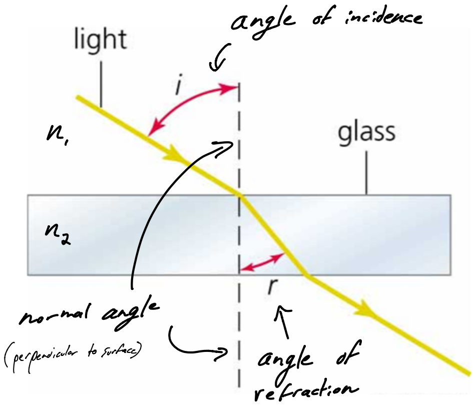
Geometric Light Optics

- Mon & Tuesday: live session
- Thursday: no live session, just video & test comments
- Honorlock practice test already active
- All tests & hw in Pearson
- Recitation grade is from Pearson hw
- Pearson link on Canvas home
- Canvas grade should be accurate
- pictures of scratch work from test will be submitted to canvas page for partial credit
- Student attendance verification is due soon
    - fill out pdf & submit to assignment
    - must be submitted or will be dropped from course
- Should now kinematics from phys I
    - mentioned at 8:50 Am

Snell's Law



Refraction: bending of light.

- A change in speed leads to a change in direction. (snell's law)

Speed of light (in a vacuum): $c = 3 \times 10^8$ m/s

n: Index of refraction (in a material) (no unit)

$$n = \frac{c}{v_m} \leftarrow \text{velocity in material } m$$

In vacuum:
$n = 1$ Vac

In Air:
$n_{air} = 1.0009 \approx 1$

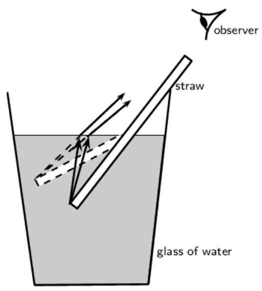
In Diamond:
$n = 2.42$

Glass:
$n = 1.52$ Crown Glass
$n = 1.66$ Flint Glass

water:

Snell's Law

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$



v_m

The speed of light in a piece of glass is measured to be 2.2 x 10^8 m / s. What is the index of refraction for this glass?

$$n = \frac{c}{v_m} = \frac{3 \times 10^8 \cancel{m/s}}{2.2 \times 10^8 \cancel{m/s}} = \frac{3}{2.2} = 1.36$$

$$n = 1.36$$

Homework problem

The index of refraction for a particular wavelength of light in water of water is 1.33. What is the speed of light in water?

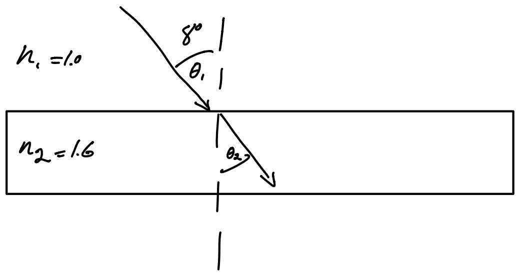
$$\boxed{n = \frac{c}{V_m}}$$

$$1.33 = \frac{3 \times 10^8 \, m/s}{V_m}$$

$$V_m = \frac{3 \times 10^8 \, m/s}{1.33}$$

$$V_m = 2.25 \times 10^8 \, m/s$$

A beam of light traveling in air of index of refraction $n_1 = 1.00$ makes an angle of 8° with the normal to a surface of a piece of glass of index of refraction 1.60. What is the angle of refraction of the beam that passes into the glass?



$$\boxed{n_1 \sin\theta_1 = n_2 \sin\theta_2}$$

$$(1)\sin(8) = (1.6)\sin\theta_2$$

$$\frac{0.139}{1.6} = \frac{\cancel{(1.6)}\sin\theta_2}{\cancel{1.6}}$$

$$\sin^{-1}\ 0.086 = \frac{\sin\theta_2}{\cancel{\sin^{-1}}}$$

$$\sin^{-1}(0.086) = \theta_2$$

$$\theta_2 = 4.99^\circ$$

If $n_2 > n_1$:

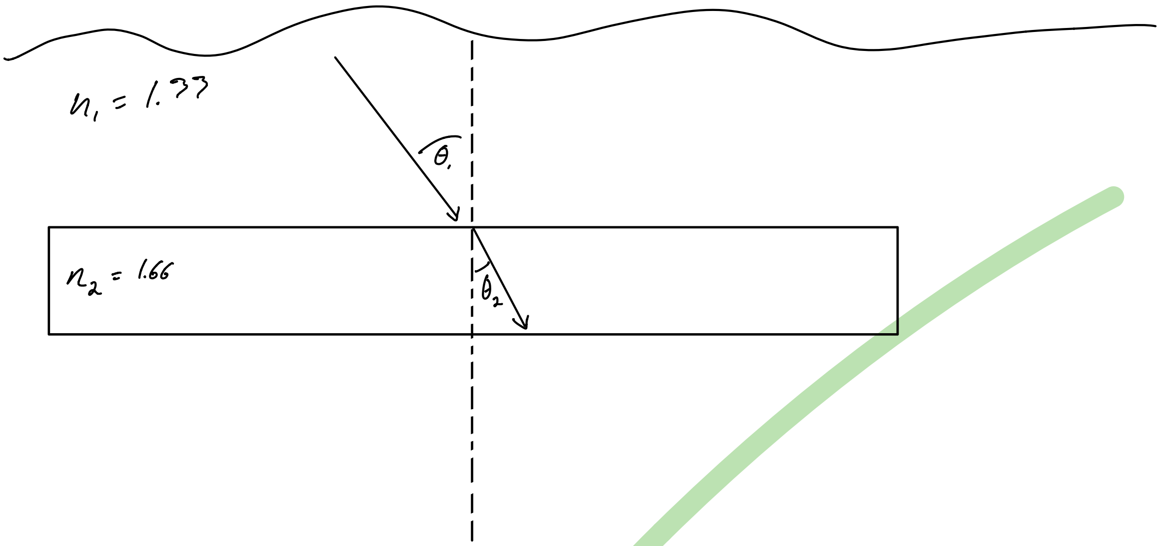
$\theta_2 < \theta_1$

If $n_1 > n_2$:

$\theta_2 > \theta_1$

homework

A ray of light strikes a thick sheet of glass (n = 1.66) submerged in water (n = 1.33) at an angle of 30° with the normal. Find the angle of the refracted ray.



$n_1 \sin\theta_1 = n_2 \sin\theta_2$

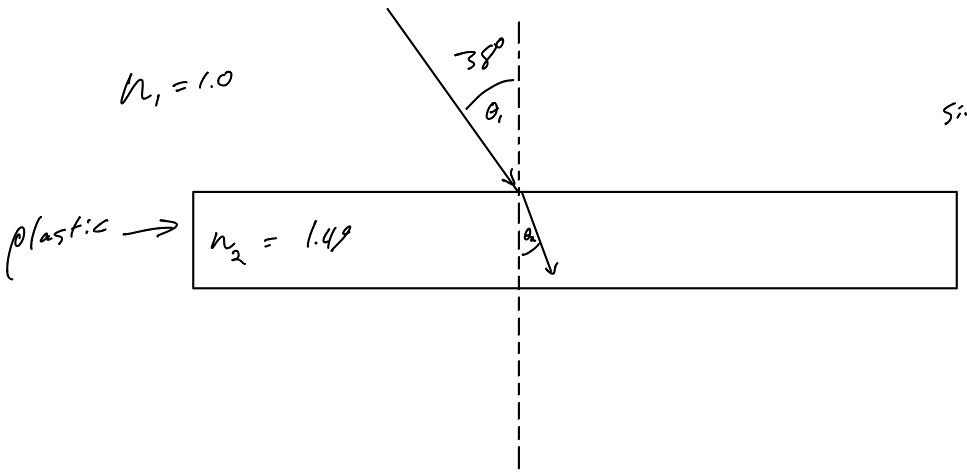
$(1.33)\sin(30°) = (1.66)\sin\theta_2$

$\theta_2 = \sin^{-1}\left[\left(\frac{1.33}{1.66}\right)\sin(30°)\right]$

$\theta_2 = \sin^{-1}\left[(0.80)(1/2)\right]$

$\theta_2 = 23.62°$

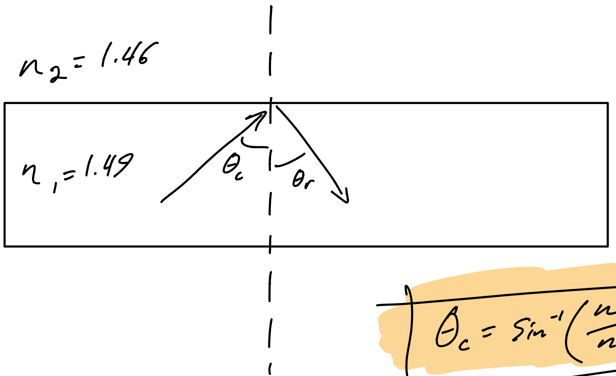
A beam of light in air is incident upon the surface of a rectangular block of clear plastic (n = 1.49) at an angle of 38°. (a) What is angle of refraction of the beam in the block? (b) What is the critical angle of the light beam in the block, if the block is submerged in Carbon Tetrachloride (n = 1.46).



(1) $\sin(38°) = (1.49)\sin\theta_2$

$\sin^{-1}\left(\frac{\sin(38°)}{1.49}\right) = \theta_2$

$\theta_2 = 24.4°$



$\boxed{\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)}$

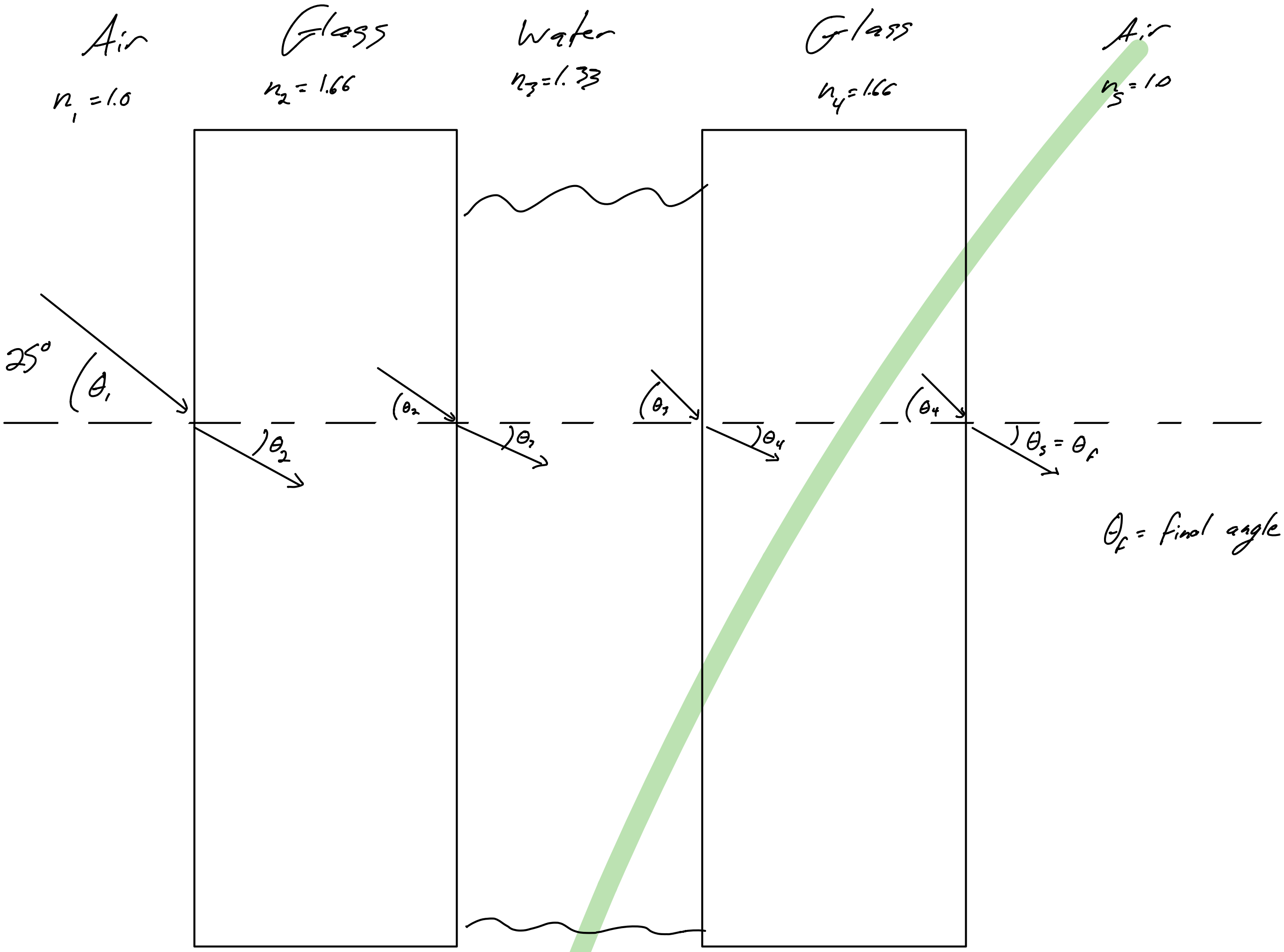
$\theta_c = \sin^{-1}\left(\frac{1.46}{1.49}\right)$

$\theta_c = 78.48°$

θ_c angles?

Critical angles

A ray of light strikes the side of a fish tank (glass n = 1.66) at an angle of 25° passes through the glass into the water (n = 1.33) through the water and glass on the other side. What is the angle of refraction of the emerging ray? Draw a diagram.



$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$\theta_2 = \sin^{-1}\left(\frac{n_1}{n_2}\sin\theta_1\right)$$

$$\theta_2 = \sin^{-1}\left(\left(\frac{1}{1.66}\right)\sin(25°)\right)$$

$$\theta_2 = 14.75°$$

$$n_2 \sin\theta_2 = n_3 \sin\theta_3$$

$$\theta_3 = \sin^{-1}\left(\frac{n_2}{n_3}\sin\theta_2\right)$$

$$\theta_3 = \sin^{-1}\left(\left(\frac{1.66}{1.33}\right)\sin(14.75)\right)$$

$$\theta_3 = 18.53°$$

$$\theta_4 = \sin^{-1}\left(\frac{n_3}{n_4}\sin\theta_3\right)$$

$$\theta_4 = \sin^{-1}\left(\left(\frac{1.33}{1.66}\right)\sin(18.53)\right)$$

$$\theta_4 = 14.75°$$

$$\theta_5 = \sin^{-1}\left(\frac{n_4}{n_5}\sin\theta_4\right)$$

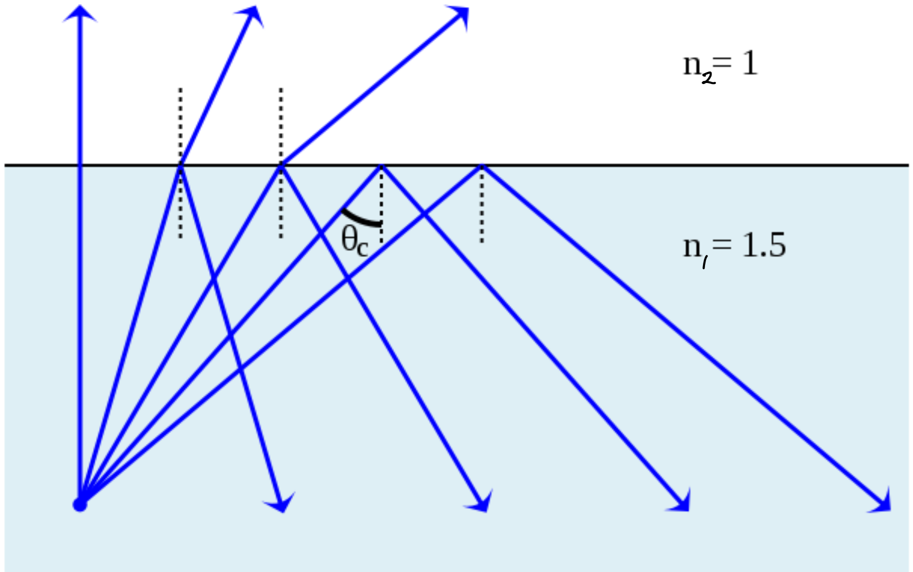
$$\theta_5 = \sin^{-1}\left(\left(\frac{1.66}{1}\right)\sin(14.75)\right)$$

$$\theta_5 = 24.999$$

$$\theta_f = \theta_5 = 25°$$

Total Internal Reflection

$$\theta_2 = \sin^{-1}\left(\frac{1.5}{1}\sin\theta_1\right)$$



θ_1	θ_2
15°	22.8°
30°	48.6°
35	59.3°
40	74.6°
41°	79.7°
41.5°	83.6°
42	undefined

$$\sin^{-1}\left(\frac{1.5}{1}\sin(42)\right)$$

why undefined?

undefined where angle would be >= 90° (along or inside surface)

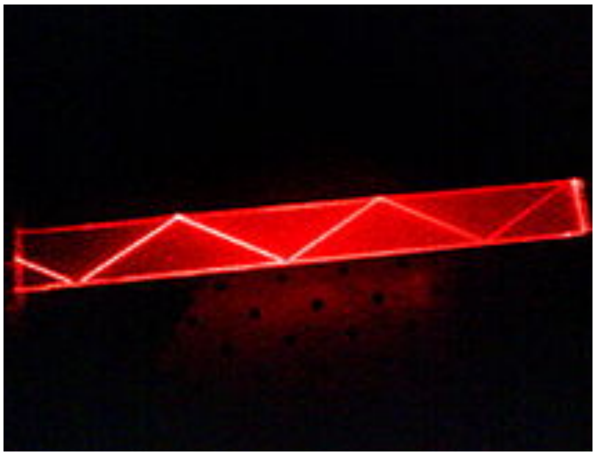
Determine critical angle of diamond $n_1 = 2.42$, in air $n_2 = 1.0$.

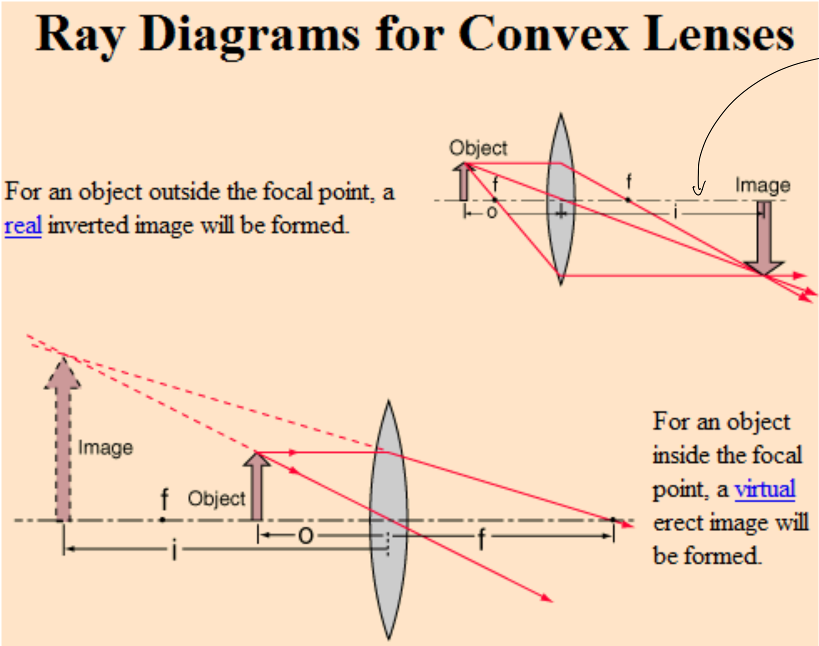
$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

$$\theta_c = \sin^{-1}\left(\frac{1}{2.42}\right)$$

$$\theta_c = 24.4^\circ$$

High index of refraction
+
low critical angle
is what makes diamonds "sparkle"
(if cut to proper angles)

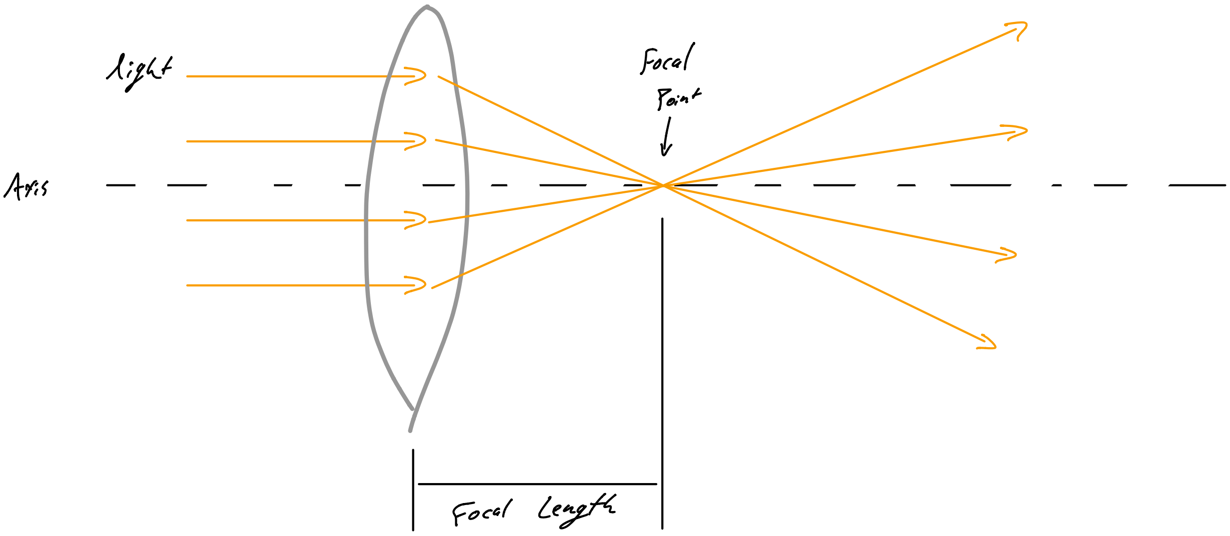


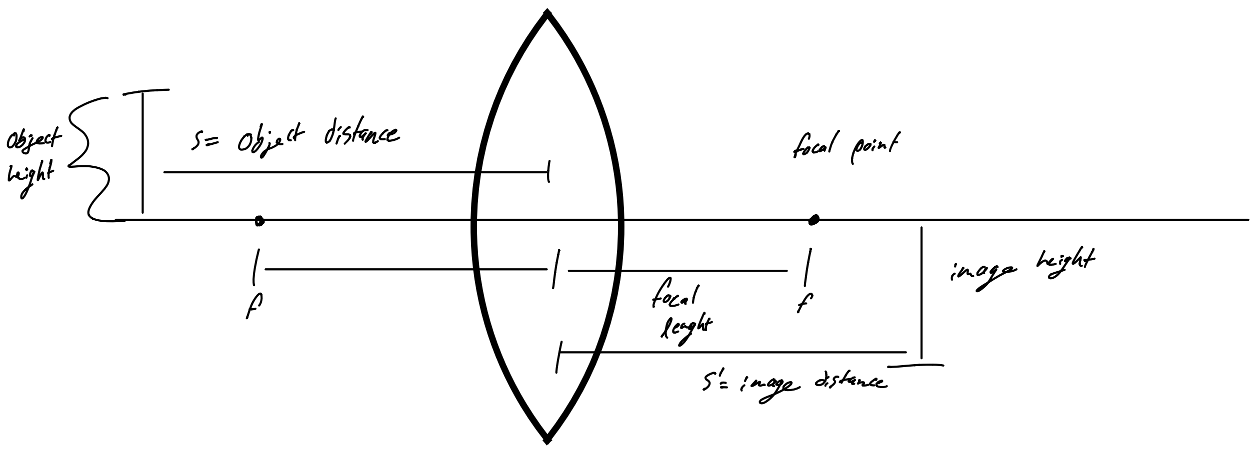


Optical Axis

Radius of curvature = R

Focal length = F





Sign Conventions:

f: positive for convex/positive/converging lenses
negative for concave/negative/diverging lenses

s: positive on left of lens
negative on right

s': positive on right of lens
negative on left

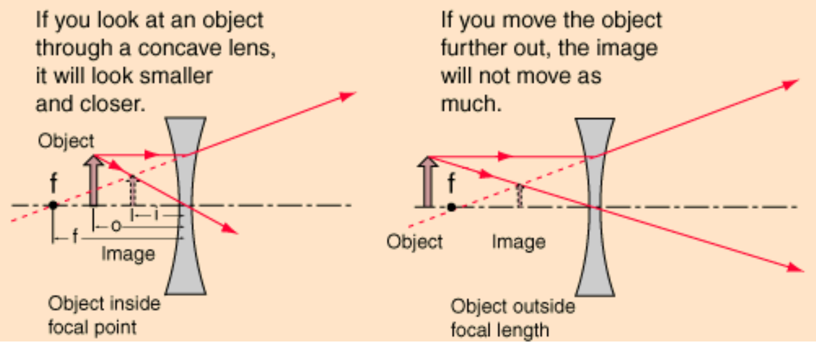
y: positive above optical axis
negative below

y': positive above the optical axis
negative below

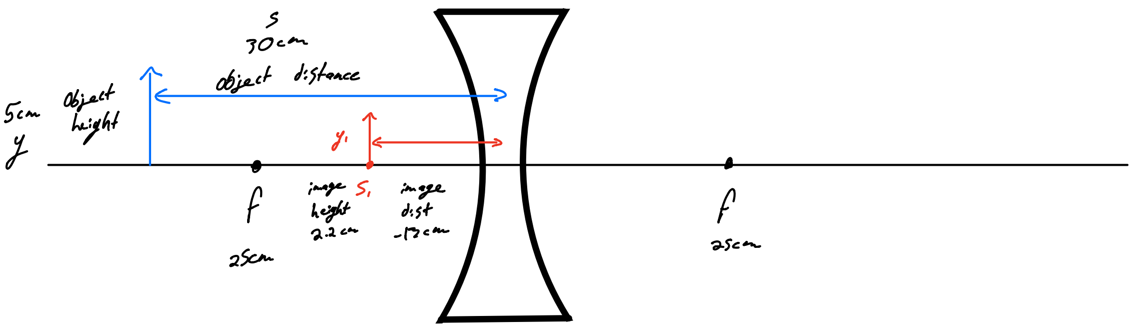
@ 10:00 am

Ray Diagrams for Concave Lenses

The ray diagrams for concave lenses inside and outside the focal point give similar results: an erect virtual image smaller than the object. The image is always formed inside the focal length of the lens.



For a concave lens with a focal length of 25 cm, describe the image formed by a 5 cm high object located 30 cm from the lens.



$$\boxed{\frac{1}{f} = \frac{1}{s} + \frac{1}{s'}}$$

s = object distance

s' = image distance

If s' < 0:
virtual image

If s' > 0:
real image

$y = 5\text{ cm}$

$s = 30\text{ cm}$

find y' & s'

$$\frac{1}{-25} = \frac{1}{30} + \frac{1}{s'}$$

$s' = -13.6$ cm

b)

$$\boxed{\begin{gathered}\frac{y'}{y} = -\frac{s'}{s} \\ y' = -\frac{s'}{s}y\end{gathered}}$$

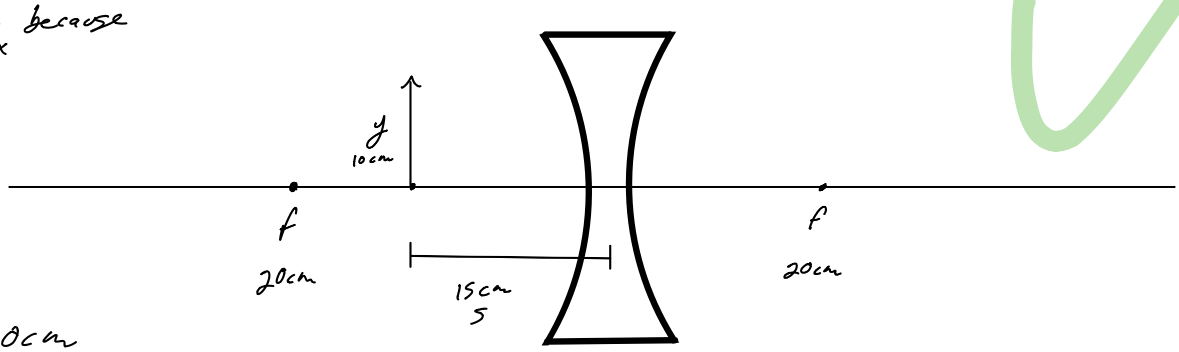
$$y' = -\frac{(-13.6\text{ cm})}{(30\text{ cm})}(5\text{ cm})$$

$y' = 2.26$ cm

Virtual
&
Upright

A 10 cm high object is placed 15 cm in front of diverging (concave) lens of focal length 20 cm. Describe the image (a) position, (b) height, (c) orientation, and (d) type of image.

Negative because convex



$f = -20\text{cm}$

$s = 15\text{cm}$

$y = 10\text{cm}$

a) $s' = \dfrac{1}{\frac{1}{f} - \frac{1}{s}}$

$= \dfrac{1}{\frac{1}{-20} - \frac{1}{15}}$

$= -8.57\text{cm}$

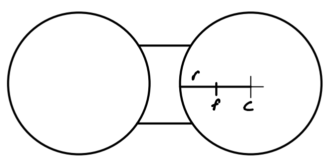
b) $y' = -\dfrac{s'}{s} y$

$y' = -\dfrac{(-8.57\text{cm})}{15\text{cm}} 10\text{cm}$

$y' = 5.71\text{cm}$

c) Upright

d) Virtual



focal point is at 1/2 of radius

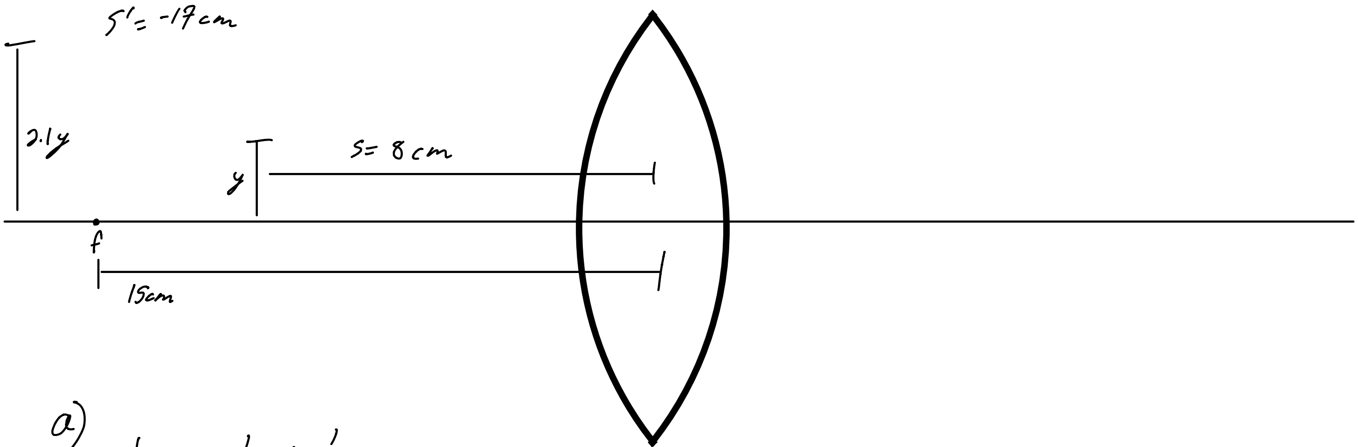
Homeworks

A lens has a focal length of 15 cm. An object is located 8 cm from the surface of the lens.

a. Calculate how far the image is from the lens.

b. Determine whether the image is real or virtual.

c. Calculate the magnification of the image (state whether the image is upright or inverted).



a)

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'}$$

$$\frac{1}{15} = \frac{1}{8} + \frac{1}{s'}$$

$$s' = -17.14 \text{ cm}$$

b) ∴ Virtual Image

c) $y' = -\frac{s'}{s} y$

$$\text{magnification} = -\frac{s'}{s}$$

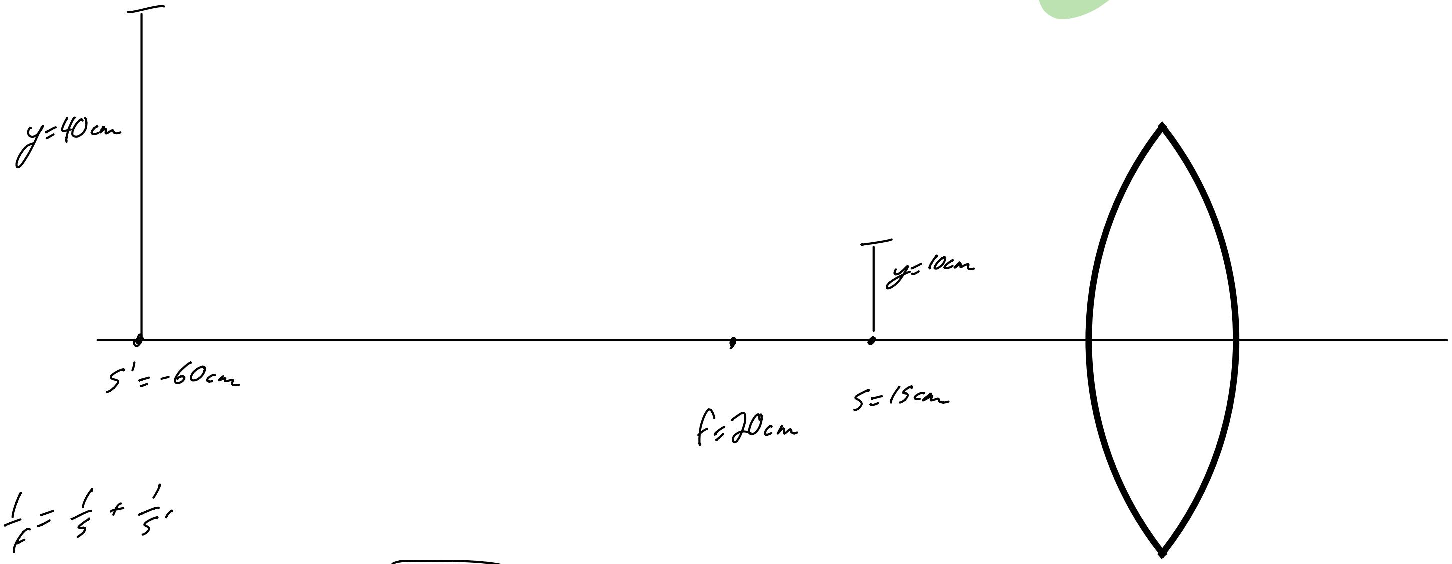
$$= -\frac{(-17.14 \text{ cm})}{8 \text{ cm}}$$

magnification = 2.1425

∴ Upright Image

homework

A 10 cm high object is placed 15 cm in front of converging (convex) lens of focal length 20 cm. Describe the image (a) position, (b) height, (c) orientation, and (d) type of image.



a) $\frac{1}{f} = \frac{1}{s} + \frac{1}{s'}$

$\frac{1}{20} - \frac{1}{15} = \frac{1}{s'}$

$s' = -60\text{ cm}$

c) Upright

d) Virtual

b) $y' = -\frac{s'}{s} y$

$y' = -\frac{-60}{15} 10$

$y' = 40\text{ cm}$

Homework

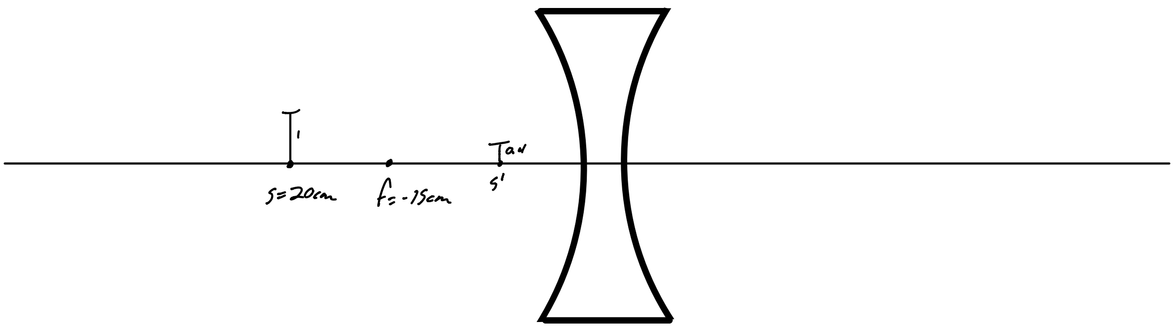
negative ∴ diverging

A lens has a focal length of -15 cm. An object is located 20 cm from the surface of the lens.

a. Calculate how far the image is from the lens.

b. Determine whether the image is real or virtual.

c. Calculate the magnification of the image (state whether the image is upright or inverted).



a)

$$\frac{1}{-15} = \frac{1}{20} + \frac{1}{s'}$$

$$s' = \frac{1}{\frac{1}{-15} - \frac{1}{20}}$$

$$s' = -8.57\,cm$$

b) ∴ Virtual

c) $$m = -\frac{s'}{s}$$

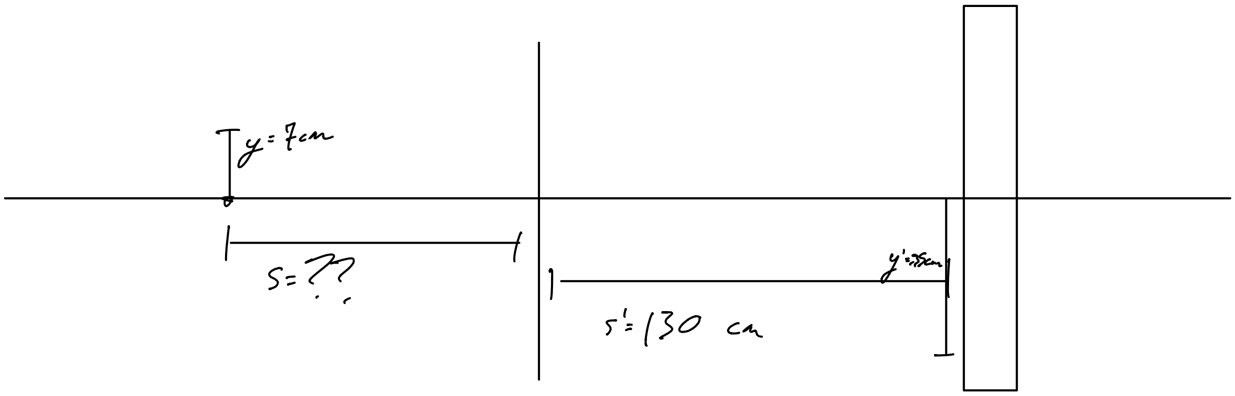
$$= -\frac{-8.57}{20}$$

$$= 0.428$$

upright

How to know inverted?

A 35 cm high image of a 7 cm high object is projected onto a screen 1.3 m from a lens. What is the (a) type of lens and (b) focal length?



a) Image right of lens

∴

Real image

∴

Convex lens

b) $-\frac{s'}{s} = \frac{y'}{y}$

$s = \frac{-s'y}{y'}$

$s = \frac{-(130cm)(7cm)}{-35cm}$

$s = 26$ cm

$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'}$

$f = \frac{1}{\frac{1}{s} + \frac{1}{s'}}$

$f = \frac{1}{\frac{1}{26cm} + \frac{1}{130cm}}$

$f = 21.7cm$

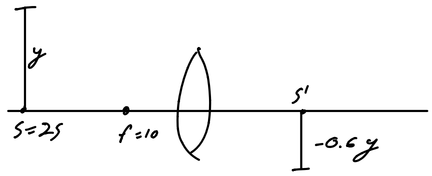
homework

A positive lens has a focal length of 10 cm. An object is located 25 cm from the lens.

a. Calculate how far the image is from the lens.

b. Determine whether the image is real or virtual.

c. Calculate the magnification of the image (state whether the image is upright or inverted).



a) $\frac{1}{f} = \frac{1}{s} + \frac{1}{s'}$

$s' = \frac{1}{\frac{1}{10} - \frac{1}{25}}$

$s' = 16.66$

b) $\therefore$ real

c) $y' = -\frac{s'}{s} y$

$y' = -\frac{16.66}{25}$

$m = -0.66$

$y' = -6.66$ cm

d) inverted

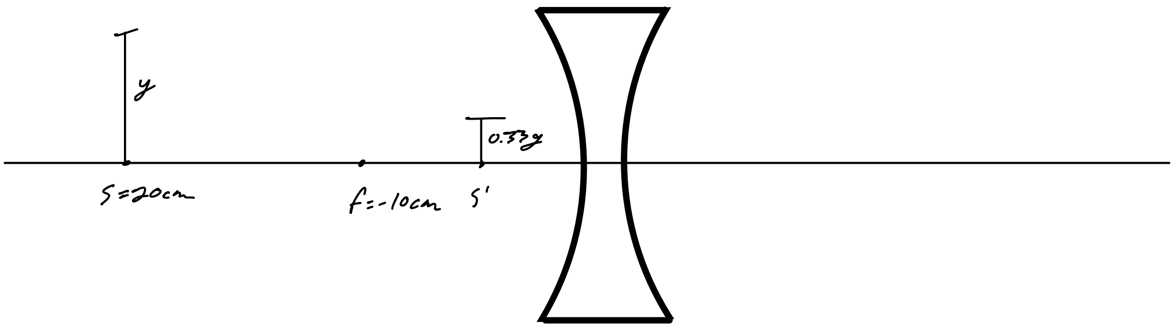
Homework

A negative lens has a focal length of - 10 cm. An object is located 20 cm from the lens.

a. Calculate how far the image is from the lens.

b. Determine whether the image is real or virtual.

c. Calculate the magnification of the image (state whether the image is upright or inverted).



a) $\frac{1}{f} = \frac{1}{s} + \frac{1}{s'}$

$s' = \frac{1}{\frac{1}{-10} - \frac{1}{20}}$

$s' = -6.66$

b) ∴ Virtual

c) $m = -\frac{s'}{s}$

$= \frac{6.66}{20}$

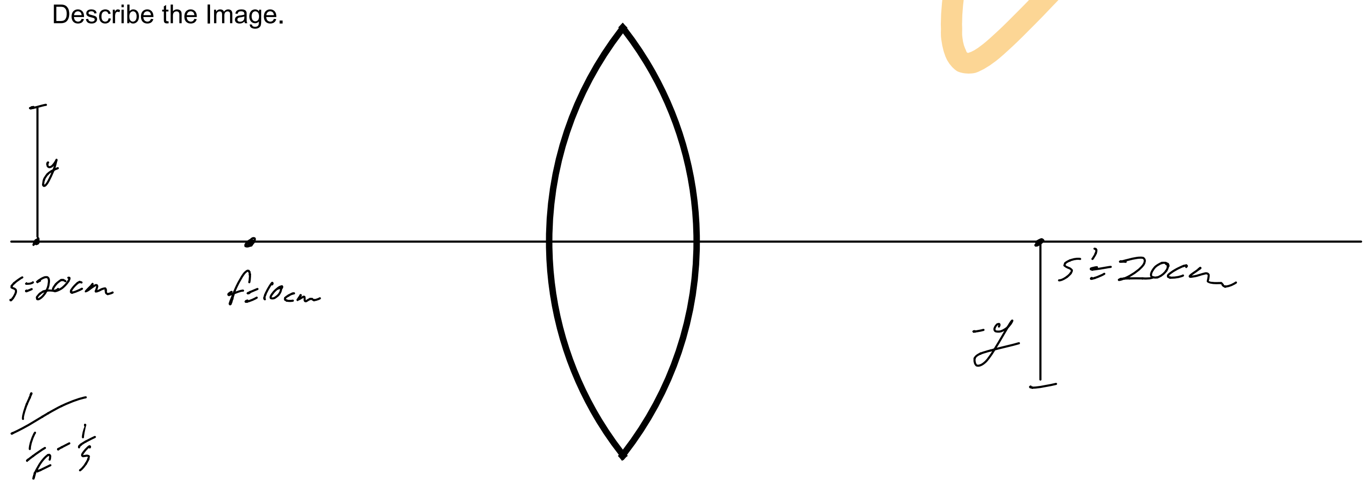
$m = 0.33$

∴ upright

homeworks

A positive lens has a focal length of 10 cm. An object is located 20 cm from the lens.

Describe the Image.



$$s' = \frac{1}{\frac{1}{f} - \frac{1}{s}}$$

$s' = 20\text{ cm}$

∴ real

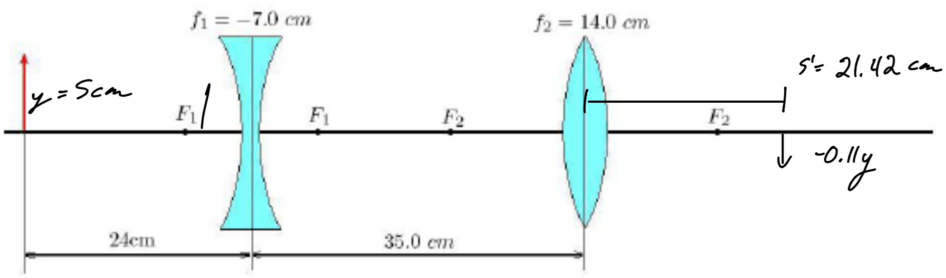
$$m = -\frac{s'}{s}$$

$$= -\frac{20}{20}$$

$\boxed{= -1}$

∴ inverted

Two lenses that are 35 cm apart are used to form an image, as shown in the figure below.



Lens 1 is diverging and has a focal length $f_1 = -7.0$ cm; lens 2 is converging and has a focal length $f_2 = 14$ cm. The object is placed 24 cm to the left of lens 1.

$s_1 = 24$

$s_1' = \dfrac{1}{\frac{1}{-7} - \frac{1}{24}}$

$s_1' = -5.42$

$35 + 5.42 = 40.42$

$s_2' = \dfrac{1}{\frac{1}{14} - \frac{1}{40.42}}$

$s_2' = 21.42$ from 2nd lens

∴ Real

$y_1' = -\dfrac{s_1'}{s_1} y_1$

$= -\dfrac{-5.42}{24} 5$

$y_1' = 1.13$ cm

$y_2 = y_1'$

$y_2' = -\dfrac{s_2'}{s_2} y_2$

$y_2' = -\dfrac{21.42}{40.42}(1.13)$

$= -0.529(1.13)$

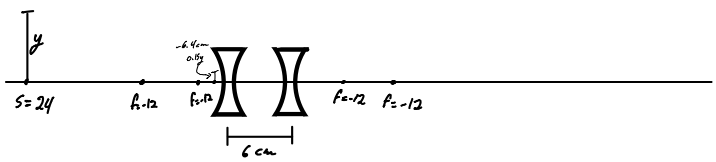
$y_2' = -0.598$ cm

∴ inverted

Two concave lenses, each with $f = -12\ cm$, are separated by 6.0 cm. An object is placed 24 cm in front of one of the lenses. Find

(a) the location and

(b) the magnification of the final image produced by this lens combination.



$$s_1' = \frac{1}{\frac{1}{f} - \frac{1}{s}}$$

$$= \frac{1}{\frac{1}{-12} - \frac{1}{24}}$$

$s_1' = -8$

$s_2 = -s_1' + 6$

$s_2 = 14$

$$s_2' = \frac{1}{\frac{1}{f} - \frac{1}{s_2}}$$

$$= \frac{1}{\frac{1}{-12} - \frac{1}{14}}$$

$s_2' = -6.46\ cm$

from 2nd lens

$$y_1' = -\frac{s_1'}{s_1} y_1$$

$$= -\frac{-8}{24}(1)$$

$y_1' = y_2 = 0.33$

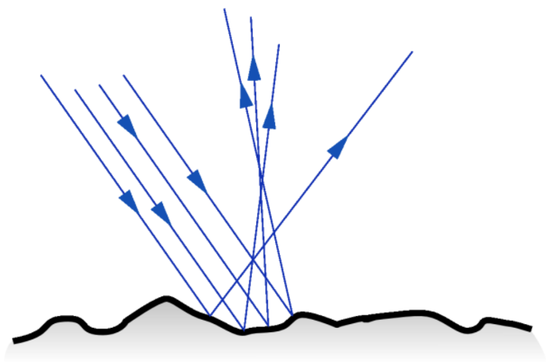
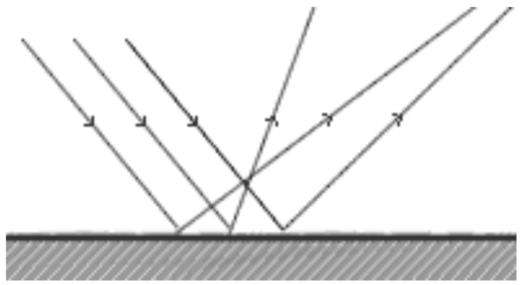
$$y_2' = -\frac{s_2'}{s_2} y_2$$

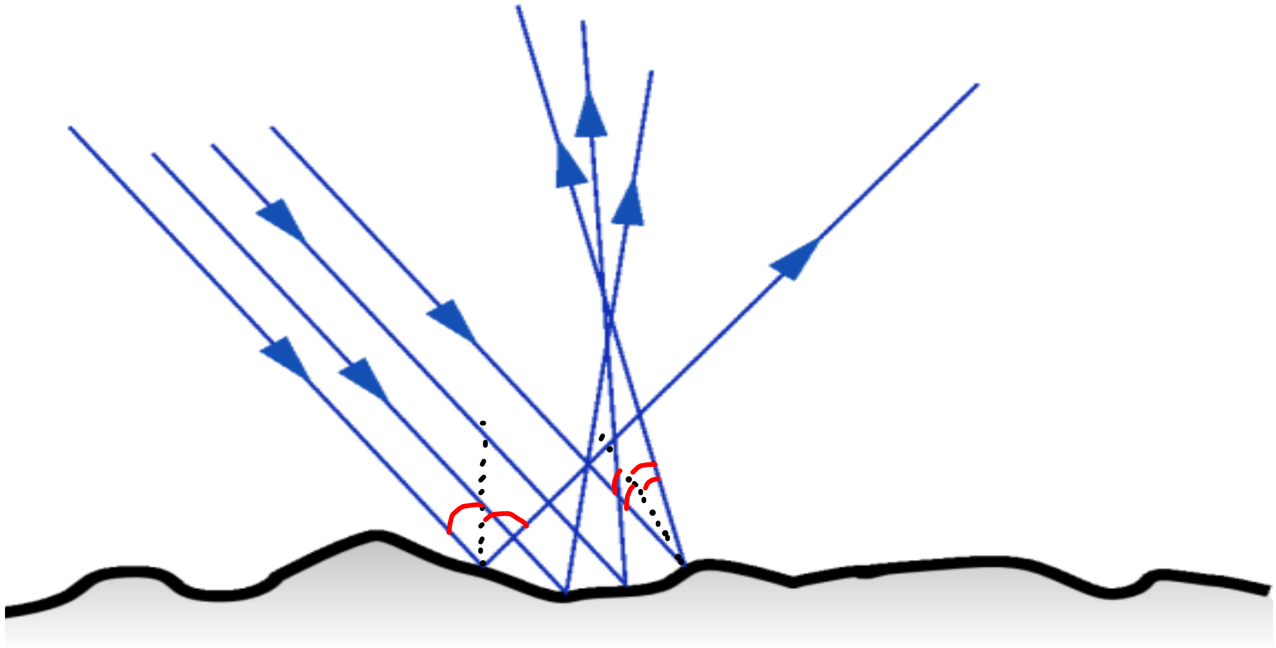
$$= -\frac{-6.46}{14}(0.33)$$

$y_2' = 0.154y$

Virtual & Upright

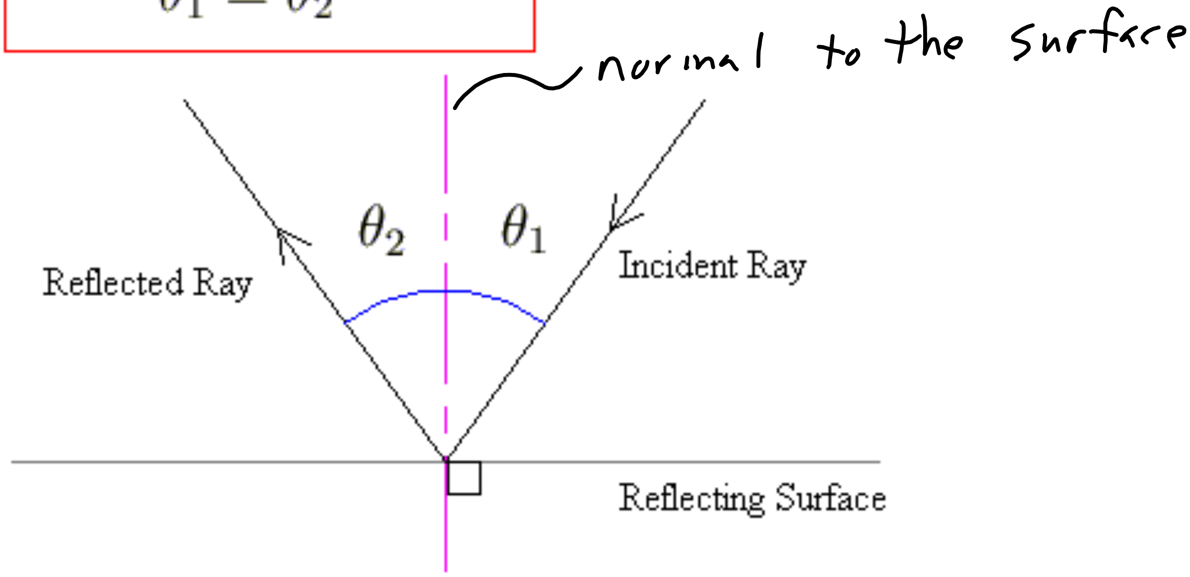
Reflection

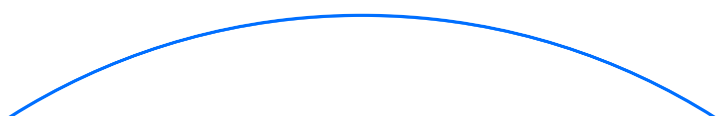
 



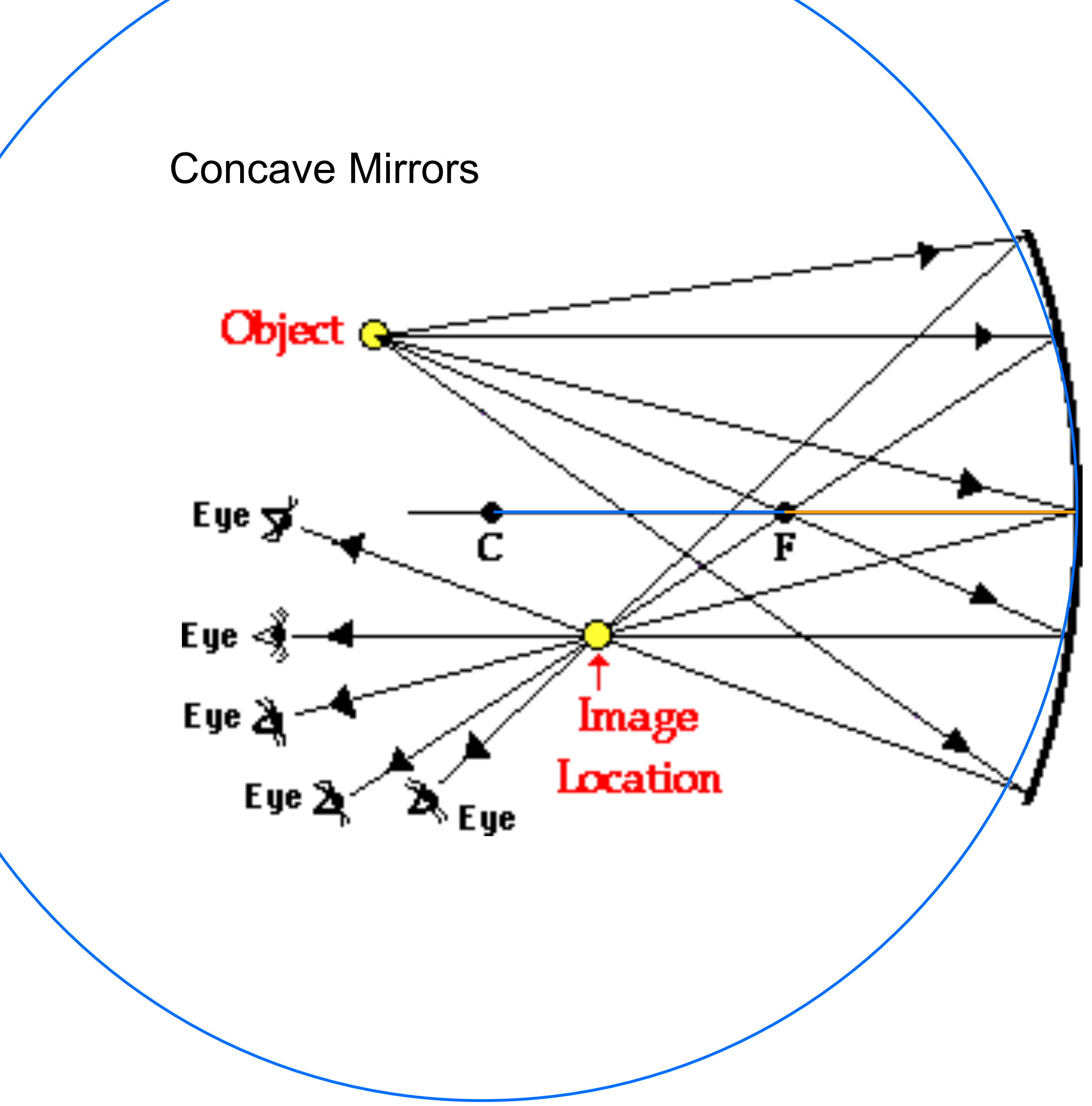
Law of Reflection:

$$\theta_1 = \theta_2$$



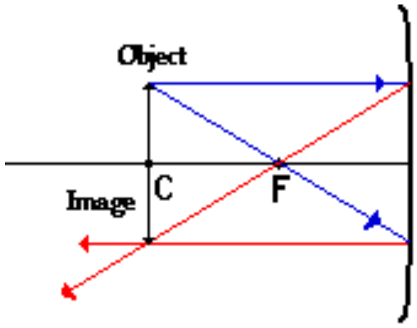


Concave Mirrors

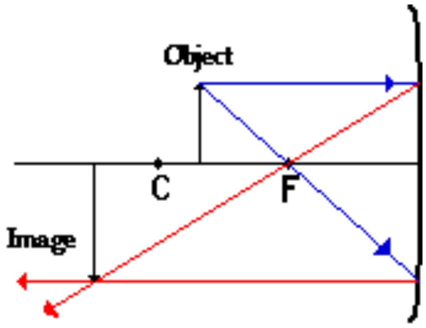


F at ½r

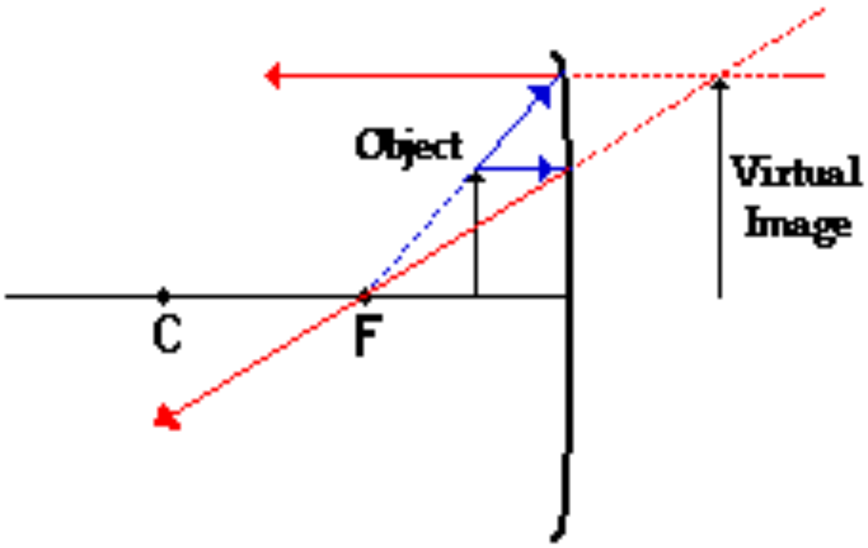
C at r ?



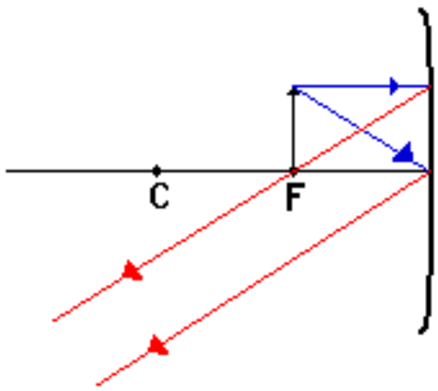
Ray Diagram for Object Located at C



Ray Diagram for Object Located Between C and F

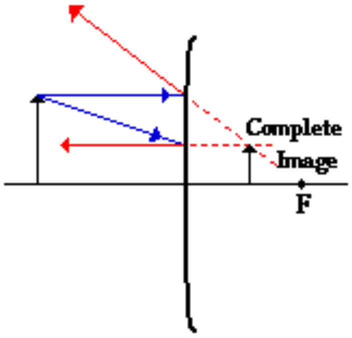


Ray Diagram for Object Located in Front of F



Ray Diagram for Object Located at F
(an image is not formed)

Convex Mirrors



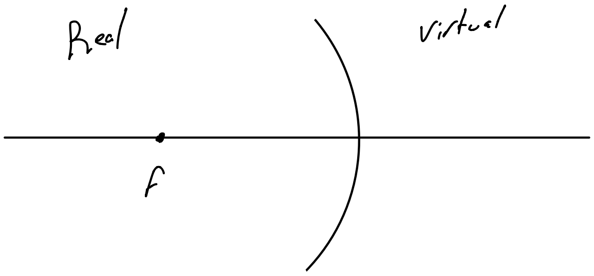
Same Formulas

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'}$$

$$M = -\frac{s'}{s} = \frac{y'}{y}$$

"In front of" "behind"

Real virtual

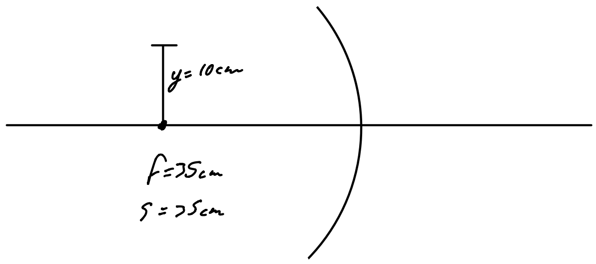


Different Sign Conventions

f: <u>positive</u> for concave mirrors
<u>negative</u> for convex mirrors

s': <u>positive</u> on left of mirror
<u>negative</u> on right of mirror

If a concave mirror has a focal length of 35 cm and an object of height 10 cm is placed at 35 cm, what is the position of the image formed?



$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'}$$

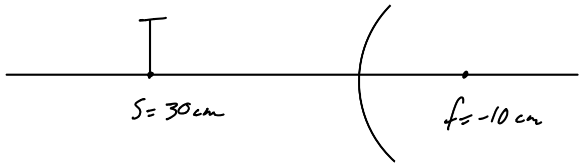
$$\frac{1}{35} = \frac{1}{35} + \frac{1}{s'}$$

$$0 = \frac{1}{s'}$$

s' = undefined NO Image

Convex

A diverging mirror of focal length 10.0 cm forms images of an object at various distances. (a) If the object is placed 30.0 cm from the mirror, describe the image. (b) If the object is placed 10.0 cm from the mirror, describe the image. (c) If the object is placed 5 cm from the mirror, describe the image. $y = 5\text{cm}$



a) $s = 30\text{ cm}$

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'}$$

$$s' = \frac{1}{\frac{1}{-10} - \frac{1}{30}}$$

$$s' = -7.5\text{ cm}$$

$$y' = -\frac{s'}{s} y$$

$$= -\frac{(-7.5)}{30} 5$$

$$y' = 1.25$$

upright & virtual

b) $s = 10\text{ cm}$

$$s' = \frac{1}{\frac{1}{-10} - \frac{1}{10}}$$

$$s' = -5\text{cm}$$

$$y' = -\frac{s'}{s} y$$

$$y' = -\frac{-5}{10} 5$$

$$y' = 2.5\text{cm}$$

Upright & virtual

c) $s = 5\text{cm}$

$$s' = \frac{1}{\frac{1}{-10} - \frac{1}{5}}$$

$$s' = -3.33\text{ cm}$$

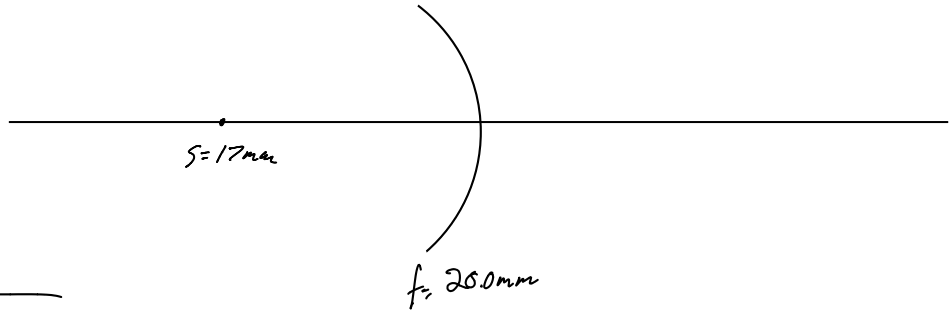
$$y' = -\frac{s'}{s} y$$

$$= -\frac{(-3.33\text{ cm})}{\cancel{5\text{cm}}} \cancel{5\text{cm}}$$

$$y' = 3.33\text{cm}$$

Upright & virtual

• A dentist uses a small concave mirror with a focal length of 20.0 mm to locate a cavity in a patient's tooth. If the mirror is held 17 mm from the tooth, what is the magnification of the image?



$$s' = \frac{1}{\frac{1}{F} - \frac{1}{s}}$$

$$= \frac{1}{\frac{1}{20} - \frac{1}{17}}$$

$$s' = -113.33$$

$$M = -\frac{s'}{s}$$

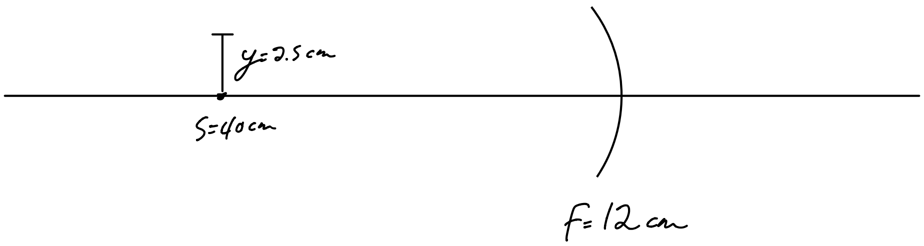
$$= -\frac{-113}{17}$$

$$M = 6.647$$

A concave mirror has a focal length of 12 cm. An object with a height of 2.5 cm is placed 40.0 cm in front of the mirror.

a. Calculate the image distance.

b. Calculate the image height



a) $s' = \frac{1}{\frac{1}{f} - \frac{1}{s}}$

$= \frac{1}{\frac{1}{12} - \frac{1}{40}}$

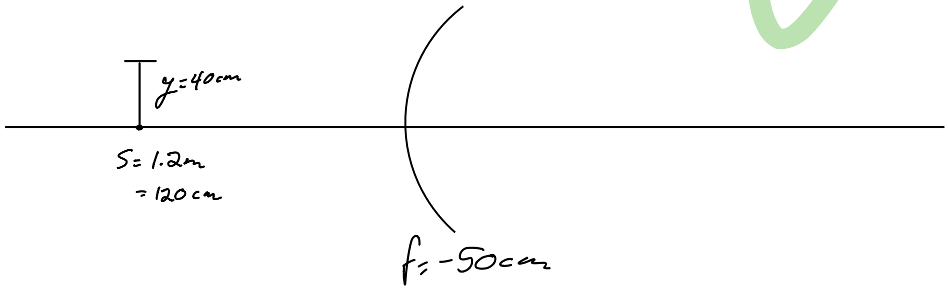
$s' = 17.42$

b) $y' = -\frac{s'}{s} y$

$= -\frac{17.4}{40} 2.5$

$y = -1.09$ cm

Describe the image formed by a 40 cm high object placed 1.2 m in front of a 50 cm focal length convex mirror. (a) position, (b) height, (c) orientation, and (d) type of image



a) $s' = \dfrac{1}{\frac{1}{f} - \frac{1}{s}}$

$= \dfrac{1}{\frac{1}{-50} - \frac{1}{120}}$

$s' = -35.3\text{cm}$

d) behind mirror
∴ virtual

b) $y' = -\dfrac{s'}{s} y$

$= -\dfrac{-35.3}{120} 40$

$y' = 11.8 \text{ cm}$

c) Upright

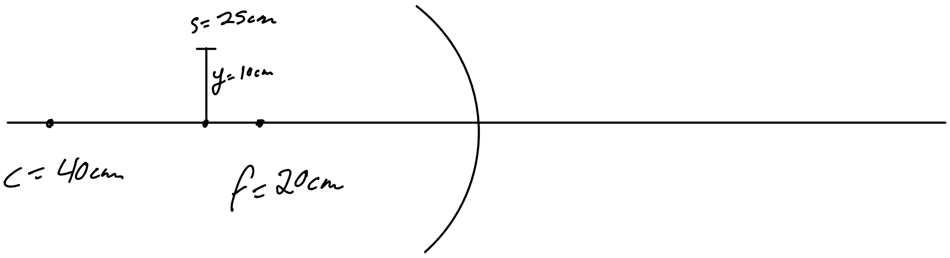
- All concave mirror images virtual?
- All mirror images virtual?

For a concave mirror with a radius of curvature of 40 cm, describe the image formed by a 10 cm high object located 25 cm from the surface of the mirror.

$c = 40cm$

$f = \frac{1}{2}c$

$\boxed{f = 20cm}$



$$s' = \frac{1}{\frac{1}{f} - \frac{1}{s}}$$

$$s' = \frac{1}{\frac{1}{20} - \frac{1}{25}}$$

$$s' = 100cm$$

~~∴ virtual~~

real

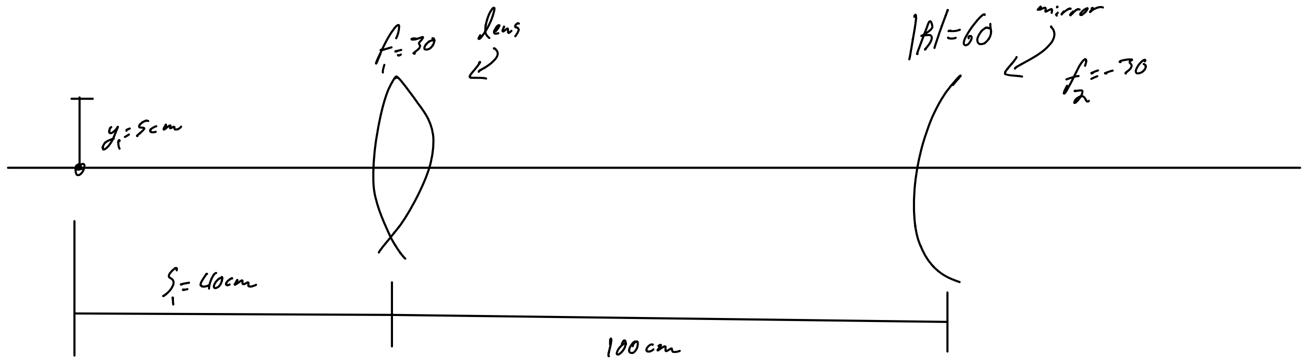
$$y' = -\frac{s'}{s}y$$

$$= -\frac{100}{25}10$$

$$y' = -40cm$$

∴ Inverted

A body is positioned 40 cm from the left of a lens (f=30) and the lens is positioned 100cm left of a convex mirror (|R|=60), where will be the image and what type and magnitude will it have?



$$s_1' = \frac{1}{\frac{1}{f_1} - \frac{1}{s_1}}$$

$$= \frac{1}{\frac{1}{30} - \frac{1}{40}}$$

$s_1' = 120$ cm right of lens

$s_2 = -20$ cm

$$s_2' = \frac{1}{\frac{1}{f_2} - \frac{1}{s_2}}$$

$$= \frac{1}{\frac{1}{-30} - \frac{1}{-20}}$$

$s_2' = 60$ cm left of mirror

virtual

$$y_1' = -\frac{s_1'}{s_1} y_1$$

$$= -\frac{120}{40} 5$$

$y_1' = -15$

$y_2 = y_1'$

$$y_2' = -\frac{s_2'}{s_2} y_2$$

$$= -\frac{60}{-20} (-15)$$

$y_2' = -45$ cm

∴ Inverted