4.4.6

AI25BTECH11010 - Dhanush Kumar

Question:
Find the equation of the plane passing through the points $A(2, 5, -3), B(-2, -3, 5)$ and $C(5, 3, -3)$.

Solution:

$$\mathbf{A} = \begin{pmatrix} 2 \\ 5 \\ -3 \end{pmatrix}, \qquad \mathbf{B} = \begin{pmatrix} -2 \\ -3 \\ 5 \end{pmatrix}, \qquad \mathbf{C} = \begin{pmatrix} 5 \\ 3 \\ -3 \end{pmatrix}. \tag{1}$$

Let the equation of the plane be

$$\mathbf{n}^T \mathbf{x} = 1. \tag{2}$$

Since $\mathbf{A}, \mathbf{B}, \mathbf{C}$ lie in the plane:

$$\mathbf{n}^T \mathbf{A} = 1, \qquad \mathbf{n}^T \mathbf{B} = 1, \qquad \mathbf{n}^T \mathbf{C} = 1, \tag{3}$$

or equivalently

$$\mathbf{A}^T \mathbf{n} = 1, \qquad \mathbf{B}^T \mathbf{n} = 1, \qquad \mathbf{C}^T \mathbf{n} = 1. \tag{4}$$

Hence,

$$\begin{pmatrix} \mathbf{A} & \mathbf{B} & \mathbf{C} \end{pmatrix}^T \mathbf{n} = 1. \tag{5}$$

$$\begin{pmatrix} 2 & 5 & -3 \\ -2 & -3 & 5 \\ 5 & 3 & -3 \end{pmatrix} \mathbf{n} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}. \tag{6}$$

Performing row operations:

$$R_2 \leftarrow R_2 + R_1, \tag{7}$$

$$\begin{pmatrix} 2 & 5 & -3 \\ 0 & 2 & 2 \\ 5 & 3 & -3 \end{pmatrix} \mathbf{n} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \tag{8}$$

$$R_3 \leftarrow 2R_3 - 5R_1, \tag{9}$$

$$\begin{pmatrix} 2 & 5 & -3 \\ 0 & 2 & 2 \\ 0 & -19 & 9 \end{pmatrix} \mathbf{n} = \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}, \tag{10}$$

$$R_3 \leftarrow 19R_2 + 2R_3, \tag{11}$$

$$\begin{pmatrix} 2 & 5 & -3 \\ 0 & 2 & 2 \\ 0 & 0 & 56 \end{pmatrix} \mathbf{n} = \begin{pmatrix} 1 \\ 2 \\ 32 \end{pmatrix}. \tag{12}$$

Thus, solving we get

$$\mathbf{n} = \begin{pmatrix} \frac{2}{7} \\ \frac{3}{7} \\ \frac{4}{7} \end{pmatrix}. \tag{13}$$

Therfore, The equation of plane is

$$\begin{pmatrix} \frac{2}{7} \\ \frac{3}{7} \\ \frac{4}{7} \end{pmatrix}^T \mathbf{x} = 1. \tag{14}$$

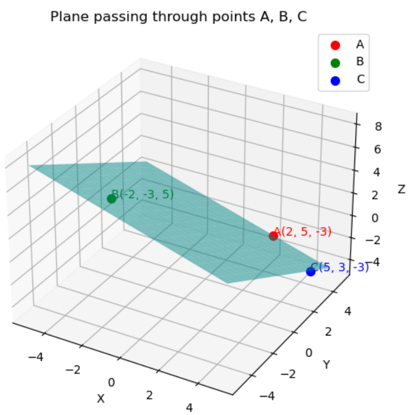

Fig. 0