

Matgeo Presentation - Problem 5.2.58

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Problem Statement

Solve the system of equations

$$x - y + z = 4$$

$$2x + y - 3z = 0$$

$$x + y + z = 2$$

Name	Equation
Equation 1	$x - y + z = 4 \iff \begin{pmatrix} 1 & -1 & 1 \end{pmatrix} \mathbf{x}_1 = 4$
Equation 2	$2x + y - 3z = 0 \iff \begin{pmatrix} 2 & 1 & -3 \end{pmatrix} \mathbf{x}_2 = 0$
Equation 3	$x + y + z = 2 \iff \begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \mathbf{x}_3 = 2$

Table : Equations

Solution

The system of equations in matrix form is :

$$\begin{pmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ 2 \end{pmatrix} \quad (0.1)$$

Forming the augmented matrix,

$$\left(\begin{array}{ccc|c} 1 & -1 & 1 & 4 \\ 2 & 1 & -3 & 0 \\ 1 & 1 & 1 & 2 \end{array} \right) \quad (0.2)$$

Using Gaussian elimination,

$$\left(\begin{array}{ccc|c} 1 & -1 & 1 & 4 \\ 2 & 1 & -3 & 0 \\ 1 & 1 & 1 & 2 \end{array} \right) \xleftrightarrow[R_2 \rightarrow R_2 - 2R_1]{R_3 \rightarrow R_3 - R_1} \left(\begin{array}{ccc|c} 1 & -1 & 1 & 4 \\ 0 & 3 & -5 & -8 \\ 0 & 2 & 0 & -2 \end{array} \right) \quad (0.3)$$

Solution

$$\left(\begin{array}{ccc|c} 1 & -1 & 1 & 4 \\ 0 & 3 & -5 & -8 \\ 0 & 2 & 0 & -2 \end{array} \right) \xleftrightarrow{R_3 \rightarrow R_3 - \frac{2}{3}R_2} \left(\begin{array}{ccc|c} 1 & -1 & 1 & 4 \\ 0 & 3 & -5 & -8 \\ 0 & 0 & \frac{10}{3} & \frac{10}{3} \end{array} \right) \quad (0.4)$$

Using back substitution we get :

$$\frac{10}{3}z = \frac{10}{3} \quad (0.5)$$

$$z = 1 \quad (0.6)$$

$$3y - 5z = -8 \quad (0.7)$$

$$3y - 5 = -8 \quad (0.8)$$

$$3y = -3 \quad (0.9)$$

$$y = -1 \quad (0.10)$$

$$x - y + z = 4 \quad (0.11)$$

$$x + 2 = 4 \quad (0.12)$$

$$x = 2 \quad (0.13)$$

Solution

Therefore the solution for the system of equations is :

$$\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} \quad (0.14)$$

Intersection of Three Planes and Solution Point P

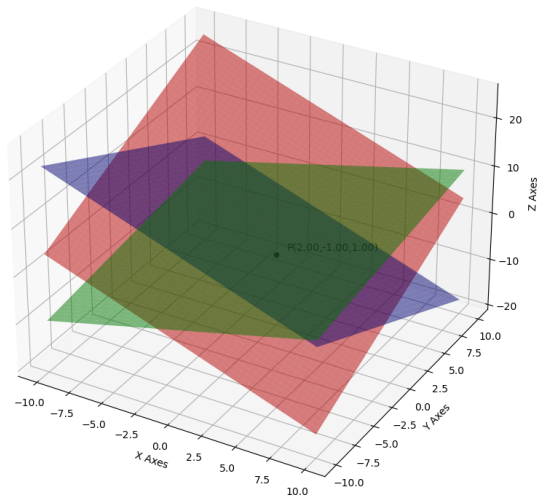


Fig : Planes