

2.7.21

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Question:

Find the values of k so that the area of the triangle with vertices $A(1, -1)$, $B(-4, 2k)$, $C(-k, -5)$ is 24 sq. units.

Solution:

The given vertices are

Point	Coordinates
A	$\begin{pmatrix} 1 \\ -1 \end{pmatrix}$
B	$\begin{pmatrix} -4 \\ 2k \end{pmatrix}$
C	$\begin{pmatrix} -k \\ -5 \end{pmatrix}$

TABLE 0: Vertices of $\triangle ABC$ before substituting k

$$\mathbf{u} = \mathbf{B} - \mathbf{A} = \begin{pmatrix} -5 \\ 2k + 1 \end{pmatrix}, \quad (0.1)$$

$$\mathbf{v} = \mathbf{C} - \mathbf{A} = \begin{pmatrix} -k - 1 \\ -4 \end{pmatrix}. \quad (0.2)$$

The area of $\triangle ABC$ is

$$\Delta = \frac{1}{2} \|\mathbf{u} \times \mathbf{v}\|. \quad (0.3)$$

Using the identity

$$\|\mathbf{u} \times \mathbf{v}\|^2 = \|\mathbf{u}\|^2 \|\mathbf{v}\|^2 - (\mathbf{u}^\top \mathbf{v})^2 \quad (0.4)$$

Hence,

$$\|\mathbf{u} \times \mathbf{v}\|^2 = (2k^2 + 3k + 21)^2 \quad (0.5)$$

$$\implies \|\mathbf{u} \times \mathbf{v}\| = |2k^2 + 3k + 21|. \quad (0.6)$$

So,

$$\Delta = \frac{1}{2} |2k^2 + 3k + 21|. \quad (0.7)$$

Given $\Delta = 24$,

$$|2k^2 + 3k + 21| = 48 \quad (0.8)$$

Case 1:

$$2k^2 + 3k + 21 = 48 \quad (0.9)$$

$$\implies 2k^2 + 3k - 27 = 0 \quad (0.10)$$

$$k = \frac{-3 \pm 15}{4} = \{3, -\frac{9}{2}\} \quad (0.11)$$

Case 2:

$$2k^2 + 3k + 21 = -48 \quad (0.12)$$

$$\implies 2k^2 + 3k + 69 = 0 \quad (0.13)$$

This has no real roots.

$$\therefore k \in \left\{3, -\frac{9}{2}\right\} \quad (0.14)$$

Point	For $k = 3$	For $k = -\frac{9}{2}$
A	$\begin{pmatrix} 1 \\ -1 \end{pmatrix}$	$\begin{pmatrix} 1 \\ -1 \end{pmatrix}$
B	$\begin{pmatrix} -4 \\ 6 \end{pmatrix}$	$\begin{pmatrix} -4 \\ -9 \end{pmatrix}$
C	$\begin{pmatrix} -3 \\ -5 \end{pmatrix}$	$\begin{pmatrix} \frac{9}{2} \\ -5 \end{pmatrix}$

TABLE 0: Vertices of $\triangle ABC$ after substituting k values

