1.8.22: Equidistant Points Problem

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Question

Find all points that are equidistant from the points

$$A = \begin{pmatrix} -5 \\ 4 \end{pmatrix}, \quad B = \begin{pmatrix} -1 \\ 6 \end{pmatrix}.$$

How many such points exist?

Solution

Let the desired point be

$$O = \begin{pmatrix} x \\ y \end{pmatrix}.$$

The equidistance condition is:

$$\|O - A\| = \|O - B\|.$$

Squaring both sides:

$$\|O - A\|^2 = \|O - B\|^2.$$

Using vector dot product,

$$(O - A)^\top (O - A) = (O - B)^\top (O - B).$$

Expanding,

$$O^\top O - 2A^\top O + A^\top A = O^\top O - 2B^\top O + B^\top B.$$

Canceling terms,

$$-2A^\top O + A^\top A = -2B^\top O + B^\top B.$$

Rearranged,

$$2(B-A)^\top O = B^\top B - A^\top A.$$

Substituting values,

$$\begin{pmatrix} 4 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \frac{37-41}{2} = -2.$$

Simplified line equation:

$$2x + y = -1.$$

Number of such points: Infinitely many points lying on the above line.

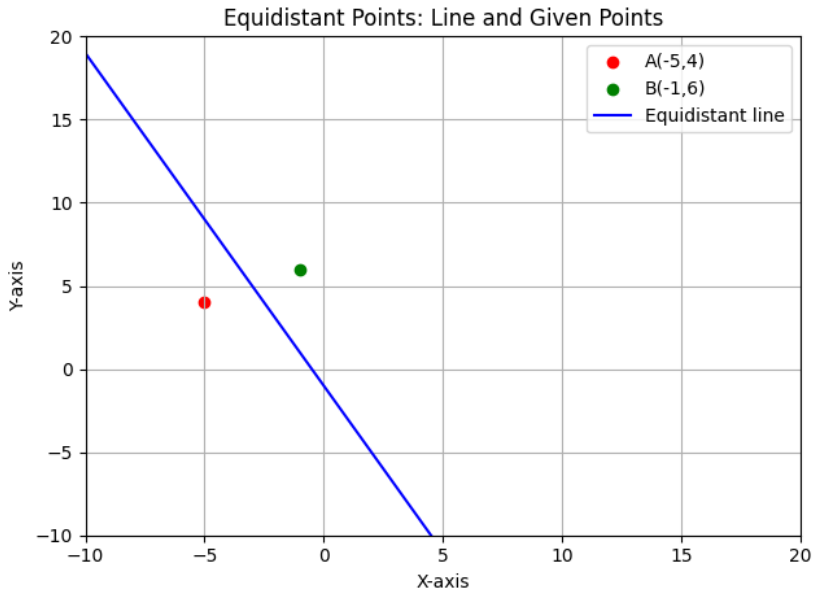

Figure 1: Points A, B and equidistant line $2x + y = -1$.