4.13.18

EE25BTECH11013 - Bhargav

Question:
The line L is given by $\frac{x}{5}+\frac{y}{b}=1$ passes through the point $(13,32)$. The line K is parallel to the line L and has the equation $\frac{x}{c}+\frac{y}{3}=1$. Then the distance between L and K is
Solution:

The equation of line:

$$\mathbf{n}^{\mathbf{T}}\mathbf{x}=1 \tag{0.1}$$

Line L:

$$\begin{pmatrix}\frac{1}{5} & \frac{1}{b}\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}=1 \tag{0.2}$$

Line L passes through $\begin{pmatrix}13\\32\end{pmatrix}$:

$$\begin{pmatrix}\frac{1}{5} & \frac{1}{b}\end{pmatrix}\begin{pmatrix}13\\32\end{pmatrix}=1 \tag{0.3}$$

$$\frac{32}{b}=1-\frac{13}{5} \implies b=-20 \tag{0.4}$$

Line K:

$$\begin{pmatrix}\frac{1}{c} & \frac{1}{3}\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}=1 \tag{0.5}$$

Since lines L and K are parallel:

$$\mathbf{n}_K=\lambda\mathbf{n}_L \tag{0.6}$$

$$\begin{pmatrix}\frac{1}{c}\\\frac{1}{3}\end{pmatrix}=\lambda\begin{pmatrix}\frac{1}{5}\\-\frac{1}{20}\end{pmatrix} \tag{0.7}$$

Thus,

$$\lambda=-\frac{20}{3} \quad c=-\frac{3}{4} \tag{0.8}$$

$$\mathbf{n}=\begin{pmatrix}4\\-1\end{pmatrix} \tag{0.9}$$

The distance between parallel lines:

$$\text{Distance}=\frac{|c_1-c_2|}{||\mathbf{n}||} \tag{0.10}$$

$$\text{Distance} = \frac{|20 - (-3)|}{\sqrt{4^2 + (-1)^2}} \tag{0.11}$$

$$\therefore \text{Distance} = \frac{23}{\sqrt{17}} \tag{0.12}$$