4.5.6

EE25BTECH11004 - Aditya Appana

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Question

Find the equations of the line that passes through the point (3,0,1) and parallel to the planes $x + 2y = 0$ and $3y - z = 0$.

Solution

We know that the normal form of a plane is $\mathbf{n}^T\mathbf{x} = 0$
The plane $x + 2y = 0$ can be expressed in vector form as:

$$\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}^T \mathbf{x} = 0 \tag{1}$$

therefore,

$$\mathbf{n}_1 = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} \tag{2}$$

The plane $3y - z = 0$ can be expressed in vector form as:

$$\begin{pmatrix} 0 \\ 3 \\ -1 \end{pmatrix}^T \mathbf{x} = 0 \tag{3}$$

therefore,

$$\mathbf{n}_2 = \begin{pmatrix} 0 \\ 3 \\ -1 \end{pmatrix} \tag{4}$$

The vector parallel to both planes will be perpendicular to the normal vectors of both planes. Therefore it can be expressed as

$$\mathbf{n}_1 \times \mathbf{n}_2 \tag{5}$$

To calculate the cross product of the two vectors a and b, we use the following determinant:

$$\begin{pmatrix} |\mathbf{A_{11}B_{23}}| \\ |\mathbf{A_{11}B_{23}}| \\ |\mathbf{A_{11}B_{23}}| \end{pmatrix} \tag{6}$$

Where $\mathrm{X}_{ij} = \begin{pmatrix} \mathrm{x}_i \\ \mathrm{x}_j \end{pmatrix}$.

Expanding the determinants, we get:

$$\begin{pmatrix} ((-2) - 0) \\ (1 - 0) \\ (3 - 0)) \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix} \tag{7}$$

Since the line passes through (3,0,1), the line can therefore be expressed as:

$$\frac{x - 3}{-2} = \frac{y}{1} = \frac{z - 1}{3} \tag{8}$$

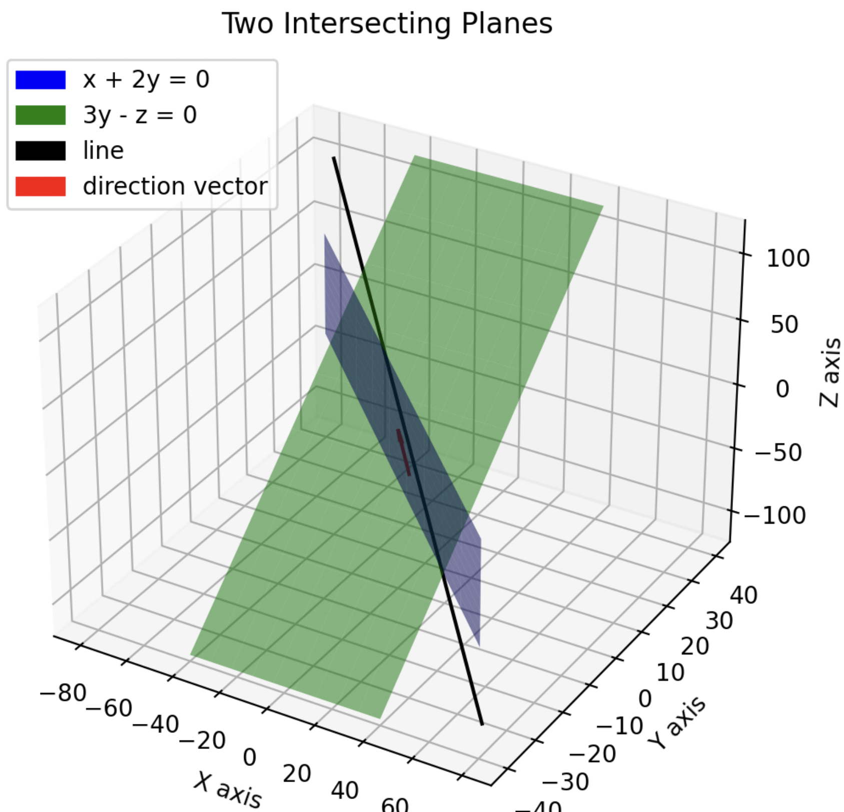

Figure 1: Plot