2.10.12

Pratik R-AI25BTECH11023

September 13, 2025

Question

A unit vector perpendicular to the plane determined by the points $P(1,-1,2), Q(2,0,-1)$ and $R(0,2,1)$ is

Solution

solution:

According to the question, Given the position vectors,

$$\mathbf{P} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}; \mathbf{Q} = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}; \mathbf{R} = \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} \tag{1}$$

Let the perpendicular vector be $\mathbf{n}^T = \begin{pmatrix} n_1 & n_2 & n_3 \end{pmatrix}$

$$\because \mathbf{n}^\top \mathbf{P} = 1 \tag{2}$$

$$\mathbf{n}^\top \mathbf{Q} = 1 \tag{3}$$

$$\mathbf{n}^\top \mathbf{R} = 1 \tag{4}$$

Solution

$$\therefore \begin{pmatrix} \mathbf{P}^\top \\ \mathbf{Q}^\top \\ \mathbf{R}^\top \end{pmatrix} \mathbf{n} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{5}$$

$$\therefore \begin{pmatrix} \mathbf{P} & \mathbf{Q} & \mathbf{R} \end{pmatrix}^\top \mathbf{n} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{6}$$

$$\begin{pmatrix} 1 & -1 & 2 \\ 2 & 0 & -1 \\ 0 & 2 & 1 \end{pmatrix} \mathbf{n} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \tag{7}$$

Solution

The augmented matrix for the above system of Equations is given by

$$\begin{pmatrix} 1 & -1 & 2 & & 1 \\ 2 & 0 & -1 & & 1 \\ 0 & 2 & 1 & & 1 \end{pmatrix} \tag{8}$$

Solving equation 0.8 we get

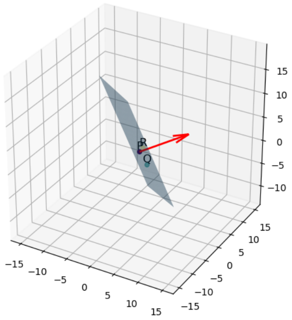
$$\mathbf{n} = \frac{1}{3}\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} \tag{9}$$

Solution

The unit vector perpendicular to the plane is given by $\mathbf{x}$

$$\mathbf{x} = \frac{\mathbf{n}}{||n||} = \frac{1}{\sqrt{6}} \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} \tag{10}$$

Plot



C Code - unit normal vector for plane

```c
#include <stdio.h>
#include <math.h>

int main() {
    // Coordinates of the points
    double P[3] = {1, -1, 2};
    double Q[3] = {2, 0, -1};
    double R[3] = {0, 2, 1};
```

C Code - unit normal vector for plane

```
    / Store points to a file (no labels, just coordinates)
FILE *fptr;
fptr = fopen("output.data", "w");
if (fptr == NULL) {
    printf("Error opening file!\n");
    return 1;
}
fprintf(fptr, "%lf %lf %lf\n", P[0], P[1], P[2]);
fprintf(fptr, "%lf %lf %lf\n", Q[0], Q[1], Q[2]);
fprintf(fptr, "%lf %lf %lf\n", R[0], R[1], R[2]);
fclose(fptr);
```

C Code - unit normal vector for plane

```
/
    // Compute vectors PQ and PR
    double PQ[3], PR[3];
    for (int i = 0; i < 3; i++) {
        PQ[i] = Q[i] - P[i];
        PR[i] = R[i] - P[i];
    }

    // Cross product PQ x PR
    double N[3];
    N[0] = PQ[1]*PR[2] - PQ[2]*PR[1];
    N[1] = PQ[2]*PR[0] - PQ[0]*PR[2];
    N[2] = PQ[0]*PR[1] - PQ[1]*PR[0];
```

C Code - unit normal vector for plane

```c
    // Magnitude of the vector N
    double mag = sqrt(N[0]*N[0] + N[1]*N[1] + N[2]*N[2]);

    // Unit vector
    double unit[3];
    for (int i = 0; i < 3; i++) {
        unit[i] = N[i] / mag;
    }

    printf("Unit vector perpendicular to the plane: (%lf, %lf, %
        lf)\n", unit[0], unit[1], unit[2]);
    return 0;
}
```

Python Code

```
import sys #for path to external scripts

import numpy as np
import numpy.linalg as LA
import matplotlib.pyplot as plt
import matplotlib.image as mpimg
from mpl_toolkits.mplot3d import Axes3D

#local imports
from line.funcs import *
#from triangle.funcs import *
#from conics.funcs import circ_gen

#if using termux
import subprocess
import shlex
#end if
```

Python Code

```python
# Read points from output.data file
points = []
with open('output.data', 'r') as file:
    for line in file:
        coords = list(map(float, line.split()))
        points.append(coords)

# Convert to numpy arrays with required shape
P = np.array(points[0]).reshape(-1, 1)
Q = np.array(points[1]).reshape(-1, 1)
R = np.array(points[2]).reshape(-1, 1)
```

Python Code

```
# Create a figure and a 3D Axes
fig = plt.figure(figsize=(8, 6))
ax = fig.add_subplot(111, projection='3d')

a, b, c, d = 2, 1, 1, -3 # coefficients of the plane equation: ax
     + by + cz + d = 0

# Generate grid points for x and y
x = np.linspace(-5, 5, 50)
y = np.linspace(-5, 5, 50)
X, Y = np.meshgrid(x, y)

# Calculate corresponding z values for each (x, y) pair to
    satisfy the plane equation
Z = (-a*X - b*Y - d) / c
```

Python Code

```
# Plot the plane
ax.plot_surface(X, Y, Z, alpha=0.5)

#Generating all lines
#x_BC = line_gen(Q,R)

#Plotting all lines
#ax.plot(x_QR[0,:],x_QR[1,:], x_QR[2,:],label='$BC$')

# Scatter plot
colors = np.arange(2, 5) # Example colors
tri_coords = np.block([P,Q,R]) # Stack P,Q,R vertically
ax.scatter(tri_coords[0, :], tri_coords[1, :], tri_coords[2, :],
    c=colors)
vert_labels = ['P', 'Q', 'R']
ax.quiver(1,-1,2, 2,1,1, color='r', linewidth=2, label='n')
```

Python Code

```python
for i, txt in enumerate(vert_labels):
    # Annotate each point with its label and coordinates
    ax.text(tri_coords[0, i], tri_coords[1, i], tri_coords[2, i],
        f'{txt}',fontsize=12, ha='center', va='bottom')
    #ax.text(tri_coords[0, i], tri_coords[1, i], tri_coords[2, i
        ], f'{txt}\n({tri_coords[0, i]:.0f}, {tri_coords[1, i]:.0
        f}, {tri_coords[2, i]:.0f})',
    # fontsize=12, ha='center', va='bottom')


ax.spines['top'].set_color('none')
ax.spines['left'].set_position('zero')
ax.spines['right'].set_color('none')
ax.spines['bottom'].set_position('zero')
```

Python Code

```
ax.set_xlim(-4, 4) # Adjust limits based on your data
ax.set_ylim(-4, 4) # Adjust limits based on your data
ax.set_zlim(-4, 4) # Adjust limits based on your data
ax.set_box_aspect([1,1,1])
'''
ax.spines['left'].set_visible(False)
ax.spines['right'].set_visible(False)
ax.spines['top'].set_visible(False)
ax.spines['bottom'].set_visible(False)
plt.xlabel('$x$')
plt.ylabel('$y$')
plt.legend(loc='best')
'''
plt.grid() # minor
plt.axis('equal')

plt.savefig('../figs/fig.png')
```