2.4.37

EE25BTECH11041 - Naman Kumar

Question:
Find a vector of magnitude 6, which is perpendicular to both the vectors $2\hat{\imath} - \hat{\jmath} + 2\hat{k}$ and $4\hat{\imath} - \hat{\jmath} + 3\hat{k}$
Solution:
Given Vectors

$$\mathbf{A} = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix} \tag{1}$$

Let required vector be,

$$\mathbf{C} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \tag{2}$$

Using Inner Product,

$$\mathbf{C}^T \cdot \mathbf{A} = 0 \text{ and } \mathbf{C}^T \cdot \mathbf{B} = 0 \tag{3}$$

$$\mathbf{C}^T \cdot \mathbf{A} = 2x - y + 2z = 0 \tag{4}$$

$$\mathbf{C}^T \cdot \mathbf{B} = 4x - y + 3z = 0 \tag{5}$$

Using Rank to Analyze the system

$$2x - y + 2z, 4x - y + 3z \tag{6}$$

$$\mathbf{A} = \begin{pmatrix} 2 & -1 & 2 \\ 4 & -1 & 3 \end{pmatrix} \tag{7}$$

Using Row Transformations to Get Row Reduced echelon Form

$$\mathbf{A} = \begin{pmatrix} 2 & -1 & 2 \\ 4 & -1 & 3 \end{pmatrix} \xrightarrow{R_2 \to R_2 - 2R_1} \begin{pmatrix} 2 & -1 & 2 \\ 0 & 1 & -1 \end{pmatrix} \tag{8}$$

$$\xrightarrow{R_1 \to \frac{1}{2}R_1} \begin{pmatrix} 1 & \frac{-1}{2} & 1 \\ 0 & 1 & -1 \end{pmatrix} \tag{9}$$

$$\xrightarrow{R_1 \to R_1 + \frac{1}{2}R_2} \begin{pmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & -1 \end{pmatrix} \tag{10}$$

$$\mathbf{A} = (\mathbf{IX}), \mathbf{I} \text{ is identity matrix} \tag{11}$$

And, $\mathbf{X}$ is

$$\begin{pmatrix} \frac{1}{2} \\ -1 \end{pmatrix} \tag{12}$$

Since rank of matrix is $2 (\leq 3)$, their are infinite many solutions $R^3 \to R^2$

From the Row Reduced Echelon form(RREF),we can write the new system of equation:

$$x + \frac{1}{2}z = 0 \tag{13}$$

$$y - z = 0 \tag{14}$$

Therefore vector **C** using equations (13) and (14) is

$$\mathbf{C} = \begin{pmatrix} x \\ -2x \\ -2x \end{pmatrix} = x \begin{pmatrix} 1 \\ -2 \\ -2 \end{pmatrix} \tag{15}$$

Now getting vector with magnitude 6

$$||C|| = 6 \tag{16}$$

$$||x|| \sqrt{(1)^2 + (-2)^2 + (-2)^2} = 6 \tag{17}$$

$$||x|| \sqrt{1 + 4 + 4} = 6 \tag{18}$$

$$||x|| \sqrt{9} = 6 \tag{19}$$

$$||x|| = 2 \tag{20}$$

Therefore final vectors are

$$C_1 = \begin{pmatrix} -2 \\ 4 \\ 4 \end{pmatrix}, C_2 = \begin{pmatrix} 2 \\ -4 \\ -4 \end{pmatrix} \tag{21}$$

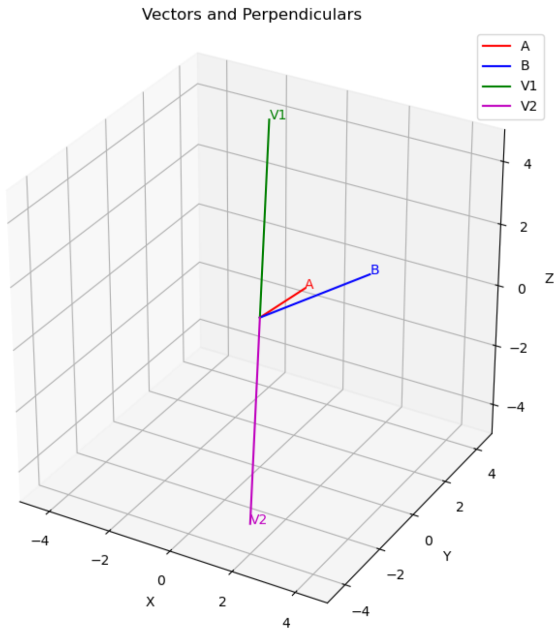

Fig. 1