4.3.56

EE25BTECH11065 - Yoshita

Question:
Find the equation of the plane with intercepts 2, 3 and 4 on the x, y and z - axis respectively.

Solution:
The intercepts define three points on the plane, which we can label A, B, and C.

Point	Vector
A	$\begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}$
B	$\begin{pmatrix} 0 \\ 3 \\ 0 \end{pmatrix}$
C	$\begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix}$

TABLE 0: Answers

We can find two direction vectors, $\mathbf{m_1}$ and $\mathbf{m_2}$, that lie in the plane:

$$\mathbf{m_1} = \mathbf{B} - \mathbf{A} = \begin{pmatrix} 0-2 \\ 3-0 \\ 0-0 \end{pmatrix} = \begin{pmatrix} -2 \\ 3 \\ 0 \end{pmatrix}$$

$$\mathbf{m_2} = \mathbf{C} - \mathbf{A} = \begin{pmatrix} 0-2 \\ 0-0 \\ 4-0 \end{pmatrix} = \begin{pmatrix} -2 \\ 0 \\ 4 \end{pmatrix}$$

The normal vector to the plane, $\mathbf{n}$, is found by the cross product of these two vectors.

$$\mathbf{n} = \mathbf{m_1} \times \mathbf{m_2} = \begin{pmatrix} -2 \\ 3 \\ 0 \end{pmatrix} \times \begin{pmatrix} -2 \\ 0 \\ 4 \end{pmatrix}$$

$$= \begin{pmatrix} (3)(4) - (0)(0) \\ (0)(-2) - (-2)(4) \\ (-2)(0) - (3)(-2) \end{pmatrix} = \begin{pmatrix} 12 \\ 8 \\ 6 \end{pmatrix}$$

We can simplify the normal vector to $\mathbf{n} = \begin{pmatrix} 6 \\ 4 \\ 3 \end{pmatrix}$. The equation of the plane is then given by:

$$\mathbf{n}^T(\mathbf{x} - \mathbf{A}) = 0$$

Substituting the numerical values, where $\mathbf{x} = [x, y, z]^T$:

$$\begin{pmatrix} 6 & 4 & 3 \end{pmatrix} \left(\begin{pmatrix} x \\ y \\ z \end{pmatrix} - \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} \right) = 0$$

$$\implies \begin{pmatrix} 6 & 4 & 3 \end{pmatrix} \begin{pmatrix} x - 2 \\ y \\ z \end{pmatrix} = 0$$

$$\implies 6(x - 2) + 4y + 3z = 0$$

$$\implies 6x - 12 + 4y + 3z = 0$$

$$\implies 6x + 4y + 3z = 12$$

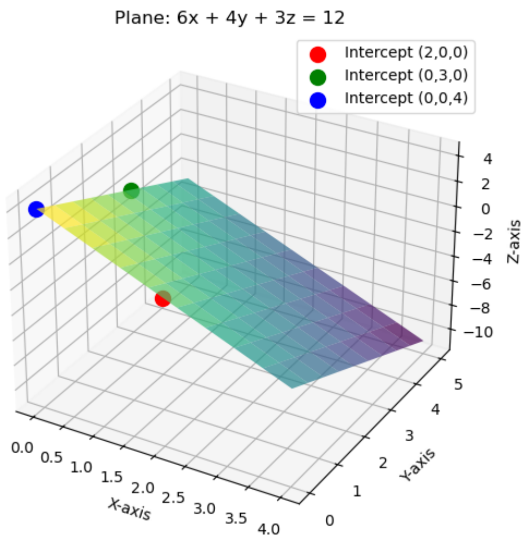

Fig. 0: A plane intersecting the x, y, and z axes.