Bonus Question

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Question

Given 3 vectors A,B,C are coplanar then show det(M) = 0 where $M=(A \ B \ C)$ Solution:

Equation of plane through 3 coplanar points is

$$\mathbf{n}^T \mathbf{x} = 0 \tag{1}$$

$$\implies \mathbf{n}^T \mathbf{A} = \mathbf{n}^T \mathbf{B} = \mathbf{n}^T \mathbf{C} = 0 \tag{2}$$

$$\mathbf{M} = (\mathbf{A} \quad \mathbf{B} \quad \mathbf{C}) \tag{3}$$

$$\implies \mathbf{n}^T \mathbf{M} = (\mathbf{n}^T \mathbf{A} \quad \mathbf{n}^T \mathbf{B} \quad \mathbf{n}^T \mathbf{C}) \tag{4}$$

$$\implies \mathbf{n}^T \mathbf{M} = (0 \quad 0 \quad 0) \tag{5}$$

$$\implies \mathbf{n}^T \mathbf{M} = \mathbf{0} \tag{6}$$

From (6) it means M has a non trivial vector in it's null space

$$\implies \operatorname{rank}(\mathbf{M}) < 3.$$
 (7)

For a 3×3 square matrix like **M** if $\det(\mathbf{M}) \neq 0$ means **M** is invertible which means **M** is a full rank matrix $\implies \operatorname{rank}(\mathbf{M}) = 3$.(if $\det(\mathbf{M}) \neq 0$)

From (7) $rank(\mathbf{M}) < 3$

 \implies **M** is not invertible

 $\implies \det(\mathbf{M}) = 0$

proof 2:

3 vectors $\mathbf{A}, \mathbf{B}, \mathbf{C}$ are coplanar means they are linearly dependent. let's assume

$$\mathbf{C} = \alpha \mathbf{A} + \beta \mathbf{B}.\tag{8}$$

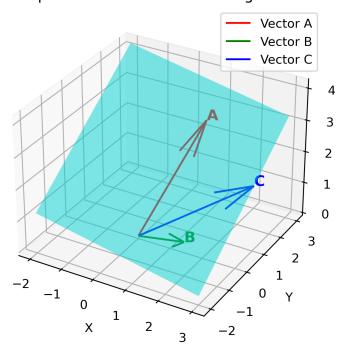
$$\det(\mathbf{M}) = \det((\mathbf{A} \quad \mathbf{B} \quad \mathbf{C}) \tag{9}$$

$$= \det((\mathbf{A} \quad \mathbf{B} \quad \alpha \mathbf{A} + \beta \mathbf{B}) \tag{10}$$

$$= \alpha \det((\mathbf{A} \quad \mathbf{B} \quad \mathbf{A}) + \beta \det((\mathbf{A} \quad \mathbf{B} \quad \mathbf{B}) = 0$$
(11)

$$\implies \det(\mathbf{M}) = 0 \tag{12}$$

Coplanar Vectors and Enclosing Plane



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