2.7.11

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Question

Q 2.7.11. Find the area of the triangle with vertices $A = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$, $B = \begin{pmatrix} -4 \\ 6 \end{pmatrix}$, $C = \begin{pmatrix} -3 \\ -5 \end{pmatrix}$.

Solution

Let $\mathrm{A} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$, $\mathrm{B} = \begin{pmatrix} -4 \\ 6 \end{pmatrix}$, $\mathrm{C} = \begin{pmatrix} -3 \\ -5 \end{pmatrix}$. then

$$\mathbf{m} = \mathbf{B} - \mathbf{A} = \begin{pmatrix} -5 \\ 7 \end{pmatrix}, \quad \mathbf{n} = \mathbf{C} - \mathbf{A} = \begin{pmatrix} -4 \\ -4 \end{pmatrix}.$$

The area of the triangle is

$$\Delta = \frac{1}{2} \left| \det \begin{pmatrix} \mathbf{m} & \mathbf{n} \end{pmatrix} \right| = \frac{1}{2} \left| (-5)(-4) - (7)(-4) \right| = \frac{1}{2} (48) = \boxed{24}.$$

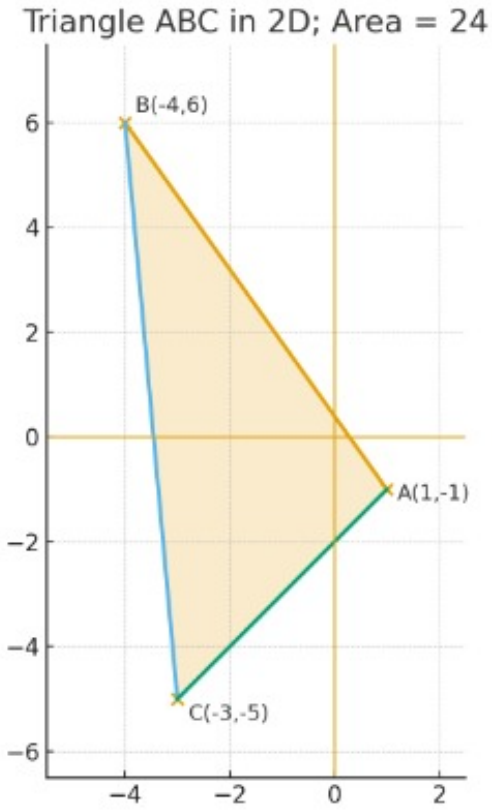

Fig. 0.1: Triangle ABC with $A(1,-1)$, $B(-4,6)$, $C(-3,-5)$; area = 24.