

2.6.23

AI25BTECH11014 - Gooty Suhas

PROBLEM

Find the area of a triangle whose vertices are given in 3D space as:

$$\mathbf{A} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$$

We will compute the area using the formula:

$$\text{Area} = \frac{1}{2} \|(\mathbf{B} - \mathbf{A}) \times (\mathbf{C} - \mathbf{A})\|$$

SOLUTION

Step 1: Compute Vectors

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$$

$$\mathbf{C} - \mathbf{A} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$$

Step 2: Cross Product

$$(\mathbf{B} - \mathbf{A}) \times (\mathbf{C} - \mathbf{A}) = \begin{pmatrix} (1)(0) - (2)(2) \\ (2)(1) - (0)(0) \\ (0)(2) - (1)(1) \end{pmatrix} = \begin{pmatrix} -4 \\ 2 \\ -1 \end{pmatrix}$$

Step 3: Magnitude

$$\|(\mathbf{B} - \mathbf{A}) \times (\mathbf{C} - \mathbf{A})\| = \sqrt{(-4)^2 + 2^2 + (-1)^2} = \sqrt{16 + 4 + 1} = \sqrt{21}$$

Step 4: Area

$$\text{Area} = \frac{1}{2} \sqrt{21}$$

$$\text{Area} = \frac{\sqrt{21}}{2}$$

Triangle ABC in 3D

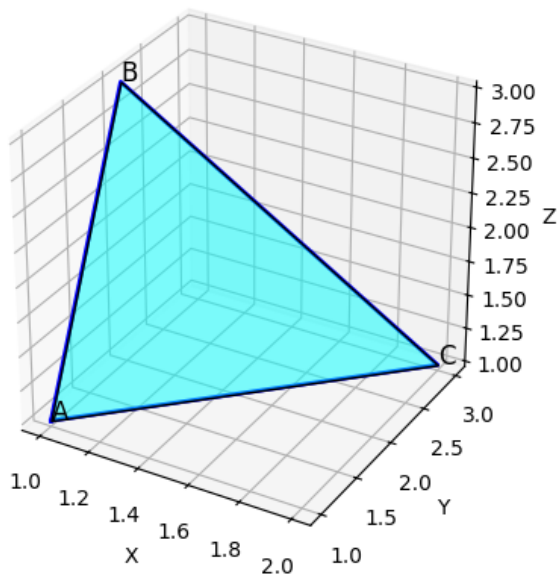


Fig. 0.1: The line