

2.3.13

EE25BTECH11010 - Arsh Dhoke

Question:

Find the angle which the line $\frac{x}{1} = \frac{y}{-1} = \frac{z}{2}$ makes with the positive direction of the Y axis.

Solution:

The line can be represented as $k \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$

Hence its direction vector is

$$\mathbf{v} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} \quad (0.1)$$

$$\mathbf{e}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad (0.2)$$

$$\mathbf{v}^T \mathbf{e}_2 = \begin{pmatrix} 1 & -1 & 2 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = -1 \quad (0.3)$$

$$\|\mathbf{v}\| = \sqrt{\mathbf{v}^T \mathbf{v}} = \sqrt{\begin{pmatrix} 1 & -1 & 2 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}} = \sqrt{6} \quad (0.4)$$

$$\|\mathbf{e}_2\| = 1 \quad (0.5)$$

$$\cos \theta = \frac{\mathbf{v}^T \mathbf{e}_2}{\|\mathbf{v}\| \|\mathbf{e}_2\|} = \frac{-1}{\sqrt{6}} \quad (0.6)$$

$$\theta = \cos^{-1} \left(-\frac{1}{\sqrt{6}} \right) \quad (0.7)$$

Therefore, $\theta = \cos^{-1} \left(-\frac{1}{\sqrt{6}} \right) \approx 114.09^\circ$

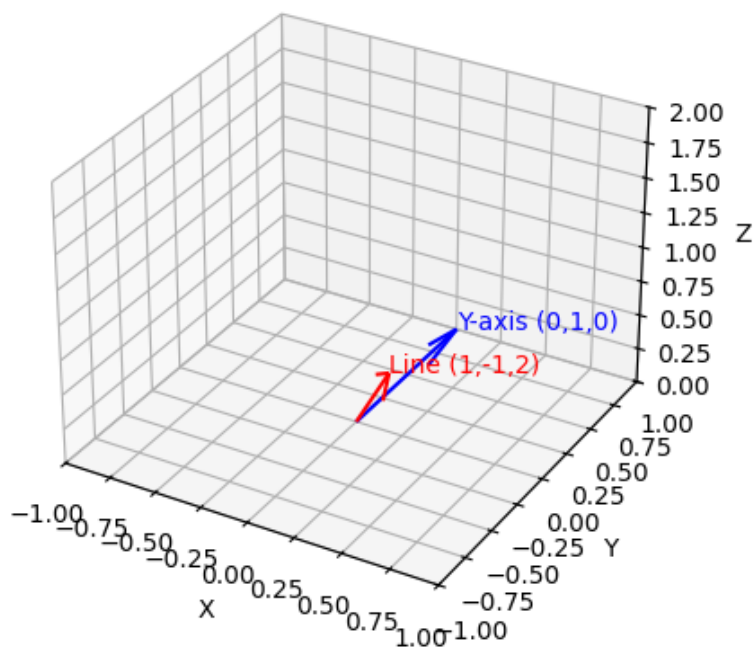


Fig. 0.1: Graph