

3.2.3

EE25BTECH11062 - Vivek K Kumar

Question:

Draw a parallelogram ABCD in which $BC = 5\text{cm}$, $AB = 3\text{cm}$ and $\angle ABC = 60^\circ$, divide it into triangles ACB and ABD by the diagonal BD

Solution:

Let **A**, **B**, **C** and **D** represent position vectors of the vertices of parallelogram.

Given information,

$$\|\mathbf{A} - \mathbf{B}\| = 3 \quad (0.1)$$

$$\|\mathbf{C} - \mathbf{B}\| = 5 \quad (0.2)$$

$$\angle B = \frac{\pi}{3} \quad (0.3)$$

The coordinates of **A**, **B**, **C** can be expressed as

$$\mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (0.4)$$

$$\mathbf{C} = \|\mathbf{C} - \mathbf{B}\| \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (0.5)$$

$$\mathbf{A} = \|\mathbf{A} - \mathbf{B}\| \begin{pmatrix} \cos B \\ \sin B \end{pmatrix} \quad (0.6)$$

Since **A**, **B**, **C**, **D** form vertices of a parallelogram,

$$\frac{\mathbf{A} + \mathbf{C}}{2} = \frac{\mathbf{B} + \mathbf{D}}{2} \quad (0.7)$$

$$\mathbf{D} = \mathbf{A} + \mathbf{C} - \mathbf{B} \quad (0.8)$$

$$\mathbf{D} = \|\mathbf{A} - \mathbf{B}\| \begin{pmatrix} \cos B \\ \sin B \end{pmatrix} + \|\mathbf{C} - \mathbf{B}\| \begin{pmatrix} 1 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (0.9)$$

Substituting values,

$$\mathbf{A} = \begin{pmatrix} 3/2 \\ 3\sqrt{3}/2 \end{pmatrix} \quad (0.10)$$

$$\mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (0.11)$$

$$\mathbf{C} = \begin{pmatrix} 5 \\ 0 \end{pmatrix} \quad (0.12)$$

$$\mathbf{D} = \begin{pmatrix} 13/2 \\ 3\sqrt{3}/2 \end{pmatrix} \quad (0.13)$$

Name	Point
A	$\begin{pmatrix} 3/2 \\ 3\sqrt{3}/2 \end{pmatrix}$
B	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$
C	$\begin{pmatrix} 5 \\ 0 \end{pmatrix}$
D	$\begin{pmatrix} 13/2 \\ 3\sqrt{3}/2 \end{pmatrix}$

TABLE 0: Coordinates of the vertices of parallelogram

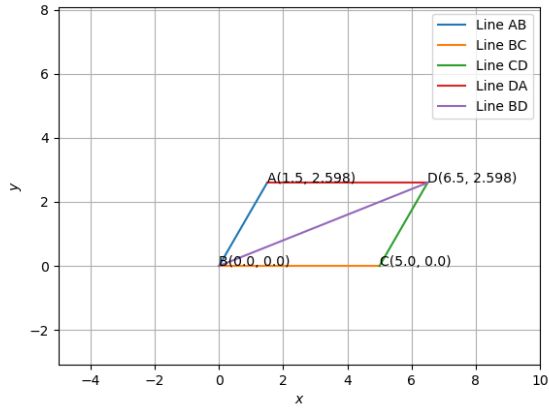


Fig. 0.1: Given parallelogram split across BD