4.2.9

EE25BTECH11006 - ADUDOTLA SRIVIDYA

Question

Find the direction vector and the normal vector of the line $x + y = 4$.

Solution

The line can be written as:

$$x + y = 2 \tag{1}$$

This equation can be expressed in terms of matrices
Let

$$\mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix} \tag{2}$$

$$\mathbf{n^T} = \begin{pmatrix} 1 & 1 \end{pmatrix} \tag{3}$$

$$c = 4 \tag{4}$$

The line equation can be written as:

$$\mathbf{n^T x} = c \tag{5}$$

Where $\mathbf{n}$ is the normal vector of the given line
The direction vector of the line can be found by observing the normal vector.

$$\mathbf{m} = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \tag{6}$$

This is true because if the director vector is represented as

$$\mathbf{m} = \begin{pmatrix} 1 \\ m \end{pmatrix} \tag{7}$$

then the normal vector can be represented as

$$\mathbf{n} = \begin{pmatrix} -m \\ 1 \end{pmatrix} \tag{8}$$

This can be verified by the following equation:

$$\mathbf{n^T m} = 0 \tag{9}$$

$$\begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} -1 \\ 1 \end{pmatrix} = 0 \tag{10}$$

The normal vector of the line is $\mathbf{n} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ The director vector of the line is $\mathbf{m} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

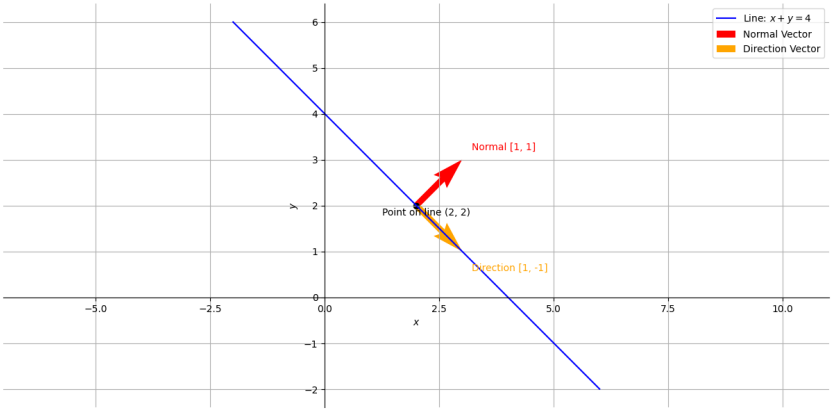

Fig. 1: Line $x + y = 4$ with its normal and direction vectors