2.10.69

EE25BTECH11041 - Naman Kumar

Question:
Determine the value of c so that for all real x, the vector $cx\hat{\imath} - 6\hat{\jmath} - 3\hat{k}$ and $x\hat{\imath} + 2\hat{\jmath} + 2cx\hat{k}$ make an obtuse angle with each other.

Solution:
We know, Inner Product of two vectors

$$\mathbf{A}^T\mathbf{B} = \|\mathbf{A}\|\|\mathbf{B}\|\cos\theta \tag{1}$$

for obtuse angle between any two vectors

$$\cos\theta < 0 \text{ or } \mathbf{A}^T\mathbf{B} < 0 \tag{2}$$

Given Vectors

$$\mathbf{A} = \begin{pmatrix} cx \\ -6 \\ -3 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} x \\ 2 \\ 2cx \end{pmatrix} \tag{3}$$

Now using these condition on given vectors

$$\mathbf{A}^T\mathbf{B} < 0 \tag{4}$$

$$\begin{pmatrix} cx & -6 & -3 \end{pmatrix}\begin{pmatrix} x \\ 2 \\ 2cx \end{pmatrix} < 0 \tag{5}$$

$$cx^2 - 12 - 6cx < 0 \text{ (quadratic in x)} \tag{6}$$

for any quadratic to be negative $\forall x \in R$, their are two conditions

$$a < 0\& D < 0 \tag{7}$$

$$\tag{8}$$

Now applying this conditions on (6) firstly on a (leading coefficient)

$$c < 0 \tag{9}$$

on D (discriminant)

$$D = b^2 - 4ac < 0 \tag{10}$$

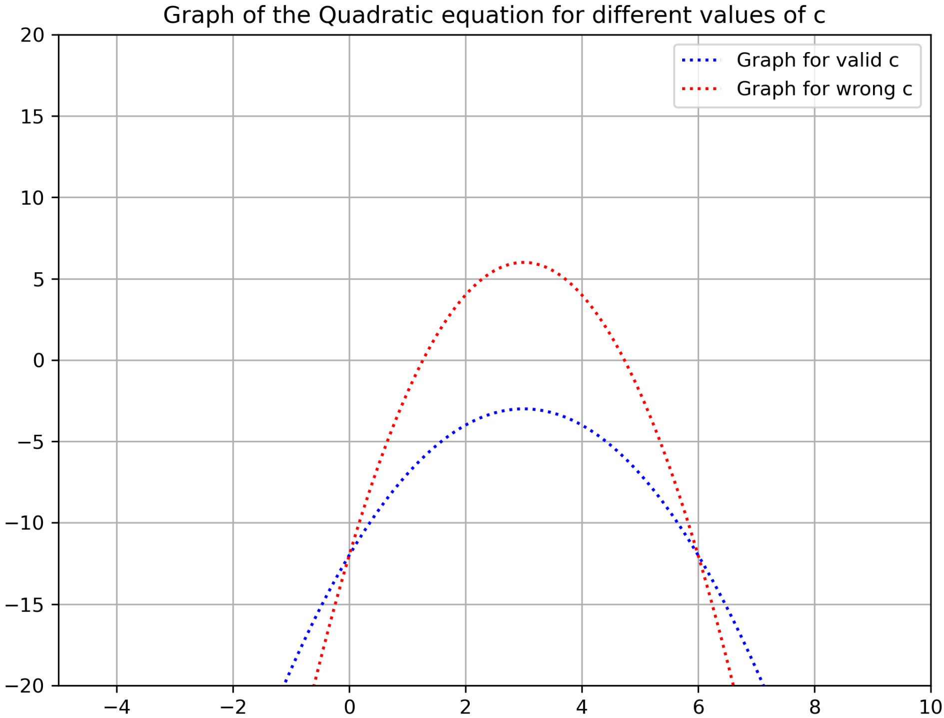
$$(-6c)^2 - 4 \times c \times (-12) < 0 \tag{11}$$

$$36c^2 + 48c < 0 \tag{12}$$

$$c(3c + 4) < 0 \tag{13}$$

therfore, by taking union of (9) and (13)

$$c \in (\frac{-4}{3}, 0) \tag{14}$$



Graph of the Quadratic equation for different values of c
Graph for valid c
Graph for wrong c
20
15
10
5
0
−5
−10
−15
−20
−4
−2
0
2
4
6
8
10