Matrices in Geometry - 4.11.15

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Sept, 2025

Problem Statement

Find the equation of the plane which contains the line of intersection of the planes $\mathbf{r} \cdot \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) - 4 = 0$, $\mathbf{r} \cdot \left(2\hat{i} + \hat{j} - \hat{k}\right) + 5 = 0$ and which is perpendicular to the plane $\mathbf{r} \cdot \left(5\hat{i} + 3\hat{j} - 6\hat{k}\right) + 8 = 0$.

Solution

We need to find an equation of the plane that contains the line of intersection of the given two planes:

$$\mathbf{P_1} \ : \ \mathbf{n_1}^\top \mathbf{x} - 4 = 0 \quad , \mathbf{n_1} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \tag{1}$$

$$\mathbf{P_2} \ : \ \mathbf{n_2}^\top \mathbf{x} + 5 = 0 \quad , \mathbf{n_2} = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} \tag{2}$$

Another plane that passes through the intersection of these two planes is

$$\mathbf{P} \ : \ \mathbf{P_1} + \lambda \mathbf{P_2} = 0 \tag{3}$$

$$\mathbf{P} \ : \ \mathbf{n_1}^\top \mathbf{x} - 4 + \lambda \left(\mathbf{n_2}^\top \mathbf{x} + 5 \right) = 0 \tag{4}$$

Solution

This can be written as

$$\mathbf{P} \ : \ \left(\mathbf{n_1}^\top + \lambda \mathbf{n_2}^\top\right)\mathbf{x} - 4 + 5\lambda = 0 \tag{5}$$

The normal to this plane is

$$\mathbf{n} = \mathbf{n_1} + \lambda \mathbf{n_2} \tag{6}$$

The plane $\mathbf{P}$ should also be perpendicular to the plane

$$\mathbf{P_3} \ : \ \mathbf{n_3}^\top \mathbf{x} + 8 = 0 \ , \ \mathbf{n_3} = \begin{pmatrix} 5 \\ 3 \\ -6 \end{pmatrix} \tag{7}$$

$$\therefore \mathbf{n_3}^\top \mathbf{n} = 0 \implies \mathbf{n_3}^\top (\mathbf{n_1} + \lambda \mathbf{n_2}) = 0 \tag{8}$$

$$\implies \mathbf{n_3}^\top \mathbf{n_1} + \lambda \mathbf{n_3}^\top \mathbf{n_2} = 0 \tag{9}$$

Solution

$$\implies \begin{pmatrix}5 & 3 & -6\end{pmatrix}\begin{pmatrix}1\\2\\3\end{pmatrix} + \lambda\begin{pmatrix}5 & 3 & -6\end{pmatrix}\begin{pmatrix}2\\1\\-1\end{pmatrix} = 0 \tag{10}$$

$$\implies -7 + 19\lambda = 0 \implies \lambda = \frac{7}{19} \tag{11}$$

Substituting this value of λ in equation of $\mathbf{P}$, we get

$$\mathbf{P} \; : \; \left(\begin{pmatrix}1 & 2 & 3\end{pmatrix} + \frac{7}{19}\begin{pmatrix}2 & 1 & -1\end{pmatrix}\right)\mathbf{x} - 4 + \frac{35}{19} = 0 \tag{12}$$

$$\mathbf{P} \; : \mathbf{n}^\top\mathbf{x} - 41 = 0 \; , \; \mathbf{n} = \begin{pmatrix}33\\45\\50\end{pmatrix} \tag{13}$$

Solution

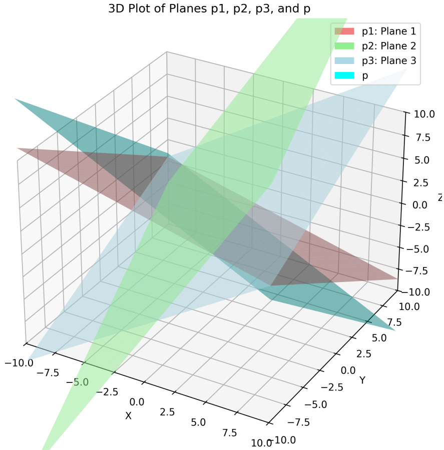

Figure: Figure for 4.11.15