

4.3.30

EE25BTECH11041 - Naman Kumar

Question:

Find the equation of the line which passes through the point $(-4, 3)$ and the portion of the line intercepted between the axes is divided internally in ratio 5:3 by this point.

Solution:

Let the intercept points be

$$\mathbf{P} = a\mathbf{e}_1, \mathbf{Q} = b\mathbf{e}_2 \quad (1)$$

$$\mathbf{e}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \mathbf{e}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, a \text{ and } b \text{ are constants} \quad (2)$$

and

$$\mathbf{R} = \begin{pmatrix} -4 \\ 3 \end{pmatrix} = -4\mathbf{e}_1 + 3\mathbf{e}_2 \quad (3)$$

be the given point.

Using

$$\mathbf{R} = \frac{k\mathbf{Q} + \mathbf{P}}{k + 1} \quad (4)$$

$$\mathbf{R} = \frac{5 \times b\mathbf{e}_2 + 3 \times a\mathbf{e}_1}{8} \quad (5)$$

$$-4\mathbf{e}_1 + 3\mathbf{e}_2 = \frac{5 \times b\mathbf{e}_2 + 3 \times a\mathbf{e}_1}{8} \quad (6)$$

$$-32\mathbf{e}_1 + 24\mathbf{e}_2 = 3a\mathbf{e}_1 + 5b\mathbf{e}_2 \quad (7)$$

General equation of line

$$\mathbf{x} = \mathbf{h} + c\mathbf{m} \quad (8)$$

Where

x	general vector on line
h	known vector of line
m	slope vector of line
c	scalar parameter

TABLE I

Slope is

$$\mathbf{m} = \mathbf{Q} - \mathbf{P} \quad (9)$$

$$(10)$$

let $\mathbf{h} = \mathbf{P}$

So, Equation of line is

$$\mathbf{x} = \mathbf{h} + c\mathbf{m} \quad (11)$$

$$\mathbf{x} = \mathbf{P} + c(\mathbf{Q} - \mathbf{P}) \quad (12)$$

Putting values of \mathbf{Q}, \mathbf{P}

$$\mathbf{x} = a\mathbf{e}_1 + c(b\mathbf{e}_2 - a\mathbf{e}_1) \quad (13)$$

Comparing terms in (7) for values of a and b

$$\mathbf{x} = \frac{-32}{3}\mathbf{e}_1 + c\left(\frac{24}{5}\mathbf{e}_2 - \frac{-32}{3}\mathbf{e}_1\right) \quad (14)$$

Therefore Final equation is

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{-32}{3} \\ 0 \end{pmatrix} + k \begin{pmatrix} \frac{32}{3} \\ \frac{24}{5} \end{pmatrix} \quad (15)$$

Graph

