

Question 2.7.12

AI25BTECH11040 - Vivaan Parashar

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Question:

Find the area of the triangle formed by joining the midpoints of the sides of the triangle ABC , whose vertices are $A(0, -1)$, $B(2, 1)$, and $C(0, 3)$

Solution:

Let us start by finding the midpoints, let's call them D, E and F.
The midpoint formula is: (Here the vectors represent position vectors of the points from the origin)

$$\mathbf{D} = \frac{\mathbf{A} + \mathbf{B}}{2} \quad (1)$$

$$\mathbf{E} = \frac{\mathbf{B} + \mathbf{C}}{2} \quad (2)$$

$$\mathbf{F} = \frac{\mathbf{C} + \mathbf{A}}{2} \quad (3)$$

$$\therefore \mathbf{D} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \mathbf{E} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \mathbf{F} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (4)$$

Now the area formula for a triangle with vertices at **P**, **Q** and **R** is given by:

$$\text{Area} = \frac{1}{2} |(\mathbf{P} - \mathbf{Q}) \times (\mathbf{P} - \mathbf{R})| \quad (5)$$

$$\therefore \text{Area of } \triangle DEF = \frac{1}{2} |(\mathbf{D} - \mathbf{E}) \times (\mathbf{D} - \mathbf{F})| \quad (6)$$

$$= \frac{1}{2} \left| \left(\begin{pmatrix} 1 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right) \times \left(\begin{pmatrix} 1 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right) \right| \quad (7)$$

$$= \frac{1}{2} \left| \begin{pmatrix} 0 \\ -2 \end{pmatrix} \times \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right| \quad (8)$$

$$= \frac{1}{2} |0 - 2| = 1 \quad (9)$$

Diagram:

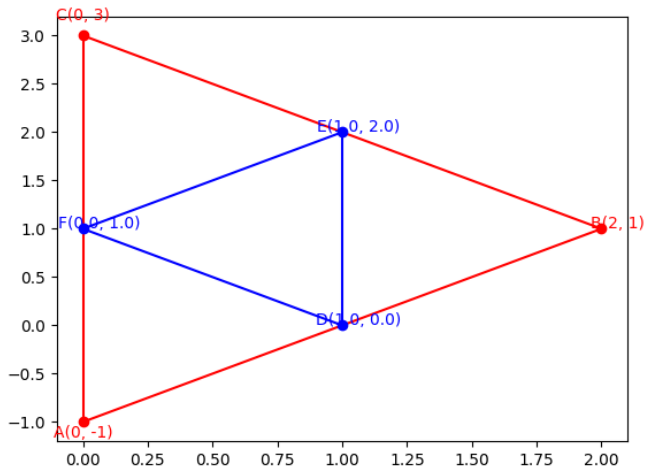


Figure: Diagram showing the triangle ABC and the triangle DEF.