2.7.3

AI25BTECH110031
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Question(2.7.3) If $\mathbf{a}$ and $\mathbf{b}$ are two vectors such that $\mathbf{a} = \hat{\imath} - \hat{\jmath} + \hat{k}$, $\mathbf{b} = 2\hat{\imath} - \hat{\jmath} - 3\hat{k}$, then find the vector $\mathbf{c}$, given that $\mathbf{a} \times \mathbf{c} = \mathbf{b}$, $\mathbf{a} \cdot \mathbf{c} = 4$.

Solution:

$$\mathbf{a} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \quad \mathbf{b} = \begin{pmatrix} 2 \\ -1 \\ -3 \end{pmatrix} \quad \mathbf{c} = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} \tag{0.1}$$

$$\mathbf{a} \times \mathbf{c} = \mathbf{b} \tag{0.2}$$

$$\begin{pmatrix} -c_2 - c_3 \\ c_1 - c_3 \\ c_1 + c_2 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ -3 \end{pmatrix} \tag{0.3}$$

$$\implies \begin{cases} -c_2 - c_3 = 2, \\ c_1 - c_3 = -1, \\ c_1 + c_2 = -3. \end{cases} \tag{0.4}$$

From the second and third equations:

$$c_1 = c_3 - 1, \qquad c_2 = -2 - c_3. \tag{0.5}$$

$$\implies \mathbf{c} = \begin{pmatrix} c_3 - 1 \\ -2 - c_3 \\ c_3 \end{pmatrix}, \quad c_3 \in \mathbb{R}. \tag{0.6}$$

Now apply the dot product condition:

$$\mathbf{a}^T \mathbf{c} = \begin{pmatrix} 1 & -1 & 1 \end{pmatrix} \begin{pmatrix} c_3 - 1 \\ -2 - c_3 \\ c_3 \end{pmatrix} = 3c_3 + 1. \tag{0.7}$$

$$3c_3 + 1 = 4 \quad \implies \quad c_3 = 1. \tag{0.8}$$

$$\therefore \quad \mathbf{c} = \begin{pmatrix} 0 \\ -3 \\ 1 \end{pmatrix} \tag{0.9}$$

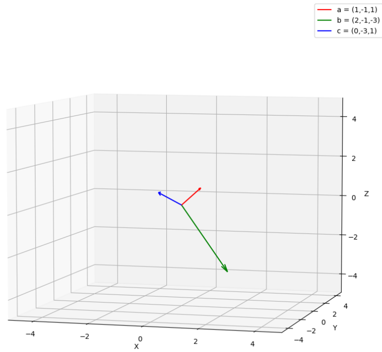

Fig. 0.1