

Question 2.3.3

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Question:

If \mathbf{a} , \mathbf{b} , \mathbf{c} are three non-zero unequal vectors such that $\mathbf{a} \cdot \mathbf{b} = \mathbf{a} \cdot \mathbf{c}$, then find the angle between \mathbf{a} and $\mathbf{b} - \mathbf{c}$.

Solution:

The dot product formula is

$$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos \theta \quad (1)$$

Where θ is the angle between vectors \mathbf{a} and \mathbf{b} .

$$\therefore \theta = \arccos\left(\frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}||\mathbf{b}|}\right) \quad (2)$$

In this case, we would need to find

$$\theta = \arccos\left(\frac{\mathbf{a} \cdot (\mathbf{b} - \mathbf{c})}{|\mathbf{a}||\mathbf{b} - \mathbf{c}|}\right) \quad (3)$$

$$\theta = \arccos\left(\frac{\mathbf{a} \cdot \mathbf{b} - \mathbf{a} \cdot \mathbf{c}}{|\mathbf{a}||\mathbf{b} - \mathbf{c}|}\right) \quad (4)$$

$$\theta = \arccos(0) = 90^\circ \quad (5)$$

$$\therefore \mathbf{a} \cdot \mathbf{b} = \mathbf{a} \cdot \mathbf{c} \text{ and } |\mathbf{a}| \neq 0, |\mathbf{b} - \mathbf{c}| \neq 0 \quad (6)$$

Therefore, the angle between the vectors \mathbf{a} and $\mathbf{b} - \mathbf{c}$ is 90° .