2.7.24

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Question:
If the vertices of a triangle are $(1,-3)$, $(4,p)$ and $(-9,7)$ and its area is 15 sq. units. Find the value(s) of p.
Solution: let $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ be the vectors such that:

Variable	value
A	$\begin{pmatrix}1\\-3\end{pmatrix}$
B	$\begin{pmatrix}4\\p\end{pmatrix}$
C	$\begin{pmatrix}-9\\7\end{pmatrix}$

TABLE 0: Variables used

given ar($\mathbf{ABC}$) = 15 sq.units

$$\text{ar}(\mathbf{ABC}) = \frac{1}{2}\|(\mathbf{B}-\mathbf{A})\times(\mathbf{C}-\mathbf{A})\| \tag{0.1}$$

$$= \frac{1}{2}\|\mathbf{B}\times(\mathbf{C}-\mathbf{A})-\mathbf{A}\times(\mathbf{C}-\mathbf{A})\| \tag{0.2}$$

$$= \frac{1}{2}\|\mathbf{B}\times\mathbf{C}-\mathbf{B}\times\mathbf{A}-\mathbf{A}\times\mathbf{C}+\mathbf{A}\times\mathbf{A}\| \tag{0.3}$$

$$= \frac{1}{2}\|\mathbf{B}\times(\mathbf{C}-\mathbf{A})-\mathbf{A}\times\mathbf{C}\| \tag{0.4}$$

$$\tag{0.5}$$

Substituting the values of $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$

$$ar(\mathbf{ABC}) = 5|p+6| = 15 \tag{0.6}$$

$$|p+6| = 3 \tag{0.7}$$

$$P = -3,-9 \tag{0.8}$$

Hence, Value of p is -3 , -9.

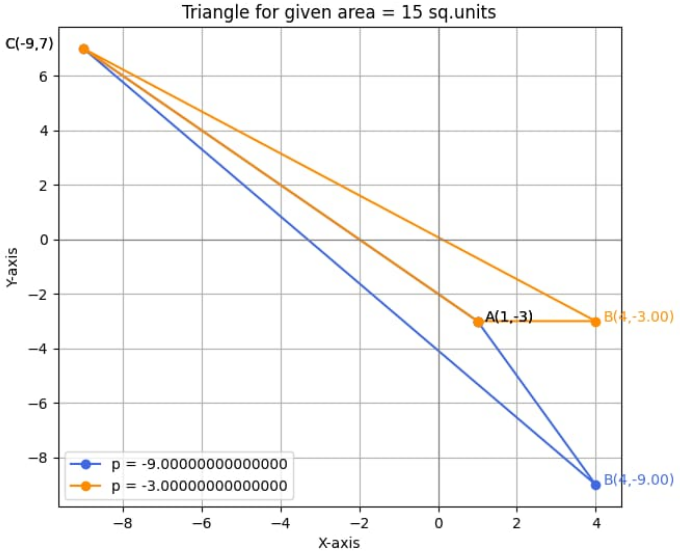

Fig. 0.1: Triangle ABC