

2.10.48

EE25BTECH11019 - Darji Vivek M.

Question:

If $\mathbf{a} = \mathbf{i} + \mathbf{j} + \mathbf{k}$, $\mathbf{a} \cdot \mathbf{b} = 1$ and $\mathbf{a} \times \mathbf{b} = \mathbf{j} - \mathbf{k}$, then \mathbf{b} is

(a) $\mathbf{i} - \mathbf{j} + \mathbf{k}$

(c) \mathbf{i}

(b) $2\mathbf{j} - \mathbf{k}$

(d) $2\mathbf{i}$

Solution:

$$\mathbf{a} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \quad (1)$$

Dot product condition:

$$\begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 1 \quad (2)$$

Cross product condition gives

$$\begin{pmatrix} 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0 \quad (3)$$

Collecting all equations:

$$\begin{pmatrix} 0 & 1 & -1 \\ 1 & 0 & -1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \quad (4)$$

Solving,

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad (5)$$

Hence,

$$\mathbf{b} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \mathbf{i} \quad (6)$$

Therefore, the correct option is (c).

Vectors a and b with coordinates

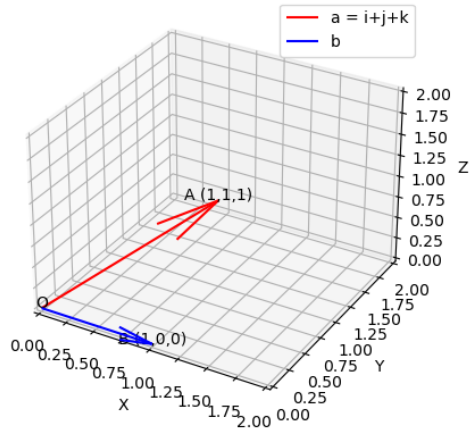


Fig. 4.1: plot