4.3.36

Puni Aditya - EE25BTECH11046

12th September, 2025

Question

The line $\mathbf{r} = \left(2\hat{i} - 3\hat{j} - \hat{k}\right) + \lambda\left(\hat{i} - \hat{j} + 2\hat{k}\right)$ lies in the plane $\mathbf{r} \cdot \left(3\hat{i} + \hat{j} - \hat{k}\right) + 2 = 0.$

Theoretical Solution

Let the line L be $\mathbf{x} = \mathbf{a} + \lambda\mathbf{b}$ and the plane P be $\mathbf{n}^\top\mathbf{x} = c$ where

$$\mathbf{a} = \begin{pmatrix} 2 \\ -3 \\ -1 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}, \mathbf{n} = \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix}, \text{c} = \text{-2}$$

$$\mathbf{n}^\top\mathbf{b} = \begin{pmatrix} 3 & 1 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} \tag{1}$$

$$= (1)(3) + (-1)(1) + (2)(-1) \tag{2}$$

$$= 3 - 1 - 2 \tag{3}$$

$$\mathbf{n}^\top\mathbf{b} = 0 \tag{4}$$

$\because$ $\mathbf{n}^\top\mathbf{b} = 0$, the line L is parallel to plane P.

Theoretical Solution

$$\mathbf{n}^\top \mathbf{a} = \begin{pmatrix} 3 & 1 & -1 \end{pmatrix} \begin{pmatrix} 2 \\ -3 \\ -1 \end{pmatrix} \tag{5}$$

$$= (2)(3) + (-3)(1) + (-1)(-1) \tag{6}$$

$$= 6 - 3 + 1 \tag{7}$$

$$= 4 \neq \text{ c} \tag{8}$$

$\because \mathbf{n}^\top \mathbf{a} \neq c$, the point $\mathbf{a}$ doesn't line in the plane P. Hence, the line L containing $\mathbf{a}$ also doesn't lie in the plane.
The given statement is **false**.

Plot

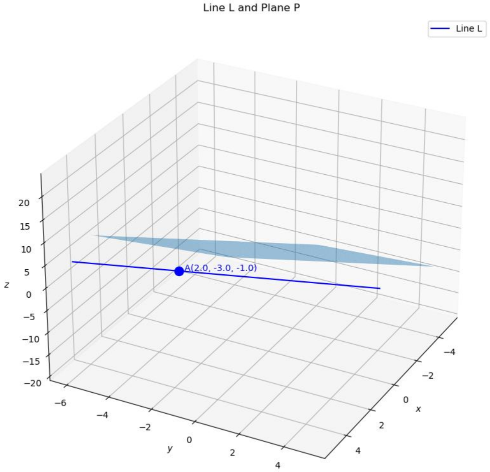

Figure: Plot