Name: Ethan Bolton Date: 8/1/20

Waves and Simple Harmonic Oscillators Test

For each of the following problems, show all work including all equations used, numbers entered into the equations, and then circle your final answer complete with units.

1. A transverse traveling wave on a taut wire has an amplitude of 200 cm and frequency of 500 Hz. It travels with a speed of 196 m/s. (a) Write an equation for this wave if y (x = 5 cm, t = 0) = 0 mm. (b) Determine the vertical position of an element of the wire at t = 1.23 s, x = 0.25 cm. (c) What is the transverse velocity of an element of the wire at at t = 1.23 s, x = 0.25 cm? (d) What is the transverse acceleration of an element of the wire at at t = 1.23 s, x = 0.25 cm?

$A = 0.2\,m$ $v = 196\ m/s$ $f = 500\,Hz$

$\omega = 2\pi f$

$\omega = 1000\pi$

$k = \frac{2\pi}{\lambda}$ $v = \omega/k$

$k = \omega/v = \frac{1000\pi}{196\ m/s}$

$k = 16.03$

a) $y(0.05, 0) = 0 = 0.2 \cos[16.03(0.05) - 1000\pi(0) + \phi]$

$0 = \cos[0.80 + \phi]$

$\frac{\pi}{2} = 0.8 + \phi$

$\phi = \frac{\pi}{2} - 0.8$

$\phi = 0.77$

$y(x,t) = 0.2 \cos[16.03(x) - 1000\pi(t) + 0.77]$

b) $y(x = 0.0025m, t = 1.23s) = 0.2 \cos[16.03(0.0025) - 1000\pi(1.23) + 0.77]$

$[0.04 - 3864 + 0.77]$

$= 0.2 \cos[3864]$

$y(x = 0.0025m, t = 1.23s) = 0.138m$

c) $v(x = 0.0025m, t = 1.23s) = -0.2(1,000\pi)\sin[3864]$

$v(x = 0.0025m, t = 1.23s) = -455\ m/s$

d) $a(x = 0.0025m, t = 1.23s) = -0.2(1,000\pi)^2 \cos[3864]$

$a(x = 0.0025m, t = 1.23s) = -1360\ m/s^2$

2. A source is moving past a sound sensor in an unknown medium. The frequency detected in front of the source was 2700 cycles per second, and the frequency detected behind the source was 1300 Hertz. The object was clocked by a radar gun to be moving at 21 m/s. What is the speed of sound in the medium?

$f'_{front} = 2700\,Hz$ $\quad f'_{behind} = 1{,}300\,Hz$ $\quad f' = f\,\frac{v \pm v_o}{v \mp v_s}$

$v_o = 0\ m/s$

$v_s = 21\ m/s$

$$\frac{1}{f'_{front}} \cdot \frac{v \pm v_o}{v \mp v_s} = \frac{1}{f'_{back}} \cdot \frac{v \pm v_o}{v \mp v_s}$$

$$\left(\frac{1}{2700\,Hz}\right)\left(\frac{v}{v - 21\,m/s}\right) = \left(\frac{1}{1{,}300\,Hz}\right)\left(\frac{v}{v + 21\,m/s}\right)$$

$$\frac{1300}{2700} = \left(\frac{v - 21}{v}\right)\left(\frac{v}{v + 21}\right)$$

$$0.482 = \frac{v - 21}{v + 21}$$

$$v = 60\ m/s$$

3. A particle moving along the x axis in simple harmonic motion starts from its equilibrium position, the origin, at t = 0 s and moves to the right. The amplitude of its motion is 2.00 cm, and the frequency is 1.50 Hz. (a) Determine the position, velocity, and acceleration equations for this particle. (b) Determine the maximum speed of this particle and the first time it reaches this speed after t=0 s.

$A = 0.02\,m$ $\quad \omega = 2\pi f$

$f = 1.5\ Hz$ $\quad \omega = 3\pi$

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a) $x(0) = 0.02\cos[3\pi(0)]$

$x(0) = 0.02\,m$

$v(0) = -0.02(3\pi)\sin[3\pi(0)]$

$v(0) = 0\ m/s$

$a(0) = -0.02(3\pi)^2\cos[3\pi(0)]$

$a(0) = -0.59\ m/s^2$

b) $V_{max} = -0.02(3\pi)\sin[3\pi(t)] = -0.02(3\pi)\sin(\pi/2)$

$\sin[3\pi(t)] = 1$

$3\pi(t) = \pi/2$

$t = \dfrac{\pi/2}{3\pi}$

$t = \frac{1}{6}\,s$

$t = 0.166\,s$

$|V_{max}| = A\omega$

$= 0.02(3\pi)$

$V_{max} = 0.188\ m/s$

4. The motion of an object moving in simple harmonic motion is given by

$$x(t) = (0.1\text{ m})[\cos(\omega t) + \sin(\omega t)]$$

where $\omega = 3\pi$. (a) Determine the velocity and acceleration equations. (b) Determine the position, velocity, and acceleration at time t = 2.4 s. **(Calculus Physics only)**

a) $v(t) = -(0.1)(3\pi)[\sin(3\pi t) - \cos(3\pi t)]$

$a(t) = -(0.1)(3\pi)^2[\cos(3\pi t) + \sin(3\pi t)]$

b) $x(2.4) = 0.1[\cos(3\pi(2.4)) + \sin(3\pi(2.4))]$

$= 0.1[-0.809 + -0.58]$

$x = -0.140\text{ m}$

$v(2.4) = -(0.1)(3\pi)[\sin(3\pi(2.4)) - \cos(3\pi(2.4))]$

$= -(0.1)(3\pi)[0.221]$

$v = -0.209\text{ m/s}$

$a(2.4) = -(0.1)(3\pi)^2[\cos(3\pi(2.4)) + \sin(3\pi(2.4))]$

$= -(0.1)(3\pi)^2[-1.397]$

$a = 12.41\text{ m/s}^2$