5.12.7 Matgeo

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Question

Solve the following equations for x and y :

$$(ax - by) + (a + 4b) = 0$$
$$(bx + ay) + (b - 4a) = 0$$

Solution

given two equations :

$$(ax - by) = -(a + 4b) \tag{1}$$
$$(bx + ay) = -(b - 4a) \tag{2}$$

these can be written as :

$$\begin{bmatrix} a & -b \\ b & a \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -a - 4b \\ 4a - b \end{bmatrix} \tag{3}$$

Solution

these can be written as :

$$\begin{bmatrix} a & -b \\ b & a \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -a-4b \\ 4a-b \end{bmatrix} = \mathbf{M} \begin{bmatrix} a \\ b \end{bmatrix} \tag{4}$$

$$\tag{5}$$

where :

$$\mathbf{M} = \begin{bmatrix} -1 & -4 \\ 4 & -1 \end{bmatrix} \tag{6}$$

Solution

Multiplying both sides by A^T :

$$\begin{bmatrix} a & b \\ -b & a \end{bmatrix} \begin{bmatrix} a & -b \\ b & a \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = A^T M \begin{bmatrix} a \\ b \end{bmatrix} \tag{7}$$

$$(a^2 + b^2) \begin{bmatrix} x \\ y \end{bmatrix} = A^T M \begin{bmatrix} a \\ b \end{bmatrix} \tag{8}$$

Solution

Hence:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{(a^2+b^2)} A^T M \begin{bmatrix} a \\ b \end{bmatrix} \tag{9}$$