5.8.31

EE25BTECH11033 - Kavin

Question:
In a $\triangle ABC, \angle C = 3\angle B = 2(\angle A + \angle B)$. Find the three angles.

Solution:
In a $\triangle ABC$, the sum of interior angles is equal to 180.

$$\angle A + \angle B + \angle C = 180 \tag{1}$$

Also,

$$\angle C - 3\angle B = 0 \tag{2}$$

$$2\angle A - \angle B = 0 \tag{3}$$

On putting the above equations in a matrix we will get,

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & -3 & 1 \\ 2 & -1 & 0 \end{pmatrix} \begin{pmatrix} \angle A \\ \angle B \\ \angle C \end{pmatrix} = \begin{pmatrix} 180 \\ 0 \\ 0 \end{pmatrix} \tag{4}$$

The augmented matrix is given by,

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 180 \\ 0 & -3 & 1 & 0 \\ 2 & -1 & 0 & 0 \end{array}\right) \tag{5}$$

$$R_3 \to R_3 - 2R_1 \implies \left(\begin{array}{ccc|c} 1 & 1 & 1 & 180 \\ 0 & -3 & 1 & 0 \\ 0 & -3 & -2 & -360 \end{array}\right) \tag{6}$$

$$R_2 \to -1/3R_2 \implies \left(\begin{array}{ccc|c} 1 & 1 & 1 & 180 \\ 0 & 1 & -1/3 & 0 \\ 0 & -3 & -2 & -360 \end{array}\right) \tag{7}$$

$$R_1 \to R_1 - R_2 \text{ and } R_3 \to R_3 + 3R_2 \implies \left(\begin{array}{ccc|c} 1 & 0 & 4/3 & 180 \\ 0 & 1 & -1/3 & 0 \\ 0 & 0 & -3 & -360 \end{array}\right) \tag{8}$$

$$R_3 \to -1/3R_3 \implies \left(\begin{array}{ccc|c} 1 & 0 & 4/3 & 180 \\ 0 & 1 & -1/3 & 0 \\ 0 & 0 & 1 & 120 \end{array}\right) \tag{9}$$

$$R_1 \rightarrow R_1 - 4/3R_3 \text{ and } R_2 \rightarrow R_2 + 1/3R_3 \implies \left(\begin{array}{ccc|c} 1 & 0 & 0 & 20 \\ 0 & 1 & 0 & 40 \\ 0 & 0 & 1 & 120 \end{array}\right) \tag{10}$$

$$\implies \begin{pmatrix} \angle A \\ \angle B \\ \angle C \end{pmatrix} = \begin{pmatrix} 20 \\ 40 \\ 120 \end{pmatrix} \tag{11}$$

Therefore,

$$\angle A = 20^\circ \qquad \angle B = 40^\circ$$

$$\angle C = 120^\circ$$