2.7.30

EE25BTECH11018 - DARISY SREETEJ

Question: Find the value of k so that the area of $\triangle ABC$ with $\mathbf{A}(k+1, 1)$, $\mathbf{B}(4, -3)$ and $\mathbf{C}(7, -k)$ is 6 square units

Solution:
Given,

$$\mathbf{A} = \begin{pmatrix} k+1 \\ 1 \end{pmatrix} \tag{0.1}$$

$$\mathbf{B} = \begin{pmatrix} 4 \\ -3 \end{pmatrix} \tag{0.2}$$

$$\mathbf{C} = \begin{pmatrix} 7 \\ -k \end{pmatrix} \tag{0.3}$$

Now, consider

$$\mathbf{AB} = \mathbf{B} - \mathbf{A} = \begin{pmatrix} 4 \\ -3 \end{pmatrix} - \begin{pmatrix} k+1 \\ 1 \end{pmatrix} = \begin{pmatrix} 3-k \\ -4 \end{pmatrix} \tag{0.4}$$

$$\mathbf{AC} = \mathbf{C} - \mathbf{A} = \begin{pmatrix} 7 \\ -k \end{pmatrix} - \begin{pmatrix} k+1 \\ 1 \end{pmatrix} = \begin{pmatrix} 6-k \\ -k-1 \end{pmatrix} \tag{0.5}$$

The Area of the $\triangle ABC$:

$$\text{Area}(\triangle ABC) = \frac{1}{2}\|(\mathbf{B} - \mathbf{A}) \times (\mathbf{C} - \mathbf{A})\|. \tag{0.6}$$

Here according to problem , the Area of $\triangle ABC$ is 6 square units
Therefore,

$$\frac{1}{2}\|(\mathbf{B} - \mathbf{A}) \times (\mathbf{C} - \mathbf{A})\| = 6 \tag{0.7}$$

$$\frac{1}{2}\left\|\begin{pmatrix} 3-k \\ -4 \end{pmatrix} \times \begin{pmatrix} 6-k \\ -k-1 \end{pmatrix}\right\| = 6 \tag{0.8}$$

$$\left|(3-k)(-k-1) - (-4)(6-k)\right| = 12 \tag{0.9}$$

$$\left|k^2 - 6k + 21\right| = 12 \tag{0.10}$$

Case 1 :

$$k^2 - 6k + 24 = 12 \tag{0.11}$$

$$k^2 - 6k + 9 = 0 \tag{0.12}$$

$$(k-3)^2 = 0 \tag{0.13}$$

$$k = 3 \tag{0.14}$$

Case 2 :

$$k^2 - 6k + 24 = -12 \quad (0.15)$$
$$k^2 - 6k + 33 = 0 \quad (0.16)$$
$$(0.17)$$

Discriminant: $D = -96$.
Since the discriminant is negative , there is no real solution for this case.

Thus , k=3

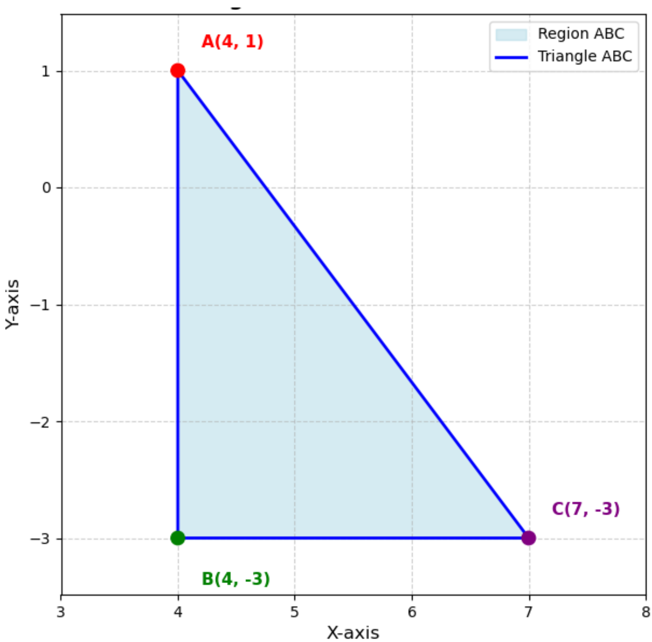

Fig: $\triangle ABC$ with shaded area