



$$\Delta x = V_{0x} t + \frac{1}{2} h_x t^2$$

$$V_x = \alpha_x + h_x t$$

$$V_x^2 = V_{0x}^2 + 2 h_x \Delta x$$

$$\Delta f = -\Delta U$$

$$\sum \vec{\rho}_i = \vec{\rho}_f$$

$$\sum \vec{F} = m \vec{a}$$

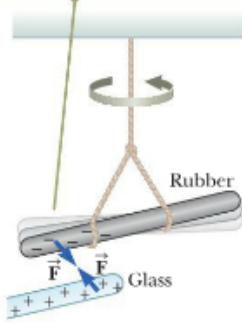
$$\sum F_x = m a_x$$

$$\sum F_y = m a_y$$

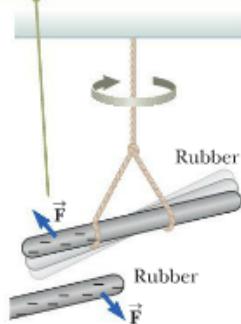
$$\sum I = I \alpha$$

$$\alpha_c = \frac{V^2}{r}$$

A negatively charged rubber rod suspended by a string is attracted to a positively charged glass rod.



A negatively charged rubber rod is repelled by another negatively charged rubber rod.



force from gravity

$$F = G \frac{m_1 m_2}{r^2}$$

force from charge

Coulomb's Law charges

$$F_c = k_e \frac{|q_1||q_2|}{r^2}$$

Coulomb's constant:

$$k_e = 8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}$$

elementary charge (charge of proton)

$$e = 1.602 \times 10^{-19} \text{ C}$$

charge of electron:  $e^-$

# Coulomb's Law

$$F_e = k_e \frac{|q_1| |q_2|}{r^2}$$

*Masses given on test?*



**Table 23.1 Charge and Mass of the Electron, Proton, and Neutron**

Particle	Charge (C)	Mass (kg)
Electron (e)	$-1.602\ 176\ 5 \times 10^{-19}$	$9.109\ 4 \times 10^{-31}$
Proton (p)	$+1.602\ 176\ 5 \times 10^{-19}$	$1.672\ 62 \times 10^{-27}$
Neutron (n)	0	$1.674\ 93 \times 10^{-27}$

The electron and proton of a hydrogen atom are separated (on the average) by a distance of approximately  $5.3 \times 10^{-11} \text{ m}$ . Find the magnitudes of the electric force and the gravitational force between the two particles.

$$F_e = k \frac{|q_1||q_2|}{r^2}$$

$$F_e = \left(8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right) \frac{|1.602 \times 10^{-19} \text{C} // -1.602 \times 10^{-19} \text{C}|}{(5.3 \times 10^{-11} \text{m})^2}$$

$$F_e = 8.21 \times 10^{-8} \text{ N}$$

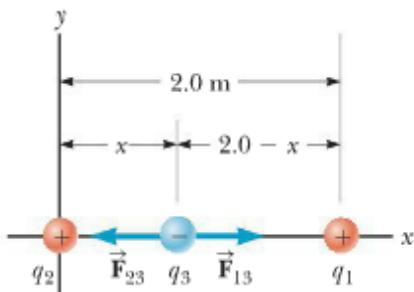
$$F_g = G \frac{m_1 m_2}{r^2}$$

$$F_g = \left(6.674 \times 10^{-11}\right) \frac{(1.67 \times 10^{-27})(1.67 \times 10^{-27})}{(5.3 \times 10^{-11})^2}$$

$$F_g = 3.61 \times 10^{-47} \text{ N}$$

gravity is nominal here

**PROBLEM** Three charges lie along the  $x$ -axis as in Figure 15.7. The positive charge  $q_1 = 15 \mu\text{C}$  is at  $x = 2.0 \text{ m}$ , and the positive charge  $q_2 = 6.0 \mu\text{C}$  is at the origin. Where must a *negative* charge  $q_3$  be placed on the  $x$ -axis so that the resultant electric force on it is zero?



$$F_{23} = k \frac{|q_2||q_3|}{x^2}$$

$$F_{13} = k \frac{|q_1||q_3|}{(2-x)^2}$$

$$F_{23} = k \frac{|q_2||q_3|}{x^2}$$

$$F_{23} - F_{13} = 0$$

$$F_{23} = F_{13}$$

$$k \frac{|q_2||q_3|}{x^2} = k \frac{|q_1||q_3|}{(2-x)^2}$$

$$\frac{|q_2|}{x^2} = \frac{|q_1|}{(2-x)^2}$$

$$|q_2|(2-x)^2 = |q_1|x^2$$

$$|q_2|(4-4x+x^2) = |q_1|x^2$$

$$4|q_2| - 4x|q_2| + x^2|q_2| - |q_1|x^2 = 0$$

$$(4|q_2| - |q_1|)x^2 - 4|q_2|x + 4|q_2| = 0$$

$$(6 \cdot 10^{-6}) - (15 \cdot 10^{-6})x^2 - 4(6 \cdot 10^{-6})x + 4(6 \cdot 10^{-6}) = 0$$

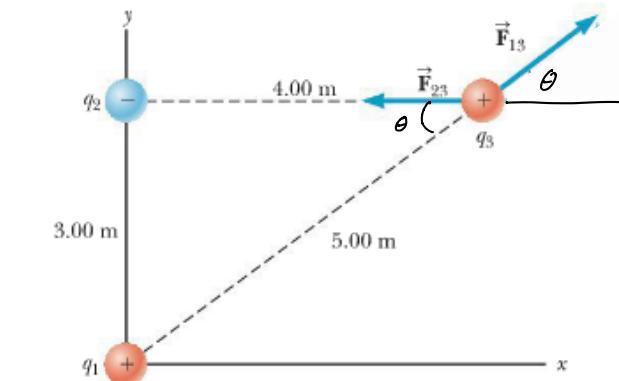
$$(-9 \cdot 10^{-6})x^2 - (24 \cdot 10^{-6})x + (24 \cdot 10^{-6}) = 0$$

$$-9x^2 - 24x + 24 = 0$$

$$x = -3.44 \quad \text{or} \quad x = 0.774 \text{ m}$$

# Chucks

**PROBLEM** Consider three point charges at the corners of a triangle, as shown in Figure 15.8, where  $q_1 = 6.00 \times 10^{-9} \text{ C}$ ,  $q_2 = -2.00 \times 10^{-9} \text{ C}$ , and  $q_3 = 5.00 \times 10^{-9} \text{ C}$ . (a) Find the components of the force  $\vec{F}_{23}$  exerted by  $q_2$  on  $q_3$ . (b) Find the components of the force  $\vec{F}_{13}$  exerted by  $q_1$  on  $q_3$ . (c) Find the resultant force on  $q_3$ , in terms of components and also in terms of magnitude and direction.



$$\sin \theta = \frac{3}{5}$$

$$\theta = \sin^{-1}\left(\frac{3}{5}\right)$$

$$\theta = 36.9^\circ$$

$$c) \vec{x} = \vec{F}_{23} + \vec{F}_{13} \cos \theta$$

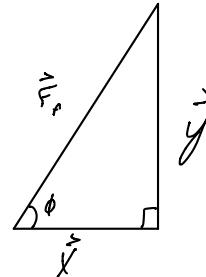
$$\vec{x} = -(5.62 \times 10^{-9} \text{ N}) + (10.8 \times 10^{-9} \text{ N}) \cos(36.9^\circ)$$

$$\vec{x} = 3.06 \times 10^{-9} \text{ N}$$

$$\vec{y} = \vec{F}_{13} \cdot \sin \theta$$

$$\vec{y} = (10.8 \times 10^{-9} \text{ N}) \sin(36.9^\circ)$$

$$\vec{y} = 6.48 \times 10^{-9} \text{ N}$$



$$\phi = \tan^{-1}\left(\frac{y}{x}\right)$$

$$\phi = \tan^{-1}\left(\frac{6.48 \times 10^{-9}}{3.06 \times 10^{-9}}\right)$$

$$\phi = 64.7^\circ$$

$$|\vec{F}_3| = \sqrt{(\vec{x}^2 + \vec{y}^2)}$$

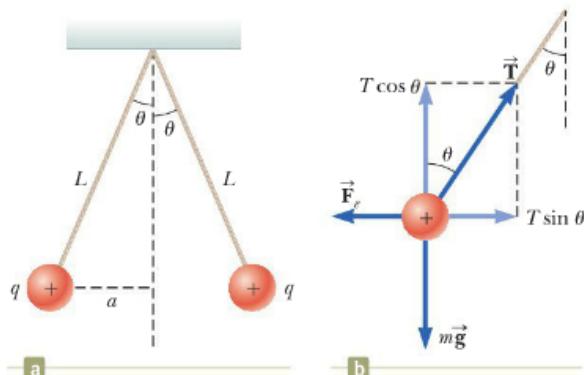
$$= \sqrt{(3.06 \times 10^{-9})^2 + (6.48 \times 10^{-9})^2}$$

$$|\vec{F}_3| = 7 \times 10^{-9} \text{ N}$$

$$\text{equilibrium: } \sum(\text{all forces}) = 0$$

0.003 kg

Two identical small charged spheres, each having a mass of  $3.00 \times 10^{-2} \text{ kg}$ , hang in equilibrium as shown in Figure 23.9a. The length  $L$  of each string is 0.150 m, and the angle  $\theta$  is  $5.00^\circ$ . Find the magnitude of the charge on each sphere.



**Figure 23.9** (Example 23.4) (a) Two identical spheres, each carrying the same charge  $q$ , suspended in equilibrium. (b) Diagram of the forces acting on the sphere on the left part of (a).

$$\begin{aligned} \sin \theta &= \frac{op}{hyp} \\ \sin \theta &= \frac{a}{L} \\ a &= L \sin \theta \\ a &= 0.15 \sin 5^\circ \\ a &= 0.0131 \text{ m} \\ r &= 2a = 0.262 \text{ m} \\ r^2 &= 0.0686 \text{ m}^2 \end{aligned}$$

$$\vec{F}_e = k_e \frac{|q_1||q_2|}{(2a)^2}$$

$$\vec{F}_e = k \frac{q^2}{r^2}$$

$$\sum F_x = 0$$

$$0 = -\vec{F}_e + T \sin \theta$$

$$0 = -k \frac{q^2}{r^2} + T \sin \theta$$

$$\frac{k q^2}{r^2} = T \sin \theta$$

$$r = \sqrt{\frac{r^2 T \sin \theta}{k}}$$

$$r = \sqrt{\frac{(0.0686)(0.029512) \sin(5^\circ)}{8.99 \times 10^9}}$$



$$\begin{aligned} q &= 1.4 \times 10^{-7} \text{ C} \\ &= 140 \times 10^{-9} \text{ C} \\ q &= 140 \text{ nC} \end{aligned}$$

T: tension

how find T?

use 4 forces = 0  
to first find T

$$\sum F_y = 0$$

$$0 = T \cos \theta - mg$$

$$T = \frac{mg}{\cos \theta}$$

$$T = \frac{(0.003)(9.8)}{\cos(5^\circ)}$$

$$T = 0.029512 \text{ N}$$

**S** Four point charges are at the corners of a square of side  $a$  as shown in Figure P15.8. Determine the magnitude and direction of the resultant electric force on  $q$ , with  $k_e$ ,  $q$ , and  $a$  left in symbolic form.

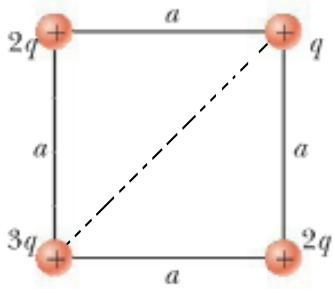
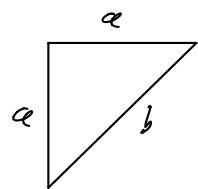
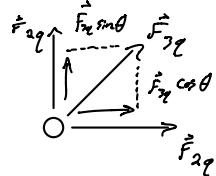


Figure P15.8



$$a^2 + a^2 = b^2$$

$$b = \sqrt{2a^2}$$



$$\sum \vec{F}_x = \frac{k_e(2q)}{a^2} + \left( \frac{k_e 3q}{2a^2} \cdot \frac{\sqrt{2}}{2} \right)$$

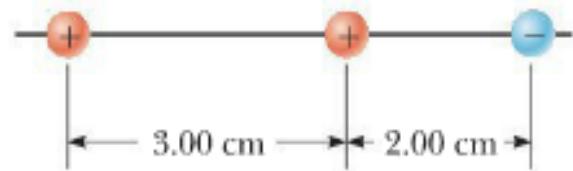
$$\sum \vec{F}_y = \frac{k_e(2q)}{a^2} + \left( \frac{k_e 3q}{2a^2} \cdot \frac{\sqrt{2}}{2} \right)$$

$$\text{Direction} = 45^\circ$$

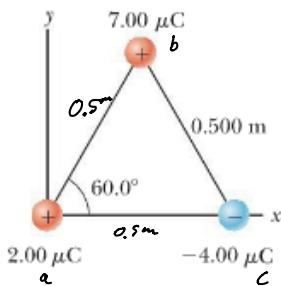
$$\text{Magnitude} = \sqrt{(\sum \vec{F}_x)^2 + (\sum \vec{F}_y)^2} \text{ N}$$

Calculate the magnitude and direction of the Coulomb force on each of the three charges shown in Figure P15.10.

$6.00 \mu\text{C}$        $1.50 \mu\text{C}$        $-2.00 \mu\text{C}$



Three point charges are located at the corners of an equilateral triangle as in Figure P15.13. Find the magnitude and direction of the net electric force on the  $2.00 \mu\text{C}$  charge.



$$\mu = 1 \times 10^{-9} ?$$

$$\vec{F}_{ac} = k_e \frac{|q_a||q_c|}{r^2}$$

$$\vec{F}_{ac} = \left( 8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2} \right) \frac{(2 \times 10^{-9} \text{C})(4 \times 10^{-9} \text{C})}{0.5^2 \text{m}^2}$$

$$\vec{F}_{ab} = \left( 8.99 \times 10^9 \right) \frac{(2 \times 10^{-9})(7 \times 10^{-9})}{0.5^2}$$

$$\vec{F}_{ab} =$$

$$\vec{x} = \vec{F}_{ac} - \vec{F}_{ab} \cos(60^\circ)$$

$$\vec{x} =$$

$$\vec{y} = \vec{F}_{ab} \sin(60^\circ)$$

$$\vec{y} =$$

**w** Two small metallic spheres, each of mass  $m = 0.20\text{ g}$ , are suspended as pendulums by light strings from a common point as shown in Figure P15.15. The spheres are given the same electric charge, and it is found that they come to equilibrium when each string is at an angle of  $\theta = 5.0^\circ$  with the vertical. If each string has length  $L = 30.0\text{ cm}$ , what is the magnitude of the charge on each sphere?

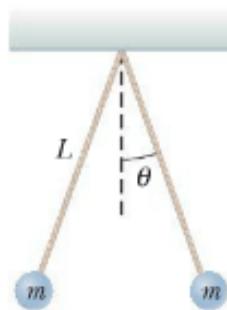


Figure P15.15

Draw other force  
diagram?

Finding T again

Two protons in an atomic nucleus are typically separated by a distance of  $2 \times 10^{-15} \text{ m}$ . The electric repulsive force between the protons is huge, but the attractive nuclear force is even stronger and keeps the nucleus from bursting apart. What is the magnitude of the electric force between two protons separated by  $2.00 \times 10^{-15} \text{ m}$ ?

$$q^+ : 1.602 \times 10^{-19} \text{ C}$$

$$F = \frac{(1.602 \times 10^{-19} \text{ C})^2}{(2.00 \times 10^{-15} \text{ m})^2}$$

Can these  
squares cancel??

$$\hat{F} =$$

# Electric Field

Resulting force from being in  
the electric field:

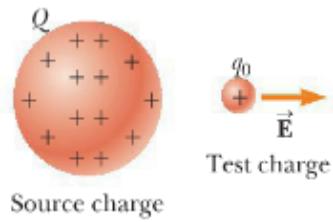
$$\vec{F}_e = k \frac{|q_1||q_2|}{r^2}$$

$$\vec{F}_e = q_0 \vec{E}$$

$$|\vec{F}_e| = |q_0| k \frac{|Q|}{r^2}$$

$$\vec{F}_e = k \frac{|q||Q|}{r^2}$$

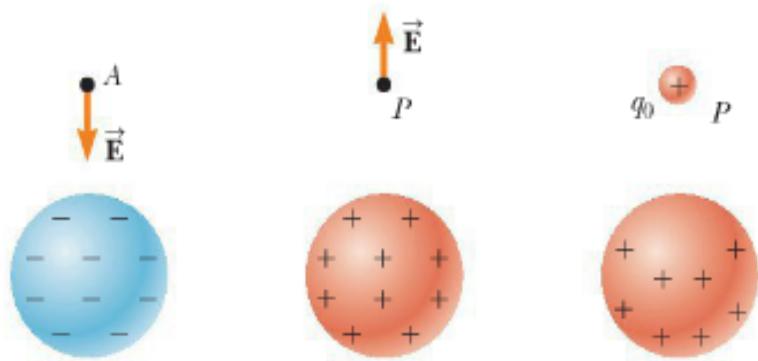
$$|F| = q_0 |E|$$



the charge  
making the field

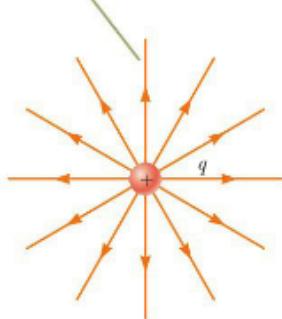
$$\vec{E}_{\text{field}} = k_e \frac{Q}{r^2}$$

Source: The charge creating the force  
Test: The particle experiencing the resulting force

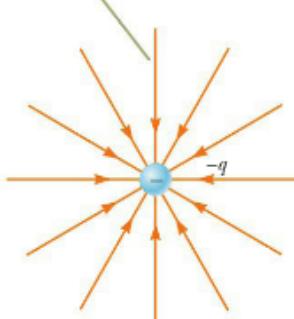


electrical fields come  
out of positive charges  
into negative charges

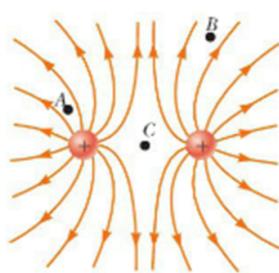
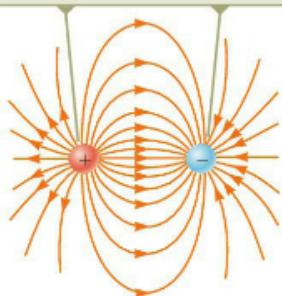
For a positive point charge, the field lines are directed radially outward.



For a negative point charge, the field lines are directed radially inward.

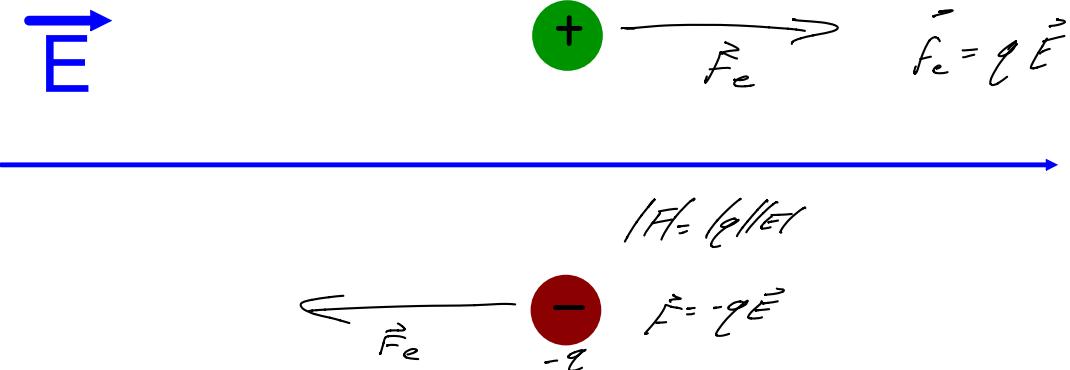


The number of field lines leaving the positive charge equals the number terminating at the negative charge.



# Forces on Charges in an Electric Field

Positive charges experience a force in the direction of the electric field.

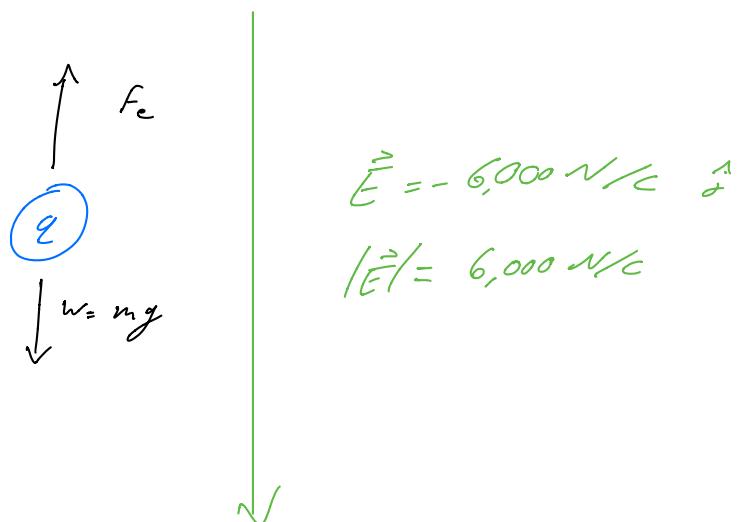


Negative charges experience a force in the direction opposite of the electric field.

A water droplet of mass  $3.00 \times 10^{-12} \text{ kg}$  is located in the air near the ground during a stormy day. An atmospheric electric field of magnitude  $6.00 \times 10^5 \text{ N/C}$  points vertically downward in the vicinity of the water droplet. The droplet remains suspended at rest in the air. What is the electric charge on the droplet?

$$m = 3.00 \times 10^{-12} \text{ kg}$$

$$|E| = 6.00 \times 10^5 \text{ N/C}$$



$$\vec{E} = -6,000 \text{ N/C} \hat{j}$$

$$|E| = 6,000 \text{ N/C}$$



$$F_e + w = 0$$

$$F_e = -w$$

$$-qE = -mg$$

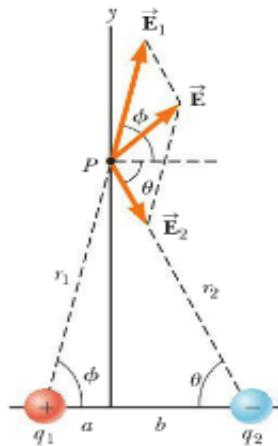
$$q = \frac{mg}{E}$$

$$q = \frac{(3 \times 10^{-12} \text{ kg})(9.8)}{6,000 \text{ N/C}}$$

$$q = -4.9 \times 10^{-15} \text{ C}$$

Charges  $q_1$  and  $q_2$  are located on the  $x$  axis, at distances  $a$  and  $b$ , respectively, from the origin as shown in Figure 23.12.

(A) Find the components of the net electric field at the point  $P$ , which is at position  $(0, y)$ .



$$a = 5\text{m} \quad b = 10\text{m} \quad y = 15\text{m}$$

$$q_1 = 5\text{nC} \quad q_2 = -2\text{nC}$$

$$|E_1| = k \frac{|q_1|}{r_1^2} = \frac{(8.99 \times 10^9)(5 \times 10^{-9})}{250}$$

$$|E_1| = 0.1798 \text{ N/C}$$

$$\phi = \tan^{-1}\left(\frac{15}{5}\right)$$

$$\phi = 71.6^\circ$$

$$\vec{E}_1 = 0.1798 \cos(71.6) \hat{i} + 0.1798 \sin(71.6) \hat{j}$$

$$E_2 = \frac{k|q_2|}{r_2^2}$$

$$E_2 = \frac{(8.99 \times 10^9)(-2 \times 10^{-9})}{325}$$

$$E_2 = 0.05532 \text{ N/C}$$

$$\theta = \tan^{-1}\left(\frac{15}{10}\right)$$

$$\theta = 56.3^\circ$$

$$\vec{E}_2 = 0.05532 \cos(56.3) \hat{i} + 0.05532 \sin(56.3) \hat{j}$$

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

$$0.1798 \cos(71.6) \hat{i} + 0.1798 \sin(71.6) \hat{j}$$

$$+ 0.05532 \cos(56.3) \hat{i} + 0.05532 \sin(56.3) \hat{j}$$

$$\vec{E}_f = 0.0874 \text{ N/C} \hat{i} + 0.125 \text{ N/C} \hat{j}$$

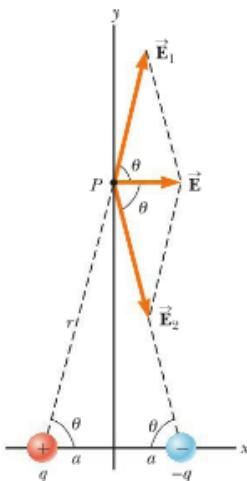
What if 5 nC charge @ P. What is the force on the charge?

$$\vec{F}_P = q_0 \vec{E}_f = (5 \text{nC}) [0.0874 \text{ N/C} \hat{i} + 0.125 \text{ N/C} \hat{j}]$$

$$\vec{F}_P = 4.24 \times 10^{-10} \text{ N} \hat{i} + 6.25 \times 10^{-10} \text{ N} \hat{j}$$

**HW**

Evaluate the electric field at point  $P$  in the special case that  $|q_1| = |q_2|$  and  $a = b$ .



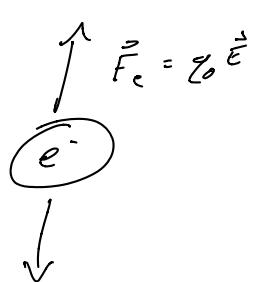
$$q_1 = 5\pi c \quad q_2 = -5\pi c \quad a = 5\pi$$

$$\vec{E} = ?$$

Show y components  
cancel out

What are the magnitude and direction of the electric field that will balance the weight of (a) an electron and (b) a proton? (You may use the data in Table 23.1.)

a)



$$\vec{F}_e = \vec{w}$$

$$q_0 \vec{E} = mg$$

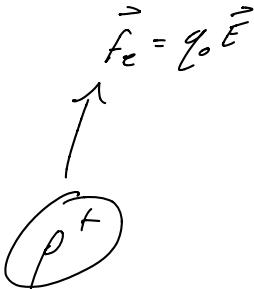
$$\vec{E} = \frac{mg}{q_0}$$

$$W = mg$$

$$\vec{E} = \frac{(9.11 \times 10^{-31} \text{ kg})(9.8)}{-1.6 \times 10^{-19} \text{ C}}$$

$$\vec{E} = 5.58 \times 10^{-11} \text{ downwards}$$

b)



$$\vec{F}_e = \vec{w}$$

$$E = \frac{mg}{q} = \frac{1.67 \times 10^{-27} (9.8)}{1.6 \times 10^{-19}}$$

$$E = 1.02 \times 10^{-7} \text{ N/C}$$

upward

$$W = mg$$

Three point charges are located on a circular arc as shown in Figure P23.31. (a) What is the total electric field at  $P$ , the center of the arc? (b) Find the electric force that would be exerted on a  $-5.00\text{-nC}$  point charge placed at  $P$ .

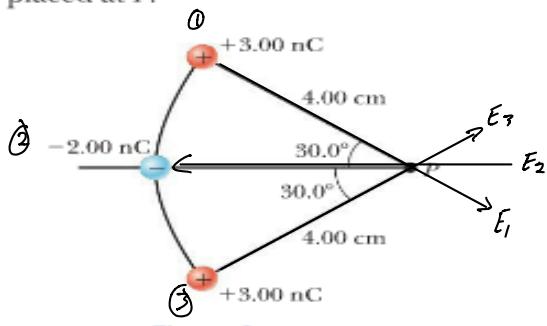


Figure P23.31

$$E_1 = E_3 = k \frac{q}{r^2} = (8.99 \times 10^9) \frac{(3 \times 10^{-9})}{(0.04)^2}$$

$$E_1 = E_3 = 16,856 \text{ N/C}$$

$$E_2 = k \frac{|-2 \text{nC}|}{(0.04)^2} = (8.99 \times 10^9) \frac{(2 \times 10^{-9})}{(0.04)^2}$$

$$E_2 = 11,238 \text{ N/C}$$

a)

$$\vec{E}_P = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$$

$$E_1 = [16,856 \cos(30) \hat{x} - 16,856 \sin(30) \hat{y}]$$

$$E_2 = [11,238 \hat{x} + \cancel{\rho \hat{z}}]$$

$$E_3 = [16,856 \cos(30) \hat{x} + 16,856 \sin(30) \hat{y}]$$

$$2[16,856 \cos(30)] + 11,238 \hat{x}$$

$$\vec{E}_P = 17,957 \hat{x}$$

$$\vec{E}_P = 18,000 \text{ N/C} \hat{x}$$

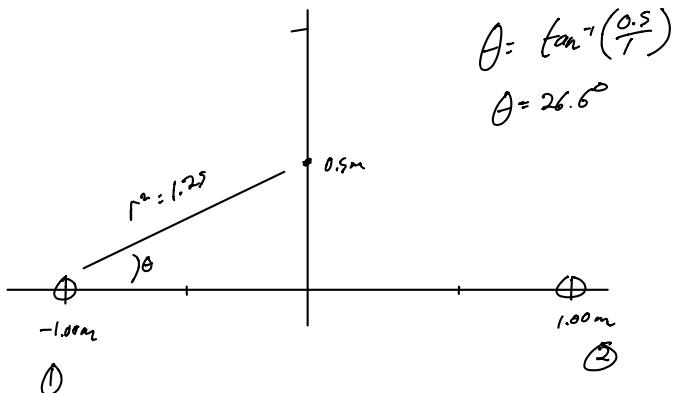
b)  $\vec{F}_P = q_P \vec{E}_P$

$$= (-5 \text{nC})(18,000 \text{ N/C} \hat{x})$$

$$= (-5 \times 10^{-9} \text{ C})(18,000 \text{ N/C} \hat{x})$$

$$\vec{F}_P = -9 \times 10^{-9} \text{ C}$$

Two  $2.00\text{-}\mu\text{C}$  point charges are located on the  $x$  axis. One is at  $x = 1.00\text{ m}$ , and the other is at  $x = -1.00\text{ m}$ .  
 (a) Determine the electric field on the  $y$  axis at  $y = 0.500\text{ m}$ . (b) Calculate the electric force on a  $-3.00\text{-}\mu\text{C}$  charge placed on the  $y$  axis at  $y = 0.500\text{ m}$ .



$$E_1 = k \frac{2\mu\text{C}}{1.25\text{m}} = \frac{(8.99 \times 10^9)(2 \times 10^{-6}\text{C})}{1.25\text{m}}$$

$$E_1 = 14,784 \text{ N/C}$$

$$E_p = E_1 + E_2$$

$$\left[ 14,784 \cos(26.5) \hat{x} + 14,784 \sin(26.5) \hat{y} \right]$$

$$\left[ 14,784 \cos(26.5) \hat{x} - 14,784 \sin(26.5) \hat{y} \right]$$

$$E_p = 12,865 \text{ N/C } \hat{x}$$

$$E_p = 12,900 \text{ N/C } \hat{y}$$

$$b) \vec{F}_p = (-3\mu\text{C})(12,900 \text{ N/C } \hat{y})$$

$$\vec{F}_p = (-3 \times 10^{-6}\text{C})(12,900 \text{ N/C } \hat{y})$$

$$\vec{F}_p = -0.0387 \text{ N } \hat{y}$$

A small, 2.00-g plastic ball is suspended by a 20.0-cm-long string in a uniform electric field as shown in Figure P23.33. If the ball is in equilibrium when the string makes a  $15.0^\circ$  angle with the vertical, what is the net charge on the ball?

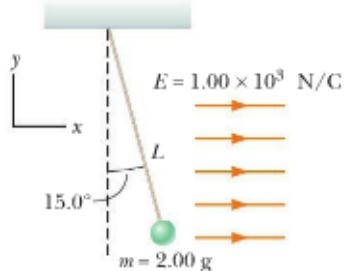
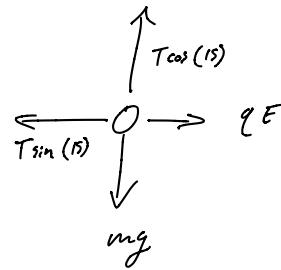


Figure P23.33

$q \text{ is positive}$

$$T \cos(15) = mg$$

$$T = \frac{mg}{\cos(15)}$$



$$qE = T \sin(15)$$

$$q = \frac{T \sin(15)}{E}$$

$$q = \frac{mg \sin(15)}{\cos(15) E}$$

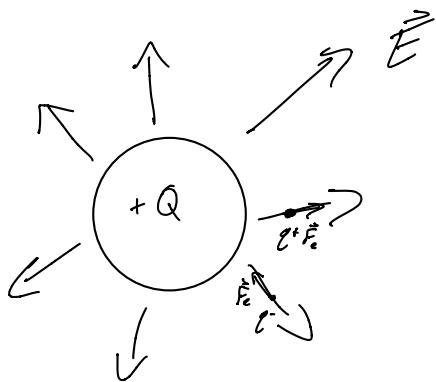
$$q = \frac{mg \tan(15)}{E}$$

$$q = \frac{(0.002)(9.8)}{1,000} \tan(15)$$

$$q = 5.25 \times 10^{-6} C$$

$$q = 5.25 \mu C$$

## Electric Potential Energy and Electric Potential



$$\vec{E} = k_c \frac{q}{r^2}$$

$V$  = Electric Potential (Volts)

$$V = k \frac{q_c}{r}$$

$$U = k \frac{q_e q_c}{r}$$

$q_e$  = Charge experiencing force from field

$q_c$  = Charge creating the field ( $Q$ )

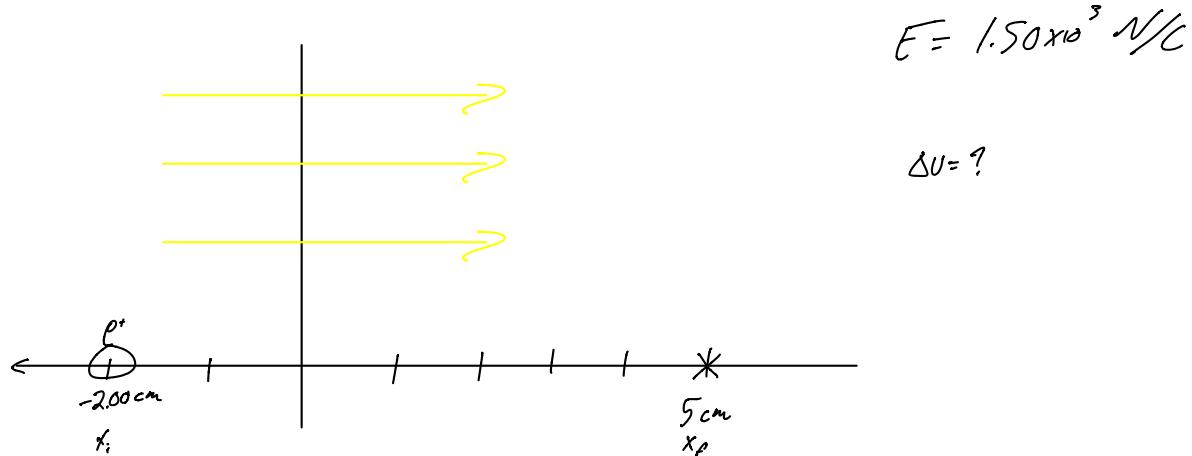
$$\Delta U = -q \Delta V$$

$$\vec{E} \perp V$$

$$\Delta V = E d$$

$$\Delta U = -q E d$$

**PROBLEM** A proton is released from rest at  $x = -2.00 \text{ cm}$  in a constant electric field with magnitude  $1.50 \times 10^3 \text{ N/C}$ , pointing in the positive  $x$ -direction. (a) Calculate the change in the electric potential energy associated with the proton when it reaches  $x = 5.00 \text{ cm}$ . (b) An electron is now fired in the same direction from the same position. What is the change in electric potential energy associated with the electron if it reaches  $x = 12.0 \text{ cm}$ ? (c) If the direction of the electric field is reversed and an electron is released from rest at  $x = 3.00 \text{ cm}$ , by how much has the electric potential energy changed when the electron reaches  $x = 7.00 \text{ cm}$ ?



a)  $\Delta U = -qEd$

positive distance because  
in direction of  $\vec{E}$

$$-\left(1.609 \times 10^{-19} \text{ C}\right)\left(1500 \text{ N/C}\right)(0.07 \text{ m})$$

$\Delta U = -1.69 \times 10^{-17} \text{ J}$

$\Delta U = -1.69 \times 10^{-17} \text{ J}$

will lose energy as  
it moves to the final point

b)  $\Delta U = -qEd$

$$-\left(-1.609 \times 10^{-19} \text{ C}\right)\left(1500 \text{ N/C}\right)(0.14 \text{ m})$$

$\Delta U = 3.38 \times 10^{-17} \text{ J}$

$\Delta U = 3.38 \times 10^{-17} \text{ J}$

c)  $\Delta U = -\left(-1.609 \times 10^{-19} \text{ C}\right)\left(1500 \text{ N/C}\right)(-0.04 \text{ m})$

$\Delta U = -9.654 \times 10^{-18} \text{ J}$

Q: What is the initial speed of the electron if it comes to rest at 12 cm?

$$\Delta K = -\Delta U$$

$$K_f - K_i = -3.38 \times 10^{-17} \text{ J}$$

$$\frac{1}{2} m v_i^2 = 3.38 \times 10^{-17}$$

$$v_i = \sqrt{\frac{2(3.38 \times 10^{-17})}{9.11 \times 10^{-31}}}$$

$v_i = 8.61 \times 10^6 \text{ m/s}$

**PROBLEM** (a) Find the speed of the proton at  $x = 0.050\text{ m}$  in part (a) of Example 16.1. (b) Find the initial speed of the electron (at  $x = -2.00\text{ cm}$ ) in part (b) of Example 16.1 given that its speed has fallen by half when it reaches  $x = 0.120\text{ m}$ .

$$a) \Delta K = -\Delta U \quad \Delta U = -1.69 \times 10^{-17} \text{ J}$$

$$k_F - k_i = \Delta U$$

$$k_F = -\Delta U$$

$$\frac{1}{2} m v_F^2 = -\Delta U$$

$$v_F = \sqrt{\frac{2(-\Delta U)}{m}}$$

$$= \sqrt{\frac{2(1.69 \times 10^{-17} \text{ J})}{1.67 \times 10^{-27} \text{ kg}}}$$

$$v_F = 142,265$$

$$v_F = 1.42 \times 10^5 \text{ m/s}$$

$$b) \Delta K = -\Delta U$$

$$k_F - k_i = -\Delta U$$

$$\frac{1}{2} m v_F^2 - \frac{1}{2} m v_i^2 = -\Delta U$$

$$\frac{1}{2} m \left[ \left( \frac{1}{2} v_i \right)^2 - v_i^2 \right] = -\Delta U$$

$$\frac{1}{2} m v_i^2 \left[ \frac{1}{4} - 1 \right] = -\Delta U$$

$$\frac{1}{2} m v_i^2 \left( -\frac{3}{4} \right) = -\Delta U$$

$$-\frac{3}{8} m v_i^2 = -\Delta U$$

$$v_i = \sqrt{\frac{8 \Delta U}{3 m}}$$

$$v_i = \sqrt{\frac{8}{3} \cdot \frac{(3.38 \times 10^{-17} \text{ J})}{(9.11 \times 10^{-31} \text{ kg})}}$$

$$v_i = 9.95 \times 10^6 \text{ m/s}$$

Speed of light:  $3 \times 10^8 \text{ m/s}$

375 N/C



## College Physics Ch. 16

1. A uniform electric field of magnitude 375 N/C pointing in the positive  $x$ -direction acts on an electron, which is initially at rest. After the electron has moved 3.20 cm, what is (a) the work done by the field on the electron, (b) the change in potential energy associated with the electron, and (c) the velocity of the electron?

a)  $w = \Delta U = -qEd$

$$= (-1.60 \times 10^{-19} C)(375 \text{ N/C})(0.032 \text{ m})$$

$$w = 1.92 \times 10^{-18} \text{ J}$$

b)  $\Delta U = 1.92 \times 10^{-18} \text{ J}$

c)  $w = \Delta K$

$$w = K_f - K_i$$

$$w = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

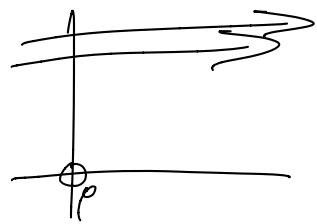
$$w = \frac{1}{2}m(v_f^2 - v_i^2)$$

$$\sqrt{\frac{2w}{m}} = v_f$$

$$v_f = \sqrt{\frac{2(1.92 \times 10^{-18} \text{ J})}{9.1 \times 10^{-31} \text{ kg}}}$$

$$v_f = 2.05 \times 10^6 \text{ m/s}$$

385 N/C



2. A proton is released from rest in a uniform electric field of magnitude 385 N/C. Find (a) the electric force on the proton, (b) the acceleration of the proton, and (c) the distance it travels in 2.00  $\mu$ s.

a)

$$\vec{F}_p = q\vec{E}$$

$$= (1.60 \times 10^{-19} C)(385 \text{ N/C})$$

$$\vec{F}_p = 6.19 \times 10^{-17} \text{ N}$$

b)

$$F = ma$$

$$a = F/m$$

$$a = \frac{(6.19 \times 10^{-17} \text{ N})}{(1.67 \times 10^{-27} \text{ kg})}$$

$$a = 3.71 \times 10^{10} \text{ m/s}^2$$

c)  $X_f = X_i + v_{ix} t + \frac{1}{2} a_x t^2$

$$X_f = \emptyset + \emptyset + \frac{1}{2} (3.71 \times 10^{10} \text{ m/s}) (2 \times 10^{-6} \text{ s})^2$$

$$X_f = 0.0742 \text{ m}$$

$$= 7.42 \text{ cm}$$

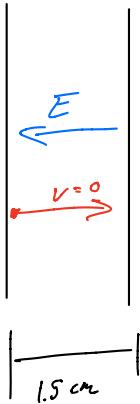
d) What is the speed at this point?

$$V_{fx} = V_i + a_x t$$

$$V_f = \emptyset + (3.71 \times 10^{10}) (2 \times 10^{-6})$$

$$V_f = 74,200 \text{ m/s}$$

5. The potential difference between the accelerating plates of a TV set is about 25 kV. If the distance between the plates is 1.5 cm, find the magnitude of the uniform electric field in the region between the plates.



$$E = ? \quad \Delta V = Ed$$

$$E = \frac{\Delta V}{d} = \frac{25000 \text{ V}}{0.015 \text{ m}}$$

$$E = 1.67 \times 10^6 \text{ N/C}$$

$$= 1.67 \text{ MN/C}$$

$$\begin{aligned}\Delta V &= 25 \text{ kV} \\ &= 25,000 \text{ V} \\ d &= 1.5 \text{ cm} \\ &= 0.015 \text{ m}\end{aligned}$$

$$V_{\text{in}} = N/C$$

College Physics Ch. 16

6. A point charge  $q = +40.0 \mu\text{C}$  moves from  $A$  to  $B$  separated by a distance  $d = 0.180 \text{ m}$  in the presence of an external electric field  $\vec{E}$  of magnitude  $275 \text{ N/C}$  directed toward the right as in Figure P16.6. Find (a) the electric force exerted on the charge, (b) the work done by the electric force, (c) the change in the electric potential energy of the charge, and (d) the potential difference between  $A$  and  $B$ .

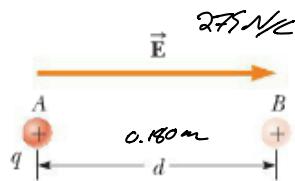


Figure P16.6

$$a) F = (40 \times 10^{-6} \text{ C})(275 \text{ N/C})$$

$$F = 0.011 \text{ N}$$

$$b) W = -\Delta U$$

$$W = 0.00198 \text{ J}$$

$$c) \Delta U = -q \Delta V$$

$$= -q Ed$$

$$= -(40 \times 10^{-6} \text{ C})(275 \text{ N/C})(0.18 \text{ m})$$

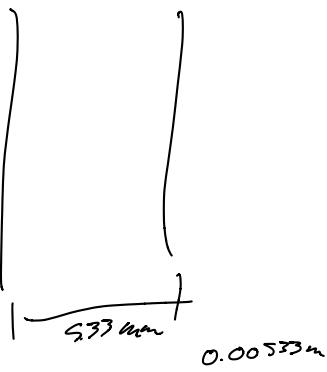
$$\Delta U = -0.00198 \text{ J}$$

$$d) \Delta V = (275)(0.18)$$

$$\Delta V = 49.5 \text{ V}$$

$$\Delta V = 600V$$

7. M Oppositely charged parallel plates are separated by 5.33 mm. A potential difference of 600 V exists between the plates. (a) What is the magnitude of the electric field between the plates? (b) What is the magnitude of the force on an electron between the plates? (c) How much work must be done on the electron to move it to the negative plate if it is initially positioned 2.90 mm from the positive plate?



$$a) \Delta V = Ed$$

$$E = \frac{\Delta V}{d}$$

$$E = \frac{600 \text{ V}}{0.00533 \text{ m}}$$

$$E = 1.12 \times 10^5 \text{ N/C}$$

$$b) F = Eq$$

$$F = (1.12 \times 10^5 \text{ N/C})(1.602 \times 10^{-19} \text{ C})$$

$$F = 1.8 \times 10^{-14} \text{ N}$$

$$c) d = 5.33 \text{ mm} - 2.90 \text{ mm}$$

$$d = 2.43 \text{ mm}$$

$$d = 0.00243 \text{ m}$$

$$w = \Delta V = -qEd$$

$$= -(1.602 \times 10^{-19} \text{ C})(1.12 \times 10^5 \text{ N/C})(0.00243 \text{ m})$$

$$w = 4.38 \times 10^{-17} \text{ J}$$

12. The two charges in Figure P16.12 are separated by  $d = 2.00 \text{ cm}$ . Find the electric potential at (a) point A and (b) point B, which is halfway between the charges.

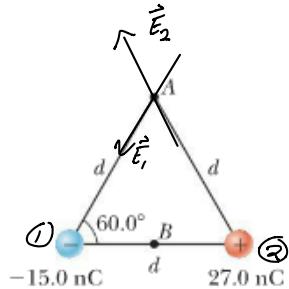


Figure P16.12

$$\text{a)} V_{1A} = k \frac{Q_1}{r_1}$$

$$= 8.99 \times 10^9 \left( \frac{-15 \times 10^{-9} \text{ C}}{0.02 \text{ m}} \right)$$

$$V_{1A} = -6,742 \text{ V}$$

$$V_{2A} = k \frac{Q_2}{r_2}$$

$$= (8.99 \times 10^9) \left( \frac{27 \times 10^{-9} \text{ C}}{0.02} \right)$$

$$V_{2A} = 12,176 \text{ V}$$

$$V_A = V_{1A} + V_{2A}$$

$$= -6,743 + 12,176$$

$$V_A = 5,393 \text{ V}$$

(= 5.39 kV)

$$\text{b)} V_B = k \frac{(-15 \text{nC})}{0.01} + k \frac{(27 \text{nC})}{0.01}$$

$$V_B = 10,788 \text{ V}$$

$$V_B = 10.8 \text{ kV}$$

# Homework

College Physics Ch. 16

- 16. QC S** Three identical point charges each of charge  $q$  are located at the vertices of an equilateral triangle as in Figure P16.16.

The distance from the center of the triangle to each vertex is  $a$ . (a) Show that the electric field at the center of the triangle is zero. (b) Find a symbolic expression for the electric potential at the center of the triangle. (c) Give a physical explanation of the fact that the electric potential is not zero, yet the electric field is zero at the center.

a)

$$\vec{F}_x = -q \cos(30^\circ) + q \cos(20^\circ)$$

$$\vec{F}_x = \emptyset$$

$$\begin{aligned} \vec{F}_y &= -q + q \sin(70^\circ) + q \sin(50^\circ) \\ &= -q + 2q^{\frac{1}{2}} \\ &= -q + q \end{aligned}$$

$$\vec{F}_y = \emptyset$$

b)  $E = k_e \frac{|q_1|}{a^2} + k_e \frac{|q_2|}{a^2} + k_e \frac{|q_3|}{a^2}$

$$E = 3k_e \frac{|q|}{a^2}$$

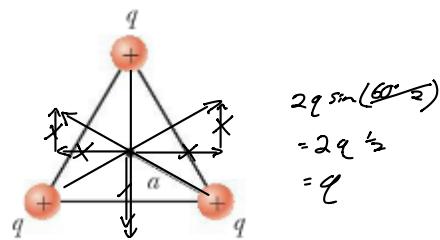
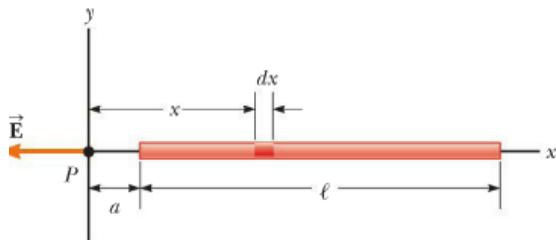


Figure P16.16

c) Potential energy at the point is the total amount of energy at that point.  
All that energy cancels out making the field  $\emptyset$ .

**Example 23.7****The Electric Field Due to a Charged Rod**

A rod of length  $\ell$  has a uniform positive charge per unit length  $\lambda$  and a total charge  $Q$ . Calculate the electric field at a point  $P$  that is located along the long axis of the rod and a distance  $a$  from one end (Fig. 23.15).

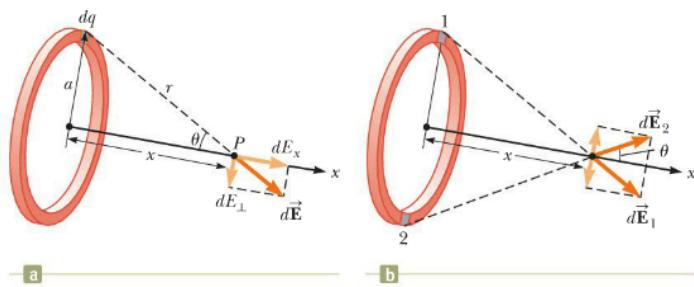


**Figure 23.15** (Example 23.7) The electric field at  $P$  due to a uniformly charged rod lying along the  $x$ -axis.

Skipped for time

**Example 23.8 The Electric Field of a Uniform Ring of Charge**

A ring of radius  $a$  carries a uniformly distributed positive total charge  $Q$ . Calculate the electric field due to the ring at a point  $P$  lying a distance  $x$  from its center along the central axis perpendicular to the plane of the ring (Fig. 23.16a).

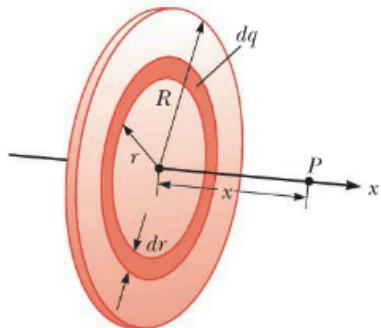


**Figure 23.16** (Example 23.8) A uniformly charged ring of radius  $a$ . (a) The field at  $P$  on the  $x$  axis due to an element of charge  $dq$ . (b) The total electric field at  $P$  is along the  $x$  axis. The perpendicular component of the field at  $P$  due to segment 1 is canceled by the perpendicular component due to segment 2.

Skipped for time

**Example 23.9 The Electric Field of a Uniformly Charged Disk**

A disk of radius  $R$  has a uniform surface charge density  $\sigma$ . Calculate the electric field at a point  $P$  that lies along the central perpendicular axis of the disk and a distance  $x$  from the center of the disk (Fig. 23.17).



Skipped for time

37. A rod 14.0 cm long is uniformly charged and has a total  
W charge of  $-22.0 \mu\text{C}$ . Determine (a) the magnitude and  
(b) the direction of the electric field along the axis of  
the rod at a point 36.0 cm from its center.

Skipped for fine

38. A uniformly charged disk of radius 35.0 cm carries charge with a density of  $7.90 \times 10^{-3} \text{ C/m}^2$ . Calculate the electric field on the axis of the disk at (a) 5.00 cm, (b) 10.0 cm, (c) 50.0 cm, and (d) 200 cm from the center of the disk.

Skipped for time

- 39.** A uniformly charged ring of radius 10.0 cm has a total charge of  $75.0 \mu\text{C}$ . Find the electric field on the axis of the ring at (a) 1.00 cm, (b) 5.00 cm, (c) 30.0 cm, and (d) 100 cm from the center of the ring.

*Skipped for time*

44. A thin rod of length  $\ell$  and uniform charge per unit length  $\lambda$  lies along the  $x$  axis as shown in Figure P23.44.  
(a) Show that the electric field at  $P$ , a distance  $d$  from the rod along its perpendicular bisector, has no  $x$

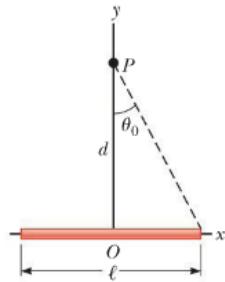


Figure P23.44

Skipped for time

46. (a) Consider a uniformly charged, thin-walled, right circular cylindrical shell having total charge  $Q$ , radius  $R$ , and length  $\ell$ . Determine the electric field at a point a distance  $d$  from the right side of the cylinder as shown in Figure P23.46. *Suggestion:* Use the result of Example 23.8 and treat the cylinder as a collection of ring charges.
- (b) **What If?** Consider now a solid cylinder with the same dimensions and carrying the same charge, uniformly distributed through its volume. Use the result of Example 23.9 to find the field it creates at the same point.

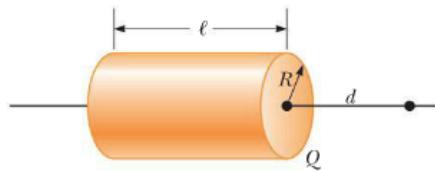


Figure P23.46

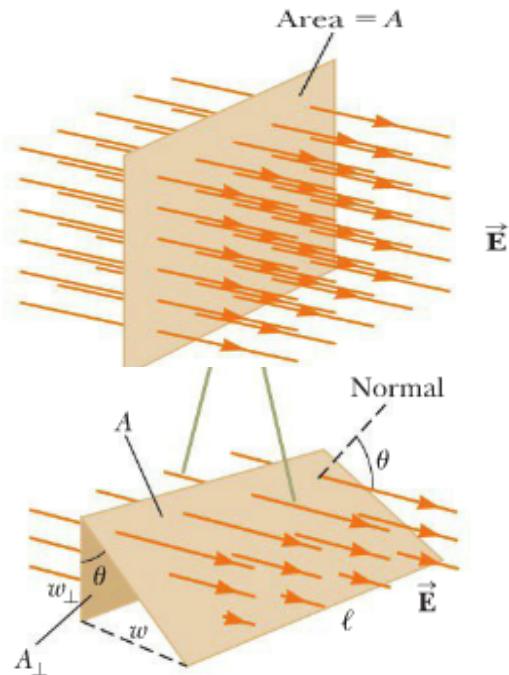
Skipped for fine

# Gauss's Law

Electric Flux - the product the magnitude of the electric field and the area perpendicular to the Electric field through which they pass.

$$\Phi_E = EA$$

$$\Phi = E \cdot A \cos(\theta)$$

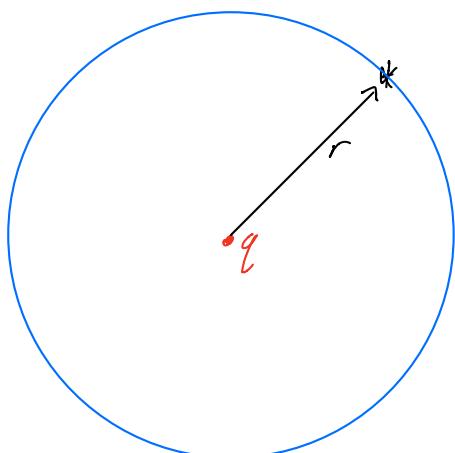


$$\Phi = \frac{Q_{\text{enclosed}}}{\epsilon_0}$$

$\epsilon_0$  = permittivity  
of free  
space

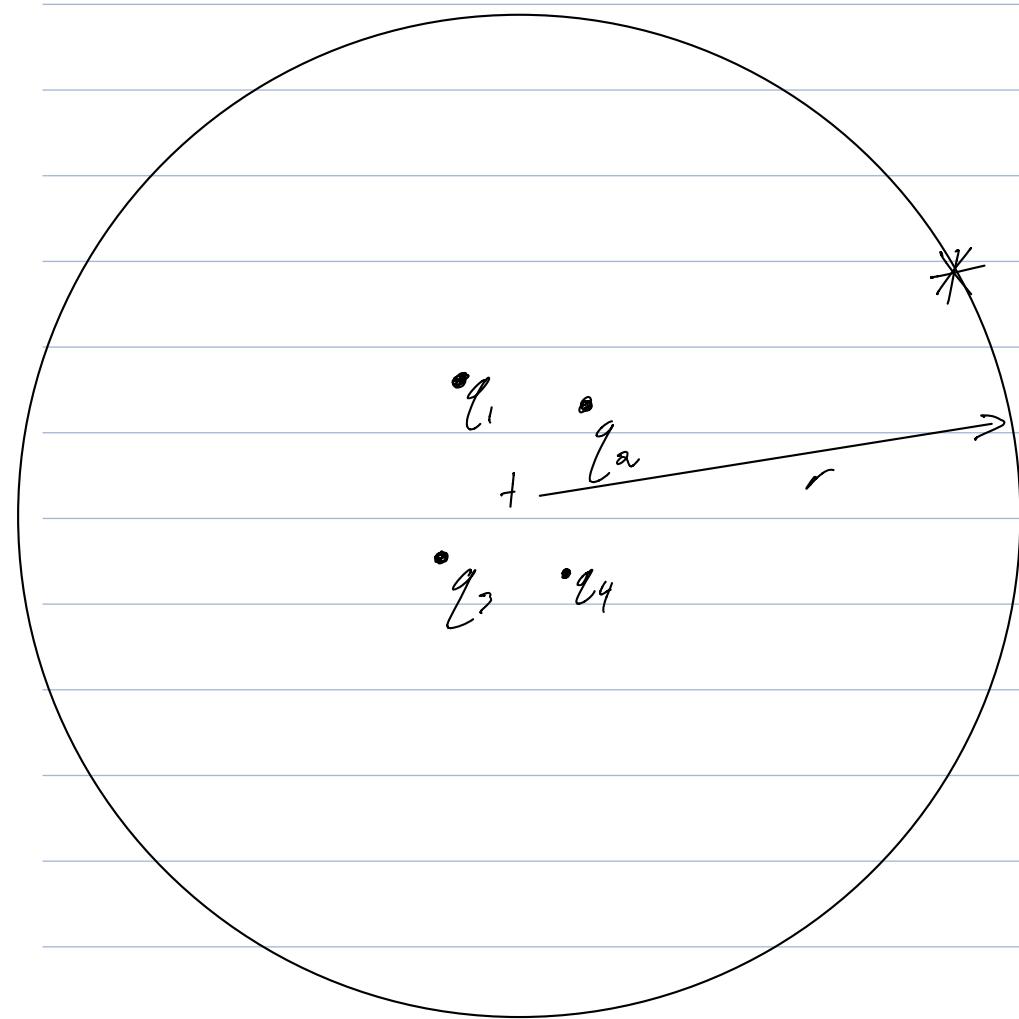
$$\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2$$

$$EA_{\perp} = \frac{Q_{\text{enclosed}}}{\epsilon_0}$$



$$A_{\text{sphere}} = 4\pi r^2$$

$$F = \frac{Q_{\text{enclosed}}}{(4\pi r^2) \epsilon_0}$$



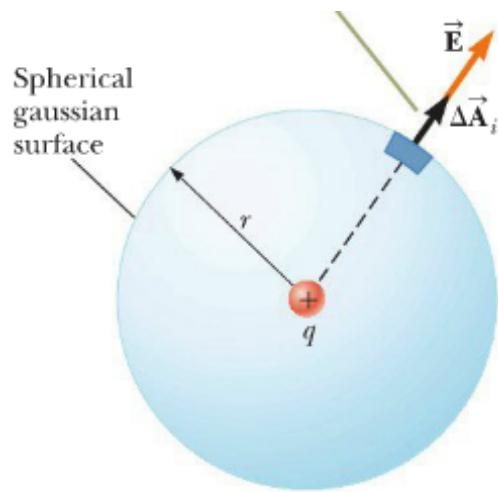
$$E = ?$$

$$E = k \frac{q}{r^2}$$

would have  
to do this  
4 times

$$E = \frac{Q_{\text{enclosed}}}{A_{\text{sphere}} \epsilon_0} = \frac{(q_1 + q_2 + q_3 + q_4)}{(4\pi r^2) \epsilon_0}$$

If all charges are  
in the sphere



$$q = 2\pi C$$

$$r = 10 \text{ cm}$$

$$\bar{E}A = \frac{Q_{enc}}{\epsilon_0}$$

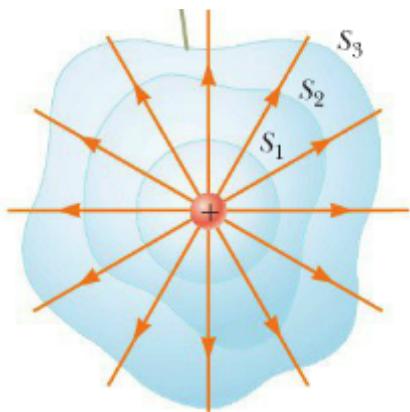
$$E = \frac{Q_{enc}}{A \epsilon_0} = \frac{q}{A \epsilon_0}$$

$$E = \frac{(2\pi C)}{(4\pi(0.1^2)) \epsilon_0}$$

$$E = 1798 \text{ N/C}$$

$$\vec{E} = 1798 \text{ N/C} \hat{r}$$

Radially outward



$$\Phi_1 = E_1 A_1 = \frac{Q_{\text{enc}1}}{\epsilon_0} = \frac{q}{\epsilon_0}$$

$$\Phi_2 = E_2 A_2 = \frac{q}{\epsilon_0}$$

$$\Phi_3 = E_3 A_3 = \frac{q}{\epsilon_0}$$

As  $E$  gets smaller  
 $A$  gets bigger  
 $\therefore$  All the same

## Charges on Objects

Insulator: charge is evenly distributed throughout

Conductor: charge is evenly distributed on the surface

Engineering Physics - Example 24.3

An insulating solid sphere of radius  $a$  has a uniform volume charge density  $\rho$  and carries a total positive charge  $Q$  (Fig. 24.10).

(A) Calculate the magnitude of the electric field at a point outside the sphere.

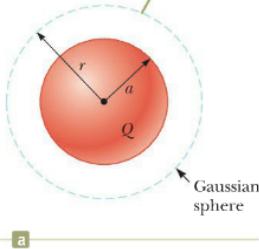
$$\rho = \frac{Q}{V} \quad r > a$$

$$\Phi = EA = \frac{Q_{\text{enc}}}{\epsilon_0}$$

$$E = \frac{Q_{\text{enc}}}{A \epsilon_0} = \frac{Q}{4\pi r^2 \epsilon_0}$$

$$\boxed{E = \frac{Q}{4\pi \epsilon_0 r^2} \hat{r}}$$

For points outside the sphere, a large, spherical gaussian surface is drawn concentric with the sphere.



a

(B) Find the magnitude of the electric field at a point inside the sphere.

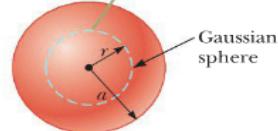
$$E = \frac{q_{\text{enc}}}{A \epsilon_0} \quad r < a$$

$$E = \frac{\left(\frac{r^3}{a^3} Q\right)}{(4\pi r^2) \epsilon_0}$$

$$\rho = \frac{Q}{V_{\text{sphere}}}$$

$$\rho = \frac{q_{\text{enc}}}{V_{\text{gaussian surface}}}$$

For points inside the sphere, a spherical gaussian surface smaller than the sphere is drawn.



b

$$E = \frac{\left(\frac{r^3}{a^3} Q\right)}{4\pi \epsilon_0}$$

$$V_{\text{sphere}} = \frac{4}{3}\pi a^3$$

$$V_{\text{gs}} = \frac{4}{3}\pi r^3$$

$$\frac{q_{\text{enc}}}{V_{\text{gs}}} = \frac{Q}{V_{\text{sphere}}}$$

$$q_{\text{enc}} = \frac{V_{\text{gs}}}{V_{\text{sphere}}} Q$$

$$= \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi a^3} Q$$

$$q_{\text{enc}} = \frac{r^3}{a^3} Q$$

### Engineering Physics - Example 24.4

Find the electric field a distance  $r$  from a line of positive charge of infinite length and constant charge per unit length  $\lambda$  (Fig. 24.12a).

$$\lambda = \frac{Q}{l}$$

$$A_{\text{gsl}} = l(2\pi r)$$

$$E = \frac{q_{\text{enc}}}{A \epsilon_0}$$

$$\begin{aligned} q_{\text{enc}} &= \lambda \cdot l \\ &= \frac{Q}{l} \cdot l \end{aligned}$$

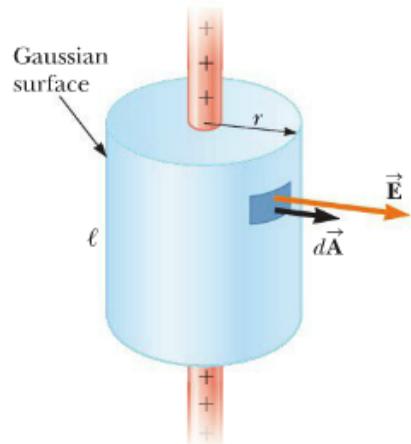
$$E = \frac{\frac{Q}{l} k}{4\pi r \epsilon_0}$$

$$E = \frac{Q}{l 2\pi r \epsilon_0}$$

if  $Q + l$  are not known, but  $\lambda$  is

$$\lambda = \frac{Q}{l}$$

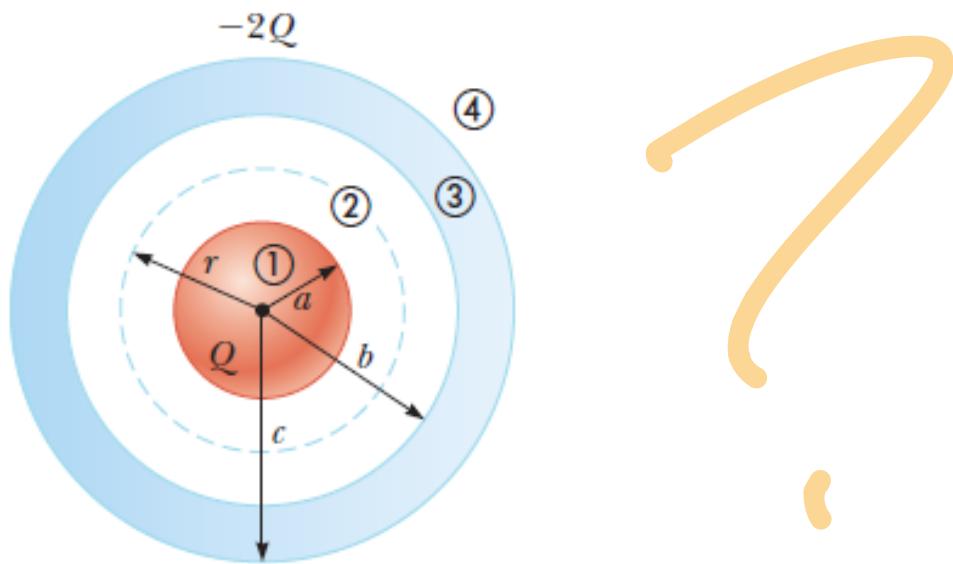
$$E = \frac{\lambda}{2\pi r \epsilon_0}$$



a

**Example 24.7 A Sphere Inside a Spherical Shell**

A solid insulating sphere of radius  $a$  carries a net positive charge  $Q$  uniformly distributed throughout its volume. A conducting spherical shell of inner radius  $b$  and outer radius  $c$  is concentric with the solid sphere and carries a net charge  $-2Q$ . Using Gauss's law, find the electric field in the regions labeled ①, ②, ③, and ④ in Figure 24.19 and the charge distribution on the shell when the entire system is in electrostatic equilibrium.



# Check

Engineering Physics - 24

4. Consider a closed triangular box resting within a horizontal electric field of magnitude  $E = 7.80 \times 10^4 \text{ N/C}$  as shown in Figure P24.4. Calculate the electric flux through (a) the vertical rectangular surface, (b) the slanted surface, and (c) the entire surface of the box.

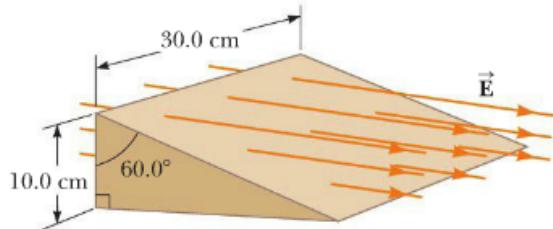


Figure P24.4

$$a) \Phi = EA$$

$$= (7.8 \times 10^4)(0.03 \text{ m}^2)$$

$$\Phi = 2340 \text{ NC}$$

$$b) \Phi = E \cdot A \cos(\theta)$$

$$= 2,340 \cdot \cos(60)$$

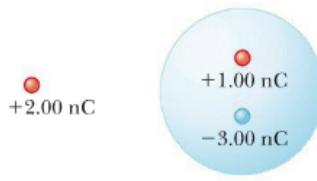
$$\Phi = 1,170 \text{ NC}$$

Q ?

## Engineering Physics - 24

# Check

8. Find the net electric flux through the spherical closed surface shown in Figure P24.8. The two charges on the right are inside the spherical surface.



$$\Phi = \frac{Q_{enc}}{\epsilon_0}$$

Figure P24.8

$$\Phi = \frac{-2 \text{ nC}}{8.85 \times 10^{-12} \frac{\text{C}^2}{\text{N} \cdot \text{m}^2}}$$

$$\Phi = -225.99$$

# Chucks

## Engineering Physics - 24

9. The following charges are located inside a submarine:  
**M** 5.00  $\mu\text{C}$ , -9.00  $\mu\text{C}$ , 27.0  $\mu\text{C}$ , and -84.0  $\mu\text{C}$ . (a) Calculate the net electric flux through the hull of the submarine. (b) Is the number of electric field lines leaving the submarine greater than, equal to, or less than the number entering it?

a) 
$$\Phi = \frac{Q_{\text{enc}}}{\epsilon_0}$$

$$= \frac{53 \mu\text{C}}{(8.85 \times 10^{-12})}$$

$$= 5,988,700$$

$$\Phi = 5.99 \times 10^6$$

b) Equal? ?

## College Physics - 15

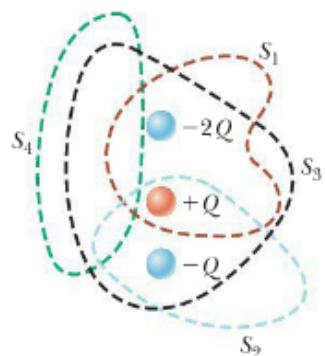
41. M An electric field of intensity  $3.50 \text{ kN/C}$  is applied along the  $x$ -axis. Calculate the electric flux through a rectangular plane  $0.350 \text{ m}$  wide and  $0.700 \text{ m}$  long if (a) the plane is parallel to the  $yz$ -plane, (b) the plane is parallel to the  $xy$ -plane, and (c) the plane contains the  $y$ -axis and its normal makes an angle of  $40.0^\circ$  with the  $x$ -axis.

42. **QC** The electric field everywhere on the surface of a charged sphere of radius 0.230 m has a magnitude of 575 N/C and points radially outward from the center of the sphere. (a) What is the net charge on the sphere? (b) What can you conclude about the nature and distribution of charge inside the sphere?



## College Physics - 15

43. **S** Four closed surfaces,  $S_1$  through  $S_4$ , together with the charges  $-2Q$ ,  $Q$ , and  $-Q$ , are sketched in Figure P15.43. (The colored lines are the intersections of the surfaces with the page.) Find the electric flux through each surface.



$$\Sigma_1 = \frac{-1}{\epsilon_0}$$

$$\Sigma_2 = \emptyset$$

$$\Sigma_3 = \frac{-2}{\epsilon_0}$$

$$\Sigma_4 = \emptyset$$

## College Physics - 15

44. A charge  $q = +5.80 \mu\text{C}$  is located at the center of a regular tetrahedron (a four-sided surface) as in Figure P15.44. Find (a) the total electric flux through the tetrahedron and (b) the electric flux through one face of the tetrahedron.

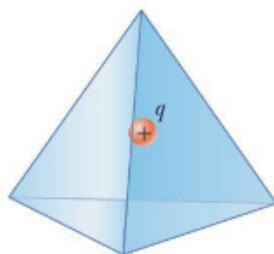


Figure P15.44

**24.** The charge per unit length on a long, straight filament **W** is  $-90.0 \mu\text{C}/\text{m}$ . Find the electric field (a) 10.0 cm, (b) 20.0 cm, and (c) 100 cm from the filament, where distances are measured perpendicular to the length of the filament.