4.7.13

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Question. Find the distance between the lines l_1 and l_2 given by

$$\overrightarrow{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$$
$$\overrightarrow{r} = 3\hat{i} + 3\hat{j} - 5\hat{k} + \mu(2\hat{i} + 3\hat{j} + 6\hat{k})$$

Solution:
Let us solve the given equation theoretically and then verify the solution computationally.
Given equation:

$$\mathbf{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k}) \tag{1}$$
$$\mathbf{r} = 3\hat{i} + 3\hat{j} - 5\hat{k} + \mu(2\hat{i} + 3\hat{j} + 6\hat{k}) \tag{2}$$
$$\tag{3}$$

From the above two equation it is clear that given two lines are parallel.
Now we'll find the distance between them.
The given lines are in the form:

$$\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ -4 \end{pmatrix} + k_1 \begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix} \tag{4}$$
$$\mathbf{r} = \begin{pmatrix} 3 \\ 3 \\ -5 \end{pmatrix} + k_2 \begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix} \tag{5}$$

Where,

$$\mathbf{A} = \begin{pmatrix} 1 \\ 2 \\ -4 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} 3 \\ 3 \\ -5 \end{pmatrix} \quad , \mathbf{M} = \begin{pmatrix} 2 & 2 \\ 3 & 3 \\ 6 & 6 \end{pmatrix} \tag{6}$$

From Least squares solution,for shortest distance:

$$\mathbf{M}^{\mathbf{T}}\mathbf{M} \begin{pmatrix} k_1 \\ -k_2 \end{pmatrix} = \mathbf{M}^{\mathbf{T}}(\mathbf{B} - \mathbf{A}) \tag{7}$$

$$\begin{pmatrix} 2 & 3 & 6 \\ 2 & 3 & 6 \end{pmatrix} \begin{pmatrix} 2 & 2 \\ 3 & 3 \\ 6 & 6 \end{pmatrix} \begin{pmatrix} k_1 \\ -k_2 \end{pmatrix} = \begin{pmatrix} 2 & 3 & 6 \\ 2 & 3 & 6 \end{pmatrix} \left(\begin{pmatrix} 3 \\ 3 \\ -5 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ -4 \end{pmatrix} \right) \tag{8}$$

$$\begin{pmatrix} 49 & 49 \\ 49 & 49 \end{pmatrix} \begin{pmatrix} k_1 \\ -k_2 \end{pmatrix} = \begin{pmatrix} 2 & 3 & 6 \\ 2 & 3 & 6 \end{pmatrix} \left(\begin{pmatrix} 3 \\ 3 \\ -5 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ -4 \end{pmatrix} \right) \tag{9}$$

$$\begin{pmatrix}49 & 49\\49 & 49\end{pmatrix}\begin{pmatrix}k_1\\-k_2\end{pmatrix}=\begin{pmatrix}2 & 3 & 6\\2 & 3 & 6\end{pmatrix}\begin{pmatrix}2\\1\\-1\end{pmatrix} \tag{10}$$

$$\begin{pmatrix}49 & 49\\49 & 49\end{pmatrix}\begin{pmatrix}k_1\\-k_2\end{pmatrix}=\begin{pmatrix}1\\1\end{pmatrix} \tag{11}$$

$$\begin{pmatrix}1 & 1\\1 & 1\end{pmatrix}\begin{pmatrix}k_1\\-k_2\end{pmatrix}=\frac{\begin{pmatrix}1\\1\end{pmatrix}}{49} \tag{12}$$

$$k_1-k_2=\frac{1}{49} \tag{13}$$

Let r_1 and r_2 be the point on the line l_1 and l_2
Now,

$$r_1-r_2=\begin{pmatrix}-2\\-1\\1\end{pmatrix}+(k_1-k_2)\begin{pmatrix}2\\3\\6\end{pmatrix} \tag{14}$$

$$r_1-r_2=\begin{pmatrix}-2\\-1\\1\end{pmatrix}+(\frac{1}{49})\begin{pmatrix}2\\3\\6\end{pmatrix} \tag{15}$$

$$r_1-r_2=\begin{pmatrix}\frac{-96}{49}\\\frac{-46}{49}\\\frac{55}{49}\end{pmatrix} \tag{16}$$

Now , Distance between two lines = $\|r_1-r_2\|$

$$\|r_1-r_2\|=\sqrt{\left(\frac{-96}{49}\right)^2+\left(\frac{-46}{49}\right)^2+\left(\frac{55}{49}\right)^2} \tag{17}$$

$$\|r_1-r_2\|=\frac{\sqrt{14357}}{49} \tag{18}$$

Therefore the distance between the lines l_1 and l_2 is $\frac{\sqrt{14357}}{49}$
From the figure it is clearly verified that the theoretical solution matches with the computational solution.

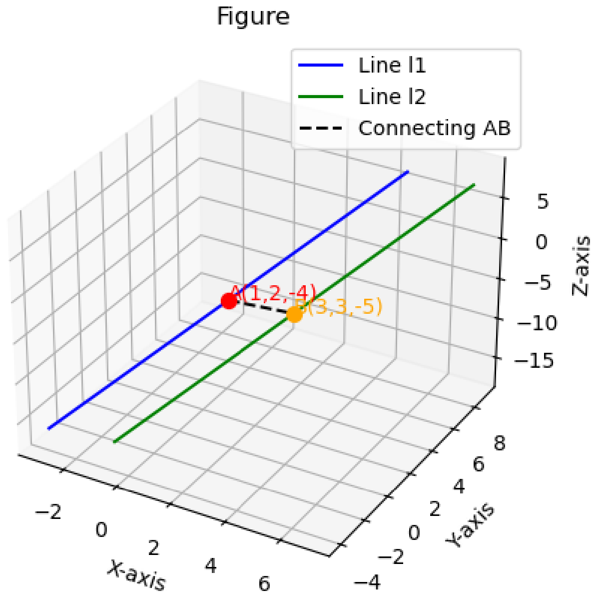

Figure
Line l1
Line l2
Connecting AB
A(1,2,-4)
B(3,3,-5)
X-axis
Y-axis
Z-axis
−2
0
2
4
6
−4
−2
0
2
4
6
8
5
0
−5
−10
−15