

1.8.23

EE25BTECH11006 - ADUDOTLA SRIVIDYA

Question:

If the point $\mathbf{A}(2, -4)$ is equidistant from $\mathbf{P}(3, 8)$ and $\mathbf{Q}(-10, y)$, find the values of y . Also find distance \mathbf{PQ} .

Solution: :

The input parameters for this problem are available in Table

Symbol	Value	Description
A	$\begin{pmatrix} 2 \\ -4 \end{pmatrix}$	equidistant point
P	$\begin{pmatrix} 3 \\ 8 \end{pmatrix}$	First point
Q	$\begin{pmatrix} -10 \\ y \end{pmatrix}$	Second point

TABLE I: Parameters for the problem

Since \mathbf{A} is equidistant from \mathbf{P} and \mathbf{Q} ,

$$\|(\mathbf{A} - \mathbf{P})\| = \|(\mathbf{A} - \mathbf{Q})\| \quad (1)$$

$$\|(\mathbf{A} - \mathbf{P})\|^2 = \|(\mathbf{A} - \mathbf{Q})\|^2 \quad (2)$$

$$(\mathbf{A} - \mathbf{P})^\top (\mathbf{A} - \mathbf{P}) = (\mathbf{A} - \mathbf{Q})^\top (\mathbf{A} - \mathbf{Q}) \quad (3)$$

$$\|\mathbf{A}\|^2 - 2\mathbf{A}^\top \mathbf{P} + \|\mathbf{P}\|^2 = \|\mathbf{A}\|^2 - 2\mathbf{A}^\top \mathbf{Q} + \|\mathbf{Q}\|^2 \quad (4)$$

$$(\mathbf{P} - \mathbf{Q})^\top \mathbf{A} = \frac{\|\mathbf{P}\|^2 - \|\mathbf{Q}\|^2}{2} \quad (5)$$

After substituting the values,

$$\begin{pmatrix} 3 - (-10) \\ 8 - y \end{pmatrix}^\top \begin{pmatrix} 2 \\ -4 \end{pmatrix} = \frac{73 - (-10)^2 - y^2}{2} \quad (6)$$

$$y^2 + 8y + 15 = 0 \quad (7)$$

Therefore,

$$y = -5, -3 \quad (8)$$

$$\mathbf{Q}_1 = \begin{pmatrix} -10 \\ -5 \end{pmatrix}, \quad \mathbf{Q}_2 = \begin{pmatrix} -10 \\ -3 \end{pmatrix} \quad (9)$$

$$\|(\mathbf{P} - \mathbf{Q}_1)\| = \left\| \begin{pmatrix} 3 \\ 8 \end{pmatrix} - \begin{pmatrix} -10 \\ -5 \end{pmatrix} \right\| \quad (10)$$

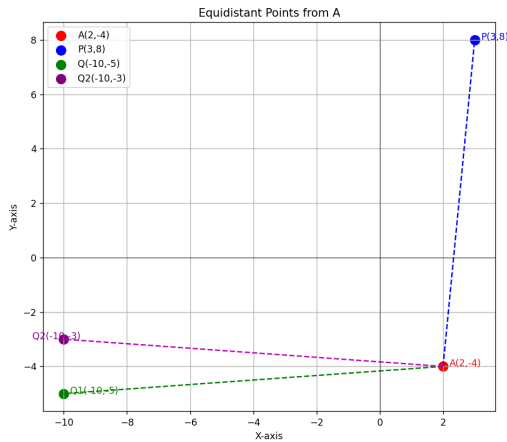
$$= \left\| \begin{pmatrix} 13 \\ 13 \end{pmatrix} \right\| \quad (11)$$

$$= 13\sqrt{2} \quad (12)$$

$$\|(\mathbf{P} - \mathbf{Q}_2)\| = \left\| \begin{pmatrix} 3 \\ 8 \end{pmatrix} - \begin{pmatrix} -10 \\ -3 \end{pmatrix} \right\| \quad (13)$$

$$= \left\| \begin{pmatrix} 13 \\ 11 \end{pmatrix} \right\| \quad (14)$$

$$= \sqrt{290} \quad (15)$$



Equidistant Points from **A** with Distances