4.3.56

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Question

Find the equation of the plane with intercepts 2, 3 and 4 on the x, y and z - axis respectively.

Theoretical Solution

The intercepts define three points on the plane, which we can label A, B, and C.

Point	Vector
A	$\begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}$
B	$\begin{pmatrix} 0 \\ 3 \\ 0 \end{pmatrix}$
C	$\begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix}$

Table: Answers

Theoretical Solution

We can find two direction vectors, $\mathbf{m_1}$ and $\mathbf{m_2}$, that lie in the plane:

$$\mathbf{m_1} = \mathbf{B} - \mathbf{A} = \begin{pmatrix} 0-2 \\ 3-0 \\ 0-0 \end{pmatrix} = \begin{pmatrix} -2 \\ 3 \\ 0 \end{pmatrix}$$

$$\mathbf{m_2} = \mathbf{C} - \mathbf{A} = \begin{pmatrix} 0-2 \\ 0-0 \\ 4-0 \end{pmatrix} = \begin{pmatrix} -2 \\ 0 \\ 4 \end{pmatrix}$$

Theoretical Solution

The normal vector to the plane, $\mathbf{n}$, is found by the cross product of these two vectors.

$$\mathbf{n} = \mathbf{m_1} \times \mathbf{m_2} = \begin{pmatrix} -2 \\ 3 \\ 0 \end{pmatrix} \times \begin{pmatrix} -2 \\ 0 \\ 4 \end{pmatrix}$$

$$= \begin{pmatrix} (3)(4) - (0)(0) \\ (0)(-2) - (-2)(4) \\ (-2)(0) - (3)(-2) \end{pmatrix} = \begin{pmatrix} 12 \\ 8 \\ 6 \end{pmatrix}$$

We can simplify the normal vector to $\mathbf{n} = \begin{pmatrix} 6 \\ 4 \\ 3 \end{pmatrix}$. The equation of the plane is then given by:

$$\mathbf{n}^T(\mathbf{x} - \mathbf{A}) = 0$$

Theoretical Solution

Substituting the numerical values, where $\mathbf{x} = [x, y, z]^T$:

$$\begin{pmatrix} 6 & 4 & 3 \end{pmatrix} \left(\begin{pmatrix} x \\ y \\ z \end{pmatrix} - \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} \right) = 0$$

$$\implies \begin{pmatrix} 6 & 4 & 3 \end{pmatrix} \begin{pmatrix} x - 2 \\ y \\ z \end{pmatrix} = 0$$

$$\implies 6(x-2) + 4y + 3z = 0$$

$$\implies 6x - 12 + 4y + 3z = 0$$

$$\implies 6x + 4y + 3z = 12$$

C Code

```c
#include <stdio.h>

typedef struct {
    double x, y, z;
} Vector;

typedef struct {
    double a, b, c, d;
} Plane;

Vector subtract_vectors(Vector v1, Vector v2) {
    Vector result;
    result.x = v1.x - v2.x;
    result.y = v1.y - v2.y;
    result.z = v1.z - v2.z;
    return result;
}
```

C Code

```c
Vector cross_product(Vector v1, Vector v2) {
    Vector result;
    result.x = v1.y * v2.z - v1.z * v2.y;
    result.y = v1.z * v2.x - v1.x * v2.z;
    result.z = v1.x * v2.y - v1.y * v2.x;
    return result;
}

double dot_product(Vector v1, Vector v2) {
    return v1.x * v2.x + v1.y * v2.y + v1.z * v2.z;
}
```

C Code

```c
Plane find_plane_equation(Vector p1, Vector p2, Vector p3) {
    Vector m1 = subtract_vectors(p2, p1);
    Vector m2 = subtract_vectors(p3, p1);
    Vector normal = cross_product(m1, m2);
    double d = dot_product(normal, p1);

    Plane result;
    result.a = normal.x;
    result.b = normal.y;
    result.c = normal.z;
    result.d = d;
    return result;
}
```

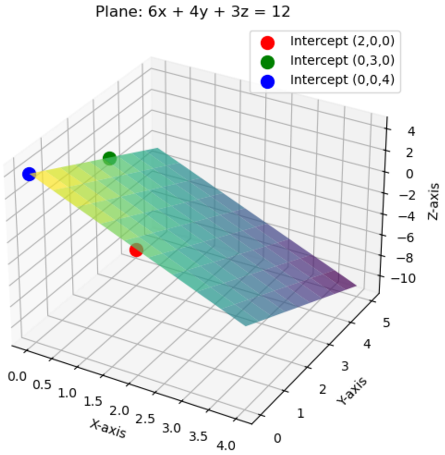
Python Code

```
import numpy as np
import matplotlib.pyplot as plt

def get_z(x, y):
    return (12 - 6*x - 4*y) / 3

x_vals = np.linspace(0, 4, 10)
y_vals = np.linspace(0, 5, 10)
x_grid, y_grid = np.meshgrid(x_vals, y_vals)
z_grid = get_z(x_grid, y_grid)

fig = plt.figure(figsize=(8, 6))
ax = fig.add_subplot(111, projection='3d')

ax.plot_surface(x_grid, y_grid, z_grid, alpha=0.7, cmap='viridis'
    )
```

Python Code

```
  ax.scatter(2, 0, 0, color='red', s=100, label='Intercept (2,0,0)')
ax.scatter(0, 3, 0, color='green', s=100, label='Intercept (0,3,0)')
ax.scatter(0, 0, 4, color='blue', s=100, label='Intercept (0,0,4)')

ax.set_xlabel('X-axis')
ax.set_ylabel('Y-axis')
ax.set_zlabel('Z-axis')
ax.set_title('Plane: 6x + 4y + 3z = 12')
ax.legend()
plt.grid(True)
plt.show()
```

Plot


Plane: 6x + 4y + 3z = 12
Intercept (2,0,0)
Intercept (0,3,0)
Intercept (0,0,4)
X-axis
Y-axis
Z-axis
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
0
1
2
3
4
5
4
2
0
−2
−4
−6
−8
−10