5.12.7

AI25BTECH11012 - GARIGE UNNATHI

Question:
Solve the following equations for x and y :

$$(ax - by) + (a + 4b) = 0$$
$$(bx + ay) + (b - 4a) = 0$$

Solution:
given two equations :

$$(ax - by) = -(a + 4b) \tag{0.1}$$
$$(bx + ay) = -(b - 4a) \tag{0.2}$$

these can be written as :

$$\begin{pmatrix} a & -b \\ b & a \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -a - 4b \\ 4a - b \end{pmatrix} = \mathbf{M}\begin{pmatrix} a \\ b \end{pmatrix} \tag{0.3}$$
$$\mathbf{M} = \begin{pmatrix} -1 & -4 \\ 4 & -1 \end{pmatrix} \tag{0.4}$$

Variable	Formula
A	$A = \begin{pmatrix} a & -b \\ b & a \end{pmatrix}$

TABLE 0: Variables Used

Multiplying both sides by A^T :

$$\begin{pmatrix} a & b \\ -b & a \end{pmatrix}\begin{pmatrix} a & -b \\ b & a \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \mathbf{A^T M}\begin{pmatrix} a \\ b \end{pmatrix} \tag{0.5}$$
$$(a^2 + b^2)\begin{pmatrix} x \\ y \end{pmatrix} = \mathbf{A^T M}\begin{pmatrix} a \\ b \end{pmatrix} \tag{0.6}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{(a^2 + b^2)}\mathbf{A^T M}\begin{pmatrix} a \\ b \end{pmatrix} \tag{0.7}$$