

2.6.12

AI25BTECH11006 - Nikhila

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Question

Find the sine of the angle between the vectors $\mathbf{a} = 3\hat{i} + \hat{j} + 2\hat{k}$ and $\mathbf{b} = 2\hat{i} + -2\hat{j} + 4\hat{k}$.

Solution

The given vectors are $\vec{a} \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}$ and $\vec{b} \begin{pmatrix} 2 \\ -2 \\ 4 \end{pmatrix}$

We know that

$$\cos \theta = \frac{\vec{a}^T \vec{b}}{\|\vec{a}\| \|\vec{b}\|} \quad (1)$$

and

$$\sin \theta = \sqrt{1 - \cos^2 \theta} \quad (2)$$

$$\vec{a}^T \vec{b} = 3(2) + 1(-2) + 2(4) = 6 - 2 + 8 = 12 \quad (3)$$

$$\|\vec{a}\| = \sqrt{(3)^2 + (1)^2 + (2)^2} = \sqrt{14}, \quad (4)$$

$$\|\vec{b}\| = \sqrt{(2)^2 + (-2)^2 + (4)^2} = \sqrt{24}. \quad (5)$$

$$\cos \theta = \frac{12}{\sqrt{14} \cdot \sqrt{24}} \quad (6)$$

$$= \frac{12}{\sqrt{336}} \quad (7)$$

$$= \frac{\sqrt{3}}{\sqrt{7}} \quad (8)$$

$$\sin \theta = \sqrt{1 - \left(\frac{\sqrt{3}}{\sqrt{7}}\right)^2} \quad (9)$$

$$= \sqrt{1 - \frac{3}{7}} \quad (10)$$

$$= \frac{2}{\sqrt{7}} \quad (11)$$

Graphical Representation

Angle Between Vectors a and b

