1\}
   };
   // Perform Gauss-Jordan elimination
   for (i = 0; i < 3; i++) {
```

C Code

```
// Make the diagonal element 1
double diag = A[i][i];
for (j = 0; j < 3; j++) {
   A[i][j] /= diag;
   I[i][j] /= diag;
}
// Make other elements in the column 0
for (k = 0; k < 3; k++) {
   if (k != i) {
       double factor = A[k][i]:
       for (j = 0; j < 3; j++) {
           A[k][j] = factor * A[i][j];
           I[k][j] -= factor * I[i][j];
```

C Code

```
// Print the inverse
printf(Inverse matrix is:\n);
for (i = 0; i < 3; i++) {</pre>
   for (j = 0; j < 3; j++) {
       printf(%8.3f , I[i][j]);
   printf(\n);
return 0;
```

Python Direct

```
import numpy as np
import libs.line.funcs as line
import libs.triangle.funcs as triangle
# Given matrix
A = np.array([[2, 1, 3],
             [4, -1, 0],
             [-7, 2, 1]], dtype=float)
# Compute inverse using numpy
A_inv = np.linalg.inv(A)
```

Python Direct

```
# Display results
print(Matrix A:)
print(A)
print(\nInverse of A:)
print(A_inv)

# Verification
I_check = A @ A_inv
print(\nVerification A * A_inv = )
print(I_check)
```

Python Shared

```
import ctypes
import numpy as np
import libs.line.funcs as line
import libs.triangle.funcs as triangle
# Load shared library
lib = ctypes.CDLL(./libinverse.so)
# Define function signature
lib.inverse.argtypes = [ctypes.POINTER(ctypes.c_double),
                      ctypes.POINTER(ctypes.c double)]
lib.inverse.restype = None
# Input matrix
A = np.array([[2, 1, 3],
             [4, -1, 0],
             [-7, 2, 1]], dtype=np.double)
```

Python Shared

```
A_inv = np.zeros((3,3), dtype=np.double)
# Call C function
lib.inverse(A.ctypes.data_as(ctypes.POINTER(ctypes.c_double)),
           A_inv.ctypes.data_as(ctypes.POINTER(ctypes.c_double)))
print(Matrix A:)
print(A)
print(\nInverse from C (via ctypes):)
print(A inv)
# Verify
print(\nVerification A * A inv =)
print(A @ A inv)
```