4.13.1

AI25BTECH11036-SNEHAMRUDULA

Question:

Q. Consider the lines given by

$$L_1 : x + 3y - 5 = 0,$$
$$L_2 : 3x - ky - 1 = 0,$$
$$L_3 : 5x + 2y - 12 = 0.$$

Match the Statements/Expressions in Column I with the Statements/Expressions in Column II.

Column I	**Column II**
(A) L_1, L_2, L_3 are concurrent, if	(a) $k = 9$
(B) One of L_1, L_2, L_3 is parallel to at least one of the other two, if	(b) $k = \frac{-6}{5}$
(C) L_1, L_2, L_3 form a triangle, if	(c) $k = \frac{5}{6}$
(D) L_1, L_2, L_3 do not form a triangle, if	(d) $k = 5$

Solution:

Solution: The general form of a line is $\mathbf{n}^\top \mathbf{x} = p$, where $\mathbf{n}$ is the normal vector.

For the given lines:

$$L_1 : \mathbf{n}_1 = \begin{pmatrix} 1 \\ 3 \end{pmatrix}, \quad p_1 = 5,$$
$$L_2 : \mathbf{n}_2 = \begin{pmatrix} 3 \\ -k \end{pmatrix}, \quad p_2 = 1,$$
$$L_3 : \mathbf{n}_3 = \begin{pmatrix} 5 \\ 2 \end{pmatrix}, \quad p_3 = 12.$$

(A) **Concurrent:** The lines are concurrent if the matrix $\begin{pmatrix} 1 & 3 & -5 \\ 3 & -k & -1 \\ 5 & 2 & -12 \end{pmatrix}$ has determinant zero.

Computing the determinant:

$$\begin{aligned} &1((-k)(-12) - (-1)(2)) - 3(3(-12) - (-1)(5)) + (-5)(3(2) - (-k)(5)) \\ &= 1(12k + 2) - 3(-36 + 5) - 5(6 + 5k) \\ &= 12k + 2 + 93 - 30 - 25k \\ &= -13k + 65. \end{aligned}$$

Setting it equal to zero: $-13k + 65 = 0 \Rightarrow k = 5$.

So concurrent if $k = 5$. Hence (A) matches with (s).

(B) **Parallel:** Two lines are parallel if their normal vectors are scalar multiples.
Checking pairs:

- $\mathbf{n}_1$ and $\mathbf{n}_2$: $\frac{3}{1} = \frac{-k}{3} \Rightarrow k = -9$.
- $\mathbf{n}_2$ and $\mathbf{n}_3$: $\frac{3}{5} = \frac{-k}{2} \Rightarrow k = \frac{-6}{5}$.
- $\mathbf{n}_1$ and $\mathbf{n}_3$: $\frac{1}{5} = \frac{3}{2}$, which is false â not parallel.

So parallel if $k = \frac{-6}{5}$. Hence (B) matches with (q).

(C) **Form a triangle:** The lines form a triangle if they are neither concurrent nor parallel.
So $k \neq 5$ and $k \neq \frac{-6}{5}$.
A typical example is $k = 9$. Hence (C) matches with (p).

(D) **Do not form a triangle:** The lines do not form a triangle if they are either concurrent or two of them are parallel.
Hence possible when $k = 5$ or $k = \frac{-6}{5}$.
Here the option is $k = 5$. Hence (D) matches with (s).