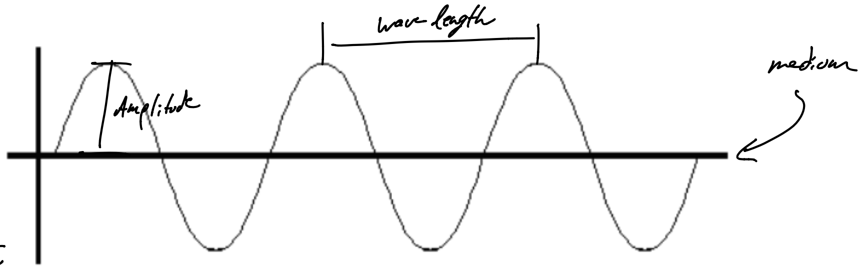
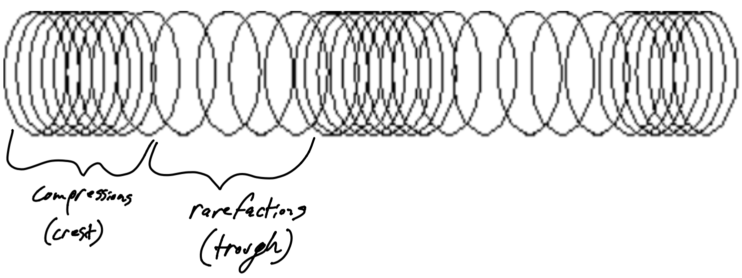
Waves

Transverse
wave: part of the
medium moves ⊥
to direction of the
wave

(water moves up
& down with waves,
but not side to side)

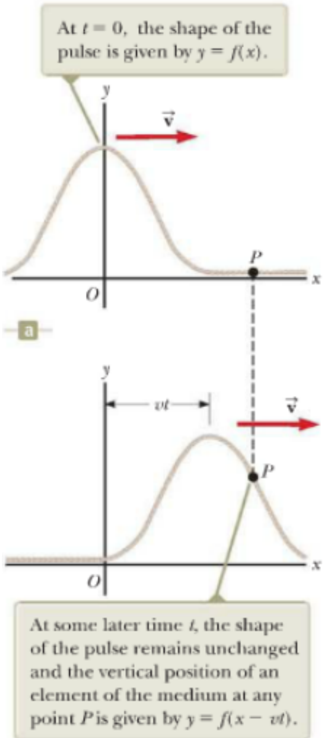


Compression/Longitudinal wave:
medium moves parallel
to the wave



∅ on y-axis would be
normal distance
(+/- from normal distance)

Wave Function



$y(x,t) = f(x,t)$

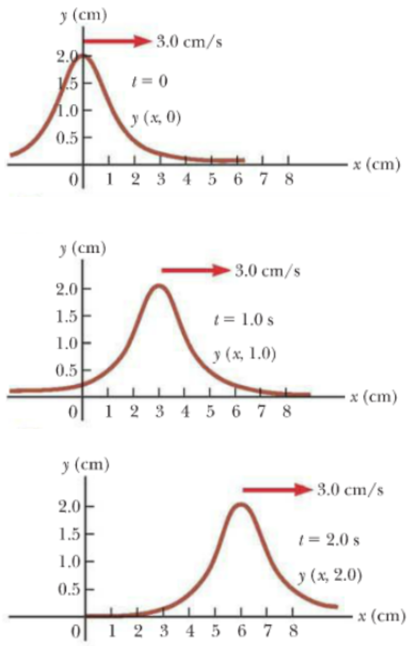
A pulse moving to the right along the x axis is represented by the wave function

$$y(x, t) = \frac{2}{(x - 3.0t)^2 + 1}$$

where x and y are measured in centimeters and t is measured in seconds. Find expressions for the wave function at $t = 0$, $t = 1.0$ s, and $t = 2.0$ s.

velocity func is the derivative of the position function

Accel func is the derivative of the velocity function



$$v(x,t) = \frac{d}{dt}\, y(x,t)$$

$$a(x,t) = \frac{d}{dt}\, v(x,t)$$

$$y(x, t) = A \sin\left[kx - \omega t + \phi\right]$$

A: Amplitude (usually in m)

k: wave number ($1/m$)

ω: angular velocity ($\frac{rad}{sec}$)

t: time (sec)

ϕ: phase constant

f: frequency (Hz)

T: period (s)

λ: wavelength (m)

V: speed of wave (m/s)

$f = \frac{1}{T}$ or $T = 1/f$

$\omega = 2\pi f = \frac{2\pi}{T}$

$k = \frac{2\pi}{\lambda}$ rad/m

$v = f\lambda = \omega/k$

Calculus

$$y(x,t) = A \sin[kx - \omega t + \phi]$$

$$v(x,t) = \frac{d}{dt} y(x,t) = A \cos[kx - \omega t + \phi](-\omega)$$

$$\boxed{v(x,t) = -A\omega \cos[kx - \omega t + \phi]}$$

velocity of a particle in the medium @ x & t

$$a(x,t) = \frac{d}{dt} v(x,t) = -A\omega\{-\sin[kx - \omega t + \phi]\}(-\omega)$$

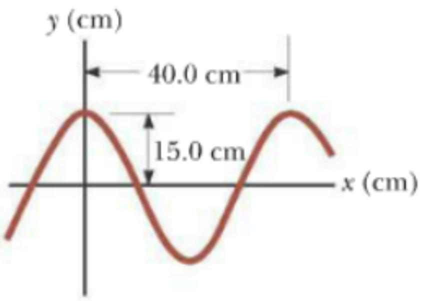
$$\boxed{a(x,t) = -A\omega^2 \sin[kx - \omega t + \phi]}$$

acceleration of a particle in the medium @ x & t

A sinusoidal wave traveling in the positive x direction has an amplitude of 15.0 cm, a wavelength of 40.0 cm, and a frequency of 8.00 Hz. The vertical position of an element of the medium at $t = 0$ and $x = 0$ is also 15.0 cm as shown in Figure 16.9.

(A) Find the wave number k, period T, angular frequency ω, and speed v of the wave.

(B) Determine the phase constant ϕ and write a general expression for the wave function.



$A = 15\text{ cm}$ $\quad \lambda = 0.4\text{ m}$ $\quad f = 8\text{ Hz}$

$y(0,0) = 15\text{cm}$

a)

$k = \frac{2\pi}{\lambda} = \frac{2\pi}{0.4}$

$k = 5\pi$

$T = 1/f = \frac{1}{8\text{Hz}}$

$T = 1/8$

$= 0.125\text{ secs}$

$\omega = 2\pi f = 2\pi(8\text{ Hz})$

$\omega = 16\pi$

$v = f\lambda = \frac{8\text{ Hz}}{0.4\text{ m}}$

$v = 3.2\text{ m/s}$

b) $y(x,t) = A\sin[kx - \omega t + \phi]$

$y(x,t) = (15\text{ cm})\sin[5\pi x - 16\pi t + \phi]$

$y(0,0) = (15\text{ cm})\sin[5\pi x - 16\pi t + \phi]$

$y(0,0) = 15\text{ cm}$

$15\text{ cm} = (15\text{ cm})\sin[\cancel{5\pi(0)} - \cancel{16\pi(0)} + \phi]$

$15\text{ cm} = (15\text{ cm})\sin\phi$

$\phi = \sin^{-1}(1)$

$\boxed{\phi = \pi/2}$

Wave function

$y(x,t) = (15\text{cm})\sin[5\pi x - 16\pi t + \frac{\pi}{2}]$

Check

A wave is described by $y = 0.0200 \sin(kx - \omega t)$, where $k = 2.11$ rad/m, $\omega = 3.62$ rad/s, x and y are in meters, and t is in seconds. Determine (a) the amplitude, (b) the wavelength, (c) the frequency, and (d) the speed of the wave.

a) $A = 0.02$ (m?)

b) $k = \frac{2\pi}{\lambda}$

$\lambda = \frac{2\pi}{k}$

$\lambda = \frac{2\pi}{2.11\ \text{rad/m}}$

$\lambda = 2.978\ \text{m}$

or $2.11\ \text{rad} = 2.11\left(\frac{180}{\pi}\right)$

$\therefore \lambda = 0.052\ \text{m}$

c) $\omega = 2\pi f$

$f = \frac{\omega}{2\pi}$

$f = \frac{3.62\ \frac{\text{rad}}{\text{s}}}{2\pi}$

$f = 5.68\ \text{Hz}$

or $\text{rad} = r\left(\frac{180}{\pi}\right)$

$\therefore f = 325.8\ \text{Hz}$

d) $v = f/\lambda$

HW

The wave function for a traveling wave on a taut string is (in SI units)

$$y(x, t) = 0.350 \sin\left(\underbrace{10\pi}_{\omega} t - \underbrace{3\pi}_{k} x + \frac{\pi}{4}\right)$$

(a) What are the speed and direction of travel of the wave? (b) What is the vertical position of an element of the string at $t = 0$, $x = 0.100$ m? What are (c) the wavelength and (d) the frequency of the wave? (e) What is the maximum transverse speed of an element of the string?

a) $v = \omega / k$

$= \frac{10\pi}{3\pi}$

$v = 3.33\ m/s$ right to left

b) $y(x=0.1m, t=0) = 0.35 \sin\left(\cancel{10\pi(0)} - 3\pi(0.1) + \frac{\pi}{4}\right)$

$y = 0.35 \sin\left(-\frac{\pi}{20}\right)$

$= 0.35(-0.156)$

$y = -0.054\ m$

$\frac{3\pi}{10} + \frac{\pi}{4}$

$-\frac{6\pi}{20} + \frac{5\pi}{20} = -\frac{\pi}{20}$

c) $\lambda = \frac{2\pi}{k}$

$= \frac{2\pi}{3\pi}$

$\lambda = 0.66\ m$

$\lambda = 2/3\ m$

d) $f = \frac{\omega}{2\pi}$

$= \frac{10\pi}{2\pi}$

$f = 5\ Hz$

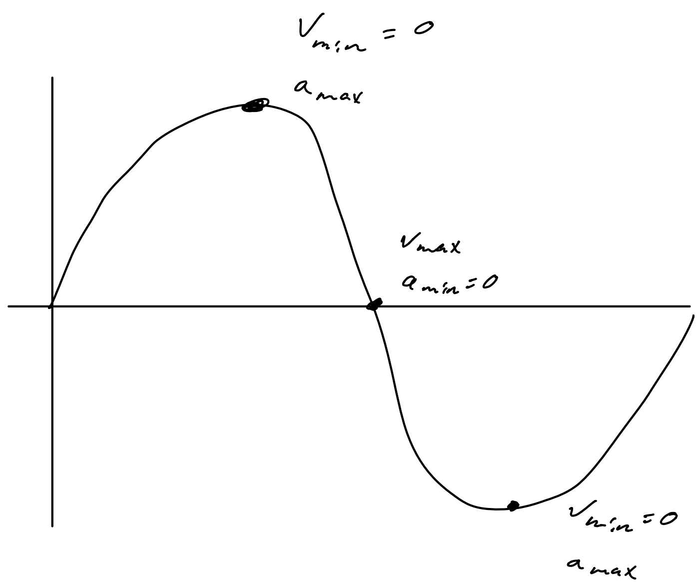
e) $v = -A\omega \cos[\ldots]$

$v_{max} = -A\omega(1)$

$|v_{max}| = A \cdot \omega$

$= (0.35)(10\pi)$

$v_{max} = 10.995\ m/s$



An 80.0-kg hiker is trapped on a mountain ledge following a storm. A helicopter rescues the hiker by hovering above him and lowering a cable to him. The mass of the cable is 8.00 kg, and its length is 15.0 m. A sling of mass 70.0 kg is attached to the end of the cable. The hiker attaches himself to the sling, and the helicopter then accelerates upward. Terrified by hanging from the cable in midair, the hiker tries to signal the pilot by sending transverse pulses up the cable. A pulse takes 0.250 s to travel the length of the cable. What is the acceleration of the helicopter? Assume the tension in the cable is uniform.

Engineering Physics Ch. 16

15. A transverse wave on a string is described by the wave **W** function

$$y = 0.120 \sin\left(\frac{\pi}{8}x + 4\pi t\right)$$

where x and y are in meters and t is in seconds. Determine (a) the transverse speed and (b) the transverse acceleration at $t = 0.200$ s for an element of the string located at $x = 1.60$ m. What are (c) the wavelength, (d) the period, and (e) the speed of propagation of this wave?

a) $v(x,t) = \frac{\partial}{\partial t} y(x,t)$

$$= (0.12)\cos\left[\frac{\pi}{8}x + 4\pi t\right](4\pi)$$

$$v(x=1.6, t=0.2) = (4\pi)(0.12)\cos\left[\frac{\pi}{8}(1.6) + 4\pi(0.2)\right]$$

17. A sinusoidal wave is described by the wave function $y = 0.25 \sin(0.30x - 40t)$ where x and y are in meters and t is in seconds. Determine for this wave (a) the amplitude, (b) the angular frequency, (c) the angular wave number, (d) the wavelength, (e) the wave speed, and (f) the direction of motion.

a) $A = 0.25\ \text{m}$

b) $\omega = 40\ \frac{\text{rad}}{\text{s}}$

c) $k = 0.3$

d) $\lambda = \frac{2\pi}{k}$

$= \frac{2\pi}{0.3}$

$\lambda = 20.94\ \text{m}$

e) $v = \frac{\omega}{k}$

$= \frac{40}{0.3}$

$v = 133.3\ \text{m/s}$

f) L to R

18. GP A sinusoidal wave traveling in the negative x direction (to the left) has an amplitude of 20.0 cm, a wavelength of 35.0 cm, and a frequency of 12.0 Hz. The transverse position of an element of the medium at $t = 0$, $x = 0$ is $y = -3.00$ cm, and the element has a positive velocity here. We wish to find an expression for the wave function describing this wave. (a) Sketch the wave at $t = 0$. (b) Find the angular wave number k from the wavelength. (c) Find the period T from the frequency. Find (d) the angular frequency ω and (e) the wave speed v. (f) From the information about $t = 0$, find the phase constant ϕ. (g) Write an expression for the wave function $y(x, t)$.

19. (a) Write the expression for *y* as a function of *x* and *t* in SI units for a sinusoidal wave traveling along a rope in the negative *x* direction with the following characteristics: $A = 8.00$ cm, $\lambda = 80.0$ cm, $f = 3.00$ Hz, and $y(0, t) = 0$ at $t = 0$. (b) **What If?** Write the expression for *y* as a function of *x* and *t* for the wave in part (a) assuming $y(x, 0) = 0$ at the point $x = 10.0$ cm.

$A = 0.08$ m

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{0.8}$$

$$\omega = 2\pi f = 2\pi(3\text{ Hz}) = 6\pi$$

$$k = 7.85 = 2.5\pi$$

$$y(x,t) = 0.08 \sin\left[\frac{5}{2}\pi x - 6\pi t\right]$$

20. A transverse sinusoidal wave on a string has a period $T = 25.0$ ms and travels in the negative x direction with a speed of 30.0 m/s. At $t = 0$, an element of the string at $x = 0$ has a transverse position of 2.00 cm and is traveling downward with a speed of 2.00 m/s. (a) What is the amplitude of the wave? (b) What is the initial phase angle? (c) What is the maximum transverse speed of an element of the string? (d) Write the wave function for the wave.

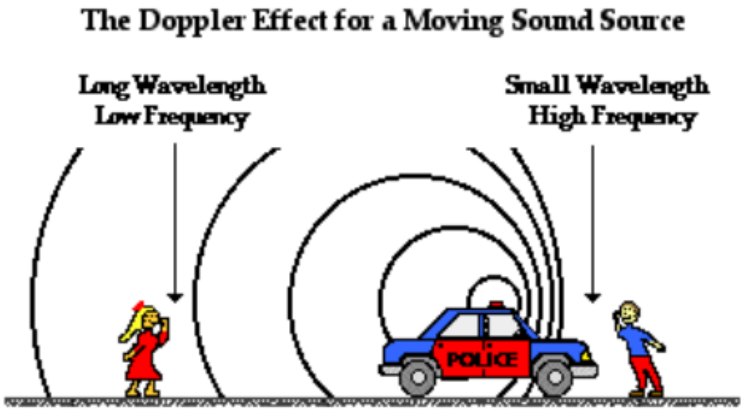
$y(0,0) = 0.02\ m$

$v(0,0) = -2\ m/s$

a) $A = 0.02\ m$

b)

Doppler Effect



$V = \lambda f$

if $\lambda \uparrow$ then $f \downarrow$

$$f' = f\,\frac{V \pm V_0}{V \mp V_s}$$

V_0 moving towards the source
V_0 moving away
V_s moving toward observer
V_s moving away

f: emitted freq
f': measured/observed freq
V: speed of wave
V_0: speed of observer
V_s: speed of the source

V_{sound}: 340 m/s
V_{light}: $c = 3 \times 10^8$ m/s

A commuter train passes a passenger platform at a constant speed of 40.0 m/s. The train horn is sounded at its characteristic frequency of 320 Hz. (a) What overall change in frequency is detected by a person on the platform as the train moves from approaching to receding? (b) What wavelength is detected by a person on the platform as the train approaches?

$V_s = 40$ m/s　　$f = 320$ Hz

$V_0 = 0$ m/s　　$V = V_{sound} = 340$ m/s

a) $\Delta f = f_r' - f_a'$

$f_a' = 320\left(\frac{340}{340-40}\right)$

$= 320\left(\frac{340}{300}\right)$

$f_a' = 363$ Hz

$f_r' = 320\left(\frac{340}{340+40}\right)$

$= 320\left(\frac{340}{380}\right)$

$f_r' = 286$ Hz

$\Delta f = 286 - 363$

$\Delta f = -77$ Hz

b) $V = f_a' \lambda_a'$

$\lambda_a' = \frac{V}{f_a'}$

$= \frac{340 \text{ m/s}}{363 \text{ Hz}}$

$\lambda_a' = 0.937$ m

Units

$\frac{\text{m/s}}{1/\text{s}}$

$= \frac{\text{m}}{\text{s}} \cdot \frac{\text{s}}{1}$

$= \frac{\text{m}}{1} = \text{m}$

$V_{sound} = 340 \ m/s$

An airplane traveling at half the speed of sound emits a sound of frequency 5.00 kHz. At what frequency does a stationary listener hear the sound (a) as the plane approaches? (b) After it passes?

$V_s = \frac{1}{2} V_{sound}$

$= \frac{1}{2}(340)$

$V_s = 170 \ m/s$

$f = 5 kHz = 5{,}000 \ Hz$

$f' = ?$

$V_0 = 0 \ m/s$

a) $f'_{approaching} = 5kHz \dfrac{340 + 0}{340 - 170}$

$= 5 \dfrac{340}{170}$

$f' = 5(2)$

$f' = 10 \ kHz$

b) $f'_{retreating} = 5kHz \dfrac{340 - 0}{340 + 170}$

$= 5\left(\dfrac{340}{510}\right)$

$= 5(0.66)$

$f' = 3.33 \ kHz$

Homework

An alert physics student stands beside the tracks as a train rolls slowly past. He notes that the frequency of the train whistle is 465 Hz when the train is approaching him and 441 Hz when the train is receding from him. Using these frequencies, he calculates the speed of the train. What value does he find?

Homework

A train approaching a station at a speed of 40 m/s sounds a 2000 Hz whistle. (a) What is the apparent frequency heard by an observer standing at the station? (b) What is the drop in frequency heard as the train passes by?

A train going 40 m/s approaches a crossing bell whose frequency is 820 Hz. (a) What is the frequency heard by the passengers of the train? (b) What is the frequency heard after the train passes the bell?

You are standing by the railroad track when a train sounding a 750 Hz whistle passes by you at 80 km/h. What is the difference in the frequencies you hear as the train approaches and departs?

$f = 440 \quad V_s = \emptyset \qquad V = V_{sound} = 340\ m/s$

$V_o = (340\ m/s)(0.9)$

A 440 Hz source has been sounding in air for a long time. (a) What frequency will you here if you move away from the source at 90% the speed of sound? (b) What frequency will you here if you move away from the source at the speed of sound?

a)

$$f' = f\,\frac{v \pm v_o}{v \mp v_s}$$

$$= 440\,\frac{340 - (0.9)(340)}{340}$$

$$= 440\left(\frac{34}{340}\right)$$

$$= 440\left(\frac{1}{10}\right)$$

$$f' = 44\ Hz$$

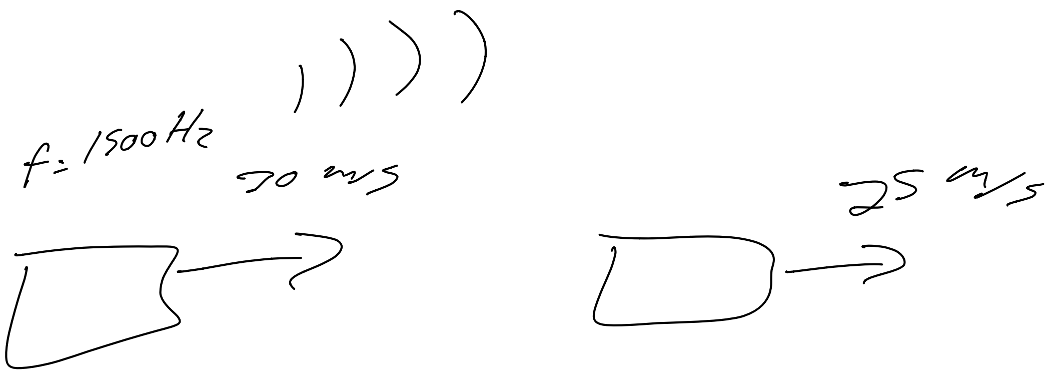
b) $V_o = 340\ m/s$

$$f' = 440\,\frac{340 - 340}{340}$$

$$= 440(\emptyset)$$

$$f' = 0\ Hz$$

No waves passing observe ∴ No sound



A car traveling at 30 m/s overtakes another car moving only at 25 m/s. When the faster car is still behind the slower car, it sounds a horn of frequency 1500 Hz. What is the frequency heard by the driver of the slower car?

$$f' = f\,\frac{v - v_o}{v - v_s}$$

$$= 1500\,\frac{340 - 25}{340 - 30}$$

$$= 1500\,(1.016)$$

$$f' = 1{,}524 \text{ Hz}$$

Hw

If you move at 15 m/s toward a 2000 Hz source that is moving toward you with a ground speed of 40 m/s, what frequency do you hear?

$f = 6000\ Hz$

$\Delta f = -30\ Hz$

If you detect a 30 Hz frequency shift in the 6000 Hz bell of a bicycle as it approaches and then leaves you, how fast is the bicycle going?

A trailer truck heading east at 30 m/s sounds a 1000 Hz horn. (a) What is the frequency heard by an approaching car heading west at 40 m/s? (b) What frequency is heard if the car is heading east?

V_s λ

A car moving at 70 km/h shines a 690 nm laser at an observer with a light detector. What frequency does the observer's detector register?

$V_s = 19.4 \ m/s$

$V = c = 3\times10^{8} \ m/s$

$\lambda_s = 690 \ nm = 690\times10^{-9} m$

$f = V/\lambda = \frac{3\times10^{8}}{690\times10^{-9}}$

$f = 4.35\times10^{14} \ Hz$

$= 435 \ THz$

$f' = f \ \frac{V \pm V_o}{V \mp V_s}$

$f' = 4.35\times10^{14}\left(\frac{3\times10^{8} \ m/s}{3\times10^{8} \ m/s - 19.4}\right)$

$f' = 4.350000281\times10^{14}$

very small undetectable diff

Using the same Doppler shift principles for light waves, what is the apparent frequency of light to an earth bound observer, if the light is coming from a star moving away from the earth at 7500 km/s as the earth moves away from the star at 29.9 km/s. The frequency of the light from the star if 600 THz.

$f = 600\ THz$

$V = c = 3\times10^{8}\ m/s$

$V_s = 7500\ km/s = 7.5\times10^{6}\ m/s$

$V_o = 29.9\ km/s = 2.99\times10^{4}\ m/s$

$$f' = f\,\frac{V \pm V_o}{V \mp V_s}$$

$$= 600\ THz\left(\frac{(3\times10^{8}) - (2.99\times10^{4})}{(3\times10^{8}) + (7.5\times10^{6})}\right)$$

$$= 600\ THz\,(0.9755)$$

$$f' = 585.3\ THz$$