# 4.13.51

EE25BTECH11047 - RAVULA SHASHANK REDDY

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**Question:**

One of the diameters of the circle circumscribing the rectangle $ABCD$ is given by

$$4y = x + 7.$$

If $\mathbf{A} = (-3, 4)$ and $\mathbf{B} = (5, 4)$, find the area of the rectangle.

**Solution:**

$$\mathbf{A} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 5 \\ 4 \end{pmatrix} \tag{1}$$

Centre $\mathbf{O}$ lies on the diameter and also on the perpendicular bisector of chord passing through $\mathbf{A}$ and $\mathbf{B}$.

Perpendicular Bisector equation:

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix}^T \mathbf{x} = 1 \tag{2}$$

Diameter equation:

$$\begin{pmatrix} 1 \\ -4 \end{pmatrix}^T \mathbf{x} = -7 \tag{3}$$

$$\therefore \begin{pmatrix} 1 & 0 \\ 1 & -4 \end{pmatrix} \mathbf{O} = \begin{pmatrix} 1 \\ -7 \end{pmatrix} \tag{4}$$

$$\begin{pmatrix} 1 & 0 & 1 \\ 1 & -4 & -7 \end{pmatrix} \xrightarrow{R_2 \to R_2 - R_1} \begin{pmatrix} 1 & 0 & 1 \\ 0 & -4 & -8 \end{pmatrix} \xrightarrow{R_2 \to (-\frac{1}{4}) R_2} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \end{pmatrix} \tag{5}$$

$$\mathbf{O} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \tag{6}$$

Radius of circumcircle

$$r = \|\mathbf{O} - \mathbf{A}\| = 2\sqrt{5} \tag{7}$$

Hence the Circumcircle equation is:

$$\|\mathbf{x}\|^2 + 2\mathbf{c}^T\mathbf{x} + f = 0 \tag{8}$$

$$\mathbf{c} = -\mathbf{O} \tag{9}$$

$$f = \|\mathbf{c}\|^2 - r^2 = -15 \tag{10}$$

$$\boxed{\|\mathbf{x}\|^2 + 2\begin{pmatrix} -1 & -2 \end{pmatrix}\mathbf{x} - 15 = 0} \tag{11}$$

$$\mathbf{C} = 2\mathbf{O} - \mathbf{A} = \begin{pmatrix} 2 \\ 4 \end{pmatrix} - \begin{pmatrix} -3 \\ 4 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \end{pmatrix}, \tag{12}$$

$$\mathbf{D} = 2\mathbf{O} - \mathbf{B} = \begin{pmatrix} 2 \\ 4 \end{pmatrix} - \begin{pmatrix} 5 \\ 4 \end{pmatrix} = \begin{pmatrix} -3 \\ 0 \end{pmatrix}. \tag{13}$$

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 8 \\ 0 \end{pmatrix}, \qquad \mathbf{D} - \mathbf{A} = \begin{pmatrix} 0 \\ -4 \end{pmatrix}. \tag{14}$$

$$\text{Area} = \left|\mathbf{B} - \mathbf{A} \times \mathbf{D} - \mathbf{A}\right| = \left|\det\begin{pmatrix} 8 & 0 \\ 0 & -4 \end{pmatrix}\right| = |8(-4) - 0| = 32. \tag{15}$$

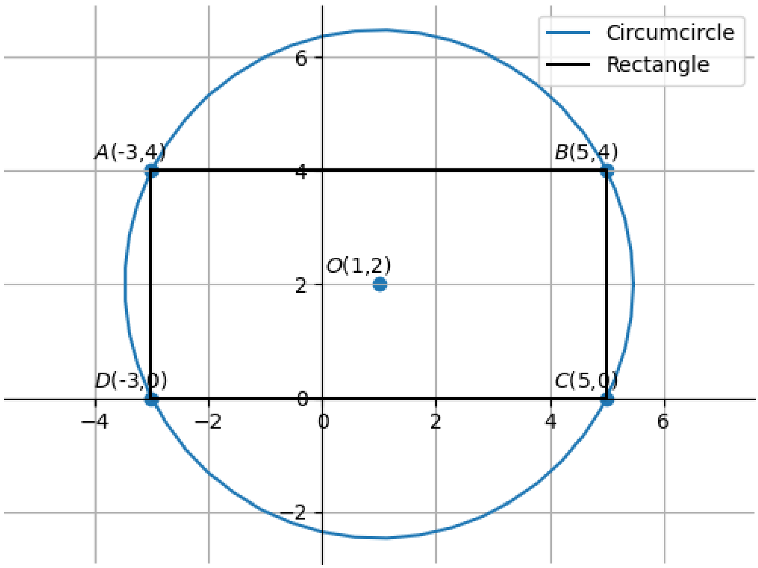


Figure 1