12.40

EE25BTECH11026-Harsha

Question:

Given $\mathbf{M} = \begin{pmatrix} 2 & 3 & 7 \\ 6 & 4 & 7 \\ 4 & 6 & 14 \end{pmatrix}$. Which of the following statements is/are correct:

1) The rank of $\mathbf{M}$ is 2
2) The rank of $\mathbf{M}$ is 3
3) The rows of $\mathbf{M}$ are linearly independent
4) The determinant of $\mathbf{M}$ is zero.

Solution:

Let us solve the given question theoretically and then verify the solution computationally.

Upon row reduction of matrix $\mathbf{M}$ to Row Echelon form (REF),

$$\begin{pmatrix} 2 & 3 & 7 \\ 6 & 4 & 7 \\ 4 & 6 & 14 \end{pmatrix} \xleftrightarrow[R_3 \leftarrow R_3 - 2\times R_1]{R_2 \leftarrow R_2 - 3\times R_1} \begin{pmatrix} 2 & 3 & 7 \\ 0 & -5 & -14 \\ 0 & 0 & 0 \end{pmatrix} \tag{4.1}$$

$$\implies (a) \text{ The rank of } \mathbf{M} \text{ is } 2$$

$$(b) \text{ The determinant of } \mathbf{M} \text{ is } 0$$