2.2.24

AI25BTECH110031
Shivam Sawarkar

Question(2.2.24) If $\hat{\imath}+\hat{\jmath}+\hat{k}$,$2\hat{\imath}+5\hat{\jmath}$,$3\hat{\imath}+2\hat{\jmath}-3\hat{k}$ and $\hat{\imath}-6\hat{\jmath}-\hat{k}$ are the position vectors of the point **A**,**B**,**C** and **D** respectively, then find the angle between **AB** and **CD**. Deduce that **AB** and **CD** are collinear.

Solution:

Given points are

$$\mathbf{A}=\begin{pmatrix}1\\1\\1\end{pmatrix},\quad \mathbf{B}=\begin{pmatrix}2\\5\\0\end{pmatrix},\quad \mathbf{C}=\begin{pmatrix}3\\2\\-3\end{pmatrix},\quad \mathbf{D}=\begin{pmatrix}1\\-6\\-1\end{pmatrix}. \tag{0.1}$$

$$\mathbf{AB}=\mathbf{B}-\mathbf{A}=\begin{pmatrix}1\\4\\-1\end{pmatrix},\qquad \mathbf{CD}=\mathbf{D}-\mathbf{C}=\begin{pmatrix}-2\\-8\\2\end{pmatrix}. \tag{0.2}$$

The angle θ between **AB** and **CD** is given by

$$\cos\theta=\frac{\mathbf{AB}^\top\mathbf{CD}}{\|AB\|\,\|CD\|}. \tag{0.3}$$

$$\mathbf{AB}^T\mathbf{CD}=\begin{pmatrix}1 & 4 & -1\end{pmatrix}\begin{pmatrix}-2\\-8\\2\end{pmatrix}=(1)(-2)+(4)(-8)+(-1)(2)=-36. \tag{0.4}$$

$$\|AB\|=3\sqrt{2},\qquad \|CD\|=6\sqrt{2}. \tag{0.5}$$

$$\cos\theta=\frac{-36}{(3\sqrt{2})(6\sqrt{2})}=\frac{-36}{36}=-1. \tag{0.6}$$

Hence,

$$\theta=\cos^{-1}(-1)=\pi\ \ (180^\circ). \tag{0.7}$$

Since, angle between vectors is 180° the given vectors are collinear

Proof of collinearity by rank method
Let,

$$\mathbf{P} = \begin{pmatrix} B - A & D - C \end{pmatrix} \tag{0.8}$$

$$\mathbf{P} = \begin{pmatrix} 1 & -2 \\ 4 & -8 \\ -1 & 2 \end{pmatrix} \tag{0.9}$$

$$\mathbf{P}^T = \begin{pmatrix} 1 & 4 & -1 \\ -2 & -8 & 2 \end{pmatrix} \tag{0.10}$$

$$R_2 \rightarrow R_2 - 2R_1 \tag{0.11}$$

$$\mathbf{P}^T = \begin{pmatrix} 1 & 4 & -1 \\ 0 & 0 & 0 \end{pmatrix} \tag{0.12}$$

$$rank\mathbf{P} = rank\mathbf{P}^T = 1 \tag{0.13}$$

$$\tag{0.14}$$

Thus the given vectors are collinear.

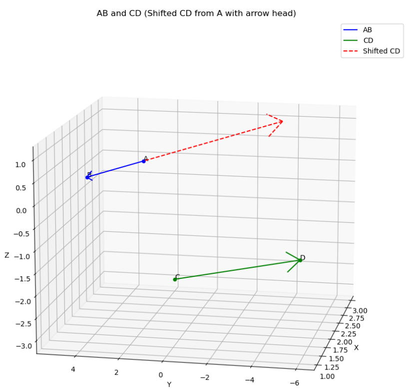

Fig. 0.1

NOTE: Parallel vectors are collinear.