1.8.25

EE25BTECH11008 - Anirudh M Abhilash

September 12, 2025

Question

Find a relation between x and y such that the point $\mathbf{P}(x, y)$ is equidistant from the points $\mathbf{A}(7, 1)$ and $\mathbf{B}(3, 5)$.

Solution

Let

$$\mathbf{P} = \begin{pmatrix} x \\ y \end{pmatrix}, \quad \mathbf{A} = \begin{pmatrix} 7 \\ 1 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}.$$

Since $\mathbf{P}$ is equidistant from $\mathbf{A}$ and $\mathbf{B}$,

$$\|\mathbf{P} - \mathbf{A}\| = \|\mathbf{P} - \mathbf{B}\|.$$

Squaring both sides and using the inner product,

$$(\mathbf{P} - \mathbf{A})^\top (\mathbf{P} - \mathbf{A}) = (\mathbf{P} - \mathbf{B})^\top (\mathbf{P} - \mathbf{B}) \tag{1}$$

$$\mathbf{P}^\top \mathbf{P} - 2\mathbf{P}^\top \mathbf{A} + \mathbf{A}^\top \mathbf{A} = \mathbf{P}^\top \mathbf{P} - 2\mathbf{P}^\top \mathbf{B} + \mathbf{B}^\top \mathbf{B}. \tag{2}$$

Cancelling $\mathbf{P}^\top \mathbf{P}$,

$$2\mathbf{P}^\top (\mathbf{B} - \mathbf{A}) = \mathbf{B}^\top \mathbf{B} - \mathbf{A}^\top \mathbf{A}. \tag{3}$$

Now,

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 3 \\ 5 \end{pmatrix} - \begin{pmatrix} 7 \\ 1 \end{pmatrix} = \begin{pmatrix} -4 \\ 4 \end{pmatrix}, \quad \mathbf{B}^\top \mathbf{B} = 3^2 + 5^2 = 34, \quad \mathbf{A}^\top \mathbf{A} = 7^2 + 1^2 = 50.$$

Thus,

$$2\begin{pmatrix} x & y \end{pmatrix}\begin{pmatrix} -4 \\ 4 \end{pmatrix} = 34 - 50 \tag{4}$$

$$-4x + 4y = -8 \tag{5}$$

$$y - x = -2. \tag{6}$$

Hence, the required relation is

$$\boxed{y = x - 2}$$

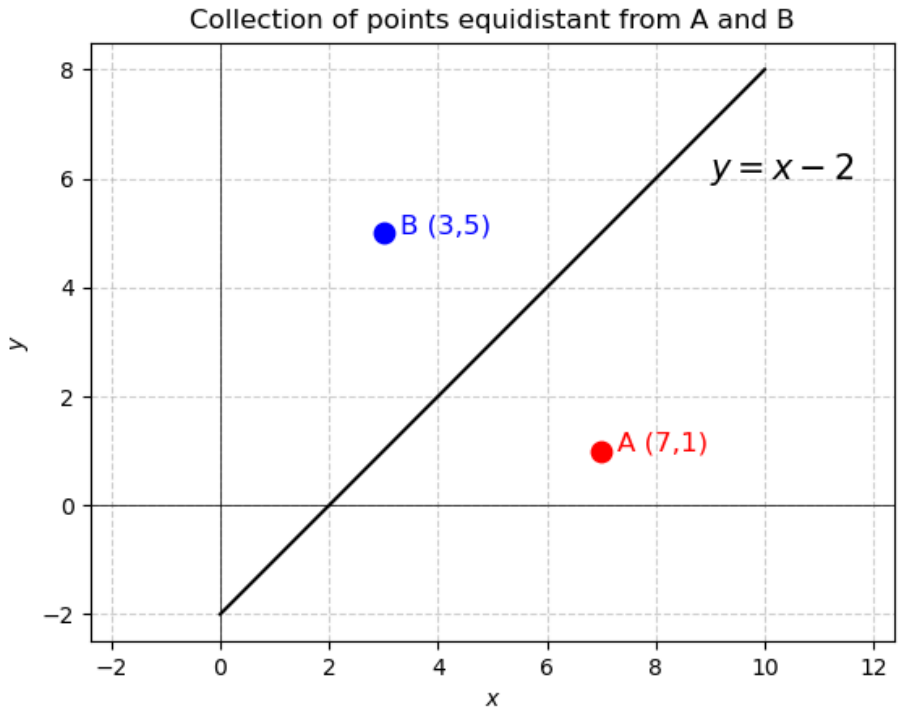

Figure 1: Locus of **P** equidistant from **A** and **B**