Problem 2.7.30

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Problem Statement

Find the value of k so that the area of $\triangle ABC$ with $\mathbf{A}\,(k+1, 1)$, $\mathbf{B}\,(4, -3)$ and $\mathbf{C}\,(7, -k)$ is 6 square units

Formula

$$\text{Area}(\triangle ABC) = \frac{1}{2}\|(\mathbf{B} - \mathbf{A}) \times (\mathbf{C} - \mathbf{A})\| \tag{1.1}$$

Solution

Here according to problem , the Area of $\triangle ABC$ is 6 square units
Therefore,

$$\frac{1}{2}\|(\mathbf{B}-\mathbf{A})\times(\mathbf{C}-\mathbf{A})\| = 6 \tag{1.2}$$

$$\frac{1}{2}\left\|\begin{pmatrix}4\\-3\end{pmatrix}-\begin{pmatrix}k+1\\1\end{pmatrix}\times\begin{pmatrix}7\\-k\end{pmatrix}-\begin{pmatrix}k+1\\1\end{pmatrix}\right\| = 6 \tag{1.3}$$

$$\frac{1}{2}\left\|\begin{pmatrix}3-k\\-4\end{pmatrix}\times\begin{pmatrix}6-k\\-k-1\end{pmatrix}\right\| = 6 \tag{1.4}$$

$$\left|(3-k)(-k-1)-(-4)(6-k)\right| = 12 \tag{1.5}$$

$$\left|k^2-6k+21\right| = 12 \tag{1.6}$$

Solution

Case 1 :

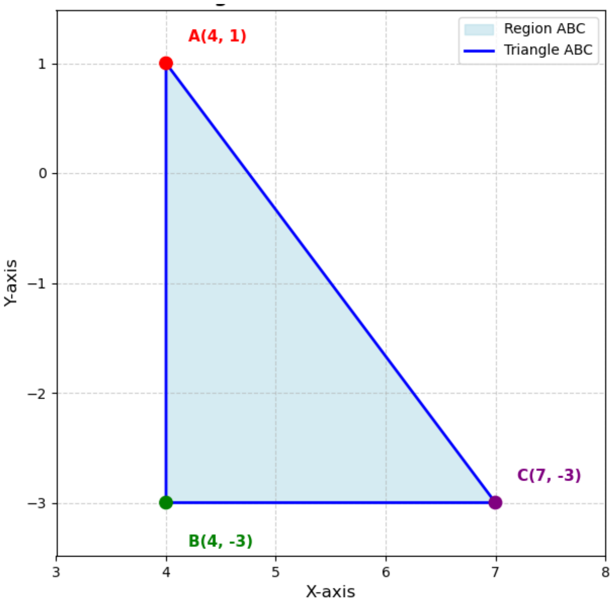
$$k^2 - 6k + 24 = 12 \tag{1.7}$$
$$k^2 - 6k + 9 = 0 \tag{1.8}$$
$$(k-3)^2 = 0 \tag{1.9}$$
$$k = 3 \tag{1.10}$$

Case 2 :

$$k^2 - 6k + 24 = -12 \tag{1.11}$$
$$k^2 - 6k + 33 = 0 \tag{1.12}$$
$$\tag{1.13}$$

Discriminant: $D = -96$.
Since the discriminant is negative , there is no real solution for this case.
Thus , k=3

Plot



Region ABC
Triangle ABC
A(4, 1)
B(4, -3)
C(7, -3)
1
0
−1
−2
−3
Y-axis
3
4
5
6
7
8
X-axis

C Code

```
// Function to calculate the value of k
double solve_for_k() {
    double k;
    // We will directly solve for k
    k = 3.0;
    return k;
}
// Function to calculate the area of the triangle for
    verification
double calculate_area(double k) {
    // Points:
    double Ax = k + 1, Ay = 1;
    double Bx = 4, By = -3;
    double Cx = 7, Cy = -k;
    // Area using determinant formula
    double area = fabs(Ax*(By-Cy) + Bx*(Cy-Ay) + Cx*(Ay-By)) /
        2.0;
    return area;
}
```

Calling C Function

```python
import ctypes

# Load the shared library
lib = ctypes.CDLL("./triangle.so")

# Define return types
lib.solve_for_k.restype = ctypes.c_double
lib.calculate_area.restype = ctypes.c_double
lib.calculate_area.argtypes = [ctypes.c_double]

# Call the function to get k
k_value = lib.solve_for_k()
print(f"Calculated value of k: {k_value}")

# Verify by calculating the area
area = lib.calculate_area(k_value)
print(f"Area of the triangle with k={k_value}: {area}")
```

Python Code for Plotting

```
import numpy as np
import matplotlib.pyplot as plt

# Given value of k
k = 3

# Coordinates of the points
A = np.array([k + 1, 1])
B = np.array([4, -3])
C = np.array([7, -k])

# Arrays for plotting
x_coords = [A[0], B[0], C[0], A[0]] # Loop back to A
y_coords = [A[1], B[1], C[1], A[1]]

# Create figure
plt.figure(figsize=(7, 7))
```

Python Code for Plotting

```python
# Shade the triangle region
plt.fill([A[0], B[0], C[0]], [A[1], B[1], C[1]], color='lightblue', alpha=0.5, label='Region ABC')

# Plot the triangle boundary
plt.plot(x_coords, y_coords, 'b-', linewidth=2, label='Triangle ABC')

# Plot points clearly
plt.scatter([A[0], B[0], C[0]], [A[1], B[1], C[1]], color=['red', 'green', 'purple'], s=80, zorder=5)

# Annotate points slightly offset for clarity
plt.text(A[0] + 0.2, A[1] + 0.2, f"A({A[0]}, {A[1]})", fontsize=11, color='red', weight='bold')
plt.text(B[0] + 0.2, B[1] - 0.4, f"B({B[0]}, {B[1]})", fontsize=11, color='green', weight='bold')
plt.text(C[0] + 0.2, C[1] + 0.2, f"C({C[0]}, {C[1]})", fontsize=11, color='purple', weight='bold')
```

Python Code for Plotting

```
# Add grid and styling
plt.title("Triangle ABC with Shaded Area", fontsize=14, weight='
    bold')
plt.xlabel("X-axis", fontsize=12)
plt.ylabel("Y-axis", fontsize=12)
plt.grid(True, linestyle='--', alpha=0.6)
plt.legend()

# Set equal aspect ratio to avoid distortion
plt.axis('equal')

# Adjust limits so points are not too close to the axes
x_min, x_max = min(x_coords) - 1, max(x_coords) + 1
y_min, y_max = min(y_coords) - 1, max(y_coords) + 1
plt.xlim(x_min, x_max)
plt.ylim(y_min, y_max)

# Show the plot
plt.show()
```