4.7.13

INDHIRESH S - EE25BTECH11027

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Question

Find the distance between the lines l_1 and l_2 given by

$$\overrightarrow{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$$
$$\overrightarrow{r} = 3\hat{i} + 3\hat{j} - 5\hat{k} + \mu(2\hat{i} + 3\hat{j} + 6\hat{k})$$

Equation

Given equation:

$$\mathbf{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k}) \tag{1}$$

$$\mathbf{r} = 3\hat{i} + 3\hat{j} - 5\hat{k} + \mu(2\hat{i} + 3\hat{j} + 6\hat{k}) \tag{2}$$

$$\tag{3}$$

From the above two equation it is clear that given two lines are parallel.
Now we'll find the distance between them.
The given lines are in the form:

$$\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ -4 \end{pmatrix} + k_1 \begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix} \tag{4}$$

$$\mathbf{r} = \begin{pmatrix} 3 \\ 3 \\ -5 \end{pmatrix} + k_2 \begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix} \tag{5}$$

Where,

$$\mathbf{A} = \begin{pmatrix} 1 \\ 2 \\ -4 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} 3 \\ 3 \\ -5 \end{pmatrix} \quad , \mathbf{M} = \begin{pmatrix} 2 & 2 \\ 3 & 3 \\ 6 & 6 \end{pmatrix} \tag{6}$$

Theoretical solution

From Least squares solution, for shortest distance:

$$\mathbf{M}^\mathbf{T}\mathbf{M}\begin{pmatrix} k_1 \\ -k_2 \end{pmatrix} = \mathbf{M}^\mathbf{T}(\mathbf{B}-\mathbf{A}) \tag{7}$$

$$\begin{pmatrix} 2 & 3 & 6 \\ 2 & 3 & 6 \end{pmatrix}\begin{pmatrix} 2 & 2 \\ 3 & 3 \\ 6 & 6 \end{pmatrix}\begin{pmatrix} k_1 \\ -k_2 \end{pmatrix} = \begin{pmatrix} 2 & 3 & 6 \\ 2 & 3 & 6 \end{pmatrix}\left(\begin{pmatrix} 3 \\ 3 \\ -5 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ -4 \end{pmatrix}\right) \tag{8}$$

$$\begin{pmatrix} 49 & 49 \\ 49 & 49 \end{pmatrix}\begin{pmatrix} k_1 \\ -k_2 \end{pmatrix} = \begin{pmatrix} 2 & 3 & 6 \\ 2 & 3 & 6 \end{pmatrix}\left(\begin{pmatrix} 3 \\ 3 \\ -5 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ -4 \end{pmatrix}\right) \tag{9}$$

$$\begin{pmatrix} 49 & 49 \\ 49 & 49 \end{pmatrix}\begin{pmatrix} k_1 \\ -k_2 \end{pmatrix} = \begin{pmatrix} 2 & 3 & 6 \\ 2 & 3 & 6 \end{pmatrix}\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} \tag{10}$$

Theoretical Solution

$$\begin{pmatrix} 49 & 49 \\ 49 & 49 \end{pmatrix} \begin{pmatrix} k_1 \\ -k_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{11}$$

$$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} k_1 \\ -k_2 \end{pmatrix} = \frac{\begin{pmatrix} 1 \\ 1 \end{pmatrix}}{49} \tag{12}$$

$$k_1 - k_2 = \frac{1}{49} \tag{13}$$

Let r_1 and r_2 be the point on the line l_1 and l_2
Now,

$$r_1 - r_2 = \begin{pmatrix} -2 \\ -1 \\ 1 \end{pmatrix} + (k_1 - k_2) \begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix} \tag{14}$$

Theoretical Solution

$$r_1 - r_2 = \begin{pmatrix} -2 \\ -1 \\ 1 \end{pmatrix} + (\frac{1}{49}) \begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix} \tag{15}$$

$$r_1 - r_2 = \begin{pmatrix} \frac{-96}{49} \\ \\ \frac{-46}{49} \\ \\ \frac{55}{49} \end{pmatrix} \tag{16}$$

Theoretical Solution

Now , Distance between two lines $= \|r_1 - r_2\|$

$$\|r_1 - r_2\| = \sqrt{\left(\frac{-96}{49}\right)^2 + \left(\frac{-46}{49}\right)^2 + \left(\frac{55}{49}\right)^2} \tag{17}$$

$$\|r_1 - r_2\| = \frac{\sqrt{14357}}{49} \tag{18}$$

Therefore the distance between the lines l_1 and l_2 is $\frac{\sqrt{14357}}{49}$

C Code

```c
#include <stdio.h>
#include <math.h>

int main() {
    double A[3] = {1,2,-4};
    double B[3] = {3,3,-5};
    double d[3] = {2,3,6};
    double AB[3], cross[3];
    double dist;

    // Compute B - A
    for(int i=0;i<3;i++) AB[i] = B[i] - A[i];

    // Cross product (AB x d)
    cross[0] = AB[1]*d[2] - AB[2]*d[1];
    cross[1] = AB[2]*d[0] - AB[0]*d[2];
    cross[2] = AB[0]*d[1] - AB[1]*d[0];
```

C Code

```
    // Norms
    double num = sqrt(cross[0]*cross[0] + cross[1]*cross[1] +
        cross[2]*cross[2]);
    double den = sqrt(d[0]*d[0] + d[1]*d[1] + d[2]*d[2]);

    dist = num / den;
    printf(Distance between lines = %lf\n, dist);

    // Output A, B, direction vector for plotting
    printf(A: %lf %lf %lf\n, A[0], A[1], A[2]);
    printf(B: %lf %lf %lf\n, B[0], B[1], B[2]);
    printf(d: %lf %lf %lf\n, d[0], d[1], d[2]);

    return 0;
}
```

Python Code

```
import ctypes
import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d.art3d import Poly3DCollection

# Load shared C library
lib = ctypes.CDLL(./distance.so)
lib.main()

# Define points and direction vector (same as in C)
A = np.array([1,2,-4])
B = np.array([3,3,-5])
d = np.array([2,3,6])

# Generate points for both lines
t = np.linspace(-2,2,10)
line1 = A[:,None] + d[:,None]*t
line2 = B[:,None] + d[:,None]*t
```

Python Code

```
# Plot
fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')

# Lines
ax.plot(line1[0], line1[1], line1[2], 'b-', label='Line l1')
ax.plot(line2[0], line2[1], line2[2], 'g-', label='Line l2')

# Points
ax.scatter(*A,color='r',s=50)
ax.text(*A,A(1,2,-4),color='red')
ax.scatter(*B,color='orange',s=50)
ax.text(*B,B(3,3,-5),color='orange')

# Dotted line AB
ax.plot([A[0],B[0]], [A[1],B[1]], [A[2],B[2]], 'k--', label='
    Connecting AB')
```

Python Code

```
ax.set_title(Figure)
ax.set_xlabel(X-axis)
ax.set_ylabel(Y-axis)
ax.set_zlabel(Z-axis)
ax.legend()
plt.savefig(/media/indhiresh-s/New Volume/Matrix/ee1030-2025/
    ee25btech11027/MATGEO/4.7.13/figs/figure1.png)
plt.show()
```

Plot

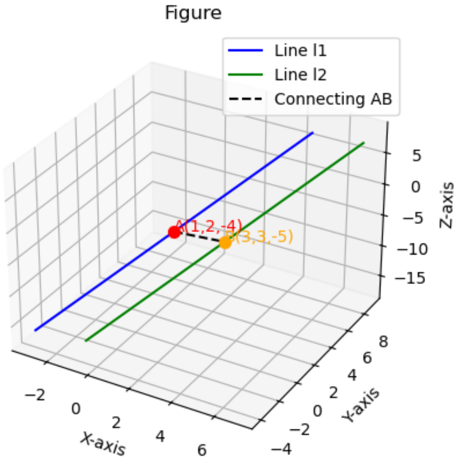

Figure
Line l1
Line l2
Connecting AB
A(1,2,-4)
B(3,3,-5)
X-axis
Y-axis
Z-axis
−2
0
2
4
6
−4
−2
0
2
4
6
8
5
0
−5
−10
−15