

**Question 4.4.22:**

Find the equation of a plane which passes through the point  $(3, 2, 0)$  and contains the line

$$\frac{x-3}{1} = \frac{y-6}{5} = \frac{z-4}{4}. \quad (1)$$

**solution:**

**Finding the plane using column vectors.**

Let the normal be  $\mathbf{n} = (a, b, c)^T$ . We use the form

$$\mathbf{n}^T \mathbf{x} = 1. \quad (2)$$

The plane passes through the point  $P = (3, 2, 0)$  and contains the line

$$\frac{x-3}{1} = \frac{y-6}{5} = \frac{z-4}{4}, \quad (3)$$

so take a point on the line  $A = (3, 6, 4)$  and the direction vector  $\mathbf{v} = (1, 5, 4)$ .

The conditions are

$$\mathbf{n}^T \mathbf{P} = 1 \quad (4)$$

$$\mathbf{n}^T \mathbf{A} = 1 \quad (5)$$

$$\mathbf{n}^T \mathbf{v} = 0. \quad (6)$$

Put these three column vectors together into a matrix (columns are the given points/vectors):

$$\mathbf{M} = \begin{pmatrix} 3 & 3 & 1 \\ 2 & 6 & 5 \\ 0 & 4 & 4 \end{pmatrix} \quad (\text{columns are } \mathbf{P}, \mathbf{A}, \mathbf{v}). \quad (7)$$

Then the three scalar conditions above read compactly as

$$\mathbf{n}^T \mathbf{M} = \begin{pmatrix} 1 & 1 & 0 \end{pmatrix}. \quad (8)$$

Transposing both sides gives a standard linear system for  $\mathbf{n}$ :

$$\mathbf{M}^T \mathbf{n} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}. \quad (9)$$

Write this out:

$$\begin{pmatrix} 3 & 2 & 0 \\ 3 & 6 & 4 \\ 1 & 5 & 4 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}. \quad (10)$$

$$\mathbf{n} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}. \quad (11)$$

Thus a convenient normal vector is  $\mathbf{n} = (1, -1, 1)^T$ , and the plane equation in the requested form is

$$\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}^T \mathbf{x} = 1 \quad (12)$$

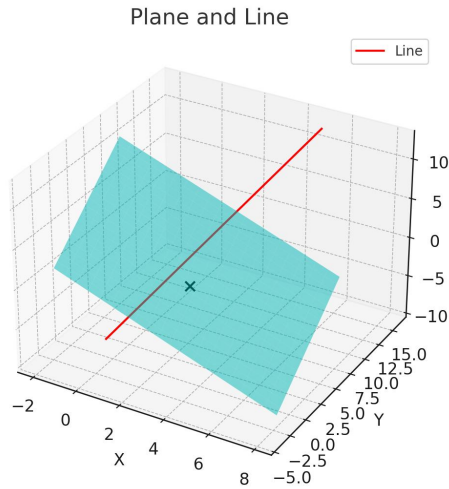


Fig. 1