4.13.51

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Question

One of the diameters of the circle circumscribing the rectangle $ABCD$ is given by

$$4y = x + 7.$$

If $\mathbf{A} = (-3, 4)$ and $\mathbf{B} = (5, 4)$, find the area of the rectangle.

Solution

$$\mathbf{A} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 5 \\ 4 \end{pmatrix} \tag{1}$$

Circle equation : Centre $\mathbf{O} = -\mathbf{u} = -\begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$

$$\|\mathbf{x}\|^2 + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{2}$$

$$\|\mathbf{A}\|^2 + 2\mathbf{u}^\top \mathbf{A} + f = 0 \tag{3}$$

$$\|\mathbf{B}\|^2 + 2\mathbf{u}^\top \mathbf{B} + f = 0 \tag{4}$$

Diameter equation:(Centre lies on diameter)

$$\begin{pmatrix} 1 \\ -4 \end{pmatrix}^T \mathbf{x} = -7 \tag{5}$$

$$\mathbf{n}^T \mathbf{u} = c \tag{6}$$

Solution

$$\begin{pmatrix} 2\mathbf{A} & 2\mathbf{B} & n \\ 1 & 1 & 0 \end{pmatrix}^T \begin{pmatrix} \mathbf{u} \\ f \end{pmatrix} = - \begin{pmatrix} ||\mathbf{A}||^2 \\ ||\mathbf{B}||^2 \\ c \end{pmatrix} \tag{7}$$

$$\begin{pmatrix} -6 & 10 & 1 \\ 8 & 8 & -4 \\ 1 & 1 & 0 \end{pmatrix}^T \begin{pmatrix} \mathbf{u} \\ f \end{pmatrix} = - \begin{pmatrix} 25 \\ 41 \\ -7 \end{pmatrix} \tag{8}$$

$$\begin{pmatrix} -6 & 8 & 1 & -25 \\ 10 & 8 & 1 & -41 \\ 1 & -4 & 0 & 7 \end{pmatrix} \xrightarrow{R_1 \leftrightarrow R_3} \begin{pmatrix} 1 & -4 & 0 & 7 \\ 10 & 8 & 1 & -41 \\ -6 & 8 & 1 & -25 \end{pmatrix} \tag{9}$$

$$R_2 \to R_2 - 10R_1,\ R_3 \to R_3 + 6R_1 : \begin{pmatrix} 1 & -4 & 0 & 7 \\ 0 & 48 & 1 & -111 \\ 0 & -16 & 1 & 17 \end{pmatrix} \tag{10}$$

Solution

$$R_3 \to R_3 + \frac{1}{3}R_2 : \begin{pmatrix} 1 & -4 & 0 & 7 \\ 0 & 48 & 1 & -111 \\ 0 & 0 & 4/3 & -20 \end{pmatrix} \tag{11}$$

$$\frac{4}{3}f = -20 \implies f = -15 \tag{12}$$

$$48u_2 + f = -111 \implies 48u_2 - 15 = -111 \implies u_2 = -2 \tag{13}$$

$$u_1 - 4u_2 = 7 \implies u_1 = -1 \tag{14}$$

$$\mathbf{u} = \begin{pmatrix} -1 \\ -2 \end{pmatrix}, \quad f = -15 \tag{15}$$

$$\mathbf{O} = -\mathbf{u} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \tag{16}$$

Solution

Equation of the Circumcircle:

$$\boxed{\|\mathbf{x}\|^2 + 2\begin{pmatrix}-1 & -2\end{pmatrix}\mathbf{x} - 15 = 0} \tag{17}$$

$$\mathbf{C} = 2\mathbf{O} - \mathbf{A} = 2\begin{pmatrix}1\\2\end{pmatrix} - \begin{pmatrix}-3\\4\end{pmatrix} = \begin{pmatrix}5\\0\end{pmatrix} \tag{18}$$

$$\mathbf{D} = 2\mathbf{O} - \mathbf{B} = 2\begin{pmatrix}1\\2\end{pmatrix} - \begin{pmatrix}5\\4\end{pmatrix} = \begin{pmatrix}-3\\0\end{pmatrix} \tag{19}$$

$$\text{Area} = |(\mathbf{B} - \mathbf{A}) \times (\mathbf{D} - \mathbf{A})| = \begin{vmatrix}8 & 0\\0 & -4\end{vmatrix} = 32 \tag{20}$$

C Code

```c
    #include <stdio.h>
#include <math.h>

int main() {
    // Given points
    double A[2] = {-3, 4};
    double B[2] = {5, 4};

    // Solve equations for center
    // x = 1
    // x - 4y = -7
    double x = 1;
    double y = (x + 7) / 4.0;

    double O[2] = {x, y};
    printf(Center O = (%.2f, %.2f)\n, O[0], O[1]);
```

C Code

```
    // Opposite vertices
    double C[2] = {2*O[0] - A[0], 2*O[1] - A[1]};
    double D[2] = {2*O[0] - B[0], 2*O[1] - B[1]};

    printf(C = (%.2f, %.2f)\n, C[0], C[1]);
    printf(D = (%.2f, %.2f)\n, D[0], D[1]);

    // Side vectors
    double AB[2] = {B[0]-A[0], B[1]-A[1]};
    double AD[2] = {D[0]-A[0], D[1]-A[1]};

    // Cross product (2D determinant)
    double area = fabs(AB[0]*AD[1] - AB[1]*AD[0]);
    printf(Area of rectangle = %.2f\n, area);

    return 0;
}
```

Python Direct

```
import numpy as np
import matplotlib.pyplot as plt
from numpy import linalg as LA


# local imports
from libs.line.funcs import *
from libs.triangle.funcs import *
from libs.conics.funcs import circ_gen

# Given points
A = np.array([-3,4]).reshape(-1,1)
B = np.array([5,4]).reshape(-1,1)

# Centre O lies on x=1 and x-4y=-7
x = 1
y = (x+7)/4
```

Python Direct

```
O = np.array([x,y]).reshape(-1,1)

# Opposite vertices
C = 2*O - A
D = 2*O - B

# Side vectors
AB = B - A
AD = D - A

# Area by cross product
area = abs(np.linalg.det(np.hstack((AB,AD))))
print(Area of rectangle =, area)

# Circumcircle radius
r = LA.norm(A - O)
```

Python Direct

```
# Circle points
x_circ = circ_gen(O.flatten(), r)

# Plotting
rect_coords = np.hstack((A,B,C,D,O))
labels = ['$A$','$B$','$C$','$D$','$O$']

plt.plot(x_circ[0,:], x_circ[1,:], label='Circumcircle')
plt.plot([A[0,0],B[0,0],C[0,0],D[0,0],A[0,0]],
         [A[1,0],B[1,0],C[1,0],D[1,0],A[1,0]],
         'k-',label='Rectangle')

plt.scatter(rect_coords[0,:], rect_coords[1,:])
```

Python Direct

```
for i, txt in enumerate(labels):
    plt.annotate(f'{txt}({rect_coords[0,i]:.0f},{rect_coords[1,i
        ]:.0f})',
                 (rect_coords[0,i], rect_coords[1,i]),
                 textcoords=offset points, xytext=(-10,5), ha='
                     center')

ax = plt.gca()
ax.spines['top'].set_color('none')
ax.spines['right'].set_color('none')
ax.spines['left'].set_position('zero')
ax.spines['bottom'].set_position('zero')

plt.grid()
plt.axis('equal')
plt.legend()
plt.show()
```

Python Shared

```
import numpy as np
import matplotlib.pyplot as plt
from numpy import linalg as LA
import ctypes

# local imports
from line.funcs import *
from triangle.funcs import *
from conics.funcs import circ_gen

# Load C library
lib = ctypes.CDLL('./rect.so')
lib.rect_area.restype = ctypes.c_double
c_area = lib.rect_area()

# Given points
A = np.array([-3,4]).reshape(-1,1)
B = np.array([5,4]).reshape(-1,1)
```

Python Shared

```
# Centre O
x = 1
y = (x+7)/4
O = np.array([x,y]).reshape(-1,1)

# Opposite vertices
C = 2*O - A
D = 2*O - B

# Side vectors
AB = B - A
AD = D - A

# Area via cross product in Python
p_area = abs(np.linalg.det(np.hstack((AB,AD))))

print(Area from C (ctypes) =, c_area)
print(Area from Python =, p_area)
```

Python Shared

```
# Circumcircle radius
r = LA.norm(A - O)
x_circ = circ_gen(O.flatten(), r)

# Plot
rect_coords = np.hstack((A,B,C,D,O))
labels = ['$A$','$B$','$C$','$D$','$O$']

plt.plot(x_circ[0,:], x_circ[1,:], label='Circumcircle')
plt.plot([A[0,0],B[0,0],C[0,0],D[0,0],A[0,0]],
         [A[1,0],B[1,0],C[1,0],D[1,0],A[1,0]],
         'k-',label='Rectangle')

plt.scatter(rect_coords[0,:], rect_coords[1,:])
```

Python Shared

```
for i, txt in enumerate(labels):
    plt.annotate(f'{txt}({rect_coords[0,i]:.0f},{rect_coords[1,i
        ]:.0f})',
                 (rect_coords[0,i], rect_coords[1,i]),
                 textcoords=offset points, xytext=(-10,5), ha='
                     center')

ax = plt.gca()
ax.spines['top'].set_color('none')
ax.spines['right'].set_color('none')
ax.spines['left'].set_position('zero')
ax.spines['bottom'].set_position('zero')

plt.grid()
plt.axis('equal')
plt.legend()
plt.show()
```

Plot

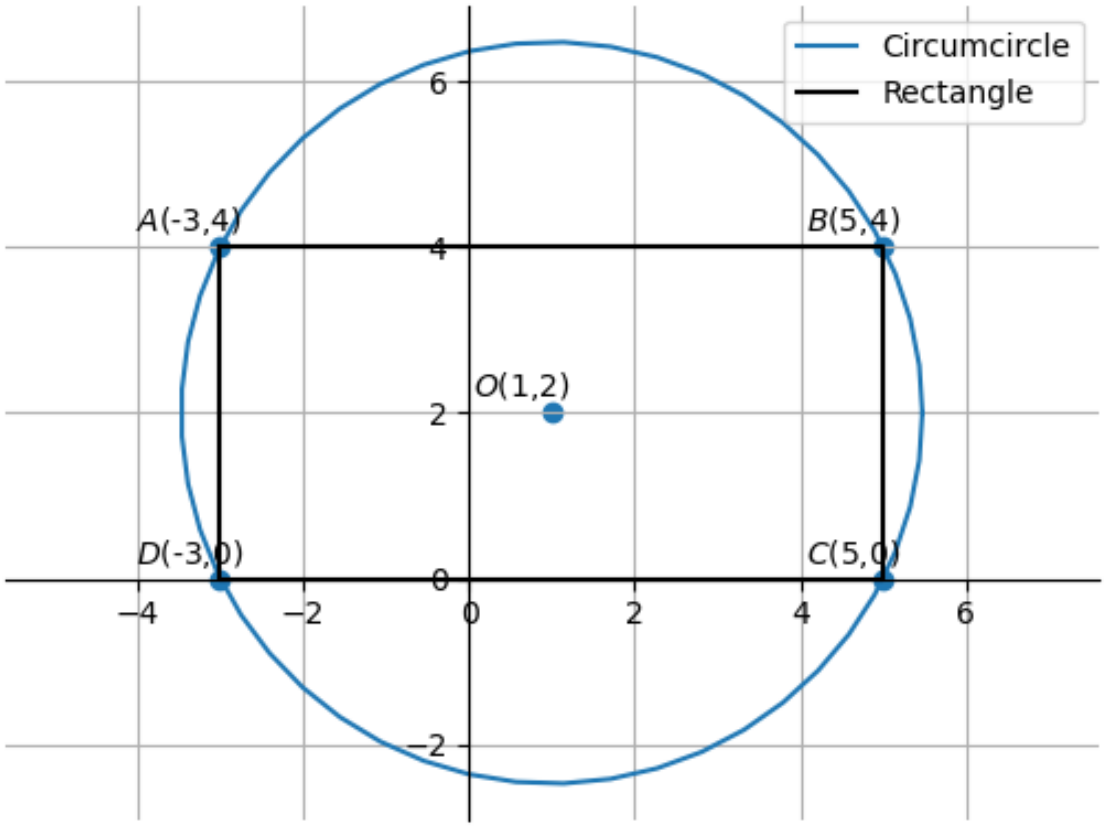

Figure: