

2.8.14

EE25BTECH11031 - Sai Sreevallabh

Question:

Three vectors \mathbf{a} , \mathbf{b} and \mathbf{c} satisfy the condition $\mathbf{a} + \mathbf{b} + \mathbf{c} = 0$. Evaluate the quantity $\mu = \mathbf{a} \cdot \mathbf{b} + \mathbf{b} \cdot \mathbf{c} + \mathbf{c} \cdot \mathbf{a}$. If $|\mathbf{a}| = 3$, $|\mathbf{b}| = 4$ and $|\mathbf{c}| = 2$.

Solution:

Given:

$$\mathbf{a} + \mathbf{b} + \mathbf{c} = 0 \text{ and } \|\mathbf{a}\| = 3, \|\mathbf{b}\| = 4, \|\mathbf{c}\| = 2 \quad (0.1)$$

To find

$$\mu = \mathbf{a}^\top \mathbf{b} + \mathbf{b}^\top \mathbf{c} + \mathbf{c}^\top \mathbf{a} \quad (0.2)$$

To find the value of μ

$$\|\mathbf{a} + \mathbf{b} + \mathbf{c}\|^2 = 0 \quad (0.3)$$

$$(\mathbf{a} + \mathbf{b} + \mathbf{c})^\top (\mathbf{a} + \mathbf{b} + \mathbf{c}) = 0 \quad (0.4)$$

$$\mathbf{a}^\top \mathbf{a} + \mathbf{b}^\top \mathbf{b} + \mathbf{c}^\top \mathbf{c} + 2(\mathbf{a}^\top \mathbf{b} + \mathbf{b}^\top \mathbf{c} + \mathbf{c}^\top \mathbf{a}) = 0 \quad (0.5)$$

By using $\mathbf{x}^\top \mathbf{x} = \|\mathbf{x}\|^2$ we get

$$(\|\mathbf{a}\|^2 + \|\mathbf{b}\|^2 + \|\mathbf{c}\|^2) + 2\mu = 0 \quad (0.6)$$

Substituting the values of $\|\mathbf{a}\|$, $\|\mathbf{b}\|$, $\|\mathbf{c}\|$ we get

$$\mu = \frac{-29}{2} \quad (0.7)$$

\therefore The value of μ is $\frac{-29}{2}$.