5.2.58

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Question : Solve the system of equations

$$x - y + z = 4$$
$$2x + y - 3z = 0$$
$$x + y + z = 2$$

Solution :

Name	Equation
Equation 1	$x - y + z = 4 \iff \begin{pmatrix} 1 & -1 & 1 \end{pmatrix} \mathbf{x}_1 = 4$
Equation 2	$2x + y - 3z = 0 \iff \begin{pmatrix} 2 & 1 & -3 \end{pmatrix} \mathbf{x}_2 = 0$
Equation 3	$x + y + z = 2 \iff \begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \mathbf{x}_3 = 2$

Table : Equations

The system of equations in matrix form is :

$$\begin{pmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ 2 \end{pmatrix} \tag{1}$$

Forming the augmented matrix,

$$\left(\begin{array}{ccc|c} 1 & -1 & 1 & 4 \\ 2 & 1 & -3 & 0 \\ 1 & 1 & 1 & 2 \end{array}\right) \tag{2}$$

Using Gaussian elimination,

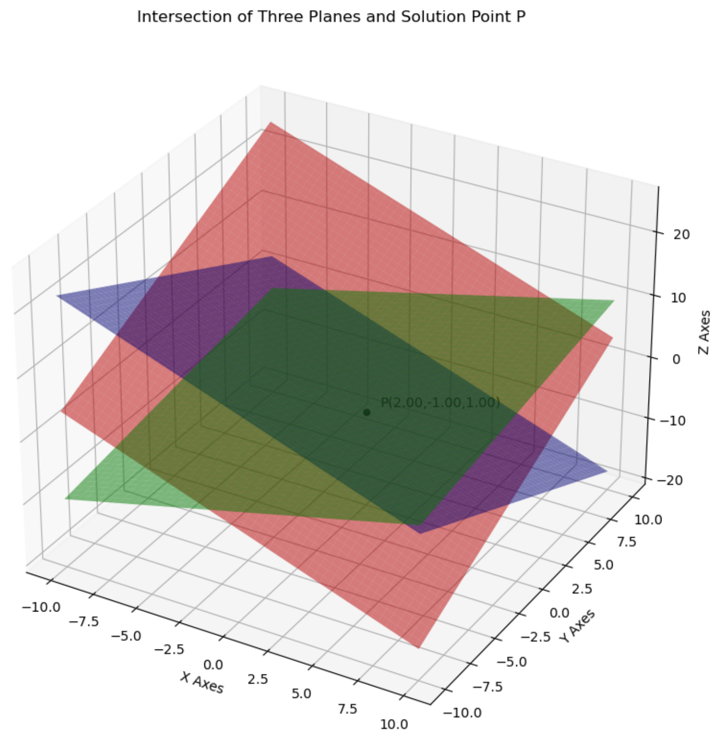
$$\left(\begin{array}{ccc|c} 1 & -1 & 1 & 4 \\ 2 & 1 & -3 & 0 \\ 1 & 1 & 1 & 2 \end{array}\right) \xleftrightarrow[R_2 \to R_2 - 2R_1]{R_3 \to R_3 - R_1} \left(\begin{array}{ccc|c} 1 & -1 & 1 & 4 \\ 0 & 3 & -5 & -8 \\ 0 & 2 & 0 & -2 \end{array}\right) \xleftrightarrow{R_3 \to R_3 - \frac{2}{3}R_2} \left(\begin{array}{ccc|c} 1 & -1 & 1 & 4 \\ 0 & 3 & -5 & -8 \\ 0 & 0 & \frac{10}{3} & \frac{10}{3} \end{array}\right) \tag{3}$$

Using back substitution we get :

$$\frac{10}{3}z = \frac{10}{3} \tag{4}$$
$$z = 1 \tag{5}$$
$$3y - 5z = -8 \tag{6}$$
$$3y - 5 = -8 \tag{7}$$
$$3y = -3 \tag{8}$$
$$y = -1 \tag{9}$$
$$x - y + z = 4 \tag{10}$$
$$x + 2 = 4 \tag{11}$$
$$x = 2 \tag{12}$$

Therefore the solution for the system of equations is :

$$\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} \tag{13}$$



Intersection of Three Planes and Solution Point P
P(2.00,-1.00,1.00)
X Axes
Y Axes
Z Axes
−10.0
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0
−20
−10
0
10
20

Fig : Planes