Problem 4.13.28

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Problem

Slope of a line passing through $\mathbf{P}\,(2,3)$ and intersecting the line $x+y=7$ at a distance of 4 units from $\mathbf{P}$, is

Solving

Given

$$\mathbf{P} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \tag{2.1}$$

Equation of a line through **P** and having slope m is

$$\mathbf{r} = \mathbf{p} + t\mathbf{b} \tag{2.2}$$

$$\mathbf{b} = \begin{pmatrix} 1 \\ m \end{pmatrix} \tag{2.3}$$

$$x + y = 7 \implies \begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{x} = 7 \tag{2.4}$$

$$\begin{pmatrix} 1 & 1 \end{pmatrix} (\mathbf{p} + t\mathbf{b}) = 7 \tag{2.5}$$

$$\begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{p} + t \begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{b} = 7 \tag{2.6}$$

$$t \begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{b} = 7 - \begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{p} \tag{2.7}$$

Solving

$$t = \frac{7 - \begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{p}}{\begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{b}} \tag{2.8}$$

$\mathbf{Q}$ be the point of intersection

$$\mathbf{q} = \mathbf{p} + t\mathbf{b} \tag{2.9}$$

$$\mathbf{q} - \mathbf{p} = t\mathbf{b} \tag{2.10}$$

$$\|\mathbf{q} - \mathbf{p}\| = |t|\|\mathbf{b}\| \implies |t| = \frac{\|\mathbf{q} - \mathbf{p}\|}{\|\mathbf{b}\|} \tag{2.11}$$

$$\left| \frac{7 - \begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{p}}{\begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{b}} \right| = \frac{\|\mathbf{q} - \mathbf{p}\|}{\|\mathbf{b}\|} \tag{2.12}$$

Substitution

Given the point is at a distance of 4 units from point **P**

$$\left| \frac{7 - \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \end{pmatrix}}{\begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ m \end{pmatrix}} \right| = \frac{4}{\sqrt{1+m^2}} \tag{2.13}$$

$$\left| \frac{7-5}{1+m} \right| = \frac{4}{\sqrt{1+m^2}} \tag{2.14}$$

$$\left(\frac{7-5}{1+m} \right)^2 = \frac{16}{1+m^2} \implies \frac{4}{(1+m)^2} = \frac{16}{1+m^2} \tag{2.15}$$

$$4\,(1+m)^2 = 1+m^2 \tag{2.16}$$

$$4\left(m^2+2m+1\right) = 1+m^2 \tag{2.17}$$

$$4m^2+8m+4 = 1+m^2 \implies 3m^2+8m+3 = 0 \tag{2.18}$$

Conclusion

$$m^2 + \frac{8m}{3} + 1 = 0 \tag{2.19}$$

$$m^2 + \frac{8m}{3} + 1 + \left(\frac{4}{3}\right)^2 = \frac{16}{9} \tag{2.20}$$

$$\left(m + \frac{4}{3}\right)^2 = \frac{16-9}{9} = \frac{7}{9} \tag{2.21}$$

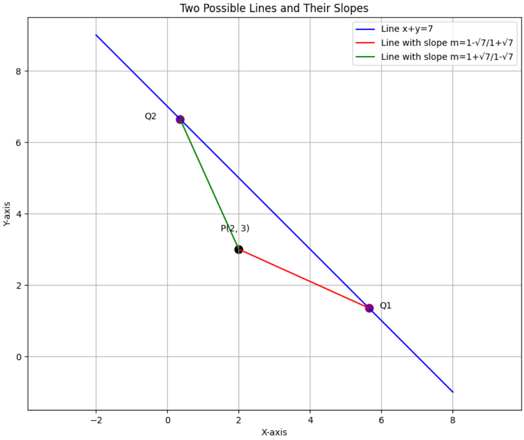
$$m + \frac{4}{3} = \pm\frac{\sqrt{7}}{3} \tag{2.22}$$

$$m = \frac{-4 - \sqrt{7}}{3} \text{ (or) } \frac{-4 + \sqrt{7}}{3} \tag{2.23}$$

According to options

$$m = \frac{-4 + \sqrt{7}}{3} = \frac{8 - 2\sqrt{7}}{-6} = \frac{\left(1 - \sqrt{7}\right)^2}{\left(1 + \sqrt{7}\right)\left(1 - \sqrt{7}\right)} = \frac{1 - \sqrt{7}}{1 + \sqrt{7}} \tag{2.24}$$

Plot



Two Possible Lines and Their Slopes
Line x+y=7
Line with slope m=1-√7/1+√7
Line with slope m=1+√7/1-√7
Q2
P(2, 3)
Q1
Y-axis
X-axis
8
6
4
2
0
−2
0
2
4
6
8

Figure:

C Code

```c
#include <math.h>
void calculate_slope_data(double* out_data) {
    double px = 2.0, py = 3.0;
    double a = 1.0, b = -6.0, c = 2.0;
    double discriminant = sqrt(b*b - 4*a*c);

    double q1_x = (-b + discriminant) / (2 * a); // 3 + sqrt(7)
    double q2_x = (-b - discriminant) / (2 * a); // 3 - sqrt(7)

    double q1_y = 7 - q1_x;
    double q2_y = 7 - q2_x;

    double slope1 = (q1_y - py) / (q1_x - px);
    double slope2 = (q2_y - py) / (q2_x - px);
    out_data[0] = px; out_data[1] = py;
    out_data[2] = q1_x; out_data[3] = q1_y;
    out_data[4] = q2_x; out_data[5] = q2_y;
    out_data[6] = slope1; out_data[7] = slope2;
}
```

Python Code for Calling

```
import ctypes
import numpy as np

def get_data_from_c():
    lib = ctypes.CDLL('./code.so')

    double_array_8 = ctypes.c_double * 8
    lib.calculate_slope_data.argtypes = [ctypes.POINTER(ctypes.
        c_double)]
    out_data_c = double_array_8()
    lib.calculate_slope_data(out_data_c)
    all_data = np.array(out_data_c)
    # Unpack the data
    point_p = all_data[0:2]
    point_q1 = all_data[2:4]
    point_q2 = all_data[4:6]
    slopes = all_data[6:8]

    return point_p, point_q1, point_q2, slopes
```

Python Code for Plotting

```
#Code by GVV Sharma
#September 12, 2023
#Revised July 21, 2024
#released under GNU GPL
import sys #for path to external scripts
sys.path.insert(0, '/workspaces/urban-potato/matgeo/codes/
    CoordGeo/')
import numpy as np
import matplotlib.pyplot as plt

from call import get_data_from_c

# Get the points and slopes from the C library
P, Q1, Q2, slopes = get_data_from_c()
slope1, slope2 = slopes
```

Python Code for Plotting

```
print(f"The two possible slopes are: {slope1:.4f} and {slope2:.4f
    }")
x_line_given = np.array([-2, 8])
y_line_given = 7 - x_line_given

# Create points for the two possible solution lines
x_line_1 = np.array([P[0], Q1[0]])
y_line_1 = np.array([P[1], Q1[1]])
x_line_2 = np.array([P[0], Q2[0]])
y_line_2 = np.array([P[1], Q2[1]])

fig, ax = plt.subplots(figsize=(10, 8))

ax.plot(x_line_given, y_line_given, 'b-', label='Line x+y=7')
ax.plot(x_line_1, y_line_1, 'r-', label=f'Line with slope m={
    slope1:.2f}')
ax.plot(x_line_2, y_line_2, 'g-', label=f'Line with slope m={
    slope2:.2f}')
```

Python Code for Plotting

```
ax.scatter(P[0], P[1], color='black', s=80)
ax.scatter(Q1[0], Q1[1], color='purple', s=80)
ax.scatter(Q2[0], Q2[1], color='purple', s=80)

ax.text(P[0] - 0.5, P[1] + 0.5, f'P({P[0]:.0f}, {P[1]:.0f})')
ax.text(Q1[0] + 0.3, Q1[1], 'Q1')
ax.text(Q2[0] - 1.0, Q2[1], 'Q2')

ax.set_title('Two Possible Lines and Their Slopes')
ax.set_xlabel('X-axis')
ax.set_ylabel('Y-axis')

ax.grid(True)
ax.axis('equal')
ax.legend()
plt.show()
plt.savefig('../figs/fig1.png')
```