4.13.51

RAVULA SHASHANK REDDY - EE25BTECH11047

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Question

One of the diameters of the circle circumscribing the rectangle *ABCD* is given by

$$4y = x + 7.$$

If **A** $= (-3, 4)$ and **B** $= (5, 4)$, find the area of the rectangle.

Solution

$$\mathbf{A} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 5 \\ 4 \end{pmatrix} \tag{1}$$

Centre $\mathbf{O} = \begin{pmatrix} x \\ y \end{pmatrix}$ satisfies

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix}^T \mathbf{x} = 1, \qquad \begin{pmatrix} 1 \\ -4 \end{pmatrix}^T \mathbf{x} = -7 \tag{2}$$

$$\begin{pmatrix} 1 & 0 \\ 1 & -4 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 1 \\ -7 \end{pmatrix} \tag{3}$$

$$\begin{pmatrix} 1 & 0 & 1 \\ 1 & -4 & -7 \end{pmatrix} \xrightarrow{R_2 \to R_2 - R_1} \begin{pmatrix} 1 & 0 & 1 \\ 0 & -4 & -8 \end{pmatrix} \xrightarrow{R_2 \to (-\frac{1}{4})R_2} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \end{pmatrix} \tag{4}$$

Solution

$$\mathbf{O} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \tag{5}$$

$$\mathbf{C} = 2\mathbf{O} - \mathbf{A} = \begin{pmatrix} 2 \\ 4 \end{pmatrix} - \begin{pmatrix} -3 \\ 4 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \end{pmatrix}, \tag{6}$$

$$\mathbf{D} = 2\mathbf{O} - \mathbf{B} = \begin{pmatrix} 2 \\ 4 \end{pmatrix} - \begin{pmatrix} 5 \\ 4 \end{pmatrix} = \begin{pmatrix} -3 \\ 0 \end{pmatrix}. \tag{7}$$

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 8 \\ 0 \end{pmatrix}, \qquad \mathbf{D} - \mathbf{A} = \begin{pmatrix} 0 \\ -4 \end{pmatrix}. \tag{8}$$

$$\text{Area} = |\mathbf{B} - \mathbf{A} \times \mathbf{D} - \mathbf{A}| = \left| \det \begin{pmatrix} 8 & 0 \\ 0 & -4 \end{pmatrix} \right| = |8(-4) - 0| = 32. \tag{9}$$

C Code

```c
    #include <stdio.h>
#include <math.h>

int main() {
    // Given points
    double A[2] = {-3, 4};
    double B[2] = {5, 4};

    // Solve equations for center
    // x = 1
    // x - 4y = -7
    double x = 1;
    double y = (x + 7) / 4.0;

    double O[2] = {x, y};
    printf(Center O = (%.2f, %.2f)\n, O[0], O[1]);
```

C Code

```c
    // Opposite vertices
    double C[2] = {2*O[0] - A[0], 2*O[1] - A[1]};
    double D[2] = {2*O[0] - B[0], 2*O[1] - B[1]};

    printf(C = (%.2f, %.2f)\n, C[0], C[1]);
    printf(D = (%.2f, %.2f)\n, D[0], D[1]);

    // Side vectors
    double AB[2] = {B[0]-A[0], B[1]-A[1]};
    double AD[2] = {D[0]-A[0], D[1]-A[1]};

    // Cross product (2D determinant)
    double area = fabs(AB[0]*AD[1] - AB[1]*AD[0]);
    printf(Area of rectangle = %.2f\n, area);

    return 0;
}
```

Python Direct

```python
import numpy as np
import matplotlib.pyplot as plt
from numpy import linalg as LA


# local imports
from libs.line.funcs import *
from libs.triangle.funcs import *
from libs.conics.funcs import circ_gen

# Given points
A = np.array([-3,4]).reshape(-1,1)
B = np.array([5,4]).reshape(-1,1)

# Centre O lies on x=1 and x-4y=-7
x = 1
y = (x+7)/4
```

Python Direct

```python
O = np.array([x,y]).reshape(-1,1)

# Opposite vertices
C = 2*O - A
D = 2*O - B

# Side vectors
AB = B - A
AD = D - A

# Area by cross product
area = abs(np.linalg.det(np.hstack((AB,AD))))
print(Area of rectangle =, area)

# Circumcircle radius
r = LA.norm(A - O)
```

```python
# Circle points
x_circ = circ_gen(O.flatten(), r)

# Plotting
rect_coords = np.hstack((A,B,C,D,O))
labels = ['$A$','$B$','$C$','$D$','$O$']

plt.plot(x_circ[0,:], x_circ[1,:], label='Circumcircle')
plt.plot([A[0,0],B[0,0],C[0,0],D[0,0],A[0,0]],
         [A[1,0],B[1,0],C[1,0],D[1,0],A[1,0]],
         'k-',label='Rectangle')

plt.scatter(rect_coords[0,:], rect_coords[1,:])
```

Python Direct

```python
for i, txt in enumerate(labels):
    plt.annotate(f'{txt}({rect_coords[0,i]:.0f},{rect_coords[1,i
        ]:.0f})',
                 (rect_coords[0,i], rect_coords[1,i]),
                 textcoords=offset points, xytext=(-10,5), ha='
                    center')

ax = plt.gca()
ax.spines['top'].set_color('none')
ax.spines['right'].set_color('none')
ax.spines['left'].set_position('zero')
ax.spines['bottom'].set_position('zero')

plt.grid()
plt.axis('equal')
plt.legend()
plt.show()
```

```python
import numpy as np
import matplotlib.pyplot as plt
from numpy import linalg as LA
import ctypes

# local imports
from line.funcs import *
from triangle.funcs import *
from conics.funcs import circ_gen

# Load C library
lib = ctypes.CDLL('./rect.so')
lib.rect_area.restype = ctypes.c_double
c_area = lib.rect_area()

# Given points
A = np.array([-3,4]).reshape(-1,1)
B = np.array([5,4]).reshape(-1,1)
```

Python Shared

```python
# Centre O
x = 1
y = (x+7)/4
O = np.array([x,y]).reshape(-1,1)

# Opposite vertices
C = 2*O - A
D = 2*O - B

# Side vectors
AB = B - A
AD = D - A

# Area via cross product in Python
p_area = abs(np.linalg.det(np.hstack((AB,AD))))

print(Area from C (ctypes) =, c_area)
print(Area from Python =, p_area)
```

```python
# Circumcircle radius
r = LA.norm(A - O)
x_circ = circ_gen(O.flatten(), r)

# Plot
rect_coords = np.hstack((A,B,C,D,O))
labels = ['$A$','$B$','$C$','$D$','$O$']

plt.plot(x_circ[0,:], x_circ[1,:], label='Circumcircle')
plt.plot([A[0,0],B[0,0],C[0,0],D[0,0],A[0,0]],
         [A[1,0],B[1,0],C[1,0],D[1,0],A[1,0]],
         'k-',label='Rectangle')

plt.scatter(rect_coords[0,:], rect_coords[1,:])
```

Python Shared

```python
for i, txt in enumerate(labels):
    plt.annotate(f'{txt}({rect_coords[0,i]:.0f},{rect_coords[1,i
        ]:.0f})',
                 (rect_coords[0,i], rect_coords[1,i]),
                 textcoords=offset points, xytext=(-10,5), ha='
                    center')

ax = plt.gca()
ax.spines['top'].set_color('none')
ax.spines['right'].set_color('none')
ax.spines['left'].set_position('zero')
ax.spines['bottom'].set_position('zero')

plt.grid()
plt.axis('equal')
plt.legend()
plt.show()
```
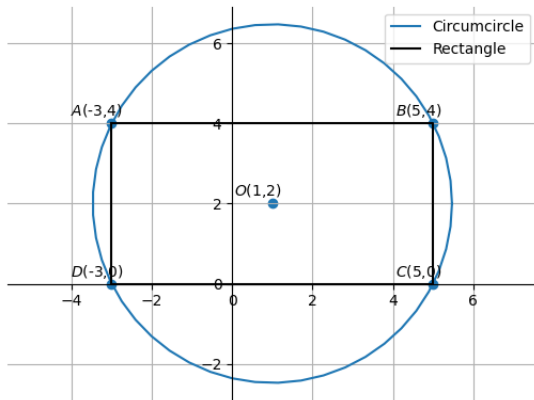
Plot

Figure: