8.2.12

EE25BTECH11047 - RAVULA SHASHANK REDDY

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Question:

Find the parameters of the conic

$$36x^2 + 4y^2 = 144.$$

Solution:

$$g(\mathbf{x}) = \mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{1}$$

$$\mathbf{V} = \begin{pmatrix} 36 & 0 \\ 0 & 4 \end{pmatrix}, \quad \mathbf{u} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad f = -144 \tag{2}$$

$$\lambda_1 = 36, \quad \lambda_2 = 4 \tag{3}$$

$$\tag{4}$$

Since $\lambda_1 > \lambda_2$, apply affine transformation

$$\mathbf{P} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \mathbf{x} = \mathbf{P}\mathbf{y} \tag{5}$$

Hence,

$$\lambda_1 = 4, \quad \lambda_2 = 36 \tag{6}$$

$$\mathbf{e}_1 = \mathbf{p}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad \mathbf{e}_2 = \mathbf{p}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{7}$$

$$f_0 = \mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f = 144 \tag{8}$$

$$e = \sqrt{1 - \frac{\lambda_1}{\lambda_2}} = \sqrt{1 - \frac{4}{36}} = \frac{2\sqrt{2}}{3} \tag{9}$$

$$\text{Major axis} = 2\sqrt{\frac{f_0}{\lambda_1}} = 12 \tag{10}$$

$$\text{Minor axis} = 2\sqrt{\frac{f_0}{\lambda_2}} = 4 \tag{11}$$

Normal vector of directrix:

$$\mathbf{n} = \sqrt{\lambda_2}\,\mathbf{p}_2 = \sqrt{36}\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}0\\6\end{pmatrix} \tag{12}$$

$$\mathbf{p}_1 = \begin{pmatrix}1\\0\end{pmatrix}, \quad \mathbf{p}_2 = \begin{pmatrix}0\\1\end{pmatrix} \tag{13}$$

$$c = \frac{e\,\mathbf{u}^\top\mathbf{n} \pm \sqrt{e^2\left(\mathbf{u}^\top\mathbf{n}\right)^2 - \lambda_2\left(e^2-1\right)\left(\|\mathbf{u}\|^2 - \lambda_2 f\right)}}{\lambda_2 e\left(e^2-1\right)} \tag{14}$$

$$c = \pm\frac{1}{e}\sqrt{\frac{\lambda_2 f_0}{\lambda_1}} = \pm\frac{1}{e}\sqrt{\frac{36\cdot 144}{4}} = \pm 27\sqrt{2} \tag{15}$$

$$\tag{16}$$

Foci:

$$\mathbf{F} = \frac{ce^2\mathbf{n} - \mathbf{u}}{\lambda_2} \tag{17}$$

$$= \pm 4\sqrt{2}\mathbf{e}_2 \tag{18}$$

Directrices:

$$\mathbf{n}^\top\mathbf{x} = c \tag{19}$$

$$\mathbf{n}^\top\mathbf{x} = \pm 27\sqrt{2} \tag{20}$$

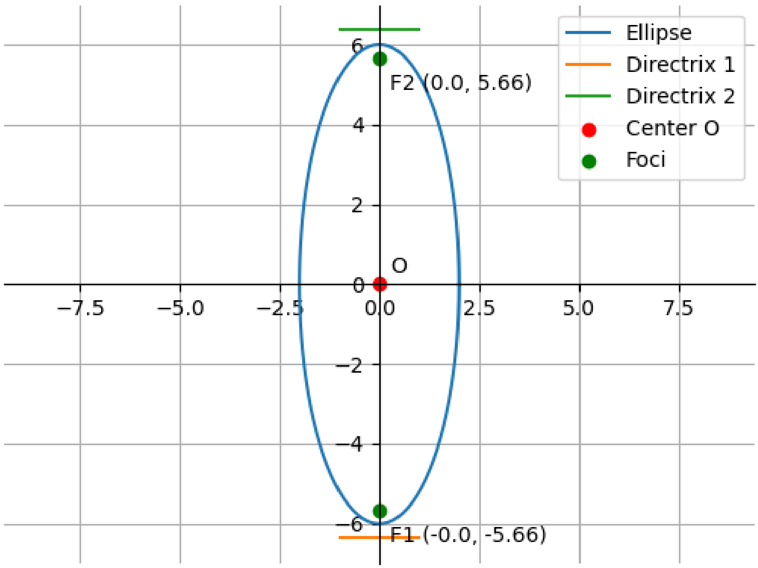
$$6\mathbf{e_2}^\top\mathbf{x} = \pm 27\sqrt{2} \tag{21}$$

$$\mathbf{e_2}^\top\mathbf{x} = \pm\frac{9\sqrt{2}}{2} \tag{22}$$

Latus rectum:

$$l = \frac{2\sqrt{|f_0\lambda_1|}}{\lambda_2} = \frac{4}{3} \tag{23}$$

Parameter	Value
$\mathbf{V},\ \mathbf{u},\ f$	$\begin{pmatrix}36 & 0\\0 & 4\end{pmatrix},\ \mathbf{0},\ -144$
λ_1, λ_2	$4,\ 36$
f_0	144
e	$\frac{2\sqrt{2}}{3}$
Major axis length	12
Minor axis length	4
Foci	$\mathbf{F} = \pm 4\sqrt{2}\,\mathbf{e}_2$
Directrices	$\mathbf{e}_2^\top\mathbf{x} = \pm\frac{9\sqrt{2}}{2}$
Latus rectum	$\frac{4}{3}$



Ellipse
Directrix 1
Directrix 2
Center O
Foci
F2 (0.0, 5.66)
F1 (-0.0, -5.66)
O
6
4
2
0
−2
−4
−6
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5

Figure 1