2.10.75

EE25BTECH11047 - RAVULA SHASHANK REDDY

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Question:

The points with position vectors $\mathbf{a} + \mathbf{b}$, $\mathbf{a} - \mathbf{b}$ and $\mathbf{a} + k\mathbf{b}$ are collinear for all real values of k.Prove

Solution:

Given:

$$\mathbf{P} = \begin{pmatrix} \mathbf{a} & \mathbf{b} \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \mathbf{Q} = \begin{pmatrix} \mathbf{a} & \mathbf{b} \end{pmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \mathbf{R} = \begin{pmatrix} \mathbf{a} & \mathbf{b} \end{pmatrix} \begin{pmatrix} 1 \\ k \end{pmatrix} \tag{1}$$

$$\mathbf{P} - \mathbf{Q} = \begin{pmatrix} \mathbf{a} & \mathbf{b} \end{pmatrix} \begin{pmatrix} 0 \\ 2 \end{pmatrix} \tag{2}$$

$$\mathbf{R} - \mathbf{P} = \begin{pmatrix} \mathbf{a} & \mathbf{b} \end{pmatrix} \begin{pmatrix} 0 \\ k-1 \end{pmatrix} \tag{3}$$

$$M = \begin{pmatrix} \mathbf{P} - \mathbf{Q} & \mathbf{R} - \mathbf{P} \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 2\mathbf{b} & (k-1)\mathbf{b} \end{pmatrix} \tag{4}$$

$$M = \mathbf{b} \begin{pmatrix} 0 & 0 \\ 2 & k-1 \end{pmatrix} \tag{5}$$

$$\text{rank}(M) \leq 1. \tag{6}$$

Therefore, the two difference vectors are linearly dependent.
Hence, the points $\mathbf{P}, \mathbf{Q}, \mathbf{R}$ are collinear for all real k.

For Example:

Take

$$\mathbf{a} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \ \mathbf{b} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}. \tag{7}$$

For $k = 0$:

$$\mathbf{R} = \begin{pmatrix} 1 + 3(0) \\ 2 + 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}. \tag{8}$$

For $k = 1$:

$$\mathbf{R} = \begin{pmatrix} 1 + 3(1) \\ 2 + 1 \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}. \tag{9}$$

For $k = 2$:

$$\mathbf{R} = \begin{pmatrix} 1 + 3(2) \\ 2 + 2 \end{pmatrix} = \begin{pmatrix} 7 \\ 4 \end{pmatrix}. \tag{10}$$

So the three points are:

$$\mathbf{Q} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}, \mathbf{P} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}, \tag{11}$$

$$\mathbf{R} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \ (k = 0), \ \begin{pmatrix} 4 \\ 3 \end{pmatrix} \ (k = 1), \ \begin{pmatrix} 7 \\ 4 \end{pmatrix} \ (k = 2). \tag{12}$$

$$M(k) = \begin{pmatrix} \mathbf{P} - \mathbf{Q} & \mathbf{R} - \mathbf{P} \end{pmatrix} = \begin{pmatrix} 6 & 3k - 3 \\ 2 & k - 1 \end{pmatrix}. \tag{13}$$

For $k = 0$:

$$M(0) = \begin{pmatrix} 6 & -3 \\ 2 & -1 \end{pmatrix}, \quad \text{rank}(M(0)) = 1. \tag{14}$$

For $k = 1$:

$$M(1) = \begin{pmatrix} 6 & 0 \\ 2 & 0 \end{pmatrix}, \quad \text{rank}(M(1)) = 1. \tag{15}$$

For $k = 2$:

$$M(2) = \begin{pmatrix} 6 & 3 \\ 2 & 1 \end{pmatrix}, \quad \text{rank}(M(2)) = 1. \tag{16}$$

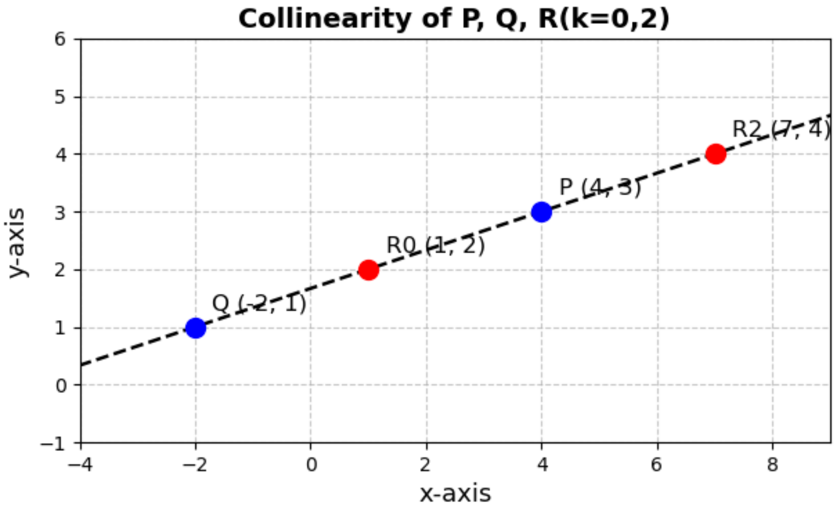

Figure 1: Caption