Presentation - Matgeo

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EE1030 - Matrix Theory

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Problem Statement

Problem (4.4.28) : The x-coordinate of a point on the line joining the points $\mathbf{P}(2, 2, 1)$ and $\mathbf{Q}(5, 1, -2)$ is 4. Find its z-coordinate.

Description of Variables used

Input variable	Value
P	$\begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$
Q	$\begin{pmatrix} 5 \\ 1 \\ -2 \end{pmatrix}$
R	$\begin{pmatrix} 4 \\ y \\ z \end{pmatrix}$

Table

Theoretical Solution

Form the column vectors $\mathbf{Q} - \mathbf{P}$ and $\mathbf{R} - \mathbf{P}$ and the matrix $\mathbf{M}$ whose columns are these vectors:

$$\mathbf{Q} - \mathbf{P} = \begin{pmatrix} 3 \\ -1 \\ -3 \end{pmatrix}, \qquad \mathbf{R} - \mathbf{P} = \begin{pmatrix} 2 \\ y-2 \\ z-1 \end{pmatrix}, \tag{2.1}$$

$$\mathbf{M} = \begin{pmatrix} 3 & 2 \\ -1 & y-2 \\ -3 & z-1 \end{pmatrix}. \tag{2.2}$$

Take the transpose $\mathbf{M}^{\mathbf{T}}$:

$$\mathbf{M}^{\mathbf{T}} = \begin{pmatrix} 3 & -1 & -3 \\ 2 & y-2 & z-1 \end{pmatrix}. \tag{2.3}$$

Theoretical Solution

Perform the row operation $R_2 \leftarrow R_2 - \frac{2}{3}R_1$. Writing the entries explicitly gives

$$R_1 = \begin{pmatrix} 3 & -1 & -3 \end{pmatrix}, \tag{2.4}$$

$$R_2 = \begin{pmatrix} 2 & y-2 & z-1 \end{pmatrix} - \frac{2}{3}\begin{pmatrix} 3 & -1 & -3 \end{pmatrix} \tag{2.5}$$

$$R_2 = \begin{pmatrix} 2 - \frac{2}{3}\cdot 3 & (y-2) - \frac{2}{3}\cdot(-1) & (z-1) - \frac{2}{3}\cdot(-3) \end{pmatrix}. \tag{2.6}$$

Thus after the row operation we have

$$\mathbf{M^T}_{\text{after}} = \begin{pmatrix} 3 & -1 & -3 \\ 2 - \frac{2}{3}\cdot 3 & (y-2) - \frac{2}{3}\cdot(-1) & (z-1) - \frac{2}{3}\cdot(-3) \end{pmatrix} \tag{2.7}$$

Theoretical Solution

Carry out the indicated multiplications to simplify the entries of the second row:

$$2 - \tfrac{2}{3} \cdot 3 = 2 - 2 = 0, \tag{2.8}$$

$$(y - 2) - \tfrac{2}{3} \cdot (-1) = y - 2 + \tfrac{2}{3} = y - \tfrac{4}{3}, \tag{2.9}$$

$$(z - 1) - \tfrac{2}{3} \cdot (-3) = z - 1 + 2 = z + 1. \tag{2.10}$$

So the fully simplified matrix after the row operation is

$$\mathbf{M^T}_{\text{after}} = \begin{pmatrix} 3 & -1 & -3 \\ 0 & y - \tfrac{4}{3} & z + 1 \end{pmatrix} \tag{2.11}$$

Theoretical Solution

For the columns of M to be linearly dependent (equivalently for P, Q, R to be collinear) we require $\text{rank}(M) = 1$. Since $\text{rank}(\mathbf{M}) = \text{rank}(\mathbf{M^T})$, and $\mathbf{M^T}_{\text{after}}$ has two rows, rank $= 1$ means the second row must be the zero row. Hence

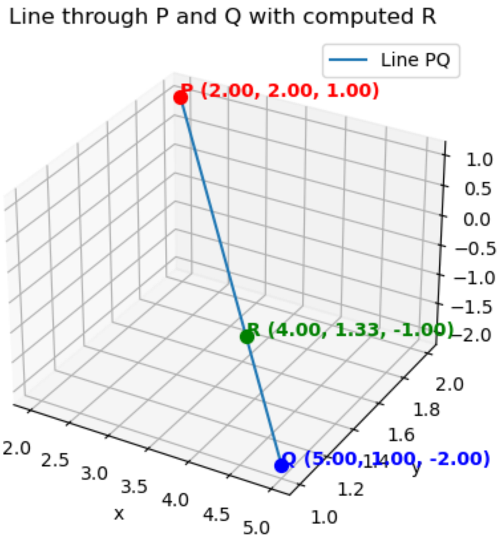
$$y - \tfrac{4}{3} = 0, \tag{2.12}$$
$$z + 1 = 0. \tag{2.13}$$

Therefore

$$y = \tfrac{4}{3}, \qquad z = -1. \tag{2.14}$$

$$\boxed{z = -1} \tag{2.15}$$

Plot


Line through P and Q with computed R
Line PQ
P (2.00, 2.00, 1.00)
R (4.00, 1.33, -1.00)
Q (5.00, 1.00, -2.00)
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
2.0
1.8
1.6
1.4
1.2
1.0
y
2.0
2.5
3.0
3.5
4.0
4.5
5.0
x

Code - C

```
#include <stdio.h>

/* Compute multiplier = a21 / a11.
   If a11 == 0, print "Error" and return 0.0 .
*/
double compute_multiplier(double a11, double a21) {
    if (a11 == 0.0) {
        printf("Error\n");
        return 0.0;
    }
    return a21 / a11;
}
```

Code - C

```c
/* Perform row op: out2[j] = r2[j] - (r2[0]/r1[0]) * r1[j] for j=0..2
   If r1[0] == 0, print "Error" and set out2 to zeros.
*/
void apply_row_op(const double r1[3], const double r2[3], double out2
    [3]) {
    if (r1[0] == 0.0) {
        printf("Error\n");
        out2[0] = out2[1] = out2[2] = 0.0;
        return;
    }
    double mult = r2[0] / r1[0];
    for (int j = 0; j < 3; ++j) {
        out2[j] = r2[j] - mult * r1[j];
    }
}
```

Code - Python(with shared C code)

```
import ctypes
from ctypes import c_double
import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D # registers 3D projection

# ----------- User inputs -----------
P = (2.0, 2.0, 1.0) # Px,Py,Pz
Q = (5.0, 1.0, -2.0) # Qx,Qy,Qz
Rx_given = 4.0 # given x-coordinate of R

# ----------- compute direction D = Q - P -----------
Px, Py, Pz = P
Qx, Qy, Qz = Q
Dx = Qx - Px
Dy = Qy - Py
Dz = Qz - Pz
```

Code - Python(with shared C code)

```
if abs(Dx) < 1e-12:
    raise SystemExit("Error: Dx = 0. This script pivots on x; please pick a
        case with Dx != 0.")
# ----------- load C shared lib -----------
lib = ctypes.CDLL('./librow.so')
lib.compute_multiplier.argtypes = (c_double, c_double)
lib.compute_multiplier.restype = c_double
lib.apply_row_op.argtypes = (ctypes.POINTER(c_double),
                             ctypes.POINTER(c_double),
                             ctypes.POINTER(c_double))
lib.apply_row_op.restype = None
# ----------- call compute_multiplier in C -----------
a11 = float(Dx) # r1[0]
a21 = float(Rx_given - Px) # r2[0]
mult = lib.compute_multiplier(c_double(a11), c_double(a21))
print("Multiplier (from C) =", mult)
```

Code - Python(with shared C code)

```
# ----------- compute y and z -----------
t = mult
y = Py + t * Dy
z = Pz + t * Dz
R = (Rx_given, y, z)
print("Computed R =", R)

# ----------- verify using apply_row_op -----------
r1 = (c_double * 3)(Dx, Dy, Dz)
r2 = (c_double * 3)(Rx_given - Px, y - Py, z - Pz)
out2 = (c_double * 3)()
lib.apply_row_op(r1, r2, out2)
out_list = [float(out2[i]) for i in range(3)]
print("Second row after elimination (from C apply_row_op):", out_list)
```

Code - Python(with shared C code)

```
# ———————————— plot P, Q, R with labels ————————————
N = 101
ts = np.linspace(0.0, 1.0, N)
points = np.array([[Px + t*Dx, Py + t*Dy, Pz + t*Dz] for t in ts])

img3d = "line_3d.png"
fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')

# line PQ
ax.plot(points[:,0], points[:,1], points[:,2], label='Line PQ')

# helper to format coordinates
def fmt_coords(pt):
    return f"({pt[0]:.2f}, {pt[1]:.2f}, {pt[2]:.2f})"
```

Code - Python(with shared C code)

```
# points + labels
ax.scatter(*P, color='red', s=50)
ax.text(*P, f"P-{fmt_coords(P)}", color='red', fontsize=10, weight='bold
    ')

ax.scatter(*Q, color='blue', s=50)
ax.text(*Q, f"Q-{fmt_coords(Q)}", color='blue', fontsize=10, weight='
    bold')

ax.scatter(*R, color='green', s=50)
ax.text(*R, f"R-{fmt_coords(R)}", color='green', fontsize=10, weight='
    bold')
```

Code - Python(with shared C code)

```
ax.set_xlabel('x')
ax.set_ylabel('y')
ax.set_zlabel('z')
ax.set_title('Line through P and Q with computed R')
plt.legend()

# save and show
plt.savefig(img3d, bbox_inches='tight')
print("Saved 3D image with P, Q, R labels ->", img3d)
plt.show()
```

Code - Python only

```
import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D # registers 3D projection

# ----------- Problem data (change if you want to test other
    examples) -----------
P = (2.0, 2.0, 1.0) # (Px, Py, Pz)
Q = (5.0, 1.0, -2.0) # (Qx, Qy, Qz)
Rx_given = 4.0 # known x-coordinate of R (we pivot on x here)
# ----------- helper: row operation in Python -----------
def apply_row_op_py(r1, r2):
    """
    Perform R2 <- R2 - mult * R1 where mult = r2[0] / r1[0].
    r1, r2 are length-3 iterables of numbers.
    Returns (mult, new_r2) where new_r2 is a list of 3 floats.
    If r1[0] == 0, raises ValueError.
    """
```

Code - Python only

```
    a11 = float(r1[0])
    if abs(a11) < 1e-15:
        raise ValueError("Pivot (r1[0]) is zero; cannot eliminate.")
    mult = float(r2[0]) / a11
    new_r2 = [r2[j] - mult * r1[j] for j in range(3)]
    return mult, new_r2

# ----------- compute direction and check pivot
    -----------
Px, Py, Pz = P
Qx, Qy, Qz = Q
Dx, Dy, Dz = Qx - Px, Qy - Py, Qz - Pz

if abs(Dx) < 1e-12:
    raise SystemExit("Pivot Dx is zero; this script is pivoting on x. Choose
        different input or pivot axis.")
```

Code - Python only

```
# ----------- build M^T rows (numeric with symbolic part in r2
    before solving) -----------
# r1 = [Dx, Dy, Dz]
# r2 (before knowing y,z) = [Rx_given - Px, (y-Py), (z-Pz)]
# we will compute t = (Rx_given - Px)/Dx via multiplier
r1 = [Dx, Dy, Dz]
r2_first = Rx_given - Px # numeric pivot entry
# ----------- get multiplier t (same as parameter)
    -----------
t = r2_first / Dx # Exactly same as compute_multiplier
print("Multiplier t = (Rx - Px)/Dx =", t)
# ----------- compute y and z from parametric form X = P + t*
    D -----------
y = Py + t * Dy
z = Pz + t * Dz
R = (Rx_given, y, z)
print("Computed R = (x, y, z) =", (R[0], R[1], R[2]))
```

Code - Python only

```
# ----------- now form numeric second row and apply row op in
    Python to verify -----------
r2 = [r2_first, y - Py, z - Pz] # numeric second row now
mult_used, new_r2 = apply_row_op_py(r1, r2)

print("\nRow operation details:")
print(" r1 =", r1)
print(" r2 (before) =", r2)
print(" multiplier used =", mult_used)
print(" r2 (after)  =", new_r2)

# check near-zero
tol = 1e-9
if all(abs(v) < tol for v in new_r2):
    print("Verification: second row reduced to zero (rank = 1) within
        tolerance.")
```

Code - Python only

```
else:
    print("Warning: second row not exactly zero; values:", new_r2)

# ------------ Plotting (3D) with labels and coordinates
    ------------
def fmt_coords(pt):
    return f"({pt[0]:.2f}, {pt[1]:.2f}, {pt[2]:.2f})"

N = 101
ts = np.linspace(0.0, 1.0, N)
points = np.array([[Px + t*Dx, Py + t*Dy, Pz + t*Dz] for t in ts])

fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')
```

Code - Python only

```
# plot the line
ax.plot(points[:,0], points[:,1], points[:,2], label='Line PQ')

# plot points
ax.scatter(*P, color='red', s=60)
ax.text(P[0], P[1], P[2], f"  P {fmt_coords(P)}", color='red', fontsize=10,
    weight='bold')

ax.scatter(*Q, color='blue', s=60)
ax.text(Q[0], Q[1], Q[2], f"  Q {fmt_coords(Q)}", color='blue', fontsize
    =10, weight='bold')

ax.scatter(*R, color='green', s=60)
ax.text(R[0], R[1], R[2], f"  R {fmt_coords(R)}", color='green', fontsize
    =10, weight='bold')
```

Code - Python only

```
# optional: draw dashed line from R down to x-y plane to help read z (
    uncomment if you like)
# ax.plot([R[0], R[0]], [R[1], R[1]], [0, R[2]], linestyle='--', linewidth=1)

ax.set_xlabel('x')
ax.set_ylabel('y')
ax.set_zlabel('z')
ax.set_title('Line through P and Q with computed R')
ax.legend()

# save and show
imgfile = "line_3d_python_only.png"
plt.savefig(imgfile, bbox_inches='tight')
print(f"Saved 3D image: {imgfile}")

plt.show()
```