4.13.85

AI25BTECH11016-Varun

Question:
If the lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$ intersect, then the value of k is
Solution:
The lines $A + K_1 m_1, B + K_2 m_2$ will intersect if

$$\text{rank}(\mathbf{M} \quad \mathbf{B} - \mathbf{A}) = 2 \tag{1}$$

$$\mathbf{M} = \begin{pmatrix} m_1 & m_2 \end{pmatrix} \tag{2}$$

$$\tag{3}$$

Here,

$$\mathbf{m_1} = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} \mathbf{m_2} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \tag{4}$$

$$\mathbf{M} = \begin{pmatrix} 2 & 1 \\ 3 & 2 \\ 4 & 1 \end{pmatrix} \tag{5}$$

$$\mathbf{A} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 3 \\ k \\ 0 \end{pmatrix} \tag{6}$$

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 3-1 \\ k-(-1) \\ -1 \end{pmatrix} = \begin{pmatrix} 2 \\ k+1 \\ -1 \end{pmatrix} \tag{7}$$

$$\text{rank}\left(\begin{pmatrix} 2 & 1 & 2 \\ 3 & 2 & k+1 \\ 4 & 1 & -1 \end{pmatrix}\right) = 2 \tag{8}$$

$$\begin{pmatrix} 2 & 1 & 2 \\ 3 & 2 & k+1 \\ 4 & 1 & -1 \end{pmatrix} \xrightarrow[R_3 \to 2R_3 - 4R_1]{R_2 \to 2R_2 - 3R_1} \begin{pmatrix} 2 & 1 & 2 \\ 0 & 1 & 2k-4 \\ 0 & -2 & -10 \end{pmatrix} \tag{9}$$

$$\tag{10}$$

$$\xrightarrow{R_3 \to R_3 + 2R_2} \begin{pmatrix} 2 & 1 & 2 \\ 0 & 1 & 2k-4 \\ 0 & 0 & 4k-18 \end{pmatrix} \tag{11}$$

For therank$(\mathbf{M} \quad \mathbf{B} - \mathbf{A})$ to be 2
the last row must be all zero implies

$$4k - 18 = 0 \tag{12}$$

$$k = \frac{9}{2} \tag{13}$$

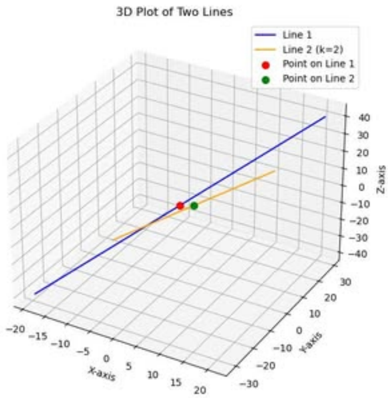

Fig. 0.1