

1.8.22: Equidistant Points Problem

Aditya Mishra - ee25btech11005

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Question

Find all points that are equidistant from

$$\mathbf{A} = \begin{pmatrix} -5 \\ 4 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} -1 \\ 6 \end{pmatrix}.$$

How many such points exist?

Given Information

Points as column vectors:

$$\mathbf{A} = \begin{pmatrix} -5 \\ 4 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} -1 \\ 6 \end{pmatrix}.$$

Desired point:

$$\mathbf{O} = \begin{pmatrix} x \\ y \end{pmatrix}.$$

Equidistant Condition

Equidistant means:

$$\|\mathbf{O} - \mathbf{A}\| = \|\mathbf{O} - \mathbf{B}\|.$$

Squaring both sides:

$$\|\mathbf{O} - \mathbf{A}\|^2 = \|\mathbf{O} - \mathbf{B}\|^2.$$

Using vector dot product:

$$(\mathbf{O} - \mathbf{A})^\top (\mathbf{O} - \mathbf{A}) = (\mathbf{O} - \mathbf{B})^\top (\mathbf{O} - \mathbf{B}).$$

Simplify the Equation

Expand both sides:

$$\mathbf{O}^\top \mathbf{O} - 2\mathbf{A}^\top \mathbf{O} + \mathbf{A}^\top \mathbf{A} = \mathbf{O}^\top \mathbf{O} - 2\mathbf{B}^\top \mathbf{O} + \mathbf{B}^\top \mathbf{B}.$$

Simplify by cancelling $\mathbf{O}^\top \mathbf{O}$:

$$-2\mathbf{A}^\top \mathbf{O} + \mathbf{A}^\top \mathbf{A} = -2\mathbf{B}^\top \mathbf{O} + \mathbf{B}^\top \mathbf{B}.$$

Rearranged:

$$2(\mathbf{B} - \mathbf{A})^\top \mathbf{O} = \mathbf{B}^\top \mathbf{B} - \mathbf{A}^\top \mathbf{A}.$$

Final Equation

$$(\mathbf{B} - \mathbf{A})^T \mathbf{O} = \frac{\mathbf{B}^T \mathbf{B} - \mathbf{A}^T \mathbf{A}}{2}.$$

Put terms explicitly:

$$\begin{pmatrix} 4 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \frac{37 - 41}{2} = -2.$$

That gives a line equation:

$$4x + 2y = -2 \implies 2x + y = -1.$$

Number of Solutions

The set of points equidistant from **A** and **B** lies on the line:

$$2x + y = -1.$$

There are infinitely many such points.

C Code: Equidistant Line Calculation

Python ctypes

Python Code: Plot Equidistant Line and Points

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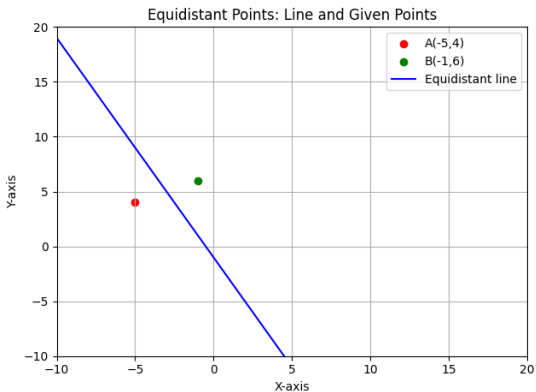


Figure: Points A , B and equidistant line $2x + y = -1$