

4.3.21

EE25BTECH11032 - Kartik Lahoti

Question:

In what ratio does the X -axis divide the line segment joining the points $(-4, -6)$ and $(-1, 7)$? Find the coordinates of the point of division.

Solution:

Given :

Symbol	Value	Description
A	$\begin{pmatrix} -4 \\ -6 \end{pmatrix}$	Given Point
B	$\begin{pmatrix} -1 \\ 7 \end{pmatrix}$	Given Point
P	$\begin{pmatrix} x \\ 0 \end{pmatrix}$	Desired Point

Let , **P** be the point on x -axis dividing the line **AB** in the ratio say k : 1.

Now, **A** , **B** and **P** are collinear ,

$$\therefore \text{rank}(\mathbf{B} - \mathbf{A} \quad \mathbf{P} - \mathbf{B}) = 1 \quad (0.1)$$

$$\begin{pmatrix} 3 & x+1 \\ 13 & -7 \end{pmatrix} \xrightarrow{R_2 \rightarrow \frac{-13}{3}R_1 + R_2} \begin{pmatrix} 3 & x+1 \\ 0 & -\frac{13x}{3} - \frac{34}{3} \end{pmatrix} \quad (0.2)$$

The number of nonzero rows in the echelon form is defined as the rank. For the above matrix to be of rank 1 ,

$$-\frac{13x}{3} - \frac{34}{3} = 0 \quad (0.3)$$

$$x = -\frac{34}{13} \quad (0.4)$$

\therefore The coordinates of the point of intersection is

$$\mathbf{P} = \begin{pmatrix} -\frac{34}{13} \\ 0 \end{pmatrix} \quad (0.5)$$

Now,

$$k = \frac{(\mathbf{A} - \mathbf{P})^\top (\mathbf{P} - \mathbf{B})}{\|\mathbf{P} - \mathbf{B}\|^2} \quad (0.6)$$

Substituting the values of **A** , **B** and **P** , we get,

$$k = \frac{\left(-\frac{18}{13} \quad -6\right) \begin{pmatrix} -\frac{21}{13} \\ -7 \end{pmatrix}}{\left\| \begin{pmatrix} -\frac{21}{13} \\ -7 \end{pmatrix} \right\|^2} \quad (0.7)$$

$$k = \frac{6}{7} \quad (0.8)$$

Thus, the ratio in which the point **P** divides the line segment **AB** is 6: 7

