Matgeo Presentation - Problem 5.9.10

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Problem Statement

A fraction becomes $\frac{1}{3}$ when 2 is subtracted from the numerator and it becomes $\frac{1}{2}$ when 1 is subtracted from the denominator. Find the fraction.

Data

Name	**Equation**
Equation 1	$3x - y = 6$
Equation 2	$2x - y = -1$

Table : Equations

Solution

Let the fraction be $\frac{x}{y}$, using the given conditions we get ,

$$\frac{x-2}{y} = \frac{1}{3} \tag{0.1}$$

$$3x - y = 6 \tag{0.2}$$

$$\frac{x}{y-1} = \frac{1}{2} \tag{0.3}$$

$$2x - y = -1 \tag{0.4}$$

The system of equations formed is :

$$3x - y = 6 \tag{0.5}$$

$$2x - y = -1 \tag{0.6}$$

Solution

Writing it in the matrix form,

$$\begin{pmatrix} 3 & -1 \\ 2 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 6 \\ -1 \end{pmatrix} \tag{0.7}$$

Forming the augmented matrix to solve the system of equations,

$$\left(\begin{array}{cc|c} 3 & -1 & 6 \\ 2 & -1 & -1 \end{array}\right) \tag{0.8}$$

Using Gaussian Elimination,

$$\left(\begin{array}{cc|c} 3 & -1 & 6 \\ 2 & -1 & -1 \end{array}\right) \xleftrightarrow{R_2 \to R_2 - \frac{2}{3}R_1} \left(\begin{array}{cc|c} 3 & -1 & 6 \\ 0 & -\frac{1}{3} & -5 \end{array}\right) \tag{0.9}$$

Solution

Using back substitution we get,

$$-\frac{y}{3} = -5 \tag{0.10}$$
$$y = 15 \tag{0.11}$$
$$3x - y = 6 \tag{0.12}$$
$$3x = 6 + 15 \tag{0.13}$$
$$x = 7 \tag{0.14}$$

The solution for the system of equations is :

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 7 \\ 15 \end{pmatrix} \tag{0.15}$$

Therefore the fraction is

$$\frac{x}{y} = \frac{7}{15} \tag{0.16}$$

Plot

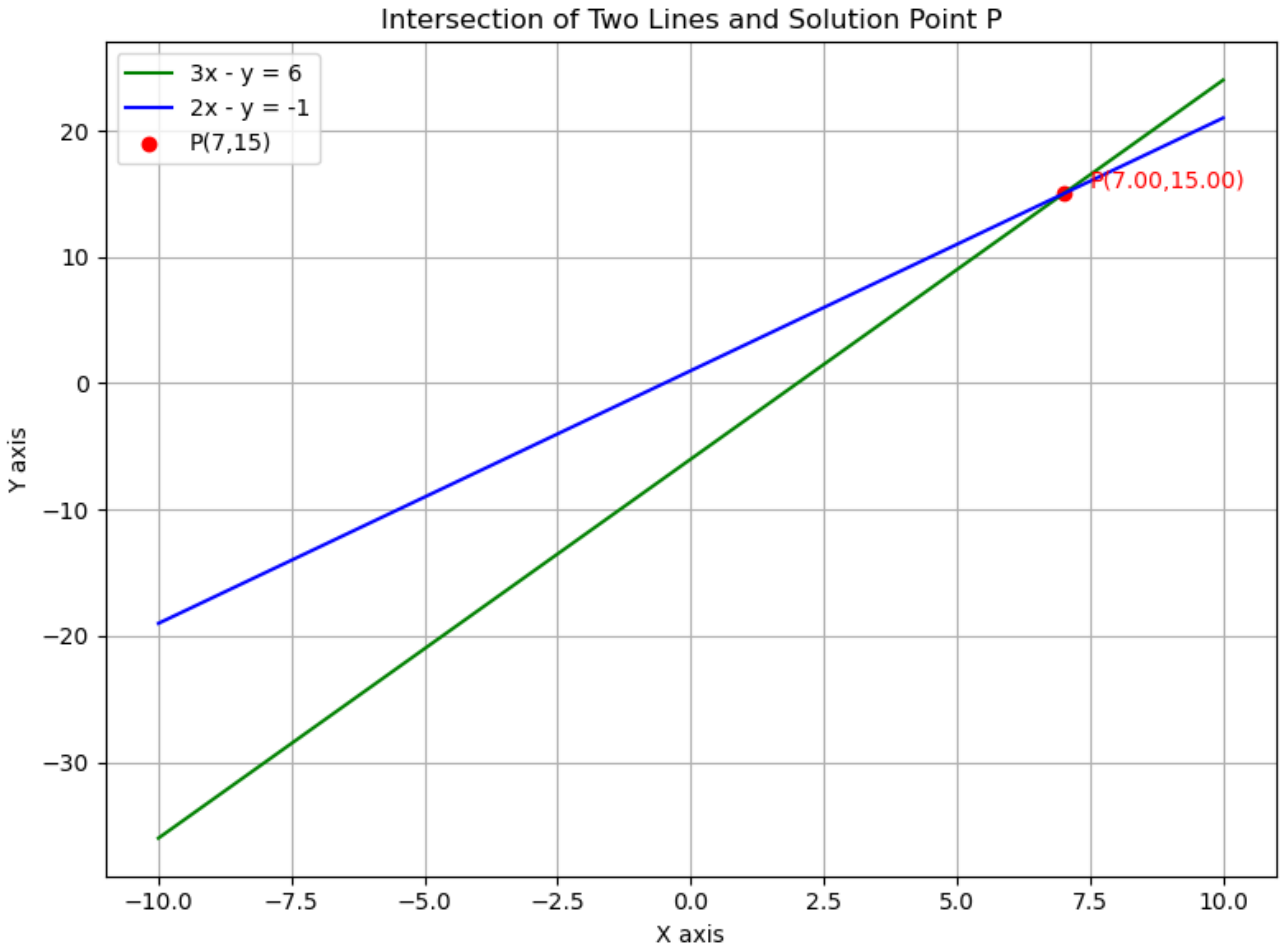

Fig : Lines