4.7.50

Shriyansh Chawda-EE25BTECH11052 August 23, 2025

Question

Find the vector equation of the line passing through (1, 2, 3) and perpendicular to the $\vec{r} \cdot (\hat{i} + 2\hat{j} - 5\hat{k}) + 9 = 0$.

Solution

The plane is given by

$$\mathbf{r} \cdot (1, 2, -5) + 9 = 0 \tag{1}$$

so the plane's normal vector is

$$\mathbf{n} = \begin{pmatrix} 1 \\ 2 \\ -5 \end{pmatrix}. \tag{2}$$

The required line is perpendicular to the plane, so its direction vector lies in the row space. Thus, the line direction vector can be chosen as

$$\mathbf{d} = \mathbf{n} = \begin{pmatrix} 1 \\ 2 \\ -5 \end{pmatrix}. \tag{3}$$

Solution

Using the point

$$\mathbf{a} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \tag{4}$$

the vector equation of the line is

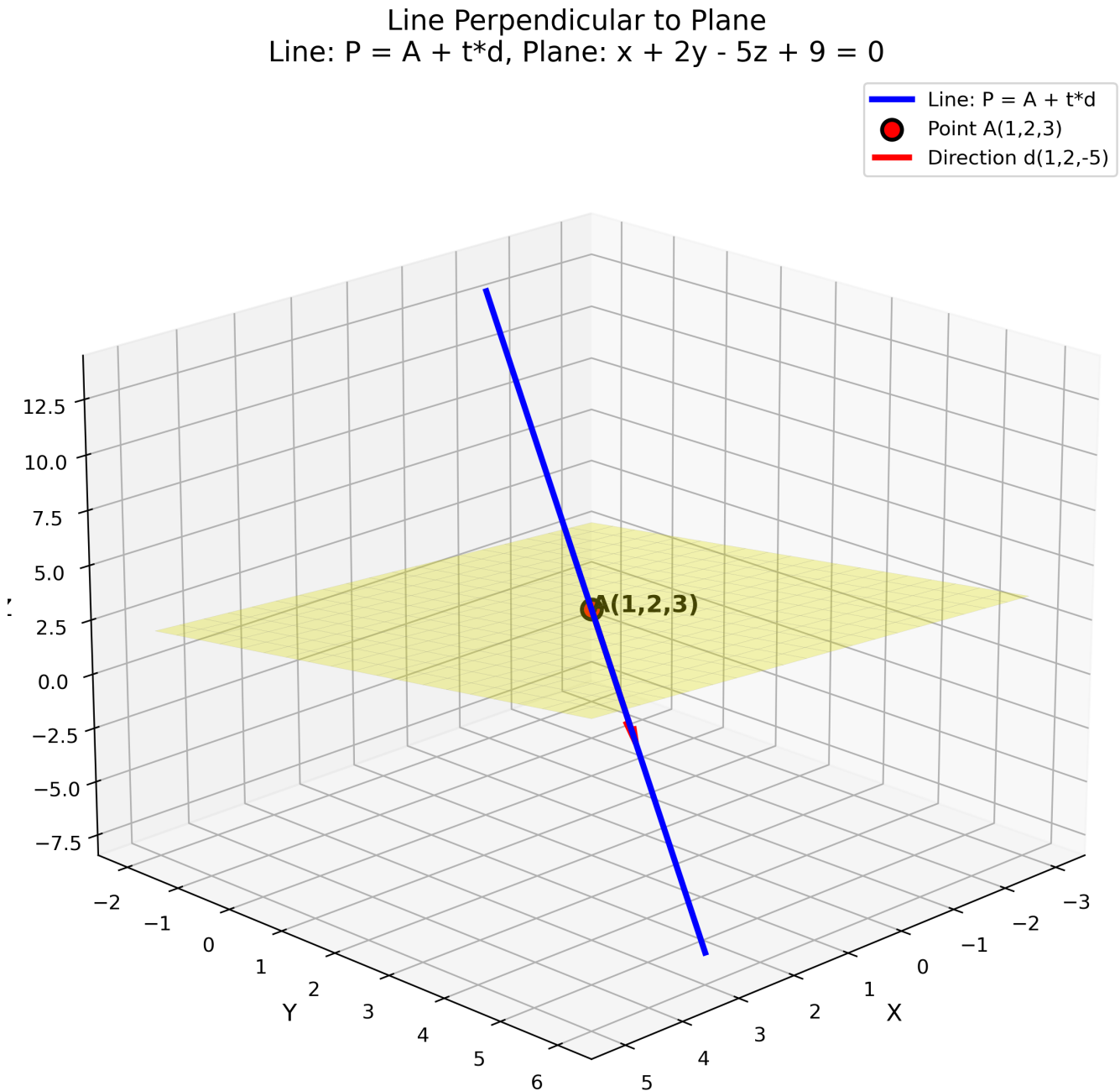
$$\mathbf{r} = \mathbf{a} + t\mathbf{d} \tag{5}$$

$$\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + t \begin{pmatrix} 1 \\ 2 \\ -5 \end{pmatrix}, \quad t \in \mathbb{R}. \tag{6}$$

In symmetric form, the line is

$$\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{-5}. \tag{7}$$

Plot


Line Perpendicular to Plane
Line: P = A + t*d, Plane: x + 2y - 5z + 9 = 0
Line: P = A + t*d
Point A(1,2,3)
Direction d(1,2,-5)
A(1,2,3)
12.5
10.0
7.5
5.0
2.5
0.0
−2.5
−5.0
−7.5
Z
−2
−1
0
1
2
3
4
5
6
Y
5
4
3
2
1
0
−1
−2
−3
X