4.7.11

EE25BTECH11025 - Ganachari Vishwambhar

Question:
Show that the path of a moving point such that its distance from two lines $3x - 2y = 5$ and $3x + 2y = 5$ are equal is a straight line.
Solution:

Given line equations can be written as:

$$\mathbf{n}_1^\top \mathbf{x} = c_1 \tag{1}$$

$$\mathbf{n}_1 = \begin{pmatrix} 3 \\ -2 \end{pmatrix}; c_1 = 5 \tag{2}$$

$$\mathbf{n}_2^\top \mathbf{x} = c_2 \tag{3}$$

$$\mathbf{n}_2 = \begin{pmatrix} 3 \\ 2 \end{pmatrix}; c_2 = 5 \tag{4}$$

let the point equidistant from the given lines be:

$$\mathbf{P} = \begin{pmatrix} x \\ y \end{pmatrix} \tag{5}$$

Proof:

From distance formula:

$$d_1 = \frac{|\mathbf{n}_1^\top \mathbf{P} - c_1|}{||\mathbf{n}_1||} \tag{6}$$

$$d_2 = \frac{|\mathbf{n}_2^\top \mathbf{P} - c_2|}{||\mathbf{n}_2||} \tag{7}$$

$$\because d_1 = d_2 \tag{8}$$

$$\frac{|\mathbf{n}_1^\top \mathbf{P} - c_1|}{||\mathbf{n}_1||} = \frac{|\mathbf{n}_2^\top \mathbf{P} - c_2|}{||\mathbf{n}_2||} \tag{9}$$

$$\because ||\mathbf{n}_1|| = ||\mathbf{n}_2|| = \sqrt{3^2 + 2^2} = \sqrt{13} \tag{10}$$

$$\mathbf{n}_1^\top \mathbf{P} - c_1 = \pm \left(\mathbf{n}_2^\top \mathbf{P} - c_2 \right) \tag{11}$$

First, by taking +:

$$\mathbf{n}_1^\top \mathbf{P} - c_1 = + \left(\mathbf{n}_2^\top \mathbf{P} - c_2 \right) \tag{12}$$

$$\mathbf{n}_1^\top \mathbf{P} - \mathbf{n}_2^\top \mathbf{P} = c_1 - c_2 \tag{13}$$

$$(\mathbf{n}_1 - \mathbf{n}_2)^\top \mathbf{P} = c_1 - c_2 \tag{14}$$

$$\begin{pmatrix} 0 & -4 \end{pmatrix} \mathbf{P} = 0 \tag{15}$$

Now by taking $-$:

$$\mathbf{n}_1^\top \mathbf{P} - c_1 = -\left(\mathbf{n}_2^\top \mathbf{P} - c_2\right) \tag{16}$$

$$\mathbf{n}_1^\top \mathbf{P} + \mathbf{n}_2^\top \mathbf{P} = c_1 + c_2 \tag{17}$$

$$\left(\mathbf{n}_1 + \mathbf{n}_2\right)^\top \mathbf{P} = c_1 + c_2 \tag{18}$$

$$\begin{pmatrix} 6 & 0 \end{pmatrix} \mathbf{P} = 10 \tag{19}$$

Since equations (15) and (19) are in the form of line equation $\mathbf{n}^\top \mathbf{x} = c$, the given path of the moving point is a line.

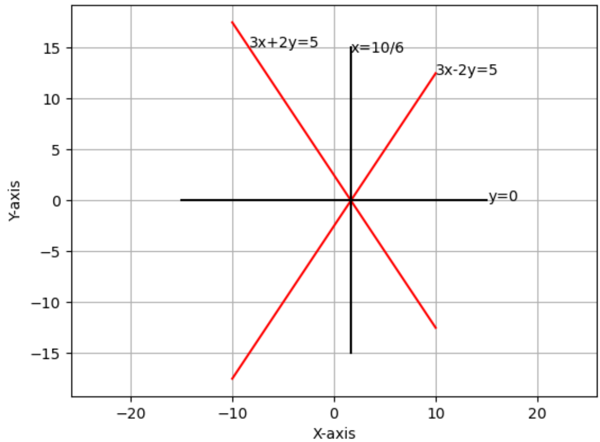

Fig. 1: Plot of the given lines and path of the moving point