2.2.24

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Question (2.2.24)

If $\hat{i}+\hat{j}+\hat{k}$,$2\hat{i}+5\hat{j}$,$3\hat{i}+2\hat{j}-3\hat{k}$ and $\hat{i}-6\hat{j}-\hat{k}$ are the position vectors of the point **A**,**B**,**C** and **D** respectively, then find the angle between **AB** and **CD**. Deduce that **AB** and **CD** are collinear.

Solution

Given points are

$$\mathbf{A} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 2 \\ 5 \\ 0 \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} 3 \\ 2 \\ -3 \end{pmatrix}, \quad \mathbf{D} = \begin{pmatrix} 1 \\ -6 \\ -1 \end{pmatrix}. \tag{1}$$

$$\mathbf{AB} = \mathbf{B} - \mathbf{A} = \begin{pmatrix} 1 \\ 4 \\ -1 \end{pmatrix}, \qquad \mathbf{CD} = \mathbf{D} - \mathbf{C} = \begin{pmatrix} -2 \\ -8 \\ 2 \end{pmatrix}. \tag{2}$$

Solution

The angle θ between **AB** and **CD** is given by

$$\cos\theta = \frac{\mathbf{AB}^{\top}\mathbf{CD}}{\|AB\|\,\|CD\|}. \tag{3}$$

$$\mathbf{AB}^{T}\mathbf{CD} = \begin{pmatrix} 1 & 4 & -1 \end{pmatrix} \begin{pmatrix} -2 \\ -8 \\ 2 \end{pmatrix} = (1)(-2) + (4)(-8) + (-1)(2) = -36. \tag{4}$$

$$\|AB\| = 3\sqrt{2}, \qquad \|CD\| = 6\sqrt{2}. \tag{5}$$

Solution

$$\cos\theta = \frac{-36}{(3\sqrt{2})(6\sqrt{2})} = \frac{-36}{36} = -1. \tag{6}$$

Hence,

$$\theta = \cos^{-1}(-1) = \pi \ \ (180^\circ). \tag{7}$$

Since, angle between vectors is 180° the given vectors are collinear

Solution

Proof of collinearity by rank method

Let,

$$\mathbf{P} = \begin{pmatrix} B - A & D - C \end{pmatrix} \tag{8}$$

$$\mathbf{P} = \begin{pmatrix} 1 & -2 \\ 4 & -8 \\ -1 & 2 \end{pmatrix} \tag{9}$$

$$\mathbf{P}^T = \begin{pmatrix} 1 & 4 & -1 \\ -2 & -8 & 2 \end{pmatrix} \tag{10}$$

$$R_2 \to R_2 - 2R_1 \tag{11}$$

$$\mathbf{P}^T = \begin{pmatrix} 1 & 4 & -1 \\ 0 & 0 & 0 \end{pmatrix} \tag{12}$$

$$rank\mathbf{P} = rank\mathbf{P}^T = 1 \tag{13}$$

$$\tag{14}$$

Thus the given vectors are collinear.

Plot

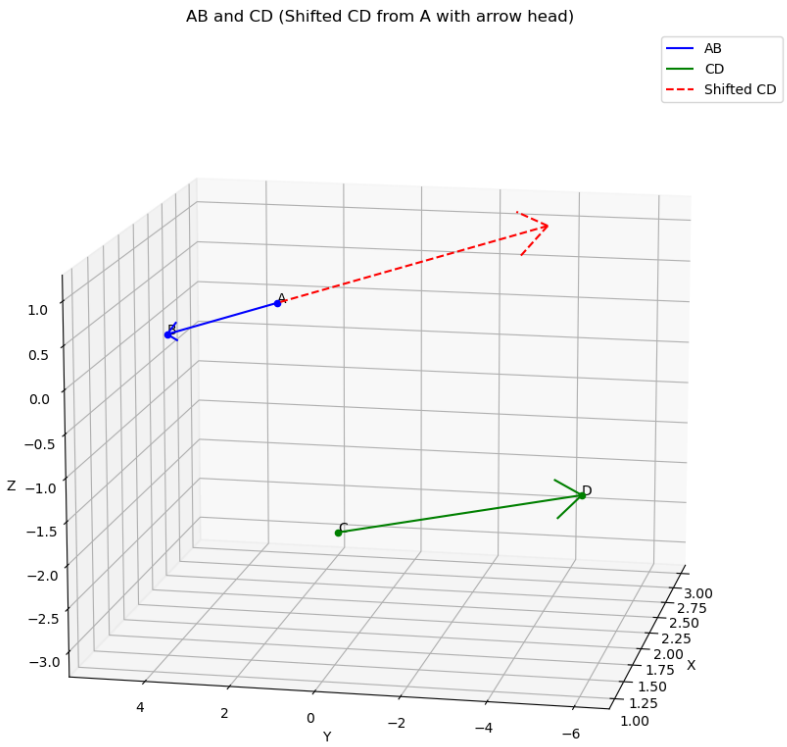
AB and CD (Shifted CD from A with arrow head)
AB
CD
Shifted CD
A
B
C
D
1.0
0.5
0.0
−0.5
Z −1.0
−1.5
−2.0
−2.5
−3.0
4
2
0
−2
−4
−6
Y
3.00
2.75
2.50
2.25
2.00
1.75
1.50
1.25
1.00
X

Figure:

C Code

```c
#include <stdio.h>
#include <math.h>

// Function to compute dot product of two vectors
double dot_product(double v1[3], double v2[3]) {
    return v1[0]*v2[0] + v1[1]*v2[1] + v1[2]*v2[2];
}

// Function to compute norm of a vector
double norm(double v[3]) {
    return sqrt(v[0]*v[0] + v[1]*v[1] + v[2]*v[2]);
}
```

C Code

```
int main() {
    double A[3], B[3], C[3], D[3];
    double AB[3], CD[3];
    int i;
    // Input points
    printf("Enter coordinates of A (x y z): ");
    scanf("%lf %lf %lf", &A[0], &A[1], &A[2]);

    printf("Enter coordinates of B (x y z): ");
    scanf("%lf %lf %lf", &B[0], &B[1], &B[2]);

    printf("Enter coordinates of C (x y z): ");
    scanf("%lf %lf %lf", &C[0], &C[1], &C[2]);

    printf("Enter coordinates of D (x y z): ");
    scanf("%lf %lf %lf", &D[0], &D[1], &D[2]);
```

C Code

```
// Compute vectors AB and CD
for (i = 0; i < 3; i++) {
    AB[i] = B[i] - A[i];
    CD[i] = D[i] - C[i];
}

// Print vectors
printf("\nVector AB = (%.2lf, %.2lf, %.2lf)\n", AB[0],
printf("Vector CD = (%.2lf, %.2lf, %.2lf)\n", CD[0], C

// Compute angle
double dot = dot_product(AB, CD);
double cos_theta = dot / (norm(AB) * norm(CD));
int x = (cos_theta * 100);
double y = x/100;
double theta = acos(y)*180/M_PI;
```

C Code

```
    printf("\nDot product = %.2lf\n", dot);
    printf("cos(theta) = %.2lf\n", cos_theta);
    printf("Angle between AB and CD = %.2lf degrees\n", th

    if (cos_theta == 1 || -1){
        printf("\n AB and CD are collinear.\n");
    } else {
        printf("\n AB and CD are not collinear.\n");
    }

    return 0;
}
```

Python Code

```
import numpy as np

# Function to read a point from user
def read_point(name):
    coords = input(f"Enter coordinates of {name} (x y z):
    return np.array(list(map(float, coords)))

# Input points
A = read_point("A")
B = read_point("B")
C = read_point("C")
D = read_point("D")

# Vectors
AB = B - A
CD = D - C
```

Python Code

```
# Step 1: Angle between AB and CD
dot_product = np.dot(AB, CD)
norms = np.linalg.norm(AB) * np.linalg.norm(CD)
cos_theta = dot_product / norms
x = cos_theta*100
y = int(x)/100
theta_deg = np.degrees(np.arccos(y))

# Step 2: Rank method for collinearity
M = np.column_stack((AB, CD))   # Matrix with AB and CD as
rank = np.linalg.matrix_rank(M)
```

Python Code

```
# Print results
print("\n--- Results ---")
print("Vector AB:", AB)
print("Vector CD:", CD)
print("Dot product =", dot_product)
print("cos(theta) =", y)
print("Angle between AB and CD =", theta_deg, "degrees")
print("Matrix M:\n", M)
print("Rank of M =", rank)

if rank == 1:
    print("Vectors AB and CD are collinear.")
else:
    print("Vectors AB and CD are not collinear.")
```