5.4.41

EE25BTECH11047 - RAVULA SHASHANK REDDY

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Question:

Using elementary transformations, find the inverse of the matrix

$$\mathbf{A} = \begin{pmatrix} 2 & 1 & 3 \\ 4 & -1 & 0 \\ -7 & 2 & 1 \end{pmatrix}.$$

Solution:

$$\mathbf{A}.\mathbf{A}^{-1} = \mathbf{I} \tag{1}$$

$$[\mathbf{A} \mid \mathbf{I}] = \begin{pmatrix} 2 & 1 & 3 & 1 & 0 & 0 \\ 4 & -1 & 0 & 0 & 1 & 0 \\ -7 & 2 & 1 & 0 & 0 & 1 \end{pmatrix} \tag{2}$$

$$\xrightarrow{R_1 \to \frac{1}{2}R_1} \begin{pmatrix} 1 & \frac{1}{2} & \frac{3}{2} & \frac{1}{2} & 0 & 0 \\ 4 & -1 & 0 & 0 & 1 & 0 \\ -7 & 2 & 1 & 0 & 0 & 1 \end{pmatrix} \tag{3}$$

$$\xrightarrow{R_2 \to R_2 - 4R_1,\ R_3 \to R_3 + 7R_1} \begin{pmatrix} 1 & \frac{1}{2} & \frac{3}{2} & \frac{1}{2} & 0 & 0 \\ 0 & -3 & -6 & -2 & 1 & 0 \\ 0 & \frac{11}{2} & \frac{23}{2} & \frac{7}{2} & 0 & 1 \end{pmatrix} \tag{4}$$

$$\xrightarrow{R_2 \to -\frac{1}{3}R_2} \begin{pmatrix} 1 & \frac{1}{2} & \frac{3}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 1 & 2 & \frac{2}{3} & -\frac{1}{3} & 0 \\ 0 & \frac{11}{2} & \frac{23}{2} & \frac{7}{2} & 0 & 1 \end{pmatrix} \tag{5}$$

$$\xrightarrow{R_1 \to R_1 - \frac{1}{2}R_2,\ R_3 \to R_3 - \frac{11}{2}R_2} \begin{pmatrix} 1 & 0 & \frac{1}{2} & \frac{1}{6} & \frac{1}{6} & 0 \\ 0 & 1 & 2 & \frac{2}{3} & -\frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{2} & -\frac{1}{6} & \frac{11}{6} & 1 \end{pmatrix} \tag{6}$$

$$\xrightarrow{R_3 \to 2R_3} \begin{pmatrix} 1 & 0 & \frac{1}{2} & \frac{1}{6} & \frac{1}{6} & 0 \\ 0 & 1 & 2 & \frac{2}{3} & -\frac{1}{3} & 0 \\ 0 & 0 & 1 & -\frac{1}{3} & \frac{11}{3} & 2 \end{pmatrix} \tag{7}$$

$$\xrightarrow{R_1 \to R_1 - \frac{1}{2}R_3,\ R_2 \to R_2 - 2R_3} \begin{pmatrix} 1 & 0 & 0 & \frac{1}{3} & -\frac{5}{3} & -1 \\ 0 & 1 & 0 & \frac{4}{3} & -\frac{23}{3} & -4 \\ 0 & 0 & 1 & -\frac{1}{3} & \frac{11}{3} & 2 \end{pmatrix} \tag{8}$$

$$\mathbf{A}^{-1} = \begin{pmatrix} \frac{1}{3} & -\frac{5}{3} & -1 \\ \frac{4}{3} & -\frac{23}{3} & -4 \\ -\frac{1}{3} & \frac{11}{3} & 2 \end{pmatrix} \tag{9}$$