2.10.6

AI25BTECH11017-BALU

Question:
If $\mathbf{A}, \mathbf{B}, \mathbf{C}$ are three non-coplanar vectors, then

$$\frac{\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C})}{(\mathbf{C} \times \mathbf{A}) \cdot \mathbf{B}} + \frac{\mathbf{B} \cdot (\mathbf{A} \times \mathbf{C})}{\mathbf{C} \cdot (\mathbf{A} \times \mathbf{B})} = \tag{0.1}$$

.

Solution:
Let us solve the given equation theoretically and then verify the solution computationally
According to the question,
Let us take three non coplanar vectors

$$\mathbf{A} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \ \mathbf{B} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \ \mathbf{C} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \tag{0.2}$$

$$\mathbf{A}^T(\mathbf{B} \times \mathbf{C}) = [\mathbf{A} \quad \mathbf{B} \quad \mathbf{C}] = \left\| \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right\| = 1 \tag{0.3}$$

$$(\mathbf{C} \times \mathbf{A})^T \mathbf{B} = [\mathbf{C} \quad \mathbf{A} \quad \mathbf{B}] = \left\| \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \right\| = 1 \tag{0.4}$$

$$\mathbf{B}^T(\mathbf{A} \times \mathbf{C}) = [\mathbf{B} \quad \mathbf{A} \quad \mathbf{C}] = \left\| \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right\| = -1 \tag{0.5}$$

$$\mathbf{C}^T(\mathbf{A} \times \mathbf{B}) = [\mathbf{C} \quad \mathbf{A} \quad \mathbf{B}] = \left\| \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \right\| = 1 \tag{0.6}$$

$$\frac{\mathbf{A}^T(\mathbf{B} \times \mathbf{C})}{(\mathbf{C} \times \mathbf{A})^T \mathbf{B}} + \frac{\mathbf{B}^T(\mathbf{A} \times \mathbf{C})}{\mathbf{C}^T(\mathbf{A} \times \mathbf{B})} = \frac{1}{1} + \frac{-1}{1} = 1 - 1 = 0 \tag{0.7}$$

By verification method we showed the result is 0