Homework 5 Solutions **hw05.zip (hw05.zip)**

Solution Files

You can find the solutions in hw05.py (hw05.py).

Required Questions

Important: Update (10/11): These questions use the function-based data abstraction `tree`. If your downloaded hw refers to the `Tree` object class, that should also be ok in terms of the autograder. If you'd like to see the updated version however, please redownload the zip and copy over your current progress. The `Tree` class will be covered on Friday, so not quite yet! Feel free to let us know on Piazza if you have any questions.

Q1: Generate Permutations

Given a sequence of unique elements, a *permutation* of the sequence is a list containing the elements of the sequence in some arbitrary order. For example, `[2, 1, 3]`, `[1, 3, 2]`, and `[3, 2, 1]` are some of the permutations of the sequence `[1, 2, 3]`.

Implement `gen_perms`, a generator function that takes in a sequence `seq` and returns a generator that yields all permutations of `seq`. For this question, assume that `seq` will not be empty.

Permutations may be yielded in any order. Note that the doctests test whether you are yielding all possible permutations, but not in any particular order. The built-in `sorted` function takes in an iterable object and returns a list containing the elements of the iterable in non-decreasing order.

Hint: If you had the permutations of all the elements in `seq` not including the first element, how could you use that to generate the permutations of the full `seq`?

Hint: Remember, it's possible to loop over generator objects because generators are iterators!

```
def gen_perms(seq):
    """Generates all permutations of the given sequence. Each permutation is a
    list of the elements in SEQ in a different order. The permutations may be
    yielded in any order.

    >>> perms = gen_perms([100])
    >>> type(perms)
    <class 'generator'>
    >>> next(perms)
    [100]
    >>> try: #this piece of code prints "No more permutations!" if calling next would caus
    ...     next(perms)
    ... except StopIteration:
    ...     print('No more permutations!')
    No more permutations!
    >>> sorted(gen_perms([1, 2, 3])) # Returns a sorted list containing elements of the ge
    [[1, 2, 3], [1, 3, 2], [2, 1, 3], [2, 3, 1], [3, 1, 2], [3, 2, 1]]
    >>> sorted(gen_perms((10, 20, 30)))
    [[10, 20, 30], [10, 30, 20], [20, 10, 30], [20, 30, 10], [30, 10, 20], [30, 20, 10]]
    >>> sorted(gen_perms("ab"))
    [['a', 'b'], ['b', 'a']]
    """
    if not seq:
        yield []
    else:
        for perm in gen_perms(seq[1:]):
            for i in range(len(seq)):
                yield perm[:i] + [seq[0]] + perm[i:]
```

Use Ok to test your code:

```
python3 ok -q gen_perms
```

Q2: Yield Paths

Define a generator function `path_yielder` which takes in a tree `t`, a value `value`, and returns a generator object which yields each path from the root of `t` to a node that has label `value`.

Each path should be represented as a list of the labels along that path in the tree. You may yield the paths in any order.

We have provided a skeleton for you. You do not need to use this skeleton, but if your implementation diverges significantly from it, you might want to think about how you can get it to fit the skeleton.

```
def path_yielder(t, value):
    """Yields all possible paths from the root of t to a node with the label
    value as a list.

    >>> t1 = tree(1, [tree(2, [tree(3), tree(4, [tree(6)]), tree(5)]), tree(5)])
    >>> print_tree(t1)
    1
      2
        3
        4
          6
        5
      5
    >>> next(path_yielder(t1, 6))
    [1, 2, 4, 6]
    >>> path_to_5 = path_yielder(t1, 5)
    >>> sorted(list(path_to_5))
    [[1, 2, 5], [1, 5]]

    >>> t2 = tree(0, [tree(2, [t1])])
    >>> print_tree(t2)
    0
      2
        1
          2
            3
            4
              6
            5
          5
    >>> path_to_2 = path_yielder(t2, 2)
    >>> sorted(list(path_to_2))
    [[0, 2], [0, 2, 1, 2]]
    """
    if label(t) == value:
        yield [value]
    for b in branches(t):
        for path in path_yielder(b, value):
                yield [label(t)] + path
```

Hint: If you're having trouble getting started, think about how you'd approach this problem if it wasn't a generator function. What would your recursive calls be? With a generator function, what happens if you make a "recursive call" within its body?

Hint: Remember, it's possible to loop over generator objects because generators are iterators!

Note: Remember that this problem should **yield items** -- do not return a list!

Use Ok to test your code:

```
python3 ok -q path_yielder
```

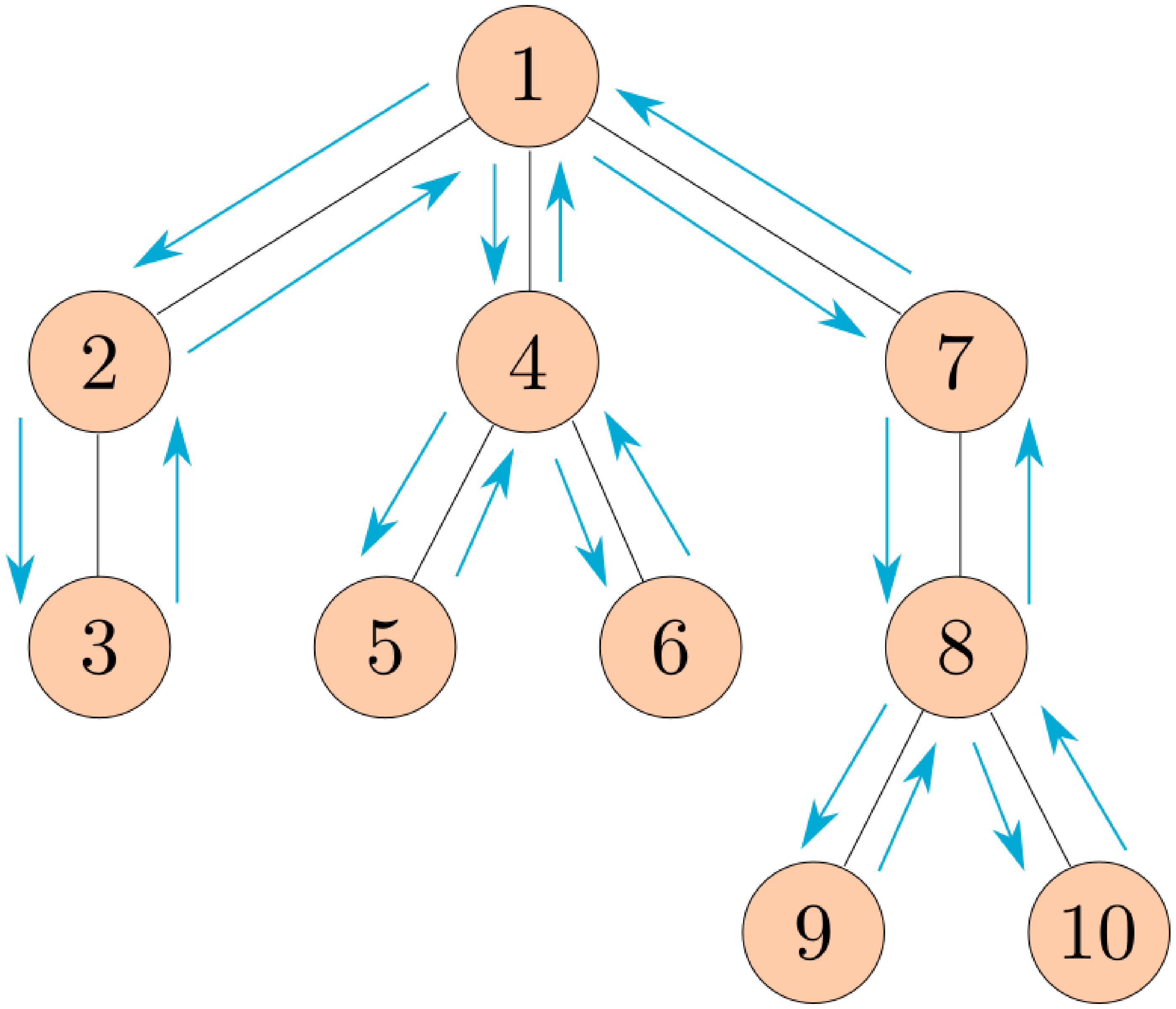
If our current label is equal to `value`, we've found a path from the root to a node containing `value` containing only our current label, so we should yield that. From there, we'll see if there are any paths starting from one of our branches that ends at a node containing `value`. If we find these "partial paths" we can simply add our current label to the beinning of a path to obtain a path starting from the root.

In order to do this, we'll create a generator for each of the branches which yields these "partial paths". By calling `path_yielder` on each of the branches, we'll create exactly this generator! Then, since a generator is also an iterable, we can iterate over the paths in this generator and yield the result of concatenating it with our current label.

Q3: Preorder

Define the function `preorder`, which takes in a tree as an argument and returns a list of all the entries in the tree in the order that `print_tree` would print them.

The following diagram shows the order that the nodes would get printed, with the arrows representing function calls.



Note: This ordering of the nodes in a tree is called a preorder traversal.

```
def preorder(t):
    """Return a list of the entries in this tree in the order that they
    would be visited by a preorder traversal (see problem description).

    >>> numbers = tree(1, [tree(2), tree(3, [tree(4), tree(5)]), tree(6, [tree(7)])])
    >>> preorder(numbers)
    [1, 2, 3, 4, 5, 6, 7]
    >>> preorder(tree(2, [tree(4, [tree(6)])]))
    [2, 4, 6]
    """
    if branches(t) == []:
        return [label(t)]
    flattened_branches = []
    for child in branches(t):
        flattened_branches += preorder(child)
    return [label(t)] + flattened_branches

# Alternate solution
from functools import reduce

def preorder_alt(t):
    return reduce(add, [preorder_alt(child) for child in branches(t)], [label(t)])
```

Use Ok to test your code:

```
python3 ok -q preorder
```

Q4: Generate Preorder

Similarly to `preorder` in Question 3, define the function `generate_preorder`, which takes in a tree as an argument and now instead `yield`s the entries in the tree in the order that `print_tree` would print them.

Hint: How can you modify your implementation of `preorder` to `yield from` your recursive calls instead of returning them?

```
def generate_preorder(t):
    """Yield the entries in this tree in the order that they
    would be visited by a preorder traversal (see problem description).

    >>> numbers = tree(1, [tree(2), tree(3, [tree(4), tree(5)]), tree(6, [tree(7)])])
    >>> gen = generate_preorder(numbers)
    >>> next(gen)
    1
    >>> list(gen)
    [2, 3, 4, 5, 6, 7]
    """
    yield label(t)
    for b in branches(t):
        yield from generate_preorder(b)
```

Use Ok to test your code:

```
python3 ok -q generate_preorder
```

Extra Questions

Q5: Remainder Generator

Like functions, generators can also be *higher-order*. For this problem, we will be writing `remainders_generator`, which yields a series of generator objects.

`remainders_generator` takes in an integer `m`, and yields `m` different generators. The first generator is a generator of multiples of `m`, i.e. numbers where the remainder is 0. The second is a generator of natural numbers with remainder 1 when divided by `m`. The last generator yields natural numbers with remainder `m - 1` when divided by `m`.

> *Hint*: To create a generator of infinite natural numbers, you can call the `naturals` function that's provided in the starter code.

> *Hint*: Consider defining an inner generator function. Each yielded generator varies only in that the elements of each generator have a particular remainder when divided by `m`. What does that tell you about the argument(s) that the inner function should take in?

```
def remainders_generator(m):
    """
    Yields m generators. The ith yielded generator yields natural numbers whose
    remainder is i when divided by m.

    >>> import types
    >>> [isinstance(gen, types.GeneratorType) for gen in remainders_generator(5)]
    [True, True, True, True, True]
    >>> remainders_four = remainders_generator(4)
    >>> for i in range(4):
    ...     print("First 3 natural numbers with remainder {0} when divided by 4:".format(i
    ...     gen = next(remainders_four)
    ...     for _ in range(3):
    ...         print(next(gen))
    First 3 natural numbers with remainder 0 when divided by 4:
    4
    8
    12
    First 3 natural numbers with remainder 1 when divided by 4:
    1
    5
    9
    First 3 natural numbers with remainder 2 when divided by 4:
    2
    6
    10
    First 3 natural numbers with remainder 3 when divided by 4:
    3
    7
    11
    """
    def gen(i):
        for e in naturals():
            if e % m == i:
                yield e
    for i in range(m):
        yield gen(i)
```

Note that if you have implemented this correctly, each of the generators yielded by `remainder_generator` will be *infinite* - you can keep calling `next` on them forever without running into a `StopIteration` exception.

Use Ok to test your code:

```
python3 ok -q remainders_generator
```