

MatGeo Assignment 2.6.13

1

AI25BTECH11007

Question:

Given that vectors **a**, **b**, **c** form a triangle such that

$$\mathbf{a} = \mathbf{b} + \mathbf{c},$$

find p, q, r, s given that

$$\mathbf{a} = p\hat{i} + q\hat{j} + r\hat{k}, \quad \mathbf{b} = s\hat{i} + 3\hat{j} + 4\hat{k}, \quad \mathbf{c} = 3\hat{i} + 1\hat{j} - 2\hat{k},$$

and the area of the triangle is $5\sqrt{6}$.

Solution:

We are given:

$$\mathbf{a} = \mathbf{b} + \mathbf{c} \tag{0.1}$$

$$\mathbf{a} = p\hat{i} + q\hat{j} + r\hat{k}, \quad \mathbf{b} = s\hat{i} + 3\hat{j} + 4\hat{k}, \quad \mathbf{c} = 3\hat{i} + 1\hat{j} - 2\hat{k} \tag{0.2}$$

and the area of the triangle formed by these vectors is:

$$\text{Area} = 5\sqrt{6} \tag{0.3}$$

Observation: For three vectors to form a triangle, they must sum to zero:

$$\mathbf{a} + \mathbf{b} + \mathbf{c} = \mathbf{0} \tag{0.4}$$

However, we are told:

$$\mathbf{a} = \mathbf{b} + \mathbf{c} \Rightarrow \mathbf{a} - \mathbf{b} - \mathbf{c} = \mathbf{0} \tag{0.5}$$

This implies:

$$\mathbf{a} + (-\mathbf{b}) + (-\mathbf{c}) = \mathbf{0} \tag{0.6}$$

So, the triangle is formed by the vectors **a**, **-b**, **-c**. For these to form a triangle, they must not lie along the same line (i.e., must not be collinear).

Now, if we assume:

$$\mathbf{a} = \mathbf{0} \Rightarrow \mathbf{b} + \mathbf{c} = \mathbf{0} \Rightarrow \mathbf{b} = -\mathbf{c} \tag{0.7}$$

Given:

$$\mathbf{c} = \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix} \Rightarrow \mathbf{b} = -\mathbf{c} = -\begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix} = \begin{pmatrix} -3 \\ -1 \\ 2 \end{pmatrix} \tag{0.8}$$

Then:

$$\mathbf{a} = \mathbf{b} + \mathbf{c} = \mathbf{0} \Rightarrow p = 0, \quad q = 0, \quad r = 0 \quad (0.9)$$

We now compute the area of the triangle using:

$$\text{Area} = \frac{1}{2} \|\mathbf{b} \times \mathbf{c}\| \quad (0.10)$$

Compute the cross product:

$$\mathbf{b} \times \mathbf{c} = \mathbf{0} \Rightarrow \text{Area} = 0 \quad (0.11)$$

If we assume $\mathbf{a} = \mathbf{0}$, then $\mathbf{b} = -\mathbf{c}$, and the triangle is degenerate (i.e., the vectors lie on a straight line). Therefore, the area is zero:

$$\boxed{\text{Area} = 0} \quad (0.12)$$

This contradicts the given area of $5\sqrt{6}$. Therefore, no solution exists such that:

$$\mathbf{a} = \mathbf{b} + \mathbf{c} \quad \text{and} \quad \text{Area} = 5\sqrt{6} \quad (0.13)$$

Degenerate case: $\mathbf{b} = -\mathbf{c}$, $\mathbf{a} = \mathbf{b} + \mathbf{c} = \mathbf{0}$ (Area = 0)

Computed area = 0.000 (should be $5\sqrt{6} \approx 12.247$)

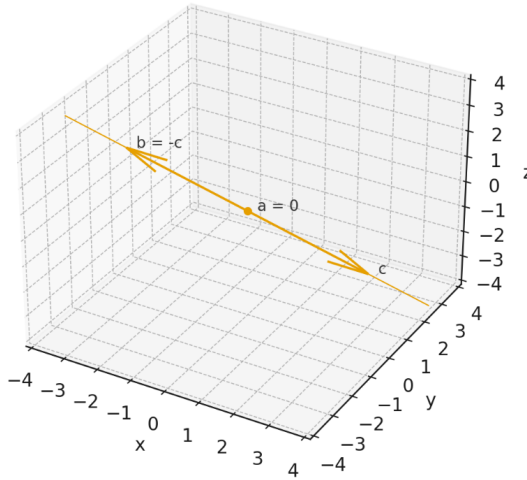


Fig. 0.1: Image Visual