4.11.24

EE25BTECH11047 - RAVULA SHASHANK REDDY

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Question:

Find the equation of the plane through the line of intersection of

$$\mathbf{r}^T(\mathbf{i}+3\mathbf{j})+6=0, \mathbf{r}^T(3\mathbf{i}-\mathbf{j}-4\mathbf{k})=0$$

which is at a unit distance from the origin.

Solution:

$$\pi_1 : \mathbf{n}_1 = \begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix}, \ c_1 = -6, \tag{1}$$

$$\pi_2 : \mathbf{n}_2 = \begin{pmatrix} 3 \\ -1 \\ -4 \end{pmatrix}, \ c_2 = 0. \tag{2}$$

Family of planes:

$$(\mathbf{n}_1 + \lambda \mathbf{n}_2)^T \mathbf{x} = c_1 + \lambda c_2 \tag{3}$$

$$\mathbf{n}^T \mathbf{x} = c \tag{4}$$

Distance condition:

$$d = \frac{|\mathbf{n}^T \mathbf{x} - c|}{\|\mathbf{n}\|} \tag{5}$$

$$\mathbf{x} = \mathbf{0} \tag{6}$$

$$\frac{|c_1 + \lambda c_2|}{\|\mathbf{n}_1 + \lambda \mathbf{n}_2\|} = 1 \quad \implies \quad (c_1 + \lambda c_2)^2 = (\mathbf{n}_1 + \lambda \mathbf{n}_2)^T (\mathbf{n}_1 + \lambda \mathbf{n}_2) \tag{7}$$

$$36 = \mathbf{n}_1^T\mathbf{n}_1 + 2\lambda\mathbf{n}_1^T\mathbf{n}_2 + \lambda^2\mathbf{n}_2^T\mathbf{n}_2 \tag{8}$$

$$\mathbf{n}_1^T\mathbf{n}_1 = 1^2 + 3^2 = 10, \tag{9}$$

$$\mathbf{n}_1^T\mathbf{n}_2 = 1.3 + 3.(-1) + 0.(-4) = 0, \tag{10}$$

$$\mathbf{n}_2^T\mathbf{n}_2 = 3^2 + (-1)^2 + (-4)^2 = 26 \tag{11}$$

Hence:

$$36 = 10 + 26\lambda^2 \quad \implies \quad 26\lambda^2 = 26 \quad \implies \quad \lambda = \pm 1 \tag{12}$$

For $\lambda = 1$:

$$\begin{pmatrix} -\frac{2}{3} & -\frac{1}{3} & \frac{2}{3} \end{pmatrix} \mathbf{x} = 1 \tag{13}$$

For $\lambda = -1$:

$$\begin{pmatrix} \frac{1}{3} & -\frac{2}{3} & -\frac{2}{3} \end{pmatrix} \mathbf{x} = 1 \tag{14}$$

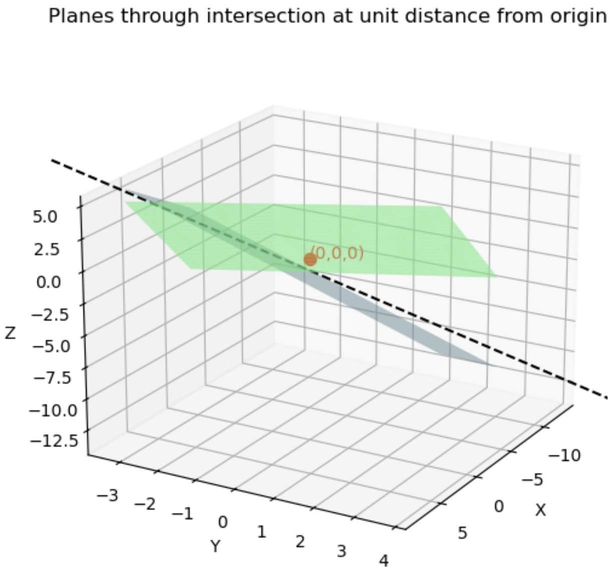
Planes through intersection at unit distance from origin
(0,0,0)
Z
5.0
2.5
0.0
−2.5
−5.0
−7.5
−10.0
−12.5
Y
−3
−2
−1
0
1
2
3
4
X
−10
−5
0
5

Figure 1