EE25BTECH11023 - Venkata Sai

Question:

Slope of a line passing through P(2,3) and intersecting the line x + y = 7 at a distance of 4 units from P, is

Solution: Given

$$\mathbf{P} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \tag{1}$$

1

Equation of a line through \mathbf{P} and having slope m is

$$\mathbf{r} = \mathbf{p} + t\mathbf{b} \tag{2}$$

$$\mathbf{b} = \begin{pmatrix} 1 \\ m \end{pmatrix} \tag{3}$$

$$x + y = 7 \implies \begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{x} = 7 \tag{4}$$

$$(1 \quad 1)(\mathbf{p} + t\mathbf{b}) = 7 \tag{5}$$

$$\begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{p} + t \begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{b} = 7 \tag{6}$$

$$t\begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{b} = 7 - \begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{p} \tag{7}$$

$$t = \frac{7 - \begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{p}}{\begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{b}} \tag{8}$$

Q be the point of intersection

$$\mathbf{q} = \mathbf{p} + t\mathbf{b} \tag{9}$$

$$\mathbf{q} - \mathbf{p} = t\mathbf{b} \tag{10}$$

$$\|\mathbf{q} - \mathbf{p}\| = |t| \|\mathbf{b}\| \implies |t| = \frac{\|\mathbf{q} - \mathbf{p}\|}{\|\mathbf{b}\|}$$

$$\tag{11}$$

$$\left| \frac{7 - \begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{p}}{\begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{b}} \right| = \frac{\|\mathbf{q} - \mathbf{p}\|}{\|\mathbf{b}\|}$$
 (12)

Given the point is at a distance of 4 units from point P

$$\left| \frac{7 - \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \end{pmatrix}}{\begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ m \end{pmatrix}} \right| = \frac{4}{\sqrt{1 + m^2}}$$
 (13)

$$\left| \frac{7-5}{1+m} \right| = \frac{4}{\sqrt{1+m^2}} \tag{14}$$

$$\left(\frac{7-5}{1+m}\right)^2 = \frac{16}{1+m^2} \implies \frac{4}{(1+m)^2} = \frac{16}{1+m^2}$$
 (15)

$$4(1+m)^2 = 1 + m^2 (16)$$

$$4(m^2 + 2m + 1) = 1 + m^2 (17)$$

$$4m^2 + 8m + 4 = 1 + m^2 \implies 3m^2 + 8m + 3 = 0 \tag{18}$$

$$m^2 + \frac{8m}{3} + 1 = 0 ag{19}$$

$$m^2 + \frac{8m}{3} + 1 + \left(\frac{4}{3}\right)^2 = \frac{16}{9} \tag{20}$$

$$\left(m + \frac{4}{3}\right)^2 = \frac{16 - 9}{9} = \frac{7}{9} \tag{21}$$

$$m + \frac{4}{3} = \pm \frac{\sqrt{7}}{3} \tag{22}$$

$$m = \frac{-4 - \sqrt{7}}{3} \text{ (or) } \frac{-4 + \sqrt{7}}{3}$$
 (23)

According to options

$$m = \frac{-4 + \sqrt{7}}{3} = \frac{8 - 2\sqrt{7}}{-6} = \frac{\left(1 - \sqrt{7}\right)^2}{\left(1 + \sqrt{7}\right)\left(1 - \sqrt{7}\right)} = \frac{1 - \sqrt{7}}{1 + \sqrt{7}}$$
(24)

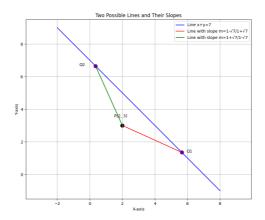


Fig. 0.1