4.4.32

AI25BTECH11036-SNEHAMRUDULA

Question:

Q. Show that the vectors $\begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$, $\begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -3 \\ 5 \end{pmatrix}$ are coplanar.

Solution:

$$\left(\mathbf{n}^T\right)\mathbf{x} = 1$$

$$\begin{bmatrix} 1 & -2 & 3 \\ 2 & 3 & -4 \\ 1 & -3 & 5 \end{bmatrix}\begin{bmatrix} l \\ m \\ n \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

The augmented matrix is

$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & 1 \\ 2 & 3 & -4 & 1 \\ 1 & -3 & 5 & 1 \end{array}\right].$$

Performing row operations:

$$R_2 \to R_2 - 2R_1, \quad R_3 \to R_3 - R_1$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & 1 \\ 0 & 7 & -10 & -1 \\ 0 & -1 & 2 & 0 \end{array}\right]$$

Swap $R_2 \leftrightarrow R_3$:

$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & 1 \\ 0 & -1 & 2 & 0 \\ 0 & 7 & -10 & -1 \end{array}\right]$$

Now,

$$R_3 \to R_3 + 7R_2$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & 1 \\ 0 & -1 & 2 & 0 \\ 0 & 0 & 4 & -1 \end{array}\right]$$

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The coefficient matrix is

$$\begin{bmatrix} 1 & -2 & 3 \\ 0 & -1 & 2 \\ 0 & 0 & 4 \end{bmatrix}, \quad \text{rank} = 3.$$

The augmented matrix is

$$\begin{bmatrix} 1 & -2 & 3 & 1 \\ 0 & -1 & 2 & 0 \\ 0 & 0 & 4 & -1 \end{bmatrix}, \quad \text{rank} = 3.$$

Since

$$\text{rank(Coefficient)} = \text{rank(Augmented)} = 3,$$

and the number of unknowns is also 3, the system has a **unique solution**.

$$\therefore \quad \text{There exists a unique vector } n \text{ such that } n^T x = 1 \text{ is the required plane.}$$