4.7.11

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Question

Show that the path of a moving point such that its distance from two lines $3x - 2y = 5$ and $3x + 2y = 5$ are equal is a straight line.

Given

Given line equations can be written as:

$$\mathbf{n}_1^\top \mathbf{x} = c_1 \tag{1}$$

$$\mathbf{n}_1 = \begin{pmatrix} 3 \\ -2 \end{pmatrix}; c_1 = 5 \tag{2}$$

$$\mathbf{n}_2^\top \mathbf{x} = c_2 \tag{3}$$

$$\mathbf{n}_2 = \begin{pmatrix} 3 \\ 2 \end{pmatrix}; c_2 = 5 \tag{4}$$

let the point equidistant from the given lines be:

$$\mathbf{P} = \begin{pmatrix} x \\ y \end{pmatrix} \tag{5}$$

Proof

From distance formula:

$$d_1 = \frac{|\mathbf{n}_1^\top \mathbf{P} - c_1|}{||\mathbf{n}_1||} \tag{6}$$

$$d_2 = \frac{|\mathbf{n}_2^\top \mathbf{P} - c_2|}{||\mathbf{n}_2||} \tag{7}$$

$$\because d_1 = d_2 \tag{8}$$

$$\frac{|\mathbf{n}_1^\top \mathbf{P} - c_1|}{||\mathbf{n}_1||} = \frac{|\mathbf{n}_2^\top \mathbf{P} - c_2|}{||\mathbf{n}_2||} \tag{9}$$

$$\because ||\mathbf{n}_1|| = ||\mathbf{n}_2|| = \sqrt{3^2 + 2^2} = \sqrt{13} \tag{10}$$

$$\mathbf{n}_1^\top \mathbf{P} - c_1 = \pm\left(\mathbf{n}_2^\top \mathbf{P} - c_2\right) \tag{11}$$

checking

First, by taking $+$:

$$\mathbf{n}_1^\top \mathbf{P} - c_1 = +\left(\mathbf{n}_2^\top \mathbf{P} - c_2\right) \tag{12}$$

$$\mathbf{n}_1^\top \mathbf{P} - \mathbf{n}_2^\top \mathbf{P} = c_1 - c_2 \tag{13}$$

$$\left(\mathbf{n}_1 - \mathbf{n}_2\right)^\top \mathbf{P} = c_1 - c_2 \tag{14}$$

$$\begin{pmatrix} 0 & -4 \end{pmatrix} \mathbf{P} = 0 \tag{15}$$

Now by taking $-$:

$$\mathbf{n}_1^\top \mathbf{P} - c_1 = -\left(\mathbf{n}_2^\top \mathbf{P} - c_2\right) \tag{16}$$

$$\mathbf{n}_1^\top \mathbf{P} + \mathbf{n}_2^\top \mathbf{P} = c_1 + c_2 \tag{17}$$

$$\left(\mathbf{n}_1 + \mathbf{n}_2\right)^\top \mathbf{P} = c_1 + c_2 \tag{18}$$

$$\begin{pmatrix} 6 & 0 \end{pmatrix} \mathbf{P} = 10 \tag{19}$$

Conclusion

Since equations (15) and (19) are in the form of line equation $\mathbf{n}^\top \mathbf{x} = c$, the given path of the moving point is a line.

C Code

```c
#include <stdio.h>

int vector1[2]={3, -2};
int constant1[1]={5};
int vector2[2]={3, 2};
int constant2[1]={5};

int get_vector1(int index){
    return vector1[index];
}
```

C Code

```c
int get_vector2(int index){
    return vector2[index];
}

int get_constant1(int index){
    return constant1[index];
}

int get_constant2(int index){
    return constant2[index];
}
```

Python Code 1

```
import ctypes

lib = ctypes.CDLL("./problem.so")

finalvector1=[0, 0]
finalvector2=[0, 0]

lib.get_vector1.argtypes = [ctypes.c_int]
lib.get_vector1.restype = ctypes.c_int

lib.get_vector2.argtypes = [ctypes.c_int]
lib.get_vector2.restype = ctypes.c_int

lib.get_constant1.argtypes = [ctypes.c_int]
lib.get_constant1.restype = ctypes.c_int
```

Python Code 1

```
lib.get_constant2.argtypes = [ctypes.c_int]
lib.get_constant2.restype = ctypes.c_int


for i in range(0,2):
    finalvector1[i]=lib.get_vector1(i)-lib.get_vector2(i)

for i in range(0,2):
    finalvector2[i]=lib.get_vector1(i)+lib.get_vector2(i)

finalconstant=lib.get_constant1(0)+lib.get_constant2(0)

print(f"The final line equations are y=0 and {finalvector2[0]}x={
    finalconstant}")
```

Python Code 2

```
import matplotlib.pyplot as plt
import numpy as np

a = np.linspace(-10, 10, 100)
b = (3*a)/2 - (5/2)

A = np.linspace(-10, 10, 100)
B = (-3*A)/2 + (5/2)

x = [10/6, 10/6]
y = [15, -15]

X = [-15, 15]
Y = [0, 0]
```

Python Code 2

```
plt.plot(a, b, 'r-')
plt.plot(A, B, 'r-')
plt.plot(x, y, 'k-')
plt.plot(X, Y, 'k-')

plt.text(10, 12.3, "3x-2y=5", fontsize=10, color='black')
plt.text(-8.3, 15, "3x+2y=5", fontsize=10, color='black')
plt.text(15.2, -0.06, "y=0", fontsize=10, color='black')
plt.text(1.6, 14.6, "x=10/6", fontsize=10, color='black')


plt.xlabel('X-axis')
plt.ylabel('Y-axis')
plt.axis('equal')
plt.grid(True)
plt.savefig("../figs/plot.png")
plt.show()
```

Plot

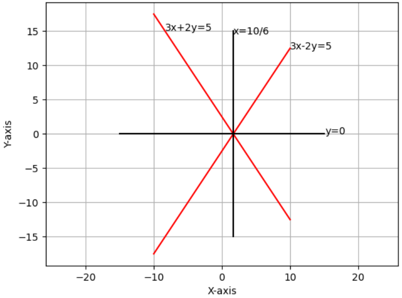

Figure: Plot of given lines and path of the moving points