2.4.5

EE25BTECH11015 - Bhoomika V

Question :-

Show that the points $\mathbf{A}(2\hat{i} - \hat{j} + \hat{k})$, $\mathbf{B}(\hat{i} - 3\hat{j} - 5\hat{k})$, $\mathbf{C}(3\hat{i} - 4\hat{j} - 4\hat{k})$ are the vertices of a right angled triangle.

Solution:

Point	Vector
A	$\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$
B	$\begin{bmatrix} 1 \\ -3 \\ -5 \end{bmatrix}$
C	$\begin{bmatrix} 3 \\ -4 \\ -4 \end{bmatrix}$

TABLE 0: Vectors

The sides of the triangle will be

$$\mathbf{B} - \mathbf{A} = \begin{bmatrix} -1 \\ -2 \\ -6 \end{bmatrix}, \quad \mathbf{C} - \mathbf{B} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}, \quad \mathbf{A} - \mathbf{C} = \begin{bmatrix} -1 \\ 3 \\ 5 \end{bmatrix} \tag{0.1}$$

In a right angled triangle

$$(\mathbf{C} - \mathbf{B})^T (\mathbf{A} - \mathbf{C}) = 0 \tag{0.2}$$

from Equation (0.1)

$$(\mathbf{C} - \mathbf{B})^T (\mathbf{C} - \mathbf{A}) = \begin{bmatrix} 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \\ 5 \end{bmatrix} = 0 \tag{0.3}$$

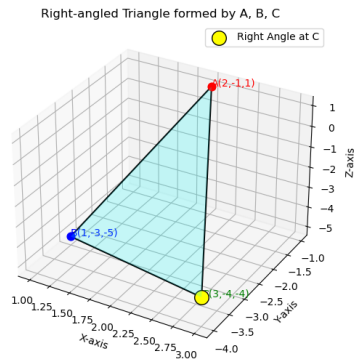Therefore the given points are vertices of a right angled triangle.

Fig. 0.1