5.2.26

EE25BTECH11023 - Venkata Sai

Question:
Solve the following system of linear equations

$$\frac{x}{a} - \frac{y}{b} = 0 \tag{1}$$

$$ax + by = a^2 + b^2 \tag{2}$$

Solution: Given

$$\frac{x}{a} - \frac{y}{b} = 0 \implies bx - ay = 0 \tag{3}$$

$$ax + by = a^2 + b^2 \tag{4}$$

The matrix equation for a line is defined as

$$\mathbf{n}^\top \mathbf{x} = c \tag{5}$$

where $\mathbf{n}$ is the coefficient matrix and $\mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix}$

$$\begin{pmatrix} b & -a \end{pmatrix} \mathbf{x} = 0 \tag{6}$$

$$\begin{pmatrix} a & b \end{pmatrix} \mathbf{x} = a^2 + b^2 \tag{7}$$

As a matrix equation

$$\begin{pmatrix} b & -a \\ a & b \end{pmatrix} \mathbf{x} = \begin{pmatrix} 0 \\ a^2 + b^2 \end{pmatrix} \tag{8}$$

$$\begin{pmatrix} b & -a \\ a & b \end{pmatrix}^\top \begin{pmatrix} b & -a \\ a & b \end{pmatrix} \mathbf{x} = \begin{pmatrix} b & -a \\ a & b \end{pmatrix}^\top \begin{pmatrix} 0 \\ a^2 + b^2 \end{pmatrix} \tag{9}$$

$$\begin{pmatrix} b & a \\ -a & b \end{pmatrix} \begin{pmatrix} b & -a \\ a & b \end{pmatrix} \mathbf{x} = \begin{pmatrix} b & a \\ -a & b \end{pmatrix} \begin{pmatrix} 0 \\ a^2 + b^2 \end{pmatrix} \tag{10}$$

$$\begin{pmatrix} a^2 + b^2 & 0 \\ 0 & a^2 + b^2 \end{pmatrix} \mathbf{x} = \begin{pmatrix} a\left(a^2 + b^2\right) \\ b\left(a^2 + b^2\right) \end{pmatrix} \tag{11}$$

$$\left(a^2 + b^2\right) \mathbf{I}\mathbf{x} = \begin{pmatrix} a\left(a^2 + b^2\right) \\ b\left(a^2 + b^2\right) \end{pmatrix} \tag{12}$$

$$\mathbf{I}\mathbf{x} = \begin{pmatrix} a\left(a^2 + b^2\right) \\ b\left(a^2 + b^2\right) \end{pmatrix} \frac{1}{a^2 + b^2} \tag{13}$$

$$\mathbf{I}\mathbf{x} = \begin{pmatrix} a \\ b \end{pmatrix} \tag{14}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} \tag{15}$$

Hence $x = a, y = b$ is the solution for given system of linear equations

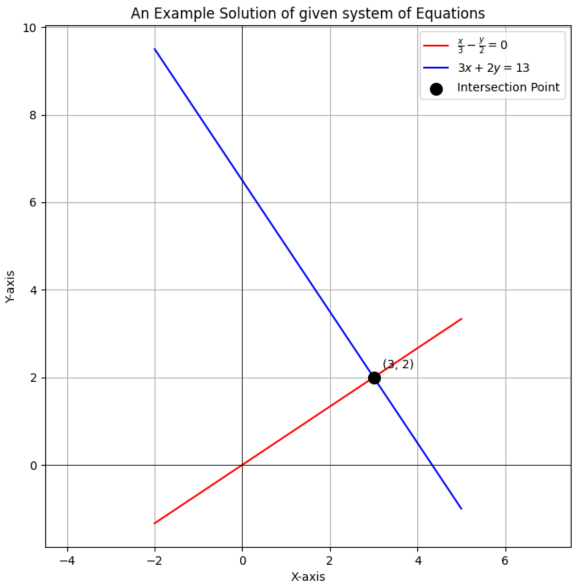

Fig. 0.1