1.7.10

AI25BTECH11024 - Pratyush Panda

Question:
Find the relation between x and y if the points $\mathbf{A}(x, y)$, $\mathbf{B}(-5, 7)$ and $\mathbf{C}(-4, 5)$ are collinear.

Solution:
Given:

$$\mathbf{A} = \begin{pmatrix} x \\ y \end{pmatrix}; \mathbf{B} = \begin{pmatrix} -5 \\ 7 \end{pmatrix}; \mathbf{C} = \begin{pmatrix} -4 \\ 5 \end{pmatrix}$$

Now we have to form the Matrix,

$$\mathbf{M} = \begin{pmatrix} A - C & B - C \end{pmatrix}^T \tag{0.1}$$

$$\mathbf{M} = \begin{pmatrix} x + 4 & y - 5 \\ -1 & 2 \end{pmatrix} \tag{0.2}$$

Now to get the RREF of this Matrix we can apply the following row operations:

$$\begin{pmatrix} x + 4 & y - 5 \\ -1 & 2 \end{pmatrix} \xrightarrow{R_2 \longleftrightarrow R_2 + \frac{1}{x+4} R_1} \begin{pmatrix} x + 4 & y - 5 \\ 0 & 2 + \frac{y-5}{x+4} \end{pmatrix} \tag{0.3}$$

For this matrix to have *rank* = 1, the second element of the second row should also be 0

Therefore, we get

$$2 + \frac{y - 5}{x + 4} = 0 \tag{0.4}$$

On simplifying,

$$y = -2x - 3 \tag{0.5}$$