4.3.30

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12 September,2025

Question

Find the equation of the line which passes through the point $(-4, 3)$ and the portion of the line intercepted between the axes is divided internally in ratio 5:3 by this point.

Solution

Let the intercept points be

$$\mathbf{P} = a\mathbf{e_1}, \mathbf{Q} = b\mathbf{e_2} \tag{1}$$

$$\mathbf{e_1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \mathbf{e_2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \text{a and b are constants} \tag{2}$$

and

$$\mathbf{R} = \begin{pmatrix} -4 \\ 3 \end{pmatrix} = -4e_1 + 3e_2 \tag{3}$$

be the given point.
Using

$$\mathbf{R} = \frac{k\mathbf{Q} + \mathbf{P}}{k+1} \tag{4}$$

$$\mathbf{R} = \frac{5 \times b\mathbf{e_2} + 3 \times a\mathbf{e_1}}{8} \tag{5}$$

Solution

$$-4\mathbf{e_1} + 3\mathbf{e_2} = \frac{5 \times b\mathbf{e_2} + 3 \times a\mathbf{e_1}}{8} \tag{6}$$

$$-32\mathbf{e_1} + 24\mathbf{e_2} = 3a\mathbf{e_1} + 5b\mathbf{e_2} \tag{7}$$

General equation of line

$$\mathbf{x} = \mathbf{h} + c\mathbf{m} \tag{8}$$

Where

$\mathbf{x}$	general vector on line
$\mathbf{h}$	known vector of line
$\mathbf{m}$	slope vector of line
c	scalar parameter

Table:

Solution

Slope is

$$\mathbf{m} = \mathbf{Q} - \mathbf{P} \tag{9}$$

$$\tag{10}$$

let $\mathbf{h} = \mathbf{P}$
So, Equation of line is

$$\mathbf{x} = \mathbf{h} + c\mathbf{m} \tag{11}$$

$$\mathbf{x} = \mathbf{P} + c(\mathbf{Q} - \mathbf{P}) \tag{12}$$

Putting values of $\mathbf{Q}, \mathbf{P}$

$$\mathbf{x} = a\mathbf{e_1} + c(b\mathbf{e_2} - a\mathbf{e_1}) \tag{13}$$

Solution

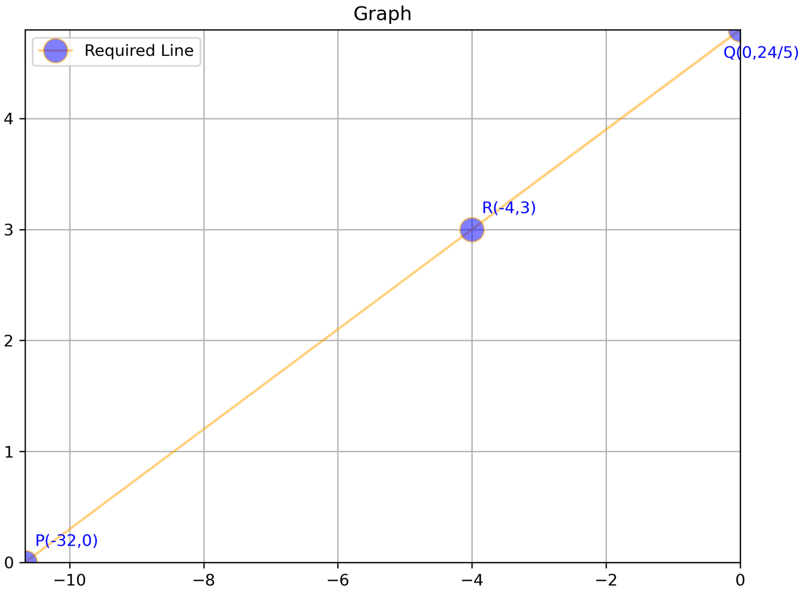
Comparing terms in (7) for values of a and b

$$\mathbf{x} = \frac{-32}{3}\mathbf{e_1} + c(\frac{24}{5}\mathbf{e_2} - \frac{-32}{3}\mathbf{e_1}) \tag{14}$$

Therefore Final equation is

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{-32}{3} \\ 0 \end{pmatrix} + k \begin{pmatrix} \frac{32}{3} \\ \frac{24}{5} \end{pmatrix} \tag{15}$$

Figure



C code

```
#include <stdio.h>

void trisec(double k, double x1, double y1, double x2, double y2,
     double* a, double* b){
    *a= (x1+k*x2)/(1+k);
    *b= (y1+k*y2)/(1+k);
}
```

Python code

```
import ctypes
import numpy as np
import matplotlib.pyplot as plt

# Load shared object
lib = ctypes.CDLL("./main.so")

# Define function signature
lib.trisec.argtypes = [
    ctypes.c_double, # k
    ctypes.c_double, # x1
    ctypes.c_double, # y1
    ctypes.c_double, # x2
    ctypes.c_double, # y2
    ctypes.POINTER(ctypes.c_double), # *a
    ctypes.POINTER(ctypes.c_double) # *b
]
```

Python code

```python
# Known intercepts from math derivation
a = -32.0/3.0
b = 24.0/5.0

# Call trisec with k = 5/3
px = ctypes.c_double()
py = ctypes.c_double()
lib.trisec(5.0/3.0, a, 0.0, 0.0, b, ctypes.byref(px), ctypes.
    byref(py))

print(f"Computed point dividing AB in 5:3 = ({px.value}, {py.
    value})")
```

Python code

```
# Now plot line
x_vals = np.linspace(a, 0, 200)
y_vals = b*(1 - x_vals/a)

plt.plot(x_vals, y_vals, label="Line through intercepts")
plt.scatter([a,0], [0,b], color="green", label="Intercepts")
plt.scatter([-4], [3], color="red", marker="x", s=100, label="Given point (-4,3)")
plt.axhline(0, color="black", linewidth=0.5)
plt.axvline(0, color="black", linewidth=0.5)
plt.legend()
plt.grid(True)
plt.show()
```

Direct Python code

```
import numpy as np
import matplotlib.pyplot as plt

plt.figure(figsize=(8, 6), dpi=100)

x = np.array([-32/3, 0, -4])
y= np.array([0, 24/5, 3])
plt.plot(x,y,'o-', color='orange',mfc='blue',ms='15',alpha=0.5,
    label="Required Line")
```

Direct Python code

```
plt.text(x[0]+0.15, y[0]+0.15, "P(-32,0)", color='blue')
plt.text(x[1]-0.25, y[1]-0.25, "Q(0,24/5)", color='blue')
plt.text(x[2]+0.15, y[2]+0.15, "R(-4,3)", color='blue')
plt.legend()
plt.title("Graph")
plt.grid()
plt.xlim(-32/3, 0)
plt.ylim(0, 24/5)

plt.savefig('figure.png', dpi=300, bbox_inches='tight')
plt.show()
```