MatGeo Assignment 2.6.13

AI25BTECH11007

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Question

Given that vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}$ form a triangle such that

$$\mathbf{a} = \mathbf{b} + \mathbf{c},$$

find p, q, r, s given that

$$\mathbf{a} = p\hat{i} + q\hat{j} + r\hat{k}, \qquad \mathbf{b} = s\hat{i} + 3\hat{j} + 4\hat{k}, \qquad \mathbf{c} = 3\hat{i} + 1\hat{j} - 2\hat{k},$$

and the area of the triangle is $5\sqrt{6}$.

Solution

We are given:

$$\mathbf{a} = \mathbf{b} + \mathbf{c} \tag{1}$$

$$\mathbf{a} = p\hat{i} + q\hat{j} + r\hat{k}, \quad \mathbf{b} = s\hat{i} + 3\hat{j} + 4\hat{k}, \quad \mathbf{c} = 3\hat{i} + 1\hat{j} - 2\hat{k} \tag{2}$$

and the area of the triangle formed by these vectors is:

$$\text{Area} = 5\sqrt{6} \tag{3}$$

Observation:

For three vectors to form a triangle, they must sum to zero:

$$\mathbf{a} + \mathbf{b} + \mathbf{c} = \mathbf{0} \tag{4}$$

However, we are told:

$$\mathbf{a} = \mathbf{b} + \mathbf{c} \Rightarrow \mathbf{a} - \mathbf{b} - \mathbf{c} = \mathbf{0} \tag{5}$$

This implies:

$$\mathbf{a} + (-\mathbf{b}) + (-\mathbf{c}) = \mathbf{0} \tag{6}$$

So, the triangle is formed by the vectors $\mathbf{a}, -\mathbf{b}, -\mathbf{c}$. For these to form a triangle, they must not lie along the same line (i.e., must not be collinear).

Now, if we assume:

$$\mathbf{a} = \mathbf{0} \Rightarrow \mathbf{b} + \mathbf{c} = \mathbf{0} \Rightarrow \mathbf{b} = -\mathbf{c} \tag{7}$$

Given:

$$\mathbf{c} = \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix} \Rightarrow \mathbf{b} = -\mathbf{c} = -\begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix} = \begin{pmatrix} -3 \\ -1 \\ 2 \end{pmatrix} \tag{8}$$

Then:

$$\mathbf{a} = \mathbf{b} + \mathbf{c} = \mathbf{0} \Rightarrow p = 0, \quad q = 0, \quad r = 0 \tag{9}$$

We now compute the area of the triangle using:

$$\text{Area} = \frac{1}{2} \|\mathbf{b} \times \mathbf{c}\| \tag{10}$$

Compute the cross product:

$$\mathbf{b} \times \mathbf{c} = \mathbf{0} \Rightarrow \text{Area} = 0 \tag{11}$$

If we assume $\mathbf{a} = \mathbf{0}$, then $\mathbf{b} = -\mathbf{c}$, and the triangle is degenerate (i.e., the vectors lie on a straight line). Therefore, the area is zero:

$$\boxed{\text{Area} = 0} \tag{12}$$

Coclusion

This contradicts the given area of $5\sqrt{6}$. Therefore, no solution exists such that:

$$\mathbf{a} = \mathbf{b} + \mathbf{c} \quad \text{and} \quad \text{Area} = 5\sqrt{6} \tag{13}$$

Plot

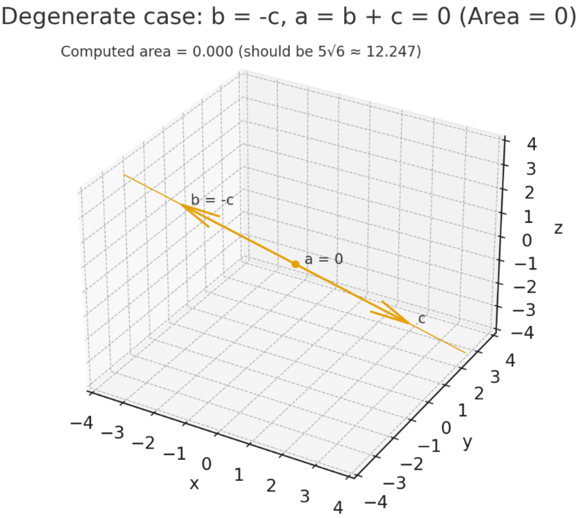

Figure: Image Visual