2.8.14

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Three vectors **a**, **b** and **c** satisfy the condition $\mathbf{a} + \mathbf{b} + \mathbf{c} = 0$. Evaluate the quantity $\mu = \mathbf{a} \cdot \mathbf{b} + \mathbf{b} \cdot \mathbf{c} + \mathbf{c} \cdot \mathbf{a}$. If $|\mathbf{a}| = 3$, $|\mathbf{b}| = 4$ and $|\mathbf{c}| = 2$.

Solution

Given:

$$\mathbf{a} + \mathbf{b} + \mathbf{c} = 0 \text{ and } \|\mathbf{a}\| = 3 \text{ , } \|\mathbf{b}\| = 4 \text{ , } \|\mathbf{c}\| = 2 \tag{1}$$

To find

$$\mu = \mathbf{a}^\top \mathbf{b} + \mathbf{b}^\top \mathbf{c} + \mathbf{c}^\top \mathbf{a} \tag{2}$$

To find the value of μ

$$\|\mathbf{a} + \mathbf{b} + \mathbf{c}\|^2 = 0 \qquad (3)$$

$$(\mathbf{a} + \mathbf{b} + \mathbf{c})^\top (\mathbf{a} + \mathbf{b} + \mathbf{c}) = 0 \qquad (4)$$

$$\mathbf{a}^\top \mathbf{a} + \mathbf{b}^\top \mathbf{b} + \mathbf{c}^\top \mathbf{c} + 2\left(\mathbf{a}^\top \mathbf{b} + \mathbf{b}^\top \mathbf{c} + \mathbf{c}^\top \mathbf{a}\right) = 0 \qquad (5)$$

Solution

By using $\mathbf{x}^\top \mathbf{x} = \|\mathbf{x}\|^2$ we get

$$\left(\|\mathbf{a}\|^2 + \|\mathbf{b}\|^2 + \|\mathbf{c}\|^2 \right) + 2\mu = 0 \tag{6}$$

Substituting the values of $\|\mathbf{a}\|, \|\mathbf{b}\|, \|\mathbf{c}\|$ we get

$$\mu = \frac{-29}{2} \tag{7}$$

$\therefore$ The value of μ is $\frac{-29}{2}$.