Matrices in Geometry 4.11.15

EE25BTECH11037 - Divyansh

Question: Find the equation of the plane which contains the line of intersection of the planes $\mathbf{r}\cdot\left(\hat{i}+2\hat{j}+3\hat{k}\right)-4=0$, $\mathbf{r}\cdot\left(2\hat{i}+\hat{j}-\hat{k}\right)+5=0$ and which is perpendicular to the plane $\mathbf{r}\cdot\left(5\hat{i}+3\hat{j}-6\hat{k}\right)+8=0$.

Solution: We need to find an equation of the plane that contains the line of intersection of the given two planes:

$$\mathbf{P_1} \;:\; \mathbf{n_1}^\top\mathbf{x}-4=0 \quad, \mathbf{n_1}=\begin{pmatrix}1\\2\\3\end{pmatrix} \tag{1}$$

$$\mathbf{P_2} \;:\; \mathbf{n_2}^\top\mathbf{x}+5=0 \quad, \mathbf{n_2}=\begin{pmatrix}2\\1\\-1\end{pmatrix} \tag{2}$$

Another plane that passes through the intersection of these two planes is

$$\mathbf{P} \;:\; \mathbf{P_1}+\lambda\mathbf{P_2}=0 \tag{3}$$

$$\mathbf{P} \;:\; \mathbf{n_1}^\top\mathbf{x}-4+\lambda\left(\mathbf{n_2}^\top\mathbf{x}+5\right)=0 \tag{4}$$

This can be written as

$$\mathbf{P} \;:\; \left(\mathbf{n_1}^\top+\lambda\mathbf{n_2}^\top\right)\mathbf{x}-4+5\lambda=0 \tag{5}$$

The normal to this plane is

$$\mathbf{n}=\mathbf{n_1}+\lambda\mathbf{n_2} \tag{6}$$

The plane $\mathbf{P}$ should also be perpendicular to the plane

$$\mathbf{P_3} \;:\; \mathbf{n_3}^\top\mathbf{x}+8=0 \;,\; \mathbf{n_3}=\begin{pmatrix}5\\3\\-6\end{pmatrix} \tag{7}$$

$$\therefore \mathbf{n_3}^\top\mathbf{n}=0 \implies \mathbf{n_3}^\top\left(\mathbf{n_1}+\lambda\mathbf{n_2}\right)=0 \tag{8}$$

$$\implies \mathbf{n_3}^\top\mathbf{n_1}+\lambda\mathbf{n_3}^\top\mathbf{n_2}=0 \tag{9}$$

$$\implies \begin{pmatrix}5 & 3 & -6\end{pmatrix}\begin{pmatrix}1\\2\\3\end{pmatrix}+\lambda\begin{pmatrix}5 & 3 & -6\end{pmatrix}\begin{pmatrix}2\\1\\-1\end{pmatrix}=0 \tag{10}$$

$$\implies -7+19\lambda=0 \implies \lambda=\frac{7}{19} \tag{11}$$

Substituting this value of λ in equation of $\mathbf{P}$, we get

$$\mathbf{P} \;:\; \left(\begin{pmatrix}1 & 2 & 3\end{pmatrix}+\frac{7}{19}\begin{pmatrix}2 & 1 & -1\end{pmatrix}\right)\mathbf{x}-4+\frac{35}{19}=0 \tag{12}$$

$$\mathbf{P} \;:\mathbf{n}^\top\mathbf{x}-41=0 \;,\; \mathbf{n}=\begin{pmatrix}33\\45\\50\end{pmatrix} \tag{13}$$

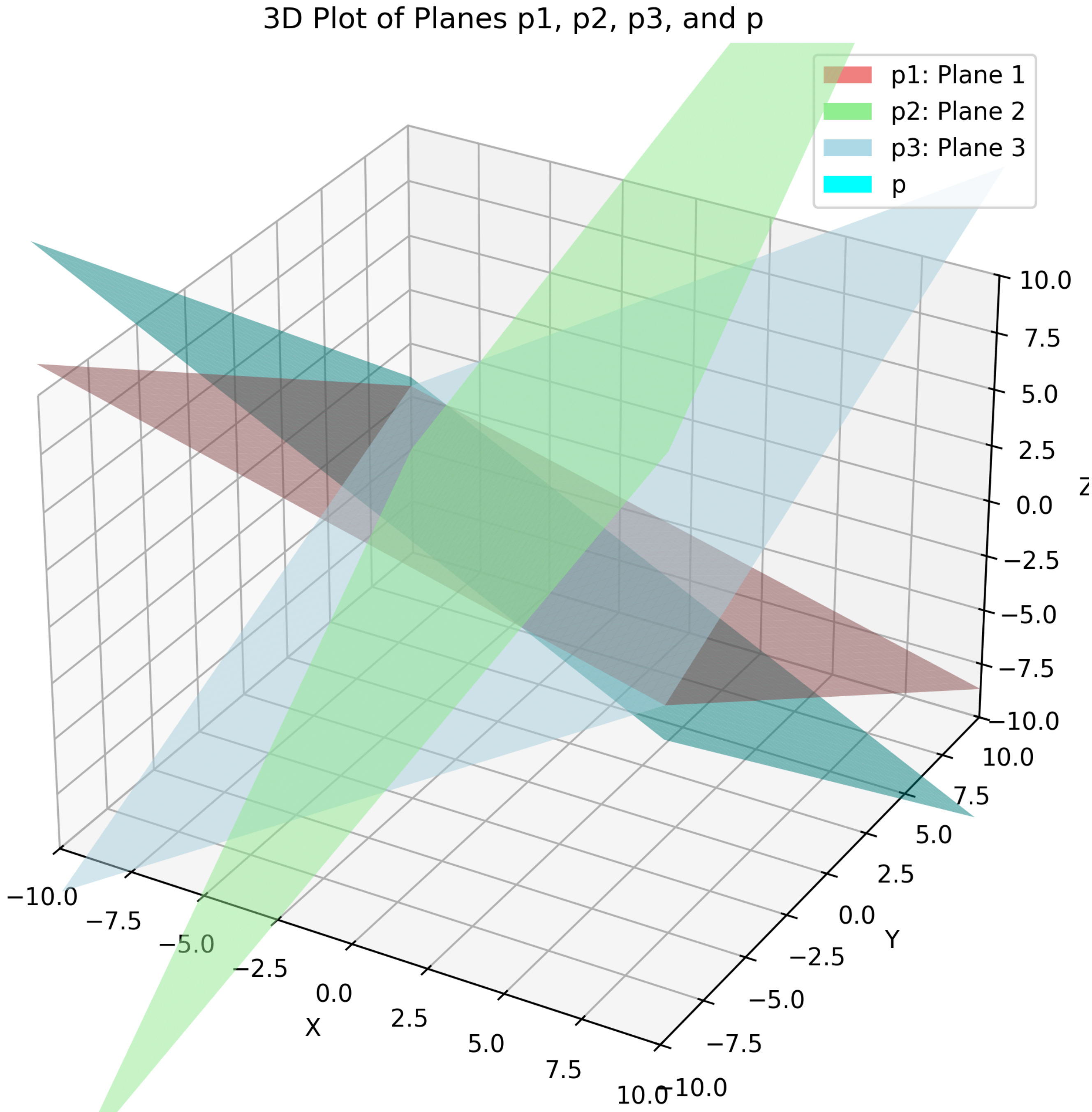

Fig. 1: Figure for 4.11.15