

4.3.37

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Question

Find the equation of the line passing through the points

$$\mathbf{A} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 3 \\ 6 \end{pmatrix}.$$

Equation

Equation of a line is:

$$\mathbf{n}^T \mathbf{x} = c$$

Theoretical Solution

$$(\mathbf{A} \quad \mathbf{B})^T \mathbf{n} = \begin{pmatrix} c \\ c \end{pmatrix} \quad (1)$$

$$\begin{pmatrix} 1 & 2 \\ 3 & 6 \end{pmatrix} \mathbf{n} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (2)$$

$$\begin{pmatrix} 1 & 2 & 0 \\ 3 & 6 & 0 \end{pmatrix} \xrightarrow{R_2 - 3R_1} \begin{pmatrix} 1 & 2 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad (3)$$

$$n_1 + 2n_2 = 0 \quad (4)$$

Theoretical Solution

Let $n_2 = 1$

$$n_1 = -2, \quad (5)$$

$$\mathbf{n} = \begin{pmatrix} -2 \\ 1 \end{pmatrix} \quad (6)$$

Equation of a Line is

$$\mathbf{n}^T \mathbf{x} = c \quad (7)$$

$$\begin{pmatrix} -2 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix} = c \quad (8)$$

$$c = 0 \quad (9)$$

$$\begin{pmatrix} -2 & 1 \end{pmatrix} \mathbf{x} = 0 \quad (10)$$

```
#include <stdio.h>

int main() {
    // Points A and B
    int Ax = 1, Ay = 2;
    int Bx = 3, By = 6;

    // Direction vector of AB
    int dx = Bx - Ax;
    int dy = By - Ay;

    // Normal vector (perpendicular to direction)
    int nx = -dy;
    int ny = dx;

    // Equation:  $\mathbf{n}^T \cdot \mathbf{x} = c$ 
    // Compute c using point A
    int c = nx * Ax + ny * Ay;
```

```
printf(Normal vector n = (%d, %d)\n, nx, ny);  
printf(Equation of line: %d*x + %d*y = %d\n, nx, ny, c);  
  
// Convert to slope-intercept form if ny != 0  
if (ny != 0) {  
    double slope = -(double)nx / ny;  
    double intercept = (double)c / ny;  
    printf(Line equation (y = mx + c): y = %.2fx + %.2f\n,  
        slope, intercept);  
}  
  
return 0;  
}
```

```
import numpy as np
import matplotlib.pyplot as plt
# local imports
from libs.line.funcs import *
from libs.triangle.funcs import *
from libs.conics.funcs import circ_gen

# Points
A = np.array([1, 2]).reshape(-1,1)
B = np.array([3, 6]).reshape(-1,1)

# Direction vector AB
d = B - A

# Normal vector n (perpendicular to AB)
n = np.array([-d[1,0], d[0,0]]).reshape(-1,1)
```



```
# Compute c using point A
c = (n.T @ A)[0,0]

# Line equation:  $y = (c - n_1 \cdot x) / n_2$ 
x_vals = np.linspace(0, 5, 100)
y_vals = (c - n[0,0]*x_vals)/n[1,0]

# Plot
plt.figure(figsize=(6,6))

# Line
plt.plot(x_vals, y_vals, 'b-', label='Line AB')

# Points with coordinates labeled
plt.plot(A[0,0], A[1,0], 'ro')
plt.text(A[0,0]+0.1, A[1,0]+0.1, f'A({A[0,0]}, {A[1,0]})',
        fontsize=12, color='red')
```

```
plt.plot(B[0,0], B[1,0], 'go')
plt.text(B[0,0]+0.1, B[1,0]+0.1, f'B({B[0,0]}, {B[1,0]})',
         fontsize=12, color='green')

# Axes and grid
plt.xlabel('x')
plt.ylabel('y')
plt.title('Line through points A and B')
plt.grid(True)
plt.axis('equal')
plt.legend()
plt.show()
```

Python Shared Output

```
import ctypes
import numpy as np
import matplotlib.pyplot as plt
# local imports
from libs.line.funcs import *
from libs.triangle.funcs import *
from libs.conics.funcs import circ_gen

# Load C shared library
lib = ctypes.CDLL('./libline.so')

# Define argument and return types
lib.line_normal.argtypes = [ctypes.c_double, ctypes.c_double,
                             ctypes.c_double, ctypes.c_double,
                             ctypes.POINTER(ctypes.c_double),
                             ctypes.POINTER(ctypes.c_double),
                             ctypes.POINTER(ctypes.c_double)]
```

Python Shared Output

```
# Points
Ax, Ay = 1.0, 2.0
Bx, By = 3.0, 6.0

# Prepare output variables
n1 = ctypes.c_double()
n2 = ctypes.c_double()
c = ctypes.c_double()

# Call the C function
lib.line_normal(Ax, Ay, Bx, By, ctypes.byref(n1), ctypes.byref(n2),
               ctypes.byref(c))
```

Python Shared Output

```
1 print(Normal vector n =, n1.value, n2.value)
2 print(Scalar c =, c.value)
3
4 # Line:  $y = (c - n1 \cdot x) / n2$ 
5 x_vals = np.linspace(0, 5, 100)
6 y_vals = (c.value - n1.value * x_vals) / n2.value
7
8 # Plot
9 plt.figure(figsize=(6,6))
10 plt.plot(x_vals, y_vals, 'b-', label='Line AB')
```

Python Shared Output

```
# Points labeled with coordinates
plt.plot(Ax, Ay, 'ro')
plt.text(Ax+0.1, Ay+0.1, f'A({Ax}, {Ay})', fontsize=12, color='
red')

plt.plot(Bx, By, 'go')
plt.text(Bx+0.1, By+0.1, f'B({Bx}, {By})', fontsize=12, color='
green')

plt.xlabel('x')
plt.ylabel('y')
plt.title('Line through points A and B (computed via C + ctypes)'
)
plt.grid(True)
plt.axis('equal')
plt.legend()
plt.show()
```

Plot

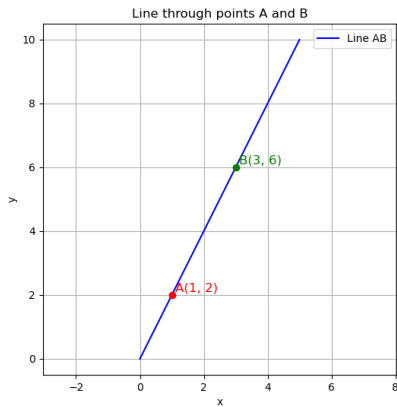


Figure: Caption