5.2.16

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Question

Solve the system of equations:

$$3x - 5y = 20 \tag{1}$$

$$6x - 10y = 40 \tag{2}$$

Line Representation

The equation of a line is

$$\mathbf{n}^{\mathsf{T}}\mathbf{x} = c \tag{3}$$

Line L:

$$\begin{pmatrix} 3 & -5 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 20 \tag{4}$$

Line K:

$$\begin{pmatrix} 6 & -10 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 40 \tag{5}$$

Matrix Form

These can be combined into matrix form:

$$\begin{pmatrix} 3 & -5 \\ 6 & -10 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 20 \\ 40 \end{pmatrix} \tag{6}$$

The following augmented matrix can be solved by Gaussian elimination

$$\left(\begin{array}{cc|c} 3 & -5 & 20 \\ 6 & -10 & 40 \end{array}\right) \xleftrightarrow{R_2 \leftarrow R_2 - 2R_1} \left(\begin{array}{cc|c} 3 & -5 & 20 \\ 0 & 0 & 0 \end{array}\right) \tag{7}$$

Rank and Reduced Equation

We end up with only one non-zero row (Rank $= 1$):

$$\begin{pmatrix} 3 & -5 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 20 \tag{8}$$

This represents a line in $\mathbb{R}^2$.

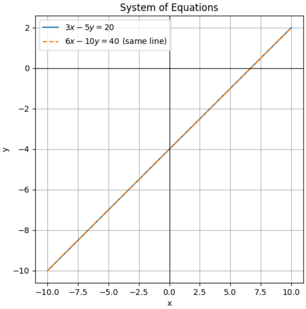
General Solution

The general solution is:

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} t \\ \frac{3t-20}{5} \end{pmatrix}, \quad t \in \mathbb{R} \tag{9}$$

Conclusion: The system has **infinitely many solutions**.

Plot


System of Equations
$3x - 5y = 20$
$6x - 10y = 40$ (same line)
y
x
2
0
−2
−4
−6
−8
−10
−10.0
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0

C Code

```c
#include <stdio.h>

// Function to compute y from first equation: y = (3x - 20)/5
double line1(double x) {
    return (3.0 * x - 20.0) / 5.0;
}

// Function to compute y from second equation: y = (6x - 40)/10
double line2(double x) {
    return (6.0 * x - 40.0) / 10.0;
}
```

Python + C Code

```
import ctypes
import numpy as np
import matplotlib.pyplot as plt


lib = ctypes.CDLL(./liblines.so)

lib.line1.argtypes = [ctypes.c_double]
lib.line1.restype = ctypes.c_double

lib.line2.argtypes = [ctypes.c_double]
lib.line2.restype = ctypes.c_double


x_vals = np.linspace(-10, 10, 400)
y1 = np.array([lib.line1(float(x)) for x in x_vals])
y2 = np.array([lib.line2(float(x)) for x in x_vals])
```

Python + C Code

```python

plt.figure(figsize=(6,6))
plt.plot(x_vals, y1, label=r'$3x - 5y = 20$')
plt.plot(x_vals, y2, '--', label=r'$6x - 10y = 40$ (same line)')

plt.axhline(0, color='black', linewidth=0.8)
plt.axvline(0, color='black', linewidth=0.8)
plt.xlabel(x)
plt.ylabel(y)
plt.title(System of Equations (C + Python))
plt.legend()
plt.grid(True)
plt.savefig(/Users/bhargavkrish/Desktop/BackupMatrix/
    ee25btech11013/matgeo/5.2.16/figs/Figure_1.png)
plt.show()
```

Python Code

```
import numpy as np
import matplotlib.pyplot as plt

A = np.array([[3, -5],
              [6, -10]])
b = np.array([20, 40])
print(The system has no solution (inconsistent).)
x_vals = np.linspace(-10, 10, 400)
y1 = (3*x_vals - 20)/5
y2 = (6*x_vals - 40)/10
plt.figure(figsize=(6,6))
plt.plot(x_vals, y1, label=r'$3x - 5y = 20$')
plt.plot(x_vals, y2, '--', label=r'$6x - 10y = 40$ (same line)')
```

Python Code

```
plt.axhline(0, color='black', linewidth=0.8)
plt.axvline(0, color='black', linewidth=0.8)
plt.xlabel(x)
plt.ylabel(y)
plt.title(System of Equations)
plt.legend()
plt.grid(True)
plt.savefig(/Users/bhargavkrish/Desktop/BackupMatrix/
    ee25btech11013/matgeo/5.2.16/figs/Figure_1.png)
plt.show()
```