4.10.15

EE25BTECH11013 - Bhargav

Question:
Show that the lines

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} \tag{0.1}$$

and

$$\frac{x-4}{5} = \frac{y-1}{2} = z \tag{0.2}$$

intersect. Also, find their point of intersection.

Solution:

The vector equations of the given lines are

$$\mathbf{r}_1 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}, \tag{0.3}$$

$$\mathbf{r}_2 = \begin{pmatrix} 4 \\ 1 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 5 \\ 2 \\ 1 \end{pmatrix}. \tag{0.4}$$

At the point of intersection,

$$\mathbf{r}_1 = \mathbf{r}_2. \tag{0.5}$$

Thus,

$$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 5 \\ 2 \\ 1 \end{pmatrix}. \tag{0.6}$$

This can be written as a matrix equation:

$$\begin{pmatrix} 2 & -5 \\ 3 & -2 \\ 4 & -1 \end{pmatrix} \begin{pmatrix} \lambda \\ \mu \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \\ -3 \end{pmatrix}. \tag{0.7}$$

The corresponding augmented matrix is

$$\left(\begin{array}{cc|c} 2 & -5 & 3 \\ 3 & -2 & -1 \\ 4 & -1 & -3 \end{array}\right) \xleftrightarrow[R_2 \leftarrow R_2 - \frac{3}{2}R_1]{R_3 \leftarrow R_3 - 2R_1} \left(\begin{array}{cc|c} 2 & -5 & 3 \\ 0 & \frac{11}{2} & \frac{-11}{2} \\ 0 & 9 & -9 \end{array}\right) \xleftrightarrow[R_3 \leftarrow 11R_3 - 9R_2]{R_2 \leftarrow 2R_2} \left(\begin{array}{cc|c} 2 & -5 & 3 \\ 0 & 11 & -11 \\ 0 & 0 & 0 \end{array}\right) \tag{0.8}$$

$$\implies \begin{pmatrix}\lambda\\ \mu\end{pmatrix} = \begin{pmatrix}-1\\ -1\end{pmatrix} \tag{0.9}$$

Substituting into $\mathbf{r}_1$:

$$\mathbf{r}_1 = \begin{pmatrix}1\\ 2\\ 3\end{pmatrix} + (-1)\begin{pmatrix}2\\ 3\\ 4\end{pmatrix} = \begin{pmatrix}-1\\ -1\\ -1\end{pmatrix}. \tag{0.10}$$

$\therefore$ the lines intersect at the point

$$\begin{pmatrix}-1\\ -1\\ -1\end{pmatrix} \tag{0.11}$$