2.9.16

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Question

Prove that three points A, B, and C with position vectors $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ respectively are collinear if and only if $(\mathbf{b} \times \mathbf{c}) + (\mathbf{c} \times \mathbf{a}) + (\mathbf{a} \times \mathbf{b}) = \mathbf{0}$.

Theoretical Solution

The three points A, B, and C are collinear if and only if the vectors **AB** and **AC** are parallel. The position vectors for these are:

$$\mathbf{A} - \mathbf{B} = \mathbf{b} - \mathbf{a}$$
$$\mathbf{A} - \mathbf{C} = \mathbf{c} - \mathbf{a}$$

If two vectors are collinear,

(b - a) $\times(\mathbf{c} - \mathbf{a}) = \mathbf{0}$

Table

Point	Vector
A	$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$
B	$\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$
C	$\begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}$

Table: Answers

Theoretical Solution

Using the determinant (matrix) form of the cross product,

$$(\mathbf{b}-\mathbf{a})\times(\mathbf{c}-\mathbf{a}) = \begin{pmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ (b_1-a_1) & (b_2-a_2) & (b_3-a_3) \\ (c_1-a_1) & (c_2-a_2) & (c_3-a_3) \end{pmatrix}$$

Rearranging the equation we get,

$$(\mathbf{a}\times\mathbf{b})+(\mathbf{b}\times\mathbf{c})+(\mathbf{c}\times\mathbf{a})=\mathbf{0} \tag{1}$$

Hence we proved that that three points A, B, and C with position vectors **a**, **b**, and **c** respectively are collinear if and only if $(\mathbf{b}\times\mathbf{c})+(\mathbf{c}\times\mathbf{a})+(\mathbf{a}\times\mathbf{b})=\mathbf{0}$

C Code

```c
#include <stdio.h>
#include <math.h>

typedef struct {
    double x, y, z;
} Vector;

Vector cross_product(Vector v1, Vector v2);
Vector add_vectors(Vector v1, Vector v2);
void print_vector(const char* name, Vector v);
void test_collinearity(const char* case_name, Vector a, Vector b,
     Vector c);
```

C Code

```
void test_collinearity(const char* case_name, Vector a, Vector b,
     Vector c) {

    Vector a_cross_b = cross_product(a, b);
    Vector b_cross_c = cross_product(b, c);
    Vector c_cross_a = cross_product(c, a);
    Vector temp_sum = add_vectors(a_cross_b, b_cross_c);
    Vector final_sum = add_vectors(temp_sum, c_cross_a);
    print_vector(Result of (a x b) + (b x c) + (c x a), final_sum
        );

    if (is_zero_vector(final_sum)) {
        printf(Result is the zero vector. The points are
            COLLINEAR.\n);
    } else {
        printf(Result is not the zero vector. The points are NOT
            collinear.\n);
    }
}
```

C Code

```c
Vector cross_product(Vector v1, Vector v2) {
    Vector result;
    result.x = v1.y * v2.z - v1.z * v2.y;
    result.y = v1.z * v2.x - v1.x * v2.z;
    result.z = v1.x * v2.y - v1.y * v2.x;
    return result;
}

Vector add_vectors(Vector v1, Vector v2) {
    Vector result;
    result.x = v1.x + v2.x;
    result.y = v1.y + v2.y;
    result.z = v1.z + v2.z;
    return result;
}
```

Python Code

```
import numpy as np
import matplotlib.pyplot as plt

def plot_vectors(ax, points, title):

    colors = ['r', 'g', 'b']

    a, b, c = points[0], points[1], points[2]

    axb = np.cross(a, b)
    bxc = np.cross(b, c)
    cxa = np.cross(c, a)

    sum_of_cross_products = axb + bxc + cxa

    ax.scatter(0, 0, 0, color='black', s=50, label='Origin')
```

Python Code

```
    for i, (point, label) in enumerate(zip(points, ['A', 'B', 'C'
        ])):
        ax.scatter(point[0], point[1], point[2], color=colors[i],
            s=50, label=f'Point {label}')
        # Draw the position vector from the origin to the point
        ax.quiver(0, 0, 0, point[0], point[1], point[2], color=
            colors[i], arrow_length_ratio=0.1, linewidth=1.5)

    all_points = np.array(points + [points[0]]) # Add first point
         to the end to close the shape
    ax.plot(all_points[:,0], all_points[:,1], all_points[:,2],
        color='grey', linestyle='--')

    ax.set_xlabel('X axis')
    ax.set_ylabel('Y axis')
    ax.set_zlabel('Z axis')
    ax.legend()
```

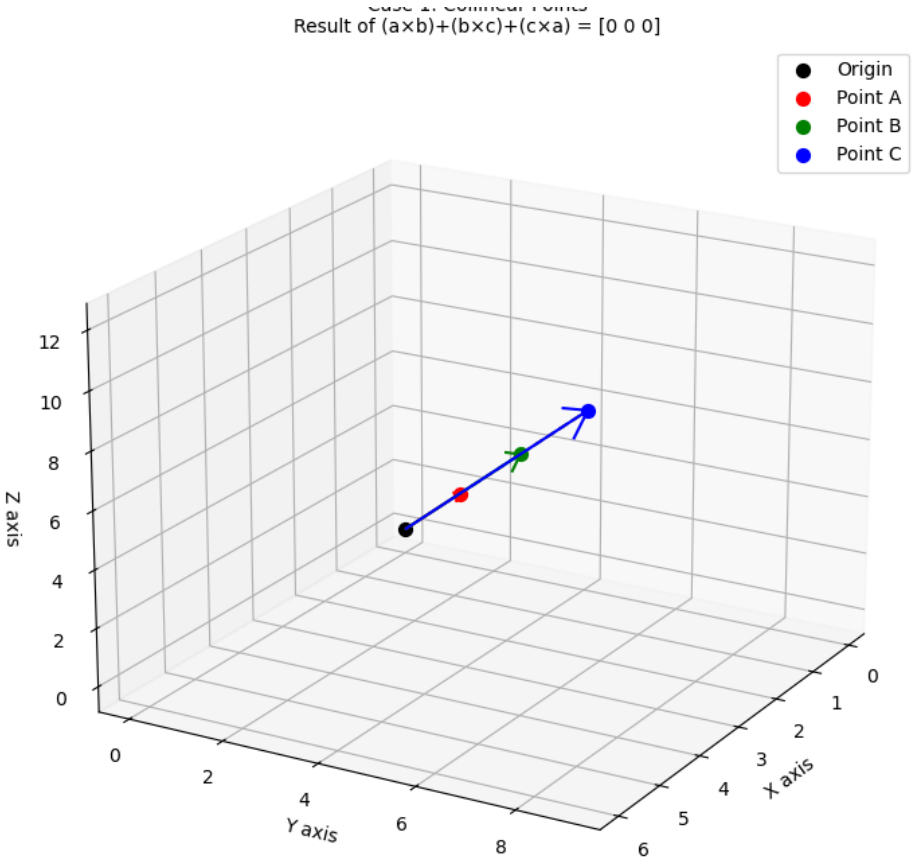
Python Code

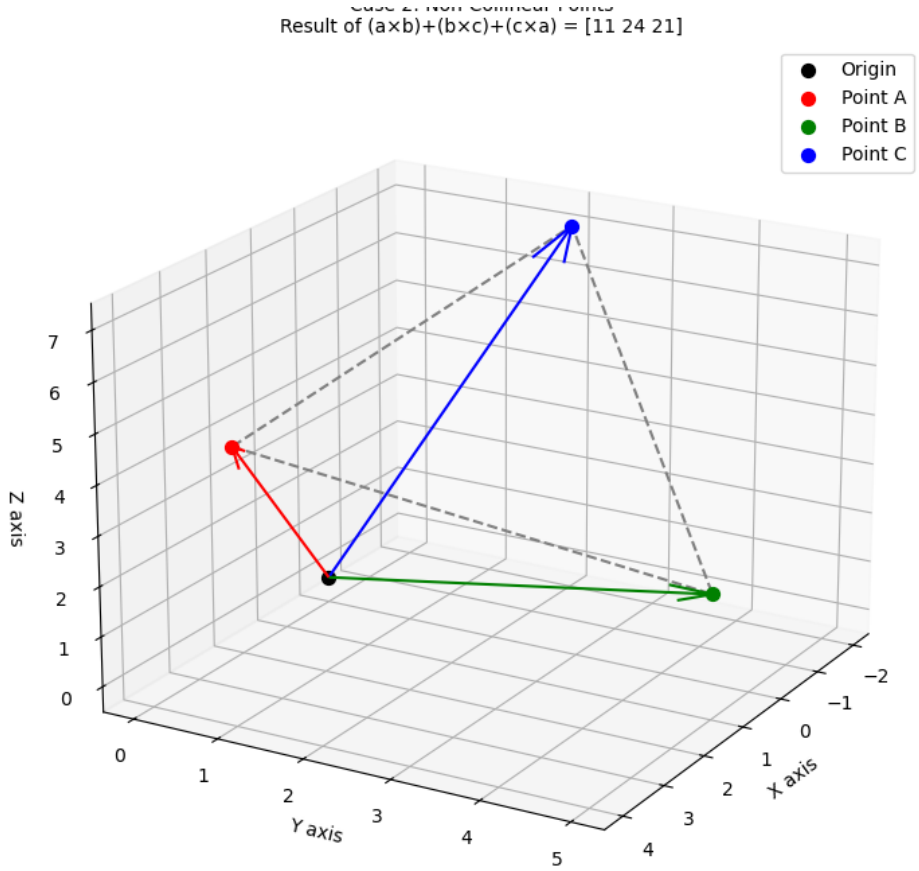
```
    result_str = np.array2string(sum_of_cross_products, formatter
        ={'float_kind':lambda x: %.1f % x})
    ax.set_title(f'{title}\nResult of (ab)+(bc)+(ca) = {
        result_str}', fontsize=10)

collinear_points = [
    np.array([2, 3, 4]),
    np.array([4, 6, 8]), # This is 2 * the first point
    np.array([6, 9, 12]) # This is 3 * the first point
]

non_collinear_points = [
    np.array([4, 1, 5]),
    np.array([1, 5, 2]),
    np.array([-2, 2, 7])
]
```

Python Code

```
fig = plt.figure(figsize=(14, 7))

ax1 = fig.add_subplot(1, 2, 1, projection='3d')
plot_vectors(ax1, collinear_points, 'Case 1: Collinear Points')
ax1.view_init(elev=20, azim=30) # Adjust viewing angle

ax2 = fig.add_subplot(1, 2, 2, projection='3d')
plot_vectors(ax2, non_collinear_points, 'Case 2: Non-Collinear
    Points')
ax2.view_init(elev=20, azim=30) # Adjust viewing angle

plt.tight_layout()
plt.show()
```

Plot


Result of (a×b)+(b×c)+(c×a) = [0 0 0]
Origin
Point A
Point B
Point C
Z axis
Y axis
X axis


Result of (a×b)+(b×c)+(c×a) = [11 24 21]
Origin
Point A
Point B
Point C
Z axis
Y axis
X axis