CS 1674: Intro to Computer Vision Visual Recognition

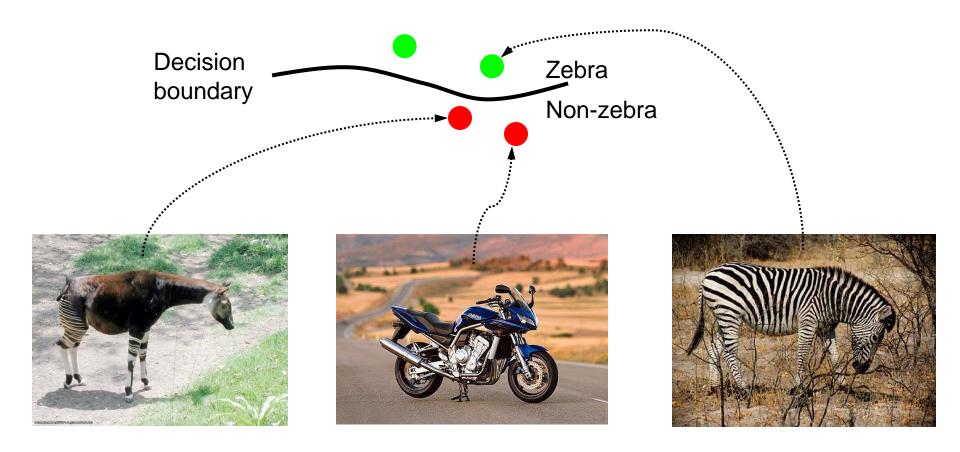
Prof. Adriana Kovashka
University of Pittsburgh
October 6, 2020

Plan for this lecture

- What is recognition?
 - a.k.a. classification, categorization
- Support vector machines
 - Separable case / non-separable case
 - Linear / non-linear (kernels)
- The importance of generalization
 - The bias-variance trade-off (applies to all classifiers)
- Example approach for scene classification

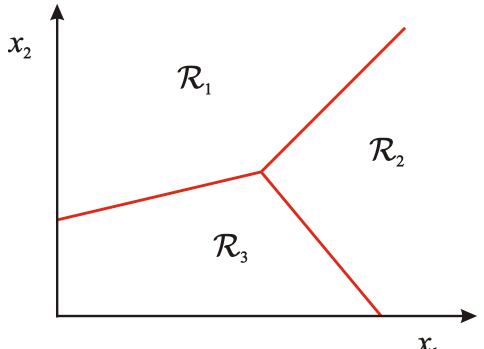
Classification

 Given a feature representation for images, how do we learn a model for distinguishing features from different classes?

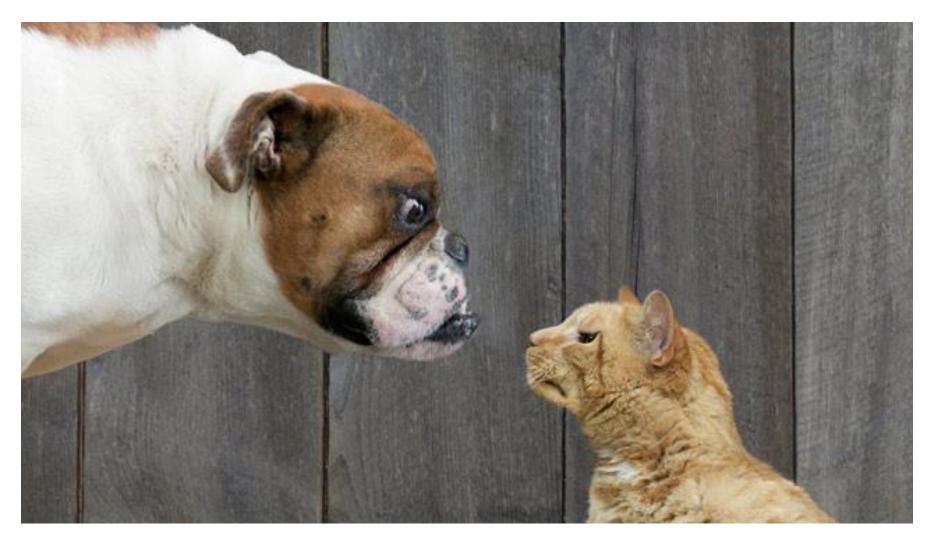


Classification

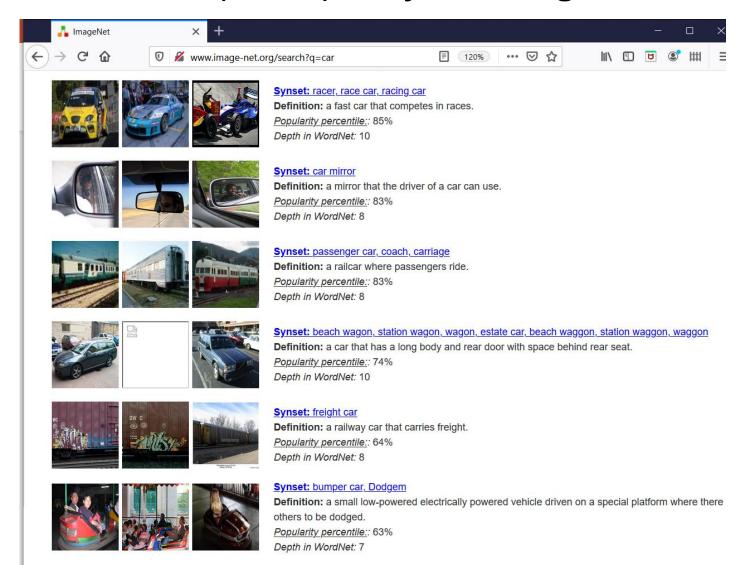
- Assign input vector to one of two or more classes
- Input space divided into decision regions separated by decision boundaries



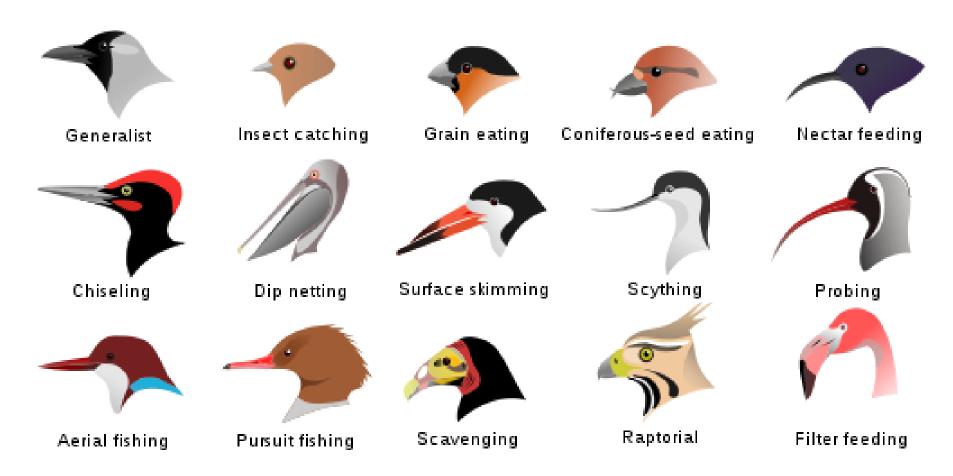
Two-class (binary): Cat vs Dog



Multi-class (often): Object recognition



Fine-grained recognition



Visipedia Project

Place recognition



Material recognition







[Bell et al. CVPR 2015]

Dating historical photos



1940

1953

1966

1977

[Palermo et al. ECCV 2012]

Image style recognition



[Karayev et al. BMVC 2014]

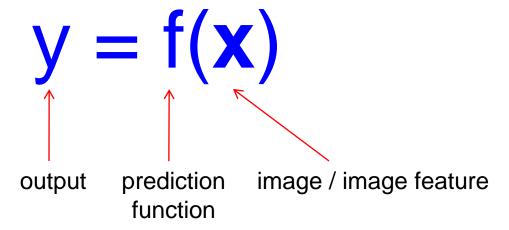
Recognition: A machine learning approach



The machine learning framework

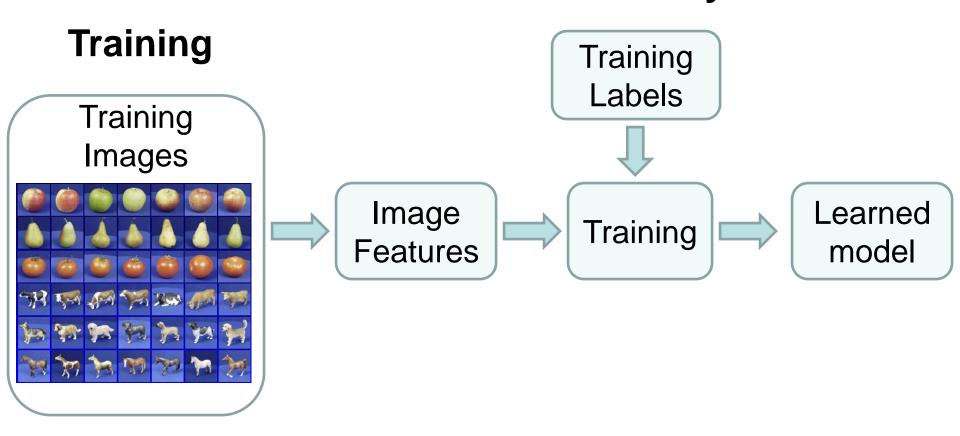
 Apply a prediction function to a feature representation of the image to get the desired output:

The machine learning framework

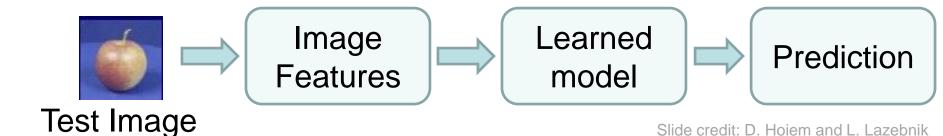


- Training: given a training set of labeled examples {(x₁,y₁), ..., (x_N,y_N)}, estimate the prediction function f by minimizing the prediction error on the training set
 - Evaluate multiple hypotheses f₁, f₂, f_H ... and pick the best one as f
- Testing: apply f to a never before seen test example x and output the predicted value y = f(x)

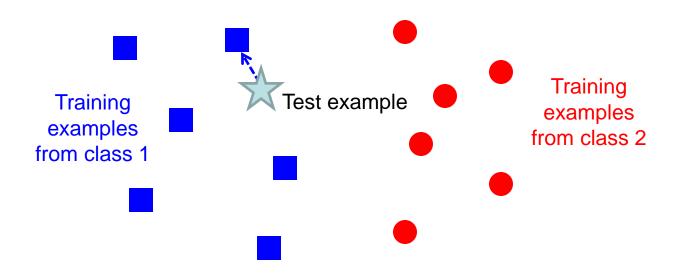
The old-school way



Testing



The simplest classifier



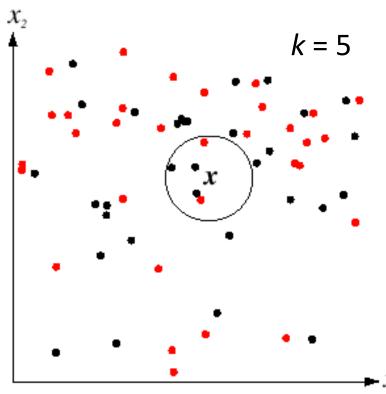
$f(\mathbf{x})$ = label of the training example nearest to \mathbf{x}

- All we need is a distance function for our inputs
- No training required!

K-Nearest Neighbors classification

- For a new point, find the k closest points from training data
- Labels of the k points "vote" to classify

Black = negative Red = positive



If query lands here, the 5 NN consist of 3 negatives and 2 positives, so we classify it as negative.

im2gps: Estimating Geographic Information from a Single Image

James Hays and Alexei Efros, CVPR 2008

Where was this image taken?



















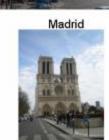










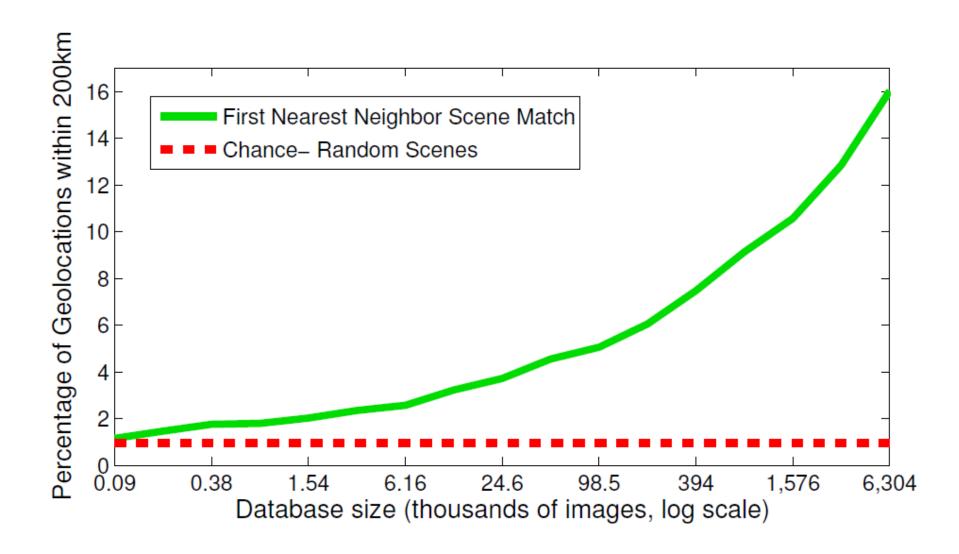




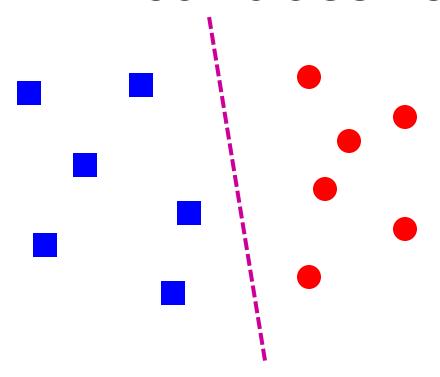
Paris

Nearest Neighbors according to bag of SIFT + color histogram + a few others

The Importance of Data



Linear classifier

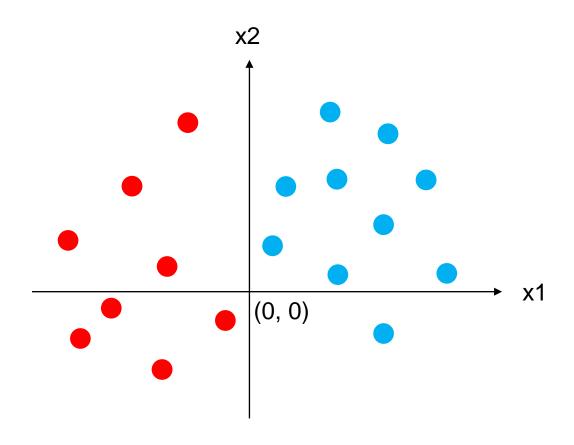


• Find a linear function to separate the classes

$$f(\mathbf{x}) = \operatorname{sgn}(w_1x_1 + w_2x_2 + \dots + w_Dx_D) = \operatorname{sgn}(\mathbf{w} \cdot \mathbf{x})$$

Linear classifier

• Decision = $sign(w^Tx) = sign(w1*x1 + w2*x2)$

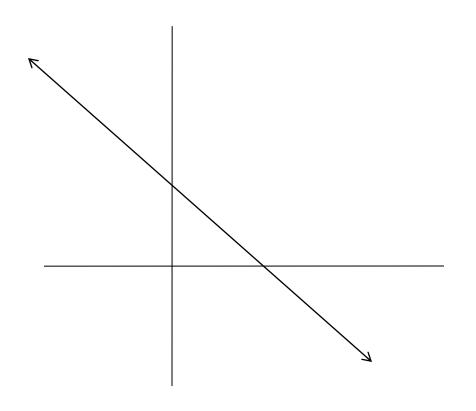


What should the weights be?

Slope: -a/c

Y-intercept: -b/c

Lines in R²



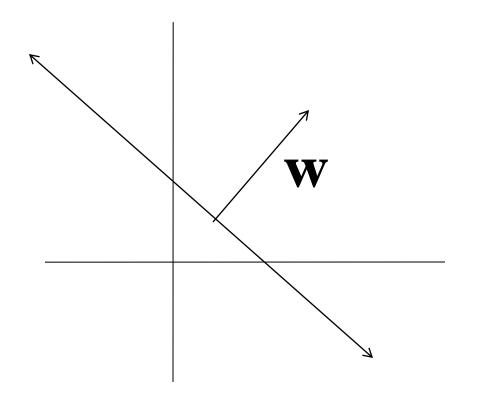
Let
$$\mathbf{w} = \begin{bmatrix} a \\ c \end{bmatrix}$$
 $\mathbf{x} = \begin{bmatrix} x \\ y \end{bmatrix}$

$$ax + cy + b = 0$$

Lines in R²

Slope: -a/c

Y-intercept: -b/c



Let
$$\mathbf{w} = \begin{bmatrix} a \\ c \end{bmatrix}$$
 $\mathbf{x} = \begin{bmatrix} x \\ y \end{bmatrix}$

$$ax + cy + b = 0$$

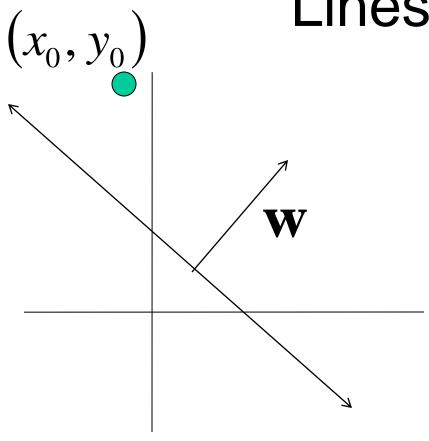
$$\mathbf{w} \cdot \mathbf{x} + b = 0$$

Slo

Slope: -a/c

Y-intercept: -b/c

Lines in R²



Let
$$\mathbf{w} = \begin{bmatrix} a \\ c \end{bmatrix}$$
 $\mathbf{x} = \begin{bmatrix} x \\ y \end{bmatrix}$

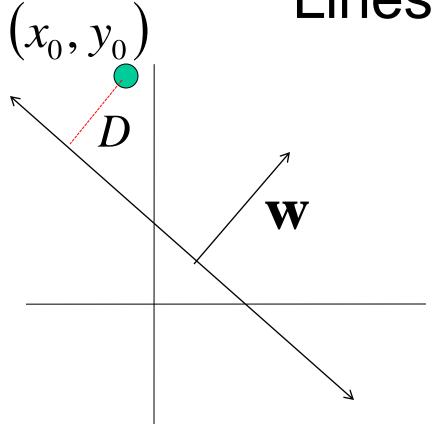
$$ax + cy + b = 0$$

$$\mathbf{w} \cdot \mathbf{x} + b = 0$$

Lines in R²

Slope: -a/c

Y-intercept: -b/c



Let
$$\mathbf{w} = \begin{bmatrix} a \\ c \end{bmatrix}$$
 $\mathbf{x} = \begin{bmatrix} x \\ y \end{bmatrix}$

$$ax + cy + b = 0$$

$$\mathbf{w} \cdot \mathbf{x} + b = 0$$

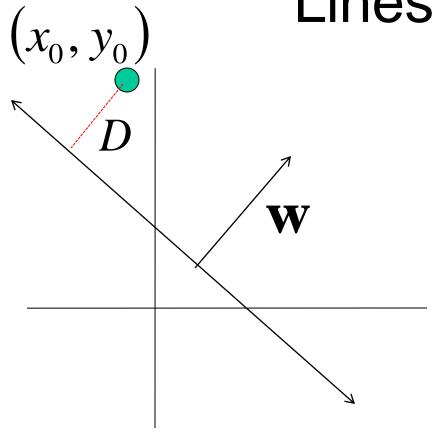
$$D = \frac{|ax_0 + cy_0 + b|}{\sqrt{a^2 + c^2}}$$

distance from point to line

Lines in R²

Slope: -a/c

Y-intercept: -b/c



Let
$$\mathbf{w} = \begin{vmatrix} a \\ c \end{vmatrix} \quad \mathbf{x} = \begin{vmatrix} x \\ y \end{vmatrix}$$

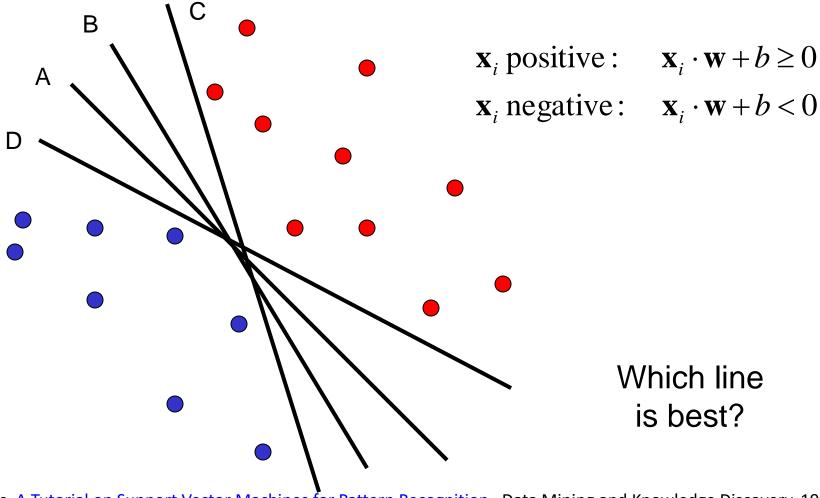
$$ax + cy + b = 0$$

$$\mathbf{w} \cdot \mathbf{x} + b = 0$$

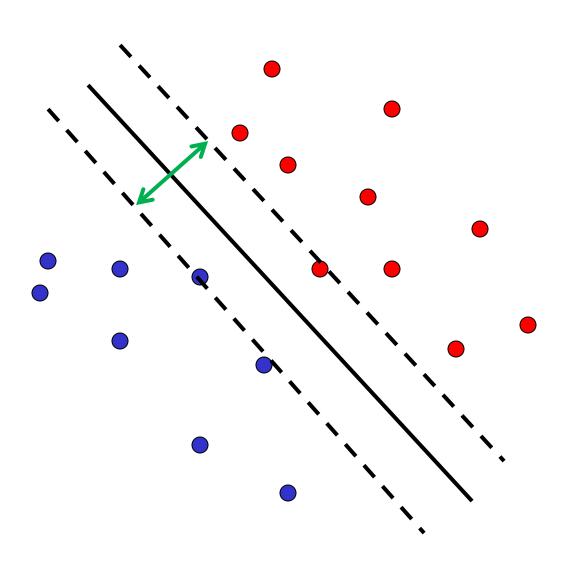
$$D = \frac{\left|ax_0 + cy_0 + b\right|}{\sqrt{a^2 + c^2}} = \frac{\left|\mathbf{w}^{\mathrm{T}}\mathbf{x} + b\right|}{\left\|\mathbf{w}\right\|} \quad \text{distance from point to line}$$

Linear classifiers

 Find linear function to separate positive and negative examples



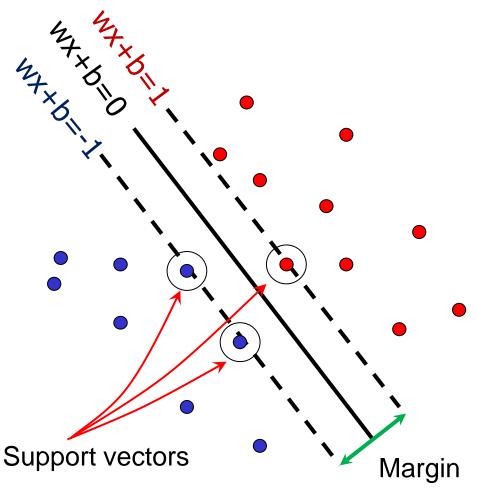
C. Burges, <u>A Tutorial on Support Vector Machines for Pattern Recognition</u>, Data Mining and Knowledge Discovery, 1998



 Discriminative classifier based on optimal separating line (for 2d case)

 Maximize the margin between the positive and negative training examples

Want line that maximizes the margin.

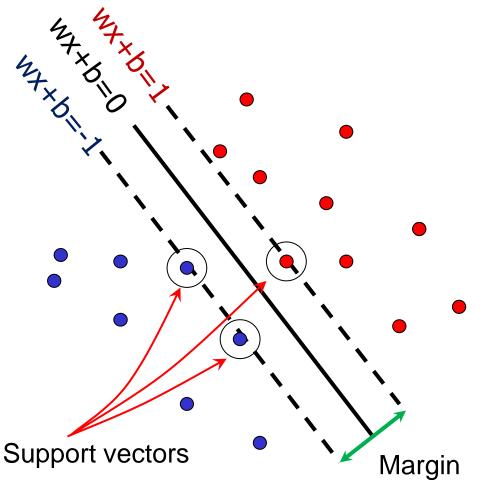


$$\mathbf{x}_i$$
 positive $(y_i = 1)$: $\mathbf{x}_i \cdot \mathbf{w} + b \ge 1$

$$\mathbf{x}_i \text{ negative}(y_i = -1): \quad \mathbf{x}_i \cdot \mathbf{w} + b \le -1$$

For support, vectors, $\mathbf{x}_i \cdot \mathbf{w} + b = \pm 1$

Want line that maximizes the margin.



$$\mathbf{x}_i$$
 positive $(y_i = 1)$: $\mathbf{x}_i \cdot \mathbf{w} + b \ge 1$

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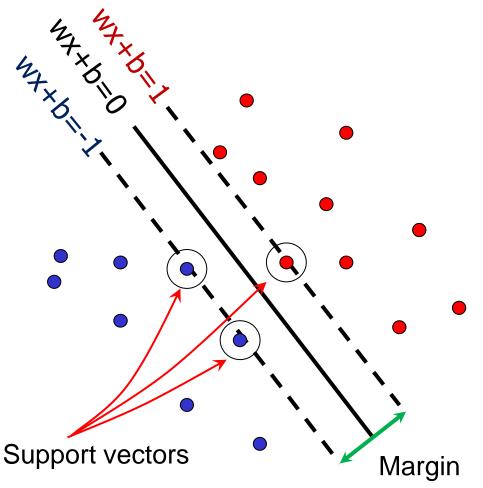
For support, vectors,
$$\mathbf{x}_i \cdot \mathbf{w} + b = \pm 1$$

Distance between point $|\mathbf{x}_i \cdot \mathbf{w} + b|$ and line: $|\mathbf{w}|$

For support vectors:

$$\frac{\mathbf{w}^T \mathbf{x} + b}{\|\mathbf{w}\|} = \frac{\pm 1}{\|\mathbf{w}\|} \qquad M = \left| \frac{1}{\|\mathbf{w}\|} - \frac{-1}{\|\mathbf{w}\|} \right| = \frac{2}{\|\mathbf{w}\|}$$

Want line that maximizes the margin.



$$\mathbf{x}_i$$
 positive $(y_i = 1)$: $\mathbf{x}_i \cdot \mathbf{w} + b \ge 1$

$$\mathbf{x}_i \text{ negative}(y_i = -1): \quad \mathbf{x}_i \cdot \mathbf{w} + b \le -1$$

For support, vectors,
$$\mathbf{x}_i \cdot \mathbf{w} + b = \pm 1$$

Distance between point
$$|\mathbf{x}_i \cdot \mathbf{w} + b|$$
 and line: $|\mathbf{w}|$

Therefore, the margin is $2 / ||\mathbf{w}||$

Finding the maximum margin line

- Maximize margin 2/||w||
- 2. Correctly classify all training data points:

$$\mathbf{x}_i$$
 positive $(y_i = 1)$: $\mathbf{x}_i \cdot \mathbf{w} + b \ge 1$

$$\mathbf{x}_i \text{ negative}(y_i = -1): \quad \mathbf{x}_i \cdot \mathbf{w} + b \le -1$$

Quadratic optimization problem:

Minimize
$$\frac{1}{2} \mathbf{w}^T \mathbf{w}$$

Subject to
$$y_i(\mathbf{w} \cdot \mathbf{x}_i + b) \ge 1$$

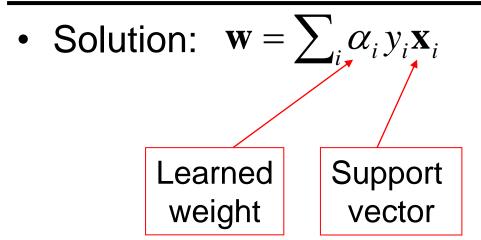
One constraint per training point.

$$w \cdot x_i + b >= 1 \text{ (if } y_i = 1)$$

Note sign trick:

$$w \cdot x_i + b >= 1$$
 (if $y_i = 1$)
 $w \cdot x_i + b <= -1$ (if $y_i = -1$)
 $(-1) w \cdot x_i + b >= 1$

Finding the maximum margin line



Finding the maximum margin line

- Solution: $\mathbf{w} = \sum_{i} \alpha_{i} y_{i} \mathbf{x}_{i}$ $b = y_{i} \mathbf{w} \cdot \mathbf{x}_{i} \text{ (for any support vector)}$
- Classification function:

$$f(x) = \text{sign} (\mathbf{w} \cdot \mathbf{x} + \mathbf{b})$$
$$= \text{sign} \left(\sum_{i} \alpha_{i} y_{i} \mathbf{x}_{i} \cdot \mathbf{x} + b \right)$$

If f(x) < 0, classify as negative, otherwise classify as positive.

- Notice that it relies on an inner product between the test point x and the support vectors x;
- (Solving the optimization problem also involves computing the inner products x_i · x_j between all pairs of training points)

Inner product

 The decision boundary for the SVM and its optimization depend on the inner product of two data points (vectors):

$$f(x) = \operatorname{sign} (\mathbf{w} \cdot \mathbf{x} + \mathbf{b})$$

$$= \operatorname{sign} \left(\sum_{i} \alpha_{i} y_{i} \mathbf{x}_{i} \cdot \mathbf{x} + \mathbf{b} \right)$$

The inner product is equal

$$(\mathbf{x}_i^T \mathbf{x}) = \|\mathbf{x}_i\| * \|\mathbf{x}_i\| \cos \theta$$

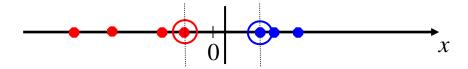
If the angle in between them is 0 then: $(\mathbf{x}_i^T \mathbf{x}) = \|\mathbf{x}_i\|^* \|\mathbf{x}_i\|^*$

If the angle between them is 90 then: $(\mathbf{x}_i^T \mathbf{x}) = 0$

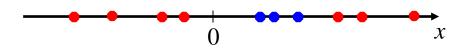
The inner product measures how similar the two vectors are

Nonlinear SVMs

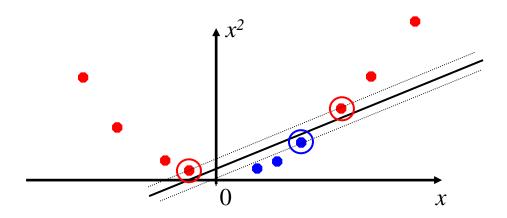
Datasets that are linearly separable work out great:



But what if the dataset is just too hard?

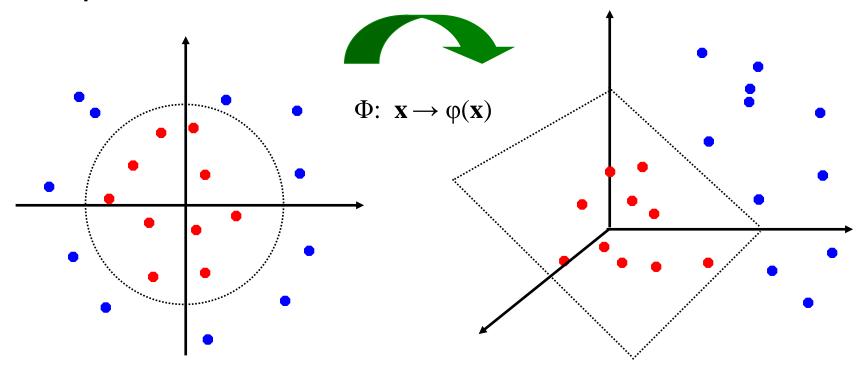


We can map it to a higher-dimensional space:



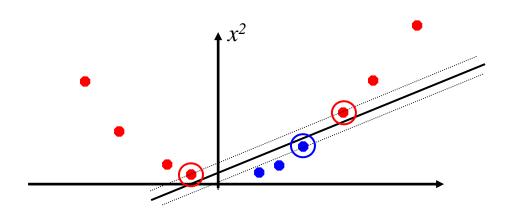
Nonlinear SVMs

 General idea: the original input space can always be mapped to some higher-dimensional feature space where the training set is separable:



Nonlinear kernel: Example

• Consider the mapping $\varphi(x) = (x, x^2)$



$$\varphi(x) \cdot \varphi(y) = (x, x^2) \cdot (y, y^2) = xy + x^2 y^2$$
$$K(x, y) = xy + x^2 y^2$$

The "Kernel Trick"

- The linear classifier relies on dot product between vectors $K(\mathbf{x}_i, \mathbf{x}_j) = \mathbf{x}_i \cdot \mathbf{x}_j$
- If every data point is mapped into high-dimensional space via some transformation Φ : $\mathbf{x}_i \to \varphi(\mathbf{x}_i)$, the dot product becomes: $K(\mathbf{x}_i, \mathbf{x}_j) = \varphi(\mathbf{x}_i) \cdot \varphi(\mathbf{x}_j)$
- A kernel function is similarity function that corresponds to an inner product in some expanded feature space
- The kernel trick: instead of explicitly computing the lifting transformation $\varphi(\mathbf{x})$, define a kernel function K such that: $K(\mathbf{x}_i, \mathbf{x}_i) = \varphi(\mathbf{x}_i) \cdot \varphi(\mathbf{x}_i)$

Examples of kernel functions

Linear: $K(x_i, x_j) = x_i^T x_j$

Polynomials of degree up to d:

$$K(x_i, x_j) = (x_i^T x_j + 1)^d$$

Gaussian RBF:

$$K(x_i, x_j) = \exp(-\frac{\|x_i - x_j\|^2}{2\sigma^2})$$

Histogram intersection:

$$K(x_i, x_j) = \sum_{k} \min(x_i(k), x_j(k))$$

The benefit of the "kernel trick"

Example: Polynomial kernel for 2-dim features

$$k(\mathbf{x}, \mathbf{z}) = (1 + \mathbf{x}^{\mathrm{T}} \mathbf{z})^{2} = (1 + x_{1}z_{1} + x_{2}z_{2})^{2}$$

$$= 1 + 2x_{1}z_{1} + 2x_{2}z_{2} + x_{1}^{2}z_{1}^{2} + 2x_{1}z_{1}x_{2}z_{2} + x_{2}^{2}z_{2}^{2}$$

$$= (1, \sqrt{2}x_{1}, \sqrt{2}x_{2}, x_{1}^{2}, \sqrt{2}x_{1}x_{2}, x_{2}^{2})(1, \sqrt{2}z_{1}, \sqrt{2}z_{2}, z_{1}^{2}, \sqrt{2}z_{1}z_{2}, z_{2}^{2})^{\mathrm{T}}$$

$$= \phi(\mathbf{x})^{\mathrm{T}}\phi(\mathbf{z}). \tag{7.42}$$

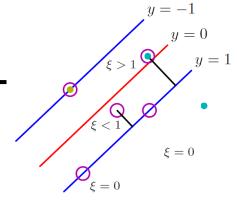
- ... lives in 6 dimensions
- With the kernel trick, we directly compute an inner product in 2-dim space, obtaining a scalar that we add 1 to and exponentiate

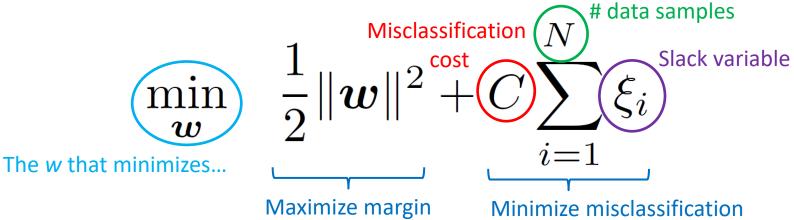
Hard-margin SVMs

$$\frac{1}{2}\|\boldsymbol{w}\|^2$$
 The \boldsymbol{w} that minimizes... Maximize margin

subject to
$$y_i \boldsymbol{w}^T \boldsymbol{x}_i \geq 1$$
 , $\forall i = 1, \dots, N$

Soft-margin SVMs





subject to
$$y_i \boldsymbol{w}^T \boldsymbol{x}_i \geq 1 - \xi_i,$$
 $\xi_i \geq 0, \ \forall i = 1, \dots, N$

What about multi-class SVMs?

- Unfortunately, there is no "definitive" multiclass SVM formulation
- In practice, we have to obtain a multi-class SVM by combining multiple two-class SVMs
- One vs. others
 - Training: learn an SVM for each class vs. the others
 - Testing: apply each SVM to the test example, and assign it to the class of the SVM that returns the highest decision value
- One vs. one
 - Training: learn an SVM for each pair of classes
 - Testing: each learned SVM "votes" for a class to assign to the test example

Multi-class problems

One-vs-all (a.k.a. one-vs-others)

- Train K classifiers
- In each, pos = data from class i, neg = data from classes other than i
- The class with the most confident prediction wins
- Example:
 - You have 4 classes, train 4 classifiers
 - 1 vs others: score 3.5
 - 2 vs others: score 6.2
 - 3 vs others: score 1.4
 - 4 vs other: score 5.5
 - Final prediction: class 2

Multi-class problems

One-vs-one (a.k.a. all-vs-all)

- Train K(K-1)/2 binary classifiers (all pairs of classes)
- They all vote for the label
- Example:
 - You have 4 classes, then train 6 classifiers
 - 1 vs 2, 1 vs 3, 1 vs 4, 2 vs 3, 2 vs 4, 3 vs 4
 - Votes: 1, 1, 4, 2, 4, 4
 - Final prediction is class 4

Using SVMs

- 1. Select a kernel function.
- 2. Compute pairwise kernel values between labeled examples.
- 3. Use this "kernel matrix" to solve for SVM support vectors & alpha weights.
- 4. To classify a new example: compute kernel values between new input and support vectors, apply alpha weights, check sign of output.

Some SVM packages

- LIBSVM http://www.csie.ntu.edu.tw/~cjlin/libsvm/
- LIBLINEAR
 https://www.csie.ntu.edu.tw/~cjlin/liblinear/
- SVM Light http://svmlight.joachims.org/

Linear classifiers vs nearest neighbors

Linear pros:

- + Low-dimensional *parametric* representation
- + Very fast at test time

Linear cons:

- Can be tricky to select best kernel function for a problem
- Learning can take a very long time for large-scale problem

NN pros:

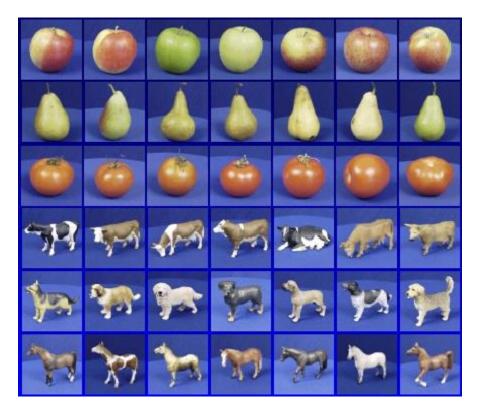
- + Works for any number of classes
- + Decision boundaries not necessarily linear
- + Nonparametric method
- + Simple to implement

NN cons:

- Slow at test time (large search problem to find neighbors)
- Storage of data
- Especially need good distance function (but true for all classifiers)

Training vs Testing

- What do we want?
 - High accuracy on training data?
 - No, high accuracy on unseen/new/test data!
 - Why is this tricky?
- Training data
 - Features (x) and labels (y) used to learn mapping f
- Test data
 - Features (x) used to make a prediction
 - Labels (y) only used to see how well we've learned f!!!
- Validation data
 - Held-out set of the training data
 - Can use both features (x) and labels (y) to tune parameters of the model we're learning



Training set (labels known)

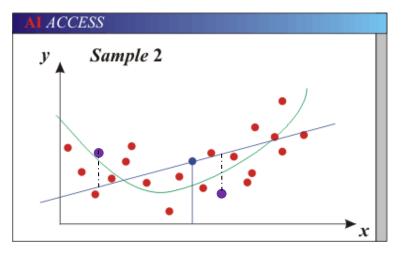


Test set (labels unknown)

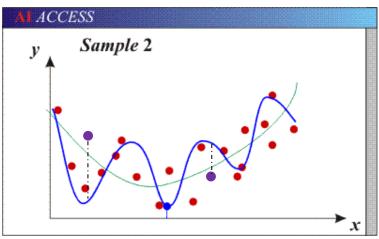
 How well does a learned model generalize from the data it was trained on to a new test set?

- Components of generalization error
 - Noise in our observations: unavoidable
 - Bias: due to inaccurate assumptions/simplifications by model
 - Variance: models estimated from different training sets differ greatly rom each other
- Underfitting: model is too "simple" to represent all the relevant class characteristics
 - High bias and low variance
 - High training error and high test error
- Overfitting: model is too "complex" and fits irrelevant characteristics (noise) in the data
 - Low bias and high variance
 - Low training error and high test error

Model 1 Dataset subset 1 Variance Dataset subset 2 Model 2 Dataset subset 3 Model 3 Model 4 Dataset subset 4 Model 5 Dataset subset 5 Average model Bias True model?



 Models with too few parameters are inaccurate because of a large bias (not enough flexibility).

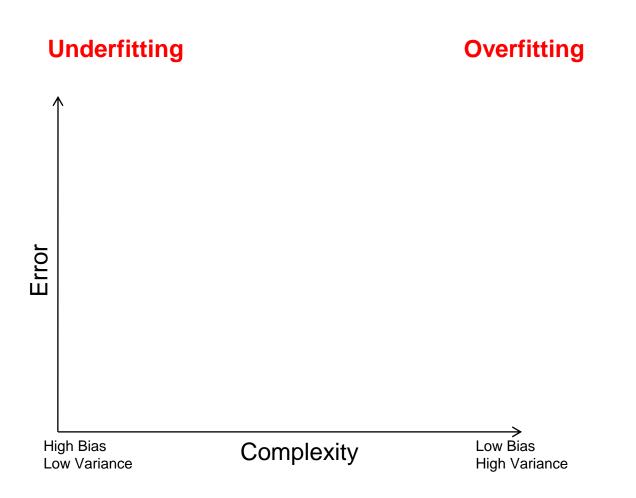


 Models with too many parameters are inaccurate because of a large variance (too much sensitivity to the sample).

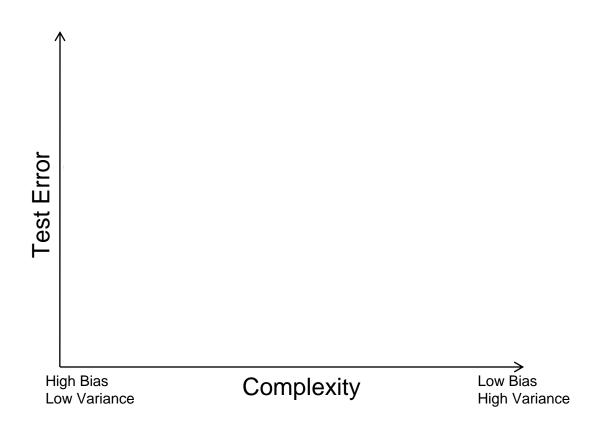
Purple dots = possible test points

Red dots = training data (all that we see before we ship off our model!)

Training vs test error

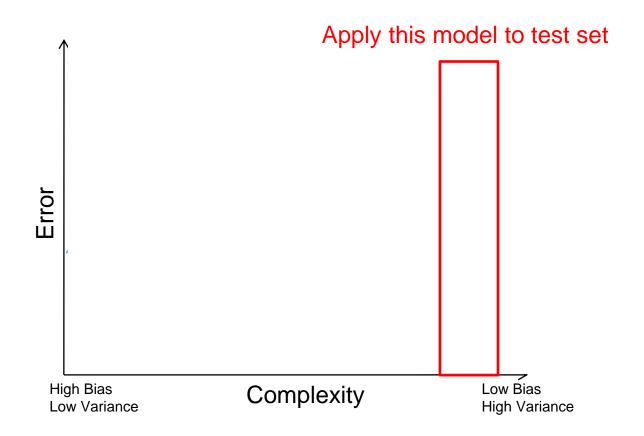


The effect of training set size



Choosing the trade-off between bias and variance

Need validation set (separate from the test set)



Generalization tips

- Try simple classifiers first
- Better to have smart features and simple classifiers than simple features and smart classifiers
- Use increasingly powerful classifiers with more training data
- As an additional technique for reducing variance, try regularizing the parameters (penalize high magnitude weights)



Beyond Bags of Features: Spatial Pyramid Matching for Recognizing Natural Scene Categories

CVPR 2006

Winner of 2016 Longuet-Higgins Prize

Svetlana Lazebnik (slazebni@uiuc.edu)
Beckman Institute, University of Illinois at Urbana-Champaign

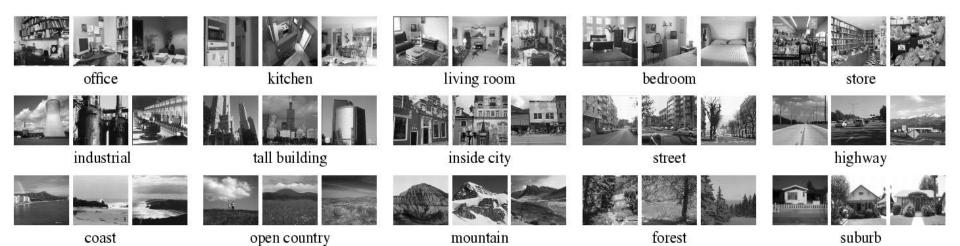
Cordelia Schmid (cordelia.schmid@inrialpes.fr)
INRIA Rhône-Alpes, France

Jean Ponce (ponce@di.ens.fr)
Ecole Normale Supérieure, France

Scene category dataset

Fei-Fei & Perona (2005), Oliva & Torralba (2001)

http://www-cvr.ai.uiuc.edu/ponce grp/data



Bag-of-words representation

- Extract local features
- 2. Learn "visual vocabulary" using clustering
- 3. Quantize local features using visual vocabulary
- 4. Represent images by frequencies of "visual words"

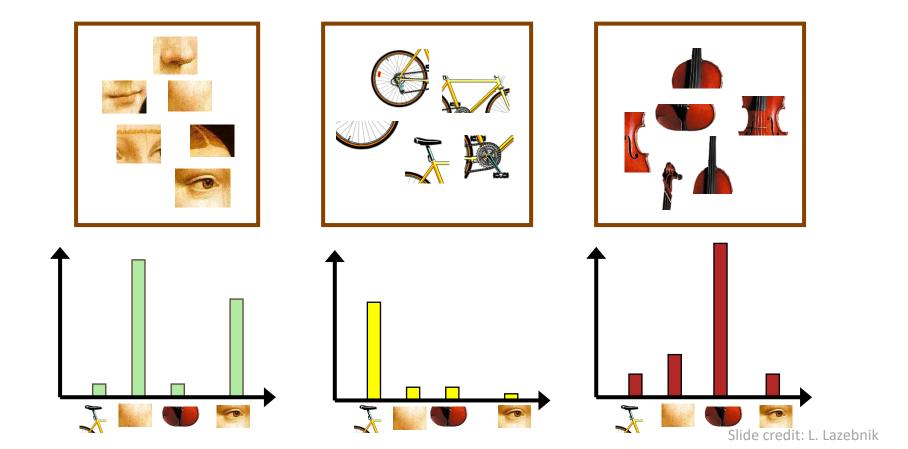


Image categorization with bag of words

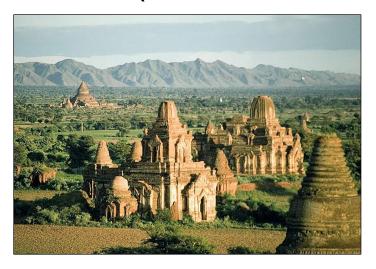
Training

- 1. Compute bag-of-words representation for training images
- 2. Train classifier on labeled examples using histogram values as features
- 3. Labels are the scene types (e.g. mountain vs field)

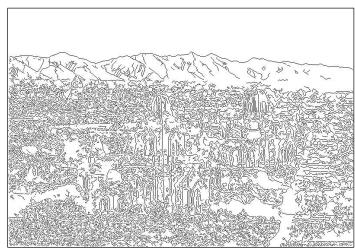
Testing

- 1. Extract keypoints/descriptors for test images
- 2. Quantize into visual words using the clusters computed at training time
- 3. Compute visual word histogram for test images
- 4. Compute labels on test images using classifier obtained at training time
- **5. Evaluation only, do only once:** Measure accuracy of test predictions by comparing them to ground-truth test labels (obtained from humans)

Feature extraction (on which BOW is based)

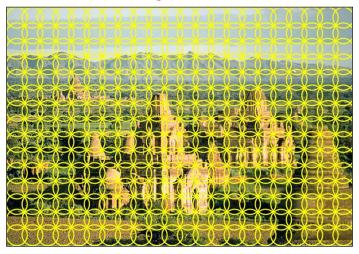


Weak features



Edge points at 2 scales and 8 orientations (vocabulary size 16)

Strong features

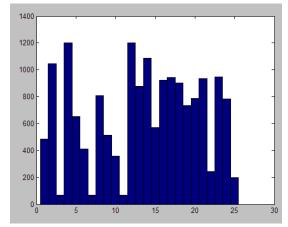


SIFT descriptors of 16x16 patches sampled on a regular grid, quantized to form visual vocabulary (size 200, 400)

What about spatial layout?



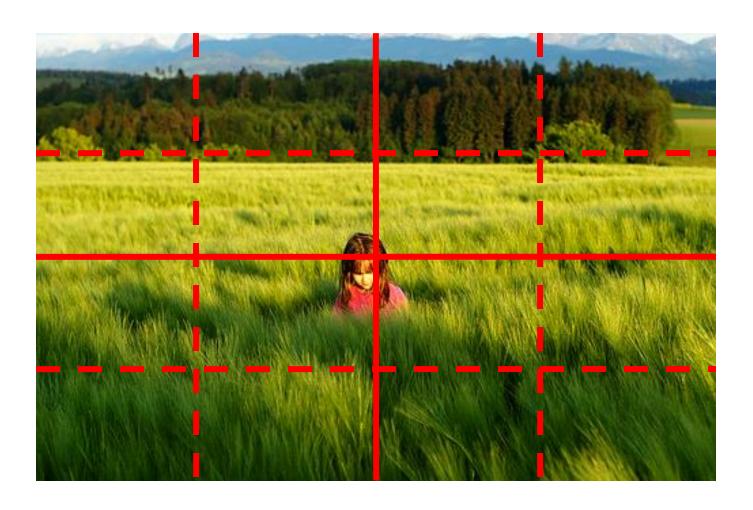






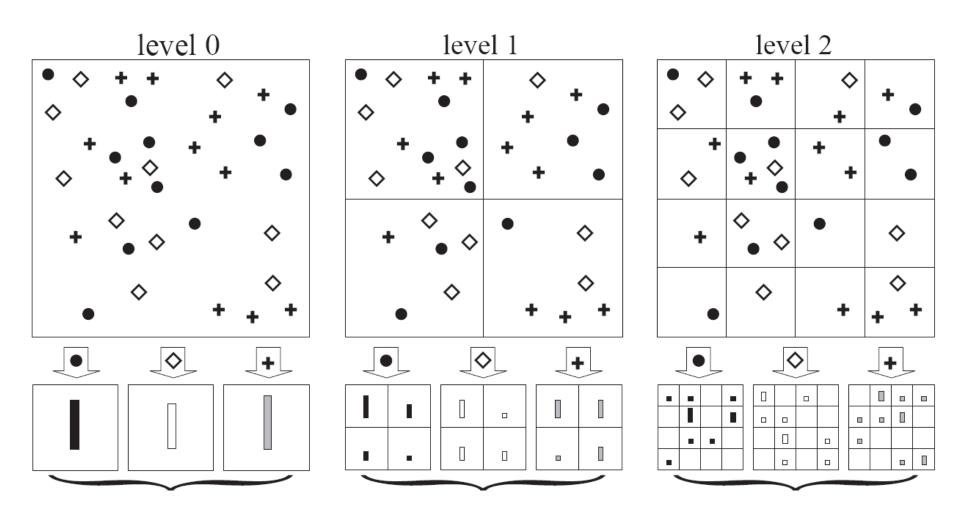
All of these images have the same color histogram

Spatial pyramid



Compute histogram in each spatial bin

Spatial pyramid



[Lazebnik et al. CVPR 2006]

Pyramid matching

Indyk & Thaper (2003), Grauman & Darrell (2005)

Matching using pyramid and histogram intersection for some particular visual word:

Original images





Feature histograms:

Level 3





Level 2

Level 1

Level 0

$$\cap$$
 = I_2

$$\cap$$
 = \mathcal{I}_1

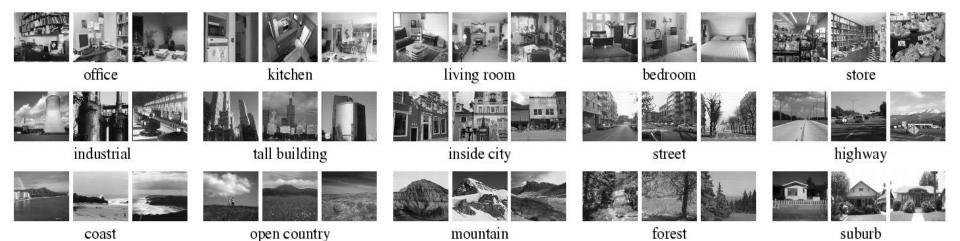
$$\square \cap \square = \mathcal{I}_0$$

 $K(x_i, x_j)$ (value of pyramid match kernel): $I_3 + \frac{1}{2}(I_2 - I_3) + \frac{1}{4}(I_1 - I_2) + \frac{1}{8}(I_0 - I_1)$

Scene category dataset

Fei-Fei & Perona (2005), Oliva & Torralba (2001)

http://www-cvr.ai.uiuc.edu/ponce grp/data

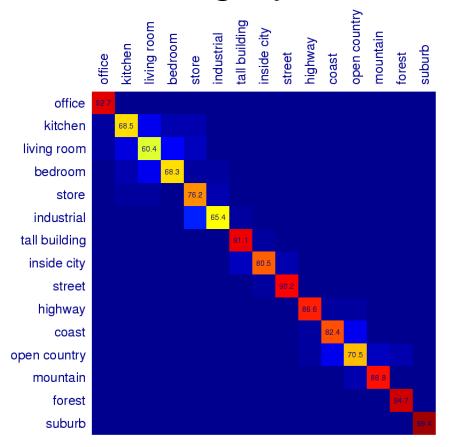


Multi-class classification results (100 training images per class)

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	Weak features		Strong features		
	(vocabulary size: 16)		(vocabulary size: 200)		
Level	Single-level	Pyramid	Single-level	Pyramid	
$0(1 \times 1)$	45.3 ± 0.5		72.2 ± 0.6		
$1(2\times2)$	53.6 ± 0.3	56.2 ± 0.6	77.9 ± 0.6	79.0 ± 0.5	
$2(4\times4)$	61.7 ± 0.6	64.7 ± 0.7	79.4 ± 0.3	81.1 ± 0.3	
$3 (8 \times 8)$	63.3 ± 0.8	66.8 ± 0.6	77.2 ± 0.4	80.7 ± 0.3	

Fei-Fei & Perona: 65.2%

Scene category confusions



Difficult indoor images







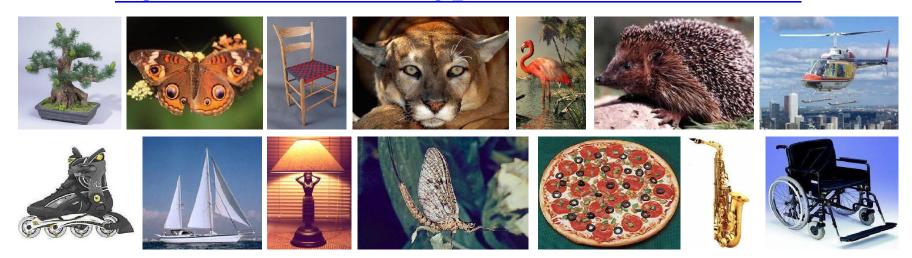
living room

bedroom

Caltech101 dataset

Fei-Fei et al. (2004)

http://www.vision.caltech.edu/Image Datasets/Caltech101/Caltech101.html



Multi-class classification results (30 training images per class)

	Weak features (16)		Strong features (200)	
Level	Single-level	Pyramid	Single-level	Pyramid
0	15.5 ± 0.9		41.2 ± 1.2	
1 1	31.4 ± 1.2	32.8 ± 1.3	55.9 ± 0.9	57.0 ± 0.8
2	47.2 ± 1.1	49.3 ± 1.4	63.6 ± 0.9	64.6 ± 0.8
3	52.2 ± 0.8	54.0 ± 1.1	60.3 ± 0.9	64.6 ± 0.7