2.9.16

EE25BTECH11065 - Yoshita

Question:
Prove that three points A, B, and C with position vectors $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ respectively are collinear if and only if $(\mathbf{b} \times \mathbf{c}) + (\mathbf{c} \times \mathbf{a}) + (\mathbf{a} \times \mathbf{b}) = \mathbf{0}$.

Solution:
The three points A, B, and C are collinear if and only if the vectors **AB** and **AC** are parallel. The position vectors for these are:

Point	Vector
A	$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$
B	$\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$
C	$\begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}$

TABLE 0: Answers

$$\mathbf{A} - \mathbf{B} = \mathbf{b} - \mathbf{a}$$
$$\mathbf{A} - \mathbf{C} = \mathbf{c} - \mathbf{a}$$

If two vectors are collinear,

$$(\mathbf{b} - \mathbf{a}) \times (\mathbf{c} - \mathbf{a}) = \mathbf{0} \tag{1}$$

Using the determinant (matrix) form of the cross product,

$$(\mathbf{b} - \mathbf{a}) \times (\mathbf{c} - \mathbf{a}) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \left(b_1 - a_1\right) & \left(b_2 - a_2\right) & \left(b_3 - a_3\right) \\ \left(c_1 - a_1\right) & \left(c_2 - a_2\right) & \left(c_3 - a_3\right) \end{vmatrix}$$

Rearranging the equation we get,

$$(\mathbf{a} \times \mathbf{b}) + (\mathbf{b} \times \mathbf{c}) + (\mathbf{c} \times \mathbf{a}) = \mathbf{0} \tag{2}$$

Hence we proved that that three points A, B, and C with position vectors **a**, **b**, and **c** respectively are collinear if and only if $(\mathbf{b} \times \mathbf{c}) + (\mathbf{c} \times \mathbf{a}) + (\mathbf{a} \times \mathbf{b}) = \mathbf{0}$

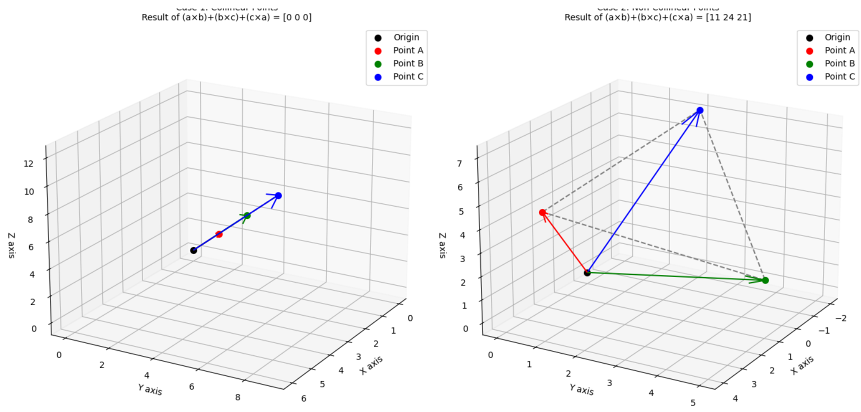

Fig. 0