1.8.20

EE25BTECH11003 - Adharvan Kshathriya Bommagani

Question:
Find a relation between x and y such that the point (x, y) is equidistant from the point (3, 6) and (-3, 4).

Solution:

$$\text{Let } \mathbf{A} = \begin{pmatrix} 3 \\ 6 \end{pmatrix} \quad , \quad \mathbf{B} = \begin{pmatrix} -3 \\ 4 \end{pmatrix} \tag{1}$$

The midpoint of the line segment $\mathbf{B} - \mathbf{A}$ is

$$\mathbf{M} = \frac{\mathbf{A} + \mathbf{B}}{2} \tag{2}$$

$$\mathbf{M} = \begin{pmatrix} 0 \\ 5 \end{pmatrix} \tag{3}$$

Let a point on the perpendicular bisector of the line segment joining $\mathbf{A}$ and $\mathbf{B}$ be $\mathbf{P}$.

$$\mathbf{P} = \begin{pmatrix} x \\ y \end{pmatrix} \tag{4}$$

$$\mathbf{P} - \mathbf{M} = \begin{pmatrix} x \\ y - 5 \end{pmatrix} \tag{5}$$

$\mathbf{P} - \mathbf{M}$ is the perpendicular bisector of line segment $\mathbf{B} - \mathbf{A}$.

$$(\mathbf{P} - \mathbf{M})^{\top} (\mathbf{B} - \mathbf{A}) = 0 \tag{6}$$

$$\begin{pmatrix} x & y - 5 \end{pmatrix} \begin{pmatrix} -6 \\ -2 \end{pmatrix} = 0 \tag{7}$$

$$-6x - 2y + 10 = 0 \tag{8}$$

$$y + 3x = 0 \tag{9}$$

$\therefore$ The relation for the values of x and y such that (x,y) is equidistant from the point (3,6) and (-3,4).

Graph of the line segment AB with midpoint M

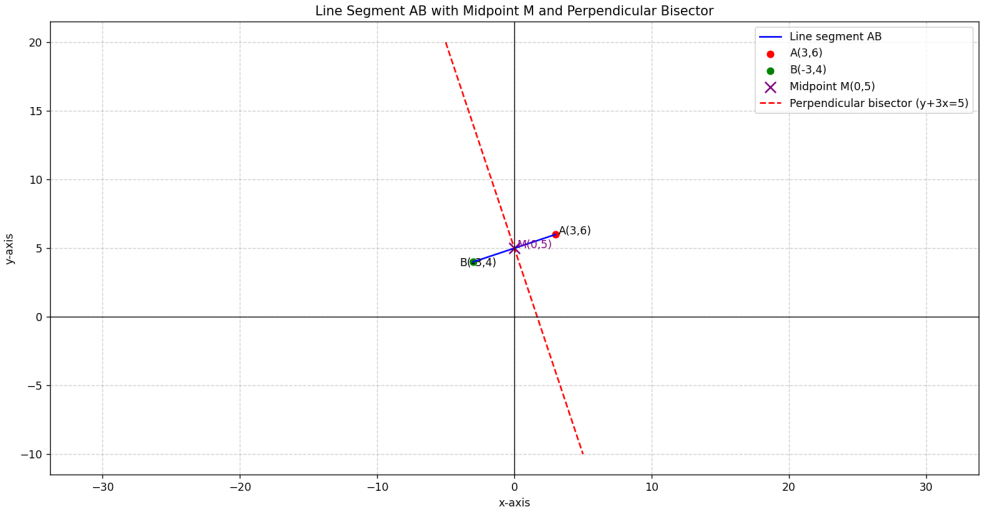

Fig. 0: Figure for 1.8.20