MatGeo Assignment 1: 1.5.12

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Problem Statement

In what ratio does the point $\mathbf{P}(-4, y)$ divide the line segment joining the points $\mathbf{A}(-6, 10)$ and $\mathbf{B}(3, -8)$? Hence, find the value of y.

Solution:

We are given three points:

$$A = \begin{pmatrix} -6 \\ 10 \end{pmatrix}, \quad P = \begin{pmatrix} -4 \\ y \end{pmatrix}, \quad B = \begin{pmatrix} 3 \\ -8 \end{pmatrix} \tag{1}$$

Step 1: Using the rank condition for collinearity

The points A, P, B are collinear if

$$\text{rank}(\begin{pmatrix} P - A & B - A \end{pmatrix}) = 1 \tag{2}$$

Thus, the matrix is

$$M = \begin{pmatrix} 2 & 9 \\ y - 10 & -18 \end{pmatrix} \tag{3}$$

Perform the row operations: $R_1 \leftarrow R_1/2$ and $R_2 \leftarrow R_2 - R_1(y-10)$ which results in

$$\begin{pmatrix} 1 & 9/2 \\ 0 & -9/2(y-6) \end{pmatrix} \tag{4}$$

If y is not equal to 6, then we perform $R_2 \leftarrow R_2/(-9/2(y-6))$ to get an identity matrix. But then the rank will be 2
For the rank to be 1, the second row must be all zeros:

$y - 6 = 0 \quad \Rightarrow \quad y = 6$

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Step 2: Finding k

Using the vector formula,

$$k = \frac{(\mathbf{A} - \mathbf{P})^\top (\mathbf{P} - \mathbf{B})}{\|\mathbf{P} - \mathbf{B}\|^2} \tag{5}$$

Substitute $y = 6$: Compute the numerator:

$$(\mathbf{A} - \mathbf{P})^\top (\mathbf{P} - \mathbf{B}) = (-2)(-7) + (4)(14) = 14 + 56 = 70 \tag{6}$$

Compute the denominator:

$$\|\mathbf{P} - \mathbf{B}\|^2 = (-7)^2 + (14)^2 = 49 + 196 = 245 \tag{7}$$

Thus,

$$k = \frac{70}{245} = \frac{2}{7} \tag{8}$$

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Final Answer:

$$\boxed{y = 6, \quad k = \frac{2}{7}} \tag{9}$$

See the graphical representation in Figure **??**.

Therefore, the point P divides the line segment AB in the ratio **2:7** and the value of y is **6**.

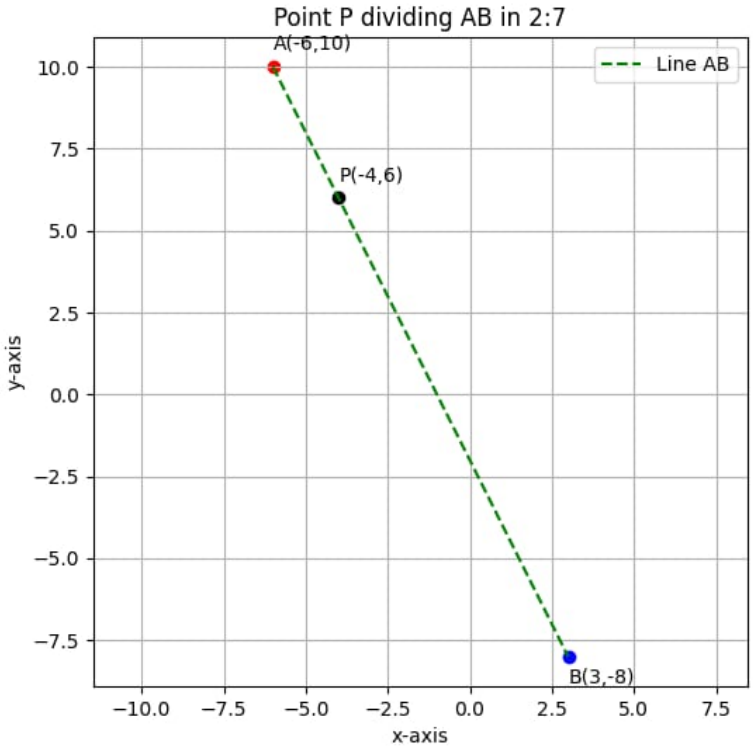

Figure 1: Visualization of point $P(-4, 6)$ dividing the line segment joining $A(-6, 10)$ and $B(3, -8)$.