2.9.4

EE25BTECH11052 - Shriyansh Kalpesh Chawda

Question:

If $\mathbf{a} = \hat{i} + \hat{j} + \hat{k}$, $\mathbf{a} \cdot \mathbf{b} = 1$, and $\mathbf{a} \times \mathbf{b} = \hat{j} - \hat{k}$, then find $|\mathbf{b}|$. (12, 2022)

Solution:

The problem can be solved using Lagrange's identity:

$$|\mathbf{a} \times \mathbf{b}|^2 + (\mathbf{a} \cdot \mathbf{b})^2 = |\mathbf{a}|^2 |\mathbf{b}|^2 \tag{0.1}$$

Given:

$$a = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \mathbf{a} \times \mathbf{b} = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$

Deriving the values needed to be substituted in the identity :

$$|\mathbf{a}|^2 = \mathbf{a}^T \mathbf{a} = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = 3 \tag{0.2}$$

$$(\mathbf{a} \cdot \mathbf{b})^2 = (1)^2 = 1 \tag{0.3}$$

$$|\mathbf{a} \times \mathbf{b}|^2 = (\mathbf{a} \times \mathbf{b})^T (\mathbf{a} \times \mathbf{b}) = \begin{pmatrix} 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} = 2 \tag{0.4}$$

Substituting values of the Components in the identity.

$$|\mathbf{a} \times \mathbf{b}|^2 + (\mathbf{a} \cdot \mathbf{b})^2 = |\mathbf{a}|^2 |\mathbf{b}|^2 \tag{0.5}$$

$$2 + 1 = 3 \cdot |\mathbf{b}|^2 \tag{0.6}$$

$$3 = 3|\mathbf{b}|^2 \tag{0.7}$$

$$|\mathbf{b}|^2 = 1 \tag{0.8}$$

$$|\mathbf{b}| = 1 \tag{0.9}$$

The magnitude of vector $\mathbf{b}$ is **1**.