2.10.12

AI25BTECH11023-Pratik R

Question:
A unit vector perpendicular to the plane determined by the points $P(1,-1,2)$,$Q(2,0,-1)$ and $R(0,2,1)$ is
solution:
According to the question,
Given the position vectors,

$$\mathbf{P} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}; \mathbf{Q} = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}; \mathbf{R} = \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} \tag{0.1}$$

Let the perpendicular vector be $\mathbf{n}^T = \begin{pmatrix} n_1 & n_2 & n_3 \end{pmatrix}$

$$\because \mathbf{n}^\top \mathbf{P} = 1 \tag{0.2}$$
$$\mathbf{n}^\top \mathbf{Q} = 1 \tag{0.3}$$
$$\mathbf{n}^\top \mathbf{R} = 1 \tag{0.4}$$

$$\therefore \begin{pmatrix} \mathbf{P}^\top \\ \mathbf{Q}^\top \\ \mathbf{R}^\top \end{pmatrix} \mathbf{n} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{0.5}$$

$$\therefore \begin{pmatrix} \mathbf{P} & \mathbf{Q} & \mathbf{R} \end{pmatrix}^\top \mathbf{n} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{0.6}$$

$$\begin{pmatrix} 1 & -1 & 2 \\ 2 & 0 & -1 \\ 0 & 2 & 1 \end{pmatrix} \mathbf{n} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \tag{0.7}$$

The augmented matrix for the above system of Equations is given by

$$\begin{pmatrix} 1 & -1 & 2 & 1 \\ 2 & 0 & -1 & 1 \\ 0 & 2 & 1 & 1 \end{pmatrix} \tag{0.8}$$

Solving equation 0.8 we get

$$\mathbf{n} = \begin{pmatrix} \frac{2}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{pmatrix} \tag{0.9}$$

The unit vector perpendicular to the plane is given by x

$$\mathbf{x} = \frac{\mathbf{n}}{\|n\|} = \frac{1}{\sqrt{6}}\begin{pmatrix}2\\1\\1\end{pmatrix} \tag{0.10}$$