2.4.8

EE25BTECH11018-Darisy Sreetej

Question: Find a unit vector perpendicular to each of the vectors $\left(\mathbf{a}+\mathbf{b}\right)$ and $\left(\mathbf{a}+\mathbf{b}\right)$ where, $\mathbf{a} = \hat{i} + \hat{j} + \hat{k}$ and $\mathbf{b} = \hat{i} + 2\hat{j} + 3\hat{k}$ is

Solution:
Given two vectors,

$$\mathbf{a} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad \mathbf{b} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \tag{0.1}$$

Let the desired vector be $\mathbf{x}$. Then, $\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$

$$\left(\mathbf{a}+\mathbf{b}\right) = \begin{pmatrix} \mathbf{a} & \mathbf{b} \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{0.2}$$

$$\left(\mathbf{a}-\mathbf{b}\right) = \begin{pmatrix} \mathbf{a} & \mathbf{b} \end{pmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{0.3}$$

According to the given question ,

$$\therefore \begin{pmatrix} \mathbf{a}+\mathbf{b} & \mathbf{a}-\mathbf{b} \end{pmatrix}^T \mathbf{x} = 0 \tag{0.4}$$

(0.4) can be expressed as

$$\left\{ \begin{pmatrix} \mathbf{a} & \mathbf{b} \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \right\}^T \mathbf{x} = 0 \tag{0.5}$$

$$\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}^T \begin{pmatrix} \mathbf{a} & \mathbf{b} \end{pmatrix}^T \mathbf{x} = 0 \tag{0.6}$$

$$\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}^T \begin{pmatrix} \mathbf{a} & \mathbf{b} \end{pmatrix}^T \mathbf{x} = 0 \tag{0.7}$$

or,

$$\begin{pmatrix} \mathbf{a} & \mathbf{b} \end{pmatrix}^T \mathbf{x} = 0 \tag{0.8}$$

which can be expressed as

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \end{pmatrix} \xleftrightarrow{R_2=R_2-R_1} \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \end{pmatrix} \tag{0.9}$$

and

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \end{pmatrix} \xleftrightarrow{R_2=R_2-2R_1} \begin{pmatrix} 1 & 1 & 1 \\ -1 & 0 & 1 \end{pmatrix} \tag{0.10}$$

yielding,

$$x_2 + 2x_3 = 0 \tag{0.11}$$

$$-x_1 + x_3 = 0 \tag{0.12}$$

$$\mathbf{x} = x_3 \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} \tag{0.13}$$

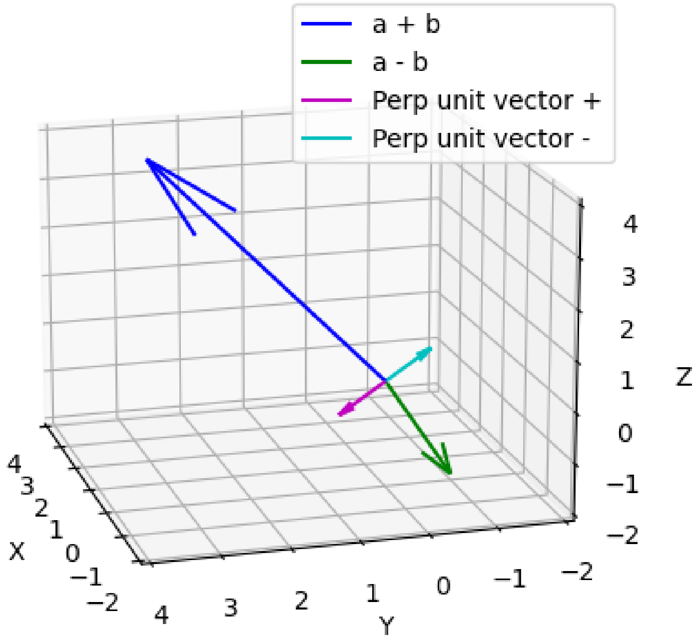
The unit vector is

$$\mathbf{x} = \frac{1}{\sqrt{6}} \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} \tag{0.14}$$

As we know that the vector can be in both the directions i.e, into and out of the plane containing **a** and **b**, so the vector perpendicular to vectors **a** and **b** would be $\pm\,(\mathbf{a} \times \mathbf{b})$.

Therefore, the desired output is

$$\mathbf{x} = \pm \frac{1}{\sqrt{6}} \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} \tag{0.15}$$



The perpendicular unit vectors