2.3.6

EE25BTECH11003 - Adharvan Kshathriya Bommagani

Question:
Find the magnitude of each of the vectors **a** and **b** , having the same magnitude such that the angle between them is 60°and their scalar product is $\frac{9}{2}$.

Solution:

We are given two vectors **a** and **b** with the same magnitude, and the angle between them is 60°. Also, their scalar product is given as:

$$\mathbf{a}^T\mathbf{b} = 2. \tag{1}$$

Let the common magnitude of the vectors be r, i.e., $\|\mathbf{a}\| = \|\mathbf{b}\| = r$. Let the magnitudes of the two vectors be equal:

$$\|\mathbf{a}\| = \|\mathbf{b}\| = r. \tag{2}$$

The cosine of the angle between the two vectors is given by:

$$\cos\theta = \frac{\mathbf{a}^T\mathbf{b}}{\|\mathbf{a}\|\,\|\mathbf{b}\|}. \tag{3}$$

Substituting $\|\mathbf{a}\| = \|\mathbf{b}\| = r$, this simplifies to:

$$\cos\theta = \frac{\mathbf{a}^T\mathbf{b}}{r^2}. \tag{4}$$

Since the angle between the vectors is 60°, we have $\cos 60^{\circ} = \frac{1}{2}$. The scalar product is given as $\mathbf{a}^T\mathbf{b} = \frac{9}{2}$. Substituting these values:

$$\frac{1}{2} = \frac{\frac{9}{2}}{r^2}. \tag{5}$$

Multiply through by $2r^2$:

$$r^2 = 9. \tag{6}$$

Taking the positive root (since magnitude cannot be negative):

$$r = +3. \tag{7}$$

$$r = 3 \tag{8}$$

Therefore the magnitude of the vectors **a** and **b** is 3 each.

Two vectors with magnitude 3 and angle 60° between them

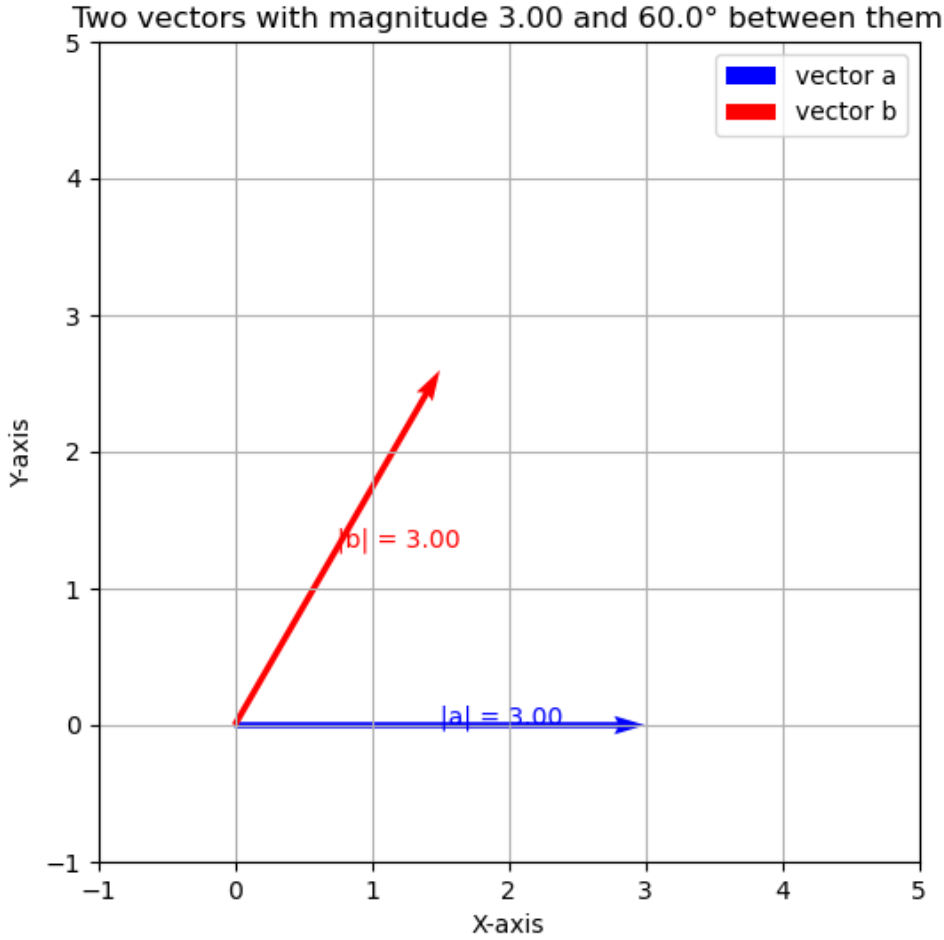

Fig. 0: Figure for 2.3.6