5.4.28

EE25BTECH11033 - Kavin

Question:
Using elementary transformations, find the inverse of the following matrix.

$$\begin{pmatrix} 2 & 4 \\ -5 & -1 \end{pmatrix}$$

Solution:
Given the matrix,

$$\mathbf{A} = \begin{pmatrix} 2 & 4 \\ -5 & -1 \end{pmatrix} \tag{1}$$

Let $\mathbf{A}^{-1}$ be the inverse of the matrix $\mathbf{A}$.

We know that ,

$$\mathbf{A}\mathbf{A}^{-1} = \mathbf{I} \tag{2}$$

The augmented matrix of $\left(\mathbf{A} \mid \mathbf{I}\right)$ is given by,

$$\left(\begin{array}{cc|cc} 2 & 4 & 1 & 0 \\ -5 & -1 & 0 & 1 \end{array}\right) \tag{3}$$

$$R_1 \to \frac{1}{2}R_1 \implies \left(\begin{array}{cc|cc} 1 & 2 & 1/2 & 0 \\ -5 & -1 & 0 & 1 \end{array}\right) \tag{4}$$

$$R_2 \to R_2 + 5R_1 \implies \left(\begin{array}{cc|cc} 1 & 2 & 1/2 & 0 \\ 0 & 9 & 5/2 & 1 \end{array}\right) \tag{5}$$

$$R_2 \to \frac{1}{9}R_2 \implies \left(\begin{array}{cc|cc} 1 & 2 & 1/2 & 0 \\ 0 & 1 & 5/18 & 1/9 \end{array}\right) \tag{6}$$

$$R_1 \to R_1 - 2R_2 \implies \left(\begin{array}{cc|cc} 1 & 0 & -1/18 & -2/9 \\ 0 & 1 & 5/18 & 1/9 \end{array}\right) \tag{7}$$

$$\implies \mathbf{A}^{-1} = \begin{pmatrix} -1/18 & -2/9 \\ 5/18 & 1/9 \end{pmatrix} \tag{8}$$