5.4.8

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Question

Using elementary transformations, find the inverse of the following matrix

$$\begin{pmatrix} -1 & 5 \\ -3 & 2 \end{pmatrix} \tag{1}$$

Inverse

We know that

$$\mathbf{A}^{-1}\mathbf{A} = \mathbf{I} \tag{2}$$

where $\mathbf{I}$ is the 2×2 identity matrix.
So we form the augmented matrix:

$$\left(\begin{array}{cc|cc} -1 & 5 & 1 & 0 \\ -3 & 2 & 0 & 1 \end{array}\right) \tag{3}$$

Row Transformations

Applying row operations:

$$\left(\begin{array}{cc|cc} -1 & 5 & 1 & 0 \\ -3 & 2 & 0 & 1 \end{array}\right) \xleftrightarrow[R_2 \leftarrow R_2+3R_1]{R_1 \leftarrow -R_1} \left(\begin{array}{cc|cc} 1 & -5 & -1 & 0 \\ 0 & -13 & -3 & 1 \end{array}\right) \tag{4}$$

$$\xleftrightarrow[R_1 \leftarrow R_1+5R_2]{R_2 \leftarrow -\frac{1}{13}R_2} \left(\begin{array}{cc|cc} 1 & 0 & \frac{2}{13} & -\frac{5}{13} \\ 0 & 1 & \frac{3}{13} & -\frac{1}{13} \end{array}\right) \tag{5}$$

Final Inverse

Therefore, the inverse is

$$\mathbf{A}^{-1} = \frac{1}{13}\begin{pmatrix} 2 & -5 \\ 3 & -1 \end{pmatrix} \tag{6}$$

This can be verified in code by showing

$$\mathbf{A}^{-1}\mathbf{A} = \mathbf{I}. \tag{7}$$

C Code

```c
#include <stdio.h>

void matmul(double* A, double* B, double* C) {
    for (int i = 0; i < 2; i++) {
        for (int j = 0; j < 2; j++) {
            double sum = 0.0;
            for (int k = 0; k < 2; k++) {
                sum += A[i*2 + k] * B[k*2 + j];
            }
            C[i*2 + j] = sum;
        }
    }
}
```

Python + C Code

```
import ctypes
import numpy as np
lib = ctypes.CDLL(./libmatmul.so)
lib.matmul.argtypes = [
    np.ctypeslib.ndpointer(dtype=np.float64, flags=C_CONTIGUOUS),
    np.ctypeslib.ndpointer(dtype=np.float64, flags=C_CONTIGUOUS),
    np.ctypeslib.ndpointer(dtype=np.float64, flags=C_CONTIGUOUS),
]
lib.matmul.restype = None
A = np.array([[-1, 5],
              [-3, 2]], dtype=np.float64)
B = (1/13.0) * np.array([[2, -5],
                         [3, -1]], dtype=np.float64)
C = np.zeros((2, 2), dtype=np.float64)
lib.matmul(A, B, C)
print(C = A*B =\n, C)
```

Python Code

```
import numpy as np


a = np.array([[-1, 5],[-3, 2]])
inverse_a = np.array([[2/13, -5/13], [3/13, -1/13]])

b = a@inverse_a
print(b)
```