Matgeo : 2.6.12

AI25BTECH11006 - Nikhila

Problem: Find the sine of the angle between the vectors $\vec{a} = 3\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} + -2\hat{j} + 4\hat{k}$

Solution:
We know that

$$\cos\theta = \frac{a \top b}{\|a\|\|b\|}.$$

Calculating each term:

$$a \top b = 3(2) + 1(-2) + 2(4) = 6 - 2 + 8 = 12, \tag{1}$$

$$\|\vec{a}\| = \sqrt{3^2 + 1^2 + 2^2} = \sqrt{14}, \tag{2}$$

$$\|\vec{b}\| = \sqrt{2^2 + (-2)^2 + 4^2} = \sqrt{24}, \tag{3}$$

$$\cos\theta = \frac{12}{\sqrt{14} \cdot \sqrt{24}} = \frac{12}{\sqrt{336}} = \frac{3}{\sqrt{21}}. \tag{4}$$

Now, to find $\sin\theta$:

$$\sin\theta = \sqrt{1 - \cos^2\theta} \tag{5}$$

$$= \sqrt{1 - \left(\frac{3}{\sqrt{21}}\right)^2} \tag{6}$$

$$= \sqrt{1 - \frac{9}{21}} \tag{7}$$

$$= \sqrt{\frac{12}{21}} \tag{8}$$

$$= \frac{2}{\sqrt{7}}. \tag{9}$$

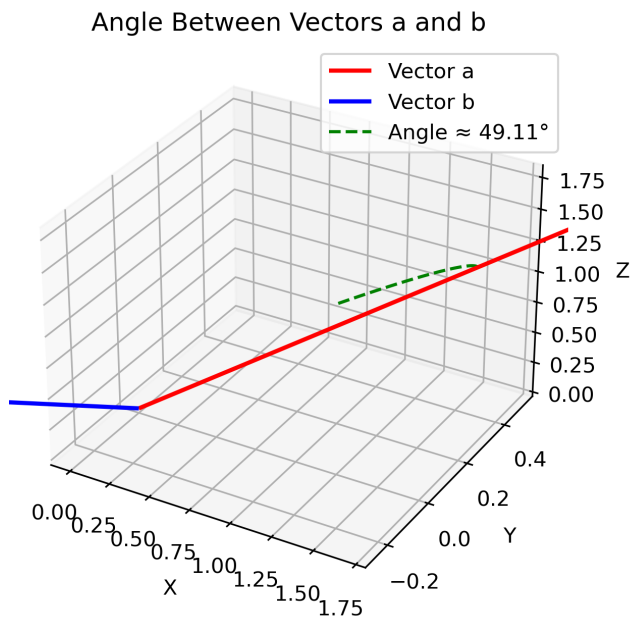The sine of the angle between the vectors $\vec{a}$ and $\vec{b}$ is $\frac{2}{\sqrt{7}}$.

Fig. 1.