

2.10.42

EE25BTECH11013 - Bhargav

Question:

If \mathbf{a} , \mathbf{b} and \mathbf{c} are unit coplanar vectors, then the scalar triple product

$$[2\mathbf{a} - \mathbf{b} \quad 2\mathbf{b} - \mathbf{c} \quad 2\mathbf{c} - \mathbf{a}] =$$

Solution:

$$\mathbf{B} = (2\mathbf{a} - \mathbf{b} \quad 2\mathbf{b} - \mathbf{c} \quad 2\mathbf{c} - \mathbf{a}) = (\mathbf{a} \quad \mathbf{b} \quad \mathbf{c}) \begin{pmatrix} 2 & 0 & -1 \\ -1 & 2 & 0 \\ 0 & -1 & 2 \end{pmatrix} \quad (0.1)$$

$$\therefore \det(\mathbf{B}) = \det(\mathbf{A}) \cdot \det(\mathbf{M}) \quad (0.2)$$

Since \mathbf{a} , \mathbf{b} , \mathbf{c} are coplanar,

$$[\mathbf{a} \quad \mathbf{b} \quad \mathbf{c}] = 0. \quad (0.3)$$

$$\Rightarrow [2\mathbf{a} - \mathbf{b} \quad 2\mathbf{b} - \mathbf{c} \quad 2\mathbf{c} - \mathbf{a}] = \det(\mathbf{M}) \cdot [\mathbf{a} \quad \mathbf{b} \quad \mathbf{c}] = 0. \quad (0.4)$$

This can be verified by taking an example of 3 coplanar unit vectors.

$$\mathbf{a} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad (0.5)$$

$$\mathbf{b} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad (0.6)$$

$$\mathbf{c} = \begin{pmatrix} 0.6 \\ 0.8 \\ 0 \end{pmatrix} \quad (0.7)$$

$$\mathbf{X} = [2\mathbf{a} - \mathbf{b} \quad 2\mathbf{b} - \mathbf{c} \quad 2\mathbf{c} - \mathbf{a}] \quad (0.8)$$

From the code, it is clear that the value of \mathbf{X} is 0