2.3.9

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Question

If vectors $\mathbf{a}$ and $\mathbf{b}$ are such that

$$|\mathbf{a}| = \frac{1}{2}, \quad |\mathbf{b}| = \frac{4}{\sqrt{3}}, \quad |\mathbf{a} \times \mathbf{b}| = \frac{1}{\sqrt{3}},$$

then find $\mathbf{a} \cdot \mathbf{b}$.

Formula

We know that

$$|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}| \sin\theta \tag{1}$$

$$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos\theta \tag{2}$$

where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$.

Solution

Substitute values:

$$\frac{1}{\sqrt{3}} = \left(\frac{1}{2}\right)\left(\frac{4}{\sqrt{3}}\right)\sin\theta \tag{3}$$

$$\sin\theta = \frac{1}{2} \implies \theta = 30^\circ \text{ or } 150^\circ \tag{4}$$

Now,

$$\mathbf{a}\cdot\mathbf{b} = \left(\frac{1}{2}\right)\left(\frac{4}{\sqrt{3}}\right)\cos\theta = \frac{2}{\sqrt{3}}\cos\theta \tag{5}$$

Final Result

For $\theta = 30°$:

$$\mathbf{a} \cdot \mathbf{b} = 1$$

For $\theta = 150°$:

$$\mathbf{a} \cdot \mathbf{b} = -1$$

Therefore, $\mathbf{a} \cdot \mathbf{b} = \pm 1$.

Vector Plot

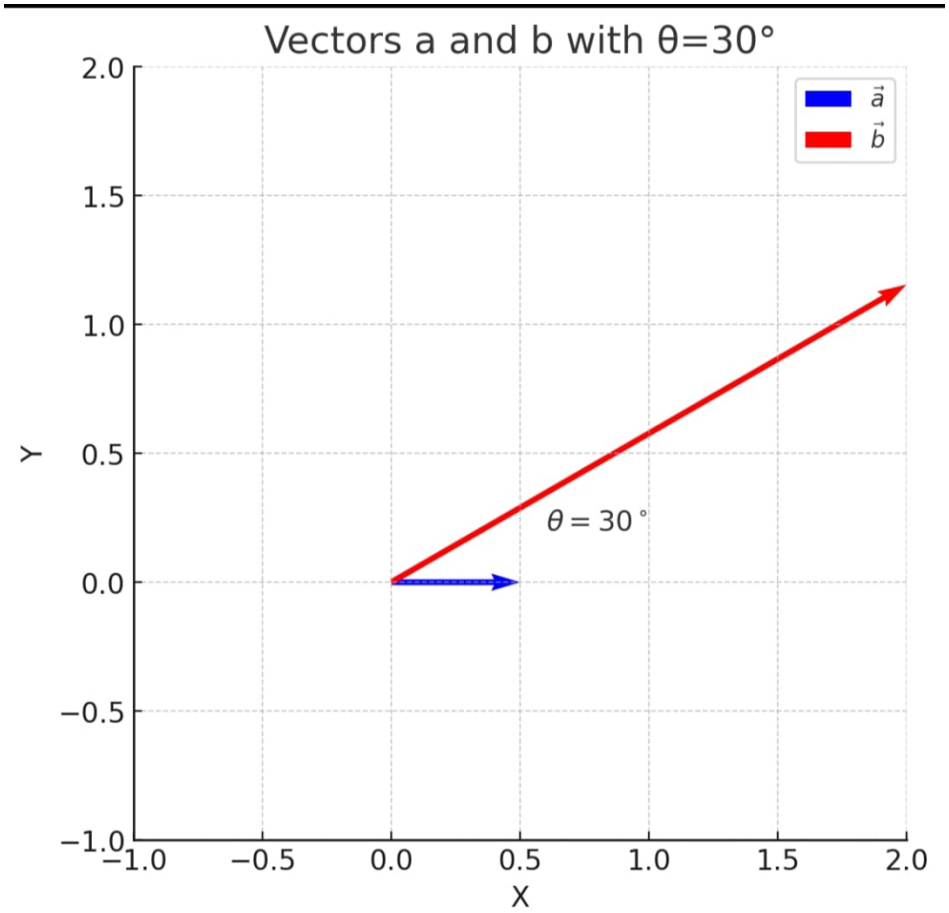

Figure: Vectors **a** and **b** with θ

C Code : Function Definition

```
#include<stdio.h>
#include<math.h>

float dotfinder(float a1, float a2, float a3,
                    float b1, float b2, float b3){

    float dot_product;
    float mod1;
    float mod2;
    float cosval;
    float angle;
    float result;
```

C Code : Function Logic

```
    dot_product = a1*b1 + a2*b2 + a3*b3;

    mod1 = sqrt(pow(a1,2) + pow(a2,2) + pow(a3,2));
    mod2 = sqrt(pow(b1,2) + pow(b2,2) + pow(b3,2));

    cosval = dot_product/(mod1 * mod2);

    angle = acos(cosval); // angle between vectors

    result = dot_product; // return ab value

    return result;
}
```

Python Code : Setup

```
import numpy as np
import math

# Given magnitudes
a_mag = 1/2
b_mag = 4/np.sqrt(3)
cross_mag = 1/np.sqrt(3)

# sin = |ab|/(|a||b|)
sin_theta = cross_mag/(a_mag*b_mag)
theta1 = math.degrees(math.asin(sin_theta))
theta2 = 180 - theta1
```

Python Code : Output

```
print("Possible angles:", theta1, "or", theta2)

# ab = |a||b|cos
dot1 = a_mag*b_mag*math.cos(math.radians(theta1))
dot2 = a_mag*b_mag*math.cos(math.radians(theta2))

print("Possible values of ab:", dot1, "or", dot2)
```