Simple Harmonic Motion – Simulation

Simulation link: **http://physics.bu.edu/~duffy/HTML5/simple_pendulum_damped.html**

In this experiment you will use a pendulum to investigate different aspects of simple harmonic motion, as well as to see that the motion of a real pendulum is not always simple harmonic, even when it is periodic. In general, oscillations follow simple harmonic motion when the equation governing the motion has the following form, which says that the acceleration (*a*) is proportional to, and opposite in direction to, the displacement (*x*) from equilibrium:

$$a = -\omega^2 x$$

A solution to such an equation is $x = A\cos(\omega t)$, where *A* is the maximum displacement from equilibrium (the **amplitude**), and ω is called the **angular frequency** – the bigger the angular frequency, the faster the oscillations.

For a ball of mass *m* attached to an ideal spring of spring constant *k*, the equation becomes:

$$a = -\frac{k}{m}x \qquad \text{which tells that } \omega = \sqrt{\frac{k}{m}}$$

For a simple pendulum (a ball on a string), it does not oscillate back and forth on a straight line, but it travels back and forth along the arc of a circle. Thus, we use angular variables, in which the general equation for simple harmonic motion looks like:

$$\alpha = -\omega^2 \theta$$

If you analyze the simple pendulum, it turns out that the equation governing the motion is not quite the above, but instead is:

$$\alpha = -\omega^2 \sin(\theta)$$

For small angles (measured in radians), it turns out that $\sin(\theta) \approx \theta$, which reduces the motion to simple-harmonic motion. This is known as the *small-angle approximation*. With this approximation, the equation for the simple pendulum becomes

$$\alpha = -\frac{g}{L}\theta \qquad \text{which tells that } \omega = \sqrt{\frac{g}{L}}$$

Interestingly, it doesn't depend on the mass of the ball on the string.

In the lab, you will compare the motion with the small-angle approximation to the actual motion of a pendulum. You will first examine qualitatively the period of a pendulum, and then do quantitative measurements of period for small and large amplitude oscillations. You will then investigate different aspects of the energy of a pendulum

The simulation can show eight different representations of the motion of a simple pendulum (four graphs plotted vs. time, and four others plotted vs. angle). On the animation, the pendulum with the red ball represents the actual motion of a pendulum. The blue ball shows the motion with only the small angle approximation taken into account. In this lab, you will be comparing these two scenarios.

Part I: Qualitative Observations

Spend a few minutes running the simulation at both small (15° or less) and large (40° or more) amplitudes. Continue to run the simulation while looking at the graphs. Make sure to read the text at the bottom of the simulation screen. The true motion of the pendulum is shown in red, and the motion with the small-angle approximation is shown in blue (unless you hit one of the other comparison buttons). The energy graphs are an exception – in that case, the colored lines are for the actual motion, and the comparison motion is shown in black. Take notes about the qualitative motion by answering the questions below.

Press the "Small-angle approx." button to make sure the blue ball shows the motion with the small-angle approximation.

Question 1: As g decreases, the frequency of the pendulum _________________

As L decreases, the frequency of the pendulum _________________

As mass decreases, the frequency of the pendulum _________________

Question 2: How does the motion of the red and blue balls compare on the animation? Are they exactly the same? Very similar? Quite different? Or does that depend on what the initial angle is? Look at the balls themselves, as well as the angle vs. time graph.

Question 3: Is the small-angle approximation consistent with energy conservation? Explain your answer using the "energy for small-angle approx." graphs in the simulation.

Part II: Quantitative Measurement

L = _______________ g = ________________ m = _______________

Which of these factors has no impact on the period of the pendulum?

[ ] L [ ] g [ ] m

Theoretical (small-angle approximation) period: $T = 2\pi\sqrt{\frac{L}{g}}$ = ________

Using the simulation, compare the periods of the actual pendulum (red) and the pendulum with the small angle approximation (blue) for different initial angles. In your trials, make sure to use at least one small angle (less than 15°), and one large angle (more than 40°). It may help to conduct your trials in order of increasing initial angle.

You can measure period by looking at the time readout for one cycle on the simulation. You can use the Step buttons to get as close as possible to one complete cycle.

Trial	1	2	3	4	5	6
θ_{max} [rad]						
T [s] (red)						
T [s] (blue)						

Question 4: Is there a difference between the period for small-amplitude and large-amplitude oscillations for an actual pendulum (red)? If so, describe the trend of periods with increasing angular amplitude, and provide and explanation for this change.

Question 5: Using the three angle graphs (angle, angular velocity, and angular acceleration vs. time), describe the difference in period between small-amplitude oscillations and large-amplitude oscillations. Sketch any differences in these graphs (try a really large amplitude).

Part III: Energy in Simple Harmonic Motion

In this part of the lab, you will be comparing a simple pendulum with and without damping. First, run the simulation at any angle without damping, and look at both the energy vs. time and the energy vs. angle graphs.

It might help to compare to "Nothing," so you don't have too many lines to follow on the graphs.

Colors

Kinetic energy:	[ ] red	[ ] blue	[ ] green
Potential energy:	[ ] red	[ ] blue	[ ] green
Mechanical energy (U + K):	[ ] red	[ ] blue	[ ] green

First, focus on the Energy vs. time graph. Try it without damping, and then with damping (move the damping slider away from zero).

Question 6: With zero damping, the pendulum would oscillate forever. The case with damping is more realistic – the amplitude of the oscillations gradually decreases until the pendulum stops. In a real-life situation, in which the pendulum slows and then stops, where does the energy go?

Question 7: With zero damping, the mechanical energy is constant. With damping, the damping causes a loss of mechanical energy in proportion to the speed. With damping on, does the total mechanical energy drop steadily? If not, when does it drop most quickly, and why?

Now, focus on the Energy vs. angle graph. Start with zero damping.

Question 8: Compare this view (energy vs. angle) to the previous view (energy vs. time). In what ways are the views the same? In what ways are they different?

Prediction (not graded) – draw your prediction of the potential energy vs. angle, kinetic energy vs. angle, and total mechanical energy vs. angle when damping is turned on. Draw your prediction for multiple cycles.

Now, turn on damping (move the damping slider away from zero)

Question 9: What do you observe about the different energy vs. angle graphs when damping is turned on (look at both large and small damping)? How does this compare to what you predicted?