# 4.13.60

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**Question :** A line through $A(5,4)$ meets the lines $x+3y+2=0$, $2x+y+4=0$ and $x-y-5=0$ at $B, C, D$ respectively. If

$$\left(\frac{15}{AB}\right)^2+\left(\frac{10}{AC}\right)^2=\left(\frac{6}{AD}\right)^2,$$

find the equation of the line.

**Solution :**

| Line | Value |
|---|---|
| $\mathbf{x_1}$ | $\begin{pmatrix}\frac{1}{3} & 1\end{pmatrix}\mathbf{x_1}=-\frac{2}{3}$ |
| $\mathbf{x_2}$ | $\begin{pmatrix}2 & 1\end{pmatrix}\mathbf{x_2}=-4$ |
| $\mathbf{x_3}$ | $\begin{pmatrix}-1 & 1\end{pmatrix}\mathbf{x_3}=-5$ |
| $\mathbf{x_4}$ | $\mathbf{x_4}=\begin{pmatrix}5\\4\end{pmatrix}+k\begin{pmatrix}1\\m\end{pmatrix}$ |

Table : Lines

Let the required line be

$$\mathbf{x_4}=\begin{pmatrix}5\\4\end{pmatrix}+k\begin{pmatrix}1\\m\end{pmatrix} \tag{1}$$

Hence the points $B, C, D$ can be written as

$$\mathbf{B}=\begin{pmatrix}5\\4\end{pmatrix}+k_1\begin{pmatrix}1\\m\end{pmatrix}=\begin{pmatrix}5+k_1\\4+k_1m\end{pmatrix} \tag{2}$$

$$\mathbf{C}=\begin{pmatrix}5\\4\end{pmatrix}+k_2\begin{pmatrix}1\\m\end{pmatrix}=\begin{pmatrix}5+k_2\\4+k_2m\end{pmatrix} \tag{3}$$

$$\mathbf{D}=\begin{pmatrix}5\\4\end{pmatrix}+k_3\begin{pmatrix}1\\m\end{pmatrix}=\begin{pmatrix}5+k_3\\4+k_3m\end{pmatrix} \tag{4}$$

Find $k_1, k_2, k_3$
Since $\mathbf{B}$ lies on $\mathbf{x_1}$

$$\begin{pmatrix}\frac{1}{3} & 1\end{pmatrix}\mathbf{B}=-\frac{2}{3} \tag{5}$$

$$\begin{pmatrix}\frac{1}{3} & 1\end{pmatrix}\begin{pmatrix}5+k_1\\4+k_1m\end{pmatrix}=-\frac{2}{3} \tag{6}$$

$$\frac{17}{3}+\left(m+\frac{1}{3}\right)k_1=-\frac{2}{3} \tag{7}$$

$$\left(m+\frac{1}{3}\right)k_1=-\frac{19}{3} \tag{8}$$

$$k_1=\frac{-19}{3m+1} \tag{9}$$

Since **C** lies on $\mathbf{x_2}$

$$\begin{pmatrix}2 & 1\end{pmatrix}\mathbf{C} = -4 \tag{10}$$

$$\begin{pmatrix}2 & 1\end{pmatrix}\begin{pmatrix}5+k_2\\4+k_2m\end{pmatrix} = -4 \tag{11}$$

$$(2+m)k_2 + 14 = -4 \tag{12}$$

$$(2+m)k_2 = -18 \tag{13}$$

$$k_2 = \frac{-18}{m+2} \tag{14}$$

Since **D** lies on $\mathbf{x_3}$

$$\begin{pmatrix}-1 & 1\end{pmatrix}\mathbf{D} = -5 \tag{15}$$

$$\begin{pmatrix}-1 & 1\end{pmatrix}\begin{pmatrix}5+k_3\\4+k_3m\end{pmatrix} = -5 \tag{16}$$

$$(m-1)k_3 - 1 = -5 \tag{17}$$

$$(m-1)k_3 = -4 \tag{18}$$

$$k_3 = \frac{-4}{m-1} \tag{19}$$

Find distances

$$||\mathbf{B}-\mathbf{A}|| = \left\|\begin{pmatrix}k_1\\k_1m\end{pmatrix}\right\| = |k_1|\sqrt{1+m^2} \tag{20}$$

$$||\mathbf{C}-\mathbf{A}|| = \left\|\begin{pmatrix}k_2\\k_2m\end{pmatrix}\right\| = |k_2|\sqrt{1+m^2} \tag{21}$$

$$||\mathbf{D}-\mathbf{A}|| = \left\|\begin{pmatrix}k_3\\k_3m\end{pmatrix}\right\| = |k_3|\sqrt{1+m^2} \tag{22}$$

Use given equation

$$\left(\frac{15}{||\mathbf{B}-\mathbf{A}||}\right)^2 + \left(\frac{10}{||\mathbf{C}-\mathbf{A}||}\right)^2 = \left(\frac{6}{||\mathbf{D}-\mathbf{A}||}\right)^2 \tag{23}$$

Substitute distances:

$$\frac{225}{k_1^2(1+m^2)} + \frac{100}{k_2^2(1+m^2)} = \frac{36}{k_3^2(1+m^2)} \tag{24}$$

Multiply throughout by $(1+m^2)$:

$$\frac{225}{k_1^2} + \frac{100}{k_2^2} = \frac{36}{k_3^2} \tag{25}$$

Substitute values of $k_1, k_2, k_3$:

$$\frac{225}{\left(\frac{-19}{3m+1}\right)^2} + \frac{100}{\left(\frac{-18}{m+2}\right)^2} = \frac{36}{\left(\frac{-4}{m-1}\right)^2} \tag{26}$$

Simplify:

$$\frac{225(3m+1)^2}{361} + \frac{100(m+2)^2}{324} = \frac{9(m-1)^2}{4} \tag{27}$$

$$\tag{28}$$

$$429031m^2 + 1108138m - 45869 = 0 \tag{29}$$

This gives a quadratic in $m$. Solving by using the quadratic formula, we get

$$m = 0.04075, \quad m = -2.62364 \tag{30}$$

**Answer :**

Final equations of the line
For $m = 0.04075$:

$$\mathbf{x_4} = \begin{pmatrix} 5 \\ 4 \end{pmatrix} + k \begin{pmatrix} 1 \\ 0.04075 \end{pmatrix} \tag{31}$$

For $m = -2.62364$ :

$$\mathbf{x_4} = \begin{pmatrix} 5 \\ 4 \end{pmatrix} + k \begin{pmatrix} 1 \\ -2.62364 \end{pmatrix} \tag{32}$$

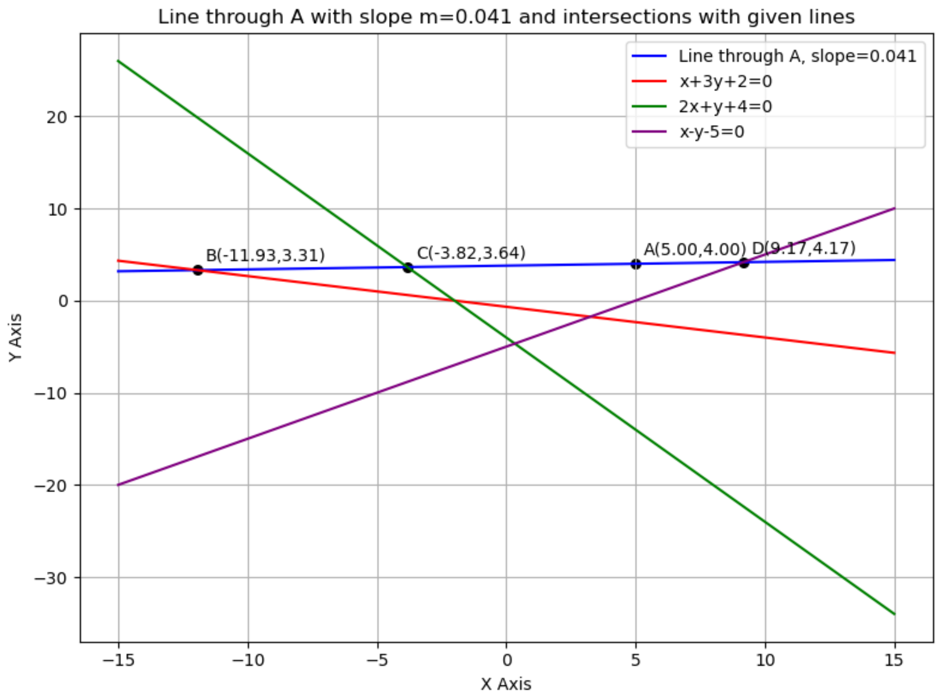


Fig : Lines 1

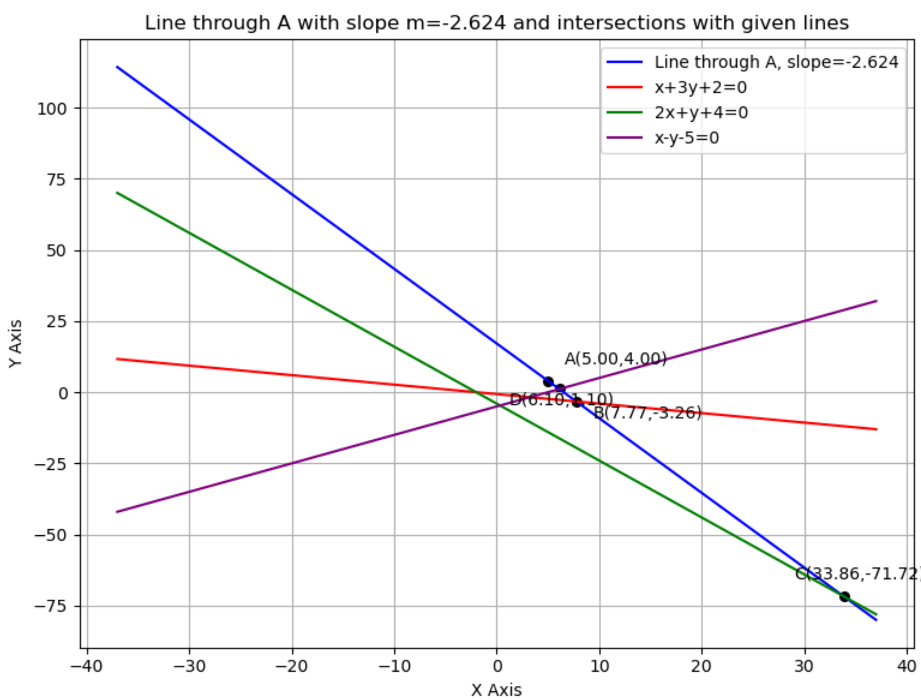


Fig : Lines 2