1.7.10

AI25BTECH11024 - Pratyush Panda

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Question:

Find the relation between x and y if the points $\mathbf{A}(x, y)$, $\mathbf{B}(-5, 7)$ and $\mathbf{C}(-4, 5)$ are collinear.

Solution:

Given:

$$\mathbf{A} = \begin{pmatrix} x \\ y \end{pmatrix}; \mathbf{B} = \begin{pmatrix} -5 \\ 7 \end{pmatrix}; \mathbf{C} = \begin{pmatrix} -4 \\ 5 \end{pmatrix}$$

Now we have to form the Matrix,

$$\mathbf{M} = \begin{pmatrix} A - C & B - C \end{pmatrix}^T \tag{0.1}$$

$$\mathbf{M} = \begin{pmatrix} x + 4 & y - 5 \\ -1 & 2 \end{pmatrix} \tag{0.2}$$

Now to get the RREF of this Matrix we can apply the following row operations:

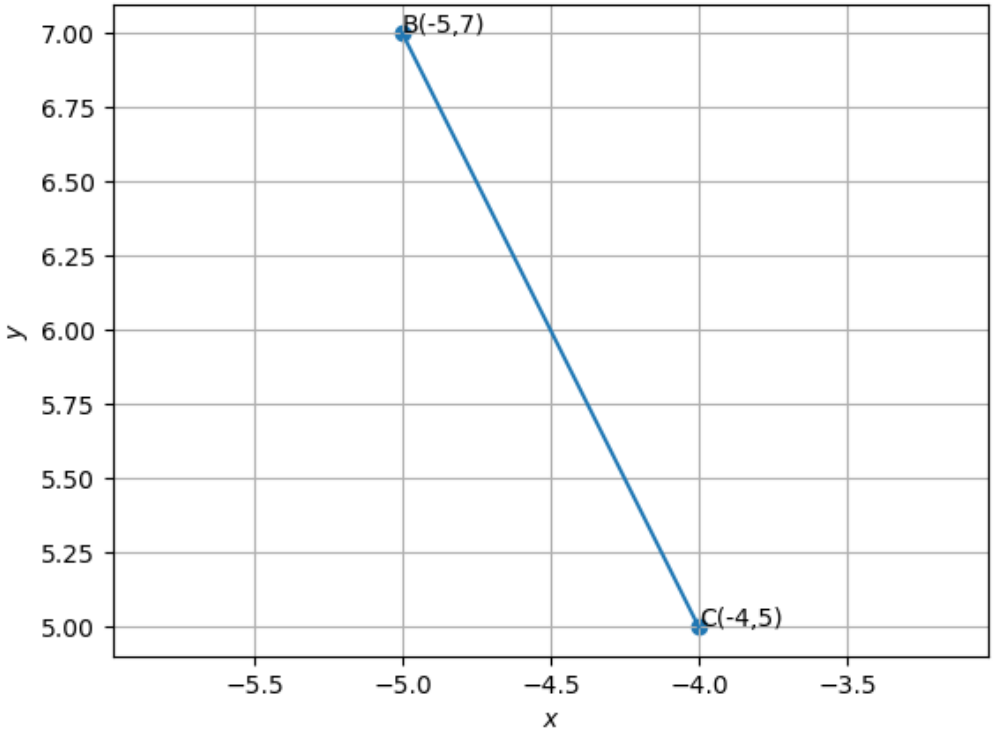
$$\begin{pmatrix} x + 4 & y - 5 \\ -1 & 2 \end{pmatrix} \xrightarrow{R_2 \longleftrightarrow R_2 + \frac{1}{x+4} R_1} \begin{pmatrix} x + 4 & y - 5 \\ 0 & 2 + \frac{y-5}{x+4} \end{pmatrix} \tag{0.3}$$

For this matrix to have $rank = 1$, the second element of the second row should also be 0
Therefore, we get

$$2 + \frac{y-5}{x+4} = 0 \tag{0.4}$$

On simplifying,

$$y = -2x - 3 \tag{0.5}$$


B(-5,7)
C(-4,5)
7.00
6.75
6.50
6.25
6.00
5.75
5.50
5.25
5.00
y
−5.5
−5.0
−4.5
−4.0
−3.5
x