2.8.23

EE25BTECH11041 - Naman Kumar

Question:
If a variable line in two adjacent positions has directions cosines l, m, n and $l+\delta l, m+\delta m, n+\delta n$, show that the small angle $\delta\theta$ between the two positions is given by

$$\delta\theta^2 = (\delta l)^2 + (\delta m)^2 + (\delta n)^2$$

Solution:
We know about direction cosine of any vector,

$$l^2 + m^2 + n^2 = 1 \tag{1}$$

and angle between two vectors

$$\cos\theta = \frac{\mathbf{A}^T\mathbf{B}}{||\mathbf{A}||||\mathbf{B}||} \tag{2}$$

We also know expansion of $\cos\delta x$ (δx represents very small x)

$$\cos\delta x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!}.... \tag{3}$$

Given Two direction cosine

$$l, m, n \text{ and } l+\delta l, m+\delta m, n+\delta n \tag{4}$$

Using (1) for both direction cosines

$$l^2 + m^2 + n^2 = 1 \tag{5}$$

and

$$(l+\delta l)^2 + (m+\delta m)^2 + (n+\delta n)^2 = 1 \tag{6}$$

$$l^2 + m^2 + n^2 + (\delta l)^2 + (\delta m)^2 + (\delta n)^2 + 2(l\delta l + m\delta m + n\delta n) = 1 \tag{7}$$

from (1)

$$1 + (\delta l)^2 + (\delta m)^2 + (\delta n)^2 + 2(l\delta l + m\delta m + n\delta n) = 1 \tag{8}$$

$$(\delta l)^2 + (\delta m)^2 + (\delta n)^2 = -2(l\delta l + m\delta m + n\delta n) \tag{9}$$

Using equation (2)

$$\cos\theta = \frac{\begin{pmatrix} l \\ m \\ n \end{pmatrix}^T \begin{pmatrix} l+\delta l \\ m+\delta m \\ n+\delta n \end{pmatrix}^T}{1\times 1} \tag{10}$$

$$\cos\theta = l(l+\delta l) + m(m+\delta m) + n(n+\delta n) \tag{11}$$

$$\cos\theta = l^2 + m^2 + n^2 + l\delta l + m\delta m + n\delta n \tag{12}$$

$$\tag{13}$$

using equation (1) (2) and (9)

$$1 - \frac{\delta\theta^2}{2!} = 1 + \frac{1}{-2}(\delta l)^2 + (\delta m)^2 + (\delta n)^2 \tag{14}$$

Where $\delta\theta$ represents very small θ

$$\delta\theta^2 = (\delta l)^2 + (\delta m)^2 + (\delta n)^2 \tag{15}$$

Hence Proved

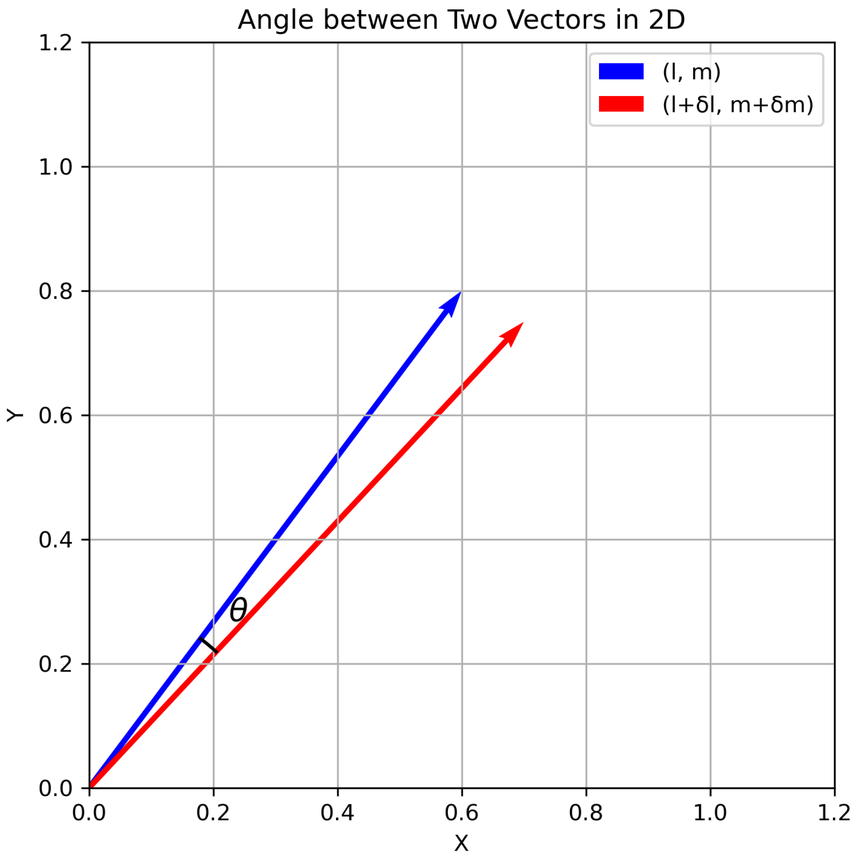

Fig. 1: Caption