

5.9.1

EE25BTECH11047 - RAVULA SHASHANK REDDY

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Question:

A fraction becomes $\frac{1}{3}$ when 1 is subtracted from its numerator and it becomes $\frac{1}{4}$ when 8 is added to its denominator. Find the fraction.

Solution:

Let fraction be $\frac{x}{y}$

$$\mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix} \quad (1)$$

$$\frac{x-1}{y} = \frac{1}{3} \quad (2)$$

$$\begin{pmatrix} 3 \\ -1 \end{pmatrix}^T \mathbf{x} = 3 \quad (3)$$

$$\frac{x}{y+8} = \frac{1}{4} \quad (4)$$

$$\begin{pmatrix} 4 \\ -1 \end{pmatrix}^T \mathbf{x} = 8 \quad (5)$$

$$\begin{pmatrix} 3 & -1 \\ 4 & -1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 3 \\ 8 \end{pmatrix} \quad (6)$$

$$\begin{pmatrix} 3 & -1 & 3 \\ 4 & -1 & 8 \end{pmatrix} \xrightarrow{R_2 \rightarrow R_2 - R_1} \begin{pmatrix} 3 & -1 & 3 \\ 1 & 0 & 5 \end{pmatrix} \xrightarrow{R_1 \rightarrow R_1 - 3R_2} \begin{pmatrix} 0 & -1 & -12 \\ 1 & 0 & 5 \end{pmatrix} \quad (7)$$

$$\mathbf{x} = \begin{pmatrix} 5 \\ 12 \end{pmatrix} \quad (8)$$

$Required \text{ Fraction} = \frac{5}{12}$
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