2.9.22

AI25BTECH11006 - Nikhila

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Question

Let $\overrightarrow{a}$, $\overrightarrow{b}$, and $\overrightarrow{c}$ be three vectors such that $|\overrightarrow{a}| = 1$, $|\overrightarrow{b}| = 2$, and $|\overrightarrow{c}| = 3$. If the projection of $\overrightarrow{b}$ along $\overrightarrow{a}$ is equal to the projection of $\overrightarrow{c}$ along $\overrightarrow{a}$, and $\overrightarrow{b}$ and $\overrightarrow{c}$ are perpendicular to each other, then find $|3\overrightarrow{a} - 2\overrightarrow{b} + 2\overrightarrow{c}|$.

Theoretical Solution using Gram Matrix

We are given:

$$\|\mathbf{a}\| = 1, \quad \|\mathbf{b}\| = 2, \quad \|\mathbf{c}\| = 3$$

with conditions

$$\mathbf{a}^T\mathbf{b} = \mathbf{a}^T\mathbf{c}, \quad \mathbf{b}^T\mathbf{c} = 0$$

Step 1: Construct Gram matrix.

$$G = \begin{bmatrix} \mathbf{a}^T\mathbf{a} & \mathbf{a}^T\mathbf{b} & \mathbf{a}^T\mathbf{c} \\ \mathbf{b}^T\mathbf{a} & \mathbf{b}^T\mathbf{b} & \mathbf{b}^T\mathbf{c} \\ \mathbf{c}^T\mathbf{a} & \mathbf{c}^T\mathbf{b} & \mathbf{c}^T\mathbf{c} \end{bmatrix} = \begin{bmatrix} 1 & x & x \\ x & 4 & 0 \\ x & 0 & 9 \end{bmatrix}$$

where $x = \mathbf{a}^T\mathbf{b} = \mathbf{a}^T\mathbf{c}$.

Theoretical Solution using Gram Matrix

Step 2: Define vector combination.

$$\mathbf{v} = 3\mathbf{a} - 2\mathbf{b} + 2\mathbf{c}, \quad \mathbf{u} = \begin{bmatrix} 3 \\ -2 \\ 2 \end{bmatrix}$$

Step 3: Use Gram matrix to compute norm.

$$\|\mathbf{v}\|^2 = \mathbf{u}^T G \mathbf{u}$$

$$= \begin{bmatrix} 3 & -2 & 2 \end{bmatrix} \begin{bmatrix} 1 & x & x \\ x & 4 & 0 \\ x & 0 & 9 \end{bmatrix} \begin{bmatrix} 3 \\ -2 \\ 2 \end{bmatrix}$$

Theoretical Solution using Gram Matrix

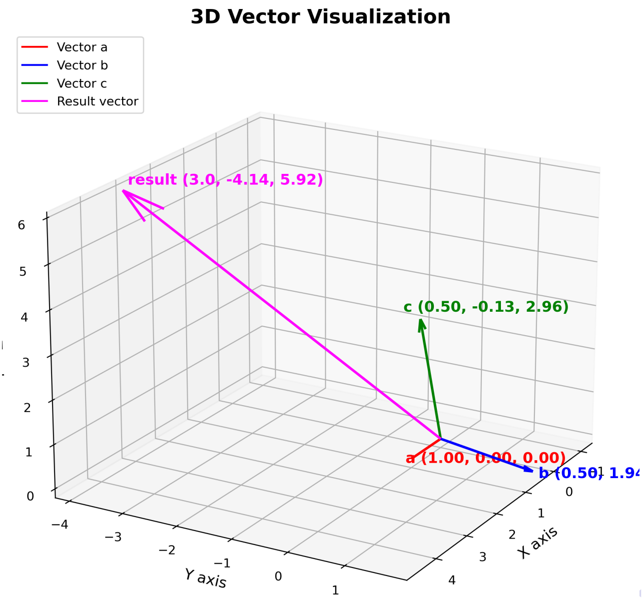
Step 4: Simplify.

$$\|\mathbf{v}\|^2 = 9 + 16 + 36 = 61$$

$$\|\mathbf{v}\| = \sqrt{61}$$

$$\boxed{\|3\mathbf{a} - 2\mathbf{b} + 2\mathbf{c}\| = \sqrt{61}}$$

Thus, the result follows directly from the Gram matrix method.



3D Vector Visualization
Vector a
Vector b
Vector c
Result vector
result (3.0, -4.14, 5.92)
c (0.50, -0.13, 2.96)
a (1.00, 0.00, 0.00)
b (0.50, 1.94
Y axis
X axis
−4
−3
−2
−1
0
1
2
3
4
5
6