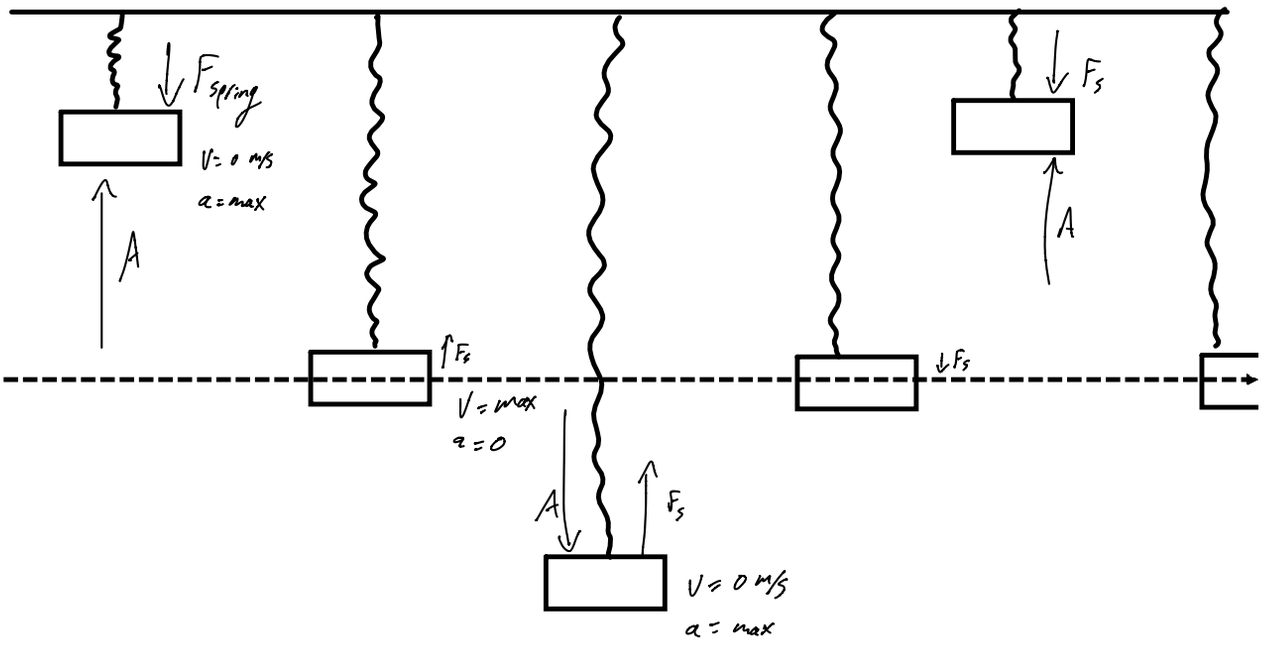
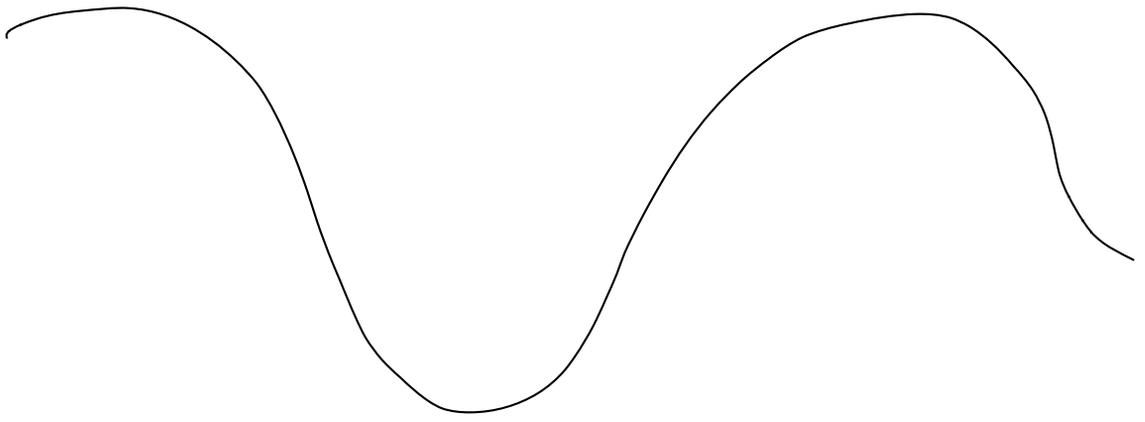
Simple Harmonic Motion





$$x(t) = A\cos[\omega t + \phi]$$

$$v(t) = -A\omega \sin[\omega t + \phi]$$

$$a(t) = -A\omega^2 \cos[\omega t + \phi]$$

$$\omega = 2\pi f = \frac{2\pi}{T}$$

f = # of times in 1 sec

T = time to do one cycle

$$\omega = \sqrt{\frac{k}{m}}$$

$F_s = kx$

k: spring constant

1. The displacement of a particle at t = 0.25 s is given by the expression
   x(t) = (4.0 m) cos (3.0π t + π),
   where x is in meters and t is in seconds. Determine (a) the frequency and period of the motion, (b) the amplitude of the motion, (c) the phase constant, and (d) the displacement of the particle at t = 0.25 s.

a) $\omega = 3\pi$

$\omega = 2\pi f$

$f = 1.5\ Hz$

$T = 1/f$

$= 1/1.5$

$T = 0.66\ s$

b) $A = 4.0\ m$

c) $\phi = \pi$

d) $x(0.25) = 4\cos(3\pi(0.25) + \pi)$

$x = 2.83\ m$

2. A simple harmonic oscillator takes 12.0 s to undergo five complete vibrations. Find (a) the period of its motion, (b) the frequency in Hz, and (c) the angular frequency in rad/s.

a) $T = \frac{12\,s}{5}$

$T = 2.4\,s$

b) $f = \frac{1}{T}$

$= \frac{1}{2.4}$

$f = 0.417\,Hz$

c) $\omega = 2\pi f$

$= 2\pi(0.417)$

$\omega = 2.62\ \frac{rad}{s}$

$= 0.834\pi$

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where $A = 2\,cm$ & where $t = 0\,s$ $x(t) = 0\,m$,

Write the eq.

$$x = 0.02\cos\left[(0.834\pi)(t) + \phi\right]$$

$$0 = \cos[\phi]$$

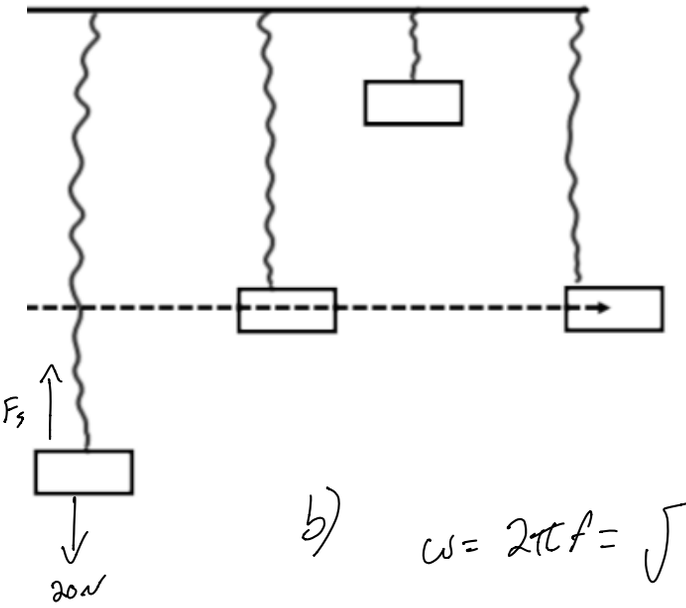
$$\phi = \cos^{-1}(0)$$

$$\phi = \pi/2$$

$$x(t) = 0.02\cos\left[2.62t + \frac{\pi}{2}\right]$$

3. A 1.5 kg block at rest on a tabletop is attached to a horizontal spring having a spring constant of 19.6 N/m. The spring is initially unstretched. A constant 20.0 N horizontal force is applied to the object causing the spring to stretch. Once stretched it is released. (a) Determine the maximum displacement, (b) the frequency and period of the mass on the spring, (c) write the equation which describes this motion, and (d) determine the velocity of the block at t = 0.015 s.

$k = 19.6\ N/m$



a) $f = kA$

$$A = \frac{(20\ N)}{(19.6\ \frac{N}{m})}$$

$A = 1.02\ m$

b) $\omega = 2\pi f = \sqrt{k/m}$

$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

$f = \frac{1}{2\pi}\sqrt{\frac{19.6}{1.5}}$

$f = 0.575\ Hz$

$T = 1/f$

$= \frac{1}{0.575}$

$T = 1.74\ s$

$\omega = 3.61$

c) $x(t) = 1.02\cos[3.61t + \phi]$

$x(0) = 1.02\ m = 1.02\cos(3.61(0) + \phi)$

$1 = \cos(\phi)$

$\phi = \cos^{-1}(1)$

$\phi = 0$

$x(t) = 1.02\cos[3.61t]$

$x(t) = 1.02\cos[3.61t]$

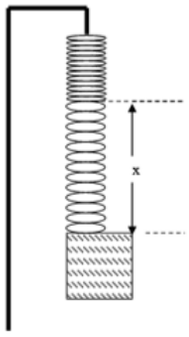
$v(t) = 1.02(3.61)\cos(3.61t)$

$v(0.015) = 1.02(3.61)\cos(3.61 \cdot 0.015)$

$v = 0.200\ m/s$

4. A mass m is oscillating freely on a vertical spring. When m = 0.810 kg, the period is 0.910 s. An unknown mass on the same spring has a period of 1.16 s. Determine (a) the spring constant k and (b) the unknown mass.

5. A spring of constant k = 100 N/m hangs at its natural length from a fixed stand. A mass of 3 kg is hung on the end of the spring, and slowly let down until the spring and mass hang at their new equilibrium position.



(a) Find the value of the quantity x in the figure above. The spring is now pulled down an additional distance x and released from rest. (b) What is the potential energy in the spring at this distance? (c) What is the speed of the mass as it passes the equilibrium position? (d) How high above the point of release will the mass rise? (e) What is the period of oscillation for the mass?

1. **(a)** A spring stretches by 0.015 m when a 1.75 kg object is suspended from its end. How much mass should be attached to the spring so that its frequency of vibration is $f = 3.0$ Hz? **(b)** An oscillating block-spring system has a mechanical energy of 1.00 J, an amplitude of 10.0 cm, and a maximum speed of 1.20 m/s. Find the spring constant, the mass of the block, and the frequency of oscillation.

A 20 g particle moves in simple harmonic motion with a frequency of 3.0 oscillations/s (3.0 Hz) and an amplitude of 5.0 cm.
(a) Through what total distance does the particle move during one cycle of its motion?
(b) What is its maximum speed? Where does that occur?
(c) Find the maximum acceleration of the particle. Where does that occur?

A block of unknown mass is attached to a spring of spring constant 6.50 N/m and undergoes simple harmonic motion with an amplitude of 10.0 cm. When the mass is halfway between its equilibrium position and the endpoint, its speed is measured to be + 30.0 cm/s.

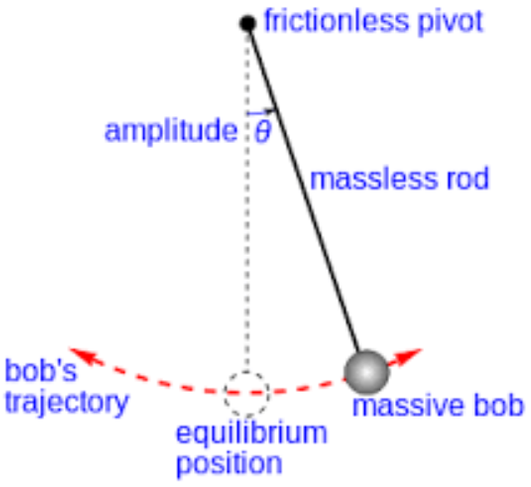
Calculate

(a) the mass of the block,

(b) the period of the motion, and

(c) the maximum acceleration of the block

Pendulum


frictionless pivot
amplitude θ
massless rod
bob's trajectory
equilibrium position
massive bob

A child swings on a playground swing with a 2.5 m long chain.

a) What is the period of the child's motion?

b) What is the frequency of the vibration?