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Matrix 2.10.4

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Question

Find the area of the triangle whose vertices are

$$A(1,-1,2), \quad B(2,0,-1), \quad C(3,-1,2).$$

Solution

Step 1: Form vectors

$$\mathbf{B}-\mathbf{A} = \begin{pmatrix} 2-1 \\ 0-(-1) \\ -1-2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ -3 \end{pmatrix}, \tag{1}$$

$$\mathbf{C}-\mathbf{A} = \begin{pmatrix} 3-1 \\ -1-(-1) \\ 2-2 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}. \tag{2}$$

Step 2: Cross product formula

The cross product of two vectors is defined as

$$\mathbf{X}\times\mathbf{Y} = \begin{pmatrix} \begin{vmatrix} \mathbf{X}_{23} & \mathbf{Y}_{23} \\ \mathbf{X}_{31} & \mathbf{Y}_{31} \\ \mathbf{X}_{12} & \mathbf{Y}_{12} \end{vmatrix} \end{pmatrix}. \tag{3}$$

Applying to $\mathbf{B}-\mathbf{A}$ and $\mathbf{C}-\mathbf{A}$:

$$(\mathbf{B}-\mathbf{A})\times(\mathbf{C}-\mathbf{A}) = \begin{pmatrix} \begin{vmatrix} \begin{pmatrix} 1 \\ -3 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \end{pmatrix} \\ \begin{pmatrix} -3 \\ 1 \end{pmatrix} & \begin{pmatrix} 0 \\ 2 \end{pmatrix} \end{vmatrix} \\ \begin{vmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} & \begin{pmatrix} 2 \\ 0 \end{pmatrix} \end{vmatrix} \end{pmatrix}. \tag{4}$$

Step 3: Simplify

$$
\begin{aligned}
(\mathbf{B}-\mathbf{A}) \times (\mathbf{C}-\mathbf{A}) &= \begin{pmatrix} (1)(0)-(-3)(0) \\ (-3)(2)-(1)(0) \\ (1)(0)-(1)(2) \end{pmatrix} && (1) \\
&= \begin{pmatrix} 0 \\ -6 \\ -2 \end{pmatrix}. && (5)
\end{aligned}
$$

Step 4: Area of triangle

$$
\begin{aligned}
\text{Area of } \triangle ABC &= \tfrac{1}{2}\,\|(\mathbf{B}-\mathbf{A}) \times (\mathbf{C}-\mathbf{A})\| && (2) \\
&= \tfrac{1}{2}\sqrt{0^2+(-6)^2+(-2)^2} && (3) \\
&= \tfrac{1}{2}\sqrt{40} && (4) \\
&= \sqrt{10}. && (6)
\end{aligned}
$$

Final Answer

$$\boxed{\text{Area of } \triangle ABC = \sqrt{10}}$$

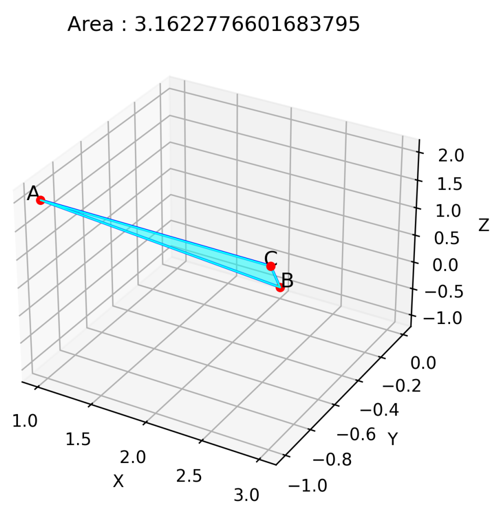

Figure 1: Triangle formed by points A, B, and C.