2.5.30

EE25BTECH11062 - Vivek K Kumar

Question:
If the two Lines

$$L_1 : x = 5, \frac{y}{3-\alpha} = \frac{z}{-2} \text{ and} \tag{0.1}$$

$$L_2 : x = 2, \frac{y}{-1} = \frac{z}{2-\alpha} \tag{0.2}$$

are perpendicular, then the value of α is _____
Solution:

Name	Point
$\mathbf{m_1}$ (Direction vector of L_1)	$\begin{pmatrix} 0 \\ 3-\alpha \\ -2 \end{pmatrix}$
$\mathbf{m_2}$ (Direction vector of L_2)	$\begin{pmatrix} 0 \\ -1 \\ 2-\alpha \end{pmatrix}$

TABLE 0: Variables Used

The lines can be represented as

$$\mathbf{x} = \begin{pmatrix} 5 \\ 0 \\ 0 \end{pmatrix} + \kappa_1 \mathbf{m_1} \tag{0.3}$$

$$= \begin{pmatrix} 5 \\ 0 \\ 0 \end{pmatrix} + \kappa_1 \begin{pmatrix} 0 \\ 3-\alpha \\ -2 \end{pmatrix} \tag{0.4}$$

and

$$\mathbf{x} = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} + \kappa_2 \mathbf{m_2} \tag{0.5}$$

$$= \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} + \kappa_2 \begin{pmatrix} 0 \\ -1 \\ 2-\alpha \end{pmatrix} \tag{0.6}$$

As the given lines are perpendicular, their direction vectors follow the relation:

$$\mathbf{m_1}^T \mathbf{m_2} = 0 \tag{0.7}$$

$$\begin{pmatrix} 0 & 3-\alpha & -2 \end{pmatrix} \begin{pmatrix} 0 \\ -1 \\ 2-\alpha \end{pmatrix} = 0 \tag{0.8}$$

$$3\alpha - 7 = 0 \tag{0.9}$$

$$\text{which gives } \alpha = \frac{7}{3} \tag{0.10}$$

$$\text{and } \mathbf{m_1} = \begin{pmatrix} 0 \\ \frac{2}{3} \\ -2 \end{pmatrix}, \mathbf{m_2} = \begin{pmatrix} 0 \\ -1 \\ \frac{-1}{3} \end{pmatrix} \tag{0.11}$$

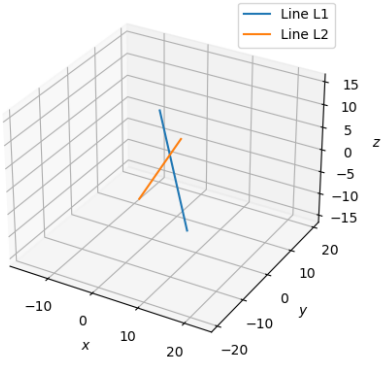

Fig. 0.1: Given vector and its direction cosines