6.4.12 Matgeo

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Find the shortest distance between the lines

$$\mathbf{r} = \hat{i} + 2\hat{j} + \hat{k} + \lambda(\hat{i} - \hat{j} + \hat{k})$$

$$\mathbf{r} = 2\hat{i} - \hat{j} - \hat{k} + \mu(2\hat{i} - \hat{j} + 2\hat{k})$$

$$x_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} + \lambda \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \tag{1}$$

$$x_2 = \begin{bmatrix} 2 \\ -1 \\ -1 \end{bmatrix} + \mu \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix} \tag{2}$$

$$\mathbf{M} = \begin{bmatrix} 1 & 2 \\ -1 & -1 \\ 1 & 2 \end{bmatrix} \tag{3}$$

$$\mathbf{B} - \mathbf{A} = \begin{bmatrix} 1 \\ -3 \\ -2 \end{bmatrix} \tag{4}$$

Solution

$$\begin{bmatrix} \mathbf{M} & \mathbf{B\text{-}A} \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 \\ -1 & -1 & -3 \\ 1 & 2 & -2 \end{bmatrix} \tag{5}$$

$$R_3 = R_3 + R_2 \quad and \quad R_2 = R_2 + R_1 \tag{6}$$

$$= \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -3 \\ 0 & 1 & -5 \end{bmatrix} \tag{7}$$

$$R_3 = R_3 - R_2 \tag{8}$$

$$= \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -3 \\ 0 & 0 & -2 \end{bmatrix} \tag{9}$$

Solution

as the rank of above matrix is 3 the lines are skew lines

$$\begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -1 & -1 \\ 1 & 2 \end{bmatrix} \kappa = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ -3 \\ -2 \end{bmatrix} \quad (10)$$

$$\begin{bmatrix} 3 & 5 \\ 5 & 9 \end{bmatrix} \kappa = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \quad (11)$$

The argumented matrix of the above matrix is

$$= \begin{bmatrix} 3 & 5 & 2 \\ 5 & 9 & 1 \end{bmatrix} \quad (12)$$

$$R_2 = R_2 - \frac{5}{3}R_1 \quad and \quad R_1 = R_1 - \frac{15}{2}R_2 \quad (13)$$

$$= \begin{bmatrix} 3 & 0 & \frac{39}{2} \\ 0 & \frac{2}{3} & -\frac{7}{3} \end{bmatrix} \quad (14)$$

Solution

yeilding

$$\begin{bmatrix} \lambda \\ -\mu \end{bmatrix} = \begin{bmatrix} \frac{13}{2} \\ -\frac{7}{2} \end{bmatrix} \tag{15}$$

$$x_1 = \frac{1}{2} \begin{bmatrix} 15 \\ -9 \\ 15 \end{bmatrix}, x_2 = \frac{1}{2} \begin{bmatrix} 18 \\ -9 \\ 12 \end{bmatrix} \tag{16}$$

The minimum distance between the lines is given by

$$\|x_2 - x_1\| = \|\frac{1}{2}\begin{bmatrix} 3 \\ 0 \\ -3 \end{bmatrix} = \frac{3\sqrt{2}}{2} \tag{17}$$

Graphical Representation

Parametric Lines in 3D